MEASURING THE DYNAMIC EFFECTS OF MONETARY POLICY SHOCKS: A BAYESIAN FAVAR APPROACH WITH SIGN RESTRICTION[1]

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We estimate the effects of monetary policy shocks in a Bayesian factor-augmented vector autoregression (BFAVAR). We propose a novel identification strategy of imposing sign restrictions directly on the impulse responses of large sets of variables. The novel feature and key strength of our approach is the additional "bite" due to the differences in factor loadings across sets of time series representing, say, "prices" or "output". Furthermore, our procedure does not require a structural interpretation of the factors themselves or adding observables to the list of factors. We impose the conventional wisdom regarding the responses of prices, monetary aggregates, spreads and interest rates. Our results are robust across different subsamples and avoid anomalies arising in Cholesky identifications.

Keywords: Bayesian FAVAR, Dynamic Factor Models, Identification, Sign Restriction, Gibbs Sampling, Monetary Policy Shocks.

1. INTRODUCTION

What are the dynamic effects of monetary policy shocks on the economy? We answer this question by combining two recent advances in empirical macroeconomics: factor-augmented VARs (FAVAR) and identification per sign restrictions. We propose a novel identification strategy of imposing sign restrictions directly on the impulse responses of large sets of variables to shocks in the core factor VAR. The novel feature and key strength of our approach is the additional "bite" due to the differences in factor loadings across sets of time series representing, say, "prices" or "output". Furthermore, our procedure does not require a structural interpretation of the factors themselves or adding observables to the list of factors. We impose the conventional wisdom regarding the responses of prices, monetary aggregates, spreads and interest rates.

By contrast, the literature has often applied FAVAR identification schemes, which require structural interpretations of the factors as well as adding observables to the list of factors. A popular procedure for the question at hand is a recursive Cholesky decomposition, with the federal funds rate and the monetary policy shock ordered last. We document that this approach leads to a number of puzzling results and subsample instability. By contrast, our results remain fairly stable across subsamples, and avoid e.g. the price puzzle by construction.

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Though prevalently analyzed in a large body of literature the real side effects of monetary policy shocks is still subject to debate. A major suspect for the dissent regarding results and conclusion is the key ingredient of identification. Depending on the assumptions underlying the identifying strategy, results are different sometimes at odds with economic theory and the class of (log-)linearized DSGE models. To grapple the task at hand we propose to employ Bayesian Factor-Augmented vector autoregression (henceforth BFAVAR) identified via a richer set of sign restrictions than currently employed in the literature. We argue that our approach is well suited to successfully pin down the correct impact of a large number of unconstrained variables of interest. To this end we can approximate an exact identification by setting restrictions grounded on widely accepted conventional wisdom derived from economic theory. This amounts to impose a negative response of a set of prices, money aggregates and a positive response for short term interest rates for a specified period following a contractionary monetary policy shock.

Following the lead of Sims [1972,1980,1986] most researchers analyze the question at hand through the lens of a vector autoregression (VAR). Most VAR studies consider a small number of variables in order to save degrees of freedom for keeping the model tractable. This means with few exceptions to employ a 6-8 variable VAR[1]. As pointed by Bernanke and Boivin [2003] monetary policy takes place in a "data-rich environment" a feature that VARs cannot accomplish appropriately due to the "curse of dimensionality" they suffer from. Thus the appealing feature of tractability also marks the limitations inherent in (monetary) VARs of small scale.

There are four key limitations of conventional VARs which dynamic factor models (henceforth DFM) can cope with, hence motivating our choice for the FAVAR framework. First, the restricted set of variables considered in VAR models is at odds with the rich set of information available to and obviously monitored by the private sector and central bankers prior to taking their decisions. Thus this restriction can entail the well known "omitted variable bias" which might lead to biased inference. Anomalies such as the "price puzzle", raised by Sims [1992] is argued to be indebted to the lack of inappropriate controlling for information that central bankers probably have about future inflation. Second, those few indicators considered in the VAR analysis are each supposed to represent a whole economic concept, e.g. GDP is supposed to reflect economic activity which is apparently restrictive. This reflects a fragmentary picture of the dynamics of the economy and hence is inept. Third, we are restricted to check the impact of only those few variables considered. However, academic researchers and practioneers alike are interested and concerned about many more economic variables affected by macroeconomic shocks. Forth, and of most interest for our identification strategy, is that the larger number of time series considered paves the way

