Bailouts, Contagion, and Risk-Taking

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Abstract

We revisit the link between bailouts and bank risk taking. The expectation of government support to failing banks (bailout) creates moral hazard and encourages risk-taking. However, when a bank's success depends on both its idiosyncratic risk and the overall stability of the banking system, a government's commitment to shield banks from contagion may increase their incentives to invest prudently. We explore these issues in a simple model of financial intermediation where a bank's survival depends on another bank's success. We show that the positive effect from systemic insurance dominates the classical moral hazard effect when the risk of contagion is high.

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1 Introduction

In the recent crisis, governments in the U.S. and elsewhere provided massive support to distressed financial institutions (directly through bailouts and indirectly through unprecedented fiscal and monetary expansions). The literature accepts that such support was essential to prevent the failure of multiple institutions that would have had devastating effects on the real economy. However it is also forceful in pointing out that, in the long run, such support carries significant costs in the form of moral hazard. First, when banks expect to be supported in a crisis, they will take more risk, because they shift some negative realizations to the taxpayer. Second, there is a strategic complementarity: banks take greater risk when other banks take greater risk, and choose correlated risks, because support is more likely when there are multiple failures. Then, the expectation of support increases the probability of the kind of systemic crisis that government wants to avoid in the first place (Acharya and Yorulmazer, 2007, Diamond and Rajan, 2009, Farhi and Tirole, 2011).

This time inconsistency is a defining theme for policy design. It suggests that governments should create conditions where banks believe that a bailout might not be forthcoming. Crisis management resembles a game of chicken: government refutes a notion of intervention until the very last moment, when the financial system is truly on the brink, and then changes its course. The threat of doom can be made credible by increasing bureaucratic or political obstacles to intervention (such as those entailed by some provisions of the Dodd-Frank Act). Like the game of chicken, the strategy is risky (support may not come in time), but the risk is compensated by the fact that it prevents a further distortion of ex-ante incentives.

In this paper, we revisit the link between bailouts and bank risk-taking. We argue that the commonly held view that government support in a financial crisis necessarily worsens banks' ex-ante incentives is not always correct. The reason is that government support, while exacerbating one inefficiency (moral hazard stemming from the implicit 'put' to the government), ameliorates another (the risk externality associated with contagion). The opposing effect of bailout rents and systemic risk insurance on banks' risk-taking incentives is the main focus of this paper.

The risk of contagion is one of the reasons that makes the banking system special. While a car company going bankrupt is an opportunity for its competitors, a bank going bankrupt is a potential threat to the industry, especially when the failing bank is large. Banks are exposed to each other directly through the interbank market, and indirectly through the real economy, reputation channels, and financial markets. While banks may control direct exposures, the indirect links are largely beyond an individual bank's control. The risk that a bank fails simply because other banks have failed is especially sharp in times of economic or financial fragility, or in particularly interconnected systems.

The threat of contagion affects bank incentives. The key mechanism is that when a bank can fail due to exogenous circumstances, it does not invest as much in ensuring safe future outcomes. Indeed, would you watch your cholesterol intake while eating on a plane that is likely to crash? Or save money for retirement when living in a war zone? Formally, we show that the threat of contagion can result in insufficient monitoring of loans, leading to an increase in bank risk. Moreover, making the threat of contagion endogenous to the monitoring choices of all interconnected banks generates a strategic complementarity: banks take more risk when other banks take more risk, because risk-taking of other banks increases the threat of contagion.

Then the government provision of systemic insurance (the promise of intervention to stem the systemic effects of bank failure) has two effects. The first is the classic moral hazard effect described in much of the literature, which amplifies risk-taking. The second is an insurance effect that increases banks' incentives to monitor loans (this is similar to the effect identified for macro shocks by Cordella and Levy-Yeyati, 2003). This reduces risk-taking through both direct (lower threat of contagion because of government intervention) and indirect (lower risk-taking of other banks) channels. Under certain conditions, the insurance effect prevails and the promise of government support reduces risk-taking of all banks despite the fact that the government may have to leave bailout rents to failing and potentially imprudent ones.

