Optimal Unemployment Insurance in a Directed Search Model*

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Abstract

Over the last few decades, several economies have experienced a reduction in the volatility of output and a rise in wage dispersion. This paper investigates the appropriate labor market policy response to these changes. We introduce unemployment benefits financed by a proportional payroll tax within a model of directed search on the job. We show that there exists a unique positive level of unemployment benefit which maximizes *ex ante* welfare of individuals. The optimal unemployment benefit level is hump-shaped as a function of the level of idiosyncratic risk. At empirically relevant levels of idiosyncratic risk, a much less generous system than in the economy without uncertainty emerges. Furthermore, the welfare costs of deviating from the optimal level are substantial, and accompanied by high unemployment rates. We also find that while the optimal generosity of the unemployment insurance system declines monotonically with the amount of aggregate risk in the economy, the welfare costs of deviating from the optimal system are rather small.

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1 Introduction

Several economies have experienced fundamental changes over the last few decades. For example, Stock and Watson (2005) document a reduction in the volatility of output growth and a moderation of business cycle fluctuation in most G7 economies over the two decades that preceded the last recession. By contrast, Heathcote et al. (2010) document an increase in wage dispersion in the U.S. over the past three decades. Other papers in the same issue of the *Review of Economic Dynamics* document similar facts for other countries. This paper investigates the impact of these fundamental changes on labor market outcomes as well as the appropriate policy response to these changes.

We study these questions within a directed search model of the labor market with on the job search. Specifically, we introduce a simple government-run unemployment insurance system—unemployment benefit financed by a proportional payroll tax—within a model of directed search along the lines of Menzio and Shi (2008, 2010). This framework is particularly well suited for our purpose, for two main reasons. First, this framework has been shown to generate several important features of the labor market.[1] Second, the model lands itself well to the introduction of both idiosyncratic and aggregate uncertainty, which are at the heart of this paper. While it is possible to introduce both types of uncertainty in a random matching model (see for example Moscarini and Postel-Vinay (2010) or Krusell et al. (2010)), directed search models are much more tractable.[2]

We begin our analysis by studying an environment which abstracts from idiosyncratic and aggregate uncertainty. In this context, we prove existence of an optimal unemployment insurance system which provides positive benefits to unemployed in-

[1] For example, these models have been shown to generate the type of residual wage inequality observed in the data, as well as positive returns to tenure and experience, appropriate worker flows between employment, unemployment, and across employers, and so on (see Delacroix and Shi (2006), Menzio and Shi (2008), Shi (2009) and Gonzalez and Shi (2010)).

[2] Technically, this is because equilibria are 'block recursive,' so that individuals' and firms' decisions do not depend directly on the distribution of workers. See Shi (2009) and Menzio and Shi (2008, 2010) for details.

dividuals without providing full insurance. The intuition for this result is simple: while an increase in unemployment benefits allows (risk-averse) individuals to better smooth consumption over time, it also raises the unemployment rate and thus lowers average consumption. The rise in the unemployment rate arises as individuals look for vacancies with higher wages, for which job finding probabilities are lower.

Next we introduce idiosyncratic uncertainty and again find a unique optimal level of unemployment benefit. Interestingly, the optimal unemployment benefit is hump-shaped as a function of the level of risk that individuals face, keeping the mean level of the shock constant at zero. Initially, an increase in idiosyncratic risk calls for a more generous unemployment insurance system in order to provide better insurance for individuals. However, as the level of risk increases further, generous unemployment benefits become unsustainable. This is because matches for workers who experience low productivity shocks are endogenously destroyed, shrinking the pool of employed workers who pay for the benefit. As a result, at high levels of idiosyncratic risk, the optimal level of unemployment benefit falls sharply as risk rises. Indeed, at empirically relevant levels of idiosyncratic risk, a much less generous system than in the economy without such risk emerges. Furthermore, the welfare costs of deviating from the optimal level are substantial, mainly because generous unemployment benefits generate high unemployment rates. These findings offer a similar interpretation of the 'European unemployment dilemma' to that of Ljungqvist and Sargent (1998): when income risk rises, generous welfare states become costly as they increase unemployment duration and the unemployment rate.

Lastly we add aggregate uncertainty to the model. To better understand the impact of aggregate uncertainty, we abstract from idiosyncratic uncertainty. In this case, the optimal generosity of the unemployment insurance system declines monotonically with the amount of aggregate risk, again keeping the average level of productivity constant. Intuitively, with a fixed benefit, unemployment becomes relatively more attractive in bad times than it becomes less attractive in good times. As a result, the unemployment rate increases, and so the pool of employed workers, who pay for the benefit, shrinks. A lower level of unemployment insurance restores the balance between consumption smoothing and the incentive to search for vacancies with relatively high job finding probabilities. For similar reasons, a pro-cyclical benefit is more desirable than a counter-cyclical (or acyclincal) one. It should be pointed out, however, that in the relevant range of aggregate risk, the welfare costs of deviating from the optimal system are rather small.

The economic mechanism at the center of our analysis is that higher unemployment benefits induce individuals to look for jobs with higher wages, which, because these jobs are harder to find, increases unemployment duration. As such, our work is related to Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), who study optimal unemployment insurance in the context of a model where individuals' search effort is private information.[3] Instead of searching for better jobs with a constant search effort, individuals in their environment search for the same job with less intensity when unemployment benefits increase. The idea is nevertheless that optimal unemployment insurance strikes the right balance between risk sharing and unemployment duration.[4]

Unemployment insurance has also been studied in search and matching models of the labor market. In a directed search environment, Acemoglu and Shimer (1999) also find a unique utility maximizing level of unemployment benefit financed by a lump-sum tax. In addition, they prove that a similar result holds for an output-maximizing level of unemployment. While the mechanism for their first result is similar to ours, their latter result is due to the way in which they model the cost of vacancies. They interpret this cost as capital, which firms choose and put in place before the vacancy is filled. Since higher unemployment benefits induce individuals to search for jobs associated with higher wages, which take longer to find, firms have an easier time filling their vacancies. The capital put in place prior to filling

[3] In the context of our model, one can think of the market (or wage) to which individuals direct their search as private information, creating a moral hazard problem which prevents a private insurer from conditioning benefits on the search strategy that individuals choose.

[4] To better relate our work to this literature, we are currently investigating an environment in which unemployment benefits comprise two parts: a fixed amount, which can be thought of as a welfare payment, and an unemployment benefit which expires with some probability at any time during an individual's unemployment spell.

the vacancy thus stays idle for a shorter period of time, inducing firms to put more capital in place.

More recently, Krusell et al. (2010) study unemployment insurance in a Bewley-Huggett-Aiyagari type model in which the labor market functions as in Diamond-Mortensen-Pissarides. One of their main findings is that unemployment insurance provides little value over and above self-insurance. The optimal replacement ratio in their benchmark economy is lower then the 40 percent they use as a proxi for the U.S. economy. Needless to say, the generosity of the optimal unemployment insurance system would clearly decline if individuals could self-insure in our model as well. Krusell et al. (2010) also speculate that the gains from running a cyclical unemployment benefit scheme are rather small, which is confirmed by our model. This is in contrast to Landais et al. (2010), who find large gains from countercyclical benefits when recessions are characterized by 'job rationing' that stems from real wage rigidities. In this context, increasing benefits in recessions improves consumption smoothing without adversely increasing unemployment duration.

Costain and Reiter (2008) document that while unemployment displays large cyclical fluctuations relative to labor productivity, the response of unemployment to labor market policies (unemployment benefit) is rather small. They argue that the standard DMP model cannot reproduce these 2 facts.[5] While we do not focus on business cycle properties *per se*, the model can generate movements in the unemployment rate relative to movements in labor productivity similar to the data. Moreover, the elasticity of the unemployment rate with respect to changes in the replacement ratio is in the range of empirical estimates reported by Costain and Reiter (2008).

