Investment and Insurance in an Economic Union*

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Abstract

The presence of private information limits the extent of risk sharing in an economic union. Studying the optimal dynamic arrangement with these impediments is particularly important because of its potential effect on investment and the distribution of power between its members. This paper studies this problem in a neoclassical growth model with two countries. One of the countries faces "demand" shocks that are privately observed. The economic union must decide how much help they should provide to this country and how to finance those transfers: Should they come from a reduction of investment or of consumption of the other member? Importantly, the viability of the Economic Union may be at risk if private information imposes large welfare losses. To address these questions, an alternative recursive method to solve for the optimal allocation in this context is developed. The results suggest that, in spite of private information, it is still (constrained) optimal to provide some insurance but at the cost of a reduction in the welfare of the country helped. The welfare costs of private information are non-monotone in the size of the country with private information.

Keywords: Private Information, Risk Sharing, Economic Union

The views expressed in this paper are those of the authors, and do not necessarily reflect those of the Federal Reserve Bank of St. Louis, or the Federal Reserve System.

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1 Introduction

What is the optimal-risk sharing arrangement among regions in a economic union while, at the same time, these regions make investment decisions? The amount of risk sharing among regions or countries, i.e. an economic union, is crucial in determining aggregate welfare. [1]

With idiosyncratic shocks, different regions may draw mutual benefits from risk-sharing arrangements; i.e. pooling resources at the regional level can help regions better deal with idiosyncratic needs as natural disasters. Capital accumulation and economic growth, on the other hand, are also key in determining aggregate welfare since they determine the stock of wealth available for consumption and investment.

The presence of private information is one of the main impediments that limits risk sharing in an economic union. Exogenous regional circumstances change over time and indeed these idiosyncratic characteristics of the regions that enter the ex-ante optimal allocation arraignment may be difficult to observe or incorporate in legislation as verifiable contingencies. For instance, as emphasized by Bordignon, M., P. Manasse and G. Tabellini (2001), "sheer lack of information is certainly relevant in the case of loose confederations such as the European Union". Therefore, there is a problem of asymmetric information: the optimal arrangement is restricted and cannot be simply contingent on all the information available to regional governments. There is a need to provide incentives to report truthfully the idiosyncratic characteristic privately observed. Studying the optimal dynamic arrangement in this situation is particularly relevant because of its potential effect on investment and growth.

This paper explores the tradeoffs between insurance and growth in a two-country economic union in which idiosyncratic needs are represented by privately observed preference

[1] This argument is often made with reference to Europe, where the single currency and tight limits on budget deficits have reduced the ability of national governments to cope with idiosyncratic shocks.

shocks. There is a neoclassical production technology to produce the consumption good. Each period, consumers in each region experience unpredictable, idiosyncratic, privately observed taste shocks affecting their marginal utility of current consumption. Efficiency dictates that more resources be allocated to those consumers who, in any given period, have a high marginal utility of current consumption, due to a high value of their taste shock. But since regional shocks are not observable, the efficient allocation of resources in this environment is impeded by the problem of incentive compatibility: if consumers in a region who report a high value of the taste shock receive more current consumption, then all the consumers in other have an incentive to misreport their current taste shock to receive the same treatment.

The problem of incentive compatibility is solved in this environment by conditioning consumption and investment on aggregate capital, on current reports of the taste shocks and on the history of members' past reports. We examine the dynamics of investment and the distribution of consumption that may be induced by the need for incentive compatibility on normative as opposed to positive grounds. To attack this problem, we develop a general recursive method for computing efficient feasible allocations in dynamic risk-sharing environments with multilateral private information. A key feature of the environments that we consider is that the evolution of the state of the economy is endogenous, since it depends on actions taken by the agents. Our approach complements the standard methods pioneered by the seminal work of Spear and Srivastava (20) and Abreu, Pearce and Stacchetti (2, APS hereafter), while it remains computationally tractable.

The study of dynamic risk-sharing problems with private information has been a fast growing area of research. Included in this literature are models with privately observed endowment shocks (Green (11), Thomas and Worrall (21), Phelan (17), among others), privately observed taste shocks (Atkeson and Lucas (5)), and privately observed effort levels

(Spear and Srivastava (20), Phelan and Townsend (18), Hopenhayn and Nicolini (12), among others). Except for some exceptions detailed below, most of the existing literature has avoided to study multilateral private information with endogenous aggregate state variables because of the computational difficulties. Since in most dynamic risk-sharing problems with multilateral private information the presence of this sort of intertemporal links is a natural feature, there is a need for sorting our this shortage.

Efficient allocations in those settings are history dependent in general, and so standard recursive methods might not apply.[2] Spear and Srivastava (20) and Abreu, Pearce and Stacchetti (2) show that, indeed, the inclusion of utility levels as endogenous state variables suffices to sort out history dependence. As the physical environment remains unchanged, this method leads to compute directly the utility possibility set. Furthermore, to study environments with endogenous aggregate state variables (i.e., the physical environment changes endogenously as time and uncertainty unfold), the generalized APS approach, provided by Espino (9), bases on an operator that iterates and computes directly on the utility possibility correspondence. Thus, the value function (and the corresponding policy functions) are recovered from the frontier of the fixed point of that operator on correspondences. The evident complexity of these models leads to enrich the set of analytical findings with numerical results and, thus, the use of computational experiments has become crucial to further understand these models. But while theoretically feasible, a method that bases on iterations on correspondences makes the computation very demanding even in simple settings. Furthermore, there is a large class of models with endogenous state variables that are computationally

[2] First best efficiency dictates that more resources must be allocated to those agents who, in a given period, value current consumption more (in our example, those agents who have a have marginal utility due to a high preference shock). But since individual shocks are unobservable, the first best efficient allocation of consumption is impeded by the problem of incentive compatibility: in our setting, if agents who report a high preference shock receive more current consumption, then all the other agents have incentives to misreport their preference shocks to receive the same treatment. See Atkeson and Lucas (1992).

untractable. For a thorough discussion of these difficulties, see Sleet and Yeltekin (19).

Our method overcomes these difficulties since it bases on iterations on the efficient frontier directly whenever the utility possibility correspondence is convex. The numerical algorithm builds on characterizing the efficient frontier of a convex utility possibility correspondence by means of welfare weights attached to each agent. The efficient allocation determines a law of motion of these parameters through continuation welfare weights that play a role that is isomorphic to the continuation utilities in APS due to convexity. Thus, these weights become endogenous state variables that summarize the history. We argue that with the methods developed in this paper, a wide range of dynamic mechanism design problems can be analyzed which were previously considered to be intractable.

