

Uncertainty Shocks in an Economy with Collateral Constraints

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ABSTRACT

We explore the implications of firm-level uncertainty shocks as an alternative or complementary source of business cycle fluctuations in a dynamic, stochastic general equilibrium setting with both real and financial frictions. In our model economy, alongside persistent shocks to aggregate and individual productivity, firms face time-varying uncertainty arising from changes in the cross-sectional dispersion of their productivities. Efficient reallocation of capital and production in response to changes in firm productivity is hindered by two frictions seen as important in explaining the dynamics of firm-level investment. First, specificity in capital implies partial investment irreversibilities that lead firms to pursue (S,s) decision rules. Given a proportional loss associated with current or future reductions in their capital stocks, firms are cautious in responding to changes in productivity, particularly in times of heightened uncertainty. Second, collateralized borrowing constraints limit the amount of investment among firms with low levels of real and financial wealth. Such firms realizing high relative productivity carry an inefficiently low share of the aggregate capital stock, which also serves to reduce the economy's average productivity.

We show that a rise in uncertainty on its own generates a substantial drop in employment, production and investment. In keeping with previous findings in the literature, we find these initial declines are larger in the presence of capital irreversibility than without, as firms experiencing relatively high productivity invest less to avoid costs associated with later reversals. Surprisingly, the inclusion of financial frictions has a slight mitigating effect on the downturn. We trace this to a reduced negative wealth effect on labor supply associated with future rises in average productivity implied by the coming greater dispersion. Absent frictions in capital reallocation, greater dispersion has a TFP enhancing effect. This positive effect associated with increased dispersion over coming dates is limited in the presence of collateralized borrowing limits hindering reallocation. Thus, we see substantially reduced rises in GDP over the episode following the uncertainty shock. In contrast to the implications of an aggregate productivity shock, an uncertainty shock generates a gradual rise in endogenous aggregate TFP in our model, alongside a nonmonotone response in the labor input. Thus, when driven by uncertainty shocks alone, the model succeeds in reproducing the near-zero correlation between hours and labor productivity in the data.

Keywords: Uncertainty shocks, financial frictions, irreversibilities, (S,s) policies, business cycles

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1 Introduction

We examine aggregate fluctuations in a general equilibrium model where firms have persistent idiosyncratic shocks to their total factor productivity, partial irreversibility in their capital adjustments and collateralized borrowing constraints. These aspects of our model generate a nontrivial distribution of production across firms, with firms differing in their total factor productivity, capital and debt levels. Further, to the extent that aggregate shocks at times increase the benefits of capital reallocation across firms (as in Eisfeldt and Rampini (2006)), our real and financial frictions impede such reallocation. In this framework, we study uncertainty shocks affecting the variability of innovations to firm-specific total factor productivity as an alternative or complementary source of aggregate fluctuations. Unlike a traditional aggregate productivity shock affecting the first moment of the firm productivity distribution, uncertainty shocks are by nature reallocative shocks. Thus, if aggregate fluctuations are driven or accompanied by such shocks, the extent to which real and financial frictions impede the flow of capital across firms has the potential to alter the overall business cycle in important ways, and perhaps to deliver the observed asymmetries in the movements of GDP over the business cycle.

We find that uncertainty shocks drive aggregate fluctuations that in several respects resemble conventional business cycles driven by aggregate shocks to total factor productivity. Over the business cycle, production is more variable than consumption, and investment is more variable than production. Moreover consumption, investment and employment are procyclical. There are also some notable differences in the effect of uncertainty shocks relative to conventional aggregate productivity shocks. First, the variability of employment is higher, which moves the model closer to the postwar U.S. data. Second, there is almost no correlation between aggregate hours worked and labor productivity. This too is consistent with the data. Third, the contemporaneous correlation between consumption and GDP is sharply reduced, leaving it understated relative to the data.

Our model economy generalizes the economy developed in Khan and Thomas (2011) to allow for uncertainty shocks. As shown in that earlier work, the misallocation arising from real and financial frictions has no implication for aggregate fluctuations when fluctuations are driven by exogenous shocks to aggregate total factor productivity. Because such shocks do not alter the overall benefits of reallocation, the handicap on such reallocation is of little consequence. Thus, we see a model with a nontrivial distribution of production and reallocative frictions behave,

in the aggregate, much like a representative firm economy. This aggregate equivalence is destroyed in the presence of uncertainty shocks. First, as is well known since the work of Bloom (2009) and Bloom et al. (2011), real frictions affect firms' responses to a rise in the level of uncertainty. Moreover, in our model, these frictions interact with collateralized borrowing limits to further dampen the reallocation of capital that characterizes the response of an economy following a persistent rise in firm-level exogenous productivity dispersion. As a result, and in contrast to the responses following an aggregate shock to TFP, there are substantive differences in our model's aggregate response following an uncertainty shock, when compared to the responses in a counterpart economy with no reallocative frictions and when compared to one simply lacking the financial friction.

The uncertainty shocks we consider here affect only the dispersion of firm specific total factor productivity; they are not accompanied by a first or second moment shock to aggregate productivity. Nonetheless, to the extent that capital can be efficiently reallocated following such a shock, there is an eventual rise in the endogenous component of aggregate total factor productivity, given increased productivity at the top end of the distribution and a relatively small fraction of productive resources devoted at the bottom end. In an economy with capital irreversibility, but no borrowing limits, a persistent rise in uncertainty delivers a nonlinear response in the economy. There is an initial recession, followed by a recovery and, to the extent the uncertainty persists, expansion. Throughout this episode, all changes in aggregate TFP are the result of movements in the distribution of production. The rise in aggregate investment that starts during the recovery following an uncertainty shock is sharply dampened by borrowing limits. These financial frictions impede the reallocation of capital across firms that is essential to transform the negative consequences of rising dispersion in firm-level total factor productivity into positive ones. The result is a smaller increase in aggregate capital and thus employment, over the recovery. This, in turn, reduces the variability of GDP. Thus we find that financial frictions alter the response of an economy to uncertainty shocks.

As mentioned above, we find that uncertainty shocks alter the business cycle implications of collateralized borrowing constraints. This is in contrast to the findings of Khan and Thomas (2011), as well as earlier work by Cordoba and Ripoll (2004) and Kocherlakota (2000). These authors argued that financial frictions did not alter the response of an economy to aggregate shocks. We confirm their finding when our model economy is driven by shocks to the exogenous

component of aggregate TFP. However, when aggregate fluctuations are driven by changes in the cross-sectional dispersion of firm productivities, there are significant differences in the aggregate response of our full model relative to a counterpart model lacking financial frictions. Such shocks amplify differences in the expected return to investment across firms, thus raising the benefits of reallocation. In both models, investment irreversibilities deter some firms from reallocating capital in response to a rise in the dispersion of expected returns. The resulting misallocation is compounded in the presence of financial frictions, since borrowing limits prevent firms with high expected marginal product from adequately increasing their investment. Thus, because uncertainty shocks in our framework are inherently reallocative, and because borrowing limits impede reallocation, the business cycle implication of collateral constraints is substantial. They dampen expansions.

Collateralized borrowing limits also, to a much lesser degree, mitigate the downturn that follows an uncertainty shock in our economy. This more minor difference in our model relative to a setting without financial frictions is explained as follows. The initial rise in uncertainty is initially a pure rise in risk, and is only later accompanied by raised dispersion. That increased dispersion in future dates will raise endogenous aggregate productivity, and thus income, so long as capital (and hence employment and production) can be appropriately reshuffled across firms. Because the representative household anticipates these benefits of the shock before the dispersion component of the uncertainty shock materializes, the result is a strong negative wealth effect on labor supply and investment. This is, in fact, the primary force driving the downturn. Recalling from above that financial frictions, by hindering reallocation, substantially reduce the ultimate expansion caused by the shock's dispersion effect, the negative wealth effects generating recession over early dates of the shock are smaller in our full model economy.

In earlier work (Khan and Thomas (2011)), we argued that heterogenous firm models with collateral constraints could explain several important aspects of the most recent U.S. recession. In particular, we showed that a credit shock in such a model could reproduce the observed magnitudes of the declines in GDP, TFP, employment and investment, alongside the decline in overall lending. Further, it could generate a gradual deterioration in employment and GDP, as well as a non-monotone response in consumption, which were also features of the recent recession. Importantly, the observed change in measured TFP was insufficient to explain the change in GDP, or the disproportionate change in investment. These differences in the economy's response

following an aggregate TFP shock, versus an exogenous reduction in collateralized borrowing limits, arose because the latter are inherently reallocative.

Uncertainty shocks, in our current environment, are also reallocative. As such, they have the potential to serve as a microeconomic foundation for something observed as a credit shock. In this paper, we explore the extent to which changes in the level of uncertainty can drive aggregate fluctuations in the presence of realistic (calibrated) ongoing real and financial frictions, and the extent to which the resulting business cycles have aspects in common with the recent U.S. recession. Despite convincing empirical evidence of a persistent rise in firm-level dispersion over the episode surrounding the 2007 recession (see Bloom et al. (2011) and Arellano, Bai and Kehoe (2011)), our findings are mixed in considering an uncertainty shock as the source of this recession. We find that uncertainty shocks increase the volatility of the response of investment and employment, relative to the changes in the endogenous component of measured aggregate TFP they imply. Further, they predict a nonlinear response in aggregate consumption. At the same time, however, a persistent rise in uncertainty does not, on its own generate a protracted recession, as would follow a credit shock.

2 Model

In our model economy, firms face both partial irreversibility of investment and collateralized borrowing limits, which together amplify the effect of persistent differences in their total factor productivities to yield substantial heterogeneity in production. These differences in their total factor productivity evolve over time in response to uncertainty shocks. We begin our description of the economy with an initial look at the optimization problem facing each firm, then follow with a brief discussion of households and equilibrium. Next, in section 3, we will use a simple implication of equilibrium alongside some immediate observations about firms' optimal allocation of profits across dividends and retained earnings to characterize the capital adjustment decisions of our firms. This analysis will show how we derive a convenient, computationally tractable algorithm to solve for equilibrium allocations in our model, despite its three-dimensional heterogeneity in production.

