

Optimal Monetary Responses to Asset Price Levels and Fluctuations: The Ramsey Problem and A Primal Approach

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Abstract

Should monetary policy react to asset prices levels and changes? In answering this question, we provide a tractable monetary Ramsey approach for a heterogeneous agents model with conventional policy (interest rate or money growth target) and unconventional policy (purchase of private illiquid assets) as instruments, in which heterogeneous agents' interaction is summarized in one implementability condition. We show that entrepreneurs hold too much liquid asset in a model with equity issuance and resale (liquidity) constraints. In the steady state, optimal policy involves paying interest on liquid assets or reducing the money supply available, leading to an equivalent increase of .40% in permanent consumption compared to the economy with no policy. In responding to liquidity shocks, the paths of macroeconomic variables under no policy and optimal policy are sharply different and suggest the need for policy on changing the rate of return on liquid assets. Finally, we prove that the unconventional policy dominates the conventional counterpart, but, quantitatively, the welfare difference of them is negligible.

1 Introduction

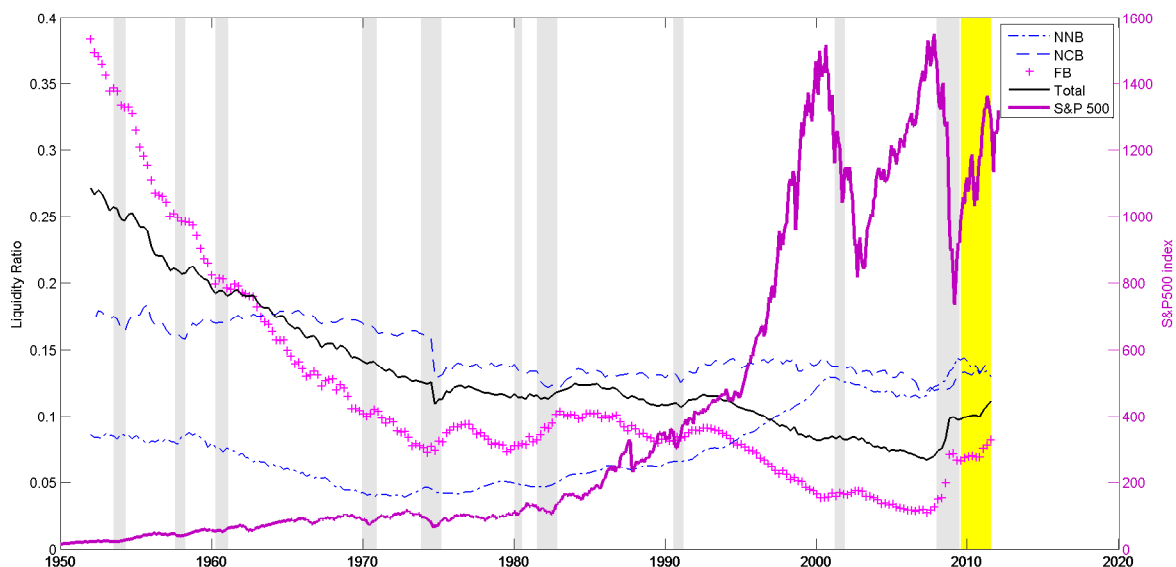
The recent financial crisis has shown that asset market liquidity fluctuations can have huge impacts on the real economy. As financial frictions widened, the economy plunged into a recession from 2007Q4 to 2009Q2. However, if one were to consider that one of the driving forces of the crisis was indeed the liquidity aspect transmitted to the real economy, it is surprising that there is no clear

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understanding on how should a central bank react or even whether it should or should not react, both in the academic literature and in the policy-making arena.

Monetary policy and asset portfolio allocation are intrinsically connected. Concerns about the interest rate, as well as forecasts about future inflation and output affect the portfolio allocation of low-return-liquid and high-return-illiquid assets. One way of seeing this is plotting liquid vs. illiquid compositions in portfolio as in Figure 1. During the last recession, the liquid share has increased, even though the equity price, as approximated by S&P 500, was already bouncing back in 2009. This change of liquidity ratio suggests that there is a large rebalancing of portfolio in all sectors during the past recession and periods after that. It is the relationship between policy, real economy activity and asset portfolio rebalancing that we investigate here.

Figure 1: Liquidity Ratio from 1952Q1



Liquidity ratio, S&P 500 Index and 3-Months Treasury Bills rate over time. Liquidity ratio is defined as the total liquid assets (check, deposit, tradable receivable and T-Bills while for financial sector with one more source, net over-night interbank lending) over total assets (for financial sectors total assets adjusted by the required reserve in central bank). NNB stands for non-farm and non-corporate business. NCB stands for non-farm and corporate business. FB stands for financial business excluding central bank. Source: 1952Q1-2011Q3 Flow of Funds Table, F102, F103, B102 and B103, Federal Reserve Z1 statistical release. S&P 500 index is obtained from Yahoo Finance and 3-months T-Bills rate is from Federal Reserve Bank at St.Louis. The grey shaded region depicts the NBER recession period and the yellow region depicts the period after 2007-2009 recession until 2011Q3.

The question we aim to tackle, therefore, is how should optimal policy behave in a liquidity constrained economy. By providing a framework to understand such question, we are also able to discuss conventional and unconventional monetary policies and some of the main trade-offs that the FED was dealing with during the crisis, providing us with positive and normative analysis.

The reason for stressing the monetary side is that the financial turmoil has shown that monetary

policy has much more discretionary ability than fiscal policy, at least in the short run. Moreover, we discuss two possible types of monetary instruments. The first one uses only liquid assets, usually called the conventional method. Examples of such are steadily increasing or decreasing the money stock, or changes in the interest rate of liquid assets. The second one is motivated by the fact that the FED has changed its balance sheet by using targeted purchases of illiquid assets, which we exemplify through open market operations on private equity and label them as unconventional policies¹. The main distinction between them is the introduction of the FED holding of partially liquid assets on its balance sheet. Such distinction enables us to tackle the recent conventional vs. unconventional debate on monetary policy with still some degree of generality.

In order to introduce portfolio balancing on liquid and illiquid assets, we introduce equity financing and resale constraints (liquidity frictions), as in Kiyotaki and Moore (2011), which generates heterogeneity in the economy and create the need for intrinsically valueless liquid assets such as money. The equity financing friction says that an investing entrepreneur can only issue new equity up to a fraction of his investment. At the same time, liquidity frictions set a bound on the fraction of equity that an agent can sell at every point in time. Such frictions affect agents in different ways, as they switch back and forth (some have investment opportunity and would like to invest as much as possible, while some do not have such opportunities). In financing new investment, equity issuance constraint limit the outside financing resources and resale frictions limit the internal financing. Therefore, agents will hold intrinsically valueless fully liquid assets for future investment. The financial frictions, in a general way, reduce the amount of resources for investment that should be transferred from non-productive agents to productive ones at every point in time. Not surprisingly, this channel reduces not only the output produced, but also consumption smoothing.

Liquid assets in the economy help lubricate funds transfer in the economy. However, by holding liquid assets, the agent does not internalize its own effect on lowering the return of it. In equilibrium, there is too much holding of liquid assets. Importantly, we ask whether, given the liquidity friction in a competitive economy, a constrained planner (who also respects the friction) can improve the social welfare. Note, however, that we are not dealing with policy that can achieve first best of the economy and restrict to the policy that cannot entirely eliminate liquidity friction.

We depart from early monetary policy literature such as Woodford (2003) by focusing on the Ramsey problem of optimal monetary policy and use the primal approach, which we can fully solve analytically. The primal approach structure of the Ramsey problem is analogous to the previous work that considers a representative agent model (Chari and Kehoe (1999)), in which one has an implementability constraint with only quantity variables for a constrained social planner, summarizing all the decentralized-market conditions. We find that the relevant constraint equals the net-worth difference of different types of agent to the total gain if non-resalable capital be-

¹We will abuse of notation calling the privately owned equity "private equity", but with a different meaning than the usual jargon.

come resalable. The implementability condition suggests that, as agents switch back and forth from being productive-type (with investment opportunity) or unproductive-type (without investment opportunity), consumption-smoothing will be harder the larger are financial frictions in the economy. It is worthwhile to consider two extreme cases emerge:

1. If there is no equity issuance friction, idiosyncratic investment opportunity risk is fully insured and equity resale friction does not matter, leading to perfect consumption smoothing;
2. If there is no equity resale friction (equity becomes fully liquid), the value gained by transforming non-resalable into resalable will be zero since all capital is by assumption resalable (again, net-worth difference is zero and we have perfect consumption smoothing).

Finally, the implementability constraint also shows that unconventional policies do weakly dominate conventional ones theoretically, since the latter can be shown to be a subset of the former.

To our best knowledge, we are the first to give an answer to what is the optimal monetary policy in the context of financing frictions. Standard New-Keynesian optimal policy uses the second order approximation to the objective function usually finding a balance between output gap and inflation gap². Such strategy is impossible in the context of financing friction, where heterogeneous agents and uninsured risks become the central theme. With the help of the implementability condition, heterogeneity can be summarized in one constraint for the policy maker.

We calibrate and structurally estimate the model, using liquid assets data in U.S. flow of funds jointly with aggregate investment from 1991 to 2007. Especially the shock to resale constraint, we want to estimate the size of the shock such that it will induce price sector to rebalance as in the data. During the sample period, liquidity ratio is relative stable and so is expected interest rate earned from liquid assets. Such treatment of data is novel and is directly linked to the new question we asked in the beginning.

First, in the steady-state, the monetary authority should “deflate” the economy, since agents have a propensity to over-save in liquid assets. Intuitively, one way of reducing such problem is by shortening constantly the supply of liquid assets, or equivalently an annual 4% real interest rate on liquid assets³. Implementing this outcome may involve different policies. If it opts for conventional policies, it may need to be coupled with other sources of revenue or inflation tax to ensure that it can pay the promised liquid interest rate. If, however, this can not be implemented, the constrained planner could use unconventional policies through buying claims of private equity and using the return from private equity to pay interest rate. In the economy where liquid assets

²As we do not follow the strategy of approximating the objective function, we also do not have welfare ranking problems, since in principle one can consider all higher order terms.

³Arguing that optimal policy is deflation abstracts from other margins that we do not consider, such as price stickiness. A more correct interpretation is that liquidity margins suggest an increase on the liquid asset. We do not incorporate sticky price consideration and the calculation of deflation could be thought of average liquid asset return after inflation adjustment. The option for doing this is exactly to highlight the monetary policy under flexible prices, which is rarely discussed.

earn higher rate of return due to policy, those who have investment opportunities and liquid assets will have a better internal financing. Wealth is then transferred from agents with funds but no investment opportunity to those with investment opportunity but not enough funds. The welfare gains by running a steady-state interest rate over zero interest rate amount to almost .4% change on permanent total consumption. Moreover, the optimal level of interest rate paying is increasing in the resale frictions.

Even though we have a somewhat similar result to "Friedman rule", the reason behind is very different. The key reason is the propensity to over-save, instead of the usual opportunity cost of holding money due to transaction needs. Our purpose, however, is not to explain data-observed inflation targets (which the New-Keynesian literature can do using price-stickiness) but how financial frictions alter the optimal level of real interest rate.

Finally, we also discuss how should monetary policy react to shocks. We examine various shocks including productivity shocks and liquidity shocks that lead to a harder resaleability of the illiquid assets.⁴ For an unexpected adverse liquidity shock, the policy should aim at help financing investment through increasing the interest rate on liquid assets. Importantly, even though theoretically we prove that the unconventional monetary policy weakly dominates conventional one, quantitatively it is negligibly if we do not constrain conventional policies to be inflating. The issue here is that we abstract from the revenue-side of the government. If, however, one considers that there is no transfer of resources to the government through conventional policy, the unconventional policy can achieve the allocation that the conventional policy is intended to, but cannot, or could even achieve better allocation.

Related Literature

The literature on financial frictions is vast, spanning mostly borrowing constraints and, more recently, equity financing constraints⁵. As we are interested in optimal policy with liquidity problems, we build upon the model of Kiyotaki and Moore (2011), since we view it as an otherwise standard business cycle model in which financial frictions are important. The important difference is that we embedded conventional and unconventional policy instruments which the market participants take as given and proceed to reach optimal policy solution. Monetary policy, therefore, is designed to influence on the return on liquid assets.

