

Optimal Fiscal and Monetary Policy With Occasionally Binding Zero Bound Constraints*

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During the Great Recession, the government provided large fiscal stimulus in an economic environment characterized by a high degree of uncertainty on the future course of the economy while the nominal interest rate was constrained at the zero lower bound. While many papers have analyzed the effects of fiscal policy at the zero lower bound, they all do so in a deterministic environment. This paper studies optimal government spending and monetary policy when the nominal interest rate is subject to the zero lower bound constraint in a stochastic environment. In the presence of uncertainty, the government chooses to increase its spending when at the zero lower bound by a substantially larger amount than it would in the deterministic environment. The welfare effect of fiscal policy is nuanced in the stochastic environment if the government cannot commit. Although the access to government spending policy increases welfare in the face of a large deflationary shock, it can decrease welfare during normal times as the government reduces the nominal interest rate less aggressively before reaching the zero lower bound.

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¹Please visit <https://files.nyu.edu/tn516/public/research.html> for the most updated version of the paper.

1 Introduction

The short-term nominal interest rate, which is widely seen as the conventional monetary policy tool, is subject to the constraint that it cannot go below zero. During the Great Recession, the Federal Reserve effectively lowered the federal funds rate to zero. The policy rate has been at the zero lower bound since then, and is expected to remain so for the near future. With the standard monetary policy tool constrained at the zero lower bound, the government has turned to alternative policy tools to stimulate the economy. One such policy tool was fiscal policy. In early 2009, Congress enacted the American Recovery and Reinvestment Act to provide large fiscal stimulus to the U.S. economy in the hope that it would create jobs and promote economic recovery.

Such policy response has generated renewed interest on the effect of fiscal policy among economists and policymakers. Over the past few years, many authors have examined the effect of fiscal policy in the context of dynamic general equilibrium models, and one theme emerged: The effect of fiscal policy is different when the economy is constrained at the zero lower bound. Some papers have shown that an exogenous increase in government spending has larger effects on consumption and output when the economy is at the zero lower bound.² Others have examined the welfare implications of fiscal policy and have found that it is optimal to increase government spending when at the zero lower bound.³ These results seem to support the active use of fiscal policy in the current economic environment.

However, there is one unsatisfactory aspect in the previous studies on fiscal policy at the zero lower bound: They all focus on deterministic environments.⁴ In these models, an exogenous force that pushes the nominal interest rate to zero at time one reverts back to its steady-state level in a deterministic manner. Such focus on the deterministic environment is unsatisfactory because the current economic environment motivating these analyses is characterized by a high degree of uncertainty. Policymakers during any economic downturn are far from certain about the severity and duration of the recession, and this might be particularly true in the recent episode. There is anecdotal evidence suggesting that uncertainty has slowed investment and hiring by firms, and many believe that it has been a key factor contributing to the slow recovery.⁵ Thus, it would be useful to examine how the presence of uncertainty alters the assessment of fiscal policy when the economy is at the zero bound.

Accordingly, this paper studies optimal fiscal and monetary policy when the nominal interest rate is subject to the zero lower bound constraint in a stochastic environment. Following the setup of the previous literature, I use an exogenous variation in the household's discount factor as the force that leads the government to lower nominal interest rate to zero. However, unlike the previous literature that studied the evolution of the economy along one specific path of the exogenous discount factor, the discount factor in this paper is stochastic. The fiscal instrument available to the government is government spending financed by lump-sum taxation.⁶ The main analysis is conducted under the timing protocol in which the government makes decisions sequentially, taking as given the future policy functions of the government, household, and

²For example, see Christiano, Eichenbaum, and Rebelo (2011), Eggertsson (2010), Erceg and Linde (2010), and Woodford (2011).

³For example, see Eggertsson (2001), Nakata (2011a), and Werning (2011).

⁴Eggertsson and Woodford (2003) considered a two-state Markov process for the natural rate of interest with an absorbing state, and many subsequent authors have adopted that process. This setup is useful because it allows one to analytically characterize some properties of the model and to use a variation of Newton-algorithm to solve the model numerically. However, it is not suited for answering the questions this paper is interested in for several reasons that will become clear later.

⁵Nicholas Bloom and Jaimovich (2010) constructs various measures of uncertainty and shows that they are countercyclical. Bloom (2009) conducts a VAR analysis using stock market volatility as a proxy for uncertainty and finds that an unexpected increase in uncertainty leads to a sharp transitory drop in industrial production by conducting VAR analyses. See Scott R. Baker and Davis (2011) and Alexopoulos and Cohen (2009) for other attempts to quantify uncertainty and its impact on real economic activities.

⁶Eggertsson (2001), Nakata (2011a), and Werning (2011) have shown that, in a deterministic environment, a transitory increase in government spending increases welfare at the zero bound.

firms. I refer to the model with this timing protocol as the model without commitment. However, I also conduct the analysis under the alternative timing protocol in which the government decides a sequence of policy variables for all states for all time periods at the beginning of time one. I refer to the model with this timing protocol as the model with commitment.

In the model without commitment, the government increases its spending at the zero bound by a substantially larger amount in the stochastic environment than it would in the deterministic environment. When the economy is subject to the zero lower bound, a mean-preserving spread in the shock distribution increases the expected real interest rate and decreases the expected real marginal cost of production. Such changes in the expectation lead forward-looking household and firms to consume less and set lower prices today. As a result, when the economy is at the zero lower bound, the declines in consumption, output, and inflation in response to a given size of the shock are larger in the stochastic environment. Faced with larger declines, the government chooses to increase its spending by a larger amount. In various parameterizations, the optimal increase in government spending in the stochastic environment is about twice as large as in the deterministic environment.

Comparing the benchmark model in which the government has both fiscal and monetary policy instruments with a constrained model in which the government has only the monetary policy instrument at its disposal, I find that access to the government spending policy not only affects allocations at the zero bound directly, but it also affects allocations away from the zero bound indirectly through an effect on the nominal interest rate policy. The indirect effect arises because, when the government does not have access to fiscal policy, it reduces the nominal interest rate more aggressively before reaching the zero lower bound. This nominal interest rate policy leads to small increases in consumption and output away from the zero lower bound, and can increase welfare.

This somewhat counterintuitive result—the fact that welfare can be larger without fiscal policy instrument—arises because the constraint on the government spending policy has two aspects in the model without commitment. On the one hand, it constrains the government’s choice on its spending today. On the other hand, this constraint represents a commitment to not rely on countercyclical fiscal policy in the future. When the economy is at the zero lower bound, the government wants to increase government spending, and the constraint on government spending will lead to larger declines in consumption, output, and inflation. However, when the economy is away from the zero lower bound, the commitment to not using government spending policy, even if the economy goes back to the zero lower bound in the future, leads the household and firms to reduce their expectations of future allocations and prices. This is because the declines in consumption, output, and inflation are larger at the zero lower bound if the government cannot use fiscal policy. Such shifts in the expectation lead the forward-looking household and firms to lower consumption and prices today by more. Today’s government tries to prevent such a reduction in consumption and inflation by lowering the nominal interest rate more aggressively. In equilibrium, there are small increases in consumption and output when the economy is away from the zero lower bound.

In the model with commitment, the presence of uncertainty also leads the government to increase its spending at the zero bound by a larger amount. However, the additional increase in government spending due to uncertainty is quantitatively small. Both in the deterministic and stochastic environments, the use of fiscal policy does not have large effects on allocations nor welfare. In the model with commitment, the government chooses to hold nominal interest rates at zero for an extended period of time. Commitment to an extended period of low nominal interest rates goes a long way in improving the allocations when the economy is at the zero lower bound and the marginal effects of fiscal policy are small even in the stochastic environment.

This paper is the first to examine the consequences of an additional policy instrument in the model with

occasionally binding zero bound constraints. Adam and Billi (2006), Adam and Billi (2007), and Nakov (2008) studied optimal policy with zero lower bound constraints in stochastic environments. However, in their models, the nominal interest rate is the only policy instrument and the government does not have any policy tool available once the nominal interest rate is at the zero bound. In reality, a myriad of policy instruments are available to the government, and they are actively used. While I focus on fiscal policy in this paper, it would be useful to consider the consequences of other policy tools as well. For example, one may want to examine the effect of financial policy in a similar environment. This paper can be seen as providing a framework within which one can conduct such analysis.

Section 2 describes the model and Section 3 formulates the government's problem. Section 4 discusses calibration and the solution method. Section 5 illustrates the importance of uncertainty at the zero bound using a simpler model with a truncated Taylor rule. Section 6 and 7 respectively discuss the results for the models without and with commitment. Section 8 discusses the implications of optimal fiscal policy on the average inflation rate. Section 9 concludes. Tables and figures follow.

2 Model

This section describes the private sector of the model and defines the equilibrium. The private sector of the economy is given by the standard New Keynesian model. The model is formulated in discrete time with infinite horizon. The economy starts at time one.

2.1 Household

The representative household chooses consumption, labor supply, and the holdings of one period risk free nominal bond to maximize the expected discounted sum of the future period utilities. The household likes consumption and government spending, and dislikes labor. The period utility is assumed to be separable. The household problem is given by

$$\max_{C, N, B} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s \right] \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} + \chi_{g,0} \frac{G_t^{1-\chi_{g,1}}}{1-\chi_{g,1}} \right]$$

subject to

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} - P_t T_t + P_t \Phi_t$$

and δ_1 is given. C_t is consumption, N_t is labor supply, and G_t is government spending. P_t is the price of consumption good, W_t is nominal wage, T_t is lump-sum taxation, and Φ_t is the profit from the intermediate goods producers. B_t is one-period risk free bond that pay one unit of money at $t+1$, and R_t is the return on the bond.

The discount rate at time t is given by $\beta \delta_t$. δ_t is the discount factor shock that alters the weight of the future utility at time $t+1$ relative to the period utility at time t . δ_t follows an AR(1) process:

$$(\delta_t - 1) = \rho(\delta_{t-1} - 1) + \epsilon_t \quad \forall \quad t \geq 2$$

where ϵ_t is a shock to the discount factor shock and is distributed as normal with mean 0 and standard deviation σ_ϵ . One can think of δ_t as a measure of the household's patience. An increase in δ_t makes the household want to save more for tomorrow and spend less today. The previous literature considered the case in which $\sigma_\epsilon = 0$. This paper analyzes the case with $\sigma_\epsilon = 0$.

2.2 Producers

There is a representative final good producer and a continuum of intermediate goods producers indexed by $i \in [0, 1]$. The representative final good producer purchases the intermediate goods, combines them into the final good using CES technology. It sells the final good to the household and government as well as to the intermediate good producers if they change the prices.

$$\max_{Y_{i,t}, i \in [0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

subject to the CES production function, $Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$.