Formally, we develop a model of financial intermediation where banks use capital and raise deposits (or debt) fund a portfolio of risky loans. The bank portfolio is subject to two sources of risk. The first is idiosyncratic and under the control of the bank. Think about this risk as dependent on the quality of a bank's borrowers, which the bank can control through costly monitoring or screening. The second source of risk is contagion. Think about this as a form of macro risk. When a bank of systemic importance fails, it has negative effects on the real economy, possibly triggering a recession. A deep enough recession can lead even the best borrowers into trouble and, as a consequence, can cause the failure of another, otherwise healthy bank independently of the quality of its portfolio. This second source of risk is exogenous to the affected bank (it cannot be managed or diversified), but is endogenous to the financial system as a whole, since it depends on risk-taking of all banks.

These two sources of risk are associated with two inefficiencies. First, banks are protected by limited liability investors cannot price risk at the margin. As a result, in equilibrium banks will tend to take excessive idiosyncratic risk. As in other models, this problem can be ameliorated through the imposition of capital requirements. The second inefficiency stems from the fact that individual banks do not take into account the effect of their risk taking on other banks. As with limited liability, this also leads to a level of risk that is too high relative to the coordinated solution. However, capital requirements are not effective in correcting this externality. Even a bank fully funded by capital will take excessive risk because it does not internalize the systemic effects of own risk.

Against this background, government intervention in support of banks has two effects on incentives. It exacerbates the moral hazard stemming from limited liability but reduces the externality associated with contagion. The latter effect dominates when the probability of contagion is high, while the rents that the government leaves to bailed out banks are low. In that case, government intervention leads to lower bank risk-taking and better ex-ante outcomes. (We do not focus on ex-post optimality, but that would depend on the difference between the economic costs of bank bankruptcy and that of the use of public funds.)

In extensions, we show that similar effects occur in settings where banks can choose correlated risks and where bank risk-taking takes the form of short-termism (such as volume-oriented lending or short-term wholesale funding) rather than insufficient monitoring.

Our paper is related to two strands of literature: on the moral hazard consequences of government intervention, and on interbank contagion. A number of recent papers emphasized the fact that bailouts create incentives for banks to excessively take and correlate risks (Acharya and Yorulmazer, 2007 and 2008A, Diamond and Rajan, 2009, Farhi and Tirole, 2011). We introduce into this framework a risk externality in the form of contagion, treating such risk as inherent to banking and never fully diversifiable (Allen and Gale, 2001, Acharya and Yorulmazer, 2007, Bebchuk and Goldstein, 2011). In such conditions, government intervention may be essential in preventing contagion (Freixas et al., 2000, Diamond and Rajan, 2005). Removing exogenous risks can improve ex-ante incentives (Holmstrom, 1979). This argument was first applied to banks by Cordella and Levy-Yeyati (2001), in the context of macroeconomic shocks. Our model relates shocks to contagion and makes them endogenous to the banking system, offering a link to risk spillovers and correlated risks.

While we focus on the risks of contagion stemming from another bank's distress, other papers offer a complementary view: that absent contagion surviving banks may benefit by buying assets liquidated by a troubled bank at firesale prices, possibly with some government support (Perotti and Suarez, 2002, Acharya and Yorulmazer, 2008A).

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 extends the model to the case of correlated risks. Section 4 concludes and discusses policy implications.

2 A Simple Model of Contagion and Risk-Taking

Consider a market with two identical risk-neutral and profit-maximizing banks. Banks are protected by limited liability and repay depositors only when successful. If they fail,

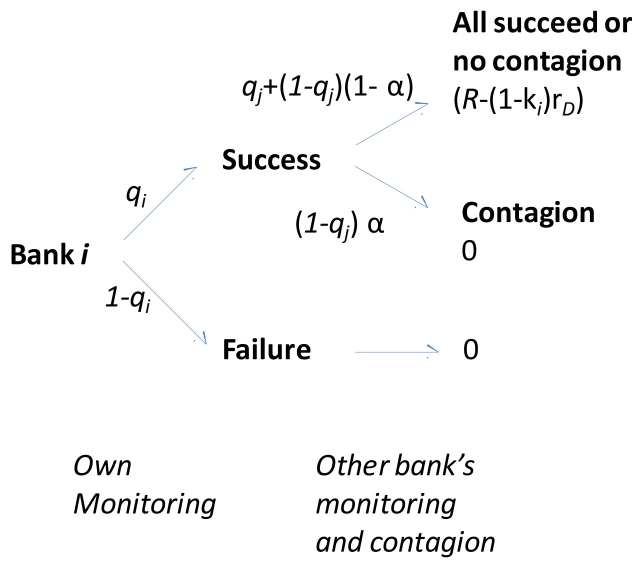

Figure 1: Bank effort choices, contagion, and payoffs.

bank owners lose the invested capital. Each bank i has a loan portfolio of size 1. The portfolio is financed by equity, k_i, and deposits (or debt), $1 - k_i$. The gross interest rate on deposits, r_D, is not risk-sensitive thanks to deposit insurance.