The most related literature to our paper is the one that merges monetary policy and financial

⁴As usual in models with only adverse unexpected liquidity shocks (Kiyotaki and Moore (2011)), flight to liquidity increases the net-worth of agents who hold liquid assets and leads to a bigger change in the supply of illiquid assets. The two effects increase illiquid asset price and increase total consumption, which we do not observe in reality. Thus, we span all the possibilities by examining shocks that provide higher asset prices (productivity shocks), roughly constant asset prices (combination of liquidity and productivity shocks) and lower asset prices (future expected liquidity shocks with current productivity shock).

⁵On borrowing constraints, the literature is very vast, but the seminal work of Kiyotaki and Moore (1997) and the recent survey of Brunnermeier, Eisenbach, and Sannikov (In Progress) are good examples of the broad literature that exists.

frictions. On one hand, some have investigated how "unconventional monetary policies", i.e., that change the central bank's balance sheet, could be used and rationalized (Gertler and Karadi (2011)). Another strand has evaluated financial frictions in a New-Keynesian model with price stickiness, as discussed in Woodford (2003) and, more specifically, in Christiano, Motto, and Rostagno (2007).

We depart from the literature, first by discussing optimal policy with financial frictions, but also by bringing the Ramsey approach to this literature. Monetary policy in our setup is non-super neutral due to distributive effects, and it is not due to the price stickiness usually assumed⁶.

The paper is structured as follows. In Section 2, we provide a canonical equity-liquidity model expanded for open market operations and helicopter drops of money, which is a variant of Kiyotaki and Moore (2011) with policy instruments and general liquid assets built in. Kiyotaki and Moore (2011) can thus be viewed as a special case with policy instruments fixed or, as we label it, "no policy". Section 3 discusses the optimal problem constrained to using conventional policies and/or unconventional policies. Section 4 provides the quantitative results under both policy instruments. Section 5 provides further discussion of the results and Section 6 concludes the paper.

2 A Canonical Model of Financial Frictions

2.1 Set-up

In this section we consider a variant of Kiyotaki and Moore (2011) in which we introduce an inelastic supply of labor and monetary policy. Conventional policies are exemplified by a helicopter drop or drain policy, but the results are entirely equivalent if we were to think about interest rate management on liquid assets if money was thought as a very general liquid asset such as the T-Bill. For unconventional policies, we consider open market operations on purchasing private equity⁷. We try to stick as much as possible to Kiyotaki and Moore (2011) model in order to evaluate the gains from optimal policy in an otherwise standard model, but some changes are needed to accommodate such policies. Therefore, we'll be brief in explaining the set-up used.

Time is discrete and infinite. The economy has two types of agents, entrepreneurs with measure 1 and household with measure L . Each agent has expected utility of

$$E_t \sum_{s=0}^{\infty} \beta^s \log(c_{t+s})$$

at time t . Only entrepreneurs have access to a constant-returns-to-scale technology for producing output from capital and labor. An entrepreneur holding k_t capital at the beginning of period t

⁶Following the jargon, neutral means money doesn't matter and super-neutral means that growth of money doesn't matter either.

⁷A more detailed explanation of such interpretations is given in Section 4.

can employ l_t in a competitive labor market to produce

$$y_t = A_t (k_t)^\alpha (l_t)^{1-\alpha}.$$

To produce output, entrepreneurs have to be involved in the production process so that their participation is necessary. Production is completed within period t , during which capital depreciates to be $(1 - \delta) k_t$, where $0 < \delta < 1$. $A_t = e^{z_t}$ is common to all entrepreneurs and z_t follows

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z.$$

Entrepreneurs hire labor at a competitive real wage w_t , the profits on capital is linear in individual entrepreneur's capital

$$y_t - w_t l_t = r_t k_t$$

where r_t is the equilibrium profits on capital and we will verify the linearity soon.. The household side is assumed to be supplying L unit of inelastic labor for simplicity⁸. After introducing labor, r_t can now be determined by clearing labor markets. For each entrepreneur with k_t , their decision on hiring labor is

$$(1 - \alpha) A_t (k_t)^\alpha l_t^{-\alpha} = w_t,$$

or

$$l_t = \left[\frac{(1 - \alpha) A_t}{w_t} \right]^{\frac{1}{\alpha}} k_t.$$

So if aggregate capital stock is K_t , the labor demand is $\left[\frac{(1 - \alpha) A_t}{w_t} \right]^{\frac{1}{\alpha}} K_t$. Wage is thus $w_t = (1 - \alpha) A_t (K_t/L)^\alpha$ and the gross profits are $A_t k_t^\alpha l_t^{1-\alpha} - w_t l_t$. Therefore we verify that profits from capital is linear in individual's capital⁹

$$\begin{aligned} r_t k_t &= A_t k_t^\alpha l_t^{1-\alpha} - w_t l_t \\ &= \alpha A_t \left(\frac{K_t}{L} \right)^{\alpha-1} k_t, \end{aligned}$$

and

$$r_t = \alpha A_t \left(\frac{K_t}{L} \right)^{\alpha-1}. \quad (1)$$

The arrival of an investment opportunity, i.e., the chance to produce new capital from general output, is independently distributed across only entrepreneurs and through time (we assume a

⁸If we specify them to have the same discount rate β in preference and allow them to buy equity for savings, they will not do so because the equilibrium rate of return will be less than β . The use of L units for workers is a normalization itself, since we keep the entrepreneurs as being 1 unit throughout.

⁹The return on individual capital is linear in their capital stock level, but decreasing in aggregate capital stock level.

Table 1: Balance Sheet of a Typical Entrepreneur

Liquid Asset (Money)	Own equity issued
Holding of Others' Equity	
Own Capital Stock	Net Worth

constant fraction at every point in time). Investment completed in period t will be available as capital in period $t + 1$:

$$k_{t+1} = (1 - \delta) k_t + i_t$$

We assume there is no insurance market against having an investment opportunity, so that the market is incomplete. In order to finance the investment opportunity, an entrepreneur can issue an equity claim to the future output from the investment, but due to the friction only θ fraction of the investment can be issued. Such friction can be motivated in a production process in which entrepreneurs have to participate in the production to produce full amount of future output. Outsiders may just be able to get $1 - \theta$ of the future output. The other friction that we introduce is the equity resale friction; entrepreneurs have difficulties in selling their capital, as they can sell only up to ϕ fraction of their own equity backed by physical capital each period. Resale friction is common especially when information asymmetry is severe. Since we abstract from different asset category while putting all assets (except fully liquid assets) together, ϕ measures the average degree of resale friction.

An entrepreneur has liquid assets, others equity, and un-mortgaged capital stock in his balance sheet as in Table 1. For simplicity, both own equity un-mortgaged initially and outside equity can be sold at most ϕ fraction and depreciate at the same rate δ . Therefore, own equity and outside equity are perfect substitutes. Entrepreneurs can remortgage their previously un-mortgaged capital stock up to ϕ_t fraction of that. There is aggregate uncertainty about resaleability ϕ_t which fluctuate over time to capture the changing of liquidity frictions. Therefore, the exogenous shocks in this economy is summarized by (z_t, ϕ_t) .

Let n_t be the equity and let m_t be money held by an entrepreneurs at the start of period t . The above discussion can be summarized as

$$n_{t+1} \geq (1 - \theta) i_t + (1 - \phi_t) (1 - \delta) n_t \quad (2)$$

$$m_{t+1} \geq 0 \quad (3)$$

The first constraint summarizes the amount of equity held in the next period. The minimum equity held would be the sum of un-mortgaged investment and equity that cannot be resold. The second constraint is just a non-negativity constraint on liquid assets (money). Private agents cannot issue very liquid assets like T-bills. Commercial papers, even some that are very liquid, are still less liquid than assets issued by the government, which is backed up by taxes and government enforcement power. Therefore, any debt that is issued by firms and commercial banks should be

thought of as the non-fully resalable assets in the model. The return on private assets in the model thus should be regarded as average return from equity and bonds in reality¹⁰.

Now, we introduce conventional policy first and leave the unconventional policy in the next subsection. Let q_t be the price of equity and let p_t be the price of liquid assets, in terms of consumption goods¹¹. From now on, we use liquid assets and money interchangeably since money is a case of liquid assets. The flow of funds constraint for an entrepreneur at time t is then given by

$$c_t + i_t + q_t (n_{t+1} - i_t - (1 - \delta) n_t) + p_t (m_{t+1} - m_t - \tilde{M}_t) = r_t n_t$$

where $\tilde{M}_t$ is the new money supply from government at time t . We model the conventional monetary policy as a helicopter drop, but, since what matters is the return of money, one could think of changing the return of liquid assets with equivalent results. The individual entrepreneurs take the new increased supply as given and think that they will not affect the equilibrium. If monetary authorities do not react at all, $\tilde{M}_t$ will always be 0. Importantly, we do not restrict the policy to be helicopter drop, i.e, $\tilde{M}_t \geq 0$. The policy can also be as a money drain, i.e, $\tilde{M}_t < 0$. In that case, the policy is equivalent to a taxation on all entrepreneurs with $p_t \tilde{M}_t$ and the government use the proceeds to pay interest rate on liquid assets since the policy will change the rate of return on liquid assets.

2.2 Recursive Equilibrium

We want to focus on an economy with valued liquid assets or money. Once money is valued, it is used as an alternative source for savings since equity financing is insufficient due to the resalability friction. Therefore, productive entrepreneurs will sell up to ϕ_t fraction of equity and their constraints are all binding¹². To reach this interesting economy equilibrium, we assume the following, as in Kiyotaki and Moore (2011)

$$\text{Assumption : } \delta\theta + \pi(1 - \delta)\phi < (\beta - 1 + \delta)(1 - \pi).$$

To understand the assumption, suppose the assumption hold and the steady state capital is K . Note that the following is impossible,

$$[\delta\theta + \pi(1 - \delta)\phi] K > \delta(1 - \pi) K.$$

To see this, the right hand side is the saving of non-investing entrepreneurs (with populations $1 - \pi$) in steady state; The left hand side is the sum of new equity issued ($\delta\theta K$, which is the investment

¹⁰

¹¹The reason to denote consumption goods as the measure is because it is convenient to think about rate of return on liquid assets

¹²We will assume that the optimal policy is also respecting the binding constraint, since we are looking only into policies that can be decentralized into a competitive equilibrium market as such.

to compensate depreciation) and existing equity sold $(\pi (1 - \delta) \phi K)$. Then the inequality says that investing entrepreneurs can transfer all the savings from non-investing entrepreneurs, which is not possible by the assumption. Thus, the first best outcome cannot be achieved by individual savings.

Entrepreneurs with investment opportunities, under the above assumption, will borrow to the limit so that constraint (2) will bind¹³. Their flow of funds constraint becomes

$$c_t^i + [1 - \theta q_t] i_t = [r_t + q_t \phi_t (1 - \delta)] n_t + p_t (m_t + \tilde{M}_t)$$

Using (2) and (3), the consumption for investing entrepreneur is $1 - \beta$ fraction of the net-worth, as we have log-utility. Therefore

$$c_t^i = (1 - \beta) \left\{ r_t n_t^i + [\phi_t q_t + (1 - \phi_t) q_t^R] (1 - \delta) n_t^i + p_t (m_t^i + \tilde{M}_t) \right\}, \quad (4)$$

where

$$q_t^R = \frac{1 - \theta q_t}{1 - \theta} < 1, \text{ as } q_t > 1.$$

Investment is thus

$$i_t = \frac{[r_t + q_t \phi_t (1 - \delta)] n_t^i + p_t (m_t^i + \tilde{M}_t) - c_t^i}{1 - \theta q_t}. \quad (5)$$

For entrepreneurs without investment opportunity

$$c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s = r_t n_t^s + q_t (1 - \delta) n_t^s + p_t (m_t^s + \tilde{M}_t),$$

where the consumption can be solved as

$$c_t^s = (1 - \beta) \left\{ r_t n_t^s + q_t (1 - \delta) n_t^s + p_t (m_t^s + \tilde{M}_t) \right\} \quad (6)$$

Meanwhile, these entrepreneurs decide on the portfolio of money and equity. A typical non-investing entrepreneur will be indifferent between money and equity. Therefore, from first order condition, we know that

$$\begin{aligned} u'(c_t^s) &= \beta E_t \left\{ \frac{p_{t+1}}{p_t} [(1 - \pi) u'(c_{t+1}^{ss}) + \pi u'(c_{t+1}^{si})] \right\} \\ &= \beta (1 - \pi) E_t \left\{ \frac{r_{t+1} + (1 - \delta) q_{t+1}}{q_t} u'(c_{t+1}^{ss}) \right\} + \\ &\quad \beta \pi E_t \left\{ \frac{r_{t+1} + (1 - \delta) \phi_{t+1} q_{t+1} + (1 - \delta) (1 - \phi_{t+1}) q_{t+1}^R}{q_t} u'(c_{t+1}^{si}) \right\}. \end{aligned} \quad (7)$$

where c_{t+1}^{si} and c_{t+1}^{ss} measures the consumption at date $t + 1$ without government transfers and subsidies.