2.1 Production, credit and capital adjustment

We assume a large number of firms, each producing a homogenous output using predetermined capital stock k and labor n , via an increasing and concave production function, $y = z\varepsilon F(k, n)$. The variable z represents exogenous stochastic total factor productivity common across firms, while ε is a firm-specific counterpart. We assume that the exogenous component of aggregate total factor productivity is a Markov Chain. Thus, $z \in \{z_1, \dots, z_{N_z}\}$, where $\Pr(z' = z_m \mid z = z_l) \equiv \pi_{lm}^z \geq 0$, and $\sum_{m=1}^{N_z} \pi_{lm}^z = 1$ for each $l = 1, \dots, N_z$.

Uncertainty shocks affect the firm-specific component of total factor productivity. These shocks are an exogenous stochastic process that take on a finite number of values. In any period, aggregate *uncertainty* is indexed by $r = \{1, \dots, N_r\}$. We assume that uncertainty follows a Markov Process, with typical element $\pi_{ij} \geq 0$, $i, j = 1, \dots, N_r$, where $\sum_{j=1}^{N_r} \pi_{ij} = 1$ for each $i = 1, \dots, N_r$.

We retain the assumption that for any uncertainty level, ε is a Markov Chain. However, the support of ε changes with the level of uncertainty in the economy. Let r_0 denote the current level of uncertainty, which takes on one of N_r distinct values; the support of ε is $\mathbf{E}_r \equiv \{\varepsilon_1^{r_0}, \dots, \varepsilon_{N_\varepsilon}^{r_0}\}$. If the level of uncertainty next period is r , which is known beforehand, then the transition probabilities for firm-specific total factor productivity is $\Pr(\varepsilon' = \varepsilon_j^r \mid \varepsilon = \varepsilon_i^{r_0}) \equiv \pi_{ij}^{r_0, r} \geq 0$, with $\sum_{j=1}^{N_\varepsilon} \pi_{ij}^{r_0, r} = 1$ for each $i = 1, \dots, N_\varepsilon$.

We assume that each firm faces a fixed probability, $\pi_d \in (0, 1)$, that it will be forced to exit the economy following production in any given period. Within a period, prior to investment, firms learn whether they will survive to produce in the next period. Exiting firms are replaced by an equal number of new firms whose initial state is described below.

At the beginning of each period, a firm is defined by its predetermined stock of capital, $k \in \mathbf{K} \subset \mathbf{R}_+$, by the level of one-period debt it incurred in the previous period, $b \in \mathbf{B} \subset \mathbf{R}$, and by its current idiosyncratic productivity level, $\varepsilon \in \{\varepsilon_1^{r_0}, \dots, \varepsilon_{N_\varepsilon}^{r_0}\}$. Immediately thereafter, the firm learns whether it will produce in the next period.¹ Given this individual state, and having observed the current aggregate state, the firm then takes a series of actions to maximize the expected discounted value of dividends. First, it chooses its current level of employment, undertakes production, and pays its wage bill. Thereafter, it repays its existing debt and, conditional

¹We have adopted this timing to ensure there is no equilibrium default in our model, so that all firms borrow at a common real interest rate. As debt is limited by a collateral constraint, firms are always able to repay their debt in the quantitative exercises to follow.

on survival, it chooses its investment, i , current dividends, and the level of debt with which it will enter into the next period, b' . For each unit of debt it incurs for the next period, a firm receives q units of output that it can use toward paying current dividends or investing in its future capital. The relative price q^{-1} reflecting the interest rate at which firms can borrow and lend is a function of the economy's aggregate state, as is the wage rate ω paid to workers. For expositional convenience, we sometimes suppress the arguments of these equilibrium price functions.

In contrast to the typical setting with firm-level capital adjustment frictions, and unlike a typical environment with financial frictions, real and financial frictions interact in our model economy. Our firms' borrowing and investment decisions are inter-related, because each firm faces a collateralized borrowing constraint inside of any period. This constraint takes the form: $b' \leq \theta_b \theta_k k$. Two external forces together determine what fraction of its capital stock a firm can borrow against - the degree of specificity in capital and enforceability of financial arrangements. Here, we simply impose both, deferring the question of their foundations.² In particular, we assume that $\theta_k \in [0, 1]$ is a parameter determining what fraction of a firm's capital stock survives when it is uninstalled and moved to another firm, and $\theta_b \in \mathbf{R}_+$ is the fraction of that collateral firms can borrow against.³ A financial shock in our model is represented by an unanticipated change in the collateral term, θ_b .

If a firm undertakes any nonnegative level of investment, then its capital stock at the start of the next period is determined by a familiar accumulation equation,

$$k' = (1 - \delta)k + i \quad \text{for } i \geq 0,$$

where $\delta \in (0, 1)$ is the rate of capital depreciation, and primes indicate one-period-ahead values. Because there is some degree of specificity in capital, the same equation does not apply when the firm undertakes negative investment. In this case, the effective relative price of investment is θ_k rather than 1, so the accumulation equation is instead:

$$\theta_k k' = \theta_k (1 - \delta)k + i \quad \text{for } i < 0.$$

In the analysis section to follow, we will show how the asymmetry that firms face in the cost of

²Our collateral constraint is reminiscent of Kiyotaki and Moore (1997). Khan, Sengupta and Thomas (2012) derive a borrowing limit by assuming non-contingent debt in an environment where firms default.

³Throughout our numerical exercises in section 5, we assume that the degree of capital irreversibility, $1 - \theta_k$, is a fixed technological parameter. In ordinary times when aggregate fluctuations arise from changes in productivity or uncertainty shocks, θ_b is also a fixed parameter. Khan and Thomas (2011) consider the effect of shocks to θ_b .

capital adjustment naturally gives rise to two-sided (S, s) investment decision rules. Firms have nonzero investment only when their capital falls outside a range of inactivity.⁴ In contrast to a nonconvexity in the capital adjustment technology, this type of adjustment friction implies not only investment inaction among firms within their (S, s) adjustment bands, but also some inertia among firms outside of their (S, s) bands.

As is clear from the discussion above, a firm's capital adjustment may also be influenced by its ability to borrow (now and in the future). This is in turn affected by the capital (collateral) it currently holds. Note also that the firm's current investment decision may influence the level of debt it carries into the next period. These observations imply that we must monitor the distinguishing features of firms along three dimensions: their capital, k , their debt, b , and their idiosyncratic productivity, ε . Thus, in contrast to models with loan market frictions, but without irreversible investment, a firm's net worth is an insufficient description of its state; capital and debt are distinct state variables.

We summarize the distribution of firms over (k, b, ε) using the probability measure μ defined on the Borel algebra, $\mathcal{S}$, generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. We follow Bloom (2009) and allow for uncertainty shocks by assuming that the level of uncertainty that will determine the distribution of shocks to firm level total factor productivity, r , is known this period. As a result, the aggregate state of the economy is described by (z, r_0, r, μ) , where r_0 is the level of uncertainty that applies today, and the distribution of firms evolves over time according to a mapping, Γ , from the current aggregate state; $\mu' = \Gamma(z, r_0, r, \mu)$. The evolution of the firm distribution is determined in part by the actions of continuing firms and in part by entry and exit. As already mentioned, fraction π_d of firms exit the economy after production in each period. These firms invest negatively to shed their remaining capital, returning the proceeds to households, and are replaced by the same number of new firms. Each new firm has zero debt and productivity $\varepsilon_0 \in \mathbf{E}^r$ drawn from an initial distribution $H(\varepsilon_0; r)$, and each enters with an initial capital stock $k_0 \in \mathbf{K}$.

We now turn to the problem solved by each firm in our economy. Let $v_0(k, b, \varepsilon; z_l, r_0, r, \mu)$ represent the expected discounted value of a firm that enters the period with (k, b) and firm-specific productivity ε , when the aggregate state of the economy is (z_l, r_0, r, μ) , just before it

⁴The problem of costly investment reversibility was originally solved by Abel and Eberly (1996). Veracierto (2002) studies a general equilibrium model with irreversible investment. Caggese (2007) characterizes a partial equilibrium problem with irreversible investment and collateral constraints.

learns whether it will survive into the next period. We state the firm's dynamic optimization problem using a functional equation defined by (1) - (4) below.

$$v_0(k, b, \varepsilon; z_l, r_0, r, \mu) = \pi_d \max_n [z_l \varepsilon F(k, n) - \omega(z_l, r_0, r, \mu) n + \theta_k (1 - \delta) k - b] \quad (1)$$

$$+ (1 - \pi_d) v(k, b, \varepsilon; z_l, r_0, r, \mu)$$

After the start of the period, the firm knows which line of (1) will prevail. If it is not continuing beyond the period, the firm simply chooses labor to maximize its current dividend payment to shareholders. Because it will carry no capital or debt into the future, an exiting firm's dividends are its output, less wage payments and debt repayment, together with the remaining capital it can successfully uninstall at the end of the period. The problem conditional on continuation is more involved, because a continuing firm must choose its current labor and dividends alongside its future capital and debt. For expositional convenience, given the partial irreversibility in investment, we begin to describe this problem by defining the firm's value as the result of a binary choice between upward versus downward capital adjustment in (2), then proceed to identify the value associated with each option in (3) and (4).

$$v(k, b, \varepsilon; z_l, r_0, r, \mu) = \max \left\{ v^u(k, b, \varepsilon; z_l, r_0, r, \mu), v^d(k, b, \varepsilon; z_l, r_0, r, \mu) \right\} \quad (2)$$

Assume that $d_{m,r,s}(z_l, r_0, r, \mu)$ is the discount factor applied by firms to their next-period expected value if the exogenous component of the aggregate state at that time is (z_m, r, s) and the current aggregate state is (z_l, r_0, r, μ) . Taking as given the evolution of ε , z and s according to the transition probabilities specified above, and given the evolution of the firm distribution, $\mu' = \Gamma(z, r_0, r, \mu)$, the firm solves the following two optimization problems to determine its values conditional on (weakly) positive and negative capital adjustment. In each case, the firm selects its current employment and production, alongside the debt and capital with which it will enter into next period and its current dividends, D , to maximize its expected discounted dividends. As above, dividends are determined by the firm's budget constraint as the residual of its current production and borrowing after its wage bill and debt repayment have been covered, net of its investment expenditures.