We compute a numerical example to illustrate the results of our model further. We find that the welfare cost of private information is non-monotone in the size of the country in need of insurance. Moreover, welfare costs, measured in consumption equivalent units, are about 0.5 percent. Studying the policy functions for consumption, we find that there is less insurance in the economy with private information. This happens because to induce truthtelling, the allocation under private information allocates less (more) consumption to reporting high (low) preference shocks. Interesting, this static feature of policy function is less important than the dynamic effect that private information imposes in our numerical examples. Specifically, to guarantee truthtelling, the allocation with private information displays history dependence: after reporting a high (low) shock, the welfare weight of Country 1 is reduced (increased). These variations in welfare weights imply that consumption of both countries is more volatile under private information. Finally, in our numerical examples, the distortion on the allocation of capital that is imposed by private information is minimal.

This paper is organized as follows. In Section 2 we describe the physical environment of the model. Next, in section 3 we define feasibility, incentive compatibility, efficiency, and show how efficient allocation can be represented with welfare weights. The recursive characterization of efficient allocations and the corresponding computational algorithm is in 4. Section 5 implements numerically our approach and provides examples that examine the welfare cost of private information. Conclusions are in section 6. Proofs are gathered in the Appendix 7.

2 Setup

Formation of the Economic Union. Consider a world with two countries indexed by $i = 1, 2$. The initial stocks of capital of each country are K_0^1 and K_0^2. They contribute their capital stocks to form the initial stock of capital of the Economic Union, $K_0 = K_0^1 + K_0^2$. In exchange, each economy was promised by the Economic Union certain level of expected discounted lifetime utility. These values of lifetime utility will be represented recursively by $w_1(K, \theta)$ and $w_2(K, \theta_1)$, where θ will represent the weight that lifetime utility of country 1 will have in the Economic Union.

Preferences. Assume each country's period utility is $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave and twice differentiable, and $\lim\limits_{c_t \to 0} u'(c_t) = +\infty$. Both countries discount the future at the rate $\beta \in (0, 1)$. Lifetime utility of country 1 is represented by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \; s_t \; u(c_1(s^t)),$$

Similarly, country 2's preferences are represented by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_2(s^t)).$$

where $s^t = (s_0, ..., s_t) \in S^t$ denotes the aggregate history of events from date 0 to date t with probability $\pi(s^t) = \pi(s_0)...\pi(s_t)$.

The key difference between these countries is that Country 1 is subject to a preference shock s_t. This idiosyncratic preference shock takes one of two values, $s_{H,t} > s_{L,t}$. This shock is assumed to be i.i.d. across time where $\pi_j > 0$ is the probability of $s_{j,t}$ for every t.

Technology. The aggregate stock of capital, K, depreciates at the rate $\delta \in (0, 1)$. Output is produced with a constant returns to scale production function, $F(K, L)$, where L denotes total labor. The aggregate endowment of labor is 1 and is supplied inelastically in the labor market. The initial stock of capital at date 0 is denoted by $K_0 > 0$. Given the technological assumptions, there exists some $\overline{K} > 0$ such that $X = [0, \overline{K}]$ denotes the sustainable levels of capital.

Information. At the beginning of each period, Country 1 privately observe shock s_t. Since the Revelation Principle applies, the problem of the Economic Union will be restricted to the allocations that induce Country 1 to truthfully report s_t. There is no way to audit or verify the answer that any agent chooses to give.

3 Finding the Optimal Allocation

To solve for the optimal arrangement, consider a fictitious planner that allocates optimally incentive compatible allocations that are feasible within the economic union.

Feasibility. At date t, country 1 has privately observed the partial history s^t. It reports $z(s^t) \in S_t$ at date t. Let $z = \{z(s^t)\}_{t=0}^{\infty}$ represent country $1's$ reporting strategy where

$z_t : S^t \to S_t$ for all t. Let z^* be the truthtelling reporting strategy for which $z^*(s^t) = s_t$ for all t and $s^t \in S^t$.

Let $K' = \{K_{t+1}\}_{t=0}^{\infty}$ be an *investment rule* which every period allocates next period capital, given a history of reports; i.e. $K_{t+1} : S^t \to \mathbb{R}_+^2$ for all t. Similarly, let $V_i = \{V_{i,t}\}_{t=0}^{\infty}$ be a *utility transfer* where $V_{i,t} : S^t \to \mathbb{R}_+$ for all t. To interpret this, consider any realization s^t up to date t and any reporting strategy z. Country $1's$ consumption at date t is given by $u^{-1}\left(V_i(z^t(s^t))\right) \geq 0$. Similarly, the aggregate stock of capital at date $t+1$ will be given by $K(z^t(s^t)) \geq 0$. Any (V, K') satisfying these properties is called a *sequential allocation*.

Definition 1 *Given K_0, a sequential allocation (V_1, V_2, K') is* ***feasible*** *if for all reporting strategies z, all t, all s^t and $K_0 = K$*

$$K(z^t(s^t)) + u^{-1}\left(V_1(z^t(s^t))\right) + u^{-1}\left(V_2(z^t(s^t))\right) \leq F(K(z^{t-1}(s^{t-1})), L) + (1-\delta)K(z^{t-1}(s^{t-1})). \tag{1}$$

Incentive compatibility. Let s^{t-1} be any arbitrary partial history reported. Let $\widehat{z}$ be an continuation reporting strategy from period t onwards. Given a sequential allocation (V, K'), Country $1's$ utility at date t is

$$\begin{aligned} & U_{1,t}(V, K', \widehat{z} \| s^{t-1}) \\ = \ & \sum_{j=0}^{\infty} \beta^j \sum_{s^{j+1}} \pi(s^{j+1}) \ s_{t+j} \ V_1(s^{t-1}, \widehat{z}^j(s^j)), \\ = \ & \sum_{s_t \in S_t} \pi(s_t) \ \left(s_t \ V_1(s^{t-1}, \widehat{z}_0(s_t) + \beta \ U_{1,t+1}(V, K', \widehat{z}'(s_t) \| (s^{t-1}, \widehat{z}_0(s_t)))\right) \end{aligned}$$

where $\widehat{z}'(s)$ is the continuation reporting strategy from period $t+1$ onwards induced by $\widehat{z}$ when the first element s is kept constant. When $t = 0$, we write $U_1(V, K', z) = U_{1,0}(V, K', z)$

for any z. Similarly,

$$\begin{aligned}
& U_{2,t}(V, K', \widehat{z} \| s^{t-1}) \\
= & \sum_{j=0}^{\infty} \beta^j \sum_{s^{j+1}} \pi(s^{j+1}) \ V_2(s^{t-1}, \widehat{z}^j(s^j)), \\
= & \sum_{s_t \in S_t} \pi(s_t) \ \left(V_2(s^{t-1}, \widehat{z}_0(s_t) + \beta \ U_{2,t+1}(V, K', \widehat{z}'(s_t) \| (s^{t-1}, \widehat{z}_0(s_t))) \right)
\end{aligned}$$

The following definition says that an allocation is *incentive compatible* if truthtelling is country 1's best response.