¹³The proof is the same as in Kiyotaki and Moore (2011).

We can do aggregation in the economy easily due to the linearity in equity and liquid assets in these equations. Before aggregation, it is appropriate now to introduce another government instrument, the purchasing and selling of private equity. More recently, the central banks have implemented a new set of policies in which they buy private equity with partial liquidity, such as mortgage backed securities. We consider, therefore, how open market operations should be used in such context. The coined term "unconventional" for open market operation is due to the fact that the asset that the FED is holding has partial resaleability. Furthermore, it pumps the liquid asset in the economy, which could be thought as money or T-Bills, to inject liquidity in the system. There are possibly indirect instruments for targeted purchases, but we will discuss the direct one for simplicity and also since it was what the Fed had actually done the most. We, therefore, introduce another instrument, which is N_t^g denoting the equity that can be purchased from private sector, as a "quantity" choice variable of the social planner. When the economy is endowed with $K_t - N_t^g$ and M_t at period t , then $\pi(K_t - N_t^g)$ capital and πM_t money is in the hands of investing entrepreneurs.

Total investment I_t can be derived from (5). Therefore, aggregate investment can be written as

$$(1 - \theta q_t) I_t = [r_t + q_t \phi_t (1 - \delta)] \pi (K_t - N_t^g) + p_t \pi (M_t + \tilde{M}_t) - C_t^i. \quad (8)$$

Goods market clearing gives (subtract labor income and labor consumption on both sides)

$$r_t K_t = C_t + I_t + G_t, \quad (9)$$

where G_t is government consumption and total consumption is defined as

$$C_t = C_t^i + C_t^s \quad (10)$$

where consumptions of investing and saving entrepreneurs are

$$C_t^i = (1 - \beta) \{ r_t \pi (K_t - N_t^g) + [\phi_t q_t + (1 - \phi_t) q_t^R] (1 - \delta) \pi (K_t - N_t^g) + p_t \pi (M_t + \tilde{M}_t) \} \quad (11)$$

$$C_t^s = (1 - \beta) \left[r_t (1 - \pi) (K_t - N_t^g) + q_t (1 - \delta) (1 - \pi) (K_t - N_t^g) + p_t (1 - \pi) (M_t + \tilde{M}_t) \right] \quad (12)$$

Then we should have an aggregate portfolio choice equation. Define the the equity held of entrepreneurs without investment opportunities at the end of period t as N_{t+1}^s , where

$$N_{t+1}^s = \theta I_t + [\phi_t \pi (1 - \delta) + (1 - \pi) (1 - \delta)] K_t.$$

Notice that in (7), the aggregate version of c_{t+1}^{ss} and c_{t+1}^{si} is

$$C_{t+1}^{ss} = (1 - \pi) \left[(1 - \beta) (r_{t+1} + (1 - \delta) q_{t+1}) N_{t+1}^s + p_{t+1} (M_t + \tilde{M}_t) \right]$$

$$C_{t+1}^{ss} = \pi \left[(r_{t+1} + \phi_{t+1} (1 - \delta) q_{t+1} + (1 - \phi_{t+1}) (1 - \delta) q_{t+1}^R) N_{t+1}^s + p_{t+1} (M_t + \tilde{M}_t) \right]$$

from which allows us to rewrite (7) as the following equation

$$\begin{aligned} & (1 - \pi) E_t \left[\frac{(r_{t+1} + (1 - \delta) q_{t+1}) / q_t - p_{t+1} / p_t}{(r_{t+1} + (1 - \delta) q_{t+1}) N_{t+1}^s + p_{t+1} (M_t + \tilde{M}_t)} \right] \\ &= \pi E_t \left[\frac{p_{t+1} / p_t - [r_{t+1} + \phi_{t+1} (1 - \delta) q_{t+1} + (1 - \phi_{t+1}) (1 - \delta) q_{t+1}^R] / q_t}{[r_{t+1} + \phi_{t+1} (1 - \delta) q_{t+1} + (1 - \phi_{t+1}) (1 - \delta) q_{t+1}^R] N_{t+1}^s + p_{t+1} (M_t + \tilde{M}_t)} \right]. \end{aligned} \quad (13)$$

When we have open market operations, we can think of the government using the money supply and return from previous equity to buy extra holding of private equity $N_{t+1}^g - (1 - \delta) N_t^g$, which translates into private sector's holding of equity of $K_t - N_t^g$ at each date t . To back out the money spent on open market operations, the government expenditure now has to satisfy:

$$G_t + q_t [N_{t+1}^g - (1 - \delta) N_t^g] + \psi (N_{t+1}^g) = r_t N_t^g + p_t \bar{M}_t \quad (14)$$

In future analysis, we tie our hand by setting $G_t = 0$ to abstract from fiscal part. We view that the marginal cost will be small once government step in to buy equity, while the marginal cost will be very high when the government holds very few or zero private equity, but the specifics of the function is discussed when presenting the parameters. Finally, the capital evolution is

$$K_{t+1} - N_{t+1} = (1 - \delta) (K_t - N_t) + I_t \quad (15)$$

Therefore, we have the following recursive equilibrium definition:

Definition. A recursive competitive equilibrium is defined as function $(z_t, \phi_t, C_t, I_t, q_t, p_t, r_t, K_{t+1}, N_{t+1}^g)$ of $(z_{t-1}, \phi_{t-1}, K_t, N_t^g)$ that satisfies (8), (10), (10), (11), (12), (13), (14), and (15), given stochastic processes of (z_t, ϕ_t) and given a sequence of money supply rule $\{\tilde{M}_t, \bar{M}_t\}_0^\infty$.

Notice that in defining the equilibrium, we already impose capital market clearing and money market clearing in deriving investment and portfolio balancing equation.

3 The Optimal Monetary Policy Problem

The approach to reach the optimal policy is in the same spirit of the public finance literature (see Chari and Kehoe (1999) for a survey) on obtaining an “implementability condition”, the so-called primal approach. After constructing the equilibrium conditions of a decentralized market, we solve out prices to depend only on allocations. The problem then becomes of a social planner choosing allocations under two constraints: one that defines a competitive equilibrium and the other that defines resources constraint.

To do so, we first describe how one can obtain an implementability condition with conventional and unconventional policies. Then we show that the conventional policy is actually a particular case of an unconventional setup.

3.1 Unconventional and Conventional Policy Together

3.1.1 Implementability Condition

Let $S_t = K_t - N_t^g$ be the holding of equity in the private sector. One can solve q_t from equations (8) and (11),

$$\frac{\beta}{1-\beta}C_t^i = (1-\theta q_t) \left[I_t + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi S_t \right] \quad (16)$$

Note that $\frac{1}{1-\beta}C_t^i$ is the net-worth of the investing agents, so $\frac{\beta}{1-\beta}C_t^i$ is the value of their equity holding (including inside and outside equity). On the left-hand side of equation (16), we have the total equity holding, on the right hand side we sum all the parts that constitute the equity holding: for I_t investment, $\theta q_t I_t$ should be subtracted and out of πS_t initial equity holding, the investing agents have $(1-\delta)(1-\phi_t) \frac{1-\theta q_t}{1-\theta} \pi S_t$ after depreciation and equity selling (note that the market value for those that cannot be sold is $\frac{1-\theta q_t}{1-\theta}$). Therefore,

$$q_t = \frac{1-d_t}{\theta} \text{ and } q_t^R = \frac{d_t}{1-\theta},$$

where

$$d_t = \frac{\frac{\beta}{1-\beta}C_t^i}{S_{t+1} - (1-\delta)S_t + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi S_t}.$$

We can interpret d_t as the down-payment rate. ($\frac{\beta}{1-\beta}C_t^i$ is the amount that investors save, while $S_{t+1} - (1-\delta)S_t + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi S_t$ is the capital that will be used in production). Hence, the price of capital can be interpreted as one minus down-payment rate over the fraction of the investment (θ) that can be initially issued. To solve p_t , again from equation (11), one can express the price of money p_t as

$$p_t = \frac{1}{\pi(M_t + \tilde{M}_t)} \left\{ \frac{C_t^i}{1-\beta} - r_t \pi S_t - \left[\frac{(1-d_t)}{\theta} \phi_t + \frac{d_t}{1-\theta} (1-\phi_t) \right] (1-\delta) \pi S_t \right\}$$

Then plug p_t and q_t into equation (12) and it yields:

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t) \frac{(1-\theta)-d_t}{\theta(1-\theta)} (1-\delta) S_t \quad (17)$$

To interpret this condition, recall that $\frac{C_t^s}{1-\beta}$ is the net worth of the saving agents and $\frac{C_t^i}{1-\beta}$ is the net worth of the investing agents, so the left hand side is the net worth difference of saving agents and investing agents normalized by the populations. Such difference is determined by the resaleability friction of the equity held after depreciation. This difference occurs since the shadow price for saving agents is $q_t = \frac{1-d_t}{\theta}$ and, for investing agents, it is valued at $q_t^R = \frac{d_t}{1-\theta}$. Hence the implementability condition states that the net-worth difference of two types of agents comes exactly from the price difference on non-resalable capital due to financing friction.

In a nutshell, the implementability condition summarizes the frictions. If we relax the financing friction, $q_t = q_t^R$, there will be no net-worth difference. If we relax the resaleability ($\phi = 1$), the net worth difference will also be equal to zero, as one would expect in a usual business cycle model.

3.1.2 Set Up Ramsey Problem

Now, suppose one wants to assign equal welfare weight to each agent in the economy. The constrained planner's problem is then given by:

Problem 1.

$$\max_{C_t^i, C_t^s, S_{t+1}, N_{t+1}^g} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi \log \left(\frac{C_t^i}{\pi} \right) + (1-\pi) \log \left(\frac{C_t^s}{1-\pi} \right) + L \log \left((1-\alpha) A_t \left(\frac{K_t}{L} \right)^\alpha / L \right) \right] \right\}$$

s.t

$$\begin{aligned} \frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} &= (1-\phi_t) \frac{(1-\theta) - d_t}{\theta(1-\theta)} (1-\delta) S_t \\ C_t^i + C_t^s + (S_{t+1} + N_{t+1}^g) + \psi(N_{t+1}^g) &= r_t K_t + (1-\delta)(S_t + N_t) \end{aligned}$$

where

$$d_t = \frac{\frac{\beta}{1-\beta} C_t^i}{(K_{t+1} - N_{t+1}^g) - (1-\delta)(K_t - N_t^g) + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi (K_t - N_t^g)}$$

Consumption can be solved as function of S_t , S_{t+1} , N_t^g and N_{t+1}^g , with detailed calculation in the appendix. Not surprisingly we have two instruments, which give rise to two first order conditions and two state variables, one on private equity holding and another on government equity holdings:

$$\begin{aligned} [S_{t+1}] : \quad & \left(\frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial S_{t+1}} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial S_{t+1}} \right) + \left(\frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial S_{t+1}} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial S_{t+1}} \right) \\ & + \beta \frac{\partial \alpha L}{\partial S_{t+1}} = 0 \end{aligned} \tag{18}$$

$$\begin{aligned}
[N_{t+1}^g] : & \left(\frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial N_{t+1}^g} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial N_{t+1}^g} \right) + \left(\frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial N_{t+1}^g} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial N_{t+1}^g} \right) \\
& + \beta \frac{\partial \alpha L}{\partial N_{t+1}} = 0
\end{aligned} \tag{19}$$

We assume the cost function for government holding private equity is a concave function ($\psi'(\cdot) > 0, \psi''(\cdot) < 0$) and satisfy that $\psi(0) = 0, \psi'(0) = 0$. For small shocks, the deviations from zero open market operation should be small since it is very costly to hold private equity; For large shocks, it becomes necessary for the government to purchase significant amount of private equity, known as unconventional monetary policy to stabilize asset price and enhance liquidity.

A full characterization of each term, as well as some further algebra that simplifies the interpretation of the results, is relegated to the appendix. Finally, the second order condition is checked numerically to ensure a maximum.

3.2 Conventional Policies Only

In this section, we restrict attention to the problem when $N_t^g = 0$, so that the central bank can only change rates of return on money, whether by a helicopter drop of money or changing interest rates paid on reserves.

