Information Acquisition in Rumor-Based Bank Runs

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Abstract

We study the endogenous information acquisition and withdrawal-redeposit decisions of individual agents when a liquidity event triggers a spreading rumor and therefore exposes a bank to a run. Uncertainty about the bank's liquidity and potential failure motivates agents who hear the rumor to acquire additional information, and in equilibrium depositors with unfavorable information run on the bank gradually. Although the bank run equilibrium is unique given the additional signal's quality, multiple equilibria emerge with endogenous information acquisition. A bank run equilibrium exists when agents aggressively acquire information. We study the threshold parameters that eliminate bank runs. Public provision of solvency information (e.g. stress tests) can eliminate bank runs by indirectly crowding-out individual depositors' effort to acquire liquidity information. However, providing too much information that slightly differentiates competing solvent-but-illiquid banks can result in inefficient runs.

JEL Classification: D8, G2

Keywords: Bank runs, learning, information acquisition, asynchronous awareness, temporal coordination, stress tests

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1 Introduction

Bank runs are unfortunately still with us. Runs occurred recently on major traditional banks such as Northern Rock and IndyMac, and on non-traditional "shadow" banks.[1] The largest ever bank failure resulted from the 2008 run on Washington Mutual (WaMu). Figure 1 shows that the run on WaMu was dynamic in nature: the bank experienced the gradual withdrawal of $16 billion in the days leading up to its takeover by the FDIC. Uncertainty regarding WaMu's liquidity and hence its eventual fate remained heightened during the run.[2] This empirical pattern that runs have an important time dimension, and that uncertainty and learning play a key role, has been found as far back as the 1857 run on the Emigrant Industrial Savings Bank.[3]

In this paper, we provide the first dynamic bank run model featuring learning and endogenous information acquisition to shed light on two important questions: Will endogenous information acquisition exacerbate the inefficient bank run equilibrium? What are the policy implications for a social planner aiming to eliminate run equilibria?

We incorporate uncertainty about the bank's liquidity into the asynchronous awareness framework of Abreu and Brunnermeier (2003).[4] At some unobservable random time, a liquidity event occurs for a fundamentally solvent bank, at which point it becomes illiquid (i.e., limited amount of cash reserves and susceptible to a run) or remains liquid (with sufficient reserves and not subject to run). The liquidity event triggers the spread of a rumor in the population that exposes the bank to a run. Agents learn about whether the bank is liquid from the passage of time without failure, and about the time the liquidity event started.

Uncertainty motivates informed agents who hear the rumor to acquire additional information

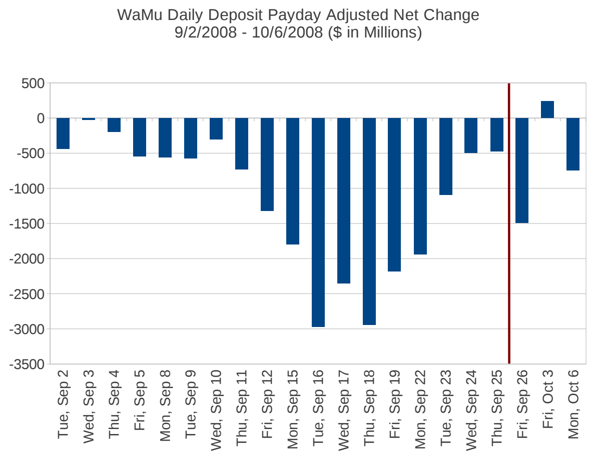
[1] Calomiris and Mason (1997), Shin (2009), and Iyer and Puri (forthcoming) document traditional bank runs, while Gorton and Metrick (2011) and Acharya, Schnabl, and Suarez (forthcoming) document "shadow" bank runs on repo and asset-backed commercial paper conduits.

[2] On September 16, 2008, "*'This week and next will be the moment of truth for WaMu,' said Fred Cannon, an analyst at Keefe Bruyette & Woods.*" See report "WaMu faces price to keep deposits," Financial Times, by Saskia Scholtes in New York.

[3] O Grada and White (2003) find that the 1857 bank run on the Emigrant Industrial Savings Bank had an important time dimension and was driven by informational shocks in the face of asymmetric information about the true condition of bank portfolios. See Kelly and O Grada (2000) for evidence on information transmission during the run.

[4] Abreu and Brunnermeier (2002, 2003) consider the asynchronous timing of awareness to study bubbles. In their model, delayed arbitrage is rational because agents do not know whether other agents are aware of this arbitrage opportunity, and more importantly, when they are taking action on it. Brunnermeier and Morgan (2010) generalize this idea to a class of "clock games" and test its main predictions in controlled experiments. We add to their model uncertainty about the capacity of the bubble (bank), allow agents to acquire additional information upon awareness, and decouple the spreading rate from the length of the awareness window.

Figure 1: The Dynamic Nature of the Run on WaMu



Daily net change in deposits as reported by Washington Mutual Bank to the Office of Thrift Supervision (OTS). We take out Friday and end-of-month fixed effects, i.e. days with automatic payday deposits estimated over the preceding 52 days. The OTS appointed the FDIC as receiver of WaMu on the evening of September 25, 2008 (the vertical line in the figure) which then sold it to JPMorgan Chase.

at an endogenously determined signal quality. The realized signal may be utterly uninformative, or reveals the bank liquidity state perfectly. The higher the signal's quality, the more likely is the signal to reveal the bank's liquidity status. If the bank is indeed illiquid, then in aggregate a higher quality translates to a greater fraction of agents who know the bank is illiquid.

The presence of agents with heterogeneous information naturally allows us to derive the unique (responsive) bank run equilibrium as an interior solution, given the signal quality about bank illiquidity.[5] Conditional on the bank being illiquid, agents with informative "low" signals perceive a high bank failure hazard rate and withdraw immediately, while agents with uninformative "medium" signals wait. When all agents with uninformative medium signals wait a bit longer, they know more agents with informative low signals have already withdrawn. As a result, the bank failure hazard rate endogenously rises because more agents run earlier. This "fear-of-low-signal-agents" generates a time-varying marginal cost of waiting for agents with the medium signal, which allows us to find the unique endogenous withdrawal time that equates the marginal benefit of waiting. Waiting is an interesting new dimension of runs that cannot be analyzed in static models.[6]

[5]In Abreu and Brunnermeier (2003), the interior equilibrium is generated under the assumption that the bubble component is exogenously decreasing over time. This time-varying attacking benefit drives arbitragers to attack only when the bubble component drops to certain level.

[6]Our model belongs to a vast literature on the role of information in static bank runs, and it is beyond the scope of our paper to have a thorough review on this topic. To name a few, Gorton (1985), Bhattacharya and Gale (1987),

We decouple the speed by which information spreads from the length of the awareness window over which information spreads and examine their effects on the survival of an illiquid bank.[7] In a static model an increase in the number of potential withdrawers intuitively exacerbates runs. By contrast, we find that when the awareness window widens so everybody knows potentially more agents run on the bank, the illiquid bank survives longer. This counter-intuitive result arises due to the novel uncertainty structure we introduce, as *the agent who hears the rumor also observes that the bank is still alive.* If the awareness window is wide, the rumor could have started spreading a long time before he heard it. Thus, conditional on the bank surviving thus far, the bank is more likely to be liquid, leading to lower running incentives.

Given the unique bank run equilibrium, the optimal signal quality for individual agents trades off the convex cost of acquiring it with a greater probability of receiving an informative signal. Moreover, the chosen quality of information affects the equilibrium survival time of the failing bank. This intricate feedback effect between bank runs and information acquisition generally leads to at most two equilibria: a *good equilibrium* where agents do not acquire information and do not run, and a *bad equilibrium* where agents acquire information aggressively and run on the illiquid bank. This result naturally leads us to examine how to eliminate the run equilibrium.

We find that in our dynamic setting the threshold parameters that eliminate bank runs altogether are nontrivial. The social planner can, for example, adjust bank reserve requirements in response to heterogeneity in the rumor spreading rate and in the length of the awareness window. Interestingly, the liquidity requirement is far less than the maximum potential withdrawal, which is the minimum reserve requirement from a static Diamond and Dybvig perspective. In fact, we can rule out pure-strategy synchronized sun-spot equilibria. Moreover, partial deposit insurance suffices in our setting to prevent runs. Though beyond the scope of the current paper, a calibration of our model to data can certainly result in quantitatively meaningful policy recommendations.

We extend our baseline model along two dimensions. First, we consider fundamentally insolvent

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Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988). For recent papers based on Diamond and Dybvig (1983) see Nikitin and Smith (2008) and Ennis and Keister (2009).

Based on the Morris and Shin (2002) global games technique, Goldstein and Pauzner (2005) study the optimal deposit contract by deriving a unique equilibrium when depositors in a Diamond and Dybvig type setting are endowed with private noisy signals about bank fundamentals. We allow for endogenous information acquisition, and show that excessive socially wasteful learning may lead to socially inefficient runs on solvent-but-illiquid banks.

[7] By contrast, in Abreu and Brunnermeier (2003) the speed by which information spreads is one over the length of the awareness window.

banks. When agents privately collect information about bank *solvency*, they inevitably learn about bank *liquidity*. Effort spent acquiring one reduces the effort needed to acquire the other. We show that public provision of solvency information can help curb the private information acquisition effort on bank liquidity. One such situation where information is key is at the disclosure of stress test results (Bernanke, 2010). Carefully constructed stress tests can therefore help prevent bank runs by crowding out information acquisition by individuals.

The planner must be careful, however, to avoid providing too much information that differentiates competing solvent but potentially illiquid banks, for such information can start a run. Competition is the subject of our second extension. In a competitive environment, small differences in liquidity can result in runs on slightly weaker banks and their subsequent failure. In this case, the planner would want to inject noise into the system, so that individual agents with less informative signals are more likely to stay in their original bank without knowing which one is more liquid. For example, by forcing all of the "Big 9" banks to take government capital on October 13, 2008, the U.S. government was in fact injecting noise about the liquidity of competing solvent banks into the economy.

Our paper contributes to a recent theoretical literature studying dynamic bank runs. Wallace (1988) and Green and Lin (2003) are earlier papers studying the sequential service constraint in the Diamond and Dybvig model. Gu (2011) is a dynamic model in nature, that studies depositors' withdrawal strategy sequentially. He and Xiong (forthcoming) develop a continuous-time debt run model of a firm with a time-varying fundamental and a staggered debt structure. Similar to the asynchronous timing structure employed in our paper, each creditor in He and Xiong coordinates his rollover decision with future maturing creditors. We emphasize the role information and learning play in runs, and how the government can eliminate runs when individuals acquire information.

The paper proceeds as follows. Section 2 describes the setting and solves the agent's learning problem. Section 3 characterizes the individual agent's optimal withdrawal policy, and Section 4 analyzes the bank run equilibrium with information acquisition. Section 5 considers extensions, and we conclude in Section 6. Proofs are in the Appendix.

2 The Model

We first describe the economy and the individual agent's problem. We then characterize the agent's belief updating, which is crucial in studying the individual agent's optimal strategy.

2.1 The Setting

2.1.1 Technology

Time is continuous on $t \in [0, \infty)$. A continuum of risk-neutral agents (depositors) with unit mass have an infinite horizon and maximize their expected utility from consumption with a zero discount rate.

Bank deposits yield a constant rate of return $r > 0$ when the bank is operating, while holding cash outside the bank earns zero return. Broadly, one can interpret the bank as some investment vehicles in the shadow banking system or even the entire financial system, and the positive relative wedge $r > 0$ reflects either a higher investment growth rate or a convenience yield for keeping funds in the institution. To avoid exploding values, we assume that the bank's growth stops at some "maturing" event modeled as a Poisson shock with intensity $\delta > r$. Following this public event, the growth-less bank liquidates without loss, and the game ends in the sense that each agent gets his deposit back for consumption. Throughout, this maturing event will be independent of any other random variables that we consider.[8]

2.1.2 Uncertainty about Bank Liquidity

In our model there are two potential types of banks that are fundamentally solvent, with one type of bank being "illiquid," and a second "liquid" bank impervious to runs. The uncertainty is crucial to our analysis. Later we will introduce insolvent banks as an extension.

Throughout, bank liquidity is defined as the amount of depositors that it takes to run down the bank. For simplicity, we assume a binary structure for the state of bank liquidity $\widetilde{\kappa}$. For the liquid bank, $\widetilde{\kappa} = \kappa_H > 1$, i.e., the bank can survive any severe run. However, when the bank is

[8]This assumption plays no role in our analysis except making the value of the one dollar inside the bank finite (as the liquid bank will grow always). Alternatively, we could assume that each individual agent suffers liquidity shocks that require immediate consumption (therefore withdrawal). Unfortunately, this brings the troublesome issue of agent replacement.

illiquid, $\widetilde{\kappa}$ takes a lower value κ_L below 1, and the bank is potentially subject to runs. In other words, the illiquid bank fails when more than a κ measure of the depositors have fully withdrawn their funds. One can literally interpret $\widetilde{\kappa}$ as the bank's cash reserves to meet withdrawals; in this paper we broadly interpret $\widetilde{\kappa}$ as the liquidity of the bank.[9]

In our economy, upon awareness, informed agents begin with a prior that $\Pr\{\widetilde{\kappa} = \kappa_L\} = p_0 \in (0, 1)$. When the illiquid bank fails, all remaining depositors in the failed bank recover a fraction $\gamma \in (0, 1)$ of their deposits. Afterwards, all agents go to autarky consuming their remaining wealth.

2.1.3 Liquidity Event and Spreading Rumors

At $t = 0$ the fundamentally solvent bank is liquid, i.e., $\widetilde{\kappa} = \kappa_H$. At some unobservable random time $\widetilde{t_0} > 0$ which we refer to as the liquidity event, the solvent bank may become illiquid, and the uncertainty is as modeled in Section 2.1.2.

Moreover, this liquidity event triggers information to spread in the population. We call those agents who have heard the information as "informed," and those who have not as "uninformed." Since at $t = 0$ the bank is liquid, we assume that the agents' beliefs are such that they expect to hold money in the bank unless they hear the information that the bank might be illiquid.

Although the exact liquidity event time t_0 is publicly unobservable, it is common knowledge that $\widetilde{t_0}$ is exponentially distributed on $[0, \infty)$ with cumulative distribution function $\Phi(t_0) \equiv 1 - e^{-\theta t_0}$, and density $\phi(t_0) \equiv \theta e^{-\theta t_0}$. This information structure is meant to capture the essence of an unverified *rumor* of uncertain origin that spreads gradually in the depositor population. Later on we will call this information about bank's liquidity status a "rumor."

Given a realization of $\widetilde{t_0} = t_0$, the rumor begins to spread over an interval $[t_0, t_0 + \eta]$ with a positive constant (exogenous) length η. Following Abreu and Brunnermeier (2003) we refer to η as the "awareness window." At any interval dt where $t \in (t_0, t_0 + \eta)$, uninformed agents become informed by receiving this signal with a probability of βdt, and this information shock is i.i.d. across the population of uninformed agents. To make the problem interesting, we assume that at time $t_0 + \eta$ the fraction of informed agents will be sufficient to take down the bank, if they decide to do so.

[9] There is not much loss in generality from assuming $\kappa_H > 1$ since we can always pick an interior $\kappa_H < 1$ such that runs do not occur in equilibrium. See Section 5 for the analysis of minimum liquidity κ to eliminate runs.

This information spreading technology is different from that of Abreu and Brunnermeier (2003) in that we allow for separation between the spreading rate β and the awareness window η. Abreu and Brunnermeier assume a linear spreading technology with rate $\frac{1}{\eta}$ so that the entire population becomes informed at $t_0 + \eta$. This way, the awareness window η is artificially tied to the spreading rate $\frac{1}{\eta}$. As we show later, because of the endogenous learning effect, the awareness window η has interesting effects opposite to common wisdom.