[1] Leeper, Sims and Zha [1996] employ a 18 Bayesian VAR imposing over identifying restrictions.

for approximating exact identification as many more indicators can be restricted according to economic theory. Hence we can pin down the effects of a random shift in monetary policy with less uncertainty as the identified responses fulfill a broader set of economically reasonable assumptions. For instance many more price series could be restricted not to increase after a contractionary monetary policy shock. Thus the space of potential identified impulse response functions can be narrowed down substantially and robustness can be monitored.

Recent advances in DFM help to overcome the aforementioned drawbacks by parsimoniously extracting few dynamic factors out of a large panel of macroeconomic data, which summarize the crucial comovements among the driving forces of the economy. Thus the rich interrelation of the monetary, financial and the real sector are detected. Bernanke and Boivin [2003] Stock and Watson [2005] and Bernanke, Boivin and Eliasz [2005] coined the FAVAR models which is a unifying framework that combines DFM with the VAR analysis. This approach has been surveyed and extended with respect to different identification strategies for the classical approach by Stock and Watson [2005]. They argue that the large set of data considered both reflects the monetary transmission mechanism better and improves the identification of shocks. For the estimation we choose the Bayesian likelihood-based estimation based on MCMC methods which is fully parametric. Thus we can explicitly exploit the factor structure of the data and the law of motion of the extracted factors. This comes at the cost of computational burden.

The key ingredient for analyzing the impact of a random shift in monetary policy is identification which is a highly controversial matter. Different assumptions produce different results sometimes at odds with economic theory. We promote to rather explicitly impose economic theory instead of implicitly expecting via the tool of sign restrictions. Key advantages are the following. First, sign restrictions, as introduced by Uhlig [2005] impose "conventional wisdom" explicitly as part of the identifying assumptions. Second, sign restrictions avoid anomalies such as the "price puzzle" by construction. Third, for identification purposes DSGE models contain a large number of adaptive sign restrictions while they rarely deliver the whole set of zero restrictions required to recover all economic shocks[2]. Hence it is a "weak" identification compared to schemes based on zero-type restrictions. Fourth, identification schemes that have been employed so far in large dimensional models often rely on some recursive identification in which observables need to be included in the factor equation as it is done in the FAVAR framework[3]. Standard identification in a general dynamic factor model in which the factor are pure statistical objects without a clear economic interpretation are not feasible. This is another key appeal of our approach as is does not pose any difficulties. Finally, it is quite easy to implement sign restrictions ex post.

As a comparison for the performance of our approach we choose the Cholesky identification applied by Bernanke, Boivin and Eliasz [2005]. They prefer the

[2]See Canova [2007] for a good discussion.

[3]See e.g. Bernanke, Bouvin and Eliasz (2005)

results provided by the nonparametric two-step estimation approach based on dynamic principal components. Amongst others they argue the results seem more reasonable to them. We show that the problem with their likelihood-based results is that the employed Cholesky identification does not identify correctly the structural shock. In the literature there is a tendency of uniformity w.r.t. the underlying assumptions in applied work towards the Cholesky identification. But this approach is unrelated to the class of DSGE models and hence results are often at odds with what economic theory predicts. Cholesky identification relies on the informational orderings about the arrival of shocks. For a discussion of the criticism of Cholesky identification see, e.g. Cooley and LeRoy [1985] who argue that the contemporaneous recursive structure is hard to obtain in general equilibrium. Canova and Pina [2005] show that DSGE models almost never recover the zero restrictions employed to identify monetary shocks and that misspecification of the feature of the model economy can be substantial.