Recall that the competitive equilibrium is defined by equations (8)-(15) and the additional constraint that $N_t^g = 0$. Then we can solve p_t and q_t from equations (8) and (11) as we did before. By plugging the prices back, with the additional constraint that the government does not buy illiquid assets, the implementability becomes:

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t) \frac{(1-\theta) - d_t}{\theta(1-\theta)} (1-\delta) K_t \tag{20}$$

The interpretation is very similar. But now since all asset is privately claimed, we do not need to distinguish between privately and publicly claimed assets. In what regards to the structure of the Ramsey problem, we only have one first order condition, since we have constrained to one instrument.

3.3 The Equivalence and Dominance Result

From the implementability conditions, one can see that conventional policy is a subset of all the allocations that can be attained through unconventional policies. Therefore, we have the following equivalence result.

Proposition. *Suppose the government has both conventional and unconventional instruments. The optimal allocation is the same as having only the unconventional instrument.*

Proof. $\tilde{M}_t$ does not show up and setting $\tilde{M}_t = 0$ will not affect the optimal S_{t+1} and, if one has unconventional policies to use. $\square$

The immediate dominance result follows:

Corollary. *Unconventional monetary policies dominate conventional ones.*

To understand the proposition and corollary, one should recall the central friction in this economy: equity resale friction. Intuitively, the imperfection on the selling equity reduces the rate of return on equity and induces a pecuniary externality, since agents do not take into account their own effect on holding the liquid asset. Furthermore, agents tend to hold liquid assets which are intrinsically valueless. Both unconventional and conventional policy are intended to correct this externality. However, unconventional policy is more accurate since it targets directly at the illiquid asset and the dominance result becomes straightforward.

3.4 Prices and Policy Instrument

Once one solves all the quantities, one can simply back out prices including asset price, return on liquid assets and policy instrument. The details can be found in the appendix. Here, we just explain how prices can be backed out in the steady-state. The steady state version of the portfolio choice equation become

$$\begin{aligned} & (1 - \pi) \frac{(r + (1 - \delta) q) / q - x}{(r + (1 - \delta) q) N^s + P^M} \\ = & \pi \frac{x - [r + \phi(1 - \delta) q + (1 - \phi)(1 - \delta) q^R] / q}{(r + \phi(1 - \delta) q + (1 - \phi)(1 - \delta) q^R) N^s + P^M} \end{aligned}$$

where $x = \frac{p_{t+1}}{p_t}$ measures the return on liquid assets and $P^M = \frac{C^i}{\pi(1-\beta)} - rK - \left[\frac{(1-d)\phi + d(1-\phi)}{\theta} \right] (1 - \delta) K$ measures the total value of liquid assets. Hence, the steady state rate of return on money can be expressed as

$$\begin{aligned} & \left[\frac{1 - \pi}{(r + (1 - \delta) q) N^s + P^M} + \frac{\pi}{[r + \phi(1 - \delta) q + (1 - \phi)(1 - \delta) q^R] N^s + P^M} \right] x \\ = & \frac{(1 - \pi) (r + (1 - \delta) q) / q}{(r + (1 - \delta) q) N^s + P^M} + \frac{\pi [r + \phi(1 - \delta) q + (1 - \phi)(1 - \delta) q^R] / q}{[r + \phi(1 - \delta) q + (1 - \phi)(1 - \delta) q^R] N^s + P^M} \end{aligned}$$

where $N^s = \theta I + \phi\pi(1 - \delta) + (1 - \pi)(1 - \delta) K$, and $q = (1 - d) / \theta$.

A final remark is that in Kiyotaki and Moore (2011) (constant money supply) economy, the net rate of return on money is always zero in the steady state ($p_{t+1}/p_t - 1 = 0$) given that the money supply does not change. But it will soon become clear that once we introduce optimal money supply, it does not need to be zero in the steady state.

4 Quantitative Examination of Optimal Policy

In this section, we highlight how important is optimal policy through a series of numerical exercises. Our benchmark is a competitive economy with no policy intervention (constant money supply). We discuss steady-state values as well as impulse response functions under no policy, policy with only conventional instruments and with unconventional instruments. The experiment exercise is to demonstrate how should the optimal policy react both qualitatively and quantitatively (or if it should react at all).

4.1 Parameters

Some of the parameters used are standard in the literature such as depreciation rate, capital share in production and discount factor, while more elaboration should be given to π , ϕ , θ and L . We assume that 6% of the entrepreneurs are productive every quarter, which is the number to match investment spikes observed from U.S. manufacturing plants in (Doms and Dunne (1998), Cooper, Haltiwanger, and Power (1999) and Negro, Eggertsson, Ferrero, and Kiyotaki (2011)). For the financial frictions, previous work by Negro, Eggertsson, Ferrero, and Kiyotaki (2011) has assumed the mean values for θ and ϕ to be 19%, matching total treasury bills over outstanding to total assets. We perform another exercise, looking at the ratio of liquid assets to total assets in the economy in the stable period (1991Q1 to 2007Q4) and we confirm these results. Later we will vary ϕ to check robustness, which directly measures the resale friction.

Finally, L should show the ratio of workers to entrepreneurs in the economy. The main difference between workers and entrepreneurs is the access to equity markets to fund the investment opportunity. Therefore, we calibrate this value to be in line with the participation rate of households from SCF in 2009 that we see in the equities market (19%), a number in line with previous studies from Mankiw and Zeldes (1991) and Heaton and Lucas (1999). Such number translates into $L = 6$.

When using unconventional monetary policy, the cost for the government of buying private equities is assumed to be

$$\psi(N_{t+1}^g) = \mu \left[\log \frac{(1 + N_{t+1}^g)}{a} \right]^2$$

Since we look for a cost on holding assets, and not on the changes of purchase, we consider a function well defined in the positive side (log). Therefore, our task is to find μ and a such that the steady state private holding of equity is the same as in the conventional policy. This requirement leads to the above policy¹⁴, together with previous parameters, is summarized in Table 2.

Finally, for the evolution of exogenous state variables z_t and ϕ_t , we follow the literature on

¹⁴The choice of using this log-function instead of the most common quadratic cost was just to ensure computational tractability to avoid negative values

Table 2: Parameters								
α	β	δ	θ	$\bar{\phi}$	L	μ	a	π
0.33	0.99	0.025	0.19	0.19	6	0.005	1.11872	0.06

assuming the productivity an AR(1) process and also take the resaleability as an AR(1). We estimate the two processes to be:

$$z_t = 0.9225z_{t-1} + \epsilon_t^z$$

$$\phi_t - \bar{\phi} = 0.895 (\phi_{t-1} - \bar{\phi}) + \epsilon_t^\phi$$

where ϵ_t^z are i.i.d zero mean normal random variable with standard deviation 0.0134, ϵ_t^ϕ are i.i.d zero mean random variable with standard deviation 0.0052 and $corr(\epsilon_t^z, \epsilon_t^\phi) = 0.495$. The productivity random process is the standard Solow residual process and is taken from estimation of Thomas (2002), in line with previous studies. The AR(1) coefficient of ϕ_t process and its residual come from estimating the model with observed rate of return on liquidity assets. Namely, we could think of not assuming that the government has already taken the optimal policy, and we use the actual rate of return on liquid assets when estimating it. We estimate the process of ϕ_t using observed policy among other direct aggregate variables (investment, liquidity assets value and total asset value) through Bayesian estimates that are detailed in the appendix.

4.2 Steady State

We discuss the effects on the steady state in which we provide three scenarios: constant money supply, optimal conventional policies and optimal unconventional policies. Macro variables are straightforward, but welfare comparison need some elaboration. To be more specific, suppose W is the welfare attained by an competitive economy. We want to compute a common μ such that

$$\pi \frac{\log\left(\mu \frac{C^i}{\pi}\right)}{1-\beta} + (1-\pi) \frac{\log\left(\mu \frac{C^s}{1-\pi}\right)}{1-\beta} + L \frac{\log\left(\mu \frac{C^L}{L}\right)}{1-\beta} = W$$

where C^i , C^s and C^L are the allocation without policy in steady state. Then

$$\mu^{1+L} = \exp\{(1-\beta)W\} \frac{1}{\left(\frac{C^i}{\pi}\right)^\pi \left(\frac{C^s}{1-\pi}\right)^{1-\pi} \left(\frac{C^L}{L}\right)^L}$$

or

$$\mu = \sqrt[1+L]{\exp\{(1-\beta)W\} \frac{1}{\left(\frac{C^i}{\pi}\right)^\pi \left(\frac{C^s}{1-\pi}\right)^{1-\pi} \left(\frac{C^L}{L}\right)^L}}$$

If W is the welfare attained by no policy, $\mu = 1$. If W is the welfare attained by policy, we should expect $\mu > 1$

Table 3: Steady State Value

	Conventional Policy (need to change)	Unconventional Policy (need to change)
Annual Interest Rate	+3.527%	+3.444%
Total Output	+0.917%	0.917%
C^i	+5.416%	+5.318%
C^s	-3.395%	-3.395%
C^L	+0.909%	+0.909%
I	+2.797%	2.803%
N_g/S	0	1%
K	+2.797%	+2.803%
Asset Price	-9.249%	-9.056%
Total Money value	+35.642%	+35.22%
Liquidity Ratio	+40.719%	+40.146%
Equivalent Consumption Increase	+0.359%	+0.361%

Most of the variables are self explanatory: C^i is consumption of investing agents, C^s is consumption of saving agents, C^L is consumption of workers, I is investment. N_g/S is the ratio of government held partially-resaleable equity, K is capital, asset price is what we labeled before q , total money value we labeled as MP and liquidity ratio is the ratio of value of liquid assets over total assets. Finally, Equivalent consumption is how much each agent would increase its consumption permanently by changing to the respective policy, which is $\mu - 1$.

Table (3) allows us to not only evaluate the gains from optimal policy, but also to compare how better is unconventional compared to conventional, since we have already showed that the former dominates the latter.

Firstly, optimal monetary policy plays a role in the steady-state, by changing the rate of return on liquid asset, as one can see in the annualized interest rate. The intuition is that saving entrepreneurs save too much in the liquid asset and use the return to finance investment. However, they create an externality on others because they reduce the return from liquid assets, inducing the others to save even more to finance future investment. A way of overcome this is by reducing the supply of the liquid asset, which increases the rate of return and it leans against the pecuniary externality. By increasing the rate of return on the liquid asset, entrepreneurs will have less incentive to hold such asset and enjoy better return from that for future new investment.

The second distinguished feature is the capital stock held in equilibrium. As one would expect, investing agents are constrained due to the financial friction, but due to redistribution policy, the capital stock increases and is closer to the economy without constraint under optimal policy. Therefore, the asset price q , which implicitly measures the degree of financing and resaleability constraint, is closer to 1, the first best outcome in which either financing friction or resaleability friction is eliminated. Thus, even though it is still constrained, the shadow value of relaxing the constraint decreases after policy intervention, leading to a higher capital and therefore a higher welfare.

The welfare gains computed suggest that the benefits from having an optimal monetary policy

in such environment are equivalent to increasing the consumption of each agent, permanently, by .36%, a sizable number since it is a permanent change in the economy.

A final comparison is on unconventional and conventional policy outcome. The quantities and prices are very similar to what conventional policy can achieve. Interest rate need not be that high since the constrained planner has another instrument to achieve desired allocation.

4.3 Simulations

In this section, we examine how monetary policy responds to shocks, through impulse response functions. We consider four cases: a pure productivity shock, a pure liquidity shock, a combination of a productivity and a liquidity shock and a combination of productivity with expected future liquidity shock. Our focus is mainly on comparing optimal policy (both conventional and unconventional policy) with a constant liquid assets supply, which we label as no-policy. In doing that, we log-linearized the model to first order to solve the expectation system¹⁵.