Loan portfolios are risky: they return R when successful and zero otherwise. There are two sources of risk. First, a portfolio can be monitored, at a cost, to reduce its endogenous risk. Each bank i can choose its monitoring effort q_i which also represents the probability of portfolio success, conditional on not being affected by contagion. This monitoring effort entails a quadratic cost $\frac{1}{2}cq_i^2$.

In addition – and this is the key feature of the model – each bank's portfolio is exposed to the risk of contagion. We assume that when a bank fails, there is a probability α that (absent government intervention) the other bank's portfolio will also become non performing, independently of its level of monitoring. Put differently, we assume that this indirect bank exposure cannot be managed or diversified.

Banks choose the monitoring effort simultaneously and cannot observe each other's choice. The game tree capturing bank effort choices and contagion is shown on Figure 1.

2.1 Contagion and Bank Risk-Taking

We start by deriving the Nash equilibrium of bank monitoring choices and showing that the risk of contagion reduces monitoring.

The expected profit of bank i can be written as:

$$E(\Pi_i) = q_i\left((1-\alpha)+\alpha q_j\right)\left(R-(1-k_i)\,r_D\right)-\frac{c}{2}q_i^2 \tag{1}$$

From the first order conditions of (1) with respect to q_i, we obtain the reaction function:

$$\widehat{q}_i\left(q_j\right) = \min\{\frac{(1-\alpha)+\alpha q_j}{c}\left(R-(1-k_i)\,r_D\right),1\} \tag{2}$$

And, imposing symmetry on (2), we obtain the Nash equilibrium:

$$\widehat{q} = \min\{\frac{(1-\alpha)\left(R-r_D\,(1-k)\right)}{c-\alpha\left(R-r_D\,(1-k)\right)},1\}.$$

Lemma 1 *When $\widehat{q}$ admits an internal solution, the externality associated with contagion reduces equilibrium monitoring effort: $d\widehat{q}/d\alpha < 0$.*

In particular $\widehat{q} = 0$ for $\alpha = 1$ (full contagion), and $\widehat{q} = \frac{1}{c}\left(R-r_D\,(1-k)\right)$ for $\alpha = 0$ (no contagion). The intuition is that the risk of contagion reduces the payoff to monitoring: relative to the no-contagion case, for a given monitoring effort, the probability that a bank i receives $\left(R-r_D\,(1-k)\right)$ is reduced by $\alpha\,(1-q_j)$. The bank, then, adjusts its monitoring effort to equalize its marginal cost to this lower marginal revenue.

The risk of contagion is exogenous to the bank but endogenous to the financial system. A bank does not internalize the positive effect that its monitoring has on another bank's expected profits. As a result, in equilibrium, banks choose an inefficiently low level of monitoring. In addition, due to contagion, each bank's payoff to own monitoring is lower (because the payoff in the form of more certain future profits is lost when another bank fails due to contagion). That prompts banks to reduce monitoring effort even further, leading to an even higher risk of failure and contagion.

2.2 Risk Externality and Bank Capital

To formally assess the externality, compare the banks' risk choices to a benchmark when the two banks are jointly owned. Then their optimal effort q would be obtained by maximizing the joint profit (similar to (1)):

$$E(\Pi_i) = 2\left[q\left((1-\alpha)+\alpha q\right)\left(R-(1-k)\,r_D\right)-\frac{c}{2}q^2\right]$$

giving:

$$\tilde{q} = \begin{cases} \min\{\frac{(1-\alpha)(R-r_D(1-k))}{c-2\alpha(R-r_D(1-k))}, 1\}, \text{if } \alpha < \frac{c}{2(R-r_D(1-k))} \\ 1, \text{ otherwise} \end{cases}$$

Note immediately that $\tilde{q} > \widehat{q}$. Put differently, the risk of contagion leads to a market equilibrium that entails a level of monitoring that is lower than what would be socially optimal.