Intermediate-good producers use labor to produce imperfectly substitutable intermediate goods using linear production function. Each firm sets the price of its own good in order to maximize the expected discounted sum of future profits.

$$\max_{P_{i,t}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\prod_{s=0}^{t-1} \delta_s \right] \lambda_t \left[P_{i,t} Y_{i,t} - W_t N_{i,t} - P_t \frac{\varphi}{2} \left[\frac{P_{i,t}}{P_{i,t-1}} - 1 \right]^2 Y_t \right]$$

subject to $Y_{i,t} = \left[\frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t$, and $Y_{i,t} = N_{i,t}$. Price changes are subject to quadratic adjustment costs. There is no heterogeneity in the time zero prices across firms. That is, $P_{i,0} = P_0$ for some given constant $P_0 > 0$.

2.3 Government's Policy Instruments

The government's problem will be introduced in the next section. Here, I describe a set of restrictions on government's policy instruments.

Throughout the paper, I assume that a lump-sum taxation is used to finance government spending and that the supply of the government bond is zero. Thus, the government budget constraint is given by $G_t = T_t$. The balanced budget assumption is made mainly in order to simplify the analysis. The more challenging model in which government debt appears as an additional state variable is the subject of ongoing research. However, for economies with large deficits, fiscal policy might be constrained by pay-as-you-go rules that limit the extent of future borrowing, and this assumption may not be too restrictive.⁷

Finally, and most importantly, I impose that the nominal interest rate cannot fall below 1.

$$R_t \geq 1$$

2.4 Market Clearing Conditions

Market clearing conditions for the final good, labor, and the bond are given by

$$\begin{aligned} Y_t &= C_t + G_t + \int \frac{\varphi}{2} \left[\frac{P_{i,t}}{P_{i,t-1}} - 1 \right]^2 Y_t di \\ N_t &= \int_0^1 N_{i,t} di \\ B_t &= 0 \end{aligned}$$

⁷While the paper focuses on the lump-sum taxation, it is straightforward to analyze the model with distortionary taxations under the balanced budget assumption.

2.5 An Implementable Symmetric Equilibrium

Given P_0 and $\{\delta_t\}_{t=1}^\infty$, an implementable symmetric equilibrium of this economy consists of allocations, $\{C_t, N_t, N_{i,t}, Y_t, Y_{i,t}\}_{t=1}^\infty$, prices $(\{W_t, P_t, P_{i,t}\}_{t=1}^\infty)$, and policies $(\{R_t, G_t, T_t\}_{t=1}^\infty)$ such that

- Allocations solve the problem of the household given prices and policies.
- $P_{i,t}$ solves the problem of firm i .
- $P_{i,t} = P_{j,t}$ for all $i \neq j$.
- All markets clear.

It is straightforward to show that a set of implementable symmetric equilibria can be characterized by $\{C_t, N_t, Y_t, w_t, \Pi_t, R_t, G_t\}_{t=1}^\infty \equiv \{d_t\}_{t=1}^\infty$ satisfying

$$C_t^{-\chi_c} = \beta \delta_t R_t E_t C_{t+1}^{-\chi_c} \Pi_{t+1}^{-1} \quad (1)$$

$$w_t = \chi_{n,0} N_t^{\chi_{n,1}} C_t^{\chi_c} \quad (2)$$

$$\frac{N_t}{C_t^{\chi_c}} [\varphi(\Pi_t - 1)\Pi_t - (1 - \theta) - \theta w_t] = \beta \delta_t E_t \frac{N_{t+1}}{C_{t+1}^{\chi_c}} \varphi(\Pi_{t+1} - 1)\Pi_{t+1} \quad (3)$$

$$Y_t = C_t + G_t + \frac{\varphi}{2} [\Pi_t - 1]^2 Y_t \quad (4)$$

$$Y_t = N_t \quad (5)$$

$$R_t \geq 1 \quad (6)$$

Eqn.(1) is consumption Euler Equation and eqn.(2) is the intratemporal optimality condition of the household. Eqn.(3) is the optimality condition of the intermediate good producing firms, often referred to as the forward looking Phillips Curve. It relates today's inflation to the real marginal cost today and expected inflation tomorrow. Eqn.(4) is the aggregate resource constraint. The last term of eqn.(4) captures the resource cost of price adjustment. Eqn.(5) is the aggregate production function.

3 Government's Problem

This section formulates the government's problem. Previous studies have documented that the government's ability to commit makes stark differences in the allocations when the nominal interest rate is constrained by the zero lower bound. Thus, I consider two alternative timing protocols. In the first protocol, the government sequentially makes decisions. Each period, the government optimizes taking as given the policy functions of future government, household, and firms. I refer to this model as the model without commitment. In the second protocol, the government decides a sequence of policy variables for all states for all time periods at the beginning of time one, announces it to agents in the private sector, and adheres to the announced policy in the future. I refer to this model as the model with commitment.

3.1 Without Commitment

For every period t , the government solves the following problem.

$$V_t(\delta_t) = \max_{\{d_t\}} \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} + \chi_{g,0} \frac{G_t^{1-\chi_{g,1}}}{1-\chi_{g,1}} \right] + \beta \delta_t E_t V_{t+1}(\delta_{t+1})$$

subject to the private sector equilibrium conditions stated above (eqn. (1) to (6)) and taking as given the next period value and policy functions $\{V_{t+1}(\cdot), C_{t+1}(\cdot), N_{t+1}(\cdot), Y_{t+1}(\cdot), w_{t+1}(\cdot), \Pi_{t+1}(\cdot), R_{t+1}(\cdot), G_{t+1}(\cdot)\}$.

A Markov-Perfect Equilibrium consists of a set of time-invariant value and policy functions $\{V(\delta_t), C(\delta_t), N(\delta_t), Y(\delta_t), w(\delta_t), \Pi(\delta_t), R(\delta_t), G(\delta_t)\}$ solving the Bellman equation above.⁸ In this paper, I focus on policy functions to depend only on the current state, δ_t . This excludes other time-consistent equilibria where policy and value functions depend on the history of states. My ongoing research analyzes those cases.

3.2 With Commitment

The government with the ability to commit chooses a sequence of policy variables for all states for all times at the beginning of time one.

$$\hat{W}(\delta_1) = \max_{\{d_t\}_{t=1}^{\infty}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \prod_{s=0}^{t-1} \delta_s \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_n}}{1+\chi_n} + \chi_{g,0} \frac{G_t^{1-\chi_{g,1}}}{1-\chi_{g,1}} \right]$$

subject to the private sector equilibrium conditions. **A Ramsey Equilibrium** consists of $\{C(\delta^t), N(\delta^t), \Pi(\delta^t), G(\delta^t), R(\delta^t)\}$ for all δ^t satisfying the FONCs of the Ramsey planner's problem, where $\delta^t \equiv [\delta_1, \dots, \delta_t]$ denotes the history of shocks.

Recursive Characterization of the Ramsey Equilibrium

Following Marcet and Marimon (2011), I characterize the Ramsey equilibrium recursively. The government's problem can be written as:

$$W(\delta_t, \phi_{1,t-1}, \phi_{2,t-1}) = \max_{\{d_t\}} \min_{\{\phi_t\}} \left[\frac{C_t^{1-\chi_c}}{1-\chi_c} - \frac{N_t^{1+\chi_{n,1}}}{1+\chi_{n,1}} + \chi_{g,0} \frac{G_t^{1-\chi_{g,1}}}{1-\chi_{g,1}} \right] + \beta \delta_t E_t W(\delta_{t+1}, \phi_{1,t}, \phi_{2,t})$$

subject to the equations characterizing the symmetric equilibrium. $\phi_t = [\phi_{1,t}, \phi_{2,t}, \phi_{3,t}, \phi_{4,t}]$ is a vector of the Lagrangian multipliers for the four constraints characterizing the symmetric equilibrium. A Ramsey Equilibrium can be characterized by a set of time-invariant policy and value functions, $[C(s_t), N(s_t), \Pi(s_t), G(s_t), R(s_t), W(s_t)]$, where $s_t \equiv [\delta_t, \phi_{1,t-1}, \phi_{2,t-1}]$.

4 Calibration and Solution Method

4.1 Calibration

Table 2 lists the parameter values selected. The values chosen for the household's preference parameters, the elasticity of substitution among intermediate goods, and the price adjustment cost are within the range considered in the literature. The discount rate β is set to $\frac{1}{1+0.0075}$, which implies the steady state real interest rate of 3 %. Two parameters describing the evolution of time-preference (ρ and σ_ϵ) have important influences on the equilibrium. For the persistence parameter, ρ , I use 0.8, which is the value considered in Adam and Billi (2006), Adam and Billi (2007), and Nakov (2008).

⁸The within-period timing assumption that leads to this optimization problem is that the government and the agents in the private sector move simultaneously. See the discussion in Eggertsson and Swanson (2008). For alternative within-period timing assumptions, see King and Wolman (2004) and van Zandweghe and Wolman (2010). For the detailed discussion on the importance of the within-period timing assumption in Markov Perfect Equilibrium, see Ortigueira (2006).

For the variance of shock, σ_ϵ , I choose a value of $\frac{0.42}{100}$ as a benchmark.⁹ This value makes the frequency of hitting the zero lower bound around 8 percent. While this may sound too large, the Federal Reserve has indicated that it is likely to keep its policy rate at exceptionally low levels until mid-2013. The frequency of being at the zero bound over post WWII periods will be about 8 % by that time. The implied unconditional standard deviation of δ_t , denoted by σ_δ , is 0.007.

4.2 Solution Method

The model is solved by a time iteration method by Coleman (1991). The time-iteration method starts from a guess of policy functions. Assuming that the guessed policy functions are in use for the next period, the FONCs of the government problem is solved to find the policy functions in the current period. This process is repeated until policy functions today become arbitrarily close to policy functions tomorrow.

For the model without commitment, I use 1001 grids on $[1 - 5\sigma_\delta, 1 + 5\sigma_\delta]$ for δ_t . For the model with commitment, I use 101 grids on $[1 - 5\sigma_\delta, 1 + 5\sigma_\delta]$ for δ_t , 11 grids on $[0, 0.05]$ for $\phi_{1,t}$, 11 grids on $[\bar{\phi}_2 - 0.02, \bar{\phi}_2 + 0.02]$ for $\phi_{2,t}$ where $\bar{\phi}_2$ is the Ramsey steady state of $\phi_{2,t}$.