4.3.1 Active Conventional Monetary Policy and No Policy

- *Pure Productivity Shock*

The first shock that we consider is a pure productivity shock (Figure 2). The shock that we investigate is a negative one standard-deviation shock to productivity:

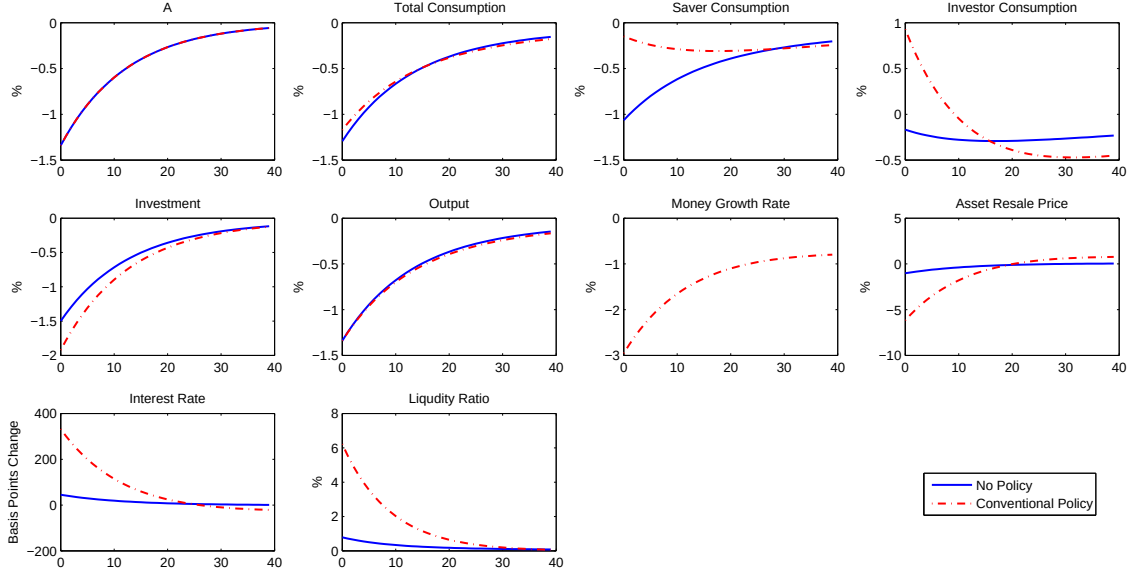
$$z_t = 0.9225z_{t-1} + \varepsilon_t^z$$

Without policy, such shock drives the price of the equity and money down under a constant money supply policy because there are less resources for agents to save. To emphasize the price level change, the initial triggering of a pure adverse productivity shock leads to inflation (money rate of return decreases). Therefore, with only a negative productivity shock, it will produce inflation pressure in recession.

With policy intervention, however, when one considers a helicopter drop (drain) type of policy, the equity price drops while the return becomes very high after the shock. To achieve this, we see a positive rate of return on liquid assets that is even higher than the steady state interest rate. The gains from having a conventional policy can be seen in the consumption of savers and investors, even though the total consumption is less affected because workers don't have their consumption much affected. Overall, aggregate consumption, investment and output do not change significantly under conventional policy and no policy, when only productivity shocks hit. Importantly, the steady state level is still higher under conventional policy. Given that the response in percentage term is similar, the conventional policy still gives a better allocation of resources.

¹⁵Detailed computation can be found in the Matlab code available in the authors' website

Figure 2: No Policy and Conventional Policy: A shock only



One standard deviation shock to ϕ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

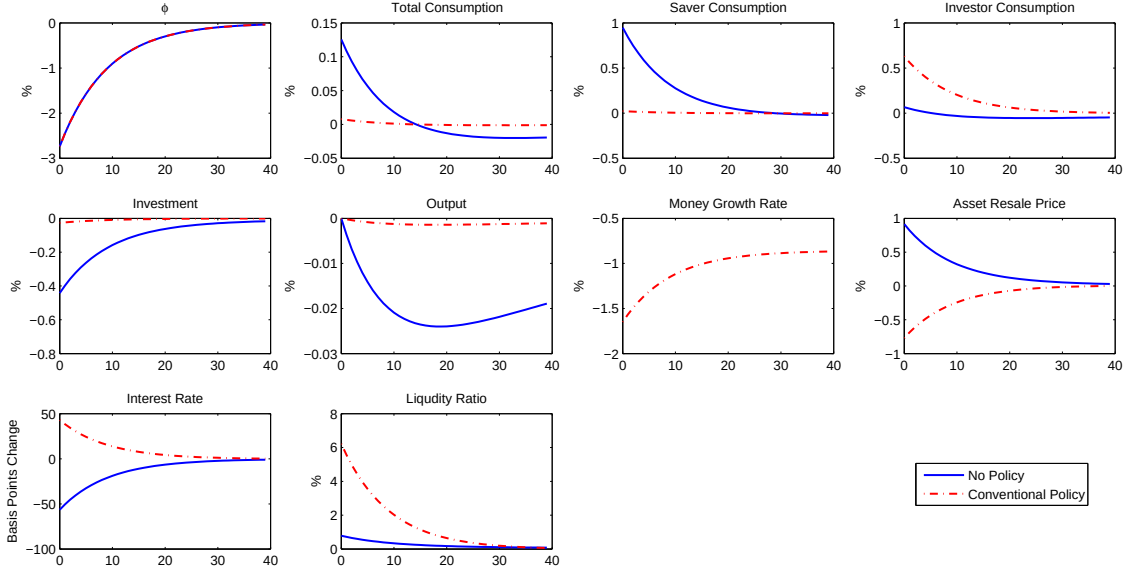
- *Pure Liquidity Shocks*

Now we consider a pure adverse liquidity shock (Figure 3). We consider, once more, an autoregressive shock with a one standard deviation shock of 0.0052 obtained from a Bayesian estimate of the model during the "great moderation", so it can be thought as a small shock during regular periods.

As documented in Kiyotaki and Moore (2011), liquidity shocks usually lead to a flight to liquidity as the seen in the figure with a lower rate of return on liquid assets. However, at the same time, the illiquid asset supply drop drastically so that asset price actually increases. Therefore, asset prices (q_t, p_t) increases lead to higher wealth which will lead to initial higher individual consumption, but lower investment (given that output will be initially the same, investment will drop). Overall, pure liquidity shock is at odds with the data too.

Under optimal policy, the optimal policy trades-off present and future as we can see from the aggregate consumption graph. Aggregate consumption rarely fluctuates. Not surprisingly, the aggregate investment and output are stable as well. This outcome is achieved by changing the liquidity asset return and then changing the consumption gap between saving and investing entrepreneurs. Note that, the larger increase in total investing entrepreneurs' consumption is a manifestation of more consumption smoothing.

Figure 3: No Policy and Conventional Policy: ϕ shock only



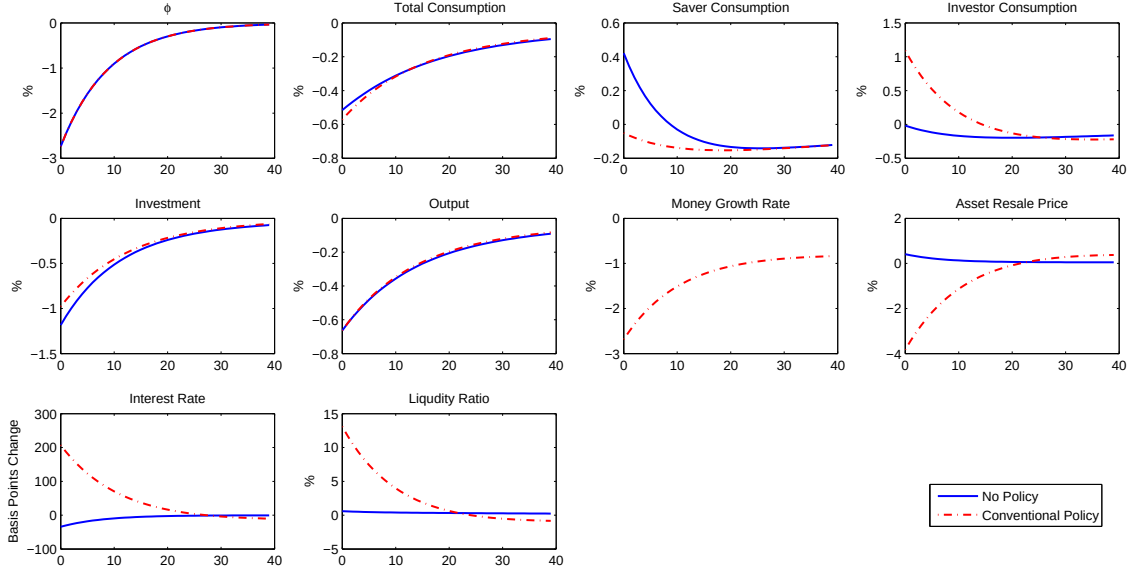
One standard deviation shock to ϕ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

- *Productivity and Liquidity Shocks*

From last two experiments, we saw that pure productivity or pure liquidity shock misses some important stylized facts observed in recession in the data, no matter whether there is optimal policy or not (mainly consumption path, rate of return on liquid assets and asset resale price). We overcome the odd behavior by considering a liquidity shock accompanied by a productivity shock and estimate its variance-covariance matrix from data (Figure 4). The shock we investigate is a liquidity shock accompanied by a simultaneous TFP as described before. The correlation between them is also obtained from Bayesian estimates and it comes to be relatively big: 0.49. The economic reason behind such experiment could be, for instance, that financing frictions lead to mis-allocation and reduce TFP, but we do not model it endogenously. Moreover, with such shock we span another possibility which is a liquidity shock that does not translate into a asset price movement in a world without policy.

Without the optimal policy, the adverse ϕ shock will reduce the demand on equity because of liquidity run, but not enough to reduce the asset price. Two forces roughly cancel out each other on this exercise: on one hand, the portfolio rebalancing to liquid assets; on the other, as productivity is auto-regressive, the economy becomes more unproductive today and in the future too, overcoming more consumption and less investment today from the liquidity run discussed before. As a result, investment will decrease drastically, while consumption should also fall because it is accompanied by a TFP loss that reduces the available resources. All these features are in line with stylized facts observed in usual recessions, particularly the portfolio rebalancing in recent years.

Figure 4: No Policy and Conventional Policy: ϕ shock and A shock



One standard deviation shock to ϕ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

With optimal policy, investment drops, but much less than without an optimal policy to counterbalance such effect. Again, the central bank is redistributing wealth through the payment of liquid assets, which helps on consumption smoothing (from the gap between saving and investing entrepreneurs consumptions).

Not surprisingly, optimal policy achieves a more stable result through redistributing resources. This can be seen from the welfare improvement, which shows that the welfare increases after a shock. Note, however, that since we are comparing to the steady-state, the optimal policy was already better than the constant money supply case and it becomes even better.

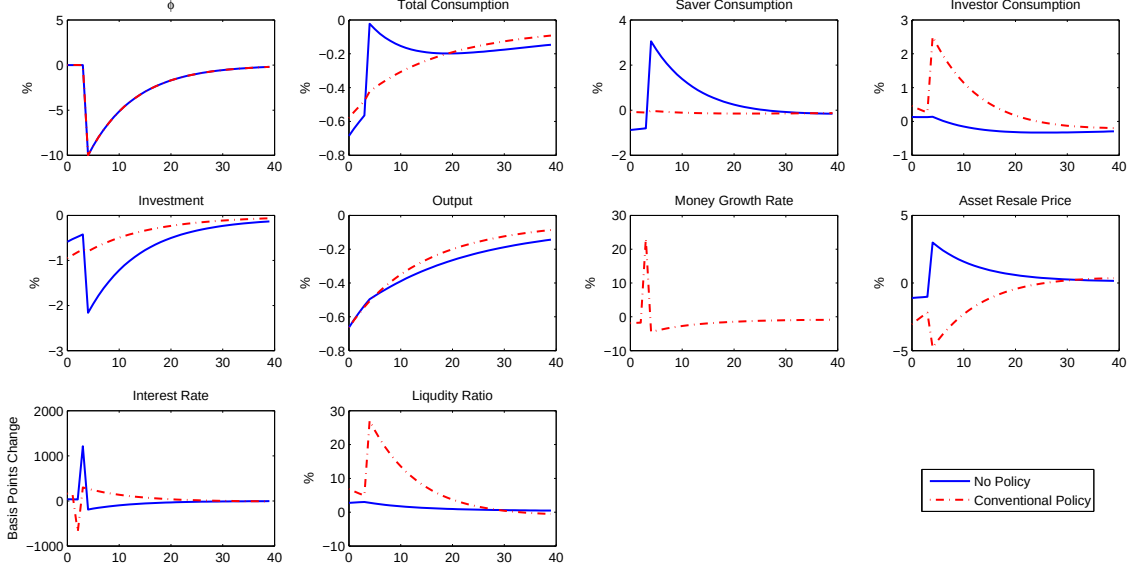
- *Expected Future Liquidity Shocks and Productivity Shocks*

The previous shocks lead to policy response, but without too much impact on stabilizing macroeconomic real variables. We consider expected future liquidity shock, say 4 quarters later, and current productivity jump and examine how much the policy could achieve (Figure 5). Such experiment is intended to partially capture the fall of Lehman Brothers in 2008Q3. The fall did not immediately stop all the business. In fact, many previous Lehman related business still ran into 2009. However, the fall may have triggered the expectation that in the near future many assets would be very illiquid, which is captured by a 4 quarters later liquidity shock. At the same time, funding froze from the banking sector, limiting efficient production, reducing total factor productivity in the economy, which is seen in the data computed by most policy paper.