In standard models, capital requirements can help discourage excessive risk taking. Higher bank capital reduces the moral hazard associated with limited liability. As shareholders' skin in the game increases, they internalize a larger share of bank risk. This is also true in this model. Indeed:

$$d\hat{q}/dk > 0 \text{ and } d\tilde{q}/dk > 0 \tag{3}$$

And, trivially, in the absence of contagion risk, a bank fully funded with equity will choose a level of monitoring consistent with the social optimum $q^* = \frac{R}{c}$.

However, capital requirements are not effective in dealing with excess risk-taking driven by the threat of contagion. The reason is that this stems from externalities that affect other banks, not the bank's own creditors. In an interconnected banking system, even a bank funded entirely with equity will take more risk than what is socially optimal. More formally, for α such that the model admits an internal solution, we have

$$\widehat{q}|_{k=1} = \frac{(1-\alpha)\,R}{c-\alpha R} < \frac{(1-\alpha)\,R}{c-2\alpha R} = \tilde{q}|_{k=1}.$$

2.3 Bailouts and Bank Risk-Taking

Now consider the case where the government can intervene to support a failing bank since its failure threatens the system's stability. Consistent with existing literature, we assume that any form of intervention will leave some rents to the failing bank (perfectly targeted bailouts would reinforce our results). What we have in mind is a situation where in response to the threat of bank failure a government engages in expansionary fiscal or monetary policy, assets purchases, or direct support. All these activities are expected to reduce the probability of default and contagion.

Formally, assume that, with some probability θ the government successfully intervenes when a bank is about to fail. Assume that θ is known in advance. The fact that θ can be less than 1 captures the notion that a government can specify the likelihood of intervention. It could do so by altering the institutional structure that makes it not certain that the intervention will be undertaken in time or that, even in the case of intervention, failure and contagion can be avoided. A successful intervention fully prevents banks from contagion. It also partially shields a bank that fails because of idiosyncratic risk: the "failing bank" gets to keep a fraction $0 < \delta \leq 1$ of its profits. The parameter δ can be thought of as bailout rents that accrue to a "saved" institution. We say that a bailout with a lower δ is "more targeted". The game tree with government intervention is shown in Figure 2.

We can re-write (1) to:

$$E(\Pi_i) = (q_i\left((1-\alpha) + \alpha\left(q_j + (1-q_j)\,\theta\right)\right) + (1-q_i)\,\theta\delta)\left(R - (1-k)\,r_D\right) - \frac{c}{2}q_i^2 \quad (4)$$

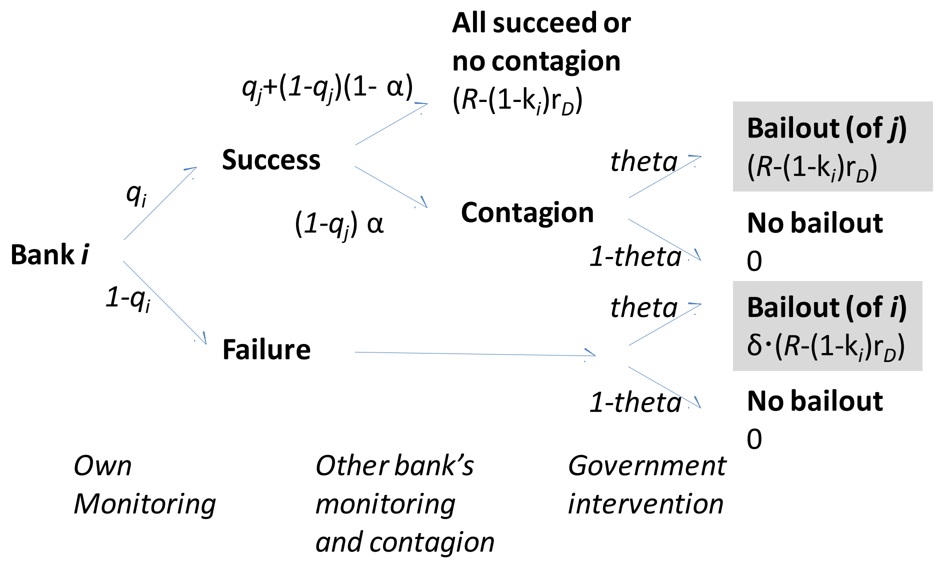

Figure 2: The effects of government intervention.