5 Simple Illustration With A Truncated Taylor Rule

Before presenting the main results on optimal policy, it is useful to understand first why the presence of uncertainty matters at the zero lower bound in a simpler setting. Thus, this section compares allocations in the deterministic and stochastic economies when the nominal interest rate is determined according to a truncated Taylor rule and government spending is constant.¹⁰ Specifically, throughout this section, I assume that the nominal interest rate and government spending are given by

$$\begin{aligned} R_t &= \max[1, \frac{1}{\beta} \Pi_t^{\phi_\pi}] \\ G_t &= 0.21 \end{aligned}$$

where I set ϕ_π , the coefficient on inflation in the truncated Taylor rule, to 2. Also, σ_ϵ is set to 0.18/100 as the equilibrium does not exist with the benchmark parameter values in the model with a truncated Taylor rule.

Figure 1 shows the policy functions in both deterministic and stochastic economies. Dashed red lines are for the deterministic economy, and solid black lines are for the stochastic economy. An increase in the discount factor shock, δ , means that the household becomes more patient. As the household becomes more patient, s/he wants to save more for tomorrow and spend less today. This decline in the demand for consumption good leads to lower output and inflation. The truncated Taylor rule dictates that a reduction in inflation be accompanied by a reduction in the nominal interest rate. In equilibrium, nominal interest rate, inflation, consumption, and output all decline as δ becomes larger.

The reduction in the nominal interest rate partially offsets the effect of the increased discount factor shock because the household would like to save less for tomorrow and spend more today when the nominal interest rate is low. However, for a sufficiently large increase in the discount factor shock, the nominal interest rate cannot be lowered further due to the ZLB. Thus, the declines in consumption, output, and

⁹The larger the σ_ϵ is, the more frequently the discount rate $\beta\delta_t$ exceeds one. The Markov equilibrium does not exist if the discount rate exceeds one sufficiently frequently.

¹⁰Nakata (2011b) examines the interaction of the zero lower bound constraint and uncertainty using the same set-up in more details.

inflation in response to an increase in the discount factor shock are larger when the nominal interest rate is constrained at the zero lower bound than otherwise. While this is true in both deterministic and stochastic economies, the declines in consumption, output, and inflation at the zero lower bound are larger in the stochastic economy than in the deterministic economy.

To understand why the presence of uncertainty reduces allocations at the ZLB, consider the following thought experiment in the deterministic economy. Suppose that there is an unexpected increase in the discount factor shock at time one so that $\delta_1 = 1.0075$. With persistence parameter $\rho = 0.8$, the discount factor shock is $\delta_2 = 1.006$ at time two. According to the policy function for consumption, consumption at time two is 0.843 if $\delta_2 = 1.006$. In the deterministic economy, the household at time one knows his/her consumption next period is 0.843, and the expected consumption tomorrow is therefore 0.843.

Now, consider asking the following hypothetical question to the household at time one. Suppose that there is uncertainty about δ_2 . That is, ϵ_2 may not be zero. There will be no uncertainty beyond $t=2$ so that $\epsilon_t = 0$ for all $t \geq 3$. What is the probability distribution of consumption tomorrow in the presence of this hypothetical one-time uncertainty?

Figure 2 shows the household's answer to this hypothetical question. The distribution is asymmetric. A negative realization of ϵ_2 reduces δ_2 and increases consumption tomorrow. However, the increase in consumption is limited because the nominal interest rate rises in response to the decline in δ_2 . On the other hand, a positive realization of ϵ_2 raises δ_2 and reduces consumption tomorrow. This increase in δ_2 will not be offset by the reduction in nominal interest rates due to the zero bound constraint. Thus, the reduction in consumption tomorrow is larger than the increase in consumption that would result with a negative realization of ϵ_2 of the same magnitude. As a result, the expected consumption tomorrow is 0.842, which is lower than in the deterministic case. Faced with lower expected consumption tomorrow, the forward-looking household consumes less today.

Similarly, an increase in uncertainty reduces the expected real wage tomorrow because the policy function for the real wage is concave due to the ZLB. Faced with lower expected real wage tomorrow, firms set lower prices today. For the one-time uncertainty case, the effect may be quantitatively small. But, this effect is amplified when there is uncertainty about δ_t for all states and for all time periods. In equilibrium, the declines in consumption, output, and inflation at the ZLB is larger in the stochastic economy than in the deterministic economy by non-trivial amount.

6 Results without Commitment

This section characterizes the equilibrium in the model without commitment. I first show how the presence of uncertainty affects allocations by comparing the stochastic economy with the deterministic economy. I then move on to study the effects of fiscal policy on allocations and welfare. To do so, I solve the model in which the government is constrained to keep its spending constant at the deterministic steady-state level for all states and for all time periods, and compare allocations and welfare in this constrained economy with those in the benchmark unconstrained economy.

6.1 Optimal Policy With and Without Uncertainty

Figure 3 shows the policy functions in the stochastic and deterministic economies. Solid black lines are for the stochastic economy ($\sigma_\epsilon = \frac{0.42}{100}$) and dashed red lines are for the deterministic economy ($\sigma_\epsilon = 0$).

In the deterministic economy, the government responds to an increase in the discount factor shock (δ) by reducing the nominal interest rate by one-for-one. An increase in the discount factor shock means that

the household becomes more patient. As the household becomes more patient, s/he want to save more for tomorrow and spend less today. The government chooses to offset this effect by offering lower nominal interest rates on the government bond. The government would like to completely neutralize the effect of an increase in δ because there is no capital or storage technology in the model and the allocations it wants to achieve do not depend on the household's discount rate. As a result, consumption, output, and inflation do not change from their deterministic steady-state levels as δ increases—as long as the government is able to reduce the nominal interest rate.

For a sufficiently large increase in the discount factor shock, the government cannot reduce the nominal interest rate further to completely offset the increase in the discount factor shock, and that leads to a decline in the demand for goods by the household. In this model with nominal price rigidity, firms respond to the reduced demand for goods by shrinking the production. This reduces the demand for labor, and that in turn puts downward pressure on the real wage. The decline in the real wage in turn leads firms to set lower prices today. Lower inflation today increases the real interest rate in the previous period. Faced with higher real interest rate, the household would like to reduce consumption even further. In equilibrium, consumption, output, and inflation all decline in response to the increase in the discount factor shock when the economy is at the zero lower bound. Faced with such declines in allocations and prices, the government chooses to increase its spending. An increase in government spending represents an increase in the demand for goods, which leads to increases in consumption, output, and inflation. Thus, by increasing its spending, the government reverses the effects of the increase in the discount factor shock.

In the stochastic environment, the reductions in consumption, output, and inflation when the economy is at the zero lower bound are larger, and the increase in government spending is also larger. Larger reductions in consumption, output, and inflation come from the same mechanism described for the simple environment in the previous section. When the economy is at the ZLB, a mean-preserving spread in the shock distribution increases the expected real interest and decreases the expected real wage, leading the forward-looking household and firms to consume less and set lower prices. Faced with larger declines in consumption, output, and inflation, the government chooses to increase its spending by larger amount. The difference in allocations between deterministic and stochastic economies is quantitatively important. For example, at $\delta = 1.021$, which is a three-standard-deviations away from the steady-state, the decline in consumption in the stochastic economy is almost twice as large as that in the deterministic economy. The increase in government spending in the stochastic environment is also twice as large as that in the deterministic environment.

Not only does the presence of uncertainty affect the allocations and government spending policy at the zero lower bound, it also affects the nominal interest rate policy. Specifically, in the presence of uncertainty, the government reduces the nominal interest rate more aggressively in response to an increase in the discount factor shock before reaching the zero lower bound than it would in the deterministic economy. In the deterministic economy, once the economy is away from the zero lower bound, the economy never goes back to the zero lower bound. However, in the stochastic economy, even if the economy is away from the zero bound, there is a positive probability that a future shock leads the economy back into the zero bound. Thus, away from the zero lower bound, the expected real interest rate and the expected real wage are respectively higher and lower in the stochastic economy than in the deterministic environment. Forward-looking household and firms therefore would like to reduce consumption and set lower prices today. The government chooses to offset this effect by reducing the nominal interest rate further.

To help us better understand these policy functions, **Figure 4** shows how differently deterministic and stochastic economies respond to a one-time large increase in the discount factor shock. Solid black lines show the evolution of the endogenous variables in the economy with $\sigma_\epsilon = \frac{0.42}{100}$ and dashed red lines show the evolution of the endogenous variables in the economy with $\sigma_\epsilon = 0$. The experiment behind the figure is as

follows. A large shock hits the economies at time one so that $\delta_1 = 1.021$, which is three standard deviations away from the steady-state. There are no shocks after time one. In the deterministic economy, the agents know that there will be no shocks in the future. On the other hand, the agents residing in the stochastic economy think that there will be additional shocks in the future and make decisions accordingly.

In the stochastic environment, the declines in consumption, output, and inflation and the increase in government spending are substantially larger at the beginning of the recession. While consumption drops by about 1 percent in the deterministic economy, it drops by about 2 percent in the stochastic economy. Inflation declines by less than 0.5 percent in the deterministic economy, but it declines by about 2 percent in the stochastic economy. While the government in the deterministic economy raises its spending by about 3 percent, the government in the stochastic economy does so by about 6 percent. The presence of uncertainty also alters the government's monetary policy responses to this shock. As discussed earlier, the government reduces nominal interest rate more aggressively before reaching the zero lower bound in the presence of uncertainty. The flip-side of this nominal interest rate policy is that the government is slower in increasing the nominal interest rate from zero out of a recession. While the government in the deterministic economy keeps the nominal interest rate at zero for 4 quarters, the government in the stochastic economy keeps the nominal interest rate at zero for 5 quarters.

6.2 Optimal Policy With and Without Fiscal Policy

This subsection and next compare the equilibria with and without government spending policy in order to understand how the access to fiscal policy alters allocations, prices, and welfare. Specifically, I solve the government problem with an additional constraint that government spending has to be constant at the deterministic steady-state level for all states and all time periods. This constrained economy corresponds to the models studied in Adam and Billi (2007) and Nakov (2008) in which the nominal interest rate is the only policy instrument.

Figure 5 shows the policy functions with and without government spending policy. Solid black lines are the policy functions in the benchmark economy with government spending policy and dashed red lines are the policy functions in the constrained economy.

Not surprisingly, the declines in output and inflation are larger at the zero bound when the government is not allowed to vary its spending. As a result of the reduced demand for goods, firms reduce the demand for labor, which puts downward pressures on real wage. A reduction in the real wage is then translated into lower inflation due to nominal price rigidities. Even though less resources are devoted for government spending, consumption is lower in the absence of government spending policy because output declines by more one-for-one in response to the reduction in government spending.

The access to fiscal policy affects nominal interest rate policy in an important way. The government without fiscal policy reduces the nominal interest rate more aggressively in response to an increase in the discount factor shock before reaching the zero lower bound. As discussed above, if the government is constrained to keep its spending constant, the declines in consumption, output, and inflation are larger at the ZLB. In the stochastic economy, even if the economy is away from the ZLB, the household and firms assign a positive probability to visiting those states in the future. Since the allocations are lower in those ZLB states if the government does not have fiscal policy, the agents' expected consumption and inflation are lower, and forward-looking agents reduce consumption and lower prices by more. The government without access to fiscal policy lowers the nominal interest rate by a larger amount to offset such larger declines.