The rest of the paper is organized as follows. The next section presents the economic environment: we introduce a simple unemployment insurance system in a directed search model of on the job search. In section 3, we characterize the optimal unemployment system in an economy which abstracts from (aggregate and idiosyncratic) uncertainty. In section 4 we present numerical results, successively

[5] We suspect that their critique also applies to Krusell et al. (2010), who need to rely on the Hagedorn and Manovskii (2008) calibration in order to generate plausible unemployment volatility.

studying the impact of introducing idiosyncratic then aggregate uncertainty. In section 5, we show that our results are robust to an alternative formulation of the unemployment insurance system where unemployment benefits are a function of past employment experience, i.e. has a replacement ratio element. We also study whether unemployment benefits should be extended for longer periods of time in recessions, a policy that was implemented in the U.S. during the last recession. A short conclusion is offered in section 6.

2 Economic Environment

The economy is populated by a continuum of risk-averse workers with measure one and a continuum of firms with positive measure.[6] Workers maximize the expected sum of per-period utilities discounted at factor $\beta \in (0, 1)$, with a periodical utility function $\nu(c)$ which is strictly increasing, strictly concave, continuously differentiable, and satisfies Inada conditions. Employed workers' consumption is equal to after-tax labor income, and unemployed workers' consumption equals unemployment benefit paid by the government.

Each firm produces output through a constant return to scale production function, $(y + z)n$, with $n \in \{0, 1\}$. When a firm is matched with a worker, its output depends on aggregate and idiosyncratic productivities. Aggregate productivity y is common across all firms, and its values lie in the set $y \in Y = \{y_1, y_2, ..., y_{N_Y}\}$, where $\underline{y} \equiv y_1 \leq y_2 \leq ... \leq y_{N_Y} \equiv \bar{y}$ and $N_Y \geq 1$ is an integer. The idiosyncratic productivity is $z \in Z = \{z_1, z_2, ..., z_{N_Z}\}$, where $\underline{z} \equiv z_1 \leq z_2 \leq ... \leq z_{N_Z} \equiv \bar{z}$ and $N_Z \geq 1$ is an integer. The firm maximizes the expected sum of profits discounted at factor β.

The labor market is organized as a continuum of submarkets $x \in X = [\underline{x}\,, \overline{x}]$, where $\underline{x} = \nu(b)/(1 - \beta)$ and $\overline{x} = \nu(\overline{y} + \overline{z})/(1 - \beta)$. If a firm meets a worker in submarket x, the firm offers the worker expected lifetime utility x, and we refer to it as the value of that submarket. The probability of finding a job and the probability

[6] Free-entry determines the measure of firms in the economy endogenously.

of filling a vacancy in a submarket depend on the tightness ratio in that submarket. The tightness ratio in submarket x is defined as the ratio of the number of vacancies created by firms to the number of individuals who search for jobs in that particular submarket, denoted $\theta(x, \psi) \geq 0$, where $\psi \in \Psi$ is the aggregate state of the economy at the beginning of each period.

The aggregate state of the economy ψ consists of aggregate productivity y and the distribution of workers among employment positions (u, e): $u \in [0, 1]$ is the measure of unemployed individuals, and the distribution of employed workers is $e : X \times Z \to [0, 1]$ with pdf $e(x, z)$, and $u + \sum_z \int_x e(x, z)dx = 1$. Menzio and Shi (2010) show that the agents' value and policy functions as well as tightness ratios depend on the aggregate state of the economy ψ only through aggregate productivity y, and not through the distribution of workers across different employment state, (u, e). Therefore, we write, hereafter, tightness ratios and values as function of aggregate productivity, not the aggregate state of economy.[7]

Time is discrete and continues forever. Each period consists of four stages: separation, search, matching, and production. In the separation stage, workers get separated from their match with probability $d \in [\delta, 1]$, where δ denotes the exogenous probability of separation. Matches can also be endogenously destroyed by setting $d = 1$. When a worker loses his job, he must remain unemployed until the start of the searching stage next period.

During the search stage, an individual who has the opportunity to search first decides in which submarket to direct his search. While all individuals who have been unemployed for at least one period have the opportunity to search, employed workers only have the opportunity to search with probability $\lambda_e \in [0, 1]$. As mentioned above, workers who's unemployment spell started this period cannot search this period. Firms also choose submarkets in which to open vacancies during the search stage.

A firm who meets a worker in submarket x during the matching stage offers

[7]Intuitively, since search is directed, each individual targets his preferred submarket regardless of the distribution of individuals. Similarly, because of free entry, vacancies are created in any submarket until firms make zero profits.

the worker a contract which delivers expected lifetime utility x to the worker. If the worker accepts the offer, he starts working at the new job in the following production stage. Otherwise, the worker stays in his previous employment position (which could be unemployment). Firms and workers cannot coordinate their actions because of search frictions in the labor market: not all workers succeed in finding a job, and not all firms succeed in hiring a worker. A worker searching in a submarket characterize by tightness ratio θ finds a job with probability $p(\theta)$, where $p : \mathbb{R}_+ \to [0, 1]$ is strictly increasing, twice-continuously differentiable and strictly concave with $p(0) = 0$ and $p(\infty) = 1$. Similarly, firms fill vacancies with probability $q(\theta)$, where $q : \mathbb{R}_+ \to [0, 1]$ is strictly decreasing, twice-continuously differentiable and strictly convex with $q(0) = 1$ and $q(\infty) = 0$. Naturally, $q(\theta) = p(\theta)/\theta$.

During the production stage, an employed worker produces $y + z$, and consumes after-tax wage $(1 - \tau)\omega$. Therefore, the firm's profits equal $y + z - \omega$ in that period. Unemployed workers receive and consume fixed unemployment benefit b.[8] At the end of the production stage, nature draws next period's aggregate productivity y' and idiosyncratic productivity z' from the probability distributions $\Phi_y(y'|y)$ and $\Phi_z(z'|z)$ respectively.

2.1 Worker's Problem

Consider an individual who's employment status can be summarized by his lifetime utility V. During the search stage, he chooses in which submarket to search in order to maximize his value of search. If the worker searches in submarket x, he finds a job with probability $p(\theta(x, y))$ and the job provides lifetime utility x. If he fails to find a job, which occurs with probability $(1 - p(\theta(x, y)))$, he retains his current employment status in the production stage. Accordingly, an individual with current lifetime utility V who has the opportunity to search chooses the submarket which maximizes his lifetime utility at the beginning of the search stage, $V + R(V, y)$, where the second term is the value of search at the beginning of the search stage.

[8] In section 5 we explore the possibility that unemployment benefits depend on past earnings.

The worker's problem at the search stage can thus be written as

$$R(V,y) = \max_{x\in X} p(\theta(x,y))(x - V), \tag{1}$$

and the solution to the above maximization is the optimal submarket in which to search, given by

$$m(V,y) = \operatorname*{argmax}_{x\in X} p(\theta(x,y))(x - V).$$

To ease notation, let $\tilde{p}(V,y)$ denote the probability of finding a job in the optimal submarket, i.e. $\tilde{p}(V,y) = p(\theta(m(V,y),y),y)$.[9]

Let $U(y)$ denote the value function of an unemployed workers at the beginning of the production stage. This lifetime utility consists of the current value of consuming unemployment benefit, b, and the value of being unemployed and searching tomorrow:

$$U(y) = \nu(b) + \beta E\{U(y') + R(U(y'),y')\} \tag{2}$$

where the value of a variable next period is denoted by a prime symbol ($'$).