Given t_0 and some future time $t \in [t_0, t_0 + \eta]$, denote by $U(t|t_0) \in [0, 1]$ the measure of uninformed agents who have not heard the rumor yet. At $t = t_0$, every agent is uninformed, so that $U(t_0|t_0) = 1$. The population of uninformed agents evolves as $dU(t|t_0) = -\beta U(t|t_0)\, dt$, which implies that

$$U(t|t_0) = e^{-\beta(t-t_0)} \text{ for } t \in [t_0, t_0 + \eta],$$

and the mass of newly informed agents within $[t, t + dt]$ is

$$\beta U(t|t_0)\, dt = \beta e^{-\beta(t-t_0)} dt \text{ for } t \in [t_0, t_0 + \eta]. \tag{1}$$

2.1.4 Information Acquisition

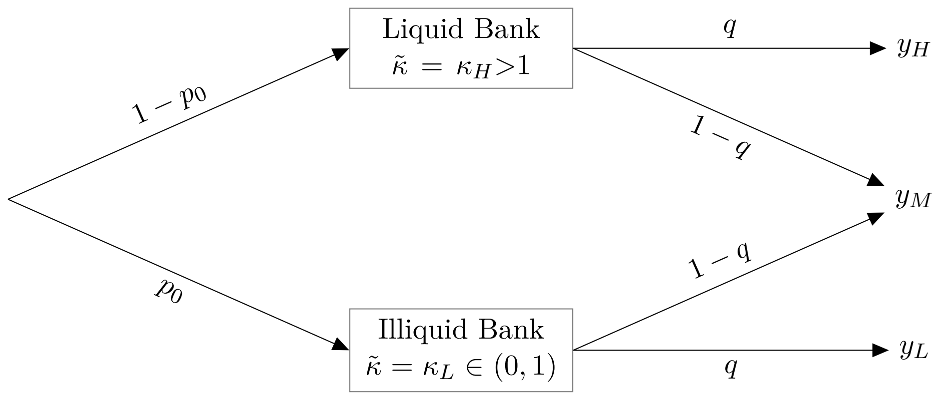
At time t_i when the agent hears the rumor, he can acquire additional information about the bank's type at some convex cost. Specifically, the agent makes an endogenous choice of information quality $q \in [0, 1]$ at the cost $\chi(q) = \alpha q^2/2$, where $\alpha > 0$ is a positive constant. For tractability, we assume that $\chi(q)$ is the per dollar information cost, so that informed agents face the same problem when they are informed at different times.

This additional signal takes three possible values $y \in \{y_L, y_M, y_H\}$ with conditional probabilities:

$$\begin{aligned}
&\Pr\{y = y_H|\tilde{\kappa} = \kappa_H\} = q, \Pr\{y = y_M|\tilde{\kappa} = \kappa_H\} = 1 - q, \text{ and } \Pr\{y = y_L|\tilde{\kappa} = \kappa_H\} = 0; \\
&\Pr\{y = y_L|\tilde{\kappa} = \kappa_L\} = q, \Pr\{y = y_M|\tilde{\kappa} = \kappa_L\} = 1 - q, \text{ and } \Pr\{y = y_H|\tilde{\kappa} = \kappa_L\} = 0.
\end{aligned} \tag{2}$$

Figure 2 summarizes this distribution. With probability q, the bank's liquidity is perfectly revealed by the signal y_H (y_L). With probability $1 - q$, the agent does not learn anything by receiving the

Figure 2: Probability Distribution of the Additional Signal $\tilde{y}$ with Quality q



medium signal y_M.[10] The realizations of these signals are i.i.d. across agents, conditional on the underlying state.

2.1.5 Information Structure

Call the agent who is informed at $t_i \geq t_0$ simply agent t_i. Denote by $\mathcal{F}_t^{t_i}$ the information set of agent t_i at $t > t_i$, and let $\mathbf{1}_t^{BF} \in \{1, 0\}$ indicate whether the bank has failed or not by time t. The agent t_i's information set is $\mathcal{F}_t^{t_i} = \left\{t, t_i, \widetilde{y}_{t_i}, \mathbf{1}_t^{BF}\right\}$, i.e. the informed agent knows the current time, the time that he hears the rumor, the additional signal that he acquired, and whether the bank has failed or not. Recall that the liquidity event t_0 is not public information.

2.1.6 Agent's Problem

Now we summarize the problem of an informed agent t_i with deposits inside the bank. He will acquire an additional signal $\widetilde{y} \in \{y_L, y_M, y_H\}$. Based on this information, he can withdraw his deposits whenever he believes bank failure is imminent, and redeposit this cash in the future if the bank's survival sufficiently improves his posterior belief about bank liquidity. To eliminate strategies with infinite transactions, we assume a constant (proportional) transaction cost k per dollar of deposits when the agent (re)deposits his cash into the bank.

The risk neutrality and the bank's superior investment technology imply that it is optimal for the

[10] Upon receiving y_M, the posterior probability of the bank being illiquid remains $\frac{p_0(1-q)}{p_0(1-q)+(1-p_0)(1-q)} = p_0$.

agent to consume only at the bank's exogenous maturing event, or when the bank endogenously failed due to runs. Also, the linearity of this problem implies that a "bang-bang" strategy, i.e. keeping the entire wealth either in or out of the bank, is optimal.

2.2 Learning

In our model informed agents need to assess the posterior probability of bank failure based on their information set. To this end we derive the bank failure hazard rate in this section, which is important for later analysis.

2.2.1 Posterior Belief about t_0

The agent t_i updates the posterior distribution of t_0 conditional on his hearing the rumor at t_i. Given t_0, for an individual uninformed agent the probability of getting informed over $[t_i, t_i + dt]$ is determined as follows. For $t_i \in [t_0, t_0 + \eta)$, the probability of becoming informed at $[t_i, t_i + dt]$ but not before t_i is $f(t_i|t_0) \equiv \beta U(t_i|t_0)\, dt = \beta e^{-\beta(t_i - t_0)} dt$. And, since the rumor stops spreading after $t_0 + \eta$, we have

$$f(t_i|t_0) = \begin{cases} \beta e^{-\beta(t_i - t_0)} dt & \text{if } t_i \in [t_0, t_0 + \eta), \\ 0 & \text{if } t_i \geq t_0 + \eta \end{cases}. \tag{3}$$

Combining with the density of $\phi(t_0)$, the informed agent t_i updates his posterior distribution about the liquidity event timing t_0.

Similar to Abreu and Brunnermeier (2003) we focus on realizations of $t_0 \geq \eta$ such that the economy is already in this stationary phase; as shown shortly agent t_i's equilibrium strategy will be independent of the exact timing of being informed.[11]

With $t_0 \geq \eta$, the informed agent $t_i \geq t_0 \geq \eta$ will update his posterior belief about t_0 as

$$\phi(t_0|t_i) \equiv \frac{f(t_i|t_0)\,\phi(t_0)}{\int_{t_i-\eta}^{t_i} f(t_i|s)\,\phi(s)\,ds} = \frac{\beta e^{-\beta(t_i - t_0)} \theta e^{-\theta t_0}}{\int_{t_i-\eta}^{t_i} \beta e^{-\beta(t_i - s)} \theta e^{-\theta s} ds} = \frac{\theta - \beta}{e^{(\theta-\beta)\eta} - 1} e^{(\theta-\beta)(t_i - t_0)}.$$

Define $\lambda \equiv \theta - \beta > 0$ so that the liquidity event intensity is greater than the rumor spreading

[11] This implies that the economy is stationary. Note that the finite awareness window η over which the rumor spreads makes the cases of $t_0 < \eta$ and $t_0 \geq \eta$ different. In the event of $t_0 \geq \eta$, it always holds that $t_i \geq \eta$, and rational agents know that $t_0 \in [t_i - \eta, t_i]$. In Appendix B we consider the equilibrium behavior for $t_0 < \eta$.

rate.[12] Then

$$\phi\left(t_0|t_i\right)=\frac{\lambda e^{\lambda\left(t_i-t_0\right)}}{e^{\lambda\eta}-1}. \tag{4}$$

Integrating (4) over t_0 we get the conditional distribution function for t_0:

$$\Phi\left(t|t_i\right)\equiv\Pr\left(\widetilde{t_0}\leq t|t_i\right)=\int_{t_i-\eta}^{t}\phi\left(s|t_i\right)ds=\begin{cases}0 & t<t_i-\eta\\ \frac{e^{\lambda\eta}-e^{\lambda\left(t_i-t\right)}}{e^{\lambda\eta}-1} & t_i-\eta\leq t\leq t_i\\ 1 & t>t_i\end{cases} \tag{5}$$

2.2.2 Bank Failure Hazard Rate

Suppose that in a symmetric equilibrium, agents believe that the illiquid bank fails at $\widetilde{t_0}+\zeta$ where ζ is a constant to be determined in equilibrium (potentially infinite). Let $F\left(t_0\right)\equiv t_0+\zeta$ denote the failure time for a given realization of t_0, conditional on the bank being illiquid, i.e., $\kappa=\kappa_L$. Obviously, the event of bank failure is $\left\{F\left(t_0\right)<t,\kappa_L\right\}$. Moreover, if the bank fails at t, then the inferred t_0 is

$$t_0=F^{-1}\left(t\right)=t-\zeta. \tag{6}$$

Denote by $p\left(t|t_i\right)\equiv\Pr\left\{\widetilde{\kappa}=\kappa_L|\mathcal{F}_t^{t_i}\right\}$ the posterior probability at time t for the bank being illiquid. Trivially, bank failure at t reveals that $p\left(t|t_i\right)=1$. Also given y_L or y_H signals, $p\left(t|t_i\right)=1$ or 0. For agent t_i with y_M signal, if the bank has not failed at t (i.e. $t<F(t_0)$), then his posterior belief of bank being illiquid is (recall that $\widetilde{\kappa}$ is independent of t_0):

$$\begin{aligned}p\left(t|t_i\right)=\Pr\left\{\kappa_L|t<F(t_0),t_i\right\}&=\frac{\Pr\left\{t<F(t_0)|\kappa_L,t_i\right\}\Pr\left\{\kappa_L\right\}}{\Pr\left\{t<F(t_0)|\kappa_L,t_i\right\}\Pr\left(\kappa_L\right)+\Pr\left\{\kappa_H\right\}}\\&=\frac{\left[1-\Phi\left(F^{-1}\left(t\right)|t_i\right)\right]p_0}{\left[1-\Phi\left(F^{-1}\left(t\right)|t_i\right)\right]p_0+1-p_0},\end{aligned} \tag{7}$$

We then use (5) and (6) to derive $p\left(t|t_i\right)$ in closed form. Moreover, as shown later in Section 4.2, in equilibrium if ζ is finite, then we must have $\zeta\leq\eta$. In this case, when agent t_i hears the rumor,

[12]The assumption of $\lambda>0$ is for exposition purpose, and the analysis goes though if $\lambda<0$. To see this, if $\lambda<0$, then the conditional density $\phi\left(t_0|t_i\right)$ has to be written as $\frac{(-\lambda)e^{\lambda\left(t_i-t_0\right)}}{1-e^{\lambda\eta}}$ which is still positive.

his posterior probability of the bank being illiquid is

$$p\left(t_i|t_i\right)=\frac{\left(e^{\lambda\zeta}-1\right)p_0}{\left(1-p_0\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0}.$$

The cumulative distribution of bank failure times $t_i+\tau$, conditional on hearing the rumor at t_i and the bank has not failed at t_i, can be derived as

$$\Pi\left(t_i+\tau|t_i\right)\equiv p\left(t_i|t_i\right)\Pr\left\{t_i<F\left(t_0\right)\le t_i+\tau|t_i,\kappa_L\right\}=p\left(t_i|t_i\right)\frac{\Phi\left(F^{-1}\left(t_i+\tau\right)|t_i\right)-\Phi\left(F^{-1}\left(t_i\right)|t_i\right)}{1-\Phi\left(F^{-1}\left(t_i\right)|t_i\right)}.$$

By calculating the density $\pi\left(t_i+\tau|t_i\right)\equiv\frac{d\Pi\left(t_i+\tau|t_i\right)}{d\tau}$, we derive the bank failure hazard rate from the perspective of the informed agent t_i as

$$h\left(t_i+\tau|t_i\right)\equiv\frac{\pi\left(t_i+\tau|t_i\right)}{1-\Pi\left(t_i+\tau|t_i\right)}. \tag{8}$$

We present the closed form expression for $h\left(t_i+\tau|t_i\right)$ in the following proposition, and in the proof (placed in Appendix) readers can find expressions for $\Pi\left(t_i+\tau|t_i\right)$ and $\pi\left(t_i+\tau|t_i\right)$.

Proposition 1. *Suppose that the illiquid bank fails at $t_0+\zeta$ where $\zeta\le\eta$ which holds in equilibrium. Then the bank failure hazard rate is*

$$h\left(\tau\right)\equiv h\left(t_i+\tau|t_i\right)=\frac{\lambda e^{\lambda(\zeta-\tau)}p_0}{\left(1-p_0\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda(\zeta-\tau)}-1\right)p_0}\ \textit{for}\ \tau\in\left[0,\zeta\right]. \tag{9}$$

For $\tau>\zeta$, $h\left(\tau\right)=0$ as the bank is revealed to be liquid.

We have three noteworthy observations. First, the hazard rate in (9) is independent of the absolute timing of agent t_i becoming informed, therefore we can denote $h\left(t_i+\tau|t_i\right)$ by $h\left(\tau\right)$. This property guarantees the stationarity of our model.

Second, the hazard rate only depends on the remaining survival time from the potential bank failure time, i.e., $\zeta-\tau$. This is intuitive: because the agents in our economy are uncertain about the exact timing of liquidity event t_0 that the rumor starts to spread, from the perspective of the agents the bank failure hazard rate will depend on how far the economy is away from the maximum potential failure time $t_0+\zeta$. The equilibrium remaining survival time is important for our later analysis.

Third, the above analysis is carried out for y_M agents who remain uncertain about the bank's type. The analysis for y_L (y_H) agents are straightforward by simply replacing p_0 with 1 (0) in equation (9).

2.3 Parameter Restrictions

We impose the following parametrization condition throughout the paper:

$$\frac{p_0}{1-p_0} > e^{\lambda\eta} - 1. \tag{10}$$

This condition implies that the bank hazard rate given in Proposition 1 is increasing with τ, i.e., the time that elapsed since hearing the rumor. Equivalently, this implies that the bank failure hazard rate is decreasing in the remaining survival time $\zeta - \tau$.

Second, since the upper bound of the measure of informed agent is $1 - U(t_0 + \eta | t_0) = 1 - e^{-\beta\eta}$, for the model to be interesting we require that the illiquid bank can potentially fail if all informed agents run immediately, i.e.,

$$1 - e^{-\beta\eta} > \kappa_L. \tag{11}$$

Third, we focus on the case where both y_M agents and y_L agents are driving the bank failure. Conditional on the bank being illiquid, there will be q measure of y_L agents. Therefore, instead of $q \in [0,1]$, we further impose that the signal quality has to satisfy

$$q \in \left[0, \frac{\kappa_L}{1 - e^{-\beta\eta}}\right]. \tag{12}$$

Under this condition, because signal quality $q < \frac{\kappa_L}{1-e^{-\beta\eta}}$, agents with y_L signal alone are not enough to take down the bank.

Fourth, we require that the maturity shock intensity δ is moderate so that

$$\frac{\delta(1-p_0)\left(e^{\lambda\eta} - 1\right)\frac{r(r-k\delta)}{\delta - r}}{\lambda(r - \lambda(1-\gamma))p_0} \in (0,1). \tag{13}$$

Note that this requirement implies that $r - \lambda(1-\gamma) > 0$. The only purpose of this condition is to guarantee the optimality of thresholds strategy even if the bank randomly matures.

Finally, we assume that

$$(\lambda (1-\gamma) - r) e^{\lambda \eta} + r > 0; \tag{14}$$

as we will see in Section 3.3, this implies that the agent with y_L signal withdraw immediately, if the bank run equilibrium exists.

3 Optimal Withdrawal Strategies

In this section, we present the key proposition that characterizes the individual agent's optimal withdrawal policy, taking both the equilibrium bank survival time ζ and information quality q as given.

