

Becker meets Ricardo: A social and cognitive skills model of human capabilities.

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Abstract

This paper studies an equilibrium model of social and cognitive skills interactions in school, work and marriage. The model uses a common team production function in each sector which integrates the complementarity concerns of Becker with the task assignment and comparative advantage concerns of Ricardo. The theory delivers full task specialization in the labor and education markets, incomplete task specialization in marriage. It rationalizes many to one matching, a common feature in labor markets. There is also occupational choice, matching by different skills in different sectors. Equilibrium is equivalent to the solution of an utilitarian social planner solving a linear programming problem.

1 Introduction

Over their lives, most individuals will participate in schooling, work and marriage. Activities in these three sectors occur within teams. Teachers match with students, workers match with managers, and husbands match with wives.

Within each sector, there is substantial variation in outcomes. For each individual, outcomes are correlated across sectors. Analyzing individual level data, a large empirical psychology literature and a much smaller one in economics have shown that both cognitive and non-cognitive factors, including social factors, affect multiple lifetime outcomes.¹ Two findings are relevant:

1. Estimates of parameters, including how many non-cognitive factors there are, differ across studies. Psychologists believe that a main cause of this heterogeneity is due to context differences across samples in different studies.²
2. With a small number of factors, estimated cognitive and non-cognitive factors are not orthogonal to each other.³

The finding of parameter instability in the empirical literature on the importance of social skills and their context specific relevance suggest that we have to model how individuals with heterogenous social skills interact. Also to be empirically salient, any model of social skills have to differentiate its effects from that of cognitive skills. To date, the theoretical literature is undeveloped. We are not aware of any equilibrium model of social interaction where individuals differ by social skills.⁴

This paper studies a social and cognitive skills model of human capabilities (hereafter SC model). Our paper studies social and cognitive skills in a team production context. Following Becker 1973 and 1974, researchers assume that the cognitive skills of team mates are complements in the production of team output. Due to complementarity, we will observe positive assortative matching (PAM) by cognitive skills in teams with frictionless matching.

¹Almlund, et. al. (2011) is a summary of the empirical psychology literature written for economists.

²E.g. Almlund, et. al. (2011); Caspi, A., and R.L. Shiner (2005); Markon, et. al. (2005); McCrae, R.R., and P.T. Costa Jr. (1999).

³Markon, et. al. (2005); DeYoung, C.G. 2006. In ongoing research, Heckman, et. al. (2011) identifies a correlated three factor model.

⁴Benhabib, et. al. 2011 is a recent comprehensive survey of social interaction models.

Now consider a team with one member who has strong social skill. Will adding another member also with strong social skill significantly increase team output? Does a team need more than one member with leadership skill? In the labor market, often the answer to both questions is no.⁵ On the other hand in many marriages, there is no leader. Also, ex-spouses often cite the lack of communication as a rationale for marital dissolution.⁶ Thus in marriage, social skills of spouses may be complementary. Can social skills rationalize why there is usually one leader per team, or many to one matching, in the labor market and not in the marriage market?

In order to answer the above questions, we posit a common team production technology for every sector: school, work and marriage. This technology integrates the complementarity concerns of team production (who matches with whom) by Becker with the task assignment and comparative advantage (who does what) concerns of Ricardo. Since the stationary frictionless matching environment is standard, all new substantive results in the paper emanate from our new team production function.

These include:

1. There is full tasks specialization at work and school. This specialization is codified by labels: teachers and students in school; managers and workers in the labor market. There is incomplete specialization in marriage modulo reproduction.⁷
2. The model rationalizes many to one matching in the labor market, a commonly observed organization design.
3. Managerial skill is not isomorphic to more cognitive skill.⁸
4. Spouses match by social and cognitive skill. Managers and workers match by cognitive skill. Students, with different social and initial cognitive skill, match with teachers of

⁵Sports teams have one head coach. Movies, plays and orchestras have one director. Firms have one CEO. Put another way, many organizations are hierarchies with one leader at the top of each sub-hierarchy.

⁶E.g., Burns 1984; Cleek, et. al. 1985; Eells, et. al. 1996.

⁷In many households, spouses may say that one of them is in charge of some decisions and the other spouse is in charge of other decisions.

⁸In one factor (cognitive) models of achievement, Larry Summers would have been president of the United States and George Bush Jr. would have been a nobody. Most observers underestimated George Bush Jr.'s significant social skills (New York Times 2000).

the same cognitive skill.⁹

5. Holding initial cognitive skill constant, the pecuniary returns to schooling for managers are higher than for workers. The labor market returns are proportional to the span of (number of workers supervised by the manager.
6. Consistent with the empirical evidence, the model generates a positive correlation between cognitive and social skills.
7. The model generates a positive correlation between earnings and the marriage rate which does not depend on rich individuals using their wealth to attract potential spouses.¹⁰

The paper also makes a methodological contribution. We show that the equilibrium of our stationary frictionless multifactor multisector matching model with endogenous occupational choice is equivalent to the solution of a utilitarian social planner solving a linear programming problem. This equivalence substantially extends known results. It also makes it easy for us to show existence and uniqueness of equilibrium; and makes equilibrium easy to compute.

We make strong assumptions about how social skill interacts with cognitive skill. Perhaps the strongest is our reduction of all non-cognitive factors which affect lifetime outcomes into one social factor. Thus the SC model is best viewed as an initial exploration of how social and cognitive skills may work in these environments.

Most components of our SC model have been studied previously in isolation. We build on and integrate their insights. So it will be convenient to defer our review of the literature until the end of the paper. For now, we acknowledge our debt to Garicano 2000 and his co-authors, whose work on task assignments, organizational design and equilibrium matching in the labor market, inspired our work.

2 The SC team production function

The new element in this paper is our team production technology. This section discusses how this technology works.

⁹In general, spouses match by attributes which are illegal to do so in the labor market or schools.

¹⁰Our frictionless marriage model assumes perfectly divisible marital output and no marriage market friction. By construction, rich individuals in the marriage market do not share their wealth in marriage.

Each adult in the society is characterized by two skills, a social skill and a cognitive skill. Let n be an adult's invariant social skill, $n \in [\underline{n}, \bar{n}]$, $1 \leq \underline{n} < \bar{n}$. We relax the assumption $1 \leq \underline{n}$ in section 3. The social skill n is fixed for the entire life of the individual and cannot be changed.¹¹ Let k be the augmentable cognitive skill of the adult, $k \in [\underline{k}, \bar{k}]$, $0 < \underline{k} < \bar{k}$. An individual, with initial cognitive skill $a \in [\underline{a}, \bar{a}]$, $0 < \underline{a} < \bar{a}$, can acquire an adult cognitive skill of k by attending the requisite amount of schooling.

In each sector, school, work or marriage, output is produced by the completion of two tasks, I (individualistic) and C (collaborative). At every time interval of team production, each task is done by a different individual. We will defer the case where one individual does both tasks until section 3.

At school, each student has one unit of time. At work or marriage, an adult also has one unit of time. Suppose individual i of type (n_i, k_i) and individual j of type (n_j, k_j) work together in a team. Let i spend $\theta_i^I \leq 1$ on task I . His effective time on task I is θ_i^I . Let j spend $\theta_j^C \leq 1$ on task C . Her effective time on task C is $n_j \theta_j^C$. The output of the team in that sector is:

$$Y(\theta_i^I, n_i, k_i, \theta_j^C, n_j, k_j) = \sqrt{k_i k_j} \min(\theta_i^I, n_j \theta_j^C) \quad (1)$$

The first term of the team production function, $\sqrt{k_i k_j}$, assumes that the cognitive skills of the two team members are complementary in producing output.¹² As is well known from Becker, all other things equal, this complementarity will induce positive assortative matching (PAM) in two member teams by cognitive skills in that sector. We employ the complementarity specification here for that reason.