Definition 2 *Given $K \in X$, a sequential allocation (V, K') is* ***incentive compatible*** *if for country 1, for all $t \geq 0$, all s^{t-1} and any continuation z'*

$$s_t \ V_1(s^{t-1}, s_t) + \beta \ U_{1,t+1}(V, K', z^* \| (s^{t-1}, s_t)) \geq s_t \ V_1(s^{t-1}, \widetilde{s}_t) + \beta \ U_{1,t+1}(V, K', z' \| (s^{t-1}, \widetilde{s}_t)) \tag{2}$$

for all $(s_t, \widetilde{s}_t)$.

Efficiency. Let $\Psi(K)$ be the utility possibility correspondence for this economy. That is,

$$\Psi(K) \equiv \left\{ w \in \mathbb{R}^2_+ : \exists (V, K') \text{ satisfying (1) and (2), and } w_i \leq U_i(U, L, K', z^*) \ i = 1, 2, \ K_0 = K \right\} \tag{3}$$

An incentive compatible, feasible sequential allocation (V^*, K'^*) is efficient if there is no alternative incentive compatible, feasible sequential allocation (V, K') such that$U_i(V, K', z^*)$ $> U_i(V^*, K'^*, z^*)$ for all i. Later, we show that $\Psi(K)$ is convex for all $K \in X$ and the set of efficient allocations can be parameterized by welfare weights $\theta \in \mathbb{R}^2_+$. Thus, (V^*, K'^*) is efficient if it solves

$$h^*(K, \theta) = \sup_{w \,\in\, \Psi(K)} \sum_{i=1,2} \theta_i \ w_i, \tag{4}$$

for some $\theta \in \mathbb{R}^2_+$. The next Lemma is one of the key steps to make our alternative approach computationally simpler.

Lemma 1 $w \in \Psi(K)$ if and only if $w \geq 0$ and

$$\min_{\widetilde{\theta} \in \Delta^{I-1}} \left[h^*(K, \widetilde{\theta}) - \sum_{i=1}^{I} \widetilde{\theta}_i \ w_i \right] \geq 0.$$

The next Lemma shows that the utility possibility correspondence is convex-valued, a result that is key to apply our alternative solution method. Notice that it is key that the incentive compatibility constraints (2) do not impose non-convexities due to the linearity of the utility function with respect to a sequential allocation.

Lemma 2 $\Psi(K)$ *is convex for all* $K \in X$.

An incentive compatible, feasible sequential allocation (V^*, K'^*) is efficient if there is no alternative incentive compatible, feasible sequential allocation (V, K') such that$U_i(V, K', z^*)$ $> U_i(V^*, K'^*, z^*)$ for all i.

Representation of Efficient Allocations. The next result shows that the set of efficient allocations can be parameterized by welfare weights. It is a direct consequence of Lemma 2.

Corollary 1 *The set of efficient allocations can be parameterized by welfare weights* $\theta \in \mathbb{R}^I_+$*. That is,* (V^*, K'^*) *is efficient if and only if* (V^*, K'^*) *is the corresponding allocation that solves*

$$h^*(K, \theta) = \sup_{w \ \in \ \Psi(K)} \sum_{i=1}^{I} \theta_i \ w_i, \tag{5}$$

for some $\theta \in \mathbb{R}^I_+$.

The next Lemma is a direct consequence of Lemma 2 and Corollary 1.

Lemma 3 $w \in \Psi(K)$ if and only if $w \geq 0$ and

$$\min_{\widetilde{\theta} \in \Delta^{I-1}} \left[h^*(K, \widetilde{\theta}) - \sum_{i=1}^{I} \widetilde{\theta}_i \; w_i \right] \geq 0.$$

Let $\mathcal{W}(k)$ denote the constraint correspondence defined by (7) - (9) at any arbitrary initial stock of capital k. Any $(v, w', k') \in \mathcal{W}(k)$ will be referred as a *feasible incentive compatible recursive allocation*. We say that $h \in F_H$ is *preserved under* T if $h(k, \theta) \leq (Th)(k, \theta)$ for all (k, θ).

Proposition 4 *If $h \in F_H$ is preserved under T, then*

$$(Th)(k, \theta) \leq h^*(k, \theta)$$

for all (k, θ).

The intuition for this result can be interpreted as follows. If a function h is preserved under T, any value of h can also be attained by some feasible incentive compatible recursive allocation. This allows one to choose the value of any feasible incentive compatible recursive allocation and transform it period-by-period into a feasible incentive compatible sequential allocation delivering those levels of utility.

4 Recursive problem

The problem of the Economic Union is to maximize the weighted sum of lifetime utility of the members. Using the recursive representation this problem can be studied with the operator

$$(Th)(k, \theta) = \max_{(v, w', k')} \sum_s \pi(s) \left\{ \theta_1 \left[s_1 v_1(s) + \beta w_1'(s) \right] + \theta_2 \left[v_2(s) + \beta w_2'(s) \right] \right\}, \tag{6}$$

subject to

$$k'(s) + u^{-1}(v_1(s)) + u^{-1}(v_2(s)) = f(k) + (1-\delta)k \tag{7}$$

$$sv_1(s) + \beta w_1'(s) \geq sv_1(\widetilde{s}) + \beta w_1'(\widetilde{s})$$

for all $(s, \widetilde{s})$ and

$$v_i(s) \geq 0, \qquad w_i'(s) \geq 0 \qquad \text{for all } s \text{ and all } i, \tag{8}$$

$$\min_{\theta' \in \Delta^{I-1}} \left[h(k'(s), \theta'(s)) - \theta_1'(s) w_1'(s) - \theta_2'(s) w_2'(s) \right] \geq 0 \qquad \text{for all } \theta' \text{ and } s. \tag{9}$$

Let $\left(\widehat{v}, \widehat{k}', \widehat{w}'\right)$ denote the set of policy functions solving (7) - (9) while θ' denotes the corresponding continuation welfare weight. Given (k_0, θ_0), we say that a set of policy functions $\left(\widehat{v}, \widehat{k}', \widehat{w}'\right)$ generates an sequential allocation $(\widehat{V}, \widehat{K}')$ if

$$\begin{aligned} \widehat{V}_i(s^t) &= \widehat{v}_i(K(s^{t-1}), \theta(s^{t-1}))(s_t), \\ \theta(s^{t-1}, s_t) &= \widehat{\theta}'(K(s^{t-1}), \theta(s^{t-1}))(s_t), \\ K(s^{t-1}, s_t) &= k'(K(s^{t-1}), \theta(s^{t-1}))(s_t), \end{aligned} \tag{10}$$

for $i = 1, 2$ and all (t, s^t) where K_0 is given. In the Appendix we prove the next result.