Identifying via Cholesky identification with the policy instrument ordered last we find that the price puzzle is still present and unreasonably high and in our benchmark model even increasing for a period up to four years. We conclude that the Cholesky identification does not recover the structural shocks. Hence the results are not identified. We find that industrial production decreases after a monetary contraction with a maximum impact after two years before reverting back. However this result is sensible with respect to the data span considered for estimation. Our subsample analysis shows that this result vanishes for the post Volcker disinflation period. Hence for the latter case the impact on output is not inconsistent with monetary neutrality. Regarding the FAVAR specification we find that our benchmark model which includes the federal funds rate and CPI as factors included in the FAVAR equation combined with our "maximal set of restrictions" works best as regards the uncertainty associated[4]. Furthermore we find that the forecast-error revision variance of output due to random shifts in monetary policy accounts for less than 20 percent on average which is consistent with results in Sims and Zha [2006] and Uhlig [2005]. Similar results have been obtained for the period of the US Great Depression in Sims [1999] and Amir Ahmadi and Ritschl [2007].

There different approaches to the sign restriction identification which are all common in the sense that they do not rely on zero-type restriction and identify via restriction on the sign and/or shape of the structural impulse response function. But they are different in their motivation. Early references are Dywer [1997], Faust [1998], Canova and Pina [2005], Canova and De Nicol [2002]. The key difference to Dywer [1997] is that in our approach we do not restrict the shape of the impulse response function. Faust [1998] imposes sign restrictions only on impact focusing on the fragility of the consensus conclusion that monetary policy shocks account for a small fraction of output fluctuations. Canova and De Nicol

[4]It is important to note that this benchmark specification is different to the one of BBE who only include the federal funds rate as a factor in the FAVAR equation. More details about our choice can be found on the section on FAVAR specification.

[2002] and Canova and Pina [2005] impose restrictions on the cross-correlation of variables in response to shocks, adding restrictions until the maximum number of shocks is uniquely identified. Uhlig [2005] imposes the restriction for several periods and leaves the variable of key interest in the analysis unrestricted, hence the term agnostic identification.

Since our first draft in 2005, Mumtaz and Surico (2009) too have introduced sign restrictions to FAVARs. There are key differences between their approach and ours, however. These authors use factors as summaries of blocks of data, constructing, for example, a real activity factor. They then apply standard (contemporaneous) sign restriction and conventional identification techniques to the small-scale VAR in these factors. The difference of their paper to the existing literature is thus the application of sign restrictions to these constructed variables, but is conventional otherwise. By contrast, we do not impose a structural interpretation to our factors, and instead impose sign restrictions directly on the responses of the various variables, exploiting the variations in factor loadings across large sets of, say, price responses. It is the latter feature which gives our procedure substantial "bite" and constitutes the major innovation compared to the existing literature. To our knowledge, ours is the first method that allows identification of shocks in dynamic factor models without requiring a structural interpretation of the factors themselves and without requiring adding observables to the list of factors. Our procedure should therefore be appealing and straightforward to use in a variety of applications.

2. THE MODEL

The model we apply is the Bayesian version of the FAVAR model introduced by Bernanke, Boivin and Eliasz [2005] which is cast in state space form. Here X_t^c denotes the $[N_c \times 1]$ vector of observable variables in period t, where $t = 1, \ldots, T$ is the time index and the superscript c refers to the panel out of which the common factors are extracted. Let f_t^c denote the $[K_c \times 1]$ vector of unobservable factors in period t and e_t^c the $[N_c \times 1]$ time t idiosyncratic component of the respective variables. Furthermore let f_t^y denote the $[N_y \times 1]$ perfectly observable vector of variables that have pervasive effects throughout the economy and are considered so important that they should be included as factors. N_c, K_c and N_y denote the number of variables in X_t^c, the number of factors to be extracted from X_t^c and the number of perfectly observable factors respectively. The model is

$$X_t^c = \lambda^c f_t^c + \lambda^y f_t^y + e_t^c \tag{2.1}$$

$$e_t^c \sim N(0, R_e) \tag{2.2}$$

Here λ^c and λ^y denote the matrix of factor loadings of the factors and the perfectly observable variables included as factors with dimension $[N_c \times K_c]$ and $[N_c \times N_y]$ respectively. The error term e_t^c has mean 0 and covariance R_e which is assumed to be diagonal. Hence the error terms of the observable variables are mutually uncorrelated. The FAVAR state equation represents the joint dynamics of factors and the observable policy variables (f_t^c, f_t^y) following a $VAR(P)$

process.

$$\begin{bmatrix} f_t^c \\ f_t^y \end{bmatrix} = \sum_{p=1}^{P} \phi_p \begin{bmatrix} f_{t-p}^c \\ f_{t-p}^y \end{bmatrix} + u_t^f \tag{2.3}$$

$$u_t^f \sim N(0, Q_u) \tag{2.4}$$

where u_t^f is the time t reduced form shock, Q_u is the factor error covariance matrix and the ϕ_p's denote the respective p-lag coefficient matrices. The dimensions are $[K \times 1]$, $[K \times 1]$ and $[K \times K]$ respectively. Note that the total number of factors is $K = K_c + N_y$.