This difference in the nominal interest rate policy between the unconstrained and constrained economies manifests itself in the difference in the frequencies of the economy being at the zero lower bound. **The**

first row of Table 3 shows that the frequencies of hitting the zero lower bound with and without fiscal policy in this model without commitment. The government without access to government spending policy lowers the nominal interest rate to zero about 9.1 percent of the time whereas the government with access to government spending policy reduces the policy rate to zero 7.8 percent of the time.

In order to understand the results from a different perspective, **Figure 6** shows how differently the two economies, one with and the other without government spending policy, evolve in response to a one-time increase in the discount factor shock. The experiment behind the figure is the same as in Figure 4, and is described in the previous subsection. At the beginning of the recession, the declines in consumption, output and inflation are substantially larger without government spending policy. The government keeps the nominal interest rate at zero for 4 quarters both with and without fiscal policy, but it is slightly slow in raising the rate back to the normal level when fiscal policy is not available.

6.3 Welfare Implications of Fiscal Policy

The difference in the nominal interest rate policy described above has important implications on welfare. **Top-left panel of Figure 7** shows the difference in the value functions between the unconstrained and constrained economies. It is shown for the range of δ_t covering four standard deviations away from the deterministic steady-state level. For a wide range of δ_t , this difference is negative, meaning that welfare in the constrained economy without government spending policy is larger than welfare in the unconstrained economy with government spending policy. Only when the discount factor shocks are very large—more than three standard deviations above the steady-state—the unconstrained economy with both fiscal and monetary policy instruments generates higher values.

As discussed in the previous subsection, the nominal interest rate is more aggressively reduced in the absence of government spending policy. This improves allocations for a range of discount factor shocks around the point where the zero bound starts binding. **Top-right panel of Figure 7** shows current period utilities associated with different discount factor shocks. Solid black lines and dashed red lines are today's utilities for the model with and without fiscal policy, respectively. For a range of discount factor shocks around 1.01 at which the zero bound constraint becomes binding, the current period utilities are larger without government spending policy. A more aggressive reduction in the nominal interest rate leads to increases in consumption and output as well as a small reduction in inflation, and it raises the current period utility. The expectation of better allocations also leads to better allocations away from the zero bound as the agents expect to visit to these states in the future. Thus, the current period utility without fiscal policy dominates the one without it for a wide range of discount factor shocks, except for very large ones. Unless δ_t exceeds 1.016, the current period utilities are larger without fiscal policy than with fiscal policy.

Since the discount factor shock gradually reverts to its steady-state level, the economy does not stay too long in the region of very large discount factor shocks where the access to fiscal policy leads to higher current period utilities. As current period utility is a small portion of the welfare, the value of the problem—the expected discounted sum of future period utilities—is larger without government spending policy for an even larger range of discount factor shocks.

This somewhat counterintuitive result on the welfare consequences of fiscal policy is driven by the fact that the constraint on government spending policy has two aspects if the government does not have a commitment technology. On the one hand, it constrains the government's choice on its spending today. On the other hand, this constraint represents a commitment to not rely on countercyclical fiscal policy in the future. If the future government uses fiscal policy to mitigate the impact of deflationary shocks in the future, the agents in the economy do not expect a large reduction in inflation and consumption. Thus, today's

government has less incentive to reduce nominal interest rates aggressively. This effect can dominate the improved allocations in those zero lower bound states with large discount factor shocks if the economy does not visit the zero lower bound sufficiently often.

In order to quantify the welfare gain from government spending policy that excludes its negative effect arising from the lack of commitment described above, I formulate a hypothetical government’s problem in which the constrained government is given an opportunity to choose its spending in the current period, but not in the future. I then calculate the amount of one-time transfer of consumption goods—measured as a percentage of the steady state consumption—required to make the constrained government as well-off as the hypothetical government with one-time control on its spending.

The bottom-left panel of Figure 7 shows the welfare gain. Solid black line shows the welfare gain in the stochastic economy and dashed red line shows the welfare gain in the deterministic economy. By construction, the welfare gain of one-time deviation is positive. One salient feature is that the welfare gains are substantially larger in the stochastic model than in the deterministic model. At $\delta_t = 1.021$, the welfare gain is close to 0.08 percent of the steady-state consumption. This number is not trivial for a one-time deviation experiment.

7 Results with Commitment

This section characterizes allocations and welfare when the government can commit to a sequence of policy variables at time one. As in the previous section, I first study the implications of uncertainty on the conduct of fiscal and monetary policy by comparing the stochastic and deterministic economies. I then study the effect of having fiscal policy as an additional policy instrument by comparing allocations and welfare in the constrained and unconstrained economies.

Policy and value functions in the recursively formulated Ramsey equilibrium are functions of three states: the discount factor shock and two Lagrangian multipliers in the previous period. Instead of directly analyzing this high-dimensional object, I use the response of the economy to a one-time deflationary shock to discuss the key features of the Ramsey equilibrium.

7.1 Optimal Commitment Policy With and Without Uncertainty

Figure 8 shows the Ramsey equilibria in response to the same experiment conducted in the Figures 4 and 6: there is a one-time shock to the discount factor shock that pushes δ_1 to 1.021. There are no further shocks after time one, and the discount factor gradually reverts back to its steady-state according to $\delta_t - 1 = \rho(\delta_{t-1} - 1)$. Solid black lines and dashed red lines respectively show the evolution of the key variables in the stochastic and deterministic economies.

In the deterministic environment, the government promises to keep the nominal interest rate for an extended period of time and increases its spending at the initial phase of the zero bound period. An extended period of low nominal interest rates creates inflation during the zero bound period, which helps to mitigate the deflationary spiral that would otherwise occur at the beginning of the recession.

In the stochastic environment, the government promises an even longer period of low nominal interest rates. In this experiment, the nominal interest rate is held at zero for 9 quarters in the stochastic setting, as opposed to 8 quarters in the deterministic setting. As in the model without commitment, the government increases its spending by a larger amount in the stochastic environment. However, the additional increase in government spending due to uncertainty is very small. The consequences of these policy responses are that consumption, inflation, and labor supply responses are little affected by the presence of uncertainty.

The allocations in the model with commitment are quite different from those in the model without commitment. In the model without commitment, the presence of uncertainty causes large changes in both fiscal and monetary policy responses, and consumption, output and inflation decline by a substantially larger amount despite such large policy responses. In the model with commitment, despite negligible changes in fiscal policy, the presence of uncertainty does not cause large drops in consumption, output, and inflation because of a more accommodative monetary policy response.

7.2 Optimal Commitment Policy With and Without Fiscal Policy

This subsection compares the Ramsey equilibria with and without government spending policy. In **Figure 9**, solid black lines show the evolution of the key variables in the unconstrained economy and dashed red lines show the evolution of the key variables in the constrained economy in which the government spending is held constant.

The access to government spending policy does not alter the nominal interest rate policy. In both constrained and unconstrained economies, the government keeps the nominal interest rate at zero for 9 quarters before gradually raising it back to the steady-state level. Consumption is essentially unchanged, and the increase in government spending leads to a one-for-one increase in output. A smoother output path leads to a slightly smoother inflation path. With government spending policy, the initial deflation is slightly contained and the inflation peaks at a slightly lower level. Overall, the limited consequences of government spending policy in the deterministic environment documented in my previous work also apply to the stochastic environment.¹¹ In the model with commitment, monetary policy does most of the work and the marginal impact of fiscal policy is small, both in the deterministic and stochastic economies.

The limited impact of government spending policy in the model with commitment can be also seen in the frequency of hitting the zero bound. **The second row of the Table 3** shows the frequencies of hitting the zero bound with and without fiscal policy. They are the same up to 2 decimal points.

7.3 Welfare Implications of Fiscal Policy

Unlike in the model without commitment, welfare is always larger with fiscal policy than without it when the government can commit to a sequence of policy variables at time one. **Top panel of Figure 10** shows the difference in the value functions between the unconstrained and constrained economies for $\phi_{2,t-1} = \bar{\phi}_2$. The difference is always positive, meaning that welfare is larger with fiscal policy than without it in any states of the economy with $\phi_{2,t-1} = \bar{\phi}_2$. Although not shown, this is true for other values of $\phi_{2,t-1}$. The constraint on government spending policy can increase welfare in the model without commitment because the constraint acts as a commitment device on the future policy. Such beneficial effect does not arise from imposing constraints on any policy instrument if the government has a commitment technology, and constraints on any policy instruments always lead to lower welfare.

Finally, to quantify the welfare effects of fiscal policy in a way that allows comparison with the model without commitment, I compute welfare gains from a one-time use of government spending policy in the constrained economy, conditional on the Lagrangian multipliers being at their steady-state values in the previous period. In **the bottom panel of Figure 10**, solid black lines and dashed red lines are welfare gains from a one-time use of fiscal policy in the stochastic and deterministic economies, respectively.

Comparing the dashed red lines in the bottom-left panel of Figure 7 and the bottom panel of Figure 10 demonstrates that welfare gains are slightly larger in the model with commitment than without commitment in the deterministic model. While this may seem to contradict with the earlier discussion that fiscal policy

¹¹See Nakata (2011a).

plays a limited role in the model with commitment, it does not. The larger welfare gain in the model with commitment comes from the fact that the steady state nominal interest rate in this model is lower than in the model without commitment. Thus, the nominal interest rate is reduced to the zero lower bound more often in the model with commitment, and the welfare gain from fiscal policy is larger at any discount factor shock compared to the one in the model without commitment.

As the previous discussion suggests, in the model with commitment, the presence of uncertainty makes the welfare gain from fiscal policy larger, but by a small amount. Solid black lines are very close to dashed red lines. This is because the government responds to the presence of uncertainty mainly by more accommodative monetary policy, not by more aggressive fiscal policy. As a result, the welfare gain of fiscal policy is little affected by the presence of uncertainty. This is in a sharp contrast to the model without commitment where the welfare gain from fiscal policy increases sharply in the presence of uncertainty (see the bottom-left panel of Figure 7).

8 The Average Inflation Rates Revisited

Although the short term nominal interest rate is still at the zero lower bound in the U.S., many economists and policymakers have already started asking the implications of the zero lower bound on the conduct of monetary policy during normal times. One issue that has been most debated is the implication of the zero lower bound on inflation and nominal interest rate targets. Some have argued that nominal interest rate (and thus inflation) should be set high during normal times so that central bank has more room to reduce it in the face of a large deflationary shock.¹² Several authors have examined this issue rigorously using dynamic stochastic general equilibrium models with occasionally binding zero bound constraints.¹³ However, these studies have focused on models that assign no role to fiscal policy, and have left for the future research the investigation of implications of fiscal or other policy instruments on this debate.