Proposition 5 *$h^*(k, \theta) = (Th^*)(k, \theta)$ for all (k, θ). Moreover, an allocation (V^*, K'^*) is efficient at K_0 if and only if it is generated by the set of policy functions as (10).*

Thus, the value of any plan that can be attained with a feasible incentive compatible sequential allocation (V, K') can also be attained by delivering to each agent current utility for today and promising to each agent some contingent levels of expected utility from tomorrow on that satisfy the incentive compatibility constraints, attached to the corresponding welfare weight.

Algorithm. How can h^* be obtained? Let h^{**} be the value function solving the recursive problem for which the incentive compatibility constraints is ignored while the corresponding operator is denoted $\widehat{T}$. Evidently, $h^*(k,\theta) \leq h^{**}(k,\theta)$ for all (k,θ). The solution algorithm is based in the result of the next proposition.

Proposition 6 *Let* $h_0 = h^{**}$ *and denote* $h_n = T^n\left(h^{**}\right)$. *Then,* $\{h_n\}$ *is a monotone decreasing sequence and* $\lim_{n\to\infty} h_n = h^*$.

The next Corollary follows because the operator T preserves concavity with respect to k and convexity with respect to θ. Differentiability follows from an application of Benveniste-Scheinkman's theorem.

Corollary 2 h^* *is concave in* k *and convex in* θ. *Moreover,* h^* *is differentiable in the interior of* $X \times \Delta^{I-1}$.

Now, since h^* is convex and differentiable in θ, then the *continuation welfare weights* can be written

$$\frac{\partial h^*}{\partial \theta_i'}(k'(s),\theta') = w_i'(s)$$

for $i = 1, 2$.

Discussion of the Method. The main gain with respect to the traditional Abreu, Pearce, and Stacchetti (1990) approach is in terms of applicability and tractability. Our approach lets to identify attainable levels of next period utility by iterating directly on the efficient utility possibility frontier with no need of knowing the utility possibility correspondence a priori. This greatly simplifies the computational burden.

To understand the difficulty in numerically solving the problem using APS approach, consider an alternative approach to state the dynamic program defined above. Instead of parameterizing allocations with welfare weights, the planner chooses current feasible consumption, capital and continuation utilities for both agents in order to maximize the utility of country 1 subject to two restrictions: (i) the utility of country 2 is above some pre-specified level (the so-called promise keeping constraint); (ii) allocations are period-by-period incentive compatible and (iii) continuation utility levels lie in the next period utility possibility correspondence. In this case, the endogenous state variable that summarizes the history stemming from private information are the utility levels itself. The issue is that condition (iii) implies that continuation utilities must belong to the feasible set defined by the utility possibility correspondence. This fact forces to compute this correspondence and it is basically identical to recover the value function (and the corresponding policy functions) from the frontier of the fixed point of that operator that iterates directly on correspondences. Perhaps due to these difficulties, there are almost no examples in the literature that actually apply APS to solve the model numerically—an exception is Ábrahám and Pavoni (2008).

Of course, the APS approach outperforms ours if one needs to consider a problem in which the utility possibility correspondence is not convex-valued. Indeed, our approach (at least as stated in this paper) is not appropriate to handle those problems.

5 Numerical Example

In this section we present the results of solving for the optimal allocation using the approach discussed above. To do so, we need to set the values for the model's parameters. The values used are described in Table 1. Most of the parameter are standard in macroeconomics. Other, like the size and probability of the demand shocks were chosen just to illustrate the model's solution.[3]

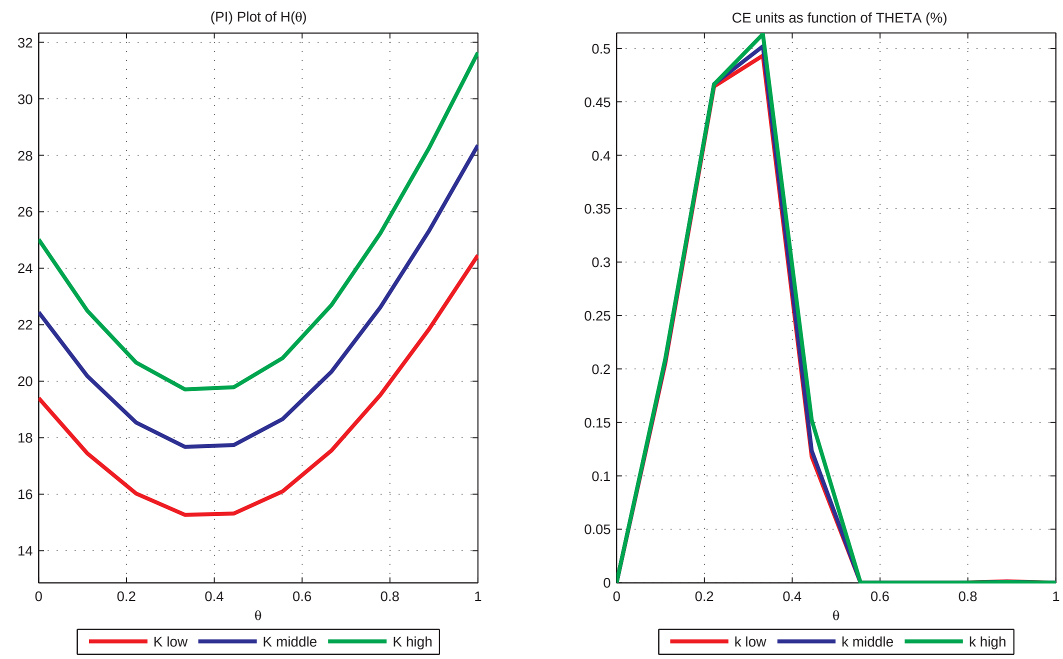
[3]We have solved the model for many other similar parameter values as robustness. The main result in this section do not depend of the choice of parameters.