Conditional on an upward capital adjustment, the firm solves the following problem constrained by (i) the fact that investment must be non-negative, (ii) a borrowing limit determined by its collateral, and (iii)-(iv) the requirements that dividends be non-negative and satisfy the firm's budget constraint.

$$v^u(k, b, \varepsilon_i^{r_0}; z_l, r_0, r, \mu) = \max_{n, k', b', D} \left[D + \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \pi_{lm}^z d_{m,r,s}(z_l, r_0, r, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{r_0,r} v_0(k', b', \varepsilon_j^r; z_m, r, s, \mu') \right] \quad (3)$$

$$\text{subject to: } k' \geq (1 - \delta)k, \quad b' \leq \theta_b \theta_k k,$$

$$0 \leq D \leq z_l \varepsilon_i^{r_0} F(k, n) - \omega(z_l, r_0, r, \mu) n + q(z_l, r_0, r, \mu) b' - b - [k' - (1 - \delta)k],$$

$$\text{and } \mu' = \Gamma(z, r_0, r, \mu)$$

The downward adjustment problem differs from that above only in that investment must be non-positive and, thus, its relative price is θ_k .

$$v^d(k, b, \varepsilon_i^{r_0}; z_l, r_0, r, \mu) = \max_{n, k', b', D} \left[D + \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \pi_{lm}^z d_m(z_l, r_0, r, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{ij}^{r_0,r} v_0(k', b', \varepsilon_j^r; z_m, r, s, \mu') \right] \quad (4)$$

$$\text{subject to: } k' \leq (1 - \delta)k, \quad b' \leq \theta_b \theta_k k,$$

$$0 \leq D \leq z_l \varepsilon_i^{r_0} F(k, n) - \omega(z_l, r_0, r, \mu) n + q(z_l, r_0, r, \mu) b' - b - \theta_k [k' - (1 - \delta)k],$$

$$\text{and } \mu' = \Gamma(z, r_0, r, \mu)$$

Notice that there is no friction associated with the firm's employment choice, since the firm pays its current wage bill after production takes place, and its capital choice for next period also has no implications for current production. Thus, irrespective of their current debt or their continuation into the next period, all firms sharing in common the same (k, ε) combination select the same employment, which we will denote by $N(k, \varepsilon; z, r_0, r, \mu)$, and hence have common production, $y(k, \varepsilon; z, r_0, r, \mu)$.⁵ The same cannot be said for the intertemporal decisions of continuing firms, given the presence of both borrowing limits and irreversibilities. Let $K(k, b, \varepsilon; z, r_0, r, \mu)$ and $B(k, b, \varepsilon; z, r_0, r, \mu)$ represent the choices of next-period capital and debt, respectively, made by firms sharing in common a complete individual type (k, b, ε) . We will characterize these decision rules below in section 3.

⁵Here forward, except where necessary for clarity, we suppress the indices for current aggregate and firm productivity, l and i , respectively.

2.2 Households

The economy is populated by a unit measure of identical households. Household wealth is held as one-period shares in firms, which we identify using the measure λ .⁶ Given the prices they receive for their current shares, $\rho_0(k, b, \varepsilon; z, r_0, r, \mu)$, and the real wage they receive for their labor effort, $\omega(z, r_0, r, \mu)$, households determine their current consumption, c , hours worked, n^h , as well as the numbers of new shares, $\lambda'(k', b', \varepsilon')$, to purchase at prices $\rho_1(k', b', \varepsilon'; z, r_0, r, \mu)$. The lifetime expected utility maximization problem of the representative household is listed below.

$$V^h(\lambda; z_l, r_0, r, \mu) = \max_{c, n^h, \lambda'} \left[U(c, 1 - n^h) + \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \pi_{lm}^z V^h(\lambda'; z_m, r, s, \mu') \right] \quad (5)$$

subject to

$$c + \int_{\mathbf{S}} \rho_1(k', b', \varepsilon'; z_l, r_0, r, \mu) \lambda'(d[k' \times b' \times \varepsilon']) \leq \omega(z_l, \mu) n^h + \int_{\mathbf{S}} \rho_0(k, b, \varepsilon; z_l, r_0, r, \mu) \lambda(d[\varepsilon \times k])$$

and $\mu' = \Gamma(z, r_0, r, \mu)$

Let $C^h(\lambda; z, r_0, r, \mu)$ describe the household decision rule for current consumption, and let $N^h(\lambda; z, r_0, r, \mu)$ be the rule determining the allocation of current available time to working. Finally, let $\Lambda^h(k', b', \varepsilon', \lambda; z, r_0, r, \mu)$ be the quantity of shares purchased in firms that will begin the next period with k' units of capital, b' units of debt, and idiosyncratic productivity ε' .

2.3 Recursive equilibrium

A *recursive competitive equilibrium* is a set of functions,

$$\left(\omega, q, (d_{m,r,s}), \rho_0, \rho_1, v_0, N, K, B, D, V^h, C^h, N^h, \Lambda^h \right),$$

that solve firm and household problems and clear the markets for assets, labor and output, as described by the following conditions.

- (i) v_0 solves (1) - (4), N is the associated policy function for exiting firms, and (N, K, B, D) are the associated policy functions for continuing firms
- (ii) V^h solves (5), and (C^h, N^h, Λ^h) are the associated policy functions for households

⁶Households also have access to a complete set of state-contingent claims. However, as there is no heterogeneity across households, these assets are in zero net supply in equilibrium. Thus, for sake of brevity, we do not explicitly model them here.

(iii) $\Lambda^h(k', b', \varepsilon_j, \mu; z, r_0, r, \mu) = \mu'(k', b', \varepsilon_j; z, r_0, r, \mu)$, for each $(k', b', \varepsilon_j) \in \mathbf{S}$

(iv) $N^h(\mu; z, r_0, r, \mu) = \int_{\mathbf{S}} \left[N(k, \varepsilon; z, r_0, r, \mu) \right] \mu(d[k \times b \times \varepsilon])$

(v) $C^h(\mu; z, r_0, r, \mu) = \int_{\mathbf{S}} \left[z\varepsilon F(k, N(\varepsilon, k; z, r_0, r, \mu)) \right.$

$$\left. - (1 - \pi_d) \mathcal{J} \left(K(k, b, \varepsilon; z, r_0, r, \mu) - (1 - \delta)k \right) \left(K(k, b, \varepsilon; z, r_0, r, \mu) \right. \right.$$

$$\left. - (1 - \delta)k \right) + \pi_d [\theta_k (1 - \delta)k - k_0] \mu(d[k \times b \times \varepsilon]),$$

$$\text{where } \mathcal{J}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ \theta_k & \text{if } x < 0 \end{cases}$$

(vi) $\mu'(A, \varepsilon_j) = (1 - \pi_d) \int_{\{(k, b, \varepsilon_i) \mid (K(k, b, \varepsilon_i; z, r_0, r, \mu), B(k, b, \varepsilon_i; z, r_0, r, \mu)) \in A\}} \pi_{ij} \mu(d[k \times b \times \varepsilon_i])$

$+ \pi_d \chi(k_0) H(\varepsilon_j, r)$, for all $(A, \varepsilon_j) \in \mathcal{S}$, defines Γ , where $\chi(k_0) =$

$\{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$

Let C and N describe the market-clearing values of household consumption and hours worked satisfying conditions (iv) and (v) above, and denote next period's equilibrium consumption and hours worked when $z' = z_m$ as C'_m and N'_m , respectively. Market-clearing requires that (a) the real wage equal the household marginal rate of substitution between leisure and consumption, $\omega(z, r_0, r, \mu) = D_2 U(C, 1 - N) / D_1 U(C, 1 - N)$, that (b) the bond price, q^{-1} , equal the expected gross real interest rate, $q(z, r_0, r, \mu) = \beta \sum_{m=1}^{N_z} \pi_{lm}^z D_1 U(C'_m, 1 - N'_m) / D_1 U(C, 1 - N)$, and that (c) firms' state-contingent discount factors agree with the household marginal rate of substitution between consumption across states $d_m(z, r_0, r, \mu) = \beta D_1 U(C'_m, 1 - N'_m) / D_1 U(C, 1 - N)$. We compute equilibrium by solving the firm-level optimization problem with these implications of household utility maximization imposed, thereby effectively subsuming households' decisions into the problems faced by firms.

Without loss of generality, we assign $p(z, r_0, r, \mu)$ as an output price at which firms value current dividends and payments and correspondingly assume that firms discount their future values by the household subjective discount factor. Given this alternative means of expressing

firms' discounting, the following three conditions ensure all markets clear in our economy.

$$p(z, r_0, r, \mu) = D_1 U(C, 1 - N) \quad (6)$$

$$\omega(z, r_0, r, \mu) = D_2 U(C, 1 - N) / p(z, r_0, r, \mu) \quad (7)$$

$$q(z, r_0, r, \mu) = \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \pi_{lm}^z p(z_m, r, s, \mu') / p(z, \mu) \quad (8)$$

Our reformulation of (1) - (4) below yields an equivalent description of the firm-level problem where each firm's value is measured in units of marginal utility, rather than output, with no change in the resulting decision rules. Suppressing the arguments of the price functions, exploiting the fact that the choice of n is independent of the k' and b' choices, and using the indicator function $\mathcal{J}(x) = \{1 \text{ if } x \geq 0 ; \theta_k \text{ if } x < 0\}$ to distinguish the relative price of nonnegative versus negative investment, we have:

$$\begin{aligned} V_0(k, b, \varepsilon_i; z_l, r_0, r, \mu) = & \pi_d \max_n p(z_l, r_0, r, \mu) \left[z_l \varepsilon_i F(k, n) - \omega(z_l, r_0, r, \mu) n \right. \\ & \left. + \theta_k (1 - \delta) k - b \right] + (1 - \pi_d) V(k, b, \varepsilon_i; z_l, r_0, r, \mu), \end{aligned} \quad (9)$$

where

$$V(k, b, \varepsilon_i; z_l, r_0, r, \mu) = \max_{n, k', b', D} \left[p(z_l, r_0, r, \mu) D + \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij}^{r_0, r} V_0(k', b', \varepsilon_j; z_m, r, s, \mu') \right] \quad (10)$$

subject to

$$0 \leq D \leq z_l \varepsilon_i F(k, n) - \omega(z_l, r_0, r, \mu) n + q(z_l, r_0, r, \mu) b' - b - \mathcal{J}(k' - (1 - \delta) k) [k' - (1 - \delta) k] \quad (11)$$

$$\text{and } b' \leq \theta_b \theta_k k. \quad (12)$$

3 Analysis

The problem listed in equations (9) - (12) forms the basis for solving equilibrium allocations in our economy, so long as the prices p, ω and q taken as given by our firms satisfy the restrictions in (6) - (8) above.⁷ From here, we begin to characterize the decision rules arising from this