The second term of the team production function, $\min(\theta_i^I, n_j \theta_j^C)$, a Leontief production function in θ_i^I and $n_j \theta_j^C$, is also familiar to economists. In our context, time is wasted if effective times in the two tasks are different. And since effective time in task C is increasing in n_j , if j has more social skill, j will need less time θ_j^C to generate the same effective time.

One view of task C is that it is the task which is used by person j to convince and/or coordinate with person i to spend time on task I to produce team output. If n_j is high, the actual time spent by j to convince and/or coordinate with i to spend time on task I to produce team output is small.

¹¹ Allowing n to be endogenous is left for future work.

¹² The Cobb Douglas form is inessential. We just require constant returns to scale and supermodularity in k_i and k_j .

n_j encompasses organizational skill. If a person has poor organizational skill, then the team will be poorly coordinated and have to spend a lot more time unravelling coordination errors.

Another aspect of n_j is that if n_j is high, person j also recognizes how to accomodate different team mates. For e.g., a teacher may reach one student in one way and another student in another way.

n_i does not enter production directly in Y . So if n_i in task I is small, it does not affect output. This aspect of the production function is central to our paper. It matters who is assigned to which task which was originally studied by Ricardo.

As will be seen later, the Leontief or fixed coefficient technology in time use is responsible for both the specialization results and the many to one matching in the labor and schooling market. It is also responsible for partial specialization in marriage. So this part of the technology is both fundamental and unique to our paper. Replacing $\min(\theta_i^I, n_j \theta_j^C)$ by 1 reduces the team production function to Becker's one factor matching technology.

We assume that output from the labor market is sold at a normalized price of one. In the marriage market, the two spouses i and j produce marital output which can be divided between the two of them.

In the schooling sector, the objective of a school is to increase the cognitive skill of students. Individual i will be a student and individual j will be a teacher. The input cognitive skills are the initial cognitive skill a_i of student i and the cognitive skill k_j of the teacher j , and output is the adult cognitive skill k_i of individual i . That is, the production function can be adapted to the schooling sector as follows

$$k_i = \sqrt{a_i k_j} \min(\theta_i^I, n_j \theta_j^C) \quad (2)$$

We now sketch two roles for task assignments implied by equation (1). The first role leads to specialization and an individual with more social skill potentially able to increase team output by interacting with more team members. The second role leads to partial specialization and an individual with more social skill being more productive in a team with fixed team size.

Concerning the first role, consider the case where i spends one unit of time on task I . j only has to spend $\theta_j^C = \frac{1}{n_j}$ to create $\sqrt{k_i k_j}$ of output. Any extra time spent on task C is wasted. Since j still have $\frac{n_j - 1}{n_j}$ units of time left, j can coordinate with $n_j - 1$ other persons

who can also spend their time on task I . In a labor market context, j as a manager doing task C , can manage n_j workers. Holding j 's cognitive skill constant, an increase in j 's social skill, n_j , will increase the span of j as a manager. We have shown that an increase in the social skill of an individual allows that individual to work productively with larger teams.

Consider a charismatic politician with large n . He goes to a local campaign office, makes a speech and get all the volunteers in the office to work hard to get him elected. His high social skill, charisma in this case, allows him to enrol more volunteers to work to get him elected relative to other politicians. Another politician with smaller n will have to spend more time trying to convince the same number of volunteers to work for him which means that in the end, he will recruit less volunteers and be less successful.

In the school context, n is the social skill of a teacher to manage his or her class and get the students to focus their attention on learning. If n is small, a teacher has to spend a lot of time getting the students' attention and therefore little time on imparting knowledge.

We can make the similar argument for a manager with large n . She is able to use less of her time with each of her workers and still be able to get them to work productively relative to another manager with less social skill.

Concerning the second role of social skill, now consider the case where time use by team members are exclusively with each other. That is, both i and j cannot work with anyone else to produce output in that sector. This exclusive time use constraint may describe a monogamous marriage where a husband and wife commits to spending time exclusively with each other to produce marital output.

In such a marriage, if i spend all his time, and is fully specialized, on task I , then j only has to spend $\theta_j^C = \frac{1}{n_j}$ to create $\sqrt{k_i k_j}$ of output. $\frac{n_j-1}{n_j}$ time of j is unused and wasted. Now let individual i transfer δ of his time from task I to task C . He will spend $1 - \delta$ time in task I and his effective time on task C is $n_i \delta$. His spouse can spend $n_i \delta$ time on task I as long as $1 - \frac{1-\delta}{n_j} > n_i \delta$ which is feasible for δ sufficiently small. Marital output under this reallocation of time for the couple is:

$$\sqrt{k_i k_j}((1 - \delta) + n_i \delta) \geq \sqrt{k_i k_j}$$

So for any monogamous marriage where two spouses spend time exclusively with each other to produce marital output,

Proposition 1 *There is partial specialization in marriage.*

In this paper, we have one common technology for all three sectors. Moreover, we consider only one social skill with particular characteristics. Thus our SC model is only an initial exploration of how social skills may operate in a society. Our rationale for this model is that even with such a restrictive formulation, the technology generates rich and new predictions about equilibrium behavior in the three sectors.

The rest of the paper works out the implications of the above team production technology for the education, labor and marriage markets.

2.1 Hedonic labor market

It is convenient for us to first analyze the labor market. Each adult consume their own labor earnings and there is no complementarity in consumption from labor earnings and marital consumption. There is no spillover from labor market earnings to the marriage market.

An adult is characterized by his or her social and cognitive skills, (n, k) . In labor market, each individual supplies labor for one unit of time. Let the equilibrium wage for an (n, k) adult for that unit of time be $\omega(n, k)$. If a firm hires the adult for θ fraction of time, then the firm will pay the adult $\theta\omega(n, k)$. The adult will supply $(1 - \theta)$ of time to other employers and earn $(1 - \theta)\omega(n, k)$ from them.

There is a perfectly elastic supply of firms and private schools in the labor market. So in equilibrium, all firms and schools make zero profit.

Schools hire adults to be teachers. A teacher of skill (n, k) will be paid $\omega(n, k)$. Thus in equilibrium the wage function $\omega(n, k)$ must satisfy the demand of firms and schools for employees. For now, we focus only on the problem which firms solve.

A firm is a collection of teams. Because the teams do not interact, we can study the problem of one team in a firm. A team is a collection of employees chosen by the firm to do tasks which are assigned by the firm subject to employees being paid their equilibrium wages.

Lemma 1 *Each team's profit is maximized by allocating every employee to a specific task, either I or C , for the entire length of the production process.*

Proof. Consider a team which employs an individual (n, k) for a short time interval Δ . The firm can allocate the employee to either task I or task C .