Table 1: Parameter values

Variable	Value	Description
$f(K)$	K^{α}	Production Function
$u(c)$	$\frac{c^{(1-\sigma)}}{1-\sigma}$	Utility Function
δ	0.07	Depreciation Rate
α	0.3	Capital share
β	0.9	Discount Factor
σ	0.5	CRRA
$\pi(L)$	0.5	Probability state Low
$\pi(H)$	0.5	Probability state High
$s(L)$	1	State Low
$s(H)$	1.5	State High

Welfare. The left panel of Figure 1 displays the value function $h*$. This function has the properties that we expected. It is convex in θ and increasing and concave in K. More interesting, the right panel of Figure 1 presents the cost of private information as a function of θ and K. To make this exercises comparable with previous literature, we translated the difference between the value functions with private information and first best into consumption equivalent (CE) units. This measures how much (in percentage terms) consumption must be increased in every period in the economy with private information to equalize welfare with the unconstrained optimal allocation. Notice that this exercise is performed for different initial values of given a particular initial state—values of θ and K. At least two features are worth mentioning. First, welfare cost of private information in non-monotonic in the size (or more precisely, in the weight in the planner problem) of the country with the privately observed shock. If this country is very tiny, toward the left of this figure, welfare costs are almost zero because providing insurance does not distort the consumption of the other country significantly. If this country is very large (or if its weight in the problem of the Economic Union is larger than 0.55) the first best allocation can be supported. This happens

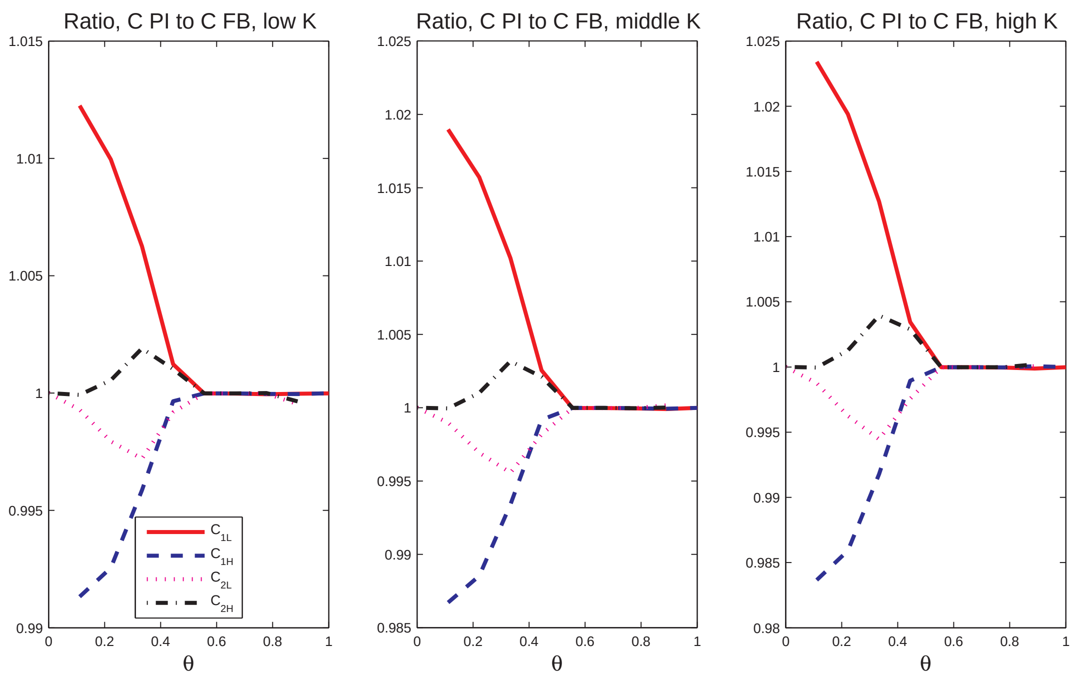
Figure 1: Value function and the welfare costs



because by misreporting this country would be harming the Economic Union of which he is the main member.

The second result of Figure 1 that is worth mentioning is that the size of the welfare cost of private information is significant. At the peak, the cost is 0.5 percent. This is larger than the cost of business cycle in the US in the post war period, estimated by Lucas (1987) to be about 0.05 and 0.1 percent. And it is smaller than the welfare gains of setting the probability of a Great Depression to zero, estimated by Chatterjee and Corbae (2007) to be between 1 and 7 percent.

Figure 2: Consumption under first best and private information



Consumption. Now focus on Figure 2. The main difference between the allocations with perfect and private information are clear in this Figure. It plots the consumption of private information relative to the consumption in the first best, as a function of the welfare weight of Country 1. The three different figures illustrate the role of capital.

First consider the red solid line. It is the consumption of Country 1 when it reports a low shock. The fact that it is greater than one means that under private information this consumption it is larger than what it would be with perfect information. This should be interpreted together with the blue dashed line: the consumption of Country 1 after it reported a high preference shock. This consumption ratio is smaller than one, meaning that under private information is lower than with perfect information. Together, this shows that under private information Country 1 cannot be helped as much as under perfect information when it reports a high demand shock. To make this report compatible with incentives, Country 1 gets more when it reports a low shocks and less when it reports a high shock, compared to the economy with perfect information.

Now focus on the consumption of Country 2 in Figure 2. The consumption in the state a high preference shock is reported by Country 1, Country 2 consumes more than un perfect information. This is just a consequence of what happen with Country 1. The non-monotonicity of Country 2 consumption in θ just reflects the non-monotonicity of the welfare cost of private information. This suggests that the share of welfare cost is a consequence of the distortions to Country 2's consumption.

Time series. To illustrate the model further we generated the model's allocation for 100,000 periods. Figure 3 presents the behavior of capital and the welfare weight of Country 1 for a selected subsample. In this case, the model is started with small θ. Remember that theta is constant in the first best allocation.

This is a period that, although the probability of high and low shocks is 1/2, there are more low shocks. Thus, this is a period in which Country 1 asks for less help than what it would be expected. The most striking consequence is on the right part of Figure 3. The weight of this country in the problem of the Economic Union increases notoriously. Notice that after 120 periods, the welfare weight that country 1 would have under perfect information is only 80 percent of the value under private information. Changes in the welfare weight of Country 1 affect the allocation of capital, too. In the left part of Figure 3 how the lager role of Country 1 in the Economic Union makes investment more responsive to shocks.