⁷Here, and in many instances below, we suppress the z, μ arguments of price functions, decision rules and firm-level state vectors to reduce notation.

problem. Each firm chooses its labor $n = N(k, \varepsilon; z, r_0, r, \mu)$ to solve $z\varepsilon D_2 F(k, n) = \omega$, which immediately returns its current production, $y(k, \varepsilon) = z\varepsilon F(k, N(k, \varepsilon; z, r_0, r, \mu))$. Let $\pi(k, b, \varepsilon)$ represent the earnings of a firm of type (k, b, ε) net of labor costs and debt.

$$\pi(k, b, \varepsilon, z, r_0, r, \mu) \equiv z\varepsilon F(k, N(k, \varepsilon; z, r_0, r, \mu)) - \omega N(k, \varepsilon; z, r_0, r, \mu) - b \quad (13)$$

The challenging objects to determine are D, k' and b' for continuing firms. Turning to these, we will use a simple observation about the implications of borrowing constraints for the value a firm places on retained earnings versus dividends. If the firm places non-zero probability weight on encountering a future state in which its borrowing constraint will bind, the shadow value of retained earnings (which includes the discounted sequence of multipliers on future borrowing constraints) will necessarily exceed the shadow value of current dividends, p .⁸ This means that it will set $D = 0$. In this case, the binding budget constraint from equation 11 establishes that the firm's choice of k' directly implies the level of debt with which it will enter into the next period. We refer to any such firm as a *constrained firm*. To be clear, a constrained firm need not currently face a binding borrowing constraint; our definition includes any firm that can now or in future encounter a binding constraint. We will return to the problem solved by a constrained firm below. It is useful to first characterize the decisions of a firm whose capital choices are never affected by borrowing limits.

3.1 Decisions among unconstrained firms

Consider a firm that has accumulated sufficient wealth (via $k > 0$ or $b < 0$) such that collateral constraints will never again affect its investment activities. In this case, the sequence of multipliers on all possible future borrowing constraints are zero, and the firm is indifferent between allocating earnings to savings versus paying dividends. We refer to any such firm as *unconstrained*. Importantly, as it is indifferent between savings and paying dividends, an unconstrained firm's marginal value of retained earnings is equal to that of households.

Let W_0 represent the beginning-of-period expected value of an unconstrained firm and W its value if it will continue beyond the current period. These functions are analogous to those defined for any firm in (1).

$$W_0(k, b, \varepsilon_i; z_l, r_0, r, \mu) = \pi_d p \left[\pi(k, b, \varepsilon_i) + \theta_k (1 - \delta) k \right] + (1 - \pi_d) W(k, b, \varepsilon_i; z_l, r_0, r, \mu). \quad (14)$$

⁸This is easily proved using a sequence approach with explicit multipliers on each constraint; see Caggese (2007).

As in (2), a continuing unconstrained firm has a binary choice involving capital adjustment. Let $W^u(k, b, \varepsilon_i; z, r_0, r, \mu)$ represent its value it chooses to undertake an upward capital adjustment, and

$W^d(k, b, \varepsilon_i; z_l, r_0, r, \mu)$ its value conditional on a downward capital adjustment.

$$W(k, b, \varepsilon_i; z_l, r_0, r, \mu) = \max\{W^u(k, b, \varepsilon_i; z_l, r_0, r, \mu), W^d(k, b, \varepsilon_i; z_l, r_0, r, \mu)\} \quad (15)$$

An unconstrained firm must never again experience a binding borrowing constraint (in any conceivable future state). We assign any such indifferent firm a savings policy just ensuring that, under all possible future paths of (ε, z, r_0, r) , it will have sufficient wealth to implement its optimal investment plan while borrowing more that is permitted by (12). Below, we will define this *minimum savings policy*. While an unconstrained firm's minimum savings policy is affected by its capital choice, its capital choice is independent of its savings or debt, b . Thus, before we derive the savings policy for unconstrained firms, we characterize their capital adjustment.

By construction, an unconstrained firm has the same marginal valuation of savings as a household. It then follows from equation 13 that, if such a firm enters a period with any non-zero debt or savings, b , its value is affected only through the change in current earnings. As current earnings are valued by p , we can express the value of a continuing unconstrained firm of type (k, b, ε) as $w(k, \varepsilon) - pb$, where $w(k, \varepsilon) \equiv W(k, 0, \varepsilon)$. The firm's beginning-of-period expected value inherits the same property; $W_0(k, b, \varepsilon_i; z_l, r_0, r, \mu) = w_0(k, \varepsilon) - pb$, where $w_0(k, \varepsilon) \equiv W_0(k, 0, \varepsilon)$. Given these observations, we have:

$$\begin{aligned} W^u(k, b, \varepsilon_i; z_l, r_0, r, \mu) &= p\pi(k, b, \varepsilon_i) + p(1 - \delta)k \\ &+ \max_{k' \geq (1 - \delta)k} \left[-pk' + \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij}^{r_0, r} w_0(k', \varepsilon_j; z_m, r, s, \mu') \right] \end{aligned} \quad (16)$$

$$\begin{aligned} W^d(k, b, \varepsilon_i; z_l, r_0, r, \mu) &= p\pi(k, b, \varepsilon_i) + p\theta_k(1 - \delta)k \\ &+ \max_{k' \leq (1 - \delta)k} \left[-p\theta_k k' + \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij}^{r_0, r} w_0(k', \varepsilon_j; z_m, r, s, \mu') \right], \end{aligned} \quad (17)$$

where (13) defines $\pi(k, b, \varepsilon)$, and $\mu' = \Gamma(z, r_0, r, \mu)$. In the above, W^u and W^d are both strictly increasing in k . This in turn implies that W and W_0 are increasing functions of the unconstrained firm's capital, as are the w and w_0 functions defined above.

We may characterize the capital decision rule for an unconstrained firm by reference to two target capital stocks, the upward and downward adjustment targets that would solve the problems

in (16) and (17), respectively, were there no sign restrictions on investment. Define the upward target, k_u^* , as the capital a firm would choose given a unit relative price of investment, and define the downward target, k_d^* , as the capital a firm would choose given a relative price at θ_k .

$$k_u^*(\varepsilon_i) = \arg \max_{k'} \left[-pk' + \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w_0(k', \varepsilon_j; z_m, r, s, \Gamma(z_l, r_0, r, \mu)) \right] \quad (18)$$

$$k_d^*(\varepsilon_i) = \arg \max_{k'} \left[-p\theta_k k' + \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} w_0(k', \varepsilon_j; z_m, r, s, \Gamma(z_l, r_0, r, \mu)) \right] \quad (19)$$

Notice that each target is independent of current capital and depends only on the aggregate state and the firm's current ε . As such, all unconstrained firms that share in common the same current productivity ε have the same upward and downward target capitals. Note also that, because $\theta_k < 1$ (and because the value function w_0 is strictly increasing in k), the upward adjustment target necessarily lies below the downward target: $k_u^* < k_d^*$.

We are now in a convenient position to retrieve the unconstrained firm's capital decision rule. Given a constant price associated with raising (lowering) its capital stock, and because w_0 is increasing in k , the firm selects a future capital as close to the upward (downward) target as its constraint set allows. Thus, the firm's decision rules conditional on upward adjustment and downward adjustment are as follow.

$$k_u(\varepsilon) = \max \{ (1 - \delta) k, k_u^*(\varepsilon) \} \quad \text{and} \quad k_d(\varepsilon) = \min \{ (1 - \delta) k, k_d^*(\varepsilon) \}$$

Given these conditional adjustment rules, we know that an unconstrained firm of type (k, b, ε) selects one of three future capital levels, $k' \in \{k_u^*(\varepsilon), k_d^*(\varepsilon), (1 - \delta)k\}$. Which one it selects depends only on where its current capital lies in relation to its two targets. Recalling that $k_u^*(\varepsilon) < k_d^*(\varepsilon)$, it is straightforward to obtain the following decision rule for an unconstrained firm.

$$K^w(k, \varepsilon; z, r_0, r, \mu) = \begin{cases} k_u^*(\varepsilon; z, r_0, r, \mu) & \text{if } k < \frac{k_u^*(\varepsilon; z, r_0, r, \mu)}{1 - \delta} \\ (1 - \delta)k & \text{if } k \in \left[\frac{k_u^*(\varepsilon; z, r_0, r, \mu)}{1 - \delta}, \frac{k_d^*(\varepsilon; z, r_0, r, \mu)}{1 - \delta} \right] \\ k_d^*(\varepsilon; z, r_0, r, \mu) & \text{if } k > \frac{k_d^*(\varepsilon; z, r_0, r, \mu)}{1 - \delta} \end{cases} \quad (20)$$

Unconstrained firms maintain sufficient assets to prevent borrowing limits affecting their future investment. In this sense, they accumulate precautionary savings. Given the decision rule for capital, we isolate a minimum level of financial savings that insures that an unconstrained firm of type (ε, k) will never be affected by borrowing constraints across all possible future $(\varepsilon'; z', r, s, \mu')$.