From the first order conditions of (4) with respect to q_i, we obtain the reaction function:

$$\widehat{q_i} = \frac{((1-\alpha) + \alpha (q_j + (1-q_j)\theta))(R-(1-k)r_D)}{c} - \frac{\theta\delta(R-(1-k)r_D)}{c} \tag{5}$$

From (5) it is immediate that

$$\frac{\partial \widehat{q_i}}{\partial \theta} = \frac{\alpha(1-q_j)\theta(R-(1-k)r_D)}{c} - \frac{\delta(R-(1-k)r_D)}{c},$$

where the first term represents the positive effect of systemic insurance (which is stronger the riskier the other bank and the greater the threat of contagion) and the second the classical moral hazard effect (which is stronger the larger the rents left to a failing bank in case of intervention). Imposing symmetry on (5), we obtain the Nash equilibrium (again assuming the parameters are such that an internal solution exists):

$$\widehat{q}(\theta) = \frac{(1-\theta\delta-\alpha(1-\theta))(R-(1-k)r_D)}{c-\alpha(1-\theta)(R-(1-k)r_D)} \tag{6}$$

We can now state the following proposition:

Proposition 1 *For $\delta < 1$, there exists $\alpha^* = \frac{c\delta}{c-(R-(1-k)r_D)(1-\delta)}$, $\alpha^* < 1$, such that for*

$\alpha > \alpha^*$, equilibrium monitoring increases with the probability of government intervention: $\frac{d\widehat{q}(\theta)}{d\theta} > 0$, and for $\alpha < \alpha^*$, the opposite occurs: $\frac{d\widehat{q}(\theta)}{d\theta} < 0$.

Proposition 1 is the key result of our paper. It shows that in states of the world where the probability of contagious is high, it may be optimal for the government to insure banks against contagion. A higher probability of a bailout increases moral hazard. But, at the same time, it corrects for the externality stemming from the threat of contagion. When this threat is high, while the rents left to a failing bank are not "too high", the second effects prevails.

3 Bailouts under Correlated Risk

Another key type of bank moral hazard is correlated risk-taking. Banks correlate risk to protect against contagion (Acharya and Yorulmazer, 2008B) and to increase the probability of a bailout in case of failure (Acharya and Yorulmazer, 2008A, and Farhi and Tirole, 2011). Correlated risk-taking is costly because it leads to a misallocation of resources (when banks lend to similar industries and forego other investment opportunities) and may increases systemic risk associated with joint failures of banks.

In the base model of Section 3, banks were choosing the risk of uncorrelated projects. We introduce correlation in steps. First, we describe bank risk choice when projects are exogenously correlated. We find that, under correlation, the risk of contagion and hence the positive "risk insurance" effect of bailouts are smaller. This means that, to be ex-ante beneficial, bailouts need to be more targeted, leaving smaller rents to banks that fail owing to their own idiosyncratic risk.

Second, we endogenize bank correlation. We show that banks have inherent incentives to correlate their projects due to the risk of contagion (as in Acharya and Yorulmazer, 2008B). Government interventions that provide systemic risk insurance can reduce bank incentives to correlate risk and thus improve ex-ante welfare.

3.1 Exogenously Correlated Risk

In this section, we explore how our results change when individual bank risks are exogenously correlated (later we examine banks' incentives to choose correlated risks).

Recall that, in the base model, bank risks were not correlated:

$$p\left(s_i = 1 | s_j = 1\right) = p\left(s_i = 1\right) = q_i,$$

where s_i is an indicator variable that takes value 1 when bank i is successful and 0 otherwise. Once we introduce correlation, however, banks will tend to succeed/fail together. More formally, $p\left(s_i = 1 | s_j = 1\right) > p\left(s_i = 1\right)$.

Consider, first, the case of the maximum correlation. Without loss of generality assume $q_i \leq q_j$, then for given q_i, q_j, the projects succeed/fail most often together when:

$$\begin{aligned} p\left(s_i = 1 | s_j = 1\right) &= \frac{q_i}{q_j} \\ p\left(s_j = 1 | s_i = 1\right) &= 1 \end{aligned}$$

That is, whenever a bank with a lower q succeeds, the bank with a higher q succeeds too. All the mass of success realizations of the bank with a lower q overlaps with the mass of success realizations of the banks with a higher q. Note that, similarly, when the bank with a higher q fails, the the bank with a lower q fails too.