Finally, Table 2 shows the main statistics for the 100,000 periods of data generated with

Figure 3: Time series, small θ

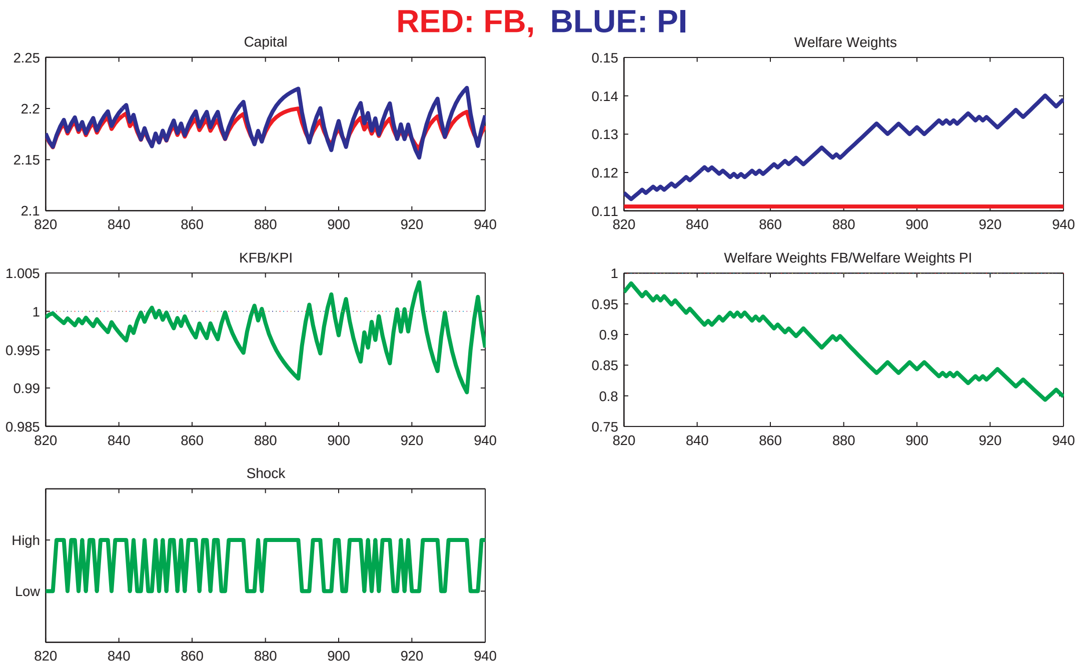


Table 2: Time series, main statistics

	Mean		Standard Deviation		Correlation with Output	
	First best	Private information	First best	Private information	First best	Private information
Output	1.31	1.31	0.025	0.025	1.00	1.00
Net investment	0.00	0.00	0.088	0.085	0.361	0.360
Consumption 1	0.31	0.26	0.102	0.161	-0.603	-0.279
Consumption 2	0.78	0.83	0.054	0.141	0.923	0.240

the model.[4] Several consistent patterns are evident.

First consider the first two columns that refer to the mean of the variables. There is clear that investment and output are not significantly affected by private information. Although the frameworks are different in several dimensions, this result is in line with the findings in Marcet and Marimon (1992).[5] The mean of consumptions are different, though. On average, about 4 percent of the aggregate output is transferred from Country 1 (the one with the shock) to Country 2. The second stylized fact is that the volatility of consumption increases for both agents. This is a consequence of varying welfare weights.

6 Conclusion

Private information may prevent insurance in an economic union, even when long term relationship are considered. Studying these arrangements in economies with capital accumulation have been proved almost untractable, though. This paper proposes an approach that can be used to characterize this problem. The approach identifies attainable levels of next period utility by iterating directly on the efficient utility possibility frontier with no need of knowing the utility possibility correspondence a priori. This greatly simplifies the

[4]Actually, the first 500 observation were eliminated before computing the statistics.

[5]They examine a two-agent model where a risk-neutral investor with unlimited resources invests in the technology of a risk-averse producer whose output is subject to privately observed productivity shocks.

computational burden.

The optimal arrangement implies that the Economic Union must decide how much help it should provide to a country in need of resources, and how to finance those transfers. The restrictions to these transfers imposed by private information may even affect the viability of the Economic Union.

A numerical example illustrates some features of the optimal arrangement. Welfare cost of private information is non-monontone in the size of the country in need of insurance. Moreover, welfare costs, measured in consumption equivalent units, are about 0.5 percent. Consumption increases less in the country with a high preference shock in the economy with private information. Moreover, history dependence in the economy with private information implies variations in welfare weights that increase consumption volatility in both countries. Finally, the distortion on the allocation of capital is minimal.

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7 Appendix

Proof of Lemma 2. Let w and $\widetilde{w} \in \Psi(K)$ and take any $\lambda \in [0,1]$. We need to show that $\lambda w + (1-\lambda)\widetilde{w} \in \Psi(K)$.

Let $\left(\widehat{V}, \widehat{K}'\right)$ and $\left(\widetilde{V}, \widetilde{K}'\right)$ be the corresponding incentive compatible, feasible sequential allocation such that

$$w \leq U\left(\widehat{V}, \widehat{K}', z^*\right) \text{ and } \widetilde{w} \leq U\left(\widetilde{V}, \widetilde{K}', z^*\right).$$

Define $\left(V^\lambda, K'^\lambda\right) = \lambda\left(\widehat{V}, \widehat{K}'\right) + (1-\lambda)\left(\widetilde{V}, \widetilde{K}'\right)$ and note that the linearity of $U(., z^*)$ with respect to (V, K') implies that

$$\lambda U\left(\widehat{V}, \widehat{K}', z^*\right) + (1-\lambda)U\left(\widetilde{V}, \widetilde{K}', z^*\right) = U\left(V^\lambda, K'^\lambda, z^*\right) \geq \lambda w + (1-\lambda)\widetilde{w}.$$

Importantly, observe that $\left(V^\lambda, K'^\lambda\right)$ satisfy (2) since these constraints are linear in V.

We only need to show that $\left(V^\lambda, K'^\lambda\right)$ is feasible. To see this, observe that $(u_i)^{-1}$ is convex given that u_i is concave and therefore, since F_i is concave, we have that

$$\begin{aligned}
& \sum_{i=1}^{I}\left(K_i^\lambda(z^t(s^t)) + u_i^{-1}\left(V_i^\lambda(z^t(s^t))\right)\right) \\
\leq \; & \sum_{i=1}^{I}\left(\lambda\,\widehat{K_i}(z^t(s^t)) + (1-\lambda)\,\widetilde{K_i}(z^t(s^t)) + \lambda u_i^{-1}\left(\widehat{V}_i(z^t(s^t))\right) + (1-\lambda)\sum_{i=1}^{I} u_i^{-1}\left(\widetilde{V}_i(z^t(s^t))\right)\right) \\
\leq \; & \lambda\,\sum_{i=1}^{I}\left(F_i\left(\widehat{K}_i(z^{t-1}(s^{t-1})), 1\right) + (1-\delta)\widehat{K}_i(z^{t-1}(s^{t-1}))\right) \\
& +(1-\lambda)\,\sum_{i=1}^{I}\left(F_i\left(\widetilde{K}_i(z^{t-1}(s^{t-1})), 1\right) + (1-\delta)\widetilde{K}_i(z^{t-1}(s^{t-1}))\right) \\
\leq \; & \sum_{i=1}^{I}\left(F\left(K_i^\lambda(z^{t-1}(s^{t-1})), 1\right) + (1-\delta)K_i^\lambda(z^{t-1}(s^{t-1}))\right)
\end{aligned}$$

where $K_0 = K$. Consequently, we can conclude that $\lambda w + (1-\lambda)\widetilde{w} \in \Psi(K)$. ■