To shed some light on this question, this section documents how the presence of government spending policy alters the average inflation rate. **Table 4** tabulates the average inflation rates with and without government spending policy. The upper and lower sections of Table 4 are respectively for the models with and without commitment. For each entry, the number on the top is the unconditional average inflation rate and the second number in brackets is the conditional average inflation rate when the nominal interest rate is away from the zero bound.

In the model without commitment, the deterministic steady-state level of inflation, or equivalently the stochastic average rate of inflation that would prevail in the absence of the zero lower bound, is positive due to inflation bias. With occasionally binding zero bound constraints, the average inflation rate decreases. This is simply because the economy experiences declines in inflation whenever the economy is at the zero lower bound. Since the expectation of visiting the zero lower bound also reduces inflation outside the zero lower bound, the conditional average inflation rate away from the zero bound is also lower than the deterministic level. As the decline in inflation is smaller at the zero lower bound if the government has the access to fiscal policy, the average inflation rate is higher with fiscal policy than without it.

In the model with commitment, the deterministic steady-state level of inflation is zero. The Ramsey planner chooses zero inflation rate in order to minimize the resource cost of non-zero inflation. As shown in Adam and Billi (2006), Billi (2011), and Nakov (2008), the presence of the zero lower bound makes the average inflation rate positive. This is because the government induces inflation during the zero bound period by promising to keep low nominal interest rates in the future. Since inflation is positive mostly during the

¹²See Blanchard, Dell’Ariccia, and Mauro (2010) for an example.

¹³See Coibion, Gorodnichenko, and Wieland (2011) and Billi (2011) for examples.

zero bound period, the conditional average inflation rate away from the zero bound is essentially zero. When the government has fiscal policy as an additional instrument, the increase in inflation during the zero bound period is slightly contained (see the Figure 9). Therefore, the average inflation rate is slightly lower with fiscal policy. However, the conditional average inflation rate is again essentially zero.

Overall, the analysis of this section shows that the access to fiscal policy partially unwinds the effect of the zero lower bound constraint on the average inflation rate. The presence of the zero lower bound reduces the average inflation rate in the economy without commitment, but the reduction is smaller when government spending policy is available. The zero lower bound increases the average inflation rate in the economy with commitment, but the increase is smaller when government spending policy is available. Even without fiscal policy, these effects of the zero bound constraints on the average inflation rate tend to be very small. The active use of fiscal policy makes them even smaller.

9 Conclusion

This paper characterized optimal government spending and monetary policy when the nominal interest rate is subject to the zero lower bound in a stochastic environment. In the model without commitment, the government increases its spending when at the zero bound by a substantially larger amount in the stochastic environment than it would in the deterministic environment. The access to government spending policy directly affects the allocation at the zero bound, but it also affects the allocations away from the zero bound indirectly through its effect on the nominal interest rate policy. When the government does not have access to fiscal policy, it reduces the nominal interest rate more aggressively before reaching the zero lower bound. Such aggressive monetary policy increases consumption away from the zero lower bound, and can increase the welfare.

This paper focused on government spending policy, but my previous research has shown that other fiscal instruments, namely labor income and consumption taxations, are more effective fiscal instruments in improving allocations at the zero bound.¹⁴ It is straightforward to extend the analysis in this paper to consider other fiscal instruments under the balanced budget assumption. Unreported analysis shows that many of the results in this paper are not only robust to these alternative fiscal instruments, but also more pronounced.

My ongoing research extends the model without commitment to allow the government to choose the amount of one period risk free nominal bond. This is an important step towards answering many interesting and policy relevant questions. How should the government conduct fiscal policy at the zero lower bound if it can borrow from the future? If the government wants to borrow from the future, how large is the welfare loss of imposing debt-ceiling? Should the debt level be kept low during normal times so that large fiscal stimulus can be provided at the zero bound without accumulating a large amount of debt? My next paper will shed light on these questions.

¹⁴See Nakata (2011a)

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Table 1: Relation to other representative works on optimal R_t and G_t

	with R_t and G_t	with R_t
Deterministic	Eggertsson (2001) Werning (2011)	Eggertsson and Woodford (2003)
Stochastic	This paper	Adam and Billi (2006 and 2007) Nakov (2008)

Table 2: Calibration

Parameter	Description	Calibrated Value
β	Discount rate	$\frac{1}{1+0.0075} \approx 0.9925$
χ_c	Inverse intertemporal elasticity of substitution for C_t	1.0
χ_n	Inverse labor supply elasticity	1.0
$\chi_{g,0}$	Utility weight on G_t	0.25
$\chi_{g,1}$	Intertemporal elasticity of substitution for G_t	1.0
θ	Elasticity of substitution among intermediate goods	10
φ	Price adjustment cost	150
ρ	AR(1) coefficient for the discount factor	0.8
σ_ϵ	The standard deviation of shocks to the discount factor	$[0, \frac{0.42}{100}]$
σ_δ	The implied unconditional standard deviation of δ	0.007

Table 3: Frequency of Hitting the Zero Bound With and Without Fiscal Policy

	with R_t only	with R_t and G_t
Without Commitment	9.1 %	7.8 %
With Commitment	14.2 %	14.3 %

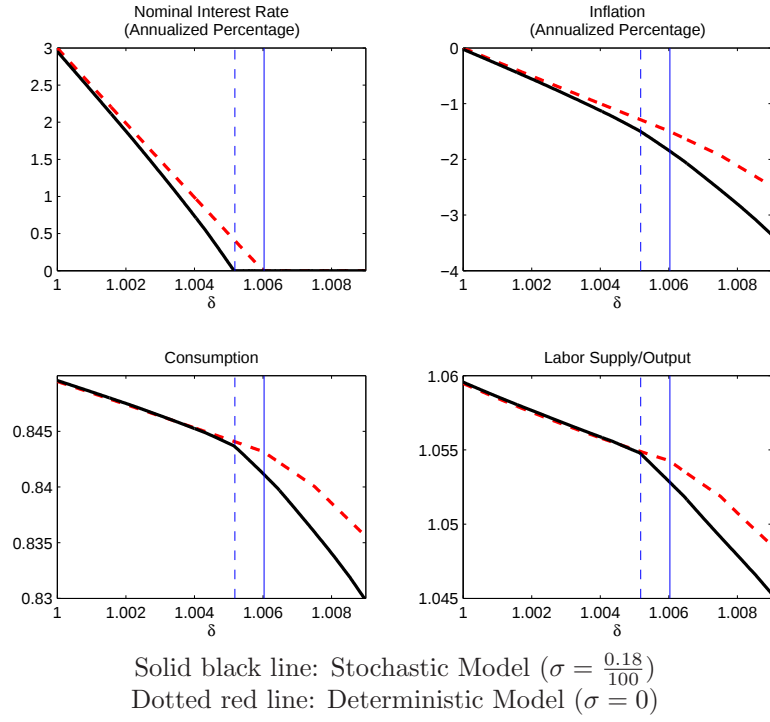
*The frequencies are computed based on 100,000 simulations.

Table 4: Average Inflation Rates With and Without Fiscal Policy

	Without Zero Bound	With Zero Bound	
		with R_t only	with R_t and G_t
Without Commitment	2.031 N/A	1.950 (2.013)	1.984 (2.020)
With Commitment	0.0 N/A	0.022 (0.005)	0.018 (0.004)

*The inflation rate is expressed as an annualized percentage. Unbracketed numbers are the unconditional averages from 100,000 simulations. The second numbers in the bracket are the averages conditional on the nominal interest rate strictly larger than zero.

Figure 1: Policy/Allocations with a Truncated Taylor Rule: Deterministic vs. Stochastic Equilibria



*Policy functions are shown for the range of δ that covers its steady-state level ($\delta = 1$) to the level that is 4 standard deviations away from the steady-state ($\delta = 1.012$).