2.2 Firm's Problem

During the matching stage, firms offer contracts $c \in C$ to workers. A contract specifies the current wage ω, the probability that the match will be destroyed in the next separation stage d', and the worker's lifetime utility V' at the beginning of the next period. This future utility will be attained by an implicit sequence of future wages. The firm chooses the contract to maximize its lifetime profits $J(V,s)$, where $s = (y,z)$, while delivering lifetime utility associated with the submarket in which they are (promise-keeping constraint) and also compatible with the worker's option to go to unemployment (individual rationality constraint).[10] The problem of a firm

[9] Menzio and Shi (2010) show that $R(V,y)$ is continuous, differentiable, and decreasing in V; that $m(V,y)$ is continuous, differentiable (over the relevant range), and increasing in V; and that $\tilde{p}(V,y)$ is continuous, differentiable, and decreasing in V.

[10] Note that it is in the best interest of the firm to endogenously separate the match when the individual rationality constraint binds. In essence, workers and firms agree on when to endogenously separate.

who meets a worker in submarket V is therefore given by

$$\begin{aligned} J(V,s) &= \max_{\omega,d',V'} \Big\{ y+z-\omega+\beta E\big[(1-d')(1-\lambda_e \tilde{p}(V',y'))J(V',s')\big]\Big\} \quad (3) \\ V &= \nu((1-\tau)\omega)+\beta E\Big\{d'U(y')+(1-d')\big[V'+\lambda_e R(V',y')\big]\Big\} \\ d' &= \begin{cases} \delta & \text{if } U(y') \leq V'+\lambda_e R(V',y') \\ 1 & \text{otherwise} \end{cases} \end{aligned}$$

where $\omega \in \mathbb{R}_+$. Let $c = (\omega, d', V')$ denote the optimal contract. Then the associated policy functions are the wage $\omega = \omega(V,s)$, next period's probability of separation $d' = d'(V,s,s')$, and the worker's lifetime utility in next period $V' = V'(V,s,s')$.[11]

2.3 Market Tightness

During the search stage, firms choose how many vacancies to open and in which submarket to open them. Free entry ensures that vacancies open until the expected value of opening a vacancy is no more than the cost of creating it, denoted κ. Given the firm's value of a match in submarket x, $J(x,y,\tilde{z})$, where $\tilde{z} \in Z$ is the idiosyncratic productivity for all new matches, and the probability of filling a vacancy in that submarket, $q(\theta(x,y))$, free entry implies that

$$\kappa \geq q(\theta(x,y))J(x,y,\tilde{z}). \quad (4)$$

While the free entry condition must hold with equality for submarkets which are open in equilibrium (i.e. submarkets in which some individuals search), such need not be the case for unvisited submarkets. Following Acemoglu and Shimer (1999) and the subsequent literature, we assume that (4) holds with equality in all submarkets in a relevant range, that is, from the lowest submarket to the submarket where firms would just cover the cost of posting a vacancy with a job filling probability equal to one. Under this assumption, market tightness is decreasing and continuous in x over the relevant range.

[11] Menzio and Shi (2010) show that $J(V,s)$ is concave, continuous, and differentiable in V.

2.4 Laws of Motion

In principle, we can compute the probability that a worker transits from one employment state to any other state using the optimal policy functions and the exogenous transition functions of the idiosyncratic productivity shock. However, since the shocks y and z take on a finite number of values, a finite number of submarket will be open (in the long run) in equilibrium. To simplify the exposition of the laws of motion, and slightly abusing notation, we redefine X to be the set of open submarkets in the long run and denote N_x the cardinality of that set. Accordingly, let $e : X \times Z \rightarrow [0, 1]$ and $u \in [0, 1]$ denote the probability distribution over employed and unemployed workers at the end of the period, respectively. To compute the transitions between employment states, fix aggregate productivity today at some state y and tomorrow at some state y'.

The measure of unemployed workers at the end of the period tomorrow, u', corresponds to the sum of unemployed workers whose search was unsuccessful plus employed workers who lost their job during the period. An unemployed worker remains unemployed with probability $(1 - \tilde{p}(U(y'), y'))$. As specified by the optimal contract, an employed worker in state (x, s) today will becomes unemployed tomorrow with probability $d'(x, s, s')$ if state $s' = (y', z')$ occurs. Given a mass $e(x, z)$ of workers today, $\Phi_z(z'|z)$ gives the fraction of workers who transit to any state z' tomorrow. The measure of unemployed workers next period can thus be written as

$$u' = (1 - \tilde{p}(U(y'), y'))u + \sum_{x \in X} \sum_{z \in Z} \sum_{z' \in Z} \Phi_z(z'|z) d'(x, s, s') e(x, z).$$

Since all workers in new matches have idiosyncratic productivity $\tilde{z}$, the transition of workers to state (x', z') when $z' \neq \tilde{z}$ is different from when $z' = \tilde{z}$. Let's start with the case where $z' \neq \tilde{z}$, which is simpler since no one can end up with idiosyncratic shock z' though successful search. For these states, the only way for a worker to end up in state (x', z') is for his current match to survive $((1 - d'(x, s, s')))$, his search to be unsuccessful $((1 - \lambda_e \tilde{p}(V'(x, s, s'), y')))$, and for today's contract to specify that tomorrow's promised utility will be x' if state z' occurs. In other words, the contract must specify that $V'(x,s,s') = x'$. Accordingly, let $\mathbb{I}[V'(x,s,s') = x']$ be equal to 1 if that is the case, and 0 otherwise. Then we have

$$e'(x',z') = \sum_{x\in X}\sum_{z\in Z}\Phi_z(z'|z)(1-d'(x,s,s'))(1-\lambda_e\tilde{p}(V'(x,s,s'),y'))\mathbb{I}[V'(x,s,s') = x']e(x,z).$$

Finally, for state $z' = \tilde{z}$, we have to take care of individuals whose search is successful in addition to those who transit to that state though their contract with the firm. First, x' could be the submarket in which unemployed workers search. Let $\mathbb{I}[m(U(y'),y') = x']$ be equal to 1 if that is the case, and 0 otherwise. Similarly, an individual who's contract specifies promised utility V' tomorrow if state z' occurs can search in market x' (note that this occur for any z'). Let $\mathbb{I}[m(V'(x,s,s')) = x']$ be equal to 1 if that is the case, and 0 otherwise. Putting it all together, we have

$$\begin{aligned} e'(x',\tilde{z}) &= \sum_{x\in X}\sum_{z\in Z}\Phi_z(\tilde{z}|z)(1-d'(x,s,y',\tilde{z}))(1-\lambda_e\tilde{p}(V'(x,s,y',\tilde{z}),y'))\mathbb{I}[V'(x,s,y',\tilde{z}) = x']e(x,z) \\ &+ \sum_{x\in X}\sum_{z\in Z}\sum_{z'\in Z}\Phi_z(z'|z)(1-d'(x,s,s'))\lambda_e\tilde{p}(V'(x,s,s'),y')\mathbb{I}[m(V'(x,s,s')) = x']e(x,z) \\ &+ \tilde{p}(U(y'),y')\mathbb{I}[m(U(y'),y') = x']u. \end{aligned}$$

The measures defined above take the aggregate states as given. We can use these measures and combine them with the transition function of the aggregate state to obtain a transition matrix $\Phi : X\times Y\times Z \to [0,1]$.[12] Below we use $\phi^e : X\times Y\times Z \to [0,1]$ and $\phi^u : Y \to [0,1]$ to denote the stationary probability distribution over employed and unemployed workers, respectively, associated with Φ.