Any such firm that maintains a level of debt not exceeding the threshold defined by the minimum savings policy will be indifferent to paying additional revenues in the form of dividends, or accumulating further savings. This, in turn, implies that the firm is willing to follow the minimum savings policy.⁹

Let $\tilde{B}\left(K^w(k, \varepsilon; z, r_0, r, \mu), \varepsilon_j; z_m, \Gamma(z, r_0, r, \mu)\right)$ define the maximum debt level at which a firm entering next period with capital $K^w(k, \varepsilon; z, r_0, r, \mu)$ and $(\varepsilon_j, z_m, r_0, r)$ will remain unconstrained. The following pair of equations recursively defines the minimum savings policy, $B^w(k, \varepsilon; z, r_0, r, \mu)$.

$$B^w(k, \varepsilon; z, r_0, r, \mu) = \min_{\{\varepsilon_j | \pi_{ij}^{r_0, r} > 0 \text{ and } z_m | \pi_{lm}^z > 0 \text{ and } s | \pi_{rs} > 0\}} \tilde{B}\left(K^w(k, \varepsilon), \varepsilon_j; z, r, s, \Gamma(z, r_0, r, \mu)\right), \quad (21)$$

$$\begin{aligned} \tilde{B}(k, \varepsilon; z, r_0, r, \mu) = & z\varepsilon F(k, N(k, \varepsilon)) - \omega N(k, \varepsilon) + q \min\left\{B^w(k, \varepsilon; z, r_0, r, \mu), \theta_b \theta_k k\right\} \\ & - \mathcal{J}\left(K^w(k, \varepsilon) - (1 - \delta)k\right) \left[K^w(k, \varepsilon) - (1 - \delta)k\right] \end{aligned} \quad (22)$$

In equation 21, $B^w(k, \varepsilon; z, r_0, r, \mu)$ is derived as the maximum level of debt with which the firm can exit this period and be certain to remain unconstrained next period, given that it adopts the unconstrained capital decision rule. Next, (22) defines the beginning of period maximum debt level under which a firm can adopt the unconstrained capital rule and debt not exceeding that identified by the minimum savings policy without paying negative dividends, and hence satisfy the definition of an unconstrained firm.

Notice that $\tilde{B}$ is increasing in the firm's current earnings, since these may be used to cover outstanding debt. The minimum operator imposes the borrowing constraint; if the firm does not have sufficient collateral to borrow to B^w , it can still be unconstrained if it has entered this period with sufficient savings to finance its investment.

Given the decision rule for capital and the minimum savings policy, we can now retrieve unconstrained firms' dividend payments.

$$\begin{aligned} D^w(k, b, \varepsilon, z, r_0, r, \mu) = & \pi(k, b, \varepsilon, z, r_0, r, \mu) - \mathcal{J}\left(K^w(k, \varepsilon) - (1 - \delta)k\right) \left[K^w(k, \varepsilon) - (1 - \delta)k\right] \\ & + q \min\left\{B^w(k, \varepsilon; z, r_0, r, \mu), \theta_b \theta_k k\right\} \end{aligned} \quad (23)$$

⁹ In effect, our minimum savings policy maximizes the dividends paid by unconstrained firms each period. We adopt this policy rather than the natural alternative minimizing unconstrained firms' period-by-period dividends so as to bound the $b < 0$ levels carried by firms that can with positive probability outlive any set finite age we consider.

The firm's value, listed above in (15), may be expressed as

$$W(k, b, \varepsilon; z, r_0, r, \mu) = pD^w(k, b, \varepsilon, z, r_0, r, \mu) \quad (24)$$

$$+ \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} W_0(K^w(k, \varepsilon; z, r_0, r, \mu), B^w(k, \varepsilon; z, r_0, r, \mu), \varepsilon_j; z_m, r, s, \mu'),$$

where $W_0(k, b, \varepsilon, z, r_0, r, \mu)$ is given by (14), and $\mu' = \Gamma(z, r_0, r, \mu)$.

3.2 Decisions among constrained firms

We now consider the decisions made by a firm that has, until now, been constrained. We begin by evaluating whether or not the firm has crossed the relevant wealth threshold to become unconstrained. If it has, the decision rules described above apply. If it has not, the collateralized borrowing constraint will continue to influence its investment decisions, and its choice of capital and debt will remain intertwined.

To ascertain whether a firm of type (k, b, ε) has become unconstrained, we need only consider whether it is feasible for the firm to adopt the capital rule $K^w(k, \varepsilon)$ and a level of debt not exceeding that implied by the rule $B^w(k, \varepsilon)$, while maintaining non-negative dividends in the current period. If the firm of type (k, b, ε) is able to adopt the decision rules in (20) and (21) without violating the non-negativity of dividends, then it achieves the value given by (24), and it exits the period indistinguishable from any other unconstrained firm that entered the period with (k, ε) .

$$\begin{aligned} V(k, b, \varepsilon; z, r_0, r, \mu) &= W(k, b, \varepsilon; z, r_0, r, \mu) \text{ iff } D^w(k, b, \varepsilon; z, r_0, r, \mu) \geq 0 \\ &= V^c(k, b, \varepsilon; z, r_0, r, \mu) \text{ otherwise} \end{aligned} \quad (25)$$

Any constrained firm that can adopt the decision rules of an unconstrained firm will always choose to do so, since $V \leq W$. However, when the inequality in the top line of (25) is not satisfied, the firm remains constrained, with value $V^c(k, b, \varepsilon; z, r_0, r, \mu)$.

We approach a continuing constrained firm's problem as follows. First, given its (k, ε) , we isolate a cutoff debt level under which non-negative investment is a feasible option. The lowest level of k' associated with non-negative investment is $(1 - \delta)k$. If this choice is not affordable given the firm's borrowing constraint in (12), it cannot undertake even a trivial upward capital adjustment. Using (11), it follows that, among any group of firms sharing a common (k, ε) , only those with $b \leq q\theta_b\theta_k k + z\varepsilon F(k, N(k, \varepsilon)) - \omega N(k, \varepsilon)$ can consider an upward capital adjustment.

Firms with higher levels of debt must choose a downward capital adjustment and repay debt by selling capital.

We identify the maximum capital stocks permitted by the borrowing constraint under upward and downward capital adjustment.

$$\begin{aligned}\bar{k}_u(k, b, \varepsilon) &\equiv (1 - \delta)k + \left[q\theta_b\theta_k k + \pi(k, b, \varepsilon) \right] \\ \bar{k}_d(k, b, \varepsilon) &\equiv (1 - \delta)k + \frac{1}{\theta_k} \left[q\theta_b\theta_k k + \pi(k, b, \varepsilon) \right]\end{aligned}$$

Next we determine the associated choice sets for upward and downward capital adjustment.

$$\begin{aligned}\Lambda^u(k, b, \varepsilon) &= [(1 - \delta)k, \bar{k}_u(k, b, \varepsilon)] \\ \Lambda^d(k, b, \varepsilon) &= [0, \max\{0, \min\{(1 - \delta)k, \bar{k}_d(k, b, \varepsilon)\}\}]\end{aligned}$$

Using these results and recalling V_0 in equation (9), we may express a continuing constrained firm's value as follows.

$$V^c(k, b, \varepsilon; z, \mu) = \max\{V^u(k, b, \varepsilon; z, r_0, r, \mu), V^d(k, b, \varepsilon; z, r_0, r, \mu)\}, \quad (26)$$

$$\begin{aligned}V^u(k, b, \varepsilon; z, r_0, r, \mu) &= \max_{k' \in \Lambda^u(k, b, \varepsilon)} \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V_0(k', b'_u(k'), \varepsilon_j; z_m, r, s, \mu') \quad (27) \\ \text{subject to } b'_u(k') &= \frac{1}{q} \left(-\pi(k, b, \varepsilon) + [k' - (1 - \delta)k] \right) \\ \text{and } \mu' &= \Gamma(z, r_0, r, \mu)\end{aligned}$$

$$\begin{aligned}V^d(k, b, \varepsilon; z, r_0, r, \mu) &= \max_{k' \in \Lambda^d(k, b, \varepsilon)} \beta \sum_{s=1}^{N_r} \pi_{r,s} \sum_{m=1}^{N_z} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^z \pi_{ij} V_0(k', b'_d(k'), \varepsilon_j; z_m, r, s, \mu') \quad (28) \\ \text{with } b'_d(k') &= \frac{1}{q} \left(-\pi(k, b, \varepsilon) + \theta_k[k' - (1 - \delta)k] \right) \\ \text{and } \mu' &= \Gamma(z, r_0, r, \mu)\end{aligned}$$

Denoting the capital stocks that solve the conditional adjustment problems in (27) and (28) above by $\hat{k}^u(k, b, \varepsilon)$ and $\hat{k}^d(k, b, \varepsilon)$, respectively, we obtain the following decision rules for capital and debt.

$$K^c(k, b, \varepsilon; z, r_0, r, \mu) = \begin{cases} \hat{k}^u(k, b, \varepsilon) & \text{if } V^c(k, b, \varepsilon; z, r_0, r, \mu) = V^u(k, b, \varepsilon; z, r_0, r, \mu) \\ \hat{k}^d(k, b, \varepsilon) & \text{if } V^c(k, b, \varepsilon; z, r_0, r, \mu) = V^d(k, b, \varepsilon; z, r_0, r, \mu) \end{cases} \quad (29)$$

$$\begin{aligned}B^c(k, b, \varepsilon; z, r_0, r, \mu) &= \frac{1}{q} \left[\mathcal{J}(K^c(k, b, \varepsilon; z, r_0, r, \mu) - (1 - \delta)k) \right] \left[K^c(k, b, \varepsilon; z, r_0, r, \mu) \right. \\ &\quad \left. - (1 - \delta)k \right] - \pi(k, b, \varepsilon; z, r_0, r, \mu)\end{aligned} \quad (30)$$

4 Calibration

The data on establishment-level investment dynamics are reported annually. As the mechanics of the reallocation of capital across firms are at the core of our model, we reproduce salient empirical regularities from this data. Accordingly, we set the length of a period to one year.

In the section to follow, we will consider how the mechanics of our model with real and financial frictions compare to those in two relevant reference models - one where there are no borrowing limits and one where there are neither financial nor real frictions ($\theta_k = 1$). These two reference models will help us to isolate how much the *interaction* between credit constraints and micro-level capital rigidities influences our economy's aggregate dynamics. Aside from the values of θ_b and θ_k , all three models share a common parameter set that is selected in our full model to best match moments drawn from postwar U.S. aggregate and firm-level data. However, as the average capital-to-output ratio and hours worked vary little across the three models, the results to follow are unaffected by our decision to maintain a fixed parameter set.