3.1 Hamilton-Jacobi-Bellman (HJB) Equations

Denote by $V_I(\tau; t_i)$ the agent's value of one dollar inside the bank at time $t_i + \tau$, where $\tau \geq 0$ is the time elapsed since agent t_i heard the rumor. Due to stationarity, $V_I(\tau; t_i)$ is a function of τ only and we suppress the argument t_i wherever appropriate. Similarly, denote by $V_O(\tau; t_i) = V_O(\tau)$ the value of one dollar outside of the bank at time $t_i + \tau$. Because withdrawal involves no transaction cost while redepositing costs k, $V_I(\tau) \geq V_O(\tau)$ and $V_O(\tau) \geq (1-k) V_I(\tau)$ for all $\tau \geq 0$.

When $\tau \geq \zeta$, the surviving bank is for sure safe. One dollar inside the bank will grow at r until the maturing event (occurs with Poisson intensity δ), which has a value of

$$V_I(\tau) = \int_0^{\infty} e^{rs} \delta e^{-\delta s} ds = \frac{\delta}{\delta - r} \text{ for } \tau \geq \zeta. \tag{15}$$

The value of one dollar outside the bank is $V_O(\zeta) = (1-k) V_I(\zeta) = \frac{(1-k)\delta}{\delta - r} > 1$, which holds when k is sufficiently small.

For $\tau < \zeta$ the agent has not fully been assured that the bank is liquid, and consider a dollar outside the bank. At any point in time, if the status-quo position (i.e. keeping the dollar outside the bank) is optimal, then the following HJB equation must hold:

$$0 = \underbrace{h(\tau)(1 - V_O(\tau))}_{\text{Bank failure}} + \underbrace{\delta (1 - V_O(\tau))}_{\text{Bank matures}} + \underbrace{V_O'(\tau)}_{\text{Time change}} \tag{16}$$

Here, the first term is the impact of bank failure: with hazard rate $h(\tau)$ the bank fails so that the agent receives 1 and loses his value $V_O(\tau)$.[13] The second term captures the bank asset maturing event, and the third term is the change due to time elapse. Combined with the option of redepositing immediately (with transaction cost k), the HJB equation for one-dollar outside the bank is

$$0 = max\left\{ \underbrace{h(\tau)(1-V_O(\tau))}_{\text{Bank failure}} + \underbrace{\delta(1-V_O(\tau))}_{\text{Bank matures}} + \underbrace{V_O'(\tau)}_{\text{Time change}}, \underbrace{(1-k)V_I(\tau) - V_O(\tau)}_{\text{Redepositing}} \right\} \tag{17}$$

Similarly, for a dollar inside the bank, its value $V_I(\tau)$ must satisfy the following HJB equation:

$$0 = max\left\{ \underbrace{rV_I(\tau)}_{\text{Interest growth}} + \underbrace{h(\tau)(\gamma - V_I(\tau))}_{\text{Bank failure}} + \underbrace{\delta(1-V_I(\tau))}_{\text{Bank matures}} + \underbrace{V_I'(\tau)}_{\text{Time change}}, \underbrace{V_O(\tau) - V_I(\tau)}_{\text{Withdrawal}} \right\} \tag{18}$$

3.2 Optimal Strategies

We solve the ordinary differential equation (16) in the interval$[\tau, \zeta]$ with the boundary condition $\widehat{V}_O(\zeta) = V_O(\zeta)$, and denote its solution as $\widehat{V}_O(\tau)$. It admits the following closed-form solution:

$$\widehat{V}_O(\tau) = \frac{e^{\lambda\eta}(1-p_0) - 1 + e^{\lambda(\zeta-\tau)}p_0 + e^{-\delta(\zeta-\tau)}(1-p_0)\left(e^{\lambda\eta}-1\right)\frac{r-k\delta}{\delta-r}}{(1-p_0)\left(e^{\lambda\eta}-1\right) + \left(e^{\lambda(\zeta-\tau)}-1\right)p_0}. \tag{19}$$

which allows us to characterize the agent's optimal strategy parsimoniously. In general $V_O(\tau) \geq \widehat{V}_O(\tau)$: $\widehat{V}_O(\tau)$ is the value at time τ by simply staying outside the bank until ζ (and redepositing at ζ^+ if the bank is liquid and survives the run), and a priori this simple continuation strategy may not be optimal.

Define the following function $g(\tau)$ which captures the first-order impact of the withdrawal decision:

$$g(\tau) \equiv h(\tau)(1-\gamma) - r\widehat{V}_O(\tau). \tag{20}$$

Here, $g(\tau)$ is difference between the instantaneous loss from one dollar due to potential bank failure (i.e. $h(\tau)(1-\gamma)$), and $r\widehat{V}_O(\tau)$ which is the instantaneous return of taking one dollar out now, keeping outside the bank until ζ, and redepositing back if the bank survives. The underlying assumption here is that a threshold strategy is optimal (i.e. redepositing is never optimal before

[13] The analysis holds for all signals as Proposition 1 gives $h(\tau)$ as a function of p_0.

ζ) which is verified in Proposition 2.

At the optimal withdrawal time τ_w, we have the first-order condition $g(\tau_w) = 0$, i.e.,

$$h(\tau_w)(1-\gamma) = r\widehat{V}_O(\tau_w) = r\left[1 + (1 - p(t_i + \tau_w|t_i))\, e^{-\delta(\zeta-\tau_w)}\frac{r-k\delta}{\delta-r}\right], \tag{21}$$

where we have used (19) and (7) in rewriting this intuitive expression. As mentioned above, the left hand side captures the marginal cost of staying, which is the hazard rate multiplying the bank failure loss. The right hand side captures the marginal benefit of staying, which is the growth rate r multiplying the agent's continuation value of one dollar by withdrawing and redepositing after ζ. For the total continuation value in the bracket of right hand side in (21), the first term 1 is the principal amount which is present in Abreu and Brunnermeier (2003). The second term consists of the option value of future redepositing. Here, $1 - p(t_i + \tau_w|t_i)$ is the probability of the bank being liquid (and surviving eventually) conditional on bank survival at τ_w, $e^{-\delta(\zeta-\tau_w)}$ is the discounting, and finally

$$\frac{r-k\delta}{\delta-r} = V_O(\zeta) - 1 = \frac{(1-k)\,\delta}{\delta-r} - 1 > 0$$

is the additional payoff from redepositing. Apparently, when $p_0 = 1$ so that there is no uncertainty about bank liquidity, this option value term vanishes.

The next proposition shows formally that a threshold strategy is optimal based on the function $g(\tau)$.

Proposition 2. *Given the equilibrium bank survival time ζ, the optimal policy for the agent with y_M is as follows:*

1. *If $g(\zeta) \leq 0$, then it is optimal to stay in the bank always.*
2. *If $g(0) \geq 0$, then it is optimal to withdraw at 0 and redeposit right after ζ.*
3. *Otherwise, we must have $g(0) < 0$ and $g(\zeta) > 0$, and there exists a unique waiting time $\tau_w \in (0, \zeta)$ so that $g(\tau_w) = 0$, and withdrawing at τ_w and redepositing at ζ^+ is optimal.*

3.3 Value Functions

In this section we derive the agent's value as a function of signals. We have studied the optimal strategy for the agent with y_M signal. It is clear that the agents with high signal y_H should keep their deposits in the bank always. And, for agents who receive low signals, if

$$g\left(0;p_0=1\right)=\frac{\left(\lambda\left(1-\gamma\right)-r\right)e^{\lambda\zeta}+r}{e^{\lambda\zeta}-1}>0 \tag{22}$$

then it is optimal to withdraw immediately. Because $\zeta<\eta$ and $\lambda\left(1-\gamma\right)<r$, (22) is implied by parameter condition (14). In fact, we will show later in Section 4.2 that condition (22) has to hold for any interior bank run equilibria. In our model, if y_L agents want to wait some positive amount of time, then generically bank run equilibria do not exist.

Proposition 3. *Suppose that $g\left(0\right)<0$ and $g\left(\zeta\right)>0$ so that the agents with y_M signals are waiting τ_w given in (21). Then upon hearing the rumor, the values conditional on signals are*

$$V_I\left(0|y_L\right)=1, V_I\left(0|y_H\right)=\frac{\delta}{\delta-r},$$

$$V_I\left(0|y_M\right)=\frac{\frac{\delta\left(e^{\lambda\eta}(1-p_0)-1\right)}{\delta-r}\left(1-e^{-(\delta-r)\tau_w}\right)+\frac{\delta+\lambda\gamma}{\lambda+\delta-r}e^{\lambda\zeta}p_0+e^{-(\delta-r)\tau_w}e^{\lambda(\zeta-\tau_w)}p_0\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)}{e^{\lambda\eta}-1-e^{\lambda\eta}p_0+e^{\lambda\zeta}p_0},$$

where τ_w satisfies the first-order condition in (21).

4 Bank Run Equilibrium

4.1 Cumulative Withdrawals Given q

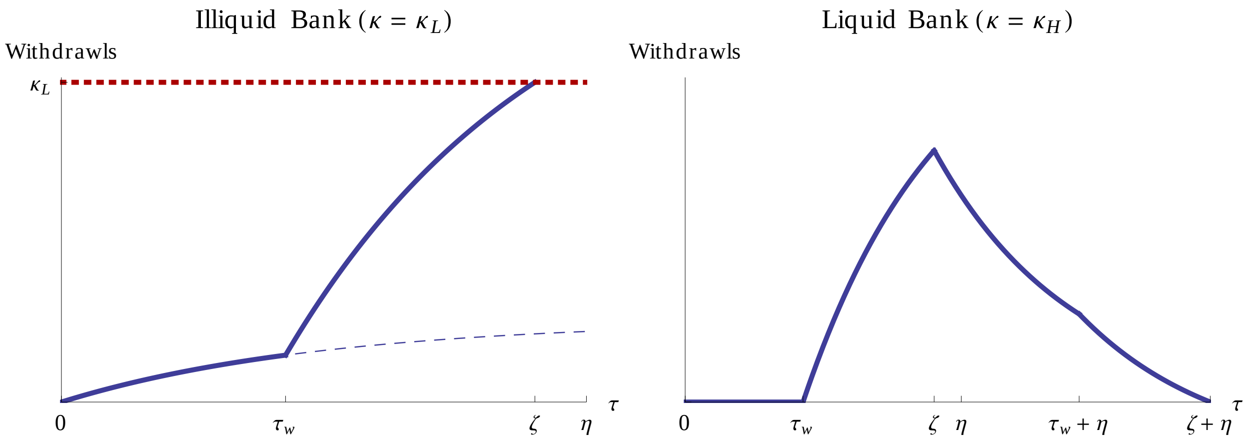
In our model, the illiquid bank fails at $t_0+\zeta$ so that aggregate cumulative withdrawals by informed agents deplete the illiquid bank's capacity κ_L. Two groups of informed agents withdraw from the illiquid bank. The first group is y_L agents with q measure in aggregate per unit of time. Therefore at $t_0+\zeta$ total withdrawals by y_L agents are

$$q\int_{t_0}^{t_0+\zeta}\beta U\left(t_i|t_0\right)dt_i=q\left(1-e^{-\beta\zeta}\right). \tag{23}$$

Agents with y_M signals wait for τ_w, and at $t_0+\zeta$ their total withdrawals are

$$\left(1-q\right)\int_{t_0}^{t_0+\zeta-\tau_w}\beta U\left(t_i|t_0\right)dt_i=\left(1-q\right)\left(1-e^{-\beta(\zeta-\tau_w)}\right). \tag{24}$$

Figure 3: Cumulative Withdrawals



Cumulative withdrawal patterns for an illiquid bank with capacity κ_L and a liquid bank with capacity $\kappa_H > 1$. At $\tau = 0$ the rumor starts to spread. Agents with a y_M signal wait in equilibrium $\tau_w = 0.85$ before withdrawing. The illiquid bank fails while the liquid bank starts experiencing redeposits at $\zeta = 1.8$. Parameter values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

Figure 3 depicts the cumulative withdrawal patterns for both banks. For illiquid banks, y_L agents begin to withdraw first right after the liquidity event t_0 as the rumor starts spreading. At $t_0+\tau_w$ to be determined shortly in equilibrium, agents with y_M signals join the force of withdrawals, until eventually cumulative withdrawals reach κ_L at $t_0 + \zeta$.

For liquid banks, things are different. No agents are withdrawing immediately after t_0, but y_M agents start withdrawing at $t_0 + \tau_w$. At $t_0 + \zeta$ those early informed y_M agents realize the bank is liquid and start redepositing, which makes aggregate withdrawals decrease over time. In this range, interestingly, at the same time late informed agents are still withdrawing while early informed agents who learned that the bank is liquid begin redepositing. The intriguing empirical pattern of simultaneous withdrawing-depositing is unique to our model with rumor-based bank runs, and we wait for future more detailed data sets to test this implication.

4.2 Bank Run Equilibrium Given q

4.2.1 Two-step Procedure

Define $\tau_r \equiv \zeta - \tau_w$ as the time period that y_M agents stay outside the bank. The label "r" captures the "remaining survival time" when they decide to stay out of the bank. The bank run equilibrium given q is determined in a straight-forward two-step procedure. Define $G(\tau_r) \equiv g(\zeta - \tau_r)$, i.e., replace $\zeta - \tau$ by τ_r in (20). From the individual y_M agent's optimal withdraw condition in (21), the equilibrium redepositing time τ_r^* must satisfy

$$G(\tau_r^*) = \frac{(\lambda(1-\gamma) - r) e^{\lambda \tau_r^*} p_0 - (1-p_0)\left(e^{\lambda\eta} - 1\right) \frac{r(r-k\delta)}{\delta - r} e^{-\delta \tau_r^*} + r\left(1 - e^{\lambda\eta}(1-p_0)\right)}{(1-p_0)(e^{\lambda\eta} - 1) + (e^{\lambda\tau_r^*} - 1) p_0} = 0. \quad (25)$$

One important observation emerge. In (25), the equilibrium survival time τ_r^* is uniquely determined, without using other endogenous variables q or ζ. This is because in the agent's first-order condition regarding optimal withdrawal, both the hazard rate $h(\tau_w)$ in (9) and the continuation value $\widehat{V}_O(\tau_w)$ in (19) only depend on the remaining survival time $\tau_r = \zeta - \tau_w$.

Once we pin down τ_r^* from (25), the equilibrium survival time ζ^* follows from the aggregate withdrawal condition. Combing (23) and (24), the illiquid bank fails when

$$\kappa_L = (1-q)\left(1 - e^{-\beta\tau_r}\right) + q\left(1 - e^{-\beta\zeta}\right). \quad (26)$$

Interestingly, the remaining survival time τ_r also determines the cumulative withdrawals by y_M agents. We can now solve for the equilibrium survival time ζ^* as

$$\zeta^* = -\frac{1}{\beta}\log\left[1 - \frac{\kappa_L - (1-q)\left(1 - e^{-\beta\tau_r^*}\right)}{q}\right]. \quad (27)$$

4.2.2 Why $\zeta^* \leq \eta$ Holds Generically?

We next show that in our model generically the illiquid bank fails before the rumor stops spreading in the equilibrium, i.e., $\zeta^* \leq \eta$. In fact, our previous analysis relies on this assumption. When we wrote down the aggregate failure condition (23), we implicitly assume that at the failure time $t_0 + \zeta^*$ there are still newly informed y_L agents withdrawing, which exactly requires that the illiquid bank fails before the rumor stops spreading, i.e., $\zeta^* \leq \eta$.

A natural question follows: Is it possible to have $\zeta^* > \eta$ in our equilibrium? The answer is negative generically,[14] unless $\zeta^* = \infty$ so that there is no bank run. The following lemma states this result formally.

[14]Here, "a statement holds generically" means that this statement may not hold only for some parameter set which is zero measure. This convention comes from the general equilibrium literature.

Lemma 1. *Generically, the equilibrium bank survival time ζ^*, if finite, cannot exceed the awareness window η, so that $\zeta^* \leq \eta$.*

Intuitively, we have seen that (25), which is the individual y_M agent's first-order condition, pins down the equilibrium remaining survival time τ_r^* based on primitives. On the other hand, from (24), the asynchronous timing structure implies that the remaining survival time $\tau_r^* = \zeta^* - \tau_w$ also determines the cumulative withdrawal of y_M agents to run down the illiquid bank. Therefore we need an extra degree of freedom to ensure that the remaining survival time derived from the individual y_M agent's optimality condition is consistent with the remaining survival time required to run down the bank.