Figure 2

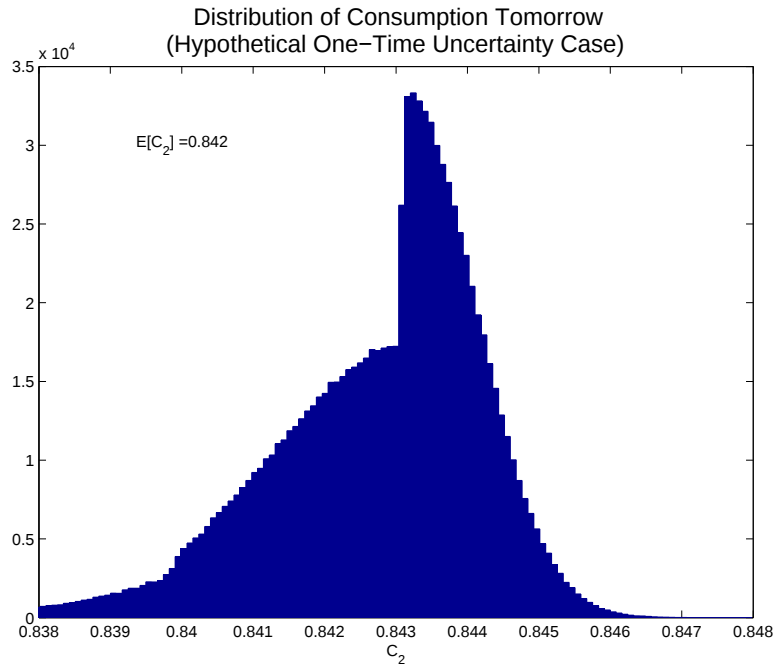
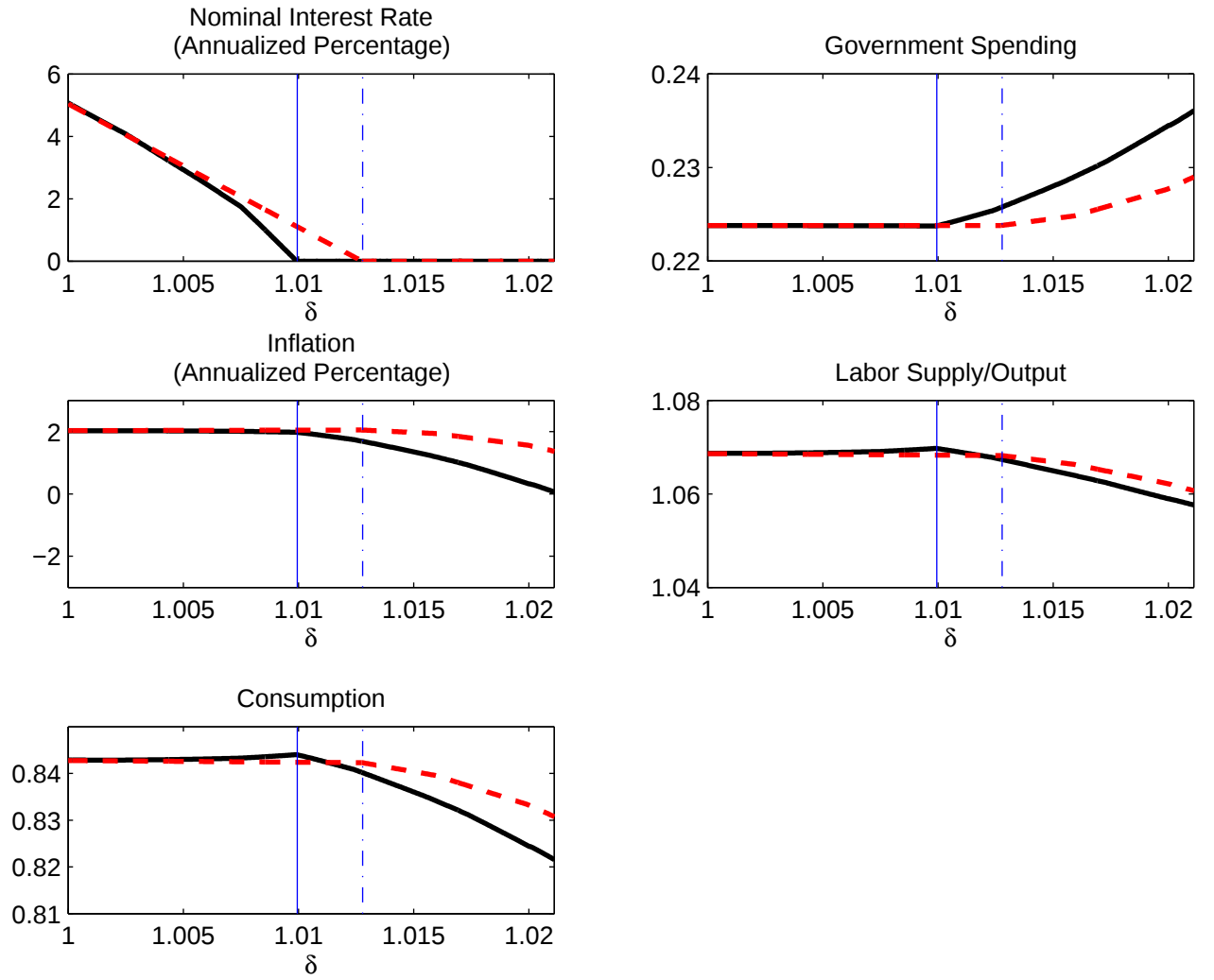


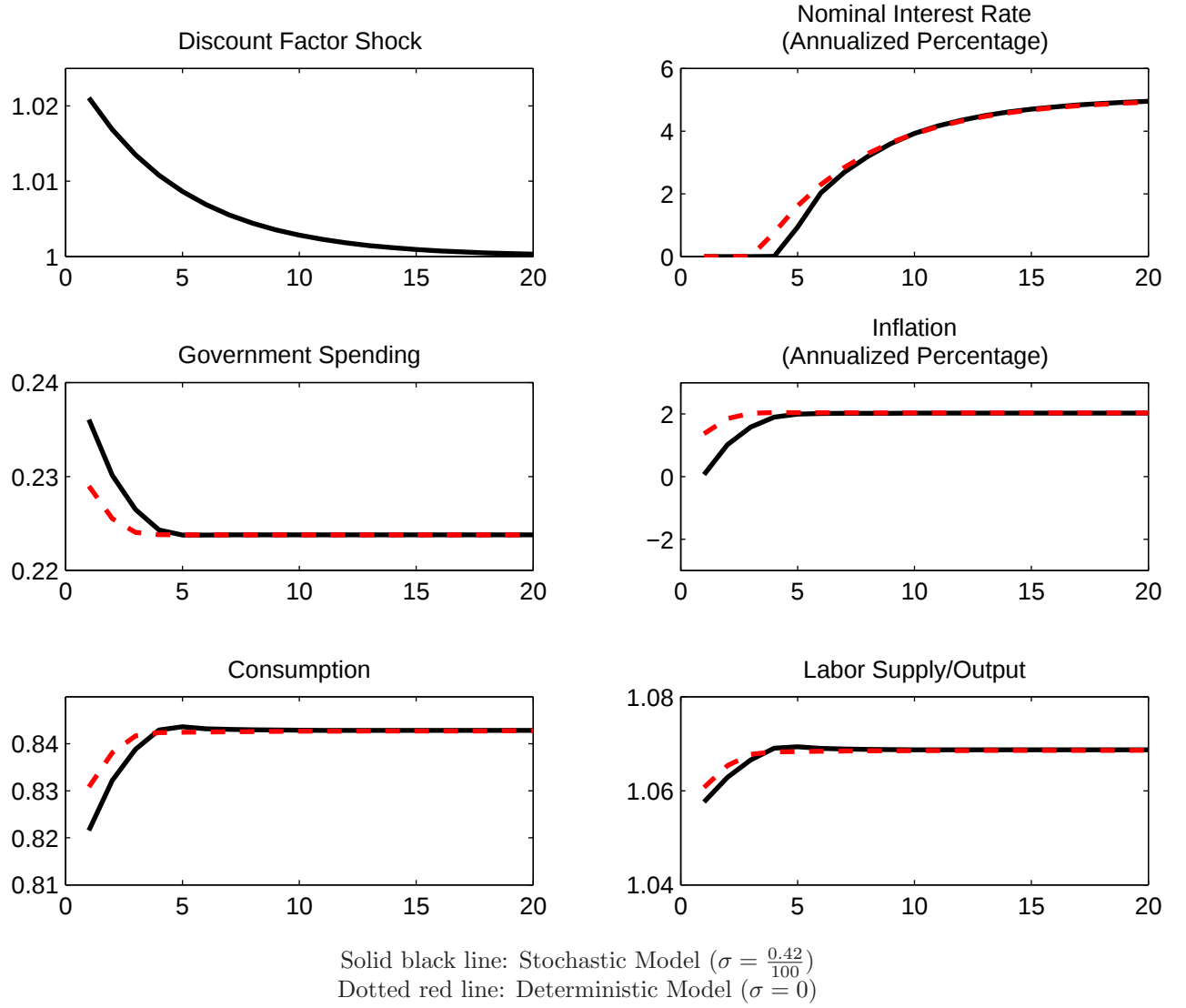
Figure 3: Optimal Policy/Allocations Without Commitment: Deterministic vs. Stochastic Equilibria



Solid black line: Stochastic Model ($\sigma = \frac{0.42}{100}$)
Dotted red line: Deterministic Model ($\sigma = 0$)

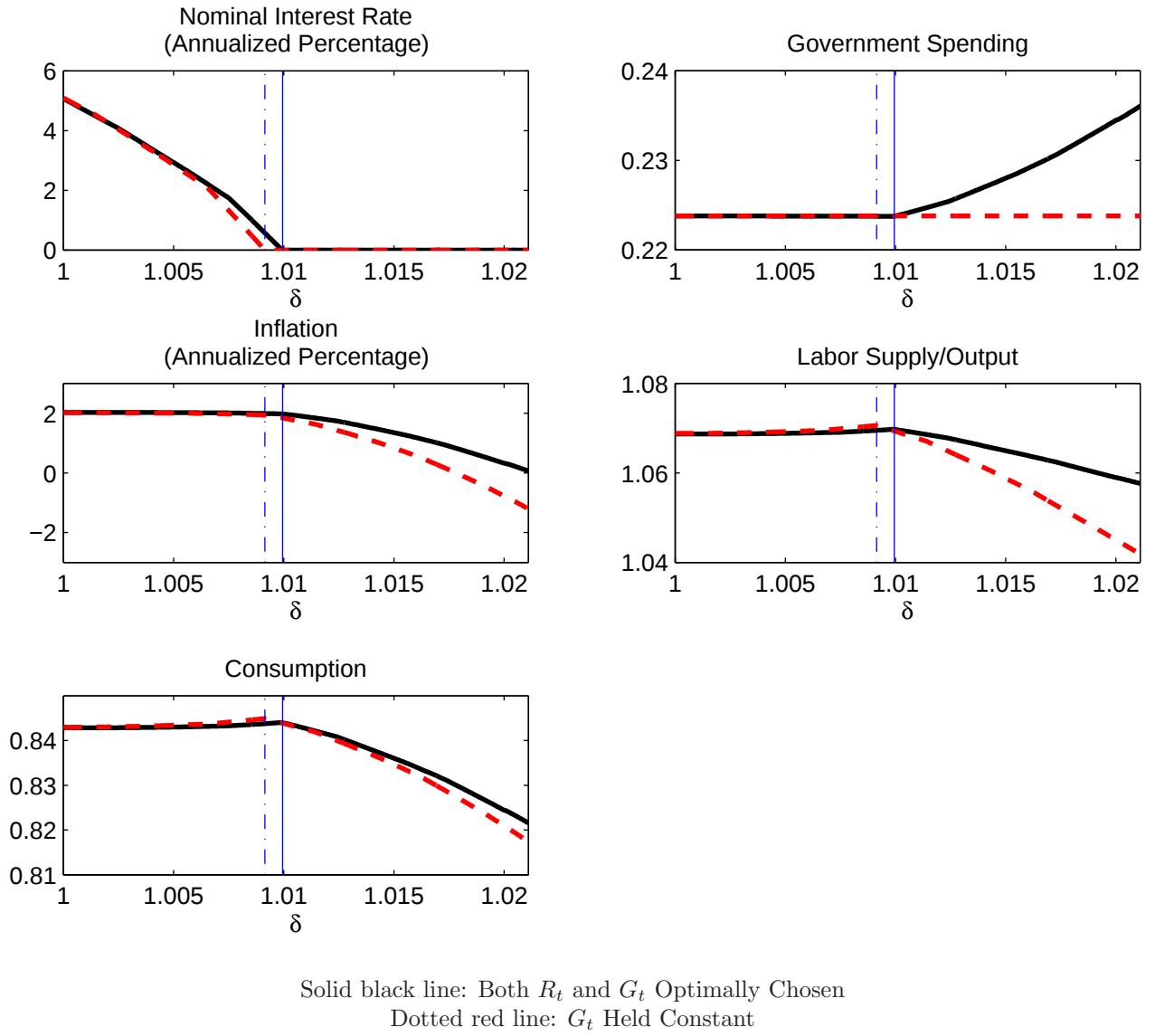
*Policy functions are shown for the range of δ that covers its steady-state level ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$).