Across our model economies, we assume that the representative household's period utility is the result of indivisible labor (Rogerson (1988)): $u(c, L) = \log c + \varphi L$. The firm-level production function is Cobb-Douglas: $z\varepsilon F(k, n) = z\varepsilon k^\alpha n^\nu$. The initial capital stock of each entering firm is a fixed χ fraction of the typical stock held across all firms in the long-run of our full economy; that is, $k_0 = \chi \int k \tilde{\mu}(d[k \times b \times \varepsilon])$, where $\tilde{\mu}$ represents the steady-state distribution therein.

4.1 Aggregate data

We determine the values of β , ν , δ , α , φ and θ_b using moments from the aggregate data as follows. First, we set the household discount factor, β , to imply an average real interest rate of 4 percent, consistent with recent findings by Gomme, Ravikumar and Rupert (2008). Next, the production parameter ν is set to yield an average labor share of income at 0.60 (Cooley and Prescott (1995)). The depreciation rate, δ , is taken to imply an average investment-to-capital ratio of roughly 0.069, which corresponds to the average value for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables, controlling for growth. Given this value, we determine capital's share, $\alpha = 0.27$, so that our model matches the average private capital-to-output ratio over the same period, at 2.3, and we set the parameter governing the preference for leisure, $\varphi = 2.15$, to imply an average of one-third of available time is spent in market work.

We calibrate our model to reproduce an aggregate measure of the indebtedness of firms in the

U.S. economy. Specifically, we set the parameter determining our collateral constraint, $\theta_b = 1.35$, to imply an average debt-to-assets ratio at 0.366, which matches that of nonfarm nonfinancial businesses over 1952-05 in the Flow of Funds. This is a moderate degree of financial frictions, with firms able to take on debt up to 135 percent of the value of their tangible assets. However, the extent to which the resulting financial frictions affect firm-level and aggregate outcomes depends on the productivity process individual firms face, as well as the extent of investment irreversibility. We will determine these aspects of the model using firm-level data below.

We consider two separate aggregate shocks, exogenous changes in the aggregate component of total factor productivity, z , and in the variability of firm-specific productivity. When determining the stochastic process for aggregate shocks to TFP, we use the result that exact aggregation obtains in the reference model without real or financial frictions; in particular, it has an aggregate production function. We use this reference model to estimate an exogenous stochastic process for aggregate productivity. We begin by assuming a continuous shock following a mean zero AR(1) process in logs: $\log z' = \rho_z \log z + \eta'_z$ with $\eta'_z \sim N(0, \sigma_{\eta_z}^2)$. Next, we estimate the values of ρ_z and σ_{η_z} from Solow residuals measured using NIPA data on US real GDP and private capital, together with the total employment hours series constructed by Prescott, Ueberfeldt, and Cociuba (2012) from CPS household survey data, over the years 1959-2002. Our estimated persistence is 0.852 and the standard deviation of innovations is 0.014. We discretize this stochastic process using a grid with 3 shock realizations ($N_z = 3$) to obtain (z_l) and (π_{lm}^z) . We apply this exogenous shock process across all three models.

When choosing the stochastic process for uncertainty shocks, we assume that $N(r) = 2$ and that the persistence of a level of uncertainty is 0.9. This is roughly equivalent to the persistence of shocks to z measured above. Next we choose the variability of innovations to firm-specific total factor productivity, when the level of uncertainty is low, to match several moments from the firm level data. This exercise is described below. The relative standard deviation of innovations to firm-specific total factor productivity when the level of uncertainty is high is set to 1.825 times the standard deviation chosen for the low level of uncertainty. This is a relatively modest uncertainty shock when compared to the literature, see for example Bloom et al. (2011). We choose this relative variability so as to generate fluctuations in aggregate output, under uncertainty shocks, that are comparable to those under productivity shocks.

4.2 Firm-level data

The costly reversibility of investment and the dispersion of firm-level total factor productivity are calibrated to reproduce microeconomic evidence on establishment-level investment dynamics. We begin by assuming that firm-specific productivity follows an AR(1) log-normal process, $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'$, with $\eta' \sim N(0, \sigma_\eta^2)$. Next we choose θ_k , ρ_ε and σ_η jointly to reproduce three aspects of establishment-level investment data documented by Cooper and Haltiwanger (2006) based on a 17-year sample drawn from the Longitudinal Research Database. These targets are (i) the average mean investment rate (i/k) across establishments: 0.122, (ii) the average standard deviation of investment rates: 0.337, and (iii) the average serial correlation of investment rates: 0.058.¹⁰

While our model has life-cycle aspects affecting firms' investments, the Cooper and Haltiwanger (2006) dataset includes only large manufacturing establishments that remain in operation throughout their sample period. Thus, in undertaking this part of our calibration, we must select an appropriate model-generated sample for comparability with their sample. This we do by simulating a large number of firms for 30 years, retaining only those firms that survive throughout, and then restricting the dates over which investment rates are measured to eliminate life-cycle effects. This restricts attention to firms whose investment decisions are unaffected by their borrowing limits. In implementing this algorithm, we discretize firms' log-normal productivity process using 7 values ($N_\varepsilon = 7$) to obtain $\{\varepsilon_i\}_{i=1}^{N_\varepsilon}$ and $(\pi_{ij})_{i,j=1}^{N_\varepsilon}$.

The idiosyncratic shock process we calibrate has a persistence of 0.653 and a standard deviation of innovations of 0.135. As firms in the model sample are unaffected by borrowing constraints, their investments would respond immediately to changes in their total factor productivities in the absence of costs of uninstalling capital. This implies a negative autocorrelation in investment rates, since capital is determined by lagged investment. The costly reversibility of capital is then essential in reproducing the investment moments reported above, and we set $\theta_k = 0.95$ to reproduce the serial correlation of investment rates in the data. Note that this is only a 5 percent loss incurred in uninstalling capital. However, firm-level shocks are far more volatile and less persistent than aggregate shocks.

If we eliminate the real friction, firm-level capital reallocation dramatically increases. Maintaining our idiosyncratic shock process but setting $\theta_k = 1$, the mean and standard deviation of

¹⁰While not a target in the calibration, our model also closely matches a fourth moment drawn from the Cooper and Haltiwanger study, the fraction of establishment-year observations wherein a positive investment spike ($i/k > 0.20$) occurs: 0.186.

firm-level investment rates rises to 0.31 and 0.85, respectively, while the serial correlation falls to -0.16 . Alternatively, if we reset $(\rho_\varepsilon, \sigma_\eta)$ to match the mean and standard deviation of $\frac{i}{k}$ in the data, the serial correlation falls to -0.2 . When we impose a cost of uninstalling capital, firms become unresponsive to moderate changes in ε . This reduces the variability of their investment, and increases its persistence.

Finally, we choose the exit rate, π_d , and the fraction of the steady-state aggregate capital stock held by each entering firm, χ . We set π_d at 0.10, so that 10 percent of firms enter and exit the economy each year. Next, we set $\chi = 0.10$ so that, in an average date, each entering firm begins with an initial capital that is one-tenth the size of the aggregate stock. If we had assumed constant returns to scale in production, this would imply an employment size of entering firms averaging one-tenth the size of the typical firm in our economy, matching the Davis and Haltiwanger (1992) data. In our model economy, where returns to scale is 0.87, the relative employment size of a new firm is 21 percent. In this sense, our choice of χ_0 is conservative.

5 Results

We begin with a description of the long-run distribution of the firms in the model in the low uncertainty steady state. Figure 1 illustrates this distribution, over capital and the ratio of debt-to-capital, for all firms that have the median level of idiosyncratic total factor productivity. Constrained firms are identified by blue, and unconstrained firms are shown in red. The large spike in the upper right of the figure is the mass of entrants. These begin with a capital stock of approximately 0.14, one-tenth of the aggregate stock held by incumbents, and no debt. As constrained firms accumulate capital, they are able to borrow more. In terms of a cohort of firms that all entered in the same period, with the median level of TFP, as long as the firm-specific component of their TFP does not change, both debt and capital grow at the same rate. We then see the echo of entrants in peaks in the distribution along the top of the figure, moving from right to left. Over time the distribution spreads out as firms with other levels of idiosyncratic productivity, and thus debt and capital, now find themselves with the median level and appear in this figure, and firms previously holding the median level of productivity experience a shock and move out of this conditional distribution.

Firms that have accumulated sufficient capital and savings are unconstrained. They have a negative relation between their debt and their capital. Those with high levels of capital may have

little savings, and are shown in red in the top left of the figure. Others that have recently had low levels of idiosyncratic TFP, but are unconstrained, have high levels of savings. They appear in the lower right.

In terms of the implied life-cycle of a firm, firms begin with a low initial capital stock and little debt. Thereafter both their debt and their capital grow for several periods. Eventually, as the level of capital nears that of an unconstrained firm with the same level of TFP, a constrained firm will begin to reduce its debt while maintaining this investment. Over this period the firm remains constrained, although it faces no binding borrowing constraint. Eventually, after the firm has reduced its debt below zero and reaches a level sufficiently negative to imply sufficiently high financial wealth, it becomes unconstrained. Across all levels of idiosyncratic TFP, 86 percent of firms are constrained by current or possible future borrowing constraints. However, only 27 percent have currently binding borrowing limits.

Given mean reversion in their productivity shocks, most firms in figure 1 with median productivity this period also shared this same productivity in common in the previous period. The overall dispersion in capital among these firms illustrates the misallocation associated with borrowing constraints. All of them have the same level of TFP this period, and most had the same expected marginal product of capital when they undertook their investments in the previous period, but they vary widely in the capital, and thus the labor, allocated to their current production. Entrants, as a group, have an expected discounted marginal product of capital that is 1.125, well above the unit marginal cost associated with positive investments. This falls to just below 1 for unconstrained firms; the fact that it is not precisely 1.0 is an implication of the real friction present in our model. The implied misallocation of capital reduces long-run aggregate production by 4 percent, and it reduces measured TFP by 1 percent, when compared to a model without the real and financial frictions.