Proof of Proposition 4. Take any arbitrary $\left(\widehat{v}_0(s_0), \widehat{k}_1'(s_0), \widehat{w}_0'(s_0)\right)_{s_0 \in S} \in \mathcal{W}(k_0)$ while $\theta'(s_0)$ denotes the corresponding vector of welfare weights such that

$$\widehat{\theta}'(s_0) = argmin_{\theta' \in \Delta^{I-1}}\left[f(\widehat{k}_1'(s_0), \theta') - \sum_{i=1}^{I}\theta_i'\; w_{i,0}'(s_0)\right]$$

Since f is preserved under T, we have that

$$\widehat{\theta}'(s_0)\widehat{w}_0'(s_0) \leq f(\widehat{k}_1'(s_0), \widehat{\theta}'(s_0)) \leq (Tf)(\widehat{k}_1'(s_0), \widehat{\theta}'(s_0)),$$

and therefore $\widehat{w}_0'(s_0)$ is a vector of utility levels that are attainable given $\widehat{\theta}'(s_0)$. This means that, for each $s_0 \in S$, there exists some $\left(\widehat{v}_1(s_0, s_1), \widehat{k}_2'(s_0, s_1), \widehat{w}_1'(s_0, s_1)\right)_{s_1 \in S} \in \mathcal{W}(\widehat{k}_1'(s_0))$ such that

$$\widehat{w}_0'(s_0) = \sum_{s_1}\pi(s_1)\left[s_i\;\widehat{v}_{i,1}(s_0, s_1) + \beta\;\widehat{w}_{i,1}'(s_0, s_1)\right].$$

Following this strategy repeatedly T times, we can conclude that for any arbitrary $\theta_0 \in \Delta^{I-1}$

$$
\begin{aligned}
& \sum_{i=1}^{I} \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) \left[s_i \ \widehat{v}_{i,0}(s_0) + \beta \ \widehat{w}'_{i,0}(s_0) \right] \right\} \\
= & \sum_{i=1}^{I} \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) s_i \ \widehat{v}_{i,0}(s_0) \right. \\
& \left. + \beta \sum_{s_0} \pi(s_0) \ \sum_{s_1} \pi(s_1) \left[s_i \ \widehat{v}_{i,1}(s_0, s_1) + \beta \ \widehat{w}'_{i,1}(s_0, s_1) \right] \right\} \\
= & \sum_{i=1}^{I} \theta_{i,0} \ E\left(\sum_{t=0}^{T} \beta^t \ s_{i,t} \widehat{v}_{i,t} \right) + \beta^{T+1} \sum_{i=1}^{I} \theta_{i,0} \ E\left(\widehat{w}'_{i,T+1} \right) \\
\leq & \sum_{i=1}^{I} \theta_{i,0} \ E\left(\sum_{t=0}^{T} \beta^t \ s_{i,t} \widehat{v}_{i,t} \right) + \beta^{T+1} \left\| f \right\| \\
\leq & \sum_{i=1}^{I} \theta_{i,0} \ E\left(\sum_{t=0}^{\infty} \beta^t \ s_{i,t} \widehat{v}_{i,t} \right)
\end{aligned}
$$

where the first inequality holds since $f \in F_H$ and (4) while the last one follows from the Dominated Convergence Theorem and $\beta \in (0, 1)$.

Notice that $(\widehat{v}, \widehat{k}')$ is sequentially feasible by construction. Now we argue that it is incentive compatible as well. To see this, denote recursively $W_{i,t}(s^t) = \widehat{w}'_{i,t}(s_0, ..., s_t)$ and observe that by construction

$$
\begin{aligned}
& \left| U_{i,t}(\widehat{v}, \widehat{k}')(s^t) - W_{i,t}(s^t) \right| \\
= & \ \beta \left| \sum_{s_{t+1}} \pi(s_{t+1}) \left(U_{i,t}(\widehat{v}, \widehat{k}')(s^t, s_{t+1}) - W_{i,t+1}(s^t, s_{t+1}) \right) \right| \\
\leq & \ \beta \sup_{s_{t+1}} \left| U_{i,t}(\widehat{v}, \widehat{k}')(s^t, s_{t+1}) - W_{i,t+1}(s^t, s_{t+1}) \right| \\
\leq & \ \beta^k \sup_{(s_{t+1}, \ldots s_{t+k})} \left| U_{i,t}(\widehat{v}, \widehat{k}')(s^t, s_{t+1}, ..., s_{t+k}) - W_{i,t+k}(s^t, s_{t+1}, ..., s_{t+k}) \right|
\end{aligned}
$$

Observe that $0 \leq W_{i,t}(s^t) \leq \|f\| < \infty$ for all i and all s^t while $\widehat{v}$ is uniformly bounded by construction. Taking the $\limsup$ as $k \to \infty$ for this last expression, we can conclude that $U_{i,t}(\widehat{v}, \widehat{k}')(s^t) = W_{i,t}(s^t)$ for all i and all s^t and, then, incentive compatibility follows immediately.

Since both $\left(\widehat{v}_0(s_0), \widehat{k}'_1(s_0), \widehat{w}'_0(s_0) \right)_{s_0} \in \mathcal{W}(k_0)$ and the corresponding sequential allocation $(\widehat{v}, \widehat{k})$ are arbitrary, we can conclude that

$$
\begin{aligned}
& \sum_{i=1}^{I} \theta_i \left\{ \sum_{s_0} \pi(s_0) \left[s_i \ \widehat{v}_{i,0}(s_0) + \beta \ \widehat{w}'_{i,0}(s_0) \right] \right\} \\
\leq & \sum_{i=1}^{I} \theta_i \ E\left(\sum_{t=0}^{\infty} \beta^t \ s_{i,t} \widehat{v}_{i,t} \right) \\
\leq & \ h^*(k, \theta)
\end{aligned}
$$

and therefore, since weak inequalities are preserved in the limit, we have that

$$
\begin{aligned}
Tf(k, \theta) & = \max_{(\widehat{v}, \widehat{k}', \widehat{w}') \in \mathcal{W}(k_0)} \sum_{i=1}^{I} \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) \left[s_i \ \widehat{v}_i + \beta \ \widehat{w}'_i \right] \right\} \\
& \leq h^*(k, \theta).
\end{aligned}
$$