Figure 4: Recovery from a Recession Without Commitment: Deterministic vs. Stochastic Equilibria



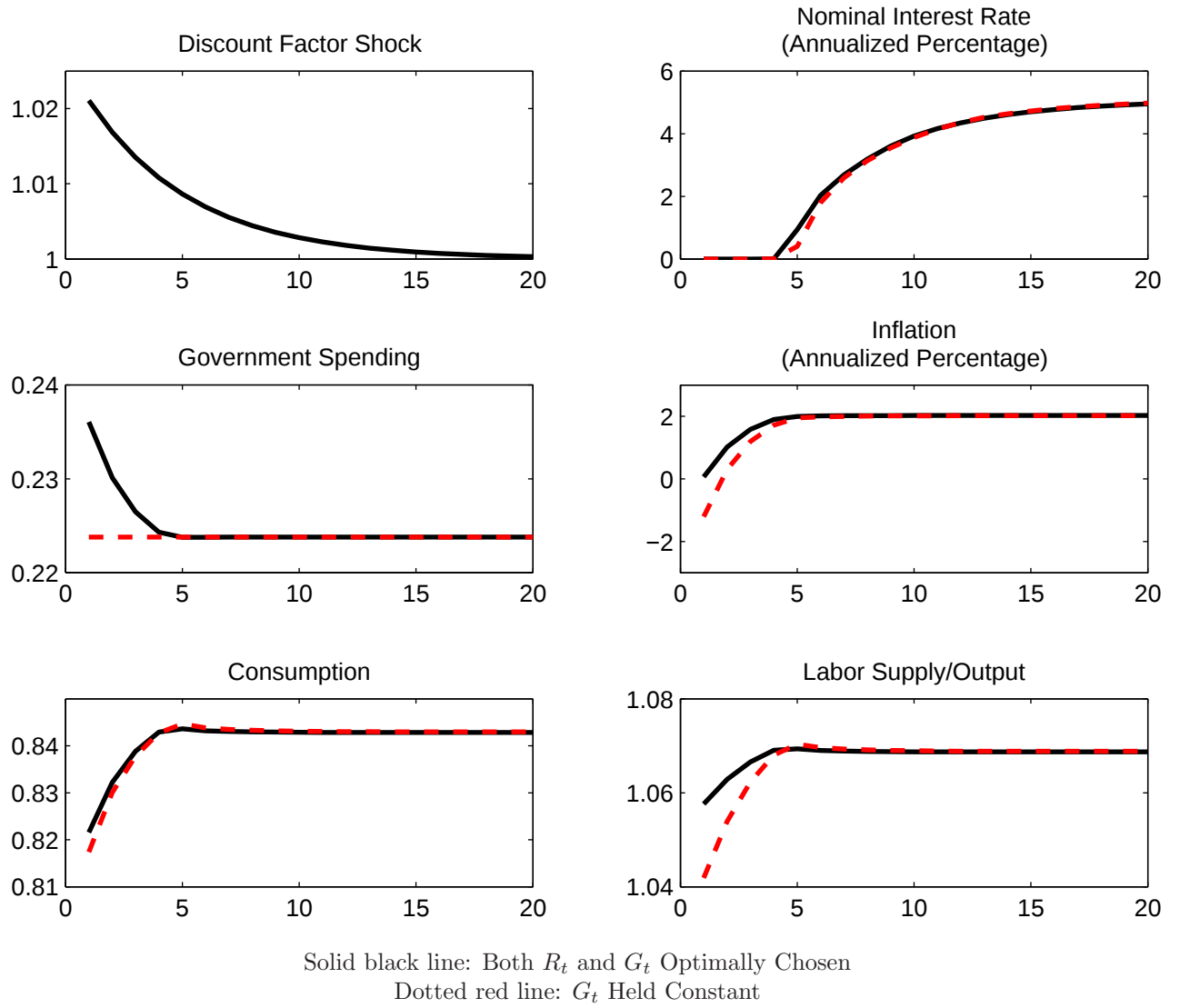
*The impulse response functions are based on the following experiment: There is a shock at time one that pushes δ_1 to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.

Figure 5: Optimal Policy/Allocations Without Commitment: With and Without Fiscal Policy



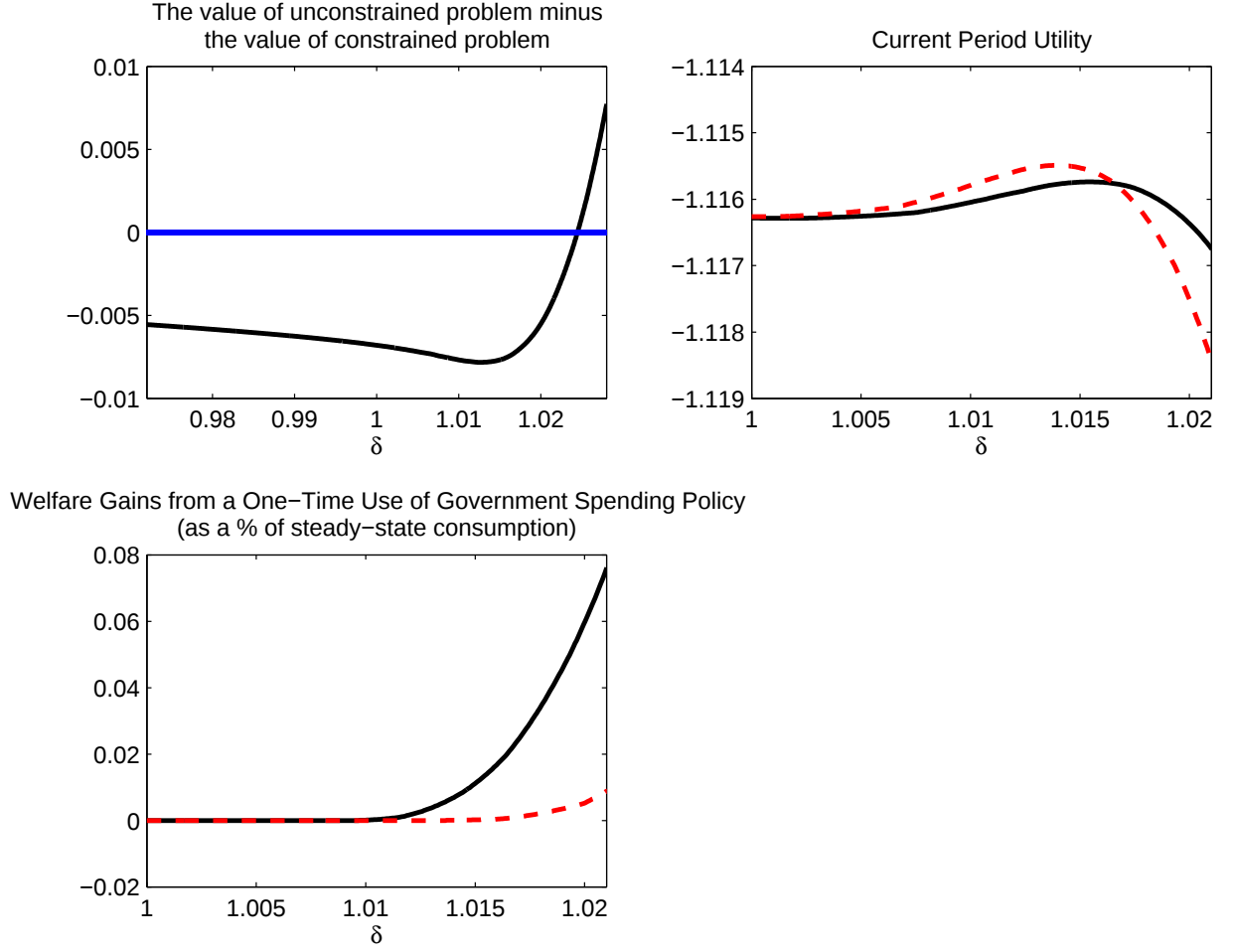
*Policy functions are shown for the range of δ that covers its steady-state level ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$).

Figure 6: Recovery from a Recession Without Commitment: With and Without Fiscal Policy



*The impulse response functions are based on the following experiment: There is a shock at time one that pushes δ_1 to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.

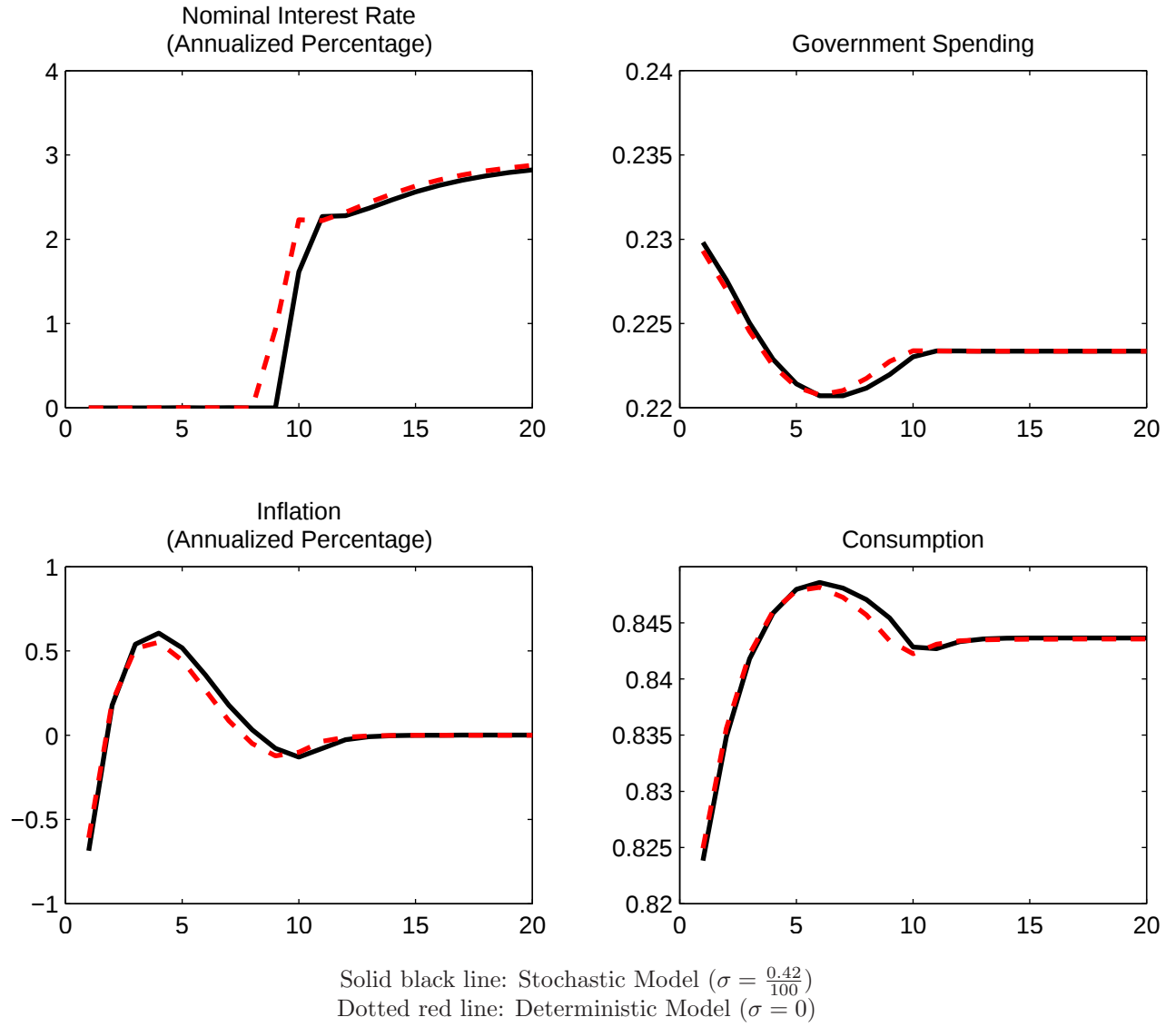
Figure 7: Welfare Consequences Of Fiscal Policy Without Commitment