For the estimation, we rely on now standard techniques. A detailed survey with several identification schemes in the classical estimation approach can be found in Stock and Watson [2005]. Bernanke, Boivin and Eliasz [2005] present two competing approaches. The second estimation approach described in their paper is the one that we employ in this paper because the likelihood based one-step estimation approach employing MCMC methods explicitly exploits the factor structure. We employ a Bayesian approach. We estimate the model with a multi-move Gibbs sampler for which we have to cast the model into its state space representation. Details matter: they are described in appendix A.

3. IDENTIFICATION

The major objective of this paper is to identify monetary policy shocks in a data rich environment through imposing sign restrictions as introduced by Uhlig [2005] for the VAR framework. The issue of how to identify structural shocks through the decomposition of the prediction error u_t^c and in particular of monetary policy shocks, has been subject of much debate in the literature. From an economic perspective it seems desirable to have an identification scheme which guarantees that the impulse responses satisfy conventional wisdom. In FAVAR models the task is actually the same with the main distinction that the structural shocks are not required to be deduced from the innovation of the reduced form observation equation, but from the FAVAR innovation, including the factors that summarize the crucial dynamics of the observed data. For comparison purposes we will employ two identifying schemes, namely the factor generalization of the aforementioned sign restriction and the factor generalization of the recursive Cholesky identification. These two identification schemes shall be explained in the following.

3.1. *Factor generalization of Sign Restrictions*

Identification of structural shocks through imposing sign restrictions is based on the assumptions about the sign of the impulse response functions for a specified period of key macroeconomic variables. Such restrictions should represent "conventional wisdom" derived from economic theory that most researchers can agree on. Conventional wisdom says that after a monetary policy contraction the federal funds rate should increase, the prices should not increase and the nonborrowed reserves should decrease. In other identification schemes that do not accomplish this wisdom, the researchers tend to call this empirical observation a "puzzle". There are even some researchers that try to build a model producing such "puzzles" out of a model as has been done by Christiano, Eichenbaum and Evans [2005]. Sims gives the advice to avoid unreasonable identification schemes. The sign restriction approach seems reasonable, especially in the context of FAVAR that can account for large cross-sectional data. Such models allow to check the robustness of identification and tighten the restriction along the lines of conventional wisdom as there are far more relevant indicators of interest.

In the context of a contractionary monetary policy shocks Uhlig [2005] set the restriction that prices and nonborrowed reserves should not increase and the federal funds rate should increase for a specified period following a contractionary monetary policy shock. We employ different versions for our empirical analysis which is summarized in table (I). The structural version of the FAVAR equation (A.3)

$$f_t = \sum_{p=1}^{P} \phi_p f_{t-p} + u_t^f$$

where the matrix A is an orthogonal invertible matrix of dimension $[K \times K]$ which satisfies $u_t^c = A\nu_t^c$. We are only interested in identifying one single shock therefore it is sufficient to identify one single row of A denoted by a_k where k refers to the respective shock or row in A. This single-equation identification is the more common approach that most of the recent literature pursue. The alternative would be to identify one row but the whole matrix A, which means to identify the full system. This approach goes back to Blanchard and Watson [1986]. The crucial step is to represent the one-step ahead prediction error u_t^c as a linear combination of orthogonalized structural shocks. We assume the fundamental innovations to be mutually independent and normalized to have unit variance. Hence $E[\nu_t^c \nu_t^{c\prime}] = I_K$. The restriction on A emerges from the covariance structure of the reduced form factor innovation which results in

$$\begin{aligned} Q_u &= E[u_t^f u_t^{f\prime}] \\ E[u_t^f u_t^{f\prime}] &= AE[\nu_t^f \nu_t^{f\prime}]A' \\ AE[\nu_t^f \nu_t^{f\prime}]A' &= AA' \end{aligned}$$

According to Uhlig [2005] we define an impulse vector as a column of the matrix A. Such a vector can be obtained from any decomposition, e.g. the Cholesky decomposition, of the covariance matrix of the factor residual matrix $\tilde{A}\tilde{A}' = Q_u$.