If employee (n, k) is allocated to task I during the time interval Δ , then the firm has to hire another individual (n', k') from the labor market to perform task C for $\Delta(n')^{-1}$ units of time in order to produce output $\sqrt{kk'}\Delta$. Choosing (n', k') optimally, the firm's profits of having (n, k) in task I for Δ time interval is given by

$$\pi^I(n, k, \Delta) = \max_{(n', k')} \sqrt{kk'}\Delta - \omega(n', k')(n')^{-1}\Delta - \omega(n, k)\Delta \quad (3)$$

If employee (n, k) is instead allocated to task C in period Δ , then the firm needs to hire another employee (n'', k'') to do task I for $n\Delta$ units of time to produce output $\sqrt{kk''}n\Delta$. The associated profits $\pi^C(n, k, \Delta)$ are

$$\pi^C(n, k, \Delta) = \max_{(k'', n'')} \sqrt{kk''}n\Delta - w(n'', k'')n\Delta - \omega(n, k)\Delta \quad (4)$$

Therefore, the firm would assign employee (n, k) to task I if and only if $\pi^I(n, k, \Delta) - \pi^C(n, k, \Delta) \geq 0$. The sign of $\pi^I(n, k) - \pi^C(n, k)$ is independent of Δ , the length of time that is available for production. Consequently, the firm's profits are maximized by allocating employee (n, k) to either I or C for the entire duration of the production process. ■

Lemma 1 indicates that in equilibrium, employees will specialize in performing either task I or task C .

Furthermore, task assignment is by comparative advantage:

Corollary 1 *Consider two team members of types (n_i, k_i) and (n_j, k_j) . Member i will be assigned to task I if and only if*

$$\frac{\omega(n_i, k_i)}{\omega(n_j, k_j)} \leq \frac{1 - n_j^{-1}}{1 - n_i^{-1}}$$

The result obtains by comparing the profits from assigning member i to task I and j to task C in the time interval Δ , versus the reverse assignment. The above corollary is known since Ricardo.

Since all firms have access to the same production technology, task assignments must be the same across firms.

Corollary 2 *If one firm strongly (weakly) prefers to assign an employee of type (n, k) to task I (C), all other firms will do the same.*

Corollary 3 *For each cognitive skill level k , there exists a cutoff value $\hat{n}(k) \in [\underline{n}, \bar{n}]$ such that individuals with social skill $n < \hat{n}(k)$ perform task I , and individuals with social skill $n \geq \hat{n}(k)$ perform task C .*

Proof. Applying the envelope theorem to (3) and (4) yields

$$\frac{d}{dn} (\pi^I(n, k, \Delta) - \pi^C(n, k, \Delta)) = - \left[\sqrt{kk''} + w(n'', k'') \right] \Delta < 0.$$

Therefore, the value of $\pi^I(n, k, \Delta) - \pi^C(n, k, \Delta)$ crosses zero only once and from above. ■

Let the employee (n, k) be optimally assigned to task I , i.e. $\pi^I(n, k, \Delta) - \pi^C(n, k, \Delta) \geq 0$. We will call these employees workers and denote their occupation by w . Note that the amount of team output produced in Δ time interval is $\sqrt{kk'}\Delta$ which is independent of n . Put another way, the firm does not value a worker's social skill and thus will not be willing to pay for it. Therefore, the equilibrium wage of workers of skill (n, k) , $\omega_w(n, k)$, is independent of n . To simplify notation, we will write $\omega(k) \equiv \omega_w(n, k)$.

On the other hand, if the employee (n, k) is assigned to task C , then output under $\pi^C(n, k, \Delta)$ depends on n . We call these employees managers and denote their occupation by m . Their wages will depend on both n and k , which is denoted by $\omega_m(n, k)$.

Given our production technology, workers with the same k are equally productive and will receive the same equilibrium wage. Therefore, the choice of a worker is equivalent to the choice of a worker's cognitive skill k .

Consider a team with a manager of type (n, k) who does task C . According to lemma 1, such an employee is only matched with other employees who does task I , i.e. workers. Let the team choose $(k_1, \dots, k_n)$ workers to match with the manager in order to maximize its profits. The team solves

$$\max_{(k_1, \dots, k_n)} \sum_{i=1}^n \left[\sqrt{kk_i} - \omega(k_i) \right] - \omega_m(n, k).$$

Given the additive separability of the production process, the optimal choices of workers satisfy $k_1^* = \dots = k_n^* = \mu(k)$ with

$$\mu(k) \in \arg \max_{k_i} \sqrt{kk_i} - \omega(k_i). \quad (5)$$

which implies

Lemma 2 *In equilibrium, it is optimal for a team to hire workers with the same cognitive skill.*

The function $\mu(k)$ depends on the manager's cognitive skill k , but not his or her social skill n . It fully captures the sorting between workers and managers in the labor market. Hence, we call $\mu(k)$ the equilibrium matching function in labor market. Given Lemma 2, we can rewrite the profits of the team with manager (n, k) as follows:

$$n \left[\sqrt{k\mu(k)} - \omega(\mu(k)) \right] - \omega_m(n, k).$$

It follows from the free-entry condition for firms that the above expression must be zero. Therefore, the manager's wage is given by

$$\omega_m(n, k) = n \left[\sqrt{k\mu(k)} - \omega(\mu(k)) \right].$$

Define $\phi(k)$ as

$$\phi(k) \equiv \sqrt{k\mu(k)} - \omega(\mu(k)) = \max_{k'} \sqrt{k k'} - \omega(k'). \quad (6)$$

We can interpret $\phi(k)$ as the profits per worker generated by a type- k manager. The equilibrium wage for the manager (n, k) can be written as

$$\omega_m(n, k) = n\phi(k). \quad (7)$$

Next we address the issue of sorting in labor market. With both workers and managers who are heterogeneous in their cognitive skill, an important question is which worker types work for which manager.

Lemma 3 *In equilibrium, the equilibrium matching function $\mu(k)$ is weakly increasing.*

Proof. We prove by contradiction. Suppose $\mu(k)$ is not weakly increasing. Then there must exist $k_1 < k_2$ such that $\mu(k_1) > \mu(k_2)$. It follows from (5) that

$$\begin{aligned} \sqrt{k_1\mu(k_1)} - \omega(\mu(k_1)) &\geq \sqrt{k_1\mu(k_2)} - \omega(\mu(k_2)) \\ \sqrt{k_2\mu(k_2)} - \omega(\mu(k_2)) &\geq \sqrt{k_2\mu(k_1)} - \omega(\mu(k_1)) \end{aligned}$$

Add these two condition to obtain

$$\sqrt{k_1\mu(k_1)} + \sqrt{k_2\mu(k_2)} \geq \sqrt{k_2\mu(k_1)} + \sqrt{k_1\mu(k_1)}$$

or equivalently

$$\left(\sqrt{k_2} - \sqrt{k_1} \right) \left(\sqrt{\mu(k_2)} - \sqrt{\mu(k_1)} \right) \geq 0$$

which is a contradiction. ■

Let us define $\mu^{-1}(\cdot)$ as the generalized inverse function of $\mu(\cdot)$:

$$\mu^{-1}(k) = \min \{k' : \mu(k') = k\}.$$

Since $\mu(k)$ is monotone, $\mu^{-1}(\cdot)$ is well-defined. Now we can link the equilibrium wage $\omega(k)$ and $\phi(k)$ with the equilibrium matching function $\mu(k)$.