2.5 Government

The government follows a fiscal policy which guaranties budget balance in the long run:

$$b\sum_{y\in Y}\phi^u(y) = \tau\sum_{x\in X}\sum_{y\in Y}\sum_{z\in Z}\omega(x,y,z)\phi^e(x,y,z). \tag{5}$$

[12] Given the discrete nature of shocks, as well as open submarkets, Φ is a Markov matrix.

The left-hand side represents transfers to unemployed workers, and the right-hand side corresponds to revenues from the payroll tax. Notice that without aggregate uncertainty, the above budget constraint reduces to period-by-period balanced-budget.

2.6 Block Recursive Equilibrium

In a Recursive Equilibrium, the functions $\{\theta, R, m, U, J, c\}$ all depend on the aggregate state of the economy. Solving for an equilibrium outside of the steady state thus requires solving a system of equations for unknowns which depend on the entire distribution of worker across employment states. Hence, solving a recursive equilibrium outside the steady state is analytically and numerically difficult because the dimension of the state space is very large.

Menzio and Shi (2010) prove existence and uniqueness of a recursive equilibrium in which the functions depend on the aggregate state of the economy only through the aggregate productivity component y and not through the distribution of workers across employment states (u, e), which they refer to as a *block recursive equilibrium.*

Definition 1 *A Block Recursive Equilibrium consists of a fiscal policy (b, τ), a market tightness function $\theta : X \times Y \to \mathbb{R}_+$, a search value function $R : X \times Y \to \mathbb{R}_+$, a policy function $m : X \times Y \to X$, an unemployment value function $U : Y \to \mathbb{R}$, a value function for firms $J : X \times Y \times Z \to \mathbb{R}$, a contract policy function $c : X \times Y \times Z \to C$, a transition probability function $\Phi : X \times Y \times Z \to [0, 1]$, a stationary probability function for employed workers $\phi^e : X \times Y \times Z \to [0, 1]$, and a stationary probability function for unemployed workers $\phi^u : Y \to [0, 1]$. These functions satisfy the following requirements:*

1. *R satisfies (1) for all $(V, y) \in X \times Y$, and m is the associated policy function;*

2. *U satisfies (2) for all $y \in Y$;*

3. *J satisfies (3) for all $(V, y, z) \in X \times Y \times Z$, and c is the associated policy function;*

4. θ satisfies (4) for all $(x, y) \in X \times Y$;

5. Φ is derived from the policy functions (m, c) and the exogenous probability distribution for y and z, and ϕ^e and ϕ^u are the associated stationary probability distributions;

6. the unemployment insurance system (b, τ) satisfies (5).

3 Optimal Unemployment Insurance: Analytics

This section establishes some results regarding the impact of unemployment insurance. Although some of the results hold more generally, we abstract from both idiosyncratic and aggregate uncertainty throughout this section. Furthermore, we assume that workers cannot search on the job, i.e. $\lambda_e = 0$. Under these assumptions, there is only one open submarket in equilibrium: all unemployed individuals search in that submarket. From the firm's problem, it is easy to show that (i) the state variable (lifetime utility of the worker) will remain constant throughout the life of the match, (ii) separation will only occur exogenously ($d = \delta$), and (iii) the wage will be constant throughout the life of the match. The latter result implies that there is a one to one relationship between x and ω, which we loosely use interchangeably below. Accordingly, the value of the firm simplifies to

$$J(\omega) = \frac{y - \omega}{1 - \beta(1 - \delta)}. \tag{6}$$

In turn, the wage in submarket x must be given by

$$x = V(\omega) = \frac{\nu((1 - \tau)\omega) + \beta\delta U}{1 - \beta(1 - \delta)}, \tag{7}$$

where the value of unemployment is

$$U = \frac{\nu(b) + \beta R(U)}{1 - \beta}. \tag{8}$$

The value of search is given by

$$R(U) = p(\theta)(x - U), \tag{9}$$

where $x = m(U)$ and θ satisfies the free entry condition

$$k = q(\theta)J(\omega). \tag{10}$$

Given the properties of the periodic utility function, it is quite trivial to show that there exists an optimal unemployment insurance system which provides positive unemployment benefits without providing full insurance to individuals: while zero benefits can be ruled out by Inada conditions, full insurance can be ruled out because it can only be achieved at zero consumption as all individuals would be unemployed. We nevertheless state that result in the following proposition.

Proposition 1 *Assume that the periodic utility function ν is strictly increasing, strictly concave, continuously differentiable, and satisfies Inada conditions. Then there exists an optimal unemployment insurance system which provides positive benefits to unemployed workers without providing full insurance: $0 < b < (1 - \tau)\omega$.*

Proof. By assumption, $\lim_{c \to 0} \nu'(c) = \infty$. It follows that a small increase in b away from zero improves welfare. To see that full insurance is not optimal, assume that the unemployment insurance system does provide full insurance, i.e. $b = (1 - \tau)w$. This implies that individuals must choose to search in the submarket which provides exactly the same utility as unemployment, that is, $m(U) = U$ and thus $R(U) = 0$. For that to be the case, it must be that $p(\theta(x)) = 0$ for all $x > U$. Since J in equation (6) is a continuous function of x (or ω), the tightness ratio is also continuous (see equation (10)). By continuity of p, the job finding probability is near zero at $m(x) = U$, and thus the unemployment rate, $\phi^u = \delta/(\delta + p(\theta(m(U)))) \approx 1$, and so consumption is again arbitrarily close to zero. It must therefore be the case that moving away from full risk sharing, i.e. $b < (1 - \tau)\omega$, improves welfare. ∎

It is worth noting that under a linear periodic utility function, unemployment insurance has no value. To see this, consider the weighted utility of unemployed and employed individuals: $W = \phi^u b + (1 - \phi^u)(1 - \tau)\omega$. Since the budget constraint of the government imposes that $\phi^u b = (1 - \phi^u)\tau\omega$, $W = (1 - \phi^u)w$ is independent of unemployment insurance.

4 Optimal Unemployment Insurance: Numerical Results

In this section we use a parameterized version of the model to study unemployment insurance in the presence of various types of risk. We first set a benchmark in an economy without idiosyncratic nor aggregate uncertainty, and successively study the impact of adding each type of risk. We also discuss the desirability for unemployment benefits to be pro- or counter-cyclical. Before doing so, we first parameterize the economy.[13]

4.1 Parameterization

We take a period to be a quarter and set the discount factor to 0.987, which implies an interest rate of 5 percent annually. The worker's periodic utility function is given by $(c^{(1-\sigma)} - 1)/(1-\sigma)$, where σ, the risk aversion coefficient, is equal to 2. The matching technology $p(\theta)$ has the functional form of $\theta(1+\theta^{\gamma})^{-1/\gamma}$, with $\gamma = 0.5$, which implies an elasticity of substitution between vacancies and applicants of 2/3.[14]

Following Menzio and Shi (2010), we set κ, δ, and λ_e equal to 0.001, 0.045, and 0.3 respectively. The aggregate productivity follows a two-state Markov process with $Y = \{0.95, 1.05\}$ and transition matrix $\Phi_y = [0.75\ 0.25; 0.25\ 0.75]$, with unconditional mean equal to 1.[15] We set the unconditional mean of idiosyncratic productivity to zero, and assume that z follows a three-state Markov process with $Z = \{-0.4, 0, 0.4\}$ and transition matrix $\Phi_z = [0.75\ 0\ 0.25; 0.5\ 0\ 0.5; 0.25\ 0\ 0.75]$. Note that idiosyncratic productivity is assume to be zero ($\tilde{z} = 0$) in all newly formed matches.

[13] The algorithm used to solve the model numerically appears in the Appendix.