We begin our study of the dynamic response of the economy to exogenous shocks by examining business cycles driven by aggregate shocks to total factor productivity, z . The results, obtained from a 5041 period simulation of a model without uncertainty shocks, are presented in table 1.1. All series are in logs, detrended using a Hodrick-Prescott filter with a weight of 100.

In an effort to isolate the effect of collateralized borrowing and irreversible investment, we develop two control models. The first is a model where firms face no borrowing limits, they have unlimited access to loans whose repayment schedule may be structured in any way such

that expected repayment is equal to the risk-free real interest rate. This is the model without collateralized borrowing in table 1.2 where the only friction is partially irreversible investment. The model in table 1.3 removes this friction, and production in that economy is effectively identical to that which would occur in a representative firm environment.

Examining table 1.1, we see that the business cycle arising from our model is consistent with the responses we see in models with a representative firm. The variability of investment exceeds that of output, while consumption is less variable. The variability of employment is a little more than half the variability of output. All series are strongly procyclical. There is some amplification in the model; the variability of z , not reported in the table, is 1.301.

While there are some differences across these first three tables, their most striking aspect is an overall similarity. To the extent that aggregate fluctuations are driven by exogenous shocks to the common component of total factor productivity, the real and financial frictions that generate considerable heterogeneity in production have little effect on aggregate fluctuations. The percentage standard deviation of output rises from 1.919, in the full model with collateralized borrowing and irreversible investment, to 1.972 in the model without frictions. Differences in other series are also small.

Aggregate shocks to TFP do not lead to a large increase in the disparities in the expected discounted marginal product of capital across firms. A persistent shock to aggregate TFP increases the expected marginal product of capital across all firms, but does little to alter the percentage differences in the distribution of marginal products. In other words, it has no real consequence for the benefits or extent of reallocation. Thus the relative disparity between unconstrained and constrained firms is unaffected. This is also true for the model with only real frictions. As a result, none of the three models experiences a substantial increase in capital reallocation following a persistent rise in aggregate TFP. Thus capital irreversibility, which dampens reallocation, and collateralized borrowing limits, which do the same, do not have first order effects on the economy's aggregate response.

Uncertainty shocks to the idiosyncratic component of firms' total factor productivity are inherently reallocative. As the variability of innovations to firm-specific TFP rises, the level of capital reallocation in the economy without frictions increases sharply. The efficient allocation of resources, with a more dispersed distribution of firm-specific TFP, implies greater inequality across firms in their levels of capital. In the absence of real and financial frictions, this real-

location occurs quickly and the expected marginal product of future capital is equated across firms. Furthermore, given any level of aggregate capital, the eventual widening of the tails of the distribution of firm-specific TFP implies a rise in the expected aggregate marginal product of capital. So long as sufficient reallocation can occur, the increased productivity at the top end of the productivity distribution is met with an increased fraction of the capital stock, while a reduced fraction is allocated at the bottom, and output per unit capital rises. If we interpret this as an economy with a representative firm that uses the aggregate stock of capital and labor, there is an increase in measured TFP for this firm. As a result, an uncertainty shock, in the absence of reallocation frictions, will ultimately be followed by an expansion of employment and investment as the distribution of firm-specific productivity becomes more dispersed over time.

Tables 2.1 - 2.3 illustrate uncertainty shocks to our full model, the model without collateralized borrowing and the model without frictions. In all three simulations reported across these tables, we suppress movements in the exogenous component of aggregate TFP, z , to examine the implications of uncertainty shocks in isolation. In each case, however, the reallocation of capital delivers an endogenous rise in measured total factor productivity following a spread in the distribution of firm-specific productivities. As a result, the variability of measured TFP is 0.945 in the full model, 1.048 in the model without borrowing limits and 1.069 in the model with no frictions. The corresponding contemporaneous correlations with output are 0.956, 0.967 and 0.96. We emphasise that these procyclical movements in measured aggregate TFP are endogenous and the result of a reallocation of capital across firms in response to increased dispersion in firm-level TFP.

Based on our results in tables 1.1 - 1.3, we argued that neither collateralized borrowing limits nor partial investment irreversibility has quantitatively important implications for the business cycles that arise from aggregate productivity shocks. The same cannot be said of uncertainty shock driven business cycles. When the financial friction is eliminated in moving from table 2.1 to table 2.2, cyclical GDP volatility rises by nearly 17 percent. Investment and employment volatility rise roughly 13 and 19 percent, respectively, and even consumption volatility rises sharply. We explore the reasons behind these large differences below by examining a specific uncertainty shock episode drawn from our simulation.

Figures 2 and 3 illustrate the dynamic response of two economies following an exogenous rise in uncertainty. In figure 2, we present the changes in measured TFP, output, employment, investment and consumption for the economy with collateralized borrowing and irreversible investment.

Figure 3 shows the same uncertainty shock in the otherwise identical economy without borrowing limits. All series are reported as a percentage deviations from their long-run average value across the full length of the simulation.

Uncertainty shocks involve an initial period characterized by a rise in risk, but with no immediate effect on firm-specific shocks. This is because we have adopted the timing of Bloom (2009), in that households and firms learn of an impending rise in firm-level productivity dispersion one period before it arrives. The economy’s response in this initial period (date 126) captures a pure risk effect of the uncertainty shock, in the sense of Takahashi (2011). Employment and output fall, investment also falls while, as in Bloom (2009) and Bloom et al (2011), consumption rises. (These observations explain the sharply reduced consumption-to-GDP correlation seen across tables 2 in comparison with tables 1.) There are two main mechanisms driving these changes at the impact of a rise in uncertainty. First, given capital irreversibility, firms are more reluctant to invest before an episode of increased variability in the returns to investment. Because this discourages household savings, it also discourages labor supply. Second, the eventual rise in endogenous aggregate productivity and output that accompanies the widening distribution of firm-level productivity implies a rise in future income. This generates a wealth effect leading households to raise their current consumption and leisure, while reducing employment and savings.

Uncertainty is low in these two figures up until period 126. At that date, households and firms learn that firm productivity dispersion will begin to rise in the next period with a rise in the volatility of productivity innovations. Because the two mechanisms described above operate in the same direction at this initial date of the shock, employment and investment fall sharply in both models, while consumption begins to rise. As will be confirmed by an examination of these two economies over subsequent dates, the ultimate rise in household income is substantially lower in the economy with financial frictions relative to that without. As a result, the wealth effect in period 126 is not as strong there, and labor supply does not fall by quite as much. This mitigates the economy’s contraction slightly. GDP falls 4.6 percent below its long-run average value in the full model economy, versus 5.1 percent in the economy without borrowing limits.

Beginning in date 127, the actual standard deviation of shocks to firms’ idiosyncratic total factor productivities increases. Over time, this results in a more dispersed distribution of firm productivity. A relatively unrestrained reallocation of capital allows the economy without borrowing limits, in figure 3, to capitalize on the resulting rise in average marginal product of capital.

There, we see a strong response in aggregate investment, which rises more than 28 percent above its average level by period 128. The consequent increase in the aggregate capital stock, alongside the endogenous rise in aggregate TFP prompting it, raises overall labor productivity and hence the real wage. This leads employment to rise 5.2 percent above its mean level, while GDP rises 4.7 percent above average.

The economy with collateralized borrowing limits, in figure 2, has more difficulty in reallocating capital across firms. Some firms that should otherwise be undertaking large investments, given the signal implied by their current productivity levels, face binding borrowing limits and cannot do so. Thus, the economy is less able to capitalize on the increase in the average marginal product of capital implied by increased dispersion in firm productivity. For this reason, aggregate investment rises 6 percentage points less in this economy relative to its counterpart in figure 3. The same restraint implies a lesser rise in wages in this economy, so that there is a 1.3 percentage point lower response in employment, and GDP rises 1 percentage point less.

Another important result of an uncertainty shock, in contrast to an aggregate shock to TFP, is that the rises in employment, output and investment are gradual. These series reach their highest levels several periods after the shock. As we have discussed above, the gradual rises there are linked to a slow rise in measured TFP, since increased dispersion raises the overall marginal product of capital, and the aggregate stock of capital rises in reaction. Despite quite different underlying mechanisms, these aspects of the response to an uncertainty shock are similar to those following a credit shock (a tightening of collateralized borrowing constraints).

Two features of our current results distinguish an uncertainty shock driven business cycle from both that generated by aggregate productivity shocks and that arising from a credit shock. First, in contrast to the gradual expansion noted above, the sharp economic downturn is immediate and short-lived. This suggests a skewness in the cyclical component of model-generated GDP that may be consistent with that of GDP in the postwar U.S. Second, while aggregate production, employment and investment are initially falling in response to an uncertainty shock, measured TFP is rising. Given the initial non-monotone response of aggregate employment, alongside the gradual but monotone rise in aggregate TFP, uncertainty shock driven business cycles deliver a very low correlation between total hours worked and labor productivity. This correlation in the full model is 0.0236, while it is -0.0348 in the model without collateralized borrowing limits. Both values are far closer to their empirical counterpart of -0.21 than the correlations generated

by aggregate shocks to TFP, which are typically close to 1.

6 Concluding remarks

To be added.