Lemma 4 *Given equilibrium matching function $\mu(k)$, the equilibrium $\phi(k)$ and $\omega(k)$ are given by*

$$\begin{aligned}\phi(k) &= \phi(\bar{k}) - \frac{1}{2} \int_k^{\bar{k}} \sqrt{\frac{\mu(x)}{x}} dx \\ \omega(k) &= \omega(\bar{k}) - \frac{1}{2} \int_k^{\bar{k}} \sqrt{\frac{\mu^{-1}(x)}{x}} dx\end{aligned}$$

Proof. We can apply the envelope theorem to (6) and obtain that

$$\frac{d\phi(k)}{dk} = \frac{1}{2} \sqrt{\frac{\mu(k)}{k}} \quad (8)$$

Furthermore, the necessary first-order condition of the maximization problem (6) is

$$\left. \frac{d\omega(k')}{dk'} \right|_{k'=\mu(k)} = \frac{1}{2} \sqrt{\frac{k}{k'}}$$

which can be rewritten as

$$\frac{d\omega(k)}{dk} = \frac{1}{2} \sqrt{\frac{\mu^{-1}(k)}{k}} \quad (9)$$

The claims then follow immediately. ■

Finally, we will characterize occupation choice in the labor market. Note that schools also compete in the labor market for teachers. Let $\omega_t(n, k)$ denote the wage paid to a type (n, k) teacher. The equilibrium wage of a teacher (n, k) is also linear in n . To see this, note that a school with a (n, k) teacher can admit n students who all pay tuition $\tau(k)$. The tuition is independent of n because it does not affect students' gain from education. Hence, the school's profit equals $n\tau(k) - \omega_t(n, k)$. Applying free entry yields the teacher's wage

$$\omega_t(n, k) = n\tau(k). \quad (10)$$

Note that an adult of skill (n, k) chooses the occupation that maximizes his or her payoff. Hence the employee's wage is

$$\omega(n, k) = \max \{\omega_m(n, k), \omega_t(n, k), \omega(k)\}$$

Since an individual (n, k) can choose to become a teacher or a manager, we must have $\tau(k) = \phi(k)$ for all k . Therefore, an individual (n, k) will become a manager/teacher if and only if

$$n\phi(k) \geq \omega(k).$$

Define

$$\hat{n}(k) \equiv \frac{\omega(k)}{\phi(k)}, \quad (11)$$

then an individual (n, k) will become a manager/teacher if $n \geq \hat{n}(k)$, and will become a worker if $n < \hat{n}(k)$. The occupation decision of an individual is summarized in the following lemma.

Lemma 5 *For each cognitive skill level k , there exists a cutoff value $\hat{n}(k)$ such that individuals with communication skill $n < \hat{n}(k)$ become workers, and individuals with communication skill $n \geq \hat{n}(k)$ become managers or teachers.*

Using equations (11), (8) and (9)

$$\frac{d\hat{n}(k)}{dk} = \frac{\omega'(k)}{\phi(k)} - \frac{\omega(k)\phi'(k)}{(\phi(k))^2} = \frac{\sqrt{\mu^{-1}(k)} - \hat{n}(k)\sqrt{\mu(k)}}{2\sqrt{k}}$$

Therefore, $\hat{n}(k)$ is decreasing in k if and only if

$$\hat{n}(k) > \sqrt{\frac{\mu^{-1}(k)}{\mu(k)}} \quad (12)$$

Remark 1 *Condition (12) has a simple economic interpretation in terms of arbitrage. Suppose $\hat{n}(k)$ is upward sloping at some neighborhood of k , with $k_1 < k < k_2$. In particular, suppose $(\hat{n}(k_1), k_1)$ is assigned as a manager while $(\hat{n}(k_1), k_2)$ is assigned as a worker. Consider a switch of role of individual $(\hat{n}(k_1), k_1)$ and $(\hat{n}(k_1), k_2)$. Before the switch, the total output involving these two individuals are*

$$\hat{n}(k_1)\sqrt{k_1\mu(k_1)} + \sqrt{\mu^{-1}(k_2)k_2}$$

After the switch, the total output involving these two individuals becomes

$$\hat{n}(k_1)\sqrt{k_2\mu(k_1)} + \sqrt{\mu^{-1}(k_2)k_1}$$

There is profitable arbitrage if

$$\hat{n}(k_1)\sqrt{k_2\mu(k_1)} + \sqrt{\mu^{-1}(k_2)k_1} > \hat{n}k_1\sqrt{k_1\mu(k_1)} + \sqrt{\mu^{-1}(k_2)k_2}$$

which is equivalent to

$$\widehat{n}(k_1) \sqrt{\mu(k_1)} > \sqrt{\mu^{-1}(k_2)}.$$

In the limit with $k_2 \rightarrow k$ and $k_1 \rightarrow k$, the above condition reduces to condition (12).

2.2 Marriage Market

A crucial feature that distinguishes the marriage market from the labor market and the education market is monogamy, i.e. all marital matches are bilateral. We formalize monogamy as each spouse in a marriage devoting all their time in the marriage market with each other.

Returning to the time allocation problem between spouses i and j , let them choose θ_i^I and θ_j^I to maximize marital output:

$$Y^M(\theta_i^I, n_i, k_i, \theta_j^I, n_j, k_j) = \sqrt{k_i k_j} (\min(\theta_i^I, n_j(1 - \theta_j^I)) + \min(\theta_j^I, n_i(1 - \theta_i^I)))$$

Lemma 6 *The optimal choices, $\widehat{\theta}_i^I$ and $\widehat{\theta}_j^I$, are*

$$\begin{aligned} \widehat{\theta}_i^I &= \frac{(n_i n_j - n_j)}{n_i n_j - 1} \\ \widehat{\theta}_j^I &= \frac{(n_i n_j - n_i)}{n_i n_j - 1} \end{aligned}$$

and optimal output is:

$$\widehat{Y^M}(n_i, k_i, n_j, k_j) = \sqrt{k_i k_j} \frac{2n_i n_j - n_j - n_i}{n_i n_j - 1}$$

Also

Corollary 4 *Marital output is increasing and supermodular in (n_i, n_j) and in (k_i, k_j) .*

Assuming that the sex ratio (ratio of men to women) for every (n, k) type of individual is one,

Lemma 7 *The marriage market equilibrium exhibits strict positive assortative matching (PAM) with respect to both cognitive skill k and social skill n . I.e. individuals only marry within their type.*

Proof. The proof follows the proof of lemma (3) extended to two factors, n and k . ■

As a result of this lemma, we can simplify notation and write the equilibrium payoff of an individual of type (n, k) in the marriage market as

$$Y^M(n, k) = \frac{n}{n+1}k. \quad (13)$$

The above equation also implies as n becomes large, marital output per spouse converges to k . What this means is that there is a limit to how much marital output can increase if a couple's cognitive skills are low. Whether this implication is empirically relevant remains to be seen.

It is convenient to summarize some of our results to date. In the labor market, employees will specialize in performing either task I or task C . Specialization in the labor market differ sharply from the marriage market where spouses will not specialize according to proposition 1. The economics is straightforward. In the labor market, a team member with large n can exploit their social skill to become a manager. A manager hires n workers to work in a team. Thus we have a model of many to one matching in the labor market. Moreover, managers and workers match assortatively only by cognitive skills. Because of PAM, individuals with high cognitive skill but poor social skill may have significant labor earnings. Put another way, the labor market is able to mitigate an individual's lack of social skill.