[14] The underlying matching function, as first introduced by den Haan et al. (2000), is given by $(v^{-\gamma} + a^{-\gamma})^{-1/\gamma}$, where v and a are the number of vacancies and applicants respectively, and $\theta = v/a$. The elasticity of substitution between vacancies and applicants is $\frac{1}{1-\gamma}$.

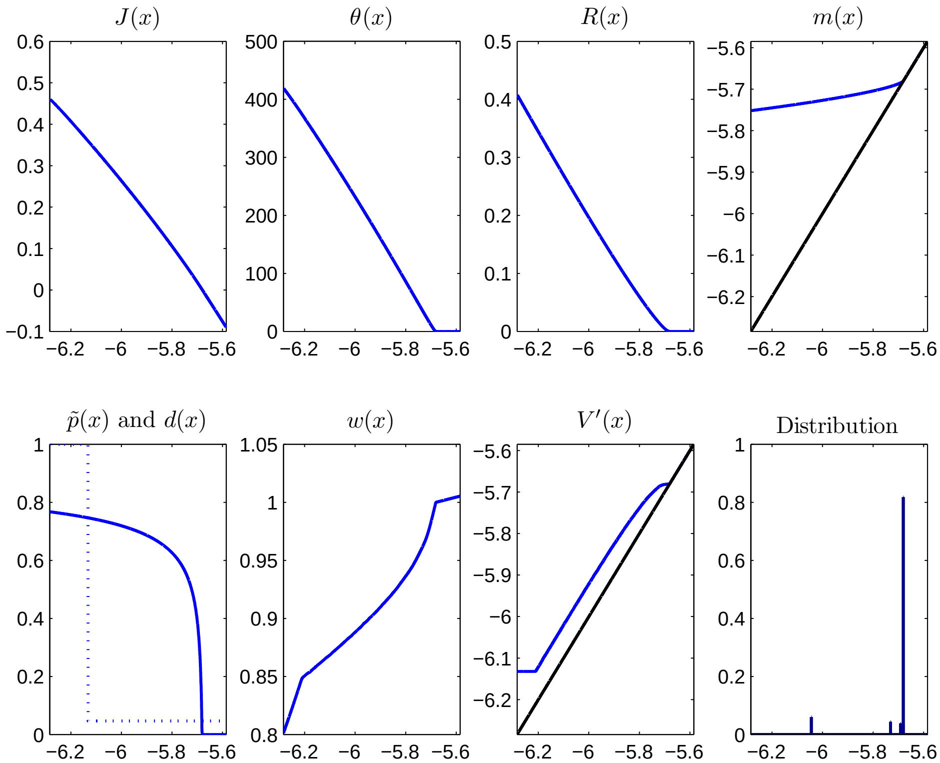
[15] We also experimented with a Markov process with $y \in \{0.95, 1.02\}$ and transition matrix $[0.875\ 0.125; 0.05\ 0.95]$, thereby making recessions harsher, less frequent, and more persistent. We omit presenting these results as the different with our benchmark results are negligible.

4.2 Benchmark Model without Uncertainty

We now assume that both aggregate and idiosyncratic productivities are constant at their unconditional means. Figure 1 displays various value and policy functions, the submarket tightness function, as well as the stationary distribution over open submarkets. Since the value of the firm ($J(x)$) is decreasing in promised utility to the worker, firms need to be compensated by a high job filling probability ($q(x)$) to open vacancies in high submarkets. This translates into a tightness ratio ($\theta(x)$) and a job finding probability ($\tilde{p}(x)$) which decrease as promised utility increases. Notice that unemployed individuals search in relatively high submarkets: this is because of the fairly generous unemployment benefit. As a result, the optimal submarket in which to search is fairly flat, and the distance between the current lifetime utility (x) and the lifetime utility associated with the submarket in which individuals search ($m(x)$) narrows down quickly. It also follows that the value of search ($R(x)$) decreases monotonically with current lifetime utility.

The wage ($\omega(x)$), future utility ($V'(x)$), and the separation probability ($d'(x)$) are all chosen by the firm as part of the contract. As one would expect, wages and future promised utility both increase with the submarket value. Because of curvature in the periodic utility function, the wage is a convex function of the submarket value over the relevant range: increasingly higher wages are necessary to provide the extra lifetime utility. Also note that the contract is designed to backload wage payments: by promising higher future utility, firms can keep current wages low while at the same time lowering the worker's job finding probability in the future. The separation probability is such that no match is ever endogenously destroyed. Finally, note that the model generates some heterogeneity among homogenous workers, as shown in the last panel of Figure 1. The first spike corresponds to the measure of unemployed individuals in the population. As dictated by the policy function $m(x)$, these individuals search in a higher submarket, which corresponds to the second spike, and so on. Of course, a limited number of submarkets can emerge without idiosyncratic nor aggregate uncertainty. Naturally, all open submarkets are located between the value of unemployment and the submarket for which firms would just

Figure 1: Economy without Uncertainty



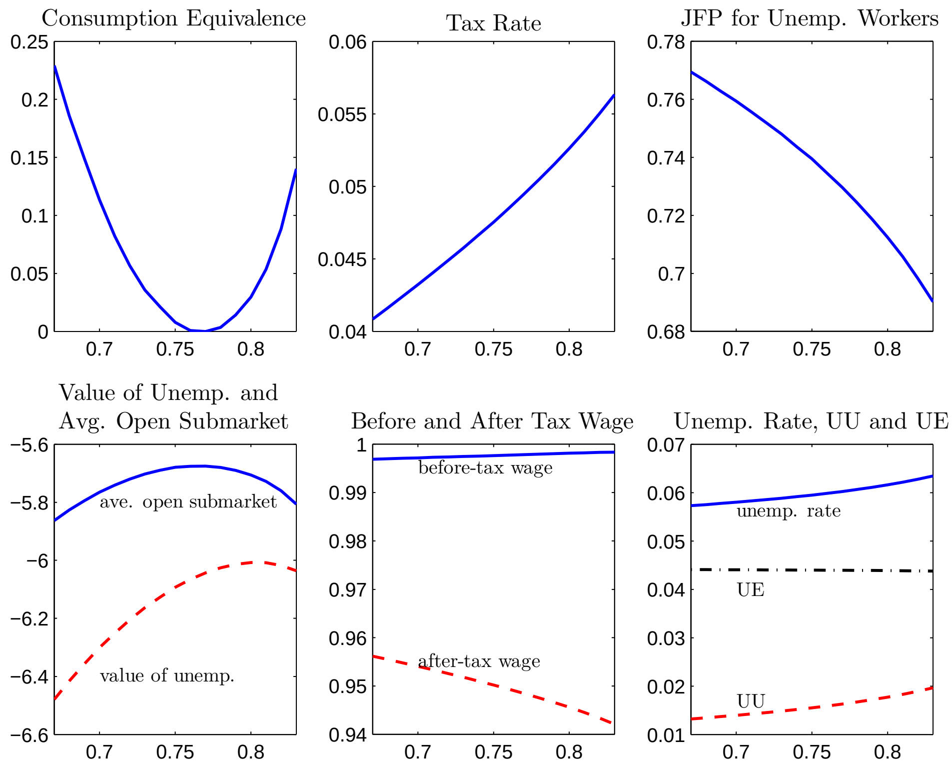
Notes: In all panels the horizontal axis is the value of submarkets x. This figure was generated under unemployment benefit $b = 0.77$, for which the replacement ratio is on average 0.812, and a labor tax rate $\tau = 0.0495$. Other parameter values are described in section 4.1.

cover the cost of posting a vacancy with a job filling probability equal to one.

Figure 2 displays how the economy behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government's budget on a period-by-period basis. The top left panel shows that there exists a unique optimum unemployment insurance configuration.[16] As discussed earlier, the intuition for this result is straightforward: while higher unemployment benefits

[16] Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one.