When comparing across economies with different correlations, it is useful to write the conditional probabilities of success as a weighted average of the full correlation and zero correlation conditional probabilities. Specifically, when projects have success probabilities q_i and q_j ($q_i \leq q_j$):

$$\begin{aligned} p\left(s_i = 1 | s_j = 1\right) &= \left(1 - \xi\right) q_i + \xi \frac{q_i}{q_j} \\ p\left(s_j = 1 | s_i = 1\right) &= \left(1 - \xi\right) q_j + \xi \end{aligned}$$

where ξ is an exogenous parameter. Note that for given q_i and q_j, their correlation is a linear function of ξ, and it is identical to ξ for $q_i = q_j$. Since we limit our analysis to the to the case of banks with symmetric monitoring costs, which results in symmetric equilibria, in what follows we will refer liberally to ξ as "correlation".[1]

Consider now how non-zero correlation $\xi > 0$ affects the result of Proposition 1.

The profit of bank i can be written as

$$E(\Pi_i) = \left(q_i\left((1-\alpha) + \alpha\left(\tilde{q}_j + (1-\tilde{q}_j)\,\theta\right)\right) + (1-q_i)\,\theta\delta\right)A - \frac{c}{2}q_i^2 \tag{7}$$

where $\tilde{q}_j$ is the probability of bank j success conditional on bank i success:

$$\tilde{q}_j = (1-\xi)\,q_j + \xi \min\left\{\frac{q_j}{q_i}; 1\right\}$$

and $A = (R - (1-k)\,r_D)$ is the interest rate margin.

We solve for the optimal response to (7) in Appendix. Imposing symmetry on the optimal response gives Nash equilibria.

Lemma 2 *Bank risk choices under correlated risks have a continuum of Nash equilibria, given by* $q_i = q_j$ *where* $q^* \leq q_{i,j} \leq q^{**}$ *and*

$$\begin{aligned} q^* &= \frac{1-\alpha+\theta\alpha-\theta\delta}{c-\alpha(1-\theta)(1-\xi)A}A \\ q^{**} &= \frac{1-\alpha+\alpha(\theta+\xi(1-\theta))-\theta\delta}{c-\alpha(1-\theta)(1-\xi)A}A \end{aligned}$$

Among the equilibria, q^{**} *maximizes each bank's profit and can be achieved through cheap talk.*

We can now establish how exogenous correlation of projects affects bank risk choices and the effect of government intervention.

[1] More precisely, we have

$$corr(s_i, s_j) = \frac{(1-\xi)\,q_iq_j + \xi q_i - q_iq_j}{\sqrt{q_i(1-q_i)}\sqrt{q_j(1-q_j)}}.$$

Observe that:

$$\frac{dq^{**}}{d\xi}=\frac{\left(c-A(1-\theta\delta)\right)A\alpha\left(1-\theta\right)}{\left(c-A\alpha+A\theta\alpha+A\alpha\xi-A\theta\alpha\xi\right)^{2}}>0$$

Therefore, higher correlation increases effort. The intuition is that correlation reduces the risk externality, so each bank is less exposed to contagion. Note that $q^{**}=\widehat{q}$ (6) for $\xi=0$.

Now consider the effects of government intervention.

Proposition 2 *For $\delta<\frac{(1-\xi)(c-A)}{c-A(1-\xi)}$, there exists $\alpha^{**}=\frac{\delta c}{(c-A(1-\delta))(1-\xi)}$ such that for $\alpha>\alpha^{**}$ equilibrium monitoring increases with the probability of government intervention: $\frac{dq^{**}(\theta)}{d\theta}>0$; for $\alpha<\alpha^{**}$ the opposite occurs. Note that a^{**} increases in ξ; and that government intervention is never optimal under full correlation $\xi=1$.*

3.2 Endogenously Correlated Risk

Next, we turn to the analysis of banks' incentives to correlate their default risks under the threat of contagion. At this stage we speculate that banks will choose maximum correlation if that comes at no cost. This is similar to Farhi and Tirole (2011) but happens for entirely different reasons: a bank wants to succeed when the competitor is more likely to succeed to minimize the risk of contagion (Acharya and Yorulmazer, 2008B). When correlating bank risk comes at a private cost (this could be because of profitable opportunities that remain unfunded if banks lend to the same sectors), banks will choose less than perfect correlation. Then, a policy of ex ante commitment to bailouts will reduce banks' incentives to correlate risks, and for certain parameters will also lead to an increase in monitoring effort.