On the other hand, monogamy restricts specialization in marriage. Furthermore, spouses match by both social and cognitive skills. So an individual with low social skill will have lower marital output. Compared with the labor market, individuals are more strongly penalized for low social skills in the marriage market. As will be shown in section 3, independent of k or labor earnings, some individuals with low social skills will remain unmarried.

2.3 Education Market

In the education market, students need to decide which school they wish to enroll. As we argued before, given our production function, the accumulation of cognitive skill depends only on a student's initial cognitive ability a_s and the cognitive skill of her teacher, k_t . Therefore, the tuition $\tau(n_t, k_t)$ charged by a school with teacher (n_t, k_t) do not depend on the social skill of the teacher, n_t , and we can write tuition as $\tau(k_t)$. Moreover since every adult can choose whether to be a manager or a teacher, the earnings in the two professions

must be the same if the same type of adult (n_t, k_t) is hired in both occupations:

$$\tau(k_t) = \phi(k_t).$$

Going to a school with a better teacher, at a higher cost of tuition, increases the adult cognitive skill of a student. Each student will choose the school that results in the highest expected payoff, given her initial ability a_s and social skill n_s . An adult with social skill n_s and cognitive skill $\sqrt{a_s k_t}$ will obtain an adult utility of:

$$\omega(n_s, \sqrt{a_s k_t}) + Y^M(n_s, \sqrt{a_s k_t})$$

For simplicity, we assumed that adult utility is the sum of labor income plus marital output. We will also assume a discount rate of zero. There is a perfect capital market and so a student can borrow against her adult earnings to pay off her tuition.

Then the lifetime utility of a student (n_s, a_s) who attends a school with a teacher with cognitive skill k_t is given by

$$\begin{aligned} & \omega(n_s, \sqrt{a_s k_t}) + Y^M(n_s, \sqrt{a_s k_t}) - \tau(k_t) \\ = & \max \left\{ \omega(\sqrt{a_s k_t}), n_s \phi(\sqrt{a_s k_t}) \right\} + \frac{n_s}{n_s + 1} a_s k_t - \phi(k_t) \end{aligned}$$

Therefore, the equilibrium school choice of student (n_s, a_s) is given by

$$\rho(n_s, a_s) \in \arg \max_{k_t} \left[\max \left\{ \omega(\sqrt{a_s k_t}), n_s \phi(\sqrt{a_s k_t}) \right\} + \frac{n_s}{n_s + 1} a_s k_t - \phi(k_t) \right]$$

In equilibrium, a student (n_s, a_s) will attend a school with teacher quality $\rho(n_s, a_s)$.

The next lemma shows how the equilibrium school choice $\rho(n_s, a_s)$ varies with respect to a student's characteristics.

Lemma 8 *Given n_s and future occupation, a student with higher ability a_s will choose a higher quality teacher:*

$$\frac{\partial \rho(n_s, a_s)}{\partial a_s} \Big|_{\text{occupation}} \geq 0.$$

Given a and future occupation, a student with a higher n_s will choose a higher quality teacher:

$$\frac{\partial \rho(n_s, a_s)}{\partial n_s} \Big|_{\text{occupation}} \geq 0.$$

Proof. First suppose student (n_s, a_s) will become a worker. Since $\omega(\sqrt{a_s k_t}) + \frac{n_s}{n_s+1} a_s k_t - \phi(k_t)$ is supermodular in k_t and a_s , and supermodular in k_t and n_s , the claim follows immediately from Topkis's Theorem. The case where student (n_s, a_s) chooses to become a manager/teacher can be proved analogously. ■

Therefore, students with higher cognitive ability a or higher communication skills n will choose to attend schools with higher quality. In equilibrium, students with different cognitive abilities may choose the same school.

*** We need a lemma which says managers have a higher incentive to invest more in schooling than workers. I.e. ***

If $\hat{n}(k)$ is increasing in k , will that lead to a contradiction in optimal investment in cognitive skill? Put another way, the student who chooses to be a manager at $\hat{n}(k)$ is different than the student who chooses to be a worker at $\hat{n}(k)$.

3 Producing alone

In this section, we relax the assumption that production occurs in teams in the labor and marriage market. We also allow $n > 0$ to be less than 1. If an individual produces output alone, that individual will do both task I and task C . Since the individual is working alone, his or her social skill n does not affect effective time in task C . If the individual works alone and spends θ^C in task C , we assume that the individual's effective time in task C is also θ^C . Let an individual of type (n, k) spend θ^I and θ^C in producing output. Then the individual's output is:

$$\tilde{Y}_A(\theta^I, \theta^C, n, k) = k \min(\theta^I, \theta^C)$$

Since total time is still 1, the individual working alone will choose $\theta^A = \theta^C = \frac{1}{2}$ and output will be:

$$Y_A(n, k) = \frac{k}{2}$$

Equilibrium marital output per spouse from a previous section, equation (13), is:

$$Y^M(n, k) = \frac{n}{n+1} k$$

When $n < 1$, the individual will choose to remain unmarried. Thus:

Proposition 2 *With a sex ratio of 1 for every (n, k) type, independent of k , there will be unmarried workers if they have $n < 1$.*

In general, there will be rich unmarried workers.¹³ On the other hand, independent of their cognitive skills, all managers are married.

In the labor market, a (n, k) type worker will earn $\omega(k)$. Since every type of individual can always choose to work alone, teams which employ type k workers exist in the labor market if and only if:

Proposition 3 $\omega(k) \geq \frac{k}{2}$

Put another way, if teams using workers with cognitive skill k exist in the labor market, there will be no adult with cognitive skill k working alone and vice versa. This contrast with the marriage market is stark where there are unmarried individuals as long as they have $n < 1$.

***Can we say more about when we will or will not have type k workers?

***Because of endogenous investments in k , is there a positive correlation between k and the marriage rate? We can simulate to see if this is feasible. If so, it will be important result. I.e. rich people have better marriages not because they are rich per se but because people with social skills become rich.

4 Existence and Uniqueness

An alternative perspective to solve the our multi-factor multi-sector matching model is to write down the utilitarian social planner's optimization problem, which turns out to be a linear program. Since markets are competitive and there is no externalities, the social planner's solution can be implemented in decentralized markets. In other words, the decentralized market equilibrium must be efficient and it is a solution to the planner's problem.

4.1 The Planner's Primal Program

Let $A \equiv [\underline{n}, \bar{n}] \times [\underline{k}, \bar{k}]$ denote the type space for adults. Let $S \equiv [\underline{n}, \bar{n}] \times [\underline{a}, \bar{a}]$ denote the type space for students. The probability measure of student types is exogenously given by $\sigma \geq 0$.

¹³The lives of successful athletes, entertainers and politicians with unsuccessful marriages are widely covered by the popular press.