Definition 1 *The vector $a \in \Re^K$ is called an **impulse vector**, iff there is some matrix A, so that $AA' = Q_u$ and so that a is a column of A*

According to the Proposition 1 of Uhlig [2005,pp. 18], any impulse vector can be characterized as follows. Let $\tilde{A}\tilde{A}' = Q_u$ be the Cholesky decomposition. Then a is an impulse vector if and only if there is some K-dimensional vector α of unit length so that

$$a = \tilde{A}\alpha$$

Given the impulse vector, let $r_k(s) \in \Re^K$ be the vector response at horizon s to the k-th shock in a Cholesky-decomposition of Q_u. Then the impulse response $r_a(s)$ for a is simply given by

$$r_a(s) = \sum_{i=1}^{K} \alpha_i r_i(s).$$

For estimation consider the companion form of the state space in (A.4)-(A.5)

$$\begin{aligned} F_t &= \Phi F_{t-p} + u_t \\ X_t &= \Lambda F_t + e_t. \end{aligned}$$

To compute the impulse response vector a, let $\mathbf{a} = [a', 0_{1,K(P-1)}]'$ and compute

$$r_{a,k}(s) = (\Phi^s \mathbf{a})_k.$$

to get the impulse response of factor k to an impulse in a at horizon s. Note that $r_a(s)$ is the vector of impulse response functions of all factors to an impulse vector a at horizon s. As a second step we have to compute the impulse response functions of the single variables by combining the respective factor loading with $r_a(s)$ accordingly. This requires to compute

$$r_a^n(s) = \Lambda_n r_a(s).$$

where Λ_n is the respective n-th row vector of the factor loading matrix. We set the sign restriction on the shape of the individual impulse response functions according to the following assumption:

Assumption 1 A (contractionary) monetary policy impulse vector *is an impulse vector* $\mathbf{a}$ *so that the individual impulse response functions to* $\mathbf{a}$ *of prices and nonborrowed reserves are not positive and the impulse responses short term interest rate is positive, for a specified horizons s=0,...,S.*

3.2. *Factor generalization of Cholesky Identification*

For comparison we also employ the Cholesky Identification following Bernanke, Boivin and Eliasz [2005] who impose a recursive structure. Let

$$\begin{aligned} u_t &= \tilde{A}\nu_t \\ \tilde{A}\tilde{A}' &= Q_u \end{aligned}$$

where $\tilde{A}$ is lower triangular Cholesky factor of the factor residual VCV matrix and where the policy instrument is ordered last in the FAVAR equation. The identifying restrictions follow in parts from the factor identification which we employ as explained in Bernanke, Boivin and Eliasz [2005] which is actually independent from the identification task regarding structural shocks. The factor identification restrict the contemporaneous channel through which monetary policy shocks can effect the first $[K_c \times K_c]$ variables of our data panel which is part of the shock identification as the first variables are output related ones.

Hence the contemporaneous reaction of the first $[K_c \times K_c]$ output related variables is restricted such that it does not react in the shock period. Furthermore the ordering of the panel is important. A detailed description of the identification assumptions can be found in Bernanke et. al. [2005] and Stock and Watson [2005].

4. A NOTE ON THE FAVAR SPECIFICATION

Bernanke, Boivin and Eliasz add the Federal Funds Rate as the key policy variable to a list of factors. The following considerations show, that one ought to also include those variables, for which the "noise" component in the factor representation is relevant for the monetary authority, when choosing its monetary policy instrument. Without it, the model is misspecified, and the identification of the monetary policy shock is incorrect.

While this may sound reasonably obvious in this rather general form, these considerations may also show, why prices might respond positively to a monetary policy shock identified in a FAVAR, when prices are not included in the core VAR. This insight may help to understand the puzzling results for the CPI index in figures IV and V of Bernanke, Boivin and Eliasz [2005].