The purpose of this exercise is twofold. First, to discuss under which conditions is the policy more relevant, and secondly to discuss a liquidity based shock in which the asset price actually

jumps. We still take the same structure discussed before, but with shock on current liquidity known 4 quarters before.

Figure 5: No Policy and Conventional Policy: future ϕ shock and A shock



One standard deviation shock to ϕ with correlated shock to $\ln A$. Money growth rate path is just its own path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state levels.

Without policy intervention, the macroeconomic variables are very unstable. For example aggregate consumption decreases by 0.7% initially, then increases a lot, decreases back to a level that is lower than the steady state and stay there persistently. The reason for that can be seen from investment, which decreases initially because of low productivity, slightly increases afterward due to less consumption and then incur a big jump because of expected liquidity shock. Persistent low investment thus leads to persistent future low consumption. Not surprisingly, output is persistently lower.

The role that government policy has is remarkable. Interest is kept almost constant until date three, when the rate reduces greatly so that there is a negative interest rate on holding liquid assets at date 3. By increasing money growth in 25% at date 3, it helps on keeping liquid asset accumulation low and a larger room for policy to increase the rate of return from date 4 to date 5. When the real shock hits on date 4, the liquid asset return was maintained high by the central bank which helps on smoothing funds transfer. This experiment demonstrate significantly that monetary policy should move fast in responding to market price and return fluctuation.

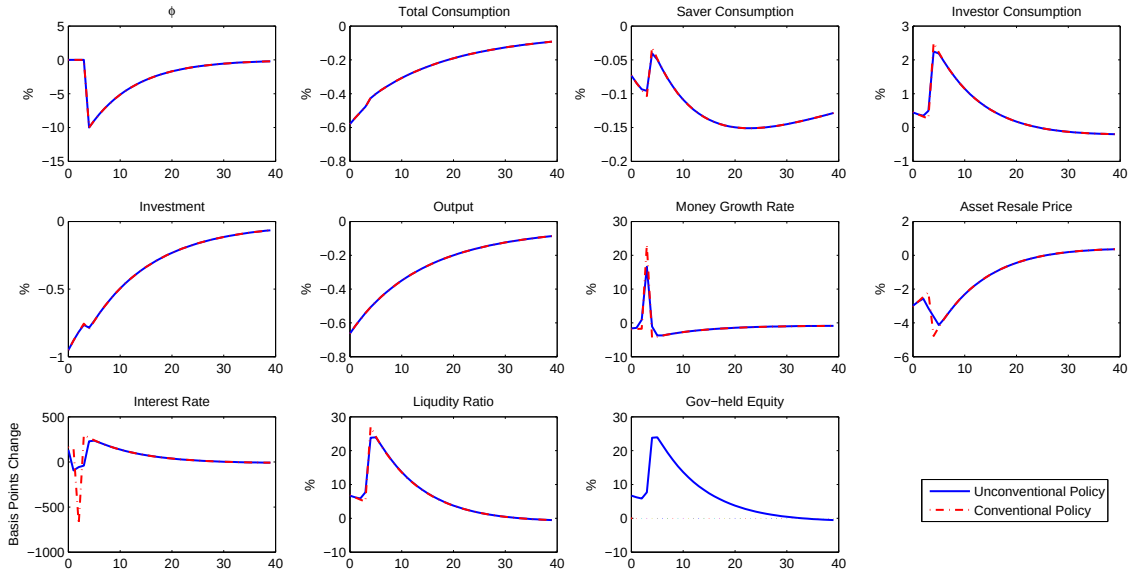
4.3.2 Unconventional Policy and Conventional Policy

Now we follow the same structure of the previous version where we have discussed productivity, liquidity and joint productivity-liquidity shocks. The difference, henceforth, is that we want to

compare the gains from using unconventional policies vs. conventional ones. We have already established that conventional policies lead to a non-negligible increase in welfare compared to a constant money supply case under steady-state. Moreover, we have theoretically established that unconventional policies are weakly better than conventional ones. A further question that one may ask is: why should we bother understanding conventional instruments if we know that unconventional ones dominate them?

The answer can be depicted from comparing conventional and unconventional policy in the “*Expected Future Liquidity Shocks and Productivity Shocks*” experiment (Figure 6): the optimal path under both policies is roughly the same, being robust to any shock. However, we have assumed that helicopter drain is feasible, which may well not be when truly implementing it. If helicopter drain is not possible, unconventional policies may help attain the desired allocations we have before. Conventional and unconventional policy under other experiments are almost exactly the same (including interest rates) and we will not show them here due to space restriction.

Figure 6: Unconventional versus Conventional



1 standard deviation shock to future ϕ with correlated shock to $\ln A$. Money growth rate path is the level path, interest rate shows the basis point change from steady state, other variables are percentage deviation from steady state.

Allowing for a new instrument to be used from the FED leads to an increase in equity purchasing of about 3%, a much bigger number than the 0.05% that we had on money growth under conventional policies. However, such difference leads roughly to the same allocation. There is no significant difference on the path of the variables using unconventional or conventional policies.

Even though the path is indistinguishable, the levels under unconventional policy are higher, since we are comparing to a higher steady state. The results for the liquidity shock case only and simultaneous shocks also give indistinguishable paths between conventional and unconventional

policies.

Finally, two comments are worth mentioning. First of all, such results do not depend on the cost function used. The intuition behind it is that the unconventional policy, if possible, almost completely reduces the pecuniary externality by changing the rate of return on liquid assets and illiquid equity. The unconventional policy leaves very few room for improvement. For roughly any concave function tested, our results persist. The changes are even smaller if we use convex cost function since marginal cost becomes higher after purchase. Besides that, these results should not be seen as a case against unconventional policies. On the contrary, even though unconventional policies cannot change the paths of variables, they do change the steady state from which we are comparing to. Therefore, the welfare remains higher during all periods after the shock in an unconventional policy. Besides that if, for instance, helicopter drain policies are not implementable, unconventional policies can substitute them.

5 Further Discussion

5.1 Inefficiency from Pecuniary Externality

There is a recent literature that studies pecuniary externality, in which the competitive market is neither efficient, nor constrained efficient. By introducing prices in an inequality constraint, the agent does not take into account her own effect on prices generating an externality. Work by Bianchi (2009), Bianchi (2010), Korinek (2009) and Lorenzoni (2008) are examples on this venue. We depart from this strand by restricting attention to competitive allocations and by looking at monetary instruments. Also, we look at constrained efficient allocation instead of first best allocation.

The fact that the competitive equilibrium is not efficient should be clear due to the frictions imposed, which translates into a price of capital that is not 1-1 to goods prices. More importantly, the competitive market is not even constrained efficient. Even if the social planner were to take into consideration all the financial frictions and constraints, the externality discussed before leads to under-investment. On one hand, the possibility of a binding borrowing constraint in the future associated with incomplete markets leads to over saving, just like in Aiyagari (1994). On the other hand, there are pecuniary externalities from not taking into account their own effects on the price. Therefore, the competitive equilibrium is not even constrained efficient.

Another remark is that we constrain the optimal allocations to instruments that can be easily understood and translated into the debate of conventional and unconventional monetary policies.

5.2 Labeling Policies

In the paper, the way we have labeled policies so far was somewhat loose. As we have argued, M is the liquid asset. Conventional policies are policies that do not target any other asset except the liquid asset that usually the FED controls or largely controls. In that sense, this can be thought as money or T-Bills. If the interpretation is money, then one can think of a helicopter drop that changes the money supply and therefore the rate of return of money or, inversely, inflation. If one interprets the liquid assets as T-Bills, the Fed is changing the real rate of return by increasing/reducing the supply accordingly.

However, as we have argued before, optimal policy involves a helicopter drain, which may be infeasible. In such cases, the social planner would be using resources from taxes not herein modeled, but the main point is still valid: paying interest on liquid assets. Meanwhile, another possibility is to do unconventional policies, which attains the optimal outcome without the need of relying on tax revenues. This could possibly help explaining the use of such monetary policy instruments during 08 recession.

Unconventional policies involve the purchase of assets with lower liquidity that are not entirely controlled by the FED. Such interpretation is in line with the massive purchases of mortgage backed securities from the FED during the crisis, such as TAF, TSLF and PDCF liquidity facilities created by NY FED. Once more, the open market operation of substituting the share of liquid/illiquid in the economy is an example of such.

5.3 Parametrization Robustness

We compute the steady-state level of capital for different parametrization to draw some comparative statics¹⁶. Importantly, since one can draw a relationship between capital and the rate of return on liquid assets, one can evaluate the relationship between the financial frictions and the return of liquid assets in the steady-state.

Fact. *The optimal steady-state rate of return on the liquid asset is decreasing in ϕ and θ*

In order to give a more concrete analysis, we can see, in Figure (7), that by relaxing the liquidity constraint from .14 to .25, for instance, the optimal rate of return on the liquid asset in the steady state would jump from around annual 6% to almost 0%¹⁷. A thorough look at how endogenous variables change as one tighten or loosen liquidity friction is in Table 4¹⁸.

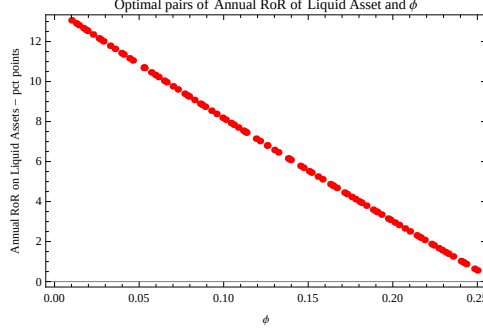
Under optimal policy, capital increases less as the friction relaxes. Not surprisingly, the interest rate change from no-policy to optimal policy is always smaller the higher is ϕ . Hence, the gains from having an optimal policy are reduced as the importance of financial friction is reduced. Moreover,

¹⁶Such results can be derived from the equations, but we present the results numerically for ease of interpretation.

¹⁷For numbers of ϕ higher than this, the assumption that ensures that the constraints are binding is not satisfied.

¹⁸The title of the column indicates which parameter are we changing compared to our benchmark model presented in Section 4.

Figure 7: Steady State Comparison

Table 4: Robustness of Steady State ϕ

	$\phi = .18$		$\phi = .19$		$\phi = .20$	
	Conventional Policy	Unconv. Policy	Conventional Policy	Unconv. Policy	Conventional Policy	Unconv. Policy
Annual Int. Rate	+4.02%	+4.06%	+3.527%	+3.444%	+3.00%	+2.95%
Total Output	+1.038%	1.043%	+0.917%	0.917%	+0.784%	0.790%
C^i	+6.148%	+6.052%	+5.416%	+5.318%	+4.667%	+4.579%
C^s	-3.837%	-3.837%	-3.395%	-3.395%	-2.949%	-2.949%
C^L	+1.047%	+1.047%	+0.909%	+0.909%	+0.789%	+0.789%
I	+3.197%	3.197%	+2.797%	2.803%	+2.407%	2.803%
K	+3.197%	+3.197%	+2.797%	+2.803%	+2.407%	+2.407%
Asset Price	-10.411%	-10.217%	-9.249%	-9.056%	-8.061%	-7.868%
Total Money value	+35.070%	+34.706%	+35.642%	+35.22%	+36.065%	+35.578%
Liquidity Ratio	+40.800%	+40.273%	+40.719%	+40.146%	+40.465%	+39.822%
Equivalent Permanent Consumption	+0.418%	+0.420%	+0.359%	+0.361%	+0.308%	+0.309%

the allocations under optimal policy and competitive equilibrium become similar as one relaxes the frictions, a result that should be expected, since there would be no role for optimal policy if there was no friction¹⁹.

6 Conclusion

We study a tractable model of optimal monetary policy instruments dealing with financial frictions, namely equity issuance and resale frictions. We provide an implementability condition that summarizes all the restrictions of a competitive equilibrium allocation in this model. The implementability condition thus allow us to derive the social optimal allocation. By doing so, we avoid the usual ambiguous welfare ranking problem in the optimal monetary policy literature.

We investigate how optimal monetary policy should react in steady state and to liquidity and productivity shocks with consideration of both conventional and unconventional policies. Due to

¹⁹Due to space restriction, we do not show the impulse response functions for different parameters, but the qualitative results discussed previously are the same, and the policy conclusions remain.

the pecuniary externality arising from the liquidity constraint in a competitive equilibrium, there will always be room for policy to improve welfare in a constrained economy. In the steady-state, permanent aggregate consumption increases by almost .4%, comparing optimal policy to non-policy. Moreover, when hit by an adverse liquidity shock, by using expansionary policy such difference increases even more. Finally, we showed that unconventional policies dominate conventional ones. But in quantitative exercises, the difference that it generates on other macroeconomic variables after a shock is very small if we don't constrain the set of allocations attained by conventional policies.