It is clear that in the above two-step procedure, the presence of y_L agents who withdraw immediately after hearing the rumor offers such flexibility. Because the required y_M withdrawal is the gap between the capacity κ_L and the cumulative y_L withdrawal $q\left(1 - e^{-\beta\zeta}\right)$, we can find such ζ so that y_M agents' individual incentives to withdraw coincide with the required aggregate y_M withdrawal. Note that this argument also implies the crucial role that heterogeneous agents play in generating the interior bank run equilibrium, which becomes more clear when we discuss the equilibrium mechanism in the next subsection.

4.2.3 Equilibrium Mechanism

Based on the aggregate bank failure condition, we can determine natural bounds for y_M agents' remaining survival time time τ_r^* in equilibrium. When y_M agents withdraw immediately as y_L agents, i.e., $\tau_w = 0$, the distance from the y_M agent's withdrawing time to potential bank failure, i.e., the remaining survival time $\zeta - \tau_w$, will assume its upper bound value:

$$\tau_r^u = \frac{1}{\beta} \ln \frac{1}{1 - \kappa_L} < \eta. \tag{28}$$

On the other hand, $\zeta \leq \eta$ in the above lemma gives the lower bound of τ_r^*:

$$\tau_r^l = \frac{1}{\beta} \ln \left(\frac{1 - q}{1 - \kappa_L - qe^{-\beta\eta}} \right). \tag{29}$$

Because of the parameter condition (12), $\tau_r^l > 0$ so that y_M withdrawals are also contributing to the bank's failure. These bounds are helpful in understanding the economic mechanism that pins down the equilibrium.

Intuitively, $G(\tau_r)$ in (25) gives the sign of marginal cost minus the marginal benefit if the y_M agent waits a bit longer. For illustration, start with the hypothetical situation where all y_M agents withdraw immediately after hearing the rumor (i.e., $\tau_w = 0$). Then remaining survival time $\tau_r = \zeta - \tau_w$ takes its upper bound τ_r^u. Then, if $G(\tau_r^u) < 0$ so that the marginal benefit of waiting exceeds the marginal cost, then it implies that each individual y_M agent wants to postpone his withdrawal.

Now suppose that every y_M agent decides to wait $\tau_w > 0$. Importantly, there are more agents with y_L signal withdrawing before y_M agents, and bank failure requires less cumulative mass of withdrawing y_M agents. This implies that the remaining survival time $\tau_r = \zeta - \tau_w$ goes down (check (26)). Thus, the closer bank failure leads to a higher failure hazard rate (check (9)), and the marginal cost of waiting goes up. To the extreme that all y_M agents are waiting for sufficiently long, then the survival time takes the upper bound η and the remaining survival time attains the lower bound$\tau_r = \tau_r^l$. Then $G\left(\tau_r^l\right) > 0$ says each individual y_M agent wants to withdraw a bit earlier. Therefore at some equilibrium withdrawal time τ_w^* we have $G(\tau_r^* = \zeta^* - \tau_w^*) = 0$ to satisfy the individual optimality condition. We have the following proposition to summarize this discussion.

Proposition 4. *Given q, the bank run equilibrium is characterized by follows:*

1. *If $G\left(\tau_r^l\right) \leq 0$, then there does not exist a bank run equilibrium.*
2. *If $G(\tau_r^u) \geq 0$, then the unique bank run equilibrium is $\tau_r^* = \zeta^* = \tau_r^u$ and $\tau_w^* = 0$.*
3. *Otherwise, we must have $G(\tau_r^u) < 0$ and $G\left(\tau_r^l\right) > 0$, and there exists a unique bank run equilibrium $\tau_r^* \in \left(\tau_r^l, \tau_r^u\right)$ so that $G(\tau_r^*) = 0$, and*

$$\zeta^* = \zeta(\tau_r^*) = \frac{1}{\beta} \ln\left[\frac{q}{1 - \kappa_L - (1-q)\, e^{-\beta\tau_r^*}}\right],$$

and $\tau_w^* = \zeta(\tau_r^*) - \tau_r^*$. The equilibrium is stable.

Before moving to the next section, we briefly discuss the possibility that y_L agents wait for some positive time. Interestingly, the argument similar to Lemma 1 implies that this cannot occur generically. If y_L agents set a positive waiting time $\tau_w^L > 0$, then it is obvious that y_M agents will set $\tau_w^M > \tau_w^L > 0$. Importantly, their respective redepositing times (τ_r^M and τ_r^L) have to satisfy the individual optimality conditions in the nature of (25). However, we require that these τ_r^M and τ_r^L satisfy the aggregate withdrawal condition as well, which generically cannot hold.[15]

4.3 Endogenous Information Acquisition

In our model, each informed agent endogenously acquires information about the bank's liquidity because of the potential bank failure. Therefore we first characterize the individual agent's optimal information acquisition condition, given the bank run equilibrium. On the other hand, we have seen that information quality q also affects the bank run equilibrium. This intricate feedback effect between bank runs and information acquisition generally leads to two equilibria: one equilibrium where agents do not acquire information and also do not run, while the other equilibrium where agents acquire information aggressively and also run on the illiquid bank.

4.3.1 The First-Order Condition of Information Acquisition

We next characterize the optimal information choice. Recall that by setting q, we have

$$\Pr\{y = y_L | t_i\} = qp(t_i|t_i), \ \Pr\{y = y_M | t_i\} = 1 - q, \text{ and } \Pr\{y = y_H | t_i\} = q(1 - p(t_i|t_i)).$$

The information cost is $\chi(q) = \frac{\alpha}{2}q^2$, which is increasing and convex in the information quality q.

Denote the signal with quality q as y^q. When agent t_i hears the rumor he spends his information collection effort to maximize the following object, taking the equilibrium survival time ζ^* as given:

$$\begin{aligned} v(0) &= \Pr\{y = y_L | t_i\} V_I(0|y_L) + \Pr\{y = y_M | t_i\} V_I(0|y_M) + \Pr\{y = y_H | t_i\} V_I(0|y_H) - \chi(q) \\ &= qp(t_i|t_i) + q(1 - p(t_i|t_i)) \frac{\delta}{\delta - r} + (1 - q) V_I(0|y_M) - \chi(q), \end{aligned} \tag{30}$$

where we have used the result in Proposition 3 which also provides an expression for $V_I(0|y_M)$. Note that we are implicitly taking the bank run equilibrium here, so that the agent with y_L finds immediate withdrawal to be optimal.

[15] The aggregate withdrawal condition becomes $\kappa_L = (1-q)\left(1 - e^{-\beta\tau_r^M}\right) + q\left(1 - e^{-\beta\tau_r^L}\right)$ if both agents are withdrawing at ζ. If y_L agents already stopped, then $\kappa_L = (1-q)\left(1 - e^{-\beta\tau_r^M}\right) + q\left(1 - e^{-\beta\eta}\right)$ and we get back to the same situation as discussed in the main text.

Therefore, take the first order condition for $v(0)$ with respect to q, the optimal q^* satisfies (recall that parameter restriction (12))[16]

$$\underbrace{p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r}}_{\mathbb{E}[V_I(0)|\text{informative signal}]} - \underbrace{V_I(0|y_M)}_{\mathbb{E}[V_I(0)|\text{uninformative signal}]} - \alpha q^* \geqslant 0, \text{ with equality if } q^* < \frac{\kappa_L}{1 - e^{-\beta\eta}} \tag{31}$$

This expression is intuitive. Raising q increases (lowers) the probability of (un)informative signals, but costs more.

Combining with the dependence of ζ^* on q^* in Proposition 4 and the individual agent's optimal information acquisition condition (31), we can solve for the endogenous information acquisition q^* and survival time ζ^* simultaneously.

4.3.2 Run and No-Run Equilibria

We show that with endogenous information acquisition, in general multiple equilibria emerge. First, we check whether $q^* = 0$ is an equilibrium. Since the marginal cost of acquiring information is zero, in order for $q^* = 0$ to hold in equilibrium, it must be that there is no bank run.[17] Of course, no bank run is also sufficient for not acquiring additional information $q^* = 0$. According to Proposition 4, the condition for existence of a no-run equilibrium in this case is

$$G\left(\tau_r^l(q)\right)|_{q=0} \leq 0. \tag{32}$$

When (32) fails, i.e., bank run occurs even fixing $q = 0$ exogenously, then bank runs with positive information acquisition must exist. The next lemma summarizes this result.

Lemma 2. *Condition (32) is the necessary and sufficient condition for the existence of an equilibrium where no bank run occurs (and therefore $q^* = 0$). If (32) fails, there exist bank run equilibria with positive information acquisition $q^* > 0$.*

The above lemma only provides sufficient conditions for the existence of bank run equilibria.

[16]The fact that we are focusing on bank run equilibrium and that the agent with y_L finds immediate withdrawal to be optimal also imply that in (31) q^* cannot bind at zero, as the Blackwell information theorem implies that $p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M) > 0$ always, i.e., information has positive value as it improves the agent's decision. On the other hand, the equilibrium without bank run must have $q^* = 0$ and is analyzed in Section 4.3.2.

[17]Otherwise, with bank run, immediate withdraw with y_L signals implies a positive value for the signal. See related argument in footnote 16.

Interestingly, bank run equilibrium (with $q^* > 0$) could exist even when (32) holds. Intuitively, although no-run-no-acquisition is an equilibrium, run-acquisition might also be an equilibrium. Once every agents raise the equilibrium q^* above zero, a bank run is possible, and this makes individual information acquisition self-enforcing.

In general, even among the class of bank run equilibria with positive endogenous information acquisition, multiplicity may occur. The next lemma shows that under certain sufficient condition provided in the Appendix, we will have at most one such equilibrium. Essentially, this condition, by guaranteeing that at the equilibrium $q^* > 0$ the marginal benefit of information acquisition has to go below the marginal cost of information acquisition for q slightly above q^*, implies that the resulting equilibria (if exist) must be unique and stable.

Lemma 3. *Under condition (39) provided in Appendix, the bank-run equilibrium with positive endogenous information acquisition, if one exists, is unique and stable.*

The next proposition follows from the two above lemmas.

Proposition 5. *Under (32) and the condition in Lemma 3, we have at most two equilibria. The first is no-run equilibrium without information acquisition, i.e., $q^* = 0$. The other is the bank run equilibrium with information acquisition so that $q^* > 0$ and $\zeta^* < \eta$.*

From now on, to facilitate analysis, we will assume that conditions imposed in Proposition 5 hold, so that we have at most two equilibria: one with active information acquisition and bank runs, and a second without information acquisition and no run. In particular, we will be interested in parameters that eliminate the bank run equilibrium.

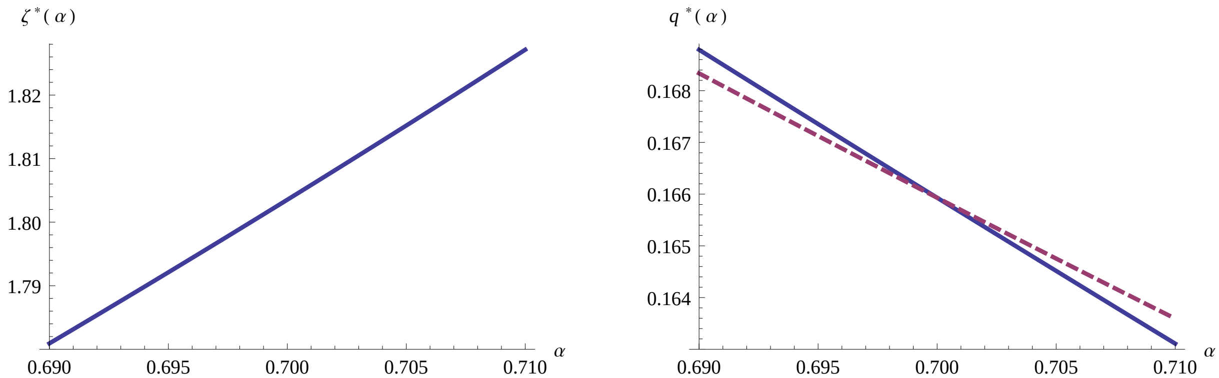
4.4 Comparative Statics

We perform comparative static analysis in this section. The following analysis focuses on the run equilibrium with endogenous information acquisition.

4.4.1 Information Acquisition Cost α

When each agent finds it easier to acquire information, naturally the precision q^* increases (the right panel in Figure 4). As shown in the left panel in Figure 4, this further leads to a shorter

Figure 4: Survival Time and Information Acquisition Response to Information Cost

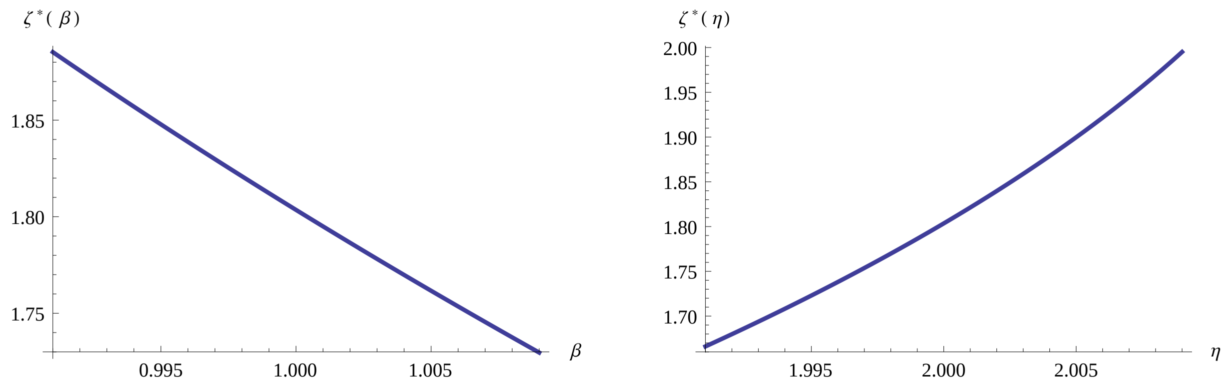


Solid lines show the equilibrium survival time of the illiquid bank ζ^*, and the information choice q^* as a function of the cost of information α. The dashed line allows the equilibrium q^* to change but holds fixed ζ^* at its baseline level. Parameter values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

survival of the illiquid bank, i.e., ζ^* becomes lower. A lower ζ^* motivates each agent to learn even more aggressively, and this feedback mechanism amplifies the initial effect of a lower α. The dashed line in Figure 4 graphs the endogenous information quality q^* by fixing the survival time at its baseline level. When α drops by one percent from the baseline level 0.7 to 0.693, the bank failure feedback mechanism leads agents to acquire 1.2 percent more information than otherwise.

This effect suggests information acquisition complementarities among depositors. Hellwig and Veldkamp (2009) show that information acquisition exhibits complementarity if and only if the actions are complementary. In our model, although running on the bank is complementary, sequential information acquisitions do not necessarily exhibit complementarity. The substitutability naturally arises in the sequential learning setting through the dependence of the posterior belief on the survival time of the bank, which is analyzed in (33). When other depositors acquire more information, the survival time ζ^* is shorter. Then, the mere survival of the bank coveys more information about the bank's liquidity so that upon hearing the rumor agents perceive the bank to be stronger. Substitutability naturally arises because this effect discourages each individual depositor's motivation for information acquisition.

Figure 5: Rumor Spreading Rate β and Awareness Window η



Solid lines show the equilibrium survival time of the illiquid bank ζ^* as a function of the rumor spreading rate β and the length of the awareness window η. Parameter values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

4.4.2 Rumor Spreading Rate β and Awareness Window η

When the rumor spreading rate β increases, all else equal, the illiquid bank will fail faster. This effect is illustrated in Figure 5. In response, each individual agent acquires more information, and the illiquid bank fails even faster. The feedback effects as discussed before are also present here, and this result is intuitive.