To provide a simple example, suppose that a price index component p_t and an interest rate component r_t follow the data generating process

$$p_t = -\alpha r_{t-1} + \gamma p_{t-1} + \epsilon_{p,t} \tag{4.1}$$

$$r_t = \phi p_t + \epsilon_{m,t} \tag{4.2}$$

where α, γ, ϕ are positive coefficients, where $\epsilon_{p,t}$ and where $\epsilon_{m,t}$ are shocks to the price level and to monetary policy respectively, assumed to be independent with each other and across time as well as normally distributed with variances $\sigma^2_{\epsilon,p}$ and $\sigma^2_{\epsilon,m}$.

It is best to think about the components p_t and r_t as the noise components in a factor structure. The model above thus makes the extreme assumption, that the monetary policy shock is entirely contained in the "noise" component of the interest rates. There are also more general versions. For example, the structure above is relevant if the monetary policy shock does not affect the price level via the other factors, but solely through the direct effect of interest rates on the "noise component" of the price level. It is also relevant, if part of the monetary policy shock component in a FAVAR without the price level included in the core VAR can actually be explained by movements in the price level, and it is the part of interest rates driven by that component, which affects the price level.

Note that the model implies

$$p_t = (\gamma - \alpha\phi)p_{t-1} + \epsilon_{p,t} - \alpha\epsilon_{m,t-1} \tag{4.3}$$

Assume that $0 \leq \gamma - \alpha\phi < 1$, and one can see that a positive monetary policy shock $\epsilon_{m,t}$ will affect the price level negatively with a one period delay, and with the price level gradually climbing back up to zero.

The empirical issue is now to correctly identify the monetary policy shock $\epsilon_{m,t}$. The approach in Bernanke-Boivin-Eliasz recommends to add the federal funds rate to the available factors. Since there are no factors in the model above (or since they have already been subtracted out), this amounts to estimating the model

$$r_t = \beta r_{t-1} + u_t \tag{4.4}$$

$$p_t = \lambda r_t + \nu_t \tag{4.5}$$

where the first equation represents the factor model (with zero factors aside from the interest rate), the second equation is the factor representation of the price level with a factor loading λ on the interest rate component as the only available factor. Usually, u_t would be interpreted as the monetary policy shock. Given the assumptions above, the coefficients can easily be calculated.

PROPOSITION 1 *Assume* $0 \leq \gamma - \alpha\phi < 1$*.*

1. *The variance of the price component is given by*

$$E[pp] = \frac{\sigma^2_{\epsilon,p} + \alpha^2 \sigma^2_{\epsilon,m}}{1 - (\gamma - \alpha\phi)^2}$$

The variance of the interest rate component is given by

$$E[rr] = \phi^2 E[pp] + \sigma^2_m$$

2. *For large samples, the estimates for* β *and* λ *converge to*

$$\beta \rightarrow \gamma - \alpha\phi - \gamma \frac{\sigma^2_m}{E[rr]} \tag{4.6}$$

$$\lambda \rightarrow \frac{1}{\phi}\left(1 - \frac{\sigma^2_m}{E[rr]}\right) \tag{4.7}$$

3. *If* $\phi > 0$*, then* $\lambda > 0$*. If* $\phi = 0$*, then* $\beta = 0$ *and* $\lambda = 0$*.*

PROOF: These results follow from straightforward calculations:

1. The equation for $E[pp]$ follows directly from (4.3). For $E[rr]$, use (4.2) to write $E[rr] = E[(\phi p + \epsilon_m)(\phi p + \epsilon_m)]$ and exploit $E[p\epsilon_m] = 0$.
2. For the covariance $E[r_t p_t]$ of r_t with p_t or the autocovariance $E[r_t r_{t-1}]$, note $E[r_t p_t] = E[(\phi p_t + \epsilon_{m,t})p_t] = \phi E[pp]$ as well as

$$E[r_t r_{t-1}] = E[\phi(-\alpha\phi r_{t-1} + \gamma p_{t-1} + \epsilon_{p,t}) r_{t-1}] = -\alpha\phi E[rr] + \gamma\phi E[r_t p_t]$$

Now, calculate $\beta \rightarrow E[r_t r_{t-1}]/E[rr]$ and $\lambda \rightarrow E[r_t p_t]/E[rr]$.
3. Direct.

Q.E.D.