Note that we do not assume sticky price in the economy but the pecuniary externality on holding liquid assets still need policy intervention. Monetary policy, therefore, mainly act like a redistribution device transferring resources from non-liquid assets holders to liquid assets holders. Whenever the economy runs into problem due to liquidity issues, firms or banks will typically hold more liquid assets, usually more than they should. A usual policy response by lowering interest rate should be reconsidered. Agents in the economy will hold much more liquid assets if other assets market persistently incur resale (liquidity) problems. In that sense, lowering the interest rate will only hurt the ability for financing future investment, since it exacerbates the incentive to hold even more liquid assets in a liquidity constrained world where liquid assets are the only possible choices for savings.

One drawback and potential future work is how the monetary policy will change illiquid asset market resaleability endogenously (in our case ϕ). We viewed it as an exogenous fluctuation but it could certainly depend on market expectation and asset quality. This possibility is left for future research work.

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7 Appendix

In this appendix, we provide the derivations for the results that we discussed in the main text. Mainly we highlight how to set-up the Ramsey problem under conventional and unconventional monetary instruments. Also, one can find estimation details in the appendix.

7.1 Appendix to Section 3

7.1.1 Setting-up the Ramsey with one instrument

From the equilibrium conditions, we have

$$C_t^i = \pi \{ r_t K_t - (1 - \pi) (1 - \beta) (1 - \phi_t) \left(\frac{(1 - \theta) - d_t}{\theta (1 - \theta)} \right) (1 - \delta) K_t - K_{t+1} + (1 - \delta) K_t - G_t \} \quad (21)$$

$$C_t^s = (1 - \pi) \{ r_t K_t + \pi (1 - \beta) (1 - \phi_t) \left(\frac{(1 - \theta) - d_t}{\theta (1 - \theta)} \right) (1 - \delta) K_t - K_{t+1} + (1 - \delta) K_t - G_t \} \quad (22)$$

$$(1 - \theta q_t) I_t = [r_t + q_t \phi_t (1 - \delta)] \pi (K_t - N_t^g) + p_t \pi (M_t + \tilde{M}_t) - C_t^i \quad (23)$$

Then we can solve p_t and q_t from equations (23) and (23):

$$\frac{\beta}{1 - \beta} C_t^i = (1 - \theta q_t) \left[I_t + \frac{(1 - \delta) (1 - \phi_t)}{1 - \theta} \pi K_t \right] \quad (24)$$

Solving the prices from such equations, we have:

$$p_t = \frac{1}{\pi (M_t + \tilde{M}_t)} \left\{ \frac{C_t^i}{1 - \beta} - r_t \pi K_t - \left[\frac{(1 - d_t)}{\theta} \phi_t + \frac{d_t}{1 - \theta} (1 - \phi_t) \right] (1 - \delta) \pi K_t \right\}$$

$$q_t = \frac{1 - d_t}{\theta} \text{ and } q_t^R = \frac{1 - \theta q_t}{1 - \theta} = \frac{d_t}{1 - \theta}$$

$$d_t = \frac{\frac{\beta}{1 - \beta} C_t^i}{I_t + \frac{(1 - \delta) (1 - \phi_t)}{1 - \theta} \pi K_t}$$

Then plug p_t and q_t into equation (22) and it yields the implementability condition:

$$\frac{C_t^s}{(1 - \beta) (1 - \pi)} - \frac{C_t^i}{(1 - \beta) \pi} = (1 - \phi_t) \frac{(1 - \theta) - d_t}{\theta (1 - \theta)} (1 - \delta) K_t$$

Therefore, one can write the problem as

Problem 2.

$$\max_{K_{t+1}, N_{t+1}^g} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi \log \left(\frac{C_t^i}{\pi} \right) + (1 - \pi) \log \left(\frac{C_t^s}{1 - \pi} \right) + L \log \left((1 - \alpha) A_t \left(\frac{K_t}{L} \right)^\alpha / L \right) \right] \right\}$$

and

$$\frac{C_t^s}{(1 - \beta)(1 - \pi)} - \frac{C_t^i}{(1 - \beta)\pi} = (1 - \phi_t) \frac{(1 - \theta) - d_t}{\theta(1 - \theta)} (1 - \delta) K_t$$

$$C_t^i + C_t^s + K_{t+1} - (1 - \delta) K_t + G_t = r_t K_t$$

where

$$d_t = \frac{\frac{\beta}{1 - \beta} C_t^i}{K_{t+1} - (1 - \delta) K_t + \frac{(1 - \delta)(1 - \phi_t)}{1 - \theta} \pi K_t}$$

Note that C_t^i and C_t^s can be expressed as functions of the endogenous variable K_t ,

$$C_t^i = \pi \{ r_t K_t - (1 - \pi)(1 - \beta)(1 - \phi_t) \left(\frac{(1 - \theta) - d_t}{\theta(1 - \theta)} \right) (1 - \delta) K_t - K_{t+1} + (1 - \delta) K_t - G_t \}$$

$$C_t^s = (1 - \pi) \{ r_t K_t + \pi(1 - \beta)(1 - \phi_t) \left(\frac{(1 - \theta) - d_t}{\theta(1 - \theta)} \right) (1 - \delta) K_t - K_{t+1} + (1 - \delta) K_t - G_t \}$$

By reorganizing the terms and replacing d_t , we can solve consumption as purely function of K_t and K_{t+1} :

$$C_t^i = \frac{\pi}{1 - B_t} \{ r_t K_t - K_{t+1} + (1 - \delta) K_t, -G_t - (1 - \pi)(1 - \beta)(1 - \phi_t)(1 - \delta) K_t / \theta \}$$

$$C_t^s = r_t K_t - K_{t+1} + (1 - \delta) K_t - G_t - C_t^i,$$

and

$$B_t = \frac{\beta \pi (1 - \pi)}{\theta \left[\frac{1 - \theta}{(1 - \phi)} \left[\frac{K_{t+1}}{(1 - \delta)} - K_t \right] + \pi K_t \right]} K_t.$$

Then the problem is transformed into only one endogenous state variable K and the planner optimally chooses a weighted welfare of savers, investors and workers. The first order condition of

the problem is then given by:

$$\begin{aligned}
[K_{t+1}] : \quad & \left(\frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial K_{t+1}} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial K_{t+1}} \right) + \left(\frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial K_{t+1}} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial K_{t+1}} \right) \\
& + \beta \frac{\alpha L}{K_{t+1}} = 0
\end{aligned} \tag{25}$$

For bookkeeping, we put the components of the first order condition below:

$$\begin{aligned}
\frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial K_{t+1}} &= \frac{\pi}{C_t^i} \left[\frac{-\pi(1-B_t) + (1-B_t) C_t^i \frac{\theta}{\beta\pi(1-\pi)K_t} \frac{(1-\theta)}{(1-\phi)(1-\delta)} B_t^2}{(1-B_t)^2} \right] \\
&= - \frac{\pi^2}{(1-B_t) C_t^i} - \frac{\theta(1-\theta) B_t^2}{\beta(1-\pi)(1-\phi)(1-\delta)(1-B_t) K_t} \\
\frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial K_{t+1}} &= \frac{(1-\pi)}{C_t^s} \left[-\frac{\partial C_t^i}{\partial K_{t+1}} - 1 \right] \\
&= \frac{\pi(1-\pi)}{(1-B_t) C_t^s} + \frac{\theta(1-\theta) B_t^2}{\beta\pi(1-\phi)(1-\delta)(1-B_t) K_t} \frac{C_t^i}{C_t^s} - \frac{(1-\pi)}{C_t^s}
\end{aligned}$$

and

$$\begin{aligned}
\beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial K_{t+1}} &= \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\pi}{(1-B_{t+1})^2} \left\{ (1-B_{t+1}) [\alpha r_{t+1} + (1-\delta) - (1-\pi)(1-\beta)(1-\phi_t)(1-\delta)/\theta] \right. \\
&\quad \left. + \frac{C_{t+1}^i (1-B_{t+1})}{\pi} \frac{\beta\pi(1-\pi) \left[\frac{\beta\pi(1-\pi)K_{t+1}}{B_{t+1}} \right] - \beta\pi(1-\pi) K_{t+1} \left[\pi - \frac{\theta(1-\theta)}{1-\phi} \right]}{\left(\frac{\beta\pi(1-\pi)K_{t+1}}{B_{t+1}} \right)^2} \right\} \\
&= \beta E_t \left\{ \frac{\pi^2 [\alpha r_{t+1} + (1-\delta) - (1-\pi)(1-\beta)(1-\phi_t)(1-\delta)/\theta]}{C_{t+1}^i (1-B_{t+1})} \right. \\
&\quad \left. + \frac{\pi}{(1-B_{t+1})} \frac{B_{t+1}}{K_{t+1}} \left[1 - \frac{B_{t+1}}{\beta\pi(1-\pi)} \left[\theta\pi - \frac{\theta(1-\theta)}{1-\phi_t} \right] \right] \right\} \\
\frac{\partial B_{t+1}}{\partial K_{t+1}} &= \frac{\beta\pi(1-\pi) \left[\frac{\beta\pi(1-\pi)K_{t+1}}{B_{t+1}} \right] - \beta\pi(1-\pi) K_{t+1} \left[\pi - \frac{\theta(1-\theta)}{1-\phi} \right]}{\left(\frac{\beta\pi(1-\pi)K_{t+1}}{B_{t+1}} \right)^2} \\
&= \frac{B_{t+1}}{K_{t+1}} \left[1 - \frac{B_{t+1}}{\beta\pi(1-\pi)} \left[\theta\pi - \frac{\theta(1-\theta)}{1-\phi_t} \right] \right]
\end{aligned}$$

$$\begin{aligned}
\beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial K_{t+1}} &= \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \left[\alpha r_{t+1} + (1-\delta) - \frac{\partial C_{t+1}^i}{\partial K_{t+1}} \right] \\
&= \beta E_t \frac{(1-\pi)}{C_{t+1}^s} [\alpha r_{t+1} + (1-\delta)] \\
&- \beta E_t \left\{ \frac{\pi (1-\pi) [\alpha r_{t+1} + (1-\delta) - (1-\pi)(1-\beta)(1-\phi_t)(1-\delta)/\theta]}{C_{t+1}^s (1-B_{t+1})} \right. \\
&+ \left. \frac{1-\pi}{(1-B_{t+1})} \frac{B_{t+1}}{K_{t+1}} \left[1 - \frac{B_{t+1}}{\beta \pi (1-\pi)} \left[\theta \pi - \frac{\theta(1-\theta)}{1-\phi_t} \right] \right] \frac{C_{t+1}^i}{C_{t+1}^s} \right\}
\end{aligned}$$

7.1.2 Setting-up Ramsey with two instruments

Let $S_t = K_t - N_t^g$ be the holding of equity in the private sector. Coupled with the competitive equilibrium equations (11), (12), (8) and portfolio equations, we can solve p_t and q_t from equations (8) and (11).

$$\frac{\beta}{1-\beta} C_t^i = (1-\theta q_t) \left[I_t + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi S_t \right]$$

Prices can be solved from 11:

$$\begin{aligned}
q_t &= \frac{1-d_t}{\theta} \text{ and } q_t^R = \frac{d_t}{1-\theta} \\
d_t &= \frac{\frac{\beta}{1-\beta} C_t^i}{S_{t+1} - (1-\delta) S_t + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi S_t} \\
p_t &= \frac{1}{\pi (M_t + \tilde{M}_t)} \left\{ \frac{C_t^i}{1-\beta} - r_t \pi S_t - \left[\frac{(1-d_t)}{\theta} \phi_t + \frac{d_t}{1-\theta} (1-\phi_t) \right] (1-\delta) \pi S_t \right\}
\end{aligned}$$

Then plug p_t and q_t into equation (12) and it yields:

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t) \frac{(1-\theta)-d_t}{\theta(1-\theta)} (1-\delta) S_t$$

Set Up Ramsey Problem - the constrained planner's problem is then given by:

Problem 3.