Relative to Abreu and Brunnermeier (2003), our model decouples the rumor spreading rate from the awareness window. When we turn to the effect of the awareness window η on the equilibrium survival time ζ^* and information precision q^*, a surprising result emerges. When η increases so that everybody knows that potentially there will be more informed agents attacking the bank, in our model each individual agent acquires information less aggressively and the illiquid bank survives longer.

It turns out that this surprising result not only comes from our decoupling of these two effects, but also relies on the novel uncertainty structure that we introduce in this framework. The intuition can be understood by investigating the posterior probability that the bank is illiquid upon hearing the rumor and *observing that the bank is still alive*:

$$p\left(t_i|t_i\right) = \frac{\Pr\left\{\text{illiquid bank survives at } t_i|\kappa_L, t_i\right\}\Pr\left\{\kappa_L\right\}}{\Pr\left\{\text{illiquid bank survives at } t_i|\kappa_L, t_i\right\}\Pr\left(\kappa_L\right) + \Pr\left\{\kappa_H\right\}} = \frac{\frac{e^{\lambda\zeta}-1}{e^{\lambda\eta}-1}p_0}{\frac{e^{\lambda\zeta}-1}{e^{\lambda\eta}-1}p_0 + 1 - p_0} \tag{33}$$

When η is large, $t_0 \in [t_i - \eta, t_i]$ could occur a long time ago, and the probability

$$\Pr\{\text{illiquid bank survives at } t_i | \kappa_L, t_i\} = \frac{e^{\lambda\zeta} - 1}{e^{\lambda\eta} - 1}$$

is lower. Consequently, conditional on the bank being alive at t_i, the bank is more likely to be liquid. Without uncertainty ($p_0 = 0$ or 1), $p(t_i|t_i)$ does not depend on the awareness window η.

This interesting result differs from that of static models where usually a greater pool of prone-to-run agents leads to more severe runs. Clearly, the casual intuition that runs are more severe with more prone-to-run agents comes from the argument that bank failure requires a sufficient mass of running depositors. However, our result shows that when cumulative informed agents are enough to run down the bank, artificially shortening the awareness window will gives rise to a novel information effect with the exact opposite direction.

4.4.3 A Short Awareness Window Example: Subprime Mortgage Crisis

On May 17, 2007 Fed Chairman Bernanke indicated in a speech about the subprime mortgage market that looser lending standards were pervasive especially in loans originated in 2006 (Bernanke, 2007). The speech took place at a time when low teaser rates on many adjustable-rate mortgages were set to expire, suggesting that the rise in defaults was just the tip of an iceberg. Subsequently, asset-backed commercial paper (ABCP) conduits experienced the modern-day equivalent of a bank "run" as ABCP outstanding dropped from \$1.3 trillion in July 2007 to \$833 billion in December 2007 (Acharya, Schnabl, and Suarez, forthcoming).

One interpretation within our model of this speech is that it signaled that the awareness window was relatively short, in which case there was not much to learn from the survival of ABCP conduits up to that point in time. Suppose that instead, the analysis revealed that looser lending standards were pervasive in loans originated since 2003. In that case, information about the resulting weakness must have been spreading for a long time. Having observed that the ABCP market kept growing despite this fact, investors would conclude that the probability of the system being liquid enough is high and a run might not occur.

5 Policy Analysis and Extensions

How much cash reserves should banks hold to prevent runs? In Section 5.1 we analyze this major question, of concern to both banks and their regulators. We then extend our base model. First, in Section 5.2 we introduce fundamentally insolvent banks, so that it is also socially efficient for individual agents to acquire information. Section 5.3 then considers a two-bank economy where competition amplifies the individual agent's socially wasteful information production. For both extensions we discuss government policy during the recent crisis.

5.1 How to Eliminate Bank Runs?

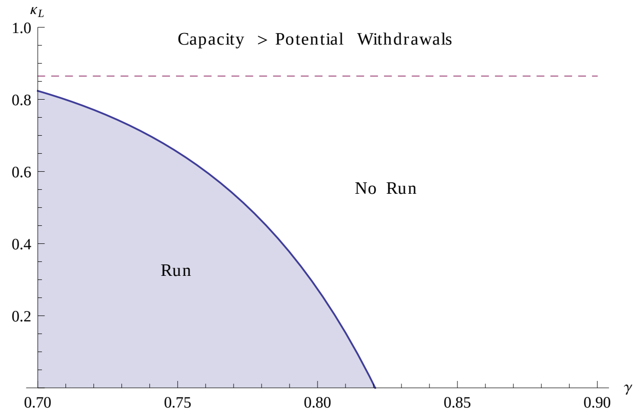
In Proposition 5 we have shown that similar to the static Diamond and Dybvig setting, generally in our model there are two equilibria: one equilibrium without learning and no bank runs, and the other with active learning and bank runs. However, the dynamic run equilibrium derived in our setting has a qualitatively different nature relative to the one in the static Diamond and Dybvig setting. This section explores this difference by focusing on the policy implications of how to eliminate the bank run equilibrium.

5.1.1 Minimum Reserve and Deposit Insurance

Unlike the typical static setting where runs occur if the bank reserve is below all potential withdrawals, our rich dynamic setting gives a non-trivial reserve threshold that eliminates the run equilibrium. In other words, due to uncertainty about the bank illiquidity and other depositors' withdrawal timings, the minimum reserve requirement that will fence off the bank run equilibrium with endogenous learning might be far below the level that is sufficient to cover all potential withdrawals. The same applies to deposit insurance: in our model, to eliminate runs, we only require a sufficiently high, but not full, deposit insurance.

In Figure 6, we plot the minimum reserve level κ_L so that the bank run equilibrium does not exist. We plot the threshold κ_L as a function of γ. Intuitively, when depositors expect to recover more of their deposit in case of failure (larger γ) the bank is less susceptible to runs. We can interpret γ as the level of deposit insurance. The figure reveals that in our dynamic setting, bank runs can be prevented even with partial deposit insurance ($\gamma < 1$).

Figure 6: Minimal Capacity Required to Eliminate Runs



The solid line is the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the recovery rate in case of failure γ. The dashed line is the potential mass of informed agents as in (11). Bank runs can occur only in the dark region. Parameter values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\alpha = 0.7$, $\gamma = 0.75$.

In Figure 6 we also show that the minimal liquidity level to eliminate the run is much lower than the potential mass of informed agents. From the view of static Diamond and Dybvig runs, the potential mass of informed agents is the relevant liquidity reserve required to eliminate runs. However, due to the asynchronous nature of our rumor-based setting, even without information acquisition, immediate withdrawal after hearing the rumor might not be an equilibrium since the bank survives for a while. Thus, eliminating dynamic rumor-based runs requires much less liquidity reserve than the one suggested by the static perspective.

Finally, it is clear that for the model to generate quantitatively meaningful policy recommendations on banks' capital reserve requirement, one needs reasonable parameters to begin with. As a first step toward this goal, our stylized model falls short on this dimension. However, though beyond the scope of the current paper, given the right data, a calibration (or even estimation) of our model can certainly achieve this more ambitious goal.

5.1.2 Comparison to Static Bank Run Models

One potential criticism to the above discussion is that we have so far only focused on responsive equilibria, i.e. only informed agents will run on the bank. Interestingly, in our dynamic setting with uncertainty, we can rule out one class of synchronized sun-spot type bank run equilibria (a la Diamond and Dybvig).

Proposition 6. *There does not exist a synchronized pure-strategy equilibrium where all agents (including the uninformed) run at some arbitrary but fixed time* $t^{SSR} \geq 0$*, where "SSR" stands for sun-spot-runs.*

The idea of this result is simple. If all agents coordinate on some positive time $t^{SSR} > 0$ to run on the bank, and running indeed dominates staying, then everyone wants to preempt to run earlier at $t^{SSR} - \epsilon$ to recover the full amount of their deposit. Hence under pure strategy equilibrium, the sun-spot-run time t^{SSR} must be zero. But nobody will run at $t^{SSR} = 0$, because at that time it is common knowledge that the bank is liquid, thus staying is the strictly dominant strategy, and the result in Proposition 6 follows. There might exist other equilibria where uninformed agents take mixed strategies, and it is an interesting future research topic to study the interaction between our responsive equilibrium where only informed agents run gradually and this class of sunspot equilibria where everyone runs.

So far we have compared our model with the Diamond and Dybvig model. To our knowledge no existing studies generate a result similar to the minimum liquidity reserve in Figure 6 based on static bank run models. While it may be possible to do so using the global games technique (e.g. Goldstein and Pauzner, 2005), the mechanism will be drastically different. To illustrate this point, note that the minimum liquidity reserve can be interpreted as liquid short-term assets held by the bank in the Diamond and Dybvig framework. Clearly, the portfolio share of liquid assets affects each individual depositor's payoff through some specific transformation technology between the liquid short-term asset and illiquid long-term asset. This effect shows up in both static and dynamic settings. However, there is one extra effect unique to the dynamic setting with learning. In the static bank run setting, withdrawals occur immediately. In contrast, in the dynamic model considered here with gradual running, a greater share of liquid holding also implies that it takes longer to run down the bank. Hence, through the endogenous learning channel, an increase in liquidity can reduce the hazard rate of failure and cause agents to *wait* longer. If we increase liquidity just enough to make the *last* agent whose withdrawal exhausts the liquid reserve to wait long enough, the bank run is averted. This practically relevant mechanism depends on both the dynamic nature of the withdrawal timing and the endogenous learning about a possible bank failure.

5.2 Insolvent Banks and Stress Tests

Our analysis thus far focuses on runs on fundamentally solvent but potentially illiquid banks. Because the first best allocation is keeping the bank alive always, information is socially "bad." Of course, in practice information may be "good" because of the presence of fundamentally insolvent banks so that runs on them are socially efficient. To address this issue, we introduce insolvency into our model. We then argue that the planner may mitigate running on fundamentally *solvent* banks by helping individual agents spot those insolvent banks more easily through stress tests.

5.2.1 Setup

Suppose that after the liquidity event that occurs at t_0, the bank in our model might be insolvent, which randomly fails (rather than matures) with intensity $\xi > r$. In this event deposits recover 0 for each dollar, and therefore $\xi > r$ implies that the bank is indeed insolvent. Of course the bank can also be solvent, and if so it can be either liquid or illiquid as we modeled before.

Naturally, there will be two layers of information. The first layer of information is regarding the bank's solvency which is both socially and individually valuable. The second is information regarding the bank's liquidity condition, which is individually valuable (and socially destructive) when agents realize that runs on the illiquid bank become a concern. We assume that although these two layers of information are different, they are inevitably related when individual agents are acquiring them. As shown shortly, this way we endogenize individuals endowed liquidity information precision via the active information collection about bank solvency.

Now when agent t_i hears the rumor, the possibility of insolvency motivates him to spend a fixed amount of effort $\underline{e} > 0$ to obtain a signal $\mathbf{1}_z \in \{0, 1\}$; for simplicity, this signal $\mathbf{1}_z$ perfectly reveals whether the bank is solvent or not.[18] We focus on situations where, in equilibrium, the agent always finds it optimal to acquire this solvency signal (which is guaranteed by a sufficiently high default intensity ξ). Therefore, conditional on the bank being insolvent, all agents who hear the rumor know that the bank is insolvent and therefore run on the bank immediately.[19]

A by-product of the agent's private learning about the bank solvency is that he also learns

[18] This assumption is for clarifying the economic channel and innocuous. See detailed discussion in footnote 20.

[19] Suppose that the insolvent bank has a capacity of κ_Z; therefore the equilibrium failure time for insolvent bank is $t_0 + \zeta_z^* = t_0 + \frac{1}{\beta} \ln \frac{1}{1-\kappa_Z}$.

something about the liquidity of the bank. Given the effort $\underline{e}$ of figuring out whether the bank is insolvent, if the bank turns out to be solvent, then the baseline quality of the bank's liquidity signal y—which is the signal we modeled in Section 2.1.4—is just $\underline{e}$. As a result, the agent's additional liquidity information precision choice is $q \geq \underline{e}$ with acquisition cost $\frac{\alpha}{2}(q-\underline{e})^2$. The interpretation is that the *process* of collecting insolvency information inevitably teaches the agent more about the bank's liquidity. The longer the process of collecting insolvency information, the more the agent knows about the bank's liquidity. Our modeling that the effort of collecting insolvency information (i.e., $\underline{e}$) becomes the baseline quality of liquidity information captures this idea in the simplest way.

5.2.2 Policy Implication: Stress Tests

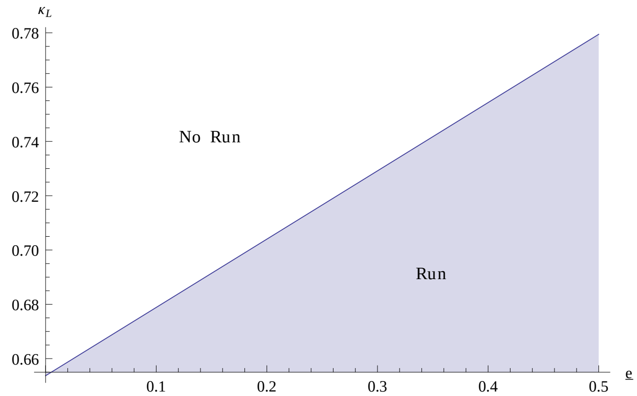
The above setting has important implications for stress tests in revealing the fundamentally problematic banks. By *providing insolvency information only*, the government can use stress tests to reduce $\underline{e}$ to eliminate runs on solvent-but-illiquid banks.

According to our model, if a great effort is required to learn about insolvent banks (say Lehman), i.e., a higher $\underline{e}$, then each agent will be automatically endowed with significant information about the liquidity of solvent banks (say Citibank). Given that, it is more likely that bank runs exist in the equilibrium, and therefore they are more likely to acquire even more information. As a result, runs on solvent banks start.

Stress tests provide transparency on potentially insolvent banks (Bernanke, 2009), and therefore reduce $\underline{e}$. Government, by providing higher quality information about banks' insolvency, can crowd out private acquisition of insolvency information. Because public information can be better targeted at insolvency alone, while the process of private acquisition of solvency information inevitably reveals liquidity information, public provision of solvency information helps all agents know that other agents do not have superior information regarding banks liquidity situation. Therefore, our model suggests that the public provision of insolvency information indirectly reduces the socially wasteful information acquisition regarding liquidity, and therefore make runs on illiquid banks less likely.

We emphasize that the perfect revelation of insolvent bank in our setting helps clarify the channel of releasing better solvency information to help illiquid banks; more precisely, it is not

Figure 7: Minimal Capacity and Information Collection Effort Required to Eliminate Runs



We plot the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information collection effort $\underline{e}$. Parameter values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $p_0 = 0.8$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.65$, $\alpha = 0.7$, $\gamma = 0.75$.

through higher average bank quality once the stress test isolates those insolvent ones.[20] Rather, the channel is through the fact that now everybody knows that everybody will wait to see the stress test and therefore not scramble to search the insolvent banks. As a result everybody will have less precise information on which solvent bank is less liquid and susceptible to a run.

This view is consistent with the Federal Reserve Board's recent break from the traditional supervisory view of opaqueness in favor of more public disclosure to restore the confidence of investors. However, our rationale is different than the one described by Bernanke (2010) who argues that the release of detailed information about the banks subjected the tests to scrutiny by outside analysts, thereby enhancing the credibility of the tests. Instead, we argue that by providing more information, the government crowds out the information collection effort by individuals about the solvency of the banks. Since liquidity and solvency are tightly related, this government policy has the useful by-product of reducing information collection about bank liquidity and therefore reducing the incidence of runs on illiquid banks.

Figure 7 plots the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information collection effort $\underline{e}$. We see from Figure 7 that as the government

[20] When the planner varies $\underline{e}$, agents can always perfectly spot insolvent banks, therefore the channel of insolvency information is shut down. Rather, the channel is the strategic interaction of individual agents who make their endogenous acquisition decision on liquidity information. In fact, our model can be handle the case that insolvent banks are imperfectly revealed. As discussed after Proposition 4, in our model generically a bank run equilibrium exists only when y_L agents withdraw immediately, and the equilibrium analysis is identical if we instead assume that agents cannot perfectly tell insolvent banks from illiquid ones (so they will withdraw immediately once receiving y_L signal or $\mathbf{1}_z = 1$). However, under this alternative assumption, it is quite obvious that better solvency information helps illiquid banks since it allows individual agents to tell them apart from insolvent ones.

provides more information about the solvency of the bank (lower $\underline{e}$), less liquid banks (lower κ_L) can avoid runs.