Figure 2: Optimal Unemployment Insurance without Uncertainty



Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

provide better consumption smoothing, it also reduces production and thus average consumption as the unemployment rate increases.[17]

[17] Production need not decrease in an economy in which firms choose the amount of capital to put in place prior to filling vacancies, as shown by Acemoglu and Shimer (1999). In their environment, firms put in place a larger amount of capital when this capital is expected to remain idle for shorter periods of time, as happens when individuals direct their search to higher submarkets.

A more generous unemployment insurance system provide better consumption smoothing not only because benefits are higher, but also because after-tax wages are lower. The lower-left panel of Figure 2 shows that despite lower consumption, employed workers work on average in higher submarkets as unemployment benefits increase, at least at relatively low unemployment benefit levels.[18] The extra utility, of course, comes from the possibility of becoming unemployed in the future, which has a higher value. But because unemployed workers search in higher submarkets, for which the job finding probability is lower, individuals spend more time in the unemployment state as unemployment insurance becomes more generous. Eventually, even the value of unemployment falls as the prospect of becoming employed not only becomes less likely but also more dire.

4.3 Introducing Idiosyncratic Uncertainty

In this section, we keep aggregate productivity fixed at its unconditional mean, but introduce idiosyncratic uncertainty in the form of a three-state Markov chain for individual productivity z. Recall that idiosyncratic productivity is assumed to be the same in all new matches, equal to its unconditional mean ($\tilde{z} = 0$). Thereafter, idiosyncratic productivity can either be low or hight, and evolves according to Φ_z.

Figure 3 illustrates the properties of the economy. Most functions look familiar following the discussion around Figure 1. The contract, however, has some new properties. First, only the dashed-line value of the firm ($J(x, \tilde{z})$) in the top panel is relevant for job creation. No new match are created for submarkets $x > \hat{x}$, where $\hat{x}$ satisfies $J(\hat{x}, \tilde{z}) = \kappa$—i.e. submarkets for which $\theta(x) = 0$. This limits the degree to which firms can backload contracts since workers cannot find jobs in submarkets higher than x. Accordingly, firms design contracts such that high productivity individuals have no incentive to search: they search in markets where the job finding probability is zero. On the flip side, firms design the contract in such a way that

[18] Note that the value of the average open submarket peaks earlier than average welfare: the value of unemployment is still rising, which, over a narrow range, dominates the rise of the weight on the value of unemployment (the unemployment rate).

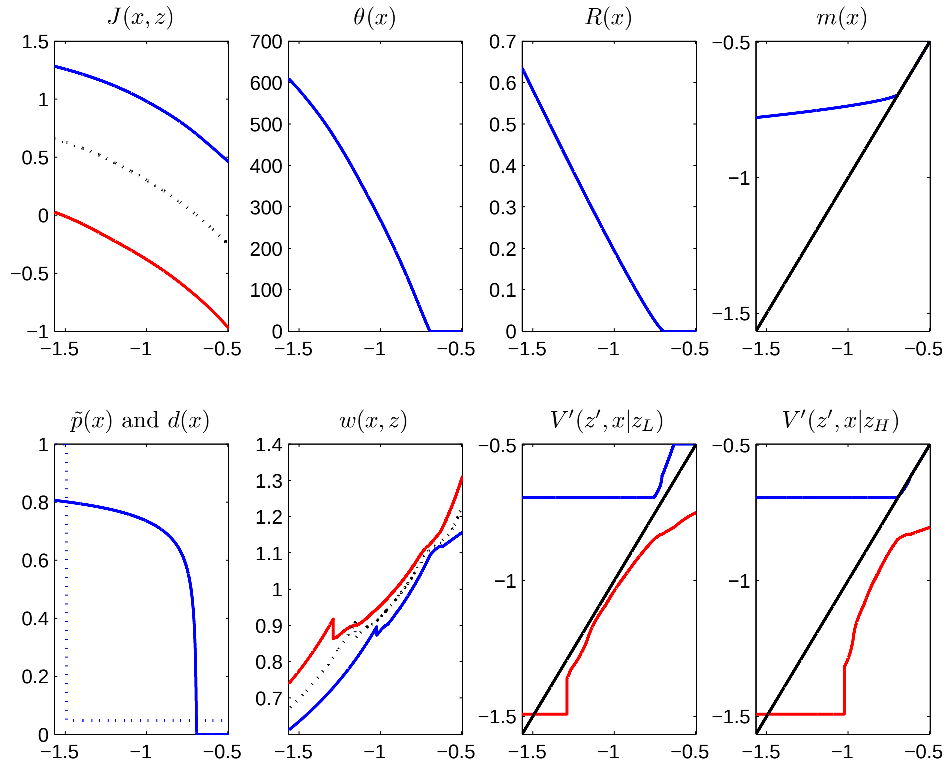
matches with some low productivity individuals are destroyed. Again, because firms cannot offer less than the value of unemployment, this limits the firm's ability to backload wages. Finally, notice that wages are higher for low productivity individuals than for high productivity individuals given a submarket x. This is due to the persistence of the shock and is most easily seen at low values of x, for which the contract specifies the same future utility for both low and high productivity individuals: since the weight on high future utility is higher for the high productivity individual ($V'(z' = z_H, x|z_H)$), a smaller wage is required to meet the promised utility x. Accordingly, the value of the firm is higher when their worker has high productivity.

Figure 4 displays how the economy with idiosyncratic risk behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government's budget on a period-by-period basis. As was the case in the economy without risk, the top left panel indicates that there exists a unique optimum unemployment insurance configuration. The same mechanism as in the benchmark model is at work: a more generous system provides better risk sharing at the cost of higher unemployment and thus lower average consumption. However, a separate mechanism is responsible for the generosity of the system to be so much lower than in the benchmark model—the average replacement ratio falls from 81 to 68 percent. The new mechanism revolves around endogenous match separation, which plays a crucial role especially at high benefit levels. This can be seen in the lower right panel, which shows that the flow from unemployment to employment rises sharply at high benefit levels.[19] This is in contrast to the benchmark model, where the rise in the unemployment rate was essentially all due to the lower job finding probability, and hence longer unemployment duration. Together, these effect translate into very high tax rates, so much so that high unemployment benefits are simply unsustainable. For the same reason, the welfare cost of running 'too generous' an unemployment insurance system is sizable.

Figure 5 illustrates how the optimal unemployment insurance system—as indexed

[19] Recall that in a stationary environment, this flow must be equal to the flow from employment to unemployment.