In the education market, we want to find a joint measure $\varepsilon \geq 0$ on $S \times A$ of many-to-one student-teacher pairings. The supply and demand constraint in the education market requires that the total number of type (n_s, a_s) students in all schools cannot exceed the total supply of type (n_s, a_s) students. Since a teacher of type (n_t, k_t) can mentor n_t students, we must have, for all (n_s, a_s) ,

$$\int_{(n_t, k_t) \in A} n_t \varepsilon(n_s, a_s; dn_t, dk_t) \leq \sigma(n_s, a_s). \quad (14)$$

Similarly, in the labor market, we want to find a joint measure $\lambda \geq 0$ on $A \times A$ of many-to-one pairings of workers to managers. As in the education market, the total demand of type (n, k) workers, type (n, k) managers, and type (n, k) teachers must not exceed the total supply of type (n, k) adults, for all (n, k) . Since a manager of type (n_m, k_m) has the capacity to supervise up to n_m workers, we must have, for all (n, k) ,

$$\begin{aligned} & \int_{(n_m, k_m) \in A} n_m \lambda(n, k; dn_m, dk_m) + \int_{(n_w, k_w) \in A} \lambda(dn_w, dk_w; n, k) + \int_{(a_s, n_s) \in S} \varepsilon(dn_s, da_s; n, k) \\ & \leq \int_{(n_t, k_t) \in A} n_t \varepsilon(n, k^2/k_t; dn_t, dk_t). \end{aligned} \quad (15)$$

Finally, in the marriage market, we have perfect assortive matching. The equilibrium payoff from marriage for a type (n, k) adult is

$$Y^M(n, k) = \frac{n}{n+1}k.$$

Given our production technology, the output Y^L for a team consisting of a type (n_m, k_m) manager and n_m type (n_w, k_w) workers is given by

$$Y^L = Y^L(n_w, k_w; n_m, k_m) = n_m \sqrt{k_w k_m},$$

independent of the worker's social skill n_w .

Thus the planner's primal linear program is given by

$$\sup_{\varepsilon, \lambda} \int_{A \times A} Y^L(n_w, k_w, n_m, k_m) \lambda(dn_w, dk_w; dn_m, dk_m) + \int_{S \times A} n_t Y^M(n_s, \sqrt{a_s k_t}) \varepsilon(dn_s, da_s; dn_t, dk_t) \quad (16)$$

given the constraints $\varepsilon \geq 0$, $\lambda \geq 0$, (14), and (15).

4.2 The Planner's Dual Program

We now focus on the dual program of the primal problem, because it is easier to work with. We first derive the dual program heuristically as follows. Let $u : S \rightarrow \mathbf{R}$ and $v : A \rightarrow \mathbf{R}$ denote the Lagrange multipliers conjugate to the constraints (14) and (15), respectively. These Lagrange multipliers $u(n_s, a_s)$ and $v(n, k)$ can be interpreted as the indirect utility (or wages) that students and adults of various types derive from their position in these markets.

We can write the Lagrangian function as

$$\begin{aligned} \mathcal{L}(\varepsilon, \lambda; u, v) &= \int_{A \times A} [Y^L(n_w, k_w; n_m, k_m) - n_m v(n_w, k_w) - v(n_m, k_m)] \lambda(dn_w, dk_w; dn_m, dk_m) \\ &\quad + \int_{S \times A} [n_t Y^M(n_s, \sqrt{a_s k_t}) + v(n_s, \sqrt{a_s k_t}) n_t - v(n_t, k_t) - u(n_s, a_s) n_t] \varepsilon(dn_s, da_s; dn_t, dk_t) \\ &\quad + \int_S u(n_s, a_s) \sigma(dn_s, da_s). \end{aligned}$$

The duality principle implies that the constrained maximum of the primal problem must be equal to the unconstrained minimax

$$\inf_{u, v} \sup_{\varepsilon, \lambda \geq 0} \mathcal{L}(\varepsilon, \lambda; u, v).$$

Let V denote the set of multipliers v satisfying the following inequality

$$n_m v(n_w, k_w) + v(n_m, k_m) \geq Y^L(n_w, k_w; n_m, k_m) = n_m \sqrt{k_w k_m}, \quad (17)$$

for all $(n_w, k_w), (n_m, k_m) \in A$. Let K denote the set of multiplier vectors (u, v) satisfying (17) and

$$v(n_t, k_t) + u(n_s, a_s) n_t \geq n_t Y^M(n_s, \sqrt{a_s k_t}) + n_t v(n_s, \sqrt{a_s k_t}) \quad (18)$$

for all $(n_s, a_s) \in S$ and $(n_w, k_w), (n_m, k_m), (n_t, k_t) \in A$. Then we can rewrite the minimax as

$$\begin{aligned} &\inf_{u, v} \sup_{\varepsilon, \lambda \geq 0} \mathcal{L}(\varepsilon, \lambda; u, v) \\ &= \inf_{(u, v) \in K} \sup_{\varepsilon, \lambda \geq 0} \mathcal{L}(\varepsilon, \lambda; u, v) \\ &= \inf_{v \in V} \int_{(n_s, a_s) \in S} \left[\sup_{(n_t, k_t) \in A} Y^M(n_s, \sqrt{a_s k_t}) + v(n_s, \sqrt{a_s k_t}) - \frac{1}{n_t} v(n_t, k_t) \right] \sigma(dn_s, da_s) \end{aligned}$$

The first equality follows because if $(u, v) \notin K$, we can always find ε and λ such that the supremum is unbounded. The last equality follows from the definition of set K .

Assuming the supremum of the primal and the infimum of the dual are attained (which is true in the discrete setting), it follows that condition (17) must hold with equality for λ -a.e. worker-manager pair and condition (18) becomes an equality for ε -a.e. teacher-student pair. These equalities reflect the fact that the equilibrium permits no opportunities for arbitrage.

Assuming the differentiability of the wages u and v , we deduce the following first-order conditions from (17): for λ -a.e. pair of worker (n_w, k_w) and manager (n_m, k_m) ,

$$\frac{1}{2}\sqrt{\frac{k_m}{k_w}} = \frac{1}{n_m} \frac{\partial Y^L}{\partial k_w} = \frac{\partial v(n_w, k_w)}{\partial k_w} \quad (19)$$

$$0 = \frac{1}{n_m} \frac{\partial Y^L}{\partial n_w} = \frac{\partial v(n_w, k_w)}{\partial n_w} \quad (20)$$

$$\frac{1}{2}n_m\sqrt{\frac{k_w}{k_m}} = \frac{\partial Y^L}{\partial k_m} = \frac{\partial v(n_m, k_m)}{\partial k_m} \quad (21)$$

$$\sqrt{k_w k_m} = \frac{\partial Y^L}{\partial n_m} = v(n_w, k_w) + \frac{\partial v(n_m, k_m)}{\partial n_m} \quad (22)$$

It follows from (20) that the worker types must be distinct from the manager types, with $v(n_w, k_w) = \omega(k_w)$ independent of n_w and semi-convex in the worker region. We also see from (19) that each worker's cognitive skill k_w determines the cognitive skill of his manager

$$\frac{1}{2}\sqrt{\frac{k_m}{k_w}} = \frac{d\omega(k_w)}{dk_w}$$

The wage constraint (17) now implies $v(n_m, k_m) = n_m\phi(k_m)$ holds for a.e. type (n_m, k_m) of manager, where $\phi(k_m)$ is defined as

$$\phi(k_m) = \sup_k \sqrt{k k_m} - \omega(k).$$

The formula $v(n_m, k_m) = n_m\phi(k_m)$ implies the worker and manager regions must be disjoint. It is easy to verify that the last two first-order conditions (21) and (22) are automatically satisfied. Condition (21) implies that

$$\frac{1}{2}\sqrt{\frac{k_w}{k_m}} = \frac{d\phi(k_m)}{dk_m}$$

so the cognitive skill of each workers k_w is determined by the human-capital of his manager k_m . Continuity of v across the worker/manager interface $\hat{n}(k)$ forces $\omega(k) = \hat{n}(k)\phi(k)$. Hence, we have

$$\frac{d\hat{n}}{dk} = \frac{\omega'(k)\phi(k) - \phi'(k)\omega(k)}{\phi(k)^2}.$$

Moreover, $\hat{n}'(k) = 0$ implies $\omega'(k) = \hat{n}(k)\phi'(k)$.