For the top-right panel
Solid black line: Both R_t and G_t Optimally Chosen, Dotted red line: G_t Held Constant
For the bottom-left panel
Solid black line: Stochastic Model ($\sigma = \frac{0.42}{100}$), Dotted red line: Deterministic Model ($\sigma = 0$)

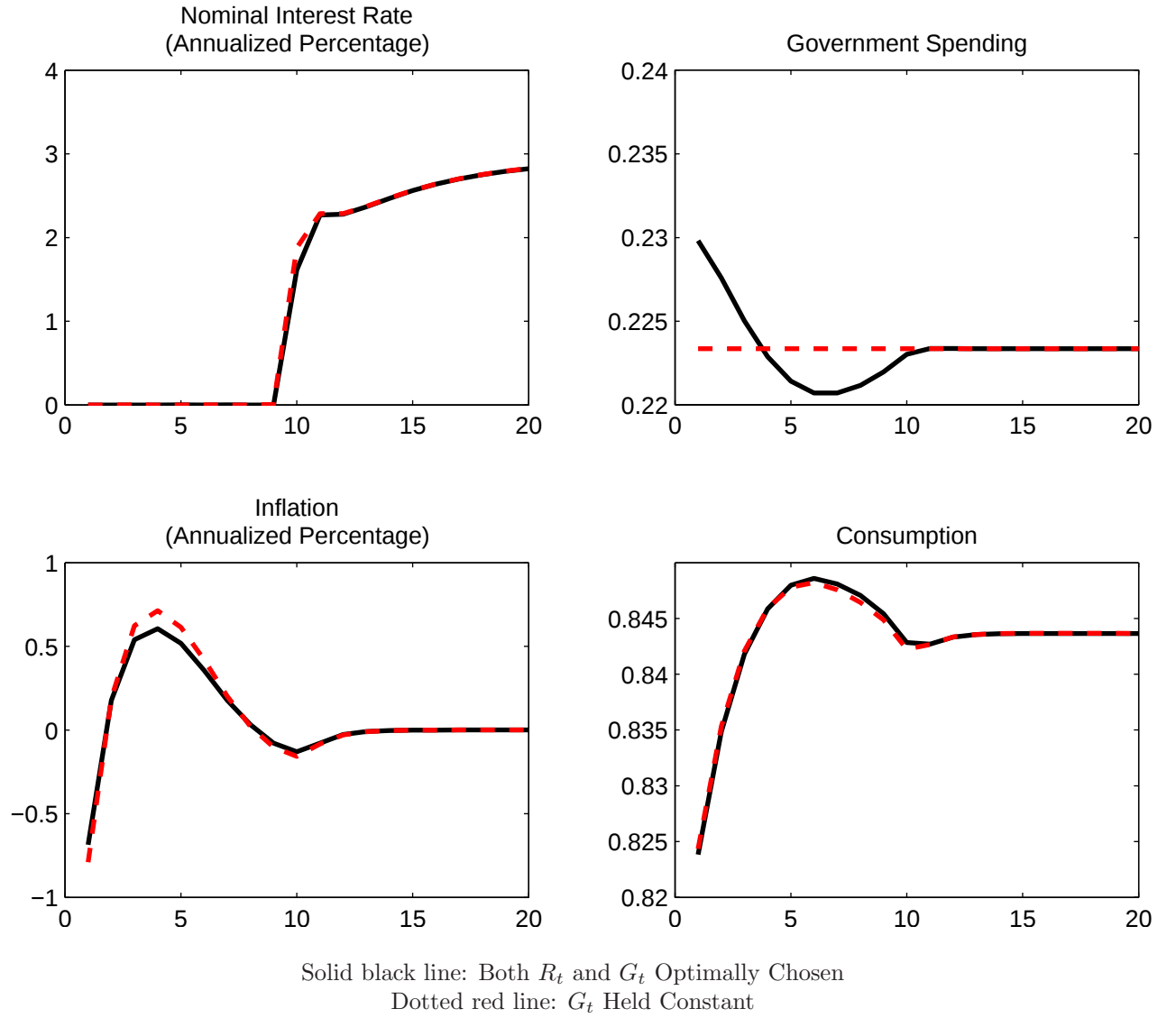
*For the top-left panel, the range of δ covers -4 to 4 standard deviations away from the steady-state ($\delta = [-0.972, 1.028]$). For the top-right and bottom-left panels, the range of δ covers its steady-state ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$).

Figure 8: Recovery from a Recession With Commitment: Deterministic vs. Stochastic Equilibria



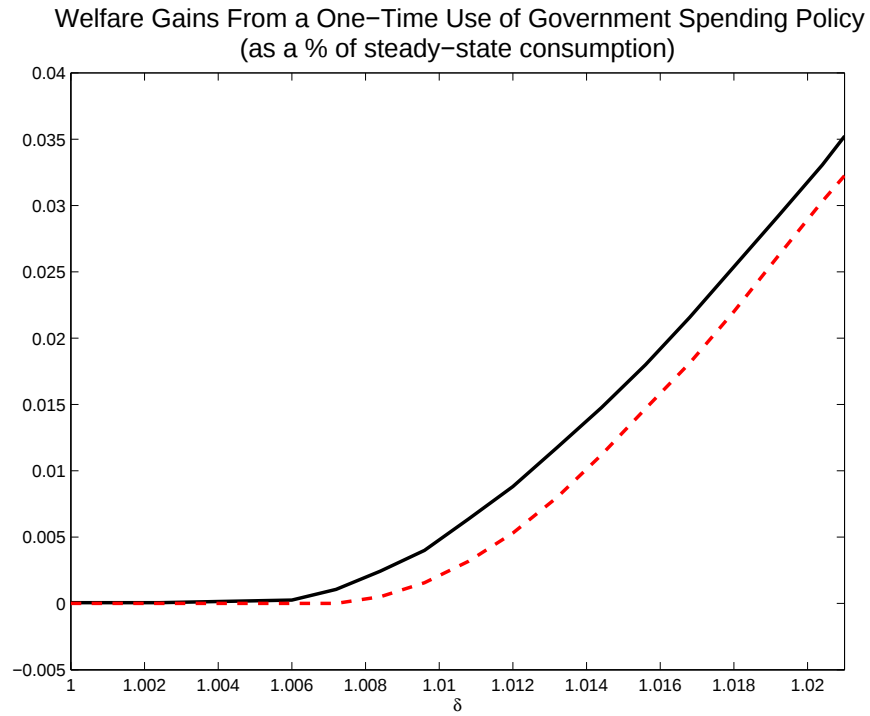
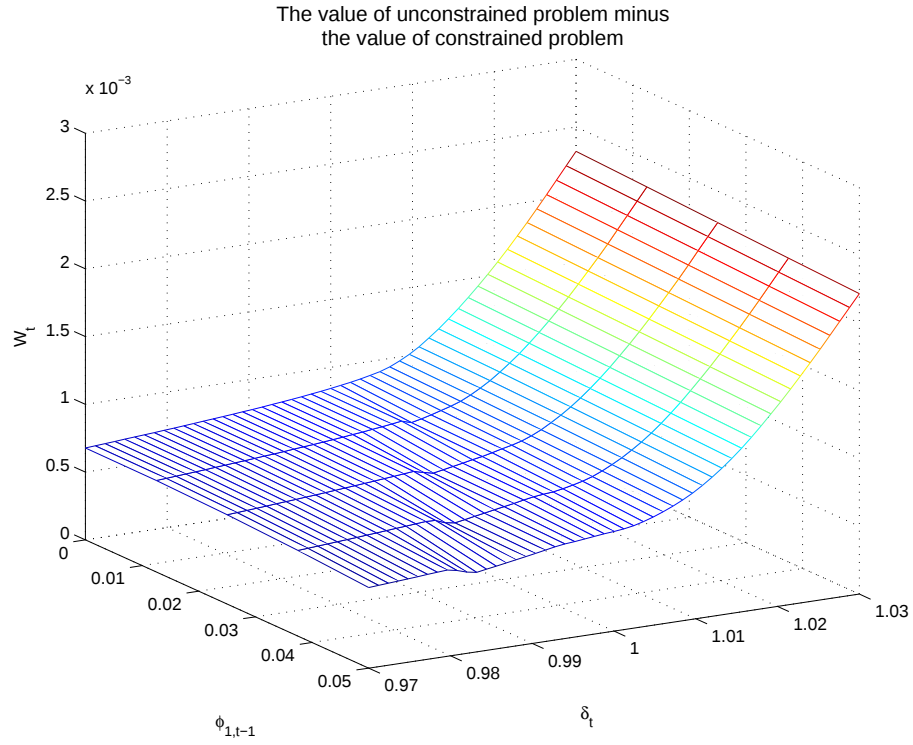
*The impulse response functions are based on the following experiment: There is a shock at time one that pushes δ_1 to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.

Figure 9: Recovery from a Recession With Commitment: With and Without Fiscal Policy



*The impulse response functions are based on the following experiment: There is a shock at time one that pushes δ_1 to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.

Figure 10: Welfare Consequences Of Fiscal Policy With Commitment



For the bottom panel: Solid black line: Stochastic Model ($\sigma = \frac{0.42}{100}$)
Dotted red line: Deterministic Model ($\sigma = 0$)

*For the bottom panel, the range of δ covers its steady-state ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$). $\phi_{1,t-1}$ and $\phi_{2,t-1}$ are set to their steady-state values.