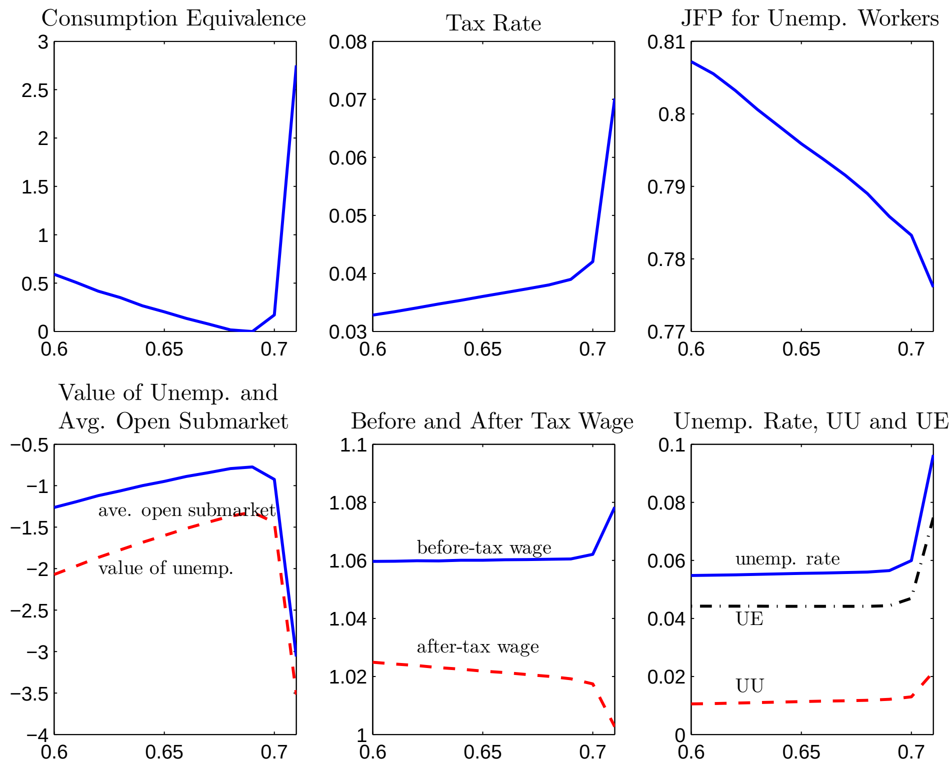
Figure 3: Economy with Idiosyncratic Uncertainty



Notes: In all panels the horizontal axis is the value of submarkets x. Red lines represent low productivity, and blue lines represent high productivity. This figure was generated under unemployment benefit $b = 0.69$, for which the replacement ratio is on average 0.677, and a labor tax rate $\tau = 0.039$. Other parameter values are described in section 4.1.

by the level of the benefit—changes with the amount of idiosyncratic risk that individuals face. As idiosyncratic risk rises, the optimal unemployment benefit initially rises smoothly but then falls rather sharply. While the initial rise allows individuals to better smooth consumption, the subsequent fall is due to endogenous separation. As the amount of risk rises, more matches with low productivity workers are endogenously destroyed. As a result, the unemployment rate rises faster, and so does the wage tax on the shirking pool of workers. These findings offer a similar interpretation of the 'European unemployment dilemma' to that of Ljungqvist and Sargent (1998):

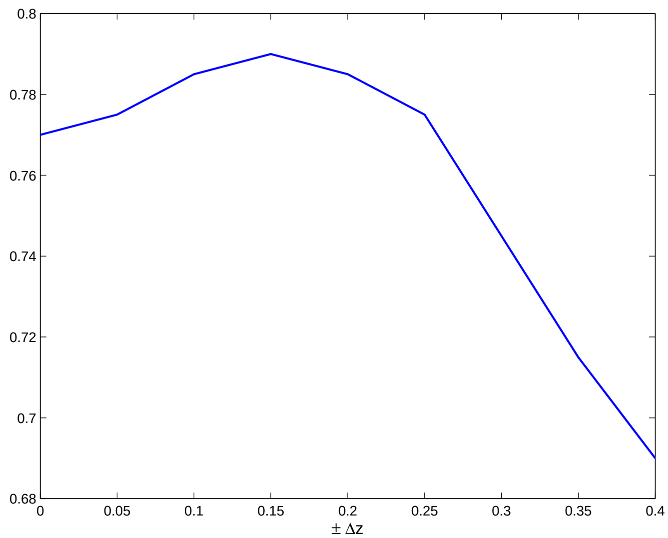
Figure 4: Optimal Unemployment Insurance with Idiosyncratic Uncertainty



Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

when income risk rises, generous welfare states become costly as they produce high unemployment rates and longer unemployment duration.

Figure 5: Optimal Unemployment Benefit and Idiosyncratic Risk

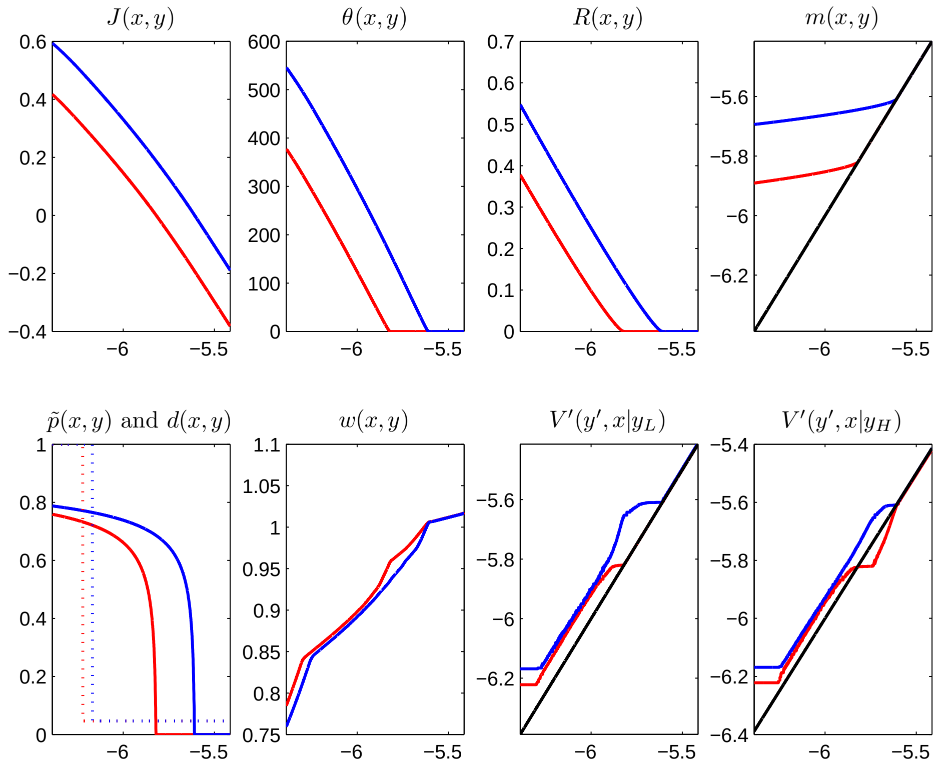


Notes: The idiosyncratic shock takes on values $z \in \{-\Delta z, 0, \Delta z\}$. For any Δz, we compute the optimal unemployment insurance system, indexed by the unemployment benefit on the vertical axis. The calibration is outlined in Section 4.1.

4.4 Introducing Aggregate Uncertainty

We now remove idiosyncratic uncertainty but introduce aggregate productivity in the form of a two-state Markov chain for aggregate productivity y. From figure 6, the same patterns as in the economy without uncertainty emerge here as well. Under high productivity, individuals search in higher submarkets. But because the value of the firm is higher, their job finding probability of unemployed workers is nevertheless higher in good times. Aggregate shocks, on their own, aren't sufficiently large for any match to be destroyed endogenously. Interestingly, a lot of consumption smoothing is built in to the contract designed by the firms: promised utility in good and bad times are fairly close to one another, at least at relatively low submarket values. This also allows the firm to backload wages. However, promised utilities in higher submarkets offer less insurance, and most open submarkets end up being in this region. This is because individuals' promise utility always rises in good time, but