5.3 Multiple Solvent Banks and Policy Implications

We now investigate the behavior of depositors and their incentives to acquire information in the presence of competing solvent banks. Instead of holding cash, a bank run in this setting involves the transfer of deposited funds from one bank to another.

We show that greater information quality leads agents to inefficient runs on the otherwise identical solvent bank.[21] Here, the two banks' difference might be minuscule, and in fact transfers between the two institutions only involve social losses (transaction fees and bank failure). The planner thus can inject noise into the system, so that individual agents with less informative signals are more likely to stay in their original bank without knowing which one is the (more) liquid one.

5.3.1 Analysis

Suppose there are two banks, ex-ante identical except that half the population deposits in bank A and half deposits in bank B. Both banks promise the same rate of return r. However, transferring funds between banks requires a transaction cost k. A liquidity event occurs at a random time t_0, and the rumor starts that exactly one of the banks is illiquid ($0 < \kappa_L < 1$) while the other is liquid ($\kappa_H > 1$). The prior probability that each bank is illiquid is $p_0 = 0.5$ since they are ex-ante identical. The learning process in the two bank set-up is simpler, because the passage of time without a failure teaches agents nothing about the relative viability of their bank.

As in the setup above, agents are allowed to acquire a costly signal $\widetilde{y} \in \{y_L, y_M, y_H\}$ about the status of their bank with probability distribution as in (2). Agents who receive the y_H signal know that their bank is the liquid one and therefore never withdraw. Agents who receive the y_M signal gain no useful information. Since there is a small transaction cost k, remaining in their original bank is optimal.[22] Finally, agents who draw the y_L signal run on their bank immediately. From

[21] Some anecdotal evidence that differences in depositor perception about bank liquidity motivates them to withdraw from illiquid banks is provided by Sidel, Enrich, and Fitzpatrick (2008): "Melody Williams, 50 years old, said in the past 30 days she has moved about $25,000 out of Washington Mutual, spreading it to other financial institutions she thought were stronger, including Wells Fargo & Co. Ms. Williams, the controller for an architecture firm, said she thought that Washington Mutual had gotten 'too big for their britches' with too many deals over the years."

[22] Formally, let $V_{-I}(0|y_M)$ be the value of a dollar in the other bank from these agents' perspective. Then $V_{-I}(0|y_M) = (1-k)\,V_I(0|y_M) < V_I(0|y_M)$, which is the value of a dollar in the original bank.

their perspective, the value of a dollar in their bank falls and the value of a dollar in the competing bank increases to a riskless $\frac{\delta}{\delta-r}$. Thus, as long as the transaction cost k is small enough, immediate withdraw is optimal.[23] Information is (privately) more valuable in this setup since the outside option is a nearly identical bank rather then holding cash.

The following proposition characterizes the equilibrium in this setting with two banks.

Proposition 7. *Under the two banks setup, the bank run equilibrium* $\{\zeta^*, q^*\}$ *is determined by the following two equations:*

$$\zeta^* = -\frac{1}{\beta}\ln\left(1 - \frac{\kappa_L}{q^*}\right), \tag{34}$$

$$\frac{1}{2}(1-k)\frac{\delta}{\delta-r} + \frac{1}{2}\frac{\delta}{\delta-r} - V_I\left(0|y_M;\zeta^*, q^*\right) = \alpha q^*, \tag{35}$$

where the expression for $V_I\left(0|y_M;\zeta^*, q^*\right)$ *is given in the Appendix.*

5.3.2 Injecting Noise

In this extension, higher information quality q about the liquidity of two solvent banks is socially undesirable, as it shortens the survival time of the illiquid bank by introducing more agents who realize that one bank strictly dominates the other. Injecting noise can alleviate the problem, and the simplest interpretation of injecting noise is to raise the information acquisition cost α.

A bank run equilibrium requires that withdrawals by the y_L agents alone can destroy a bank, i.e. $\kappa_L < q\left(1 - e^{-\beta\eta}\right)$.[24] The threshold $\overline{q}$ so that no run would occur is $\overline{q} \equiv \frac{\kappa_L}{1-e^{-\beta\eta}}$. If the planner raises α so that the marginal benefit of acquiring information is below its marginal cost, i.e.,

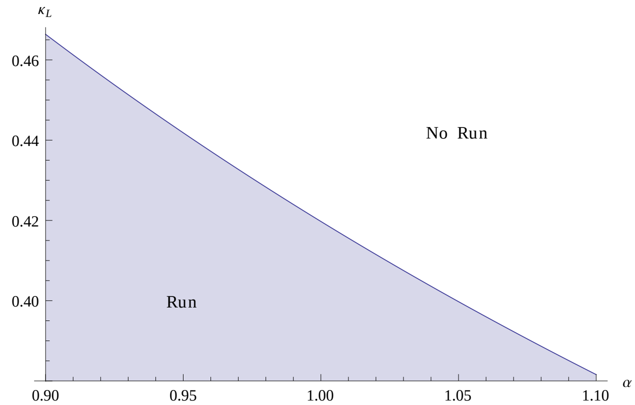
$$\frac{1}{2}(1-k)\frac{\delta}{\delta-r} + \frac{1}{2}\frac{\delta}{\delta-r} - V_I\left(0|y_M\right) \le \alpha\overline{q},$$

then the illiquid bank is always liquid enough to sustain a run. Figure 8 plots the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information cost α for the two bank setup. The injection of noise into the economy (higher α) blurs the differences between the competing solvent banks. The noise reduces the equilibrium information quality q, and as a result, a run is easier to eliminate and requires less reserves by the illiquid bank.

[23] More specifically, we require k is sufficiently small that $V_I\left(0|y_M\right) < (1-k)\,V_{-I}\left(0|y_H\right) = (1-k)\,\frac{\delta}{\delta-r}$.

[24] In this two-bank setting we no long impose the restriction of $q \le \frac{\kappa_L}{1-e^{-\beta\eta}}$ as in condition (12), because now only y_L agents are withdrawing to potentially take down the illiquid bank.

Figure 8: Minimal Capacity and Information Cost Required to Eliminate Runs



We plot the minimal illiquid bank capacity κ_L that eliminates the bank run equilibrium as a function of the information cost α for the two bank setup. Parameter values are $r = 0.09$, $\beta = 1$, $\theta = 1.03$, $\eta = 2$, $\delta = 0.12$, $k = 10^{-6}$, $\kappa_L = 0.3$, $\alpha = 1$, $\gamma = 0.75$.

Consider the recent financial crisis in 2008. A fear that some banks were insolvent prompted the Capital Purchase Program commonly known as the bailout of the nine largest U.S. financial institutions on October 13, 2008. When presenting the program to the CEOs of the 9 banks, Secretary of Treasury Henry M. Paulson was concerned that the strongest banks, e.g., JPMorgan, would not participate.[25] To make sure that they do, government officials suggested that if a bank refused the funds, its regulator would later force it to raise capital anyway and under worse terms.[26] The government was in fact injecting noise about the liquidity of competing solvent banks into the economy. By pooling banks together the incentive to transfer funds between them was kept low enough so that none of the nine banks suffered a run.

6 Conclusion

We study above new dimensions of bank runs previously unexplored by focusing on information acquisition in rumor-based runs. These dimensions include the spreading rate of information and the awareness window over which it spreads. We show that individuals acquire information excessively about the liquidity of banks subject to runs. We then analyze the role government information

[25] "I was concerned about Jamie Dimon, because JPMorgan appeared to be in the best shape of the group, and I wanted to be sure he would accept the capital." - Paulson (2010)

[26] "Look, we're making you an offer," I said, jumping in. "If you don't take it and sometime later your regulator tells you that you are undercapitalized and you have to raise private-sector capital but you are unable to do so, you may not like the terms if you have to come back to me." - Paulson (2010)

policy can play in bank run prevention.

Empirically, our model can shed new light on both traditional bank runs and debt runs more broadly. Our model makes new predictions linking the likelihood of a run to the cost of acquiring information, the spreading rate of the rumor and its awareness window. It also predicts that for banks that survive a run we should observe a period of simultaneous withdrawals and redeposits by heterogeneously informed agents. Our model is rich enough for structural estimation or calibration of its parameters given detailed withdrawals data, which can result in meaningful policy implications.

Finally, the dynamic bank run model we provide above can shed new light on other economic settings such as arbitrageur behavior, currency attacks, and R&D investment games.

References

Abreu, Dilip, and Markus K. Brunnermeier, 2002, Synchronization risk and delayed arbitrage, *Journal of Financial Economics* 66, 341 – 360.

———, 2003, Bubbles and crashes, *Econometrica*.

Acharya, Viral V., Philipp Schnabl, and Gustavo Suarez, forthcoming, Securitization without risk transfer, *Journal of Financial Economics*.

Bernanke, Ben S., 2007, The subprime mortgage market, *Speech at the Federal Reserve Bank of Chicago 43rd Annual Conference on Bank Structure and Competition.*

———, 2009, The supervisory capital assessment program, *Speech at the Federal Reserve Bank of Atlanta 2009 Financial Markets Conference, Jekyll Island, Georgia.*

———, 2010, The supervisory capital assessment program–one year later, *Speech at the Federal Reserve Bank of Chicago 46th Annual Conference on Bank Structure and Competition, Chicago, Illinois.*

Bhattacharya, Sudipto, and Douglas Gale, 1987, Preference shocks, liquidity, and central bank policy, *in W. Barnett and K. Singleton eds., New Approaches to Monetary Policy, Cambridge University Press* pp. 69–88.

Brunnermeier, Markus K., and John Morgan, 2010, Clock games: Theory and experiments, *Games and Economic Behavior* 68, 532 – 550.

Calomiris, Charles W., and Joseph R. Mason, 1997, Contagion and bank failures during the great depression: The june 1932 chicago banking panic, *The American Economic Review* 87, pp. 863–883.

Chari, V. V., and Ravi Jagannathan, 1988, Banking panics, information, and rational expectations equilibrium, *The Journal of Finance* 43, pp. 749–761.

Diamond, Douglas W., and Philip H. Dybvig, 1983, Bank runs, deposit insurance, and liquidity, *The Journal of Political Economy* 91, pp. 401–419.

Ennis, Huberto M., and Todd Keister, 2009, Bank runs and institutions: The perils of intervention, *The American Economic Review* 99, 1588–1607.

Goldstein, Itay, and Ady Pauzner, 2005, Demand-deposit contracts and the probability of bank runs, *Journal of Finance* 60, pp. 1293–1327.

Gorton, Gary, 1985, Bank suspension of convertibility, *Journal of Monetary Economics* 15, 177 – 193.

Gorton, G., and A. Metrick, 2011, Securitized banking and the run on repo, *Journal of Financial Economics*.

Green, Edward J., and Ping Lin, 2003, Implementing efficient allocations in a model of financial intermediation, *Journal of Economic Theory* 109, 1 – 23.

Gu, Chao, 2011, Herding and bank runs, *Journal of Economic Theory* 146, 163–188.

He, Zhiguo, and Wei Xiong, forthcoming, Dynamic debt runs, *Review of Financial Studies*.

Hellwig, Christian, and Laura Veldkamp, 2009, Knowing what others know: Coordination motives in information acquisition, *Review of Economic Studies* 76, 223–251.

Iyer, Rajkamal, and Manju Puri, forthcoming, Understanding bank runs: The importance of depositor-bank relationships and networks, *American Economic Review*.

Jacklin, Charles J., and Sudipto Bhattacharya, 1988, Distinguishing panics and information-based bank runs: Welfare and policy implications, *Journal of Political Economy* 96, pp. 568–592.

Kelly, Morgan, and Cormac O Grada, 2000, Market contagion: Evidence from the panics of 1854 and 1857, *The American Economic Review* 90, pp. 1110–1124.

Morris, Stephen, and Hyun Song Shin, 2002, Social value of public information, *The American Economic Review* 92, 1521–1534.

Nikitin, Maxim, and R. Todd Smith, 2008, Information acquisition, coordination, and fundamentals in a financial crisis, *Journal of Banking and Finance* 32, 907 – 914.

O Grada, Cormac, and Eugene N. White, 2003, The panics of 1854 and 1857: A view from the emigrant industrial savings bank, *The Journal of Economic History* 63, pp. 213–240.

Paulson, Henry M., 2010, *On the Brink: Inside the Race to Stop the Collapse of the Global Financial System* (Business Plus).

Shin, Hyun Song, 2009, Reflections on northern rock: The bank run that heralded the global financial crisis, *Journal of Economic Perspectives* 23, 101–119.

Sidel, Robin, David Enrich, and Dan Fitzpatrick, 2008, WaMu is seized, sold off to J.P. Morgan, in largest failure in U.S. banking history, *The Wall Street Journal* p. A1.

Wallace, Neil, 1988, Another attempt to explain an illiquid banking: the diamond-dybvig model with sequential service taken seriously, *Fed Reserve Bank Minneapolis Quraterly Review* 12, 2–16.

A Appendix A

A.1 Proof of Proposition 1

We have two cases to consider for the hazard rate. First, suppose $\zeta > \eta$. Then an agent informed at t_i learns nothing from the fact that the bank has not failed by t_i. His distribution of failure dates $t_i + \tau$ is

$$\Pi(t_i + \tau | t_i) = \begin{cases} 0 & \tau < \zeta - \eta \\ p_0 \frac{e^{\lambda\eta} - e^{\lambda(\zeta-\tau)}}{e^{\lambda\eta}-1} & \zeta - \eta \leq \tau < \zeta \\ p_0 & \zeta \leq \tau. \end{cases}$$

with non-zero density $\pi(t_i + \tau | t_i) = p_0 \frac{\lambda e^{\lambda(\zeta-\tau)}}{e^{\lambda\eta}-1}$ for $\zeta - \eta < \tau < \zeta$. On the other hand, if $\zeta \leq \eta$, then agent t_i's distribution of failure dates $t_i + \tau$ is

$$\Pi(t_i + \tau | t_i) = \begin{cases} 0 & \tau < 0 \\ p(t_i | t_i) \frac{1-e^{-\lambda\tau}}{1-e^{-\lambda\zeta}} & 0 \leq \tau < \zeta \\ p(t_i | t_i) & \zeta \leq \tau. \end{cases}$$

non-zero density $\pi(t_i + \tau | t_i) = p(t_i | t_i) \frac{\lambda e^{-\lambda\tau}}{1-e^{-\lambda\zeta}}$ for $0 < \tau < \zeta$. Plugging either of the pairs into the definition of the hazard rate (8) yields after some algebraic manipulation the same expression (9) for any $\tau \geq \max\{0, \zeta - \eta\}$ and zero elsewhere.

A.2 Proof of Proposition 2

We first establish the following lemma.

Lemma 4. *The function $g(\tau)$ crosses zero from below at most once in the interval $[0, \zeta]$.*

Proof. Since it is the change in the numerator that dominates around $g(\tau) = 0$, it suffices to show that the numerator of $g(\tau)$ (ignoring the constant)

$$(\lambda(1-\gamma) - r) e^{\lambda(\zeta-\tau)} p_0 - (1-p_0)\left(e^{\lambda\eta} - 1\right) \frac{r(r-k\delta)}{\delta - r} e^{-\delta(\zeta-\tau)} \tag{36}$$

is increasing over the interval $[0, \zeta]$. Furthermore, since $\lambda(1-\gamma) - r < \lambda(1-\gamma) - r(1-k) < 0$ from (13), it follows that (36) is concave in τ. Let $\overline{\tau}$ be the unique maximizer. At the maximum

$$\zeta - \overline{\tau} = \frac{1}{\lambda + \delta} \ln \frac{\delta(1-p_0)\left(e^{\lambda\eta} - 1\right) \frac{r(r-k\delta)}{\delta - r}}{\lambda(r - \lambda(1-\gamma)) p_0} < 0$$

due to (13). Thus, the function in (36) attains its maximum to the right of ζ and is therefore increasing over $[0, \zeta]$.