Note now, that the impulse response of the price component to u_t, the "identified" monetary policy shock, is simply λ times the impulse response of r_t to a monetary policy shock. Since $\lambda > 0$ for $\phi > 0$, there will be a price puzzle on impact. If furthermore $\beta > 0$, a condition which is easily met, the price component appears to react positively to a monetary policy shock throughout. Put differently, if the "noise" price component matters to monetary policy in setting interest rates, and if the researcher fails to include prices in the "core" VAR, one easily obtains a price puzzle.

On the other hand, note the shocks could be correctly identified from a VAR in (r_t, p_t) with one lag and a Cholesky-decomposition. Thus adding both, the interest rate as well as the CPI index, to the factor specification, would resolve this problem and result in correct identification of the monetary policy shock.

5. EMPIRICAL RESULTS

In this section we lay down the results of the paper. First we describe the data and the respective model specifications chosen. Next we present the models fit followed by the results for the convergence diagnostics. Finally we present our results for our Baseline model and 6 further Versions with the specification summarized in Table (I).

5.1. *Data and Model Specification*

We follow the empirical approach of Bernanke, Boivin and Eliasz [2005] who have used the data set of Stock and Watson [1998,1999] which consists of a panel of 120 macroeconomic variables in monthly frequency transformed to induce stationarity. For comparison purposes we use a the same data span from January 1959 through August 2001. For our analysis we use an updated version as employed in Stock and Watson [2005][5]. The data set is listed in Appendix A. The "core VAR" consists in our preferred baseline model of the policy instrument[6] and the CPI index, which is necessary to avoid misspecification as explained in the previous section. We add 4 factors, which turns out to be sufficient to capture the crucial dynamics driving the economy. We experimented with changing the number of factors, and found little information was added by increasing the dimension of the system by analyzing the marginal contribution of further factor for the explanation of the variation in the data based on R^2. For our Baseline model we report results based on a lag length of 12 where the federal funds rate and CPI are included as factors in Y_t. We tried several versions with different lag length, but the results compared to the ones reported here do not differ much.

[5] We thank Mark Watson for making his data and computer codes available on his webpage http://www.princeton.edu/ mwatson/wp.html.

[6] In our case the Federal Funds rate

TABLE I

Different model specifications analyzed.

The table gives an overview of the different model versions analyzed in the paper with respect to the specification of the core VAR and the sign restrictions imposed.

Model Version	Core VAR	Identification Scheme
Model A *[Baseline Model]*	[F_4,CPI, FFR]	Sign Restriction
Model B *[Pure FAVAR]*	[F_4,FFR]	Sign Restriction
Model C *[Max. Restriction]*	[F_4,CPI, FFR]	Sign Restriction
Model D *[Output Restr.]*	[F_4,CPI, FFR]	Sign Restriction
Model E *[Recursive Model]*	[F_4,CPI, FFR]	Recursive Cholesky Identification FFR ordered last
Model F *[Minimal Baseline]*	[F_4,CPI, FFR]	Sign Restriction
Model G *[Recursive Pure FAVAR]*	[F_4,FFR]	Recursive Cholesky Identification FFR ordered last

The horizon for the sign restriction to hold is set to $H = 6$ which is based on Uhlig (2005). Not reported results confirm the tendency that the impact and intensity of the responses of restricted variables increases with the horizon H. Our different model specifications are:

1. **Model A** is the *Baseline Model* which includes as the Core VAR FFR and CPI[7].
2. **Model B** is the *Pure FAVAR*. In this model version the specification is the same as in **Model A** except that we exclude the CPI variable in the core VAR. We will see that the potential misspecification by excluding CPI is less severe when employing the sign restriction identification than in the competing recursive scheme.
3. **Model C** is the *Maximal Price Restrictions* version. It turns out that restricting prices are key to identifying monetary policy shocks. They deliver the crucial "*byte*" in the space of possible reaction of the variables and

[7]For the analysis of the identification of monetary policy shocks the ordering of the FAVAR equation is irrelevant in the case of sign restriction. When we identify the shocks in the recursive case the identifying assumption is that the factors are ordered first before the CPI and the policy instrument last.

TABLE II

Sign restrictions imposed for identification

This table summarizes the sign restriction to hold for the contemporaneous period and the specified period following the shock for the different models analyzed.