Figure 6: Economy with Aggregate Uncertainty

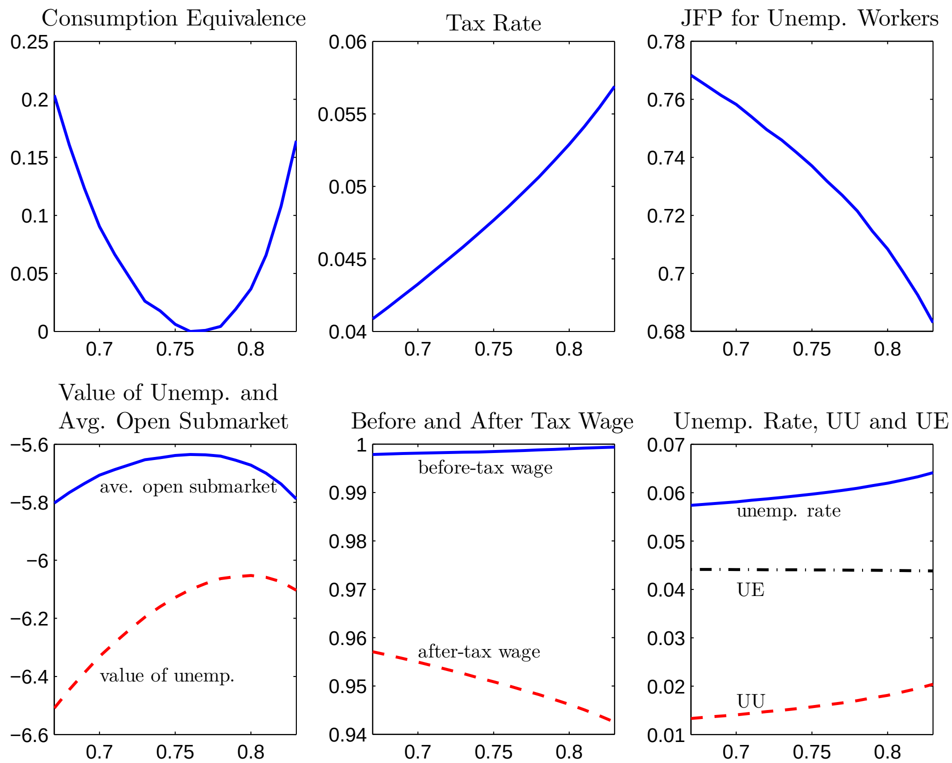


Notes: In all panels the horizontal axis is the value of submarkets x. Red lines represent low productivity, and blue lines represent high productivity. This figure was generated under unemployment benefit $b = 0.76$, for which the replacement ratio is on average 0.80, and a labor tax rate $\tau = 0.049$. Other parameter values are described in section 4.1.

does not necessarily go down in bad times except in the period of the shock. But because of the downgrade in the period of the shock, wages are lower in bad times. Furthermore, the average wage goes up in good times because new matches occur in higher submarkets. As a result, with a constant unemployment insurance system, the government runs a surplus in good times and deficits in bad times.

Figure 7 displays how the economy with aggregate uncertainty behaves under various levels of unemployment benefits, each financed through a labor income tax which balances the government's budget on average in the long run. Comparing this

Figure 7: Optimal Unemployment Insurance with Aggregate Uncertainty

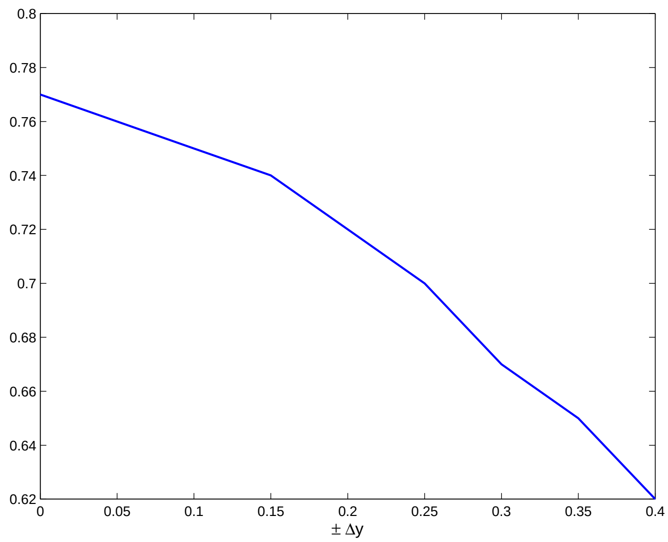


Notes: In all panels the horizontal axis is the value of unemployment benefits b. The tax rate is such that the government budget constraint holds on a period-by-period basis in the stationary equilibrium given b. Wages refer to average before- and after-tax wages of employed workers. 'UU' refers to the flow of individuals who remain unemployed, and 'UE' the flow of unemployed who transit to employment. Consumption Equivalence refers to the amount of extra consumption per period, in percentage terms, which individuals would require in order to be indifferent between a given unemployment insurance system and the optimal one. The calibration is outlined in Section 4.1.

Figure to its counterpart without uncertainty (Figure 2), we conclude that aggregate uncertainty does not alter how unemployment insurance affects the economy in general. Indeed even the optimal level of benefits is very similar, albeit a little smaller. It follows that the cost of deviating from the optimal system are rather small.

Figure 8 illustrates how the optimal unemployment insurance system—as indexed

Figure 8: Optimal Unemployment Benefit and Aggregate Risk



Notes: The aggregate shock takes on values $y \in \{-\Delta y, \Delta y\}$. For any Δy, we compute the optimal unemployment insurance system, indexed by the unemployment benefit on the vertical axis. The calibration is outlined in Section 4.1.

by the level of the benefit—changes with the amount of aggregate risk in the economy.[20] This Figure shows that the generosity of unemployment insurance should decline monotonically as the economy becomes riskier. Our interpretation of this finding is that because the unemployment benefit is the same in good and bad times, it has a larger detrimental effect on unemployment duration during bad times, when wages are low, than a beneficial on in good times, when wages are high. On average, then, the unemployment rate is higher, and so the tax rate necessary to finance any level of benefit is higher.

The reasoning above is that a flat benefit provides 'too much' consumption smoothing in bad times, and 'too little' in good times. As such, it points to the idea that the unemployment benefit should be pro-cyclical, that is, benefits should be reduced in recessions. We investigated this issue to find that this is indeed the

[20]Note that the relevant range of aggregate risk is below 0.05.

case. The optimal benefits in bad and good times turn out to be 0.76 and 0.80, respectively, which translate into replacement ratios of 0.80 and 0.84.

4.5 Aggregate and Idiosyncratic Uncertainty Together

We now investigate the full model, with both aggregate and idiosyncratic uncertainty. Since idiosyncratic uncertainty is much more important than aggregate uncertainty, this economy behaves essentially in the same way as the economy with only idiosyncratic uncertainty. The most interesting aspect of combining the two types of uncertainty is that the amount of endogenous destruction increases in recessions. This effect is strong enough to essentially double the reduction in the optimal benefit from adding aggregate uncertainty an economy with idiosyncratic risk relative to one with no risk. For similar reasons, the highest level of benefit that is sustainable is smaller in the full model.

5 Extensions

To be written.

5.1 Replacement Ratio

To be written.

5.2 Stochastic Unemployment Benefit Duration

To be written.

6 Conclusion

To be written.

A Algorithm

For any given level of unemployment benefit b:

1. Set an initial guess for tax rate τ;
2. Compute the firm's value function J, submarket tightness function θ, the policy function m and the optimal contract c by iterating on the following steps:
   (a) Set an initial guess for J, and U;
   (b) Compute the market tightness θ using (4) and the implied job finding probability p from the matching function;
   (c) Compute the value of search R, and associated policy function m using (1);
   (d) Compute the new firm's value function J' and associated optimal contract $c(\omega, d', V')$ using (3);
   (e) Construct the new value of unemployment U' using (2).
   (f) If $J \approx J'$, done, otherwise use the J' and U' as new guesses and go back to step 2b;
3. Calculate the stationary distributions ϕ^e and ϕ^u associated with the policy function m the optimal contract c, and the tightness ratio θ;
4. Compute the government's budget deficit: if the budget deficit is zero, stop; otherwise set a new guess of the tax rate and go back to step 2.

The optimal unemployment insurance system is found by repeating these steps for several values of unemployment benefits.

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