□

Lemma 4 implies that if $g(\zeta) \leq 0$ ($g(0) \geq 0$) then $g(\tau) < 0$ ($g(\tau) \geq 0$) always for $\tau \in [0, \zeta]$. We next consider the optimal strategy for the three cases of the proposition:

Case 1. *If $g(\zeta) \leq 0$, then it is optimal to stay in the bank always.* To prove our claim, it suffices to show that $V_I(\tau) > V_O(\tau)$ for $\tau \in [0,\zeta]$. Suppose not, then there must exist some τ_w so that $V_I(\tau_w) = V_O(\tau_w)$ and $V_I'(\tau_w) > V_O'(\tau_w)$ because $V_I(\zeta) > V_O(\zeta)$. From HJB equations, we have

$$h(\tau_w)(1-\gamma) - rV_I(\tau_w) = V_I'(\tau_w) - V_O'(\tau_w) > 0$$

However, since $V_I(\tau_w) \geq V_O(\tau_w) \geq \widehat{V}_O(\tau_w)$ by definition (the first inequality is because there is no transaction cost to take one dollar out, and the second inequality is because $\widehat{V}_O(\tau_w)$ may be derived under suboptimal policy), we have

$$h(\tau_w)(1-\gamma) - rV_I(\tau_w) < h(\tau_w)(1-\gamma) - r\widehat{V}_O(\tau_w) = g(\tau_w) \leq 0,$$

a contradiction.

□

Case 2. *If $g(0) \geq 0$, then it is optimal to withdraw at 0 and redeposit right after ζ.* Well, if $g(0) \geq 0$, then $g(\tau) \geq 0$ always for $\tau \in [0,\zeta]$, and $g(\zeta) > 0$. Using $g(\zeta) > 0$, we first show that since k is arbitrarily small, there exists some $\widehat{\tau}$ close to ζ so that $V_O(\widehat{\tau}) = V_I(\widehat{\tau})$. To show this, we show that $V_O'(\zeta) - V_I'(\zeta)$ is strictly below zero when k is arbitrarily small. To see this, from the HJB equations we know that

$$\begin{aligned} V_O'(\zeta) - V_I'(\zeta) &= h(\zeta)(V_O(\zeta) - V_I(\zeta) + \gamma - 1) + \delta(V_I(\zeta) - V_O(\zeta)) + rV_I(\zeta) \\ &= -g(\zeta) + (h(\zeta) + \delta - r)(V_O(\zeta) - V_I(\zeta)) \end{aligned}$$

The first is strictly negative while the second term converges to zero as $k \to 0$. Therefore, when k is arbitrary small there exists some ε so that $V_I(\zeta - \varepsilon) < V_O(\zeta - \varepsilon)$. Due to continuity and the fact that $V_I(\zeta) = \frac{V_O(\zeta)}{1-k} > V_O(\zeta)$, there exists some $\widehat{\tau}$ close to ζ so that $V_I(\widehat{\tau}) = V_O(\widehat{\tau})$. Note that $V_O(\widehat{\tau}) = \widehat{V}_O(\widehat{\tau})$.
Now to prove that "it is optimal to withdraw at 0 and redeposit right after ζ," we only need to show that $V_I(\tau) = V_O(\tau)$ holds for all $\tau \in [0,\widehat{\tau}]$ (intuitively, at any point of time a dollar inside the bank has the value of taking outside, it is always optimal to keep the money outside). Suppose that this does not hold; since $V_I(\tau) \geq V_O(\tau)$ in general, there must exits some point $\tau_w \in [0,\widehat{\tau}]$ so that $V_I(\tau_w) = V_O(\tau_w)$ and $V_I'(\tau_w) < V_O'(\tau_w)$. Choosing the largest value τ_w, so that $V_O(\tau_w) = \widehat{V}_O(\tau_w)$ holds (i.e., the optimal continuation strategy is wait outside the bank until ζ). Similar to the argument before, we have

$$h(\tau_w)(1-\gamma) - rV_O(\tau_w) = V_I'(\tau_w) - V_O'(\tau_w) < 0,$$

but this contradicts the fact that $h(\tau_w)(1-\gamma) - rV_O(\tau_w) = h(\tau_w)(1-\gamma) - r\widehat{V}_O(\tau_w) \geq 0$ since $g(\tau) \geq 0$ always.

□

Case 3. It follows from the Lemma 4 that $g(\zeta) > 0$ and $g(0) < 0$ imply that there exists a unique $\tau_w \in (0,\zeta)$ so that $g(\tau_w) = 0$, $g(\tau) > 0$ for $\tau \in (\tau_w,\zeta)$ and $g(\tau) < 0$ for $\tau \in (0,\tau_w)$. Following the same argument in the second part by replacing 0 with τ_w, we know that it is optimal to withdraw at τ_w and redeposit at $\zeta+$, and $V_I(\tau_w) = V_O(\tau_w) = \widehat{V}_O(\tau_w)$. Then to prove our claim we only need to show that $V_I(\tau) > V_O(\tau)$ for $\tau \in (0,\tau_w)$.

Let $H(\tau) \equiv V_I(\tau) - V_O(\tau)$ with $H(\tau_w) = 0$, we need to show that $H(\tau) > 0$ for $\tau \in (0, \tau_w)$, as $H(\tau) = V_I(\tau) - V_O(\tau) \geqslant 0$ in general. First, we show that it is impossible to have $H(\tau) = 0$ uniformly on any interval $(\tau_w - \Delta, \tau_w)$ where $\Delta > 0$; if it is true then it must be that $V_I(\tau) = V_O(\tau) = \widehat{V}_O(\tau)$ on that interval so that

$$\begin{aligned} 0 &= rV_I(\tau) + h(\tau)(\gamma - V_I(\tau)) + \delta(1 - V_I(\tau)) + V_I'(\tau) \\ &= r\widehat{V}_O(\tau) + h(\tau)\left(\gamma - \widehat{V}_O(\tau)\right) + \delta\left(1 - \widehat{V}_O(\tau)\right) + \widehat{V}_O'(\tau) \\ &= r\widehat{V}_O(\tau) - h(\tau)(1-\gamma) = -g(\tau) > 0 \end{aligned}$$

where the first equality is (18) and third equality is using the ODE for $\widehat{V}_O(\tau)$ with $0 = \; h(\tau)\left(1 - \widehat{V}_O(\tau)\right) + \delta\left(1 - \widehat{V}_O(\tau)\right) + \widehat{V}_O'(\tau)$. This contradiction implies that we must have $V_I(\tau) > V_O(\tau)$ for some τ close to τ_w. Now suppose that there exists another point $\tau_w^1 < \tau_w$ so that $V_I(\tau_w^1) = V_O(\tau_w^1)$. Take τ_w^1 that is closet to τ_w so that $V_I'(\tau_w^1) \geq V_O'(\tau_w^1)$. At τ_w^1, $V_O(\tau_w^1)$ must satisfy the HJB in (18) (because τ_w^1 is in the inaction region around the neighborhood, i.e., $(1-k)V_I(\tau) < V_O(\tau)$ for τ close to τ_w). Then

$$\begin{aligned} 0 &= rV_I\left(\tau_w^1\right) + h\left(\tau_w^1\right)\left(\gamma - V_I\left(\tau_w^1\right)\right) + \delta\left(1 - V_I\left(\tau_w^1\right)\right) + V_I'\left(\tau_w^1\right) \\ &\geq rV_O\left(\tau_w^1\right) + h\left(\tau_w^1\right)\left(\gamma - V_O\left(\tau_w^1\right)\right) + \delta\left(1 - V_O\left(\tau_w^1\right)\right) + V_O'\left(\tau_w^1\right) \\ &= rV_O\left(\tau_w^1\right) - h\left(\tau_w^1\right)(1-\gamma) \geq -g\left(\tau_w^1\right) > 0 \end{aligned}$$

where we have used the HJB for $V_O(\tau_w^1)$, and the fact that $V_O(\tau_w^1) \geq \widehat{V}_O(\tau_w^1)$ in general. Again we get a contradiction with (18).

□

A.3 Proof of Proposition 3

Proof. Given τ_w simple integration yields

$$V_I(0|y_M) = \frac{\left[\begin{array}{c} \frac{\delta\left(e^{\lambda\eta}(1-p_0)-1\right)}{\delta-r}\left(1 - e^{-(\delta-r)\tau_w}\right) + \frac{\delta+\lambda\gamma}{\lambda+\delta-r}e^{\lambda\zeta}p_0\left(1 - e^{-(\lambda+\delta-r)\tau_w}\right) \\ +e^{-(\delta-r)\tau_w}\left(e^{\lambda\eta} - 1 - \left(e^{\lambda\eta} - e^{\lambda(\zeta-\tau_w)}\right)p_0\right)V_O(\tau_w) \end{array}\right]}{e^{\lambda\eta} - 1 - e^{\lambda\eta}p_0 + e^{\lambda\zeta}p_0}$$

Note that $h(\tau_w)(1-\gamma) = rV_O(\tau_w)$ implies that

$$\left(e^{\lambda\eta} - 1 - \left(e^{\lambda\eta} - e^{\lambda(\zeta-\tau_w)}\right)p_0\right)V_O(\tau_w) = \frac{\lambda(1-\gamma)e^{\lambda(\zeta-\tau_w)}p_0}{r}$$

where we used the definition of h in (8). Then note that

$$\frac{\lambda(1-\gamma)}{r} - \frac{\delta+\lambda\gamma}{\lambda+\delta-r} = \frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}$$

which gives our expression.

□

A.4 Proof of Lemma 1

Proof. Suppose that $\zeta > \eta$, so that at ζ the cumulative withdrawal from y_L agents is $q\left(1-e^{-\beta\eta}\right)$. Then using the aggregate condition (26), we can back out the equilibrium τ_r for y_M agents as

$$\tau_r = -\frac{1}{\beta}\ln\left(1-\frac{\kappa_L - q\left(1-e^{-\beta\eta}\right)}{1-q}\right). \tag{37}$$

However, unless parameters are such that the above τ_r happens to satisfy $G(\tau_r) = 0$ which is the y_M agents' optimal waiting decision, generically this cannot occur.

□

A.5 Proof of Proposition 4

First note that the G function is the mirror image of individual FOC condition function, i.e., $G(\tau_r) = g(\zeta - \tau_r)$, and it shares the same (but opposite) property of $g(\cdot)$ shown in Lemma 4:

Corollary 1. *$G(\tau_r)$ crosses zero from above at most once on $\tau_r \in [0, \zeta]$.*

This result implies that the following holds for the three cases of the proposition:

Case 1. If $G\left(\tau_r^l\right) \leq 0$, then $G(\tau_r) \leq 0$ for all $\tau_r \geq \tau_r^l$. Thus if all other agents strategy is to redeposit after any $\tau_r \geq \tau_r^l$, it is optimal for the individual agent to deviate and wait a bit longer. Therefore, $\zeta^* \to \infty$ and no run equilibrium exists.

□

Case 2. If $G(\tau_r^u) \geq 0$, then $G(\tau_r) \geq 0$ for all $\tau_r \leq \tau_r^u$. Thus, if all other agents' strategy is to withdraw at some interior $\tau_r \leq \tau_r^u$, it is optimal for the individual agent to deviate and withdraw earlier. Therefore, agents withdraw immediately in the only symmetric equilibrium.

□

Case 3. Finally, if $G(\tau_r^u) < 0$ and $G\left(\tau_r^l\right) > 0$ then by continuity of G and Corollary 1, there exists a unique bank run equilibrium $\tau_r^* \in \left(\tau_r^l, \tau_r^u\right)$ so that $G(\tau_r^*) = 0$. Plugging into (26) we get the equilibrium survival time ζ^* and waiting time τ_w^*. A second implication of Corollary 1 is that $G'(\tau_r^*) < 0$. Therefore the equilibrium is stable.

□

A.6 Proof of Lemma 2

Proof. The proof of the first statement is given in the text. To show the second statement, we need to show that the agent's FOC in information acquisition

$$A(q) \equiv p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M) - \alpha q \geqslant 0 \text{ with equality if } q < \frac{\kappa_L}{1 - e^{-\beta\eta}},$$

combined with bank-run equilibrium condition, has a solution. Note that both $p(t_i|t_i)$ and $V_I(0|y_M)$ (given in (33) and Proposition 3) depend on ζ that is determined in Proposition 4 about bank run equilibrium given q. Now because (32) fails, the bank run equilibrium exists even with exogenously given $q = 0$. From (22) it is optimal to withdraw given y_L. The agent's strategy depends on his signal, and therefore information has a positive value, i.e. $A(0) = p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M) > 0$. Now suppose that q takes its upper bound $\frac{\kappa_L}{1-e^{-\beta\eta}}$; if $A\left(q = \frac{\kappa_L}{1-e^{-\beta\eta}}\right) > 0$ then the upper bound information quality and associated bank run equilibrium is just the equilibrium that we are after. If instead $A\left(q = \frac{\kappa_L}{1-e^{-\beta\eta}}\right) < 0$, then an interior equilibrium exists because of the continuity of $A(q)$.

□

A.7 Proof of Lemma 3

Proof. First, when the bank run equilibrium occurs with corner solution $\zeta = \tau_r^u = \frac{1}{\beta} \ln \frac{1}{1-\kappa_L}$, then the marginal benefit of information $MB = p(t_i|t_i) + (1 - p(t_i|t_i)) \frac{\delta}{\delta - r} - V_I(0|y_M)$ is independent of q, and the equilibrium q^* equates $MC = \alpha q^* = MB$. Therefore the equilibrium is unique and stable (MB is constant while MC increases with q). Also if q^* takes the upper bound corner value, the associated run equilibrium is unique as well. So the rest of proof focus on the case where both the information quality of equilibrium survival time take interior solutions.

From now on we focus on interior bank run equilibrium. Importantly, this implies that τ_r is determined in (25) which only depends on primitives. Therefore we treat τ_r as a primitive parameter. The FOC (31) when the agent sets q^* is

$$\frac{\left[\begin{array}{c} \left(e^{\lambda\zeta} - 1\right) p_0 + (1 - p_0)\left(e^{\lambda\eta} - 1\right) \frac{\delta}{\delta - r} - \frac{\delta\left(e^{\lambda\eta}(1-p_0)-1\right)}{\delta - r}\left(1 - e^{-(\delta - r)\tau_w}\right) \\ -\frac{\delta+\lambda\gamma}{\lambda+\delta-r} e^{\lambda\zeta} p_0 - e^{-(\delta-r)\tau_w} e^{\lambda\tau_r} p_0 \left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \alpha q\left((1 - p_0)\left(e^{\lambda\eta} - 1\right) + \left(e^{\lambda\zeta} - 1\right) p_0\right) \end{array}\right]}{(1 - p_0)\left(e^{\lambda\eta} - 1\right) + \left(e^{\lambda\zeta} - 1\right) p_0} = 0 \quad (38)$$

where ζ and $\tau_w = \zeta - \tau_r$ depend on q through (27). We have

$$\zeta'(q) = \tau_w'(q) = \frac{e^{-\beta\zeta} - e^{-\beta\tau_r}}{q\beta e^{-\beta\zeta}} = \frac{1 - e^{\beta\tau_w}}{q\beta} < 0.$$

The derivative at the point where (38) takes zero value yields:

$$\begin{aligned}
&\lambda\zeta' e^{\lambda\zeta} p_0 - \delta\left(e^{\lambda\eta}(1-p_0)-1\right)e^{-(\delta-r)\tau_w}\tau_w' - \frac{\delta+\lambda\gamma}{\lambda+\delta-r} p_0 \lambda e^{\lambda\zeta}\zeta' + (\delta-r)e^{-(\delta-r)\tau_w}e^{\lambda\tau_r}p_0\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)\tau_w' \\
&-\alpha\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right)-\alpha q e^{\lambda\zeta}\lambda\zeta' p_0 \\
&=\lambda\zeta' e^{\lambda\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r}-\alpha q\right]+e^{-(\delta-r)(\zeta-\tau_r)}\tau_w'\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)-\delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right] \\
&-\alpha\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right) \\
&=\zeta'(q)e^{-(\delta-r)\zeta}\left[\lambda e^{(\lambda+\delta-r)\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r}-\alpha q\right]+e^{(\delta-r)\tau_r}\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)-\delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right]\right] \\
&-\alpha\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right)
\end{aligned}$$

The second line is clearly negative. Assume that (note that $e^{\lambda\eta}(1-p_0)<1$)

$$\begin{aligned}
& e^{(\delta-r)\tau_r}\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)-\delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right] \\
&+\lambda e^{(\lambda+\delta-r)\eta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r}-\alpha\frac{\kappa_L}{1-e^{-\beta\eta}}\right]>0.
\end{aligned}\tag{39}$$

Then, since $\zeta<\eta$, $q^*<\frac{\kappa_L}{1-e^{-\beta\eta}}$, and $\zeta'(q)<0$, the first line is also negative. As a result, the derivative of (38) is always negative. As a result, when (38) equals zero, it must go down. Combined with differentiability of (38), this result rules out multiple solutions, because if there exist, then there must have one solution with the local slope being nonnegative. Therefore, (38) crosses zero at most once from above, which implies that the bank run equilibrium, if exists, is unique and stable.