Turning now to the education market, we know that condition (18) must hold with equality

$$\frac{n_s}{n_s + 1} n_t \sqrt{a_s k_t} + n_t v(n_s, \sqrt{a_s k_t}) = n_t u(n_s, a_s) + v(n_t, k_t)$$

for α -a.e. pairing of student type (n_s, a_s) with teacher type (n_t, k_t) . This implies

$$\begin{aligned} \frac{n_s}{n_s + 1} \frac{1}{2} \sqrt{\frac{k_t}{a_s}} + \frac{1}{2} \sqrt{\frac{k_t}{a_s}} \frac{\partial v(n_s, \sqrt{a_s k_t})}{\partial k} &= \frac{\partial u(n_s, a_s)}{\partial a_s} \\ \frac{1}{(n_s + 1)^2} \sqrt{a_s k_t} + \frac{\partial v(n_s, \sqrt{a_s k_t})}{\partial n_s} &= \frac{\partial u(n_s, a_s)}{\partial n_s} \\ \frac{n_s}{n_s + 1} n_t \frac{1}{2} \sqrt{\frac{a_s}{k_t}} + \frac{1}{2} n_t \sqrt{\frac{a_s}{k_t}} \frac{\partial v(n_s, \sqrt{a_s k_t})}{\partial k} &= \frac{\partial v(n_t, k_t)}{\partial k_t} \\ \frac{\partial v(n_t, k_t)}{\partial n_t} &= \frac{1}{n_t} v(n_t, k_t). \end{aligned}$$

From the last inequality and positivity of v , we see teacher and worker types cannot overlap.

It follows from the first and third equality that

$$n_t a_s \frac{\partial u(n_s, a_s)}{\partial a_s} = k_t \frac{\partial v(n_t, k_t)}{\partial k_t} = k_t \phi'(k_t),$$

which shows that each student's type (n_s, a_s) determines his or her teacher's cognitive skill $k_t = \rho(n_s, a_s)$.

The solution to the first and second equations depends on whether or not the student is destined to be a worker or a teacher/manager:

$$\begin{aligned} \frac{\partial u(n_s, a_s)}{\partial a_s} &= \begin{cases} \frac{n_s}{n_s + 1} \frac{1}{2} \sqrt{\frac{k_t}{a_s}} + k_t \omega'(\sqrt{a_s k_t}) & \text{if worker} \\ \frac{n_s}{n_s + 1} \frac{1}{2} \sqrt{\frac{k_t}{a_s}} + k_t n_s \phi'(\sqrt{a_s k_t}) & \text{if manager} \end{cases}, \\ \frac{\partial u(n_s, a_s)}{\partial n_s} &= \begin{cases} \frac{1}{(n_s + 1)^2} \sqrt{a_s k_t} & \text{if worker} \\ \frac{1}{(n_s + 1)^2} \sqrt{a_s k_t} + \phi(\sqrt{a_s k_t}) & \text{if manager} \end{cases}. \end{aligned}$$

Therefore,

$$u(n_s, a_s) = \sup_{k_t} v(n_s, \sqrt{a_s k_t}) - \phi(k_t).$$

If $n_s < \hat{n}(\sqrt{a_s k_t})$, then $v(n_s, \sqrt{a_s k_t}) = \omega(\sqrt{a_s k_t})$. Hence, we have

$$u(n_s, a_s) = \sup_{k_t: n_s < \hat{n}(\sqrt{a_s k_t})} \omega(\sqrt{a_s k_t}) - \phi(k_t).$$

Otherwise, $v(n_s, \sqrt{a_s k_t}) = n_s \phi(\sqrt{a_s k_t})$, and we have

$$u(n_s, a_s) = \sup_{k_t: n_s \geq \hat{n}(\sqrt{a_s k_t})} n_s \phi(\sqrt{a_s k_t}) - \phi(k_t).$$

5 Simulation

6 Related literature

We provide some links, albeit inadequately, to the several large literatures from which this paper builds on.

First, there is a large empirical psychology literature and a much smaller one in economics which have shown that both cognitive and non-cognitive factors, including social factors, affect multiple lifetime outcomes.¹⁴ Almlund, et. al. (2011) is a summary of the empirical psychology literature written for economists. It summarizes both the substantive findings and also the identification problems associated with estimating empirical factor models based on individual data.

Two findings from the above studies are particularly relevant. First, researchers do not currently agree on how many non-cognitive factors there are. Psychologists believe that much of the disagreements are due to context differences across samples in different studies.¹⁵ For example, it difficult to compare two persons' agreeableness by the number of fights they have gotten into without controlling for the exogenous and endogenous environments in which they operate.¹⁶

The second finding is that in empirical models with a small number of factors, the empirical cognitive and non-cognitive factors which they recover are not orthogonal to each other.¹⁷ Some researchers expand the number of factors until the estimated factors are othogonal to each other. The SC model suggests that this is probably not a useful approach. Instead, our behavioral model supports a small number of non-orthogonal factors. In complementary re-

¹⁴E.g. Almlund, et. al. (2011); Caspi, A., and R.L. Shiner (2005); Markon, et. al. (2005); McCrae, R.R., and P.T. Costa Jr. (1999).

¹⁵John, et. al. 2008 provide an intellectual history of the "Big Five" model which is a popular empirical model of non-cognitive skills. Most of this history has to do with reconciling disparate findings across studies.

¹⁶Part of the problem is due to the fact that some questions are not consistent across surveys, and also some questions are about about exogenous and others about endogenous variables. Since empirical factor models are not behavioral models, there is a limit to agreement about how different factors affect the responses to different questions.

¹⁷Markon, et. al. (2005); DeYoung, C.G. 2006.

search, Cunha and Heckman (2007); Cunha, et. al. (2010) showed that optimal single agent investments in multidimensional skills may help identify such models using individual data. In addition to skill accumulation, Heckman, et. al. (2011) found a three factor structure in their reanalysis of the Perry pre-school experiment data. Two of these correspond to our cognitive and social factors. They also have a factor related to the marginal utility of effort which we ignored due to our assumption of inelastic labor supply in each sector.

While there is much work on social interaction models, as discussed in the introduction, we are not aware of any equilibrium model of social interactions where individuals differ by social skills. Benhabib, et. al. 2011 provide a recent comprehensive survey on research on social interaction models. There are one sector models of endogenous group formation which are focused on complementary issues (E.g. Jackson 2008; Kelso and Crawford 1982; Roth and Sotomayor 1990).

Becker (1973, 1974; summarized in 1991) began a large theoretical literature on frictionless one factor transferable utility model of marriage and positive assortative matching (See Roth and Sotomayor (1990), Part 3; and Weiss (1997) for surveys). Chiappori, et. al. 2010; extends the framework to multiple factors. The empirical relevance of this model has been demonstrated by Choo and Siow 2006; Brandt, Siow and Vogel 2011; Siow 2010; Chiappori, Salanie and Weiss 2011.

Becker's matching model has been extended in different directions including the labor market (E.g. Eeckhout and Kicher 2011, Legros and Newman 2002; Shimer and Smith 2000).