$$\max_{C_t^i, C_t^s, S_{t+1}, N_{t+1}^g} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi \log \left(\frac{C_t^i}{\pi} \right) + (1-\pi) \log \left(\frac{C_t^s}{1-\pi} \right) + L \log \left((1-\alpha) A_t \left(\frac{K_t}{L} \right)^\alpha / L \right) \right] \right\}$$

s.t

$$\frac{C_t^s}{(1-\beta)(1-\pi)} - \frac{C_t^i}{(1-\beta)\pi} = (1-\phi_t) \frac{(1-\theta)-d_t}{\theta(1-\theta)} (1-\delta) S_t$$

$$C_t^i + C_t^s + (S_{t+1} + N_{t+1}^g) + G_t + \psi(N_{t+1}^g) = r_t K_t + (1 - \delta)(S_t + N_t)$$

where

$$d_t = \frac{\frac{\beta}{1-\beta} C_t^i}{(K_{t+1} - N_{t+1}^g) - (1 - \delta)(K_t - N_t^g) + \frac{(1-\delta)(1-\phi_t)}{1-\theta} \pi (K_t - N_t^g)}$$

From the implementability condition and resources constraint, we have

$$\begin{aligned} C_t^i &= \pi \{r_t K_t - K_{t+1} + (1 - \delta) K_t - G_t - \psi(N_{t+1}^g) \\ &\quad - (1 - \pi)(1 - \beta)(1 - \phi_t) \left(\frac{(1 - \theta) - d_t}{\theta(1 - \theta)} \right) (1 - \delta)(K_t - N_t^g)\} \end{aligned}$$

$$\begin{aligned} C_t^s &= (1 - \pi) \{r_t K_t - K_{t+1} + (1 - \delta) K_t - G_t - \psi(N_{t+1}^g) \\ &\quad + \pi(1 - \beta)(1 - \phi_t) \left(\frac{(1 - \theta) - d_t}{\theta(1 - \theta)} \right) (1 - \delta)(K_t - N_t^g)\} \end{aligned}$$

Also, we can represent $S_t = K_t - N_t^g$. Therefore, one can rewrite the problem as

Problem 4.

$$\max_{S_{t+1}, N_{t+1}^g} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\pi \log \left(\frac{C_t^i}{\pi} \right) + (1 - \pi) \log \left(\frac{C_t^s}{1 - \pi} \right) + L \log \left((1 - \alpha) A_t \left(\frac{S_t + N_t^g}{L} \right)^\alpha / L \right) \right] \right\}$$

where

$$\begin{aligned} C_t^i(S_t, S_{t+1}, N_{t+1}^g) &= \frac{\pi}{1 - B_t} \{r_t(S_t + N_t^g) - (S_{t+1} + N_{t+1}^g) + (1 - \delta)(S_t + N_t^g) - G_t - \psi(N_{t+1}^g) \\ &\quad - (1 - \pi)(1 - \beta)(1 - \phi_t)(1 - \delta)S_t / \theta\} \end{aligned}$$

$$C_t^s = r_t(S_t + N_t^g) - (S_{t+1} + N_{t+1}^g) + (1 - \delta)(S_t + N_t^g) - G_t - \psi(N_{t+1}^g) - C_t^i,$$

and

$$B_t = \frac{\beta \pi (1 - \pi)}{\theta \left[\frac{1 - \theta}{(1 - \phi)} \left[\frac{S_{t+1}}{(1 - \delta)} - S_t \right] + \pi S_t \right]} S_t.$$

Consumption can be solved as function of S_t , S_{t+1} , N_t^g and N_{t+1}^g . Not surprisingly we now have two instruments, which give rise to two first order conditions and two state variables, one on private equity holding and another on government equity holdings:

Rewriting

$$[S_{t+1}] : \left(\frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial S_{t+1}} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial S_{t+1}} \right) + \left(\frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial S_{t+1}} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial S_{t+1}} \right) + \beta \frac{\partial \alpha L}{\partial S_{t+1}} = 0$$

$$[N_{t+1}^g] : \left(\frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial N_{t+1}^g} + \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial N_{t+1}^g} \right) + \left(\frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial N_{t+1}^g} + \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial N_{t+1}^g} \right) + \beta \frac{\partial \alpha L}{\partial N_{t+1}} = 0$$

where

$$\frac{\partial B_t}{\partial S_{t+1}} = - \frac{1-\theta}{(1-\phi)(1-\delta)} \frac{\theta}{\beta \pi (1-\pi) S_t} B_t^2$$

The first order conditions can be decomposed into:

$$\begin{aligned} \frac{\pi}{C_t^i} \frac{\partial C_t^i}{\partial S_{t+1}} &= \frac{\pi}{C_t^i} \left[\frac{-\pi(1-B_t) + (1-B_t) C_t^i \frac{\theta}{\beta \pi (1-\pi) S_t} \frac{(1-\theta)}{(1-\phi)(1-\delta)} B_t^2}{(1-B_t)^2} \right] \\ &= - \frac{\pi^2}{(1-B_t) C_t^i} - \frac{\theta(1-\theta) B_t^2}{\beta(1-\pi)(1-\phi)(1-\delta)(1-B_t) S_t} \\ \frac{(1-\pi)}{C_t^s} \frac{\partial C_t^s}{\partial S_{t+1}} &= \frac{(1-\pi)}{C_t^s} \left[-\frac{\partial C_t^i}{\partial S_{t+1}} - 1 \right] \\ &= \frac{\pi(1-\pi)}{(1-B_t) C_t^s} + \frac{(1-\pi)\theta(1-\theta) B_t^2}{\beta \pi (1-\phi)(1-\delta)(1-B_t) S_t} \frac{C_t^i}{C_t^s} - \frac{(1-\pi)}{C_t^s} \end{aligned}$$

and

$$\begin{aligned} \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\partial C_{t+1}^i}{\partial S_{t+1}} &= \beta E_t \frac{\pi}{C_{t+1}^i} \frac{\pi}{(1-B_{t+1})^2} \{ (1-B_{t+1}) [\alpha r_{t+1} + (1-\delta) - (1-\pi)(1-\beta)(1-\phi_t)(1-\delta)/\theta] \\ &\quad + \frac{C_{t+1}^i (1-B_{t+1})}{\pi} \frac{\beta \pi (1-\pi) \left[\frac{\beta \pi (1-\pi) S_{t+1}}{B_{t+1}} \right] - \beta \pi (1-\pi) S_{t+1} \left[\pi - \frac{\theta(1-\theta)}{1-\phi} \right]}{\left(\frac{\beta \pi (1-\pi) S_{t+1}}{B_{t+1}} \right)^2} \} \\ &= \beta E_t \{ \frac{\pi^2 [\alpha r_{t+1} + (1-\delta) - (1-\pi)(1-\beta)(1-\phi_t)(1-\delta)/\theta]}{C_{t+1}^i (1-B_{t+1})} \\ &\quad + \frac{\pi}{(1-B_{t+1})} \frac{B_{t+1}}{S_{t+1}} \left[1 - \frac{B_{t+1}}{\beta \pi (1-\pi)} \left[\theta \pi - \frac{\theta(1-\theta)}{1-\phi_t} \right] \right] \} \end{aligned}$$

$$\begin{aligned}
\beta E_t \frac{(1-\pi)}{C_{t+1}^s} \frac{\partial C_{t+1}^s}{\partial S_{t+1}} &= \beta E_t \frac{(1-\pi)}{C_{t+1}^s} \left[\alpha r_{t+1} + (1-\delta) - \frac{\partial C_{t+1}^i}{\partial S_{t+1}} \right] \\
&= \beta E_t \frac{(1-\pi)}{C_{t+1}^s} [\alpha r_{t+1} + (1-\delta)] \\
&\quad - \beta E_t \left\{ \frac{\pi(1-\pi)[\alpha r_{t+1} + (1-\delta) - (1-\pi)(1-\beta)(1-\phi_t)(1-\delta)/\theta]}{C_{t+1}^s(1-B_{t+1})} \right. \\
&\quad \left. + \frac{1-\pi}{(1-B_{t+1})} \frac{B_{t+1}}{S_{t+1}} \left[1 - \frac{B_{t+1}}{\beta\pi(1-\pi)} \left[\theta\pi - \frac{\theta(1-\theta)}{1-\phi_t} \right] \right] \right\} \frac{C_{t+1}^i}{C_{t+1}^s}
\end{aligned}$$

7.2 Details on the Estimation

In this section, we estimate the previous model using Bayesian methods. The purpose of the exercise is to obtain the distribution of the shock of ϕ and how it correlates with productivity shocks on A . In order to do so, we estimate the previous dynamic stochastic general equilibrium model with measurement errors since some margins of our model are not stressed, but focusing on the margins related to portfolio since rebalancing is a key mechanism in our work that has not been much addressed in previous macroeconomic studies.

As usual, to have identification, we consider the number of shock/measurement errors to be the same as the number of observed variables. We introduce 5 shocks in the estimation: resaleability, productivity shock, resaleability and productivity correlation, as well as measurement errors on the total liquid asset value and the expected interest rate on liquid assets.. We have already calibrated the productivity shock from previous literature, leaving 4 shocks to be estimated, with two of them being measurement error.

The data used is roughly standard. We consider deviations from the HP trend for aggregate investment. The other variables that we consider are related to portfolio rebalancing. For the total liquid asset value, as defined in 1, we considered check, deposit, tradable receivable and T-Bills. Total assets value also come from the Flow of Funds table and is also defined in 1. For the rate of return of liquid assets, we considered the 3-Month Treasury bill rate adjusted for expected inflation from the Michigan survey. The sample period used is from 1991 to 2007, in order to consider a stationary and stable period, that we can be sure to be dealing with "normal" times.

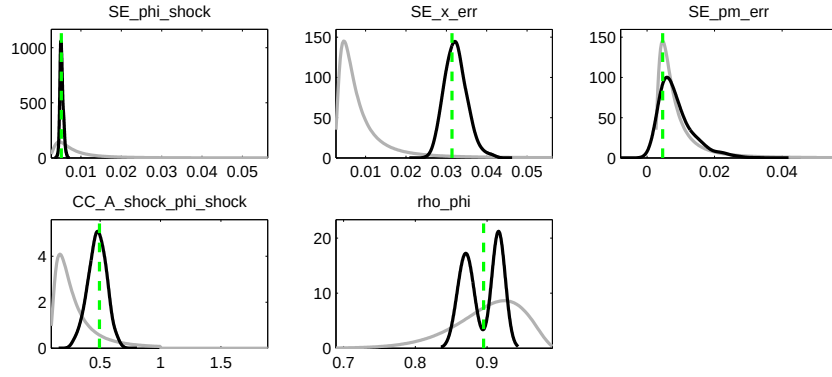
As we are doing Bayesian estimation, we have to set priors. Following the literature, we consider inverse gamma distributions on the standard error of the shocks that we are interested in to have a conjugate prior. Table 5 summarizes the prior and posterior information. We have tried many different priors and the posteriors are very robust. Interested reader can directly check the code available on the authors' website.

For the standard deviation on ϕ , we consider it as being small because ϕ itself is already small and we want to have a high probability of staying the positive domain. The measurement errors on liquid asset expected returns and money value come from the fact that these returns are expected,

Table 5: Prior and Posterior of the Parameters

Parameters	Prior distribution			Posterior distribution			
	Distribution	Prior Mean	Std	Posterior Mean	Posterior Mode	5%	95%
σ_ϕ	Inverse Gamma	0.01	1	0.0052	0.0051	0.0046	0.0058
$\sigma_{ln(x)}$	Inverse Gamma	0.01	1	0.0323	0.0314	0.0280	0.0362
$\sigma_{ln(P^M)}$	Inverse Gamma	0.01	1	0.0086	0.0046	0.0158	0.0026
$\sigma_{z,\phi}$	Inverse Gamma	0.3	1	0.4746	0.4950	0.3382	0.5765
ρ_ϕ	Beta	0.90	0.05	0.8929	0.8951	0.8626	0.9201

Figure 8: Prior and Posterior Distribution



Prior distributions are shown in gray color, while posterior distributions are shown in black color. The header of each subplot stands for the parameters estimated. SE_phi_shock: σ_ϕ . SE_x_err: $\sigma_{ln(x)}$. SE_pm_err: $\sigma_{ln(P^M)}$. CC_A_shock_phi_shock: $\sigma_{z,\phi}$. rho_phi: ρ_ϕ

so subject to some error, and the money value may have some accounting error, so we provide a rather flat prior but with a small mean on the standard error. Finally, for the auto-persistent term of liquidity, we leave it to be somewhat persistent .9 and we start with a somewhat large cross-correlation to induce a sizable productivity shock simultaneous to the liquidity one.

The posterior mode of $\sigma_\phi, \sigma_{z,\phi}$ and ρ_ϕ estimated are used in our numerical analysis. The posterior shape of the 3 parameters are very concentrated, given that we have a relatively flat prior.