□

A.8 Proof of Proposition 5

Proof. Now suppose that $q^*=\frac{\kappa_L}{1-e^{-\beta\eta}}$ which is the upper bound, so that $\tau_r^l=0$. Suppose that

$$(\lambda(1-\gamma)-r)p_0-(1-p_0)\left(e^{\lambda\eta}-1\right)\frac{r(r-k\delta)}{\delta-r}+r\left(1-e^{\lambda\eta}(1-p_0)\right)\geq 0$$

so that run equilibrium occurs. There must be another smallest $\underline{q}$ so that run could occur. The lower bound $\underline{q}$ satisfies $G\left(\tau_r^l\left(\underline{q}\right)\right)=0$, i.e., $\underline{q}$ solves

$$\begin{aligned}
&(\lambda(1-\gamma)-r)\left(\frac{1-\underline{q}}{1-\kappa_L-\underline{q}e^{-\beta\eta}}\right)^{\frac{\lambda}{\beta}}p_0- \\
(1-p_0)\left(e^{\lambda\eta}-1\right)\frac{r(r-k\delta)}{\delta-r}&\left(\frac{1-\underline{q}}{1-\kappa_L-\underline{q}e^{-\beta\eta}}\right)^{-\frac{\delta}{\beta}}+r\left(1-e^{\lambda\eta}(1-p_0)\right) = 0.
\end{aligned}$$

At this $\underline{q}$, $\zeta = \eta$ and $\tau_w = \eta - \frac{1}{\beta}\ln\left(\frac{1-\underline{q}}{1-\kappa_L-\underline{q}e^{-\beta\eta}}\right)$, and we need to check if agents has sufficient incentives to improve signal quality (note that in this case $p(t_i|t_i) = p_0$ because $\zeta = \eta$)

$$p_0 + (1-p_0)\frac{\delta}{\delta - r} - V_I(0|y_M) > \alpha q^*$$

If this is true, then we will have another point $q^* > \underline{q}$ to be the bank run equilibrium. Otherwise, each individual agent would like to lower their information q below $\underline{q}$—but we know lower q cannot trigger bank run, contradiction. Moreover, it implies that at $\underline{q}$, $FOC = MB - MC < 0$. Given the following conditions, FOC has to goes down once it touches zero, so there will be no $q > \underline{q}$ to generate bank run equilibrium. The FOC in (31) is

$$\frac{\left[\begin{array}{c}\left(e^{\lambda\zeta}-1\right)p_0 + (1-p_0)\left(e^{\lambda\eta}-1\right)\frac{\delta}{\delta-r} - \frac{\delta\left(e^{\lambda\eta}(1-p_0)-1\right)}{\delta-r}\left(1-e^{-(\delta-r)\tau_w}\right) \\ -\frac{\delta+\lambda\gamma}{\lambda+\delta-r}e^{\lambda\zeta}p_0 - e^{-(\delta-r)\tau_w}e^{\lambda\tau_r}p_0\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \alpha q^*\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right)\end{array}\right]}{(1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0}$$

we know that that τ_r is independent of q, and

$$\begin{aligned}\zeta &\in \left[\frac{1}{\beta}\ln\frac{1}{1-\kappa_L}, \eta\right] \\ \tau_w &\in [0, \eta - \tau_r]\end{aligned}$$

and

$$\zeta'(q) = \tau_w'(q) = \frac{e^{-\beta\zeta} - e^{-\beta\tau_r}}{q\beta e^{-\beta\zeta}} = \frac{1-e^{\beta\tau_w}}{q\beta} > \frac{1-e^{\beta(\eta-\tau_r)}}{\underline{q}\beta}$$

derivative at FOC=0

$$\begin{aligned}
& \lambda\zeta' e^{\lambda\zeta}p_0 - \delta\left(e^{\lambda\eta}(1-p_0)-1\right)e^{-(\delta-r)\tau_w}\tau_w' - \frac{\delta+\lambda\gamma}{\lambda+\delta-r}p_0\lambda e^{\lambda\zeta}\zeta' + (\delta-r)e^{-(\delta-r)\tau_w}e^{\lambda\tau_r}p_0\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right)\tau_w' \\
& -\alpha\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right) - \alpha q^* e^{\lambda\zeta}\lambda\zeta' p_0 \\
= \ & \lambda\zeta' e^{\lambda\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \alpha q^*\right] + e^{-(\delta-r)\tau_w}\tau_w'\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right] \\
& -\alpha\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right) \\
= \ & \zeta'(q)\left[\lambda e^{\lambda\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \alpha q^*\right] + e^{-(\delta-r)\tau_w}\left[e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \delta\left(e^{\lambda\eta}(1-p_0)-1\right)\right]\right] \\
& -\alpha\left((1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right)
\end{aligned}$$

Assume that (note that $e^{\lambda\eta}(1-p_0) < 1$)

$$e^{\lambda\tau_r}p_0(\delta-r)\left(\frac{(\lambda+\delta)(\lambda(1-\gamma)-r)}{r(\lambda+\delta-r)}\right) - \delta\left(e^{\lambda\eta}(1-p_0)-1\right) > 0$$

Now we put bounds on each term. we have

$$0 > \lambda e^{\lambda\zeta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \alpha q^*\right] > \lambda e^{\lambda\eta}p_0\left[\frac{\lambda(1-\gamma)-r}{\lambda+\delta-r} - \alpha\right]$$

and

$$e^{-(\delta-r)\tau_w}\left[e^{\lambda\tau_d}p_0\left(\delta-r\right)\left(\frac{\left(\lambda+\delta\right)\left(\lambda\left(1-\gamma\right)-r\right)}{r\left(\lambda+\delta-r\right)}\right)-\delta\left(e^{\lambda\eta}\left(1-p_0\right)-1\right)\right]$$
$$> \quad e^{-(\delta-r)(\eta-\tau_d)}\left[e^{\lambda\tau_d}p_0\left(\delta-r\right)\left(\frac{\left(\lambda+\delta\right)\left(\lambda\left(1-\gamma\right)-r\right)}{r\left(\lambda+\delta-r\right)}\right)-\delta\left(e^{\lambda\eta}\left(1-p_0\right)-1\right)\right]$$

and finally the last term is

$$-\alpha\left(\left(1-p_0\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0\right)<-\alpha\left(\left(1-p_0\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\frac{1}{\beta}\ln\frac{1}{1-\kappa_L}}-1\right)p_0\right)$$

therefore the FOC are bounded above by

$$\frac{1-e^{\beta(\eta-\tau_d)}}{\underline{q}\beta}\left\{\lambda e^{\lambda\eta}p_0\left[\frac{\lambda\left(1-\gamma\right)-r}{\lambda+\delta-r}-\alpha\right]+e^{-(\delta-r)(\eta-\tau_d)}\left[\left(\delta-r\right)\frac{\lambda\left(1-\gamma\right)}{r}e^{\lambda\tau_d}p_0-\delta\left(e^{\lambda\eta}\left(1-p_0\right)-1\right)-\frac{\delta+\lambda\gamma}{\lambda+\delta-r}p_0\left(\delta-r\right)e^{\lambda\tau_d}\right]\right\}$$
$$-\alpha\left(\left(1-p_0\right)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\frac{1}{\beta}\ln\frac{1}{1-\kappa_L}}-1\right)p_0\right)$$

if this quantity is negative, then there is only one interior solution.

□

A.9 Proof of Proposition 6

Proof. First, suppose to the contrary that everyone runs at some $t^{SSR}>0$, which must be because the bank has some strictly positive probability of being illiquid at t^{SSR} (otherwise waiting is optimal). But if the bank is indeed potentially illiquid at t^{SSR}, then every agent would like to preempt. It is because conditional on the bank being illiquid, running at t^{SSR} (with others) gives the agent $\kappa_L+\gamma\left(1-\kappa_L\right)<1$, while he will receive his entire deposit if he preempts the run and withdraws at $t^{SSR}-\epsilon$ for some $\epsilon>0$. This argument implies that the only possible synchronized run occurs at $t^{SSR}=0$. However, at $t^{SSR}=0$, it is common knowledge that the bank is liquid almost surely, as the bank starts liquid and only becomes illiquid at some liquidity event $t_0>0$ (recall Section 2.1.3), i.e. with probability 1, at $t^{SSR}=0$ we have and $\kappa=\kappa_H>1$. Thus, even if everyone else withdraws immediately, an individual agent would remain in the bank and this behavior is not an equilibrium.

□

A.10 Proof of Proposition 7

Proof. At the time of hearing the rumor, the value of a dollar in the bank for agents with y_L and y_H signals are respectively $V_I\left(0|y_L\right)=\frac{(1-k)\delta}{\delta-r}$ and $V_I\left(0|y_H\right)=\frac{\delta}{\delta-r}$. In order to calculate the value for agents with the y_M signal, note that with probability 1/2 the original bank is the illiquid one, but the agent can deposit his funds (after the liquidation cost $1-\gamma$) to the liquid one. As a result, the value with y_M signal is

$$
\begin{aligned}
V_I(0|y_M) &= \frac{1}{2}\frac{1}{1-\Pi(0|\kappa_L)}\int_0^{\zeta}\left[\delta e^{-(\delta-r)s}(1-\Pi(s|\kappa_L))+e^{-(\delta-r)s}\pi(s|\kappa_L)\gamma(1-k)\frac{\delta}{\delta-r}\right]ds+\frac{1}{2}\frac{\delta}{\delta-r} \\
&= \frac{1}{2}\int_0^{\zeta}\left[\delta e^{-(\delta-r)s}\left(1-\frac{1-e^{-\lambda s}}{1-e^{-\lambda\zeta}}\right)+e^{-(\delta-r)s}\frac{\lambda e^{-\lambda s}}{1-e^{-\lambda\zeta}}\gamma(1-k)\frac{\delta}{\delta-r}\right]ds+\frac{1}{2}\frac{\delta}{\delta-r} \\
&= \frac{1}{2}\frac{1}{1-e^{-\lambda\zeta}}\int_0^{\zeta}\left[-e^{-\lambda\zeta}\delta e^{-(\delta-r)s}+\delta e^{-(\delta-r+\lambda)s}+\lambda\gamma(1-k)\frac{\delta}{\delta-r}e^{-(\delta-r+\lambda)s}\right]ds+\frac{1}{2}\frac{\delta}{\delta-r} \\
&= \frac{1}{2}\frac{1}{1-e^{-\lambda\zeta}}\left[-e^{-\lambda\zeta}\frac{\delta}{\delta-r}\left(1-e^{-(\delta-r)\zeta}\right)+\frac{\delta}{\delta-r+\lambda}\left(1-e^{-(\delta-r+\lambda)\zeta}\right)+\frac{\lambda\gamma(1-k)}{\delta-r+\lambda}\frac{\delta}{\delta-r}\left(1-e^{-(\delta-r+\lambda)\zeta}\right)\right]+\frac{1}{2}\frac{\delta}{\delta-r} \\
&= \frac{1}{2}\frac{1}{1-e^{-\lambda\zeta}}\left[-e^{-\lambda\zeta}\frac{\delta}{\delta-r}\left(1-e^{-(\delta-r)\zeta}\right)+\frac{\delta}{\delta-r+\lambda}\left(1+\frac{\lambda\gamma(1-k)}{\delta-r}\right)\left(1-e^{-(\delta-r+\lambda)\zeta}\right)\right]+\frac{1}{2}\frac{\delta}{\delta-r}.
\end{aligned}
$$

□

B Appendix B

We consider the non-stationary part of the model here. If $t_0 < \eta$, then some early informed agents with $t_i < \eta$ knows that $t_0 \in [0, t_i]$, and this truncation implies strategy may be t_i-dependent. However, as shown in Abreu and Brunnermeier (2003), those early agents will be bunching together to eliminate the non-stationarity. We modify their results to our setting.

Focus on the bank being illiquid. To be precise, follow Abreu and Brunnermeier (2003) in our model the agents who hears the rumor before $\zeta - \tau_w = \tau_r$ will behave as if the agent who hears the rumor exactly at τ_r. The strategy of the agent who hears rumor at τ_r is that, independent of signal (y_L or y_M) he will withdraw at $\tau_r + \tau_w = \zeta$. Moreover, for agents who hear rumor at $t_i \in [\tau_r, \zeta]$, they take the following strategy. If they receive y_L signal then he will withdraw at ζ, while if they get y_M signal then they withdraw at $t_i + \tau_w$. This additional modification is because relative to Abreu and Brunnermeier (2003) agents may have different signals in our model.

For illustration, suppose that $t_0 = 0$ so the bank should fail at ζ. Recall that there are q measure of y_L signals and $1-q$ measure of y_M signals. Since the information keeps spreading at ζ (recall that $\eta < \zeta$) and all agents hears the rumor before ζ will withdraw at ζ, there are $q\left(1-e^{-\beta\zeta}\right)$ measure of y_L agents withdrawing. On the other hand, y_M agents who hear the rumor in the interval $[0, \tau_r]$ are withdrawing at ζ, with a total mass of $(1-q)\left(1-e^{-\beta\tau_r}\right)$. Therefore, we have

$$
(1-q)\left(1-e^{-\beta\tau_r}\right)+q\left(1-e^{-\beta\zeta}\right)=\kappa_L, \tag{40}
$$

which is exactly (26). A similar argument can be applied to the case of $t_0 > 0$ so that the bank failure time is $t_0 + \zeta$: this is because endogenously there are less agents bunching at the physical time ζ, so the bank failure time is postponed to $t_0 + \zeta > \zeta$.

There is one issue that our richer (than Abreu and Brunnermeier (2003)) setting leads to potential non-stationarity. Although withdraw behavior can be stationary, the endogenous learning about bank liquidity is non-stationary when $\eta < \zeta$. In fact, initially when $t_0 = 0$, agents have no other information so $p(t_i|t_i) = p_0$ must holds. In stationary state, $p(t_i|t_i) = \widehat{p}_0 \equiv \frac{\left(e^{\lambda\zeta}-1\right)p_0}{(1-p_0)\left(e^{\lambda\eta}-1\right)+\left(e^{\lambda\zeta}-1\right)p_0} < p_0$. This difference potentially alters the optimal withdraw strategies for agents with different timings. To resolve this issue, we simply assume that for $t_i < \eta$, the prior is time-varying

$$p_0(t_i) = \frac{\left(e^{\lambda t_i} - 1\right) p_0}{\left[(1 - p_0)\left(e^{\lambda\eta} - 1\right) + p_0\left(e^{\lambda t_i} - 1\right)\right]},$$

and one can show that with this specification, the resulting posterior upon hearing the rumor, $p(t_i|t_i)$, is always $\widehat{p}_0$. One can presumably achieve this by a more structural way; for instance, introduce other shocks so that, conditional on survival and hearing the rumor, the posterior of the bank being illiquid is always $\widehat{p}_0$. Also, we have to fix the signal quality structure the q in (2) is the same as in the stationary phase. We deem these technical issue non-essential for the economic questions that we are after in this paper.