The empirical marriage matching literature shows that spouses sort by age, ethnicity, immigration status, language, religion, socioeconomic status, height, weight and many other characteristics. While there is no comparable studies of workers in a firm, many of the characteristics which spouses sort by will be illegal in the labor or education market. There is an empirical literature on positive assortative matching by workers and firms (E.g. Abowd, Kramarz, and Margolis 1999; Lise, et. al. 2008; Mendes, et. al. 2006). Also see the empirical literature on firm size and wages (E.g. Groshen 1991).

Building on one factor models of hierarchies by Lucas (1978) and Rosen(1982), Garicano (2000) and his co-authors (e.g. Garicano and Rossi-Hansberg (2006)) study one factor optimal task assignments, organization design and equilibrium matching in the labor market. Although we use a two factor model and we ended up with a different formulation and study

many sectors, our concerns were motivated by their models.¹⁸

In general, the Garicano class of models predict many to many matching (managers and workers) in a firm. The SC model predicts many to one matching in the labor market. To the best of our knowledge, ours is the first theory of this commonly observed organizational form. In other applications such as matching schools with students, and hospitals with interns, the many to one matching is assumed and researchers focus on other issues. Roth 2008 surveys the rapid progress in this literature.

Roy (1950) is the classic two factor model of occupational choice. Also see Willis and Rosen (1979); Heckman and Sedlacek (1995); Heckman and Honore (1990); Dahl 2002. The use of the Roy model beyond the study of selection is modest. We suspect that this lack of progress is that researchers have not imposed much behavioral structure on the two factors. Our SC model does exactly that. Pitt, Rosenzweig and Hassan 2010 is another exception.

Building on Rosen (1978), Becker (1991) studied the division of labor within the household. He argued that at least one spouse will be fully specialized in a task. Pollak (2011) argues that Becker's full specialization is a special case based on different concerns than those discussed here.

Starting from Ricardo, there is of course a large literature on task assignment based on comparative advantage. To the best of our knowledge, our matching concerns about complementarity and task assignments by comparative advantage in one model is new. The SC model of incomplete specialization due to the limits to trade in spousal time due to monogamy is an example of Adam Smith's claim that the division of labor is limited by the extent of the market.

Chiappori, McCann, Nesheim (2010) showed that the frictionless multifactor marriage matching model is equivalent to a utilitarian social planner's linear programming problem. We extend this equivalence to a frictionless multisector multifactor many to many matching framework with endogenous occupational choice.

Smeets and Warzynski 2008 showed that within one corporation, when managers with larger teams are promoted to a higher level, they continue to manage larger teams after their promotions.

There is substantial evidence that employees in higher levels of a corporate hierarchy have more education (E.g. Ortín-Ángel, etl. al. 2007; Smeets and Warzynski 2008; Gibbons

¹⁸One nice feature of their models is that they extend to multi level hierarchies which we do not have.

and Waldman 1999 is a survey of careers in hierarchies).

This paper predicts that the rate of return education for managers is proportional to the span of the manager. There is no empirical evidence on this prediction. There is evidence that managers have a higher rate of return to schooling than non-managers (E.g. Hirsch 1978). Building on the Roy model, Keane and Wolpin 1997 showed that the rate of return to schooling is higher for white collar workers compared with blue collar workers after controlling for a one factor unobserved ability of the individual. More evidence is definitely on this issue.

Previously, economists have used signalling and screening models of education to explain endogenous bunching of educational attainment (E.g. Bedard 2001; Wiess 1995). The SC model generates endogenous bunching in a frictionless full information environment. Our bunching mechanism is the mirror image of the Merlitz model of international trade where he generates bunching in quality of goods by exporting versus non-exporting firms by assuming a common revenue function for quality in domestic and foreign markets, and different fixed costs for exporting versus non-exporting firms.

Pre-marital investments and marital matching have been discussed by Peters and Siow 2002; Iyigun and Walsh 2007.

The impact of education on marital output is also well documented (E.g. Behrman, et. al. 1994; Chaiporri, Salanie and Weiss 2011; Isen and Stevenson 2010; Siow 2010). Isen and Stevenson also reports that college educated couples self report that they are happier with their marriage than non-college couples.

While researchers have not systematically studied the social skills of spouses in affecting divorce, lack of communication within the marriage is one of the most often cited rationale for marital dissolution by ex-spouses (E.g. Burns 1984; Cleek, et. al. 1985; Eells, et. al. 1996).

According to Burda, et. al. 2007, there is gender equality in the total of non-work time consumed in rich nations that lack a Catholic cultural background. That is, the sum of labor market time and housework is essentially the same for both genders. The specialization by husbands into market work in some marriages can be explained by a fixed time cost of work which makes it efficient for one spouse to specialize in market work and the other in house work (E.g. Van Soest 1995). There is no evidence of increasing or decreasing specialization by gender in leisure time activities (Ramey and Francis 2009).

There is an empirical literature on the matching of students and colleges (Hoxby 2009 is a survey). A common answer in this literature about why high and low cognitive ability students are in the same school is that there are student peer effects (E.g. Epple and Romano 2011). Low cognitive ability students subsidize high ability students to interact with them. In their surveys on peer effects among college students, Epple and Romano, and Sarcedote 2011 do not find quantitatively large academic student peer effects. There is more evidence to support student peer effects in non-academic behavior such as drinking.

7 Conclusion

Motivated by the empirical literature on the importance of social skills for lifetime outcomes, this paper presents a social and cognitive skills model of human capabilities. The new results in this paper are primarily due to our new output technology production function which integrates Becker's concern for complementarity with Ricardo's concern for task assignments.

Since we aggregate all non-cognitive factors into one social factor and also make strong functional form assumptions, the SC model is best viewed as an initial exploration of how cognitive and social skills may work in these environments.

We now provide some avenues for further research. The current model assumes that there is inelastic labor supply to the marriage and labor market. One unit of time to be divided between the labor and marriage market is a useful extension. A further extension is to allow different individuals to value marital output and labor market income differently.

The many to one matching in this paper assume no interaction between workers in a team. Thus our many to one model is a first step towards more general models of many to one matching.

To avoid dealing with gender issues, we assumed that the sex ratio in the marriage market by types is unity. Since there are gender differences in cognitive and non-cognitive skills, this assumption should be relaxed for any analysis of gender roles in this class of models (E.g. Siow 1998).

There is substantial empirical evidence that social skills can be accumulated. Thus there is room for a model where individuals can accumulate both social and cognitive skills.

Currently, all firms have access to the same production technologies and therefore all firms earn zero profit. It will be useful extend to the case where firms have different productivities,

match with different teams of employees, and earn different profits.

Our labor market model generates two level hierarchies. Extensions to multi level hierarchies are needed.

The room for empirical research is wide open. A first order problem is to separately identify social and cognitive skills in different sectors. For example, what are the testable implications of differing social skills within a marriage?

Motivated by this paper, ongoing research by Kamborov, Siow and Turner show that individuals with more previous divorces are more likely to separate from their current employer, and individuals with more previous separations from employers are more likely to divorce their current spouse.

Another avenue for empirical research is to study non-cognitive factors which affect the schooling decision and subsequent occupational choices. In particular, the model predicts that the rate of return to schooling is increasing in the span of a manager.

Finally, there is need to integrate the large non-behavioral empirical literature on cognitive and non-cognitive skills, and the recent empirical behavioral models on cognitive and non-cognitive skill acquisitions with the equilibrium matching concerns studied here.

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