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TABLE 1.1. Business Cycles in the Full Economy: Aggregate Shocks

$x =$	Y	C	I	N	K
σ_x	1.919	0.987	7.358	1.050	0.915
$corr(x, Y)$	1.000	0.939	0.968	0.946	0.066

TABLE 1.2. Business Cycles without Financial Frictions: Aggregate Shocks

$x =$	Y	C	I	N	K
σ_x	1.955	0.971	7.368	1.111	0.921
$corr(x, Y)$	1.000	0.930	0.969	0.948	0.062

TABLE 1.3. Business Cycles without Financial or Real Frictions: Aggregate Shocks

$x =$	Y	C	I	N	K
σ_x	1.972	0.944	7.960	1.159	0.889
$corr(x, Y)$	1.000	0.918	0.968	0.950	0.047

TABLE 2.1. Business Cycles in the Full Economy: Uncertainty Shocks

$x =$	Y	C	I	N	K
σ_x	1.864	0.589	10.856	1.754	1.098
$corr(x, Y)$	1.000	0.241	0.968	0.955	-0.096

TABLE 2.2. Business Cycles without Financial Frictions: Uncertainty Shocks

$x =$	Y	C	I	N	K
σ_x	2.176	0.666	12.247	2.095	1.267
$corr(x, Y)$	1.000	0.274	0.967	0.952	-0.101

TABLE 2.3. Business Cycles without Financial or Real Frictions: Uncertainty Shocks

$x =$	Y	C	I	N	K
σ_x	2.236	0.665	13.422	2.171	1.304
$corr(x, Y)$	1.000	0.241	0.968	0.955	-0.096

FIGURE 1. Steady state distribution in the full model

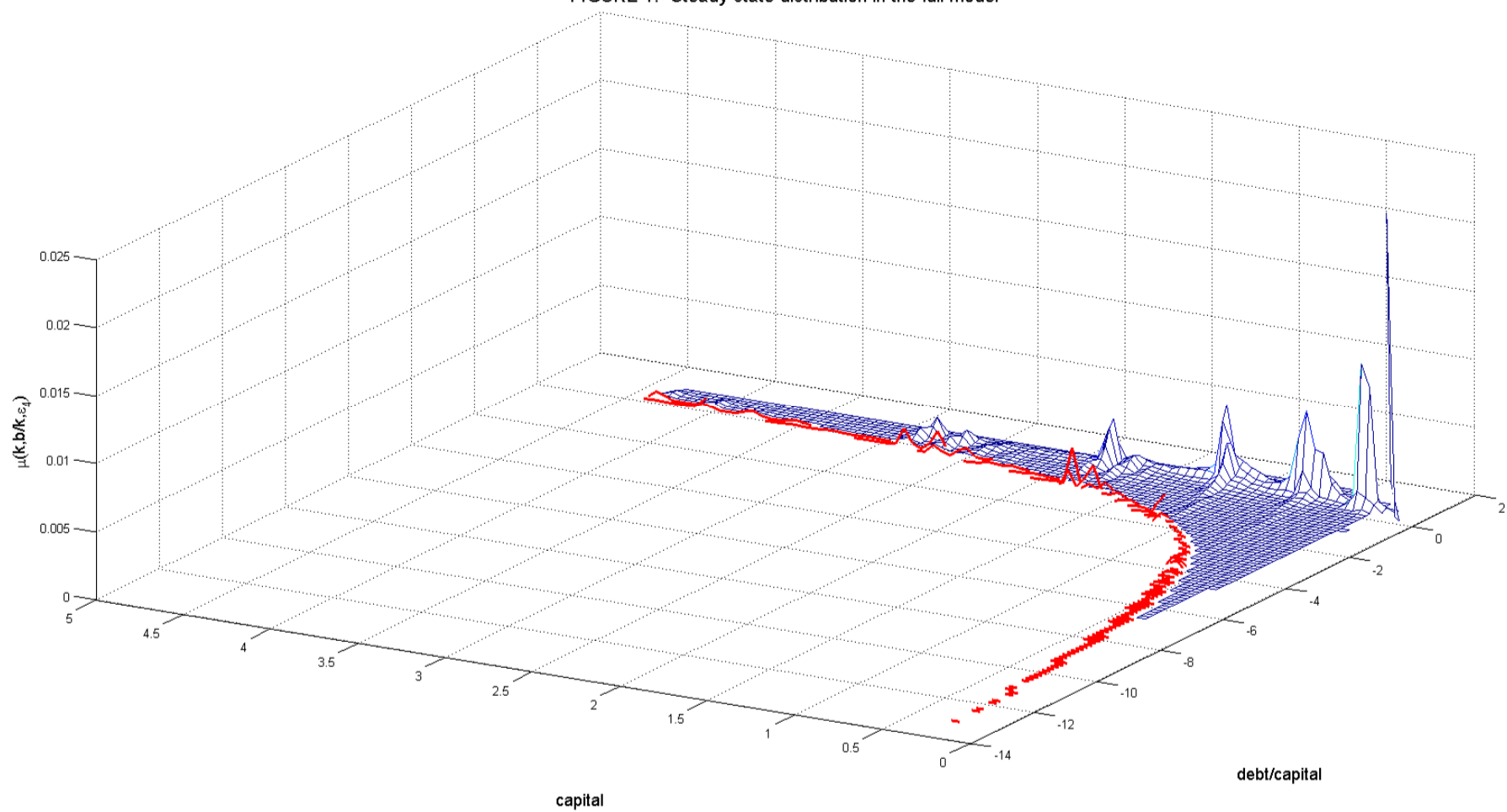


Figure 2 Uncertainty Shocks in the full model

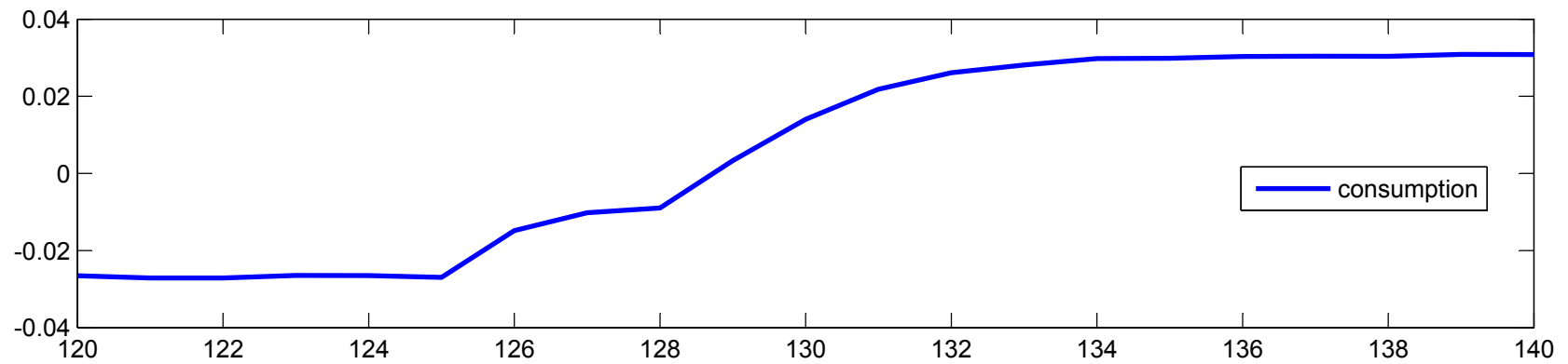
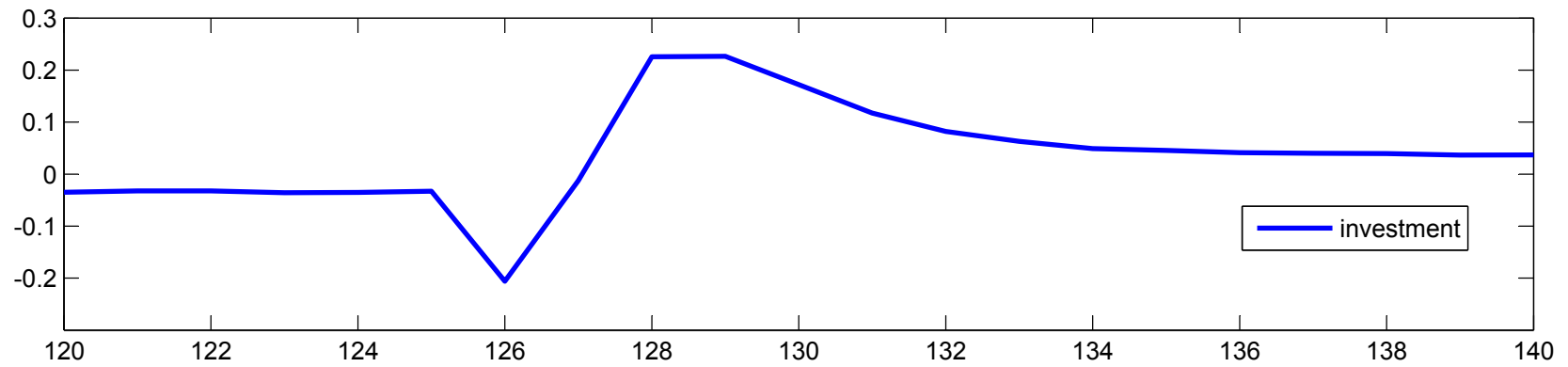
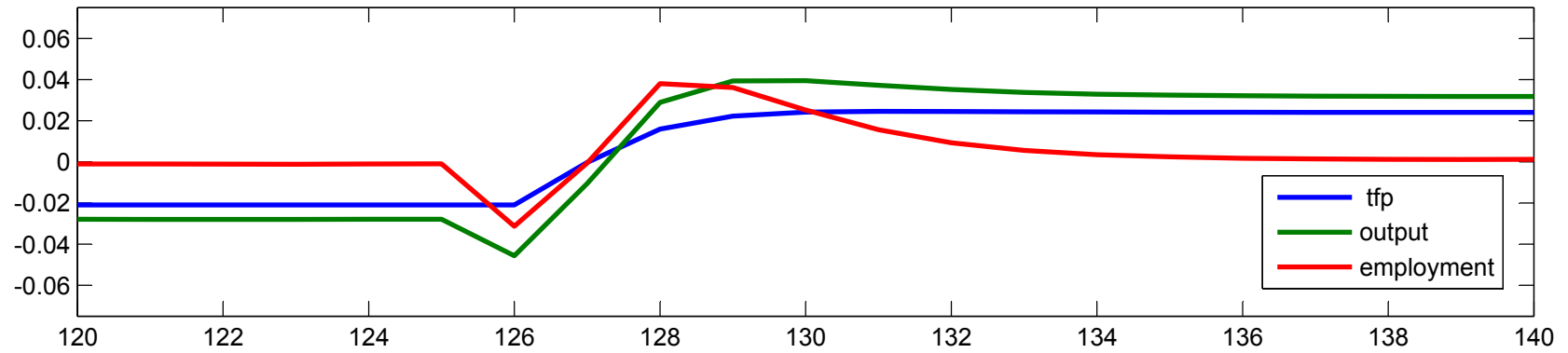


Figure 3 Uncertainty Shocks in the model without collateral constraints