■

Proof of Proposition 5. Given (K,θ), take any $w \in \Psi(K)$ for which (V,K') denotes the corresponding feasible incentive compatible allocation. Observe that

$$\sum_{i=1}^{I} \theta_i \; U_i(V,K') = \sum_{i=1}^{I} \theta_i \sum_{s_0 \in S_0} \pi(s_0) \; [s_{i,0} \; V_i(s_0) + \beta \; U_{i,1}(V,K' \| (s_0))]$$

Notice that $(U_{i,1}(V,K'\|(s_0)))_{i=1}^{I} \in \Psi(K(s_0))$ for all s_0. It follows by Lemma 3 that

$$h^*(K'(s_0),\theta') \geq \sum_{i=1}^{I} \theta_i' \; U_{i,1}(V,K'\|(s_0))$$

for all $\theta' \in \Delta^{I-1}$ and all s_0. Therefore, $(V_i, K', U_{i,1}(V,K'))_{i=1}^{I} \in \mathcal{W}(K_0)$ and then

$$\sum_{i=1}^{I} \theta_i \; U_i(V,K') \leq (Th^*)(K,\theta)$$

Since weak inequalities are preserved in the limit, we can conclude that

$$h^*(K,\theta) = \sup_{(V,K')} \sum_{i=1}^{I} \theta_i U_i(V,K') \leq (Th^*)(K,\theta),$$

for all (K,θ) (i.e. h^* is preserved under T) and thus $h^*(K,\theta) = (Th^*)(K,\theta)$ for all (K,θ). ■

Proof of Proposition 6. It is a routine exercise to show that T is a monotone operator (i.e., if $f \geq g$ then $Tf \geq Tg$). Also, observe that $h^* = Th^* \leq \widehat{T}h^* \leq \widehat{T}h^{**} = h^{**}$ by Proposition 5. Monotonicity implies that $T^n(h^{**}) \leq T^{n-1}(h^{**})$ and thus $h_n \geq h_{n+1} \geq h^*$. Since this implies that $\{h_n\}$ is a monotone decreasing sequence of uniformly bounded functions, then there exists a function $h_\infty \geq h^*$ such that $\lim_{n\to\infty} h_n = h_\infty$. It remains to show that $h_\infty \leq h^*$, for which it is sufficient that h_∞ is preserved under T due to Proposition 4.

Given (k,θ), $h_\infty(k,\theta) \leq h_n(k,\theta)$ implies that, for all n, there exists $(v^n, k'^n, w'^n) \in \mathcal{W}^n(k)$ that attains $h_\infty(k,\theta)$. Observe that (v^n, k'^n, w'^n) lies in a compact set and, thus, it has a convergent subsequence with limit point $\left(\widehat{v}, \widehat{k}', \widehat{w}'\right)$. Notice that incentive compatibility is preserved in the limit; also, we have that for all n and all s_0

$$h_n(\widehat{k}'(s_0),\theta') \geq \sum_{i=1}^{I} \theta_i' \; \widehat{w}_i'^n(s_0), \quad \text{for all } \theta'$$

and then

$$h_\infty(\widehat{k}'(s_0),\theta') \geq \sum_{i=1}^{I} \theta_i' \; \widehat{w}_i'(s_0),$$

Therefore, $\left(\widehat{v}, \widehat{k}', \widehat{w}'\right) \in \mathcal{W}^\infty(k)$ for which

$$h_\infty(k,\theta) = \sum_{i=1}^{I} \theta_i \sum_{s_0} \pi(s_0) \left[\widehat{v}_i(s_0) + \beta \; \widehat{w}_i'(s_0)\right].$$

Finally, since $\left(\widehat{v}, \widehat{k}', \widehat{w}'\right) \in \mathcal{W}^\infty(k)$ is arbitrary, we can conclude that

$$\begin{aligned} (Th_\infty)(k,\theta) &\geq \sum_{i=1}^{I} \theta_i \sum_{s_0} \pi(s_0) \left[\widehat{v}_i(s_0) + \beta \; \widehat{w}_i'(s_0)\right] \\ &= h_\infty(k,\theta), \end{aligned}$$

for all (k,θ) and thus h_∞ is preserved under T by definition. ■

Proof of Corollary convexity of h*. We only sketch convexity.

Let $\lambda \in (0,1)$, $\theta, \widetilde{\theta} \in \Delta^{I-1}$ and denote $\theta(\lambda) = \lambda\theta + (1-\lambda)\widetilde{\theta}$. Let $\left[(v(\theta) w'(\theta)), \left(v(\widetilde{\theta}), w'(\widetilde{\theta})\right), (v(\theta(\lambda)), w'(\theta(\lambda)))\right]$ be the corresponding feasible incentive compatible recursive allocation. Observe that

$$
\begin{aligned}
h^*(k,\theta(\lambda)) &= \sum_{i=1}^{I} \left(\lambda\theta_i + (1-\lambda)\widetilde{\theta}_i\right) \left\{\sum_s \pi(s) \left[s_i \; v_i(\theta(\lambda))(s) + \beta \; w_i'(\theta(\lambda))(s)\right]\right\} \\
&= \lambda \left(\sum_{i=1}^{I} \theta_i \left\{\sum_s \pi(s) \left[s_i \; v_i(\theta(\lambda))(s) + \beta \; w_i'(\theta(\lambda))(s)\right]\right\} + (1-\lambda) \sum_{i=1}^{I} \widetilde{\theta}_i \left\{\sum_s \pi(s) \left[s_i \; v_i(\theta(\lambda))(s) + \beta \; w_i'(\theta(\lambda))(s)\right]\right\}\right) \\
&\leq \lambda \sum_{i=1}^{I} \theta_i \left\{\sum_s \pi(s) \left[s_i \; v_i(\theta)(s) + \beta \; w_i'(\theta)(s)\right]\right\} + (1-\lambda) \sum_{i=1}^{I} \widetilde{\theta}_i \left\{\sum_s \pi(s) \left[s_i \; v_i(\widetilde{\theta})(s) + \beta \; w_i'(\widetilde{\theta})(s)\right]\right\} \\
&= \lambda h^*(k,\theta) + (1-\lambda) h^*(k,\widetilde{\theta})
\end{aligned}
$$

since $(v(\theta(\lambda)), w'(\theta(\lambda)))$ is incentive compatible and feasible for θ and $\widetilde{\theta}$. ■