4 Conclusion

This paper revisits the link between bailouts and bank risk-taking. It is accepted that bailouts have a moral hazard effect that encourages risk-taking. However, we also show that, when there are risk externalities across banks, this effect coexists with an opposite one: bailouts protect prudent banks against contagion. This encourages monitoring effort and reduces risk-taking. A government's commitment to save systemic banks when the threat of contagion is high can reduce risk-taking by all banks, even if a bailout leaves banks with some rents.

While our model is set as a general model of bank effort choice, one could rewrite it for the specific context of banks' short-term behavior. Indeed, observe that insufficient ex-ante monitoring can be interpreted as a variety of business practices that generate short-term return at expense of higher long-term risk. Many practices that banks engaged in the run-up to the recent crisis – fee- and volume-based banking, lending with teaser rates, or the use of cheaper but unstable short-term funding are all associated with higher short-term returns but higher long-term risk. Our analysis explains that have intrinsic incentives to engage in such behavior when they operate in an interconnected financial system, especially in periods when the risk of contagion is high and when other banks are also engaging in similar behavior.

Overall, our results caution that the recent policy initiatives that create impediments to timely and targeted intervention may in effect destabilize financial system by creating an expectation of more macro, less targeted bailouts (bail outs that leave grater rents to failing banks). The results suggest that a more promising policy direction might be to focus on the efficiency of interventions: create environment (legal and implementation) where intervention is designed to leave the stakeholders of distressed systemically important banks as little rents as possible.

A Proofs

A.1 Optimal response under (7)

Consider (7). There are two cases. When $q_i \geq q_j$, (7) takes form:

$$E^{*}(\Pi_i) = \left(q_i \left((1-\alpha) + \alpha \left(\left((1-\xi)\, q_j + \xi \frac{q_j}{q_i}\right) + \left(1 - \left((1-\xi)\, q_j + \xi \frac{q_j}{q_i}\right)\right)\theta\right)\right) + (1-q_i)\,\theta\delta\right) A - \frac{c}{2} q_i^2$$

Which, by using FOC, is maximized on $q_i \geq q_j$ in:

$$\begin{aligned} q_i &= q_i^{*}(q_j) \\ &= \frac{(1-\alpha) + \alpha\left((1-\xi)\left(q_j + (1-q_j)\theta\right) + \xi\theta\right) - \theta\delta}{c} A \end{aligned}$$

for

$$q_j \leq q_j^{*} = \frac{1 - \alpha + \theta\alpha - \theta\delta}{c - \alpha\,(1-\theta)\,(1-\xi)\,A} A$$

and in $q_i = q_j$ for $q_j \geq q_j^{*}$.

When $q_i \leq q_j$, (7) takes form:

$$E^{**}(\Pi_i) = \left(q_i \left((1-\alpha) + \alpha \left(\left((1-\xi)\, q_j + \xi\right) + \left(1 - \left((1-\xi)\, q_j + \xi\right)\right)\theta\right)\right) + (1-q_i)\,\theta\delta\right) A - \frac{c}{2} q_i^2 \tag{8}$$

Which, by using FOC, is maximized on $q_i \leq q_j$ in:

$$\begin{aligned} q_i &= q_i^{**}(q_j) \\ &= \frac{(1-\alpha) + \alpha\left((1-\xi)\left(q_j + (1-q_j)\,\theta\right) + \xi\right) - \theta\delta}{c} A \end{aligned}$$

for

$$q_j \geq q_j^{**} = \frac{1 - \alpha + \alpha\left(\theta + \xi\,(1-\theta)\right) - \theta\delta}{c - \alpha\,(1-\theta)\,(1-\xi)\,A} A$$

and in $q_i = q_j$ for $q_j \leq q_j^{**}$

Note that $q_j^* \leq q_j^{**}$ and $q_j^* = q_j^{**}$ for $\xi = 0$. Overall, the optimal response is:

$$\begin{aligned} q_i &= q_i^*(q_j) \text{ for } q_j \leq q_j^* \\ q_i &= q_j \text{ for } q_j^* \leq q_j \leq q_j^{**} \\ q_i &= q_i^{**}(q_j) \text{ for } q_j \geq q_j^{**} \end{aligned}$$

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