	Model A	Model B	Model C	Model D	Model F
Industrial Production			≤ 0	≤ 0	
Federal Funds Rate	$0 \geq$	$0 \geq$	$0 \geq$	$0 \geq$	$0 \geq$
3-m Treasury Bills			$0 \geq$		
6-m Treasury Bills			$0 \geq$		
1-yr Treasury Bills			$0 \geq$		
5yr-FF spread			≤ 0		
10yr-FF spread			≤ 0		
M1	≤ 0	≤ 0	≤ 0	≤ 0	
Monetary Base	≤ 0	≤ 0	≤ 0	≤ 0	
Total Reserves	≤ 0	≤ 0	≤ 0	≤ 0	
Nonborrowed Reserves	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0
Commodity Price Index	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0
CPI Index	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0
ALL PRICE INDECES			≤ 0		

hence narrow down the space of the potential impact of monetary policy shock approximating an exact identification. In this model we restrict the maximal number of prices that serve correct responses.

4. <u>**Model D**</u> is the *Output* version. Some might argue that the restrictions are not sufficient because output is left unrestricted. Therefore we added here to the **Baseline Model** the restriction output to react negative for some periods following the shock. Apart from that the model is identical to the first case.
5. <u>**Model E**</u> is the *Baseline Model with Recursive Identification*. The Specification is identical to **Baseline Model**. The identification scheme is the recursive Cholesky identification restricting the channels through which the policy instrument affects the variables in the shock period via the factors. The factors are ordered first before CPI and the policy instrument is ordered last.
6. <u>**Model F**</u> is the *Baseline Model with Minimal Restriction*. The Specification is identical to **Baseline Model**. The main distinction is for the restrictions imposed on the shape of the impulse response functions are according to the ones set in Uhlig [2005] for the VAR framework.
7. <u>**Model G**</u> is the *Baseline Model without CPI and Recursive Identification*. This version is the same as **Model E** except that we exclude CPI from the core VAR in order to analyze the impact of the misspecification discussed in section 5.

5.2. *Model Fit*

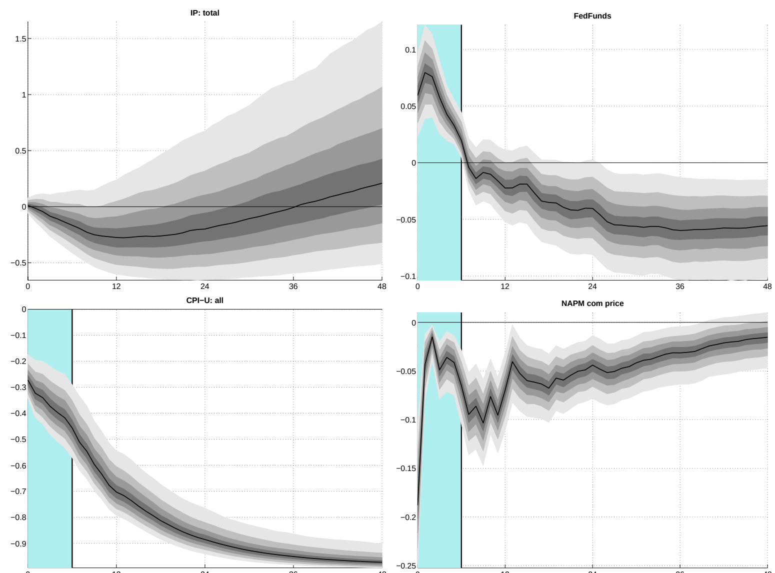
We want to assure that our methodology can represent the data in an adequate manner. In this section we want to report results that pursue to assess the models fit to the data. The first obvious check is to see how well the factors represent our panel of data series. To this end, we estimated the R^2s from regressing the respective series onto the five factors. Results are listed in the Table (IV) below. These results give an report to what extend the individual series are driven by common components. Those with a low R^2 are more driven by idiosyncratic forces. Hence the factor model is informative in the sense that it describes the aggregate common components that we are interested in. Thus a VAR in these factors or common components should not suffer from omitted variable bias, which implies that adding individual series to the VAR in eq. (3) above does not alter the shape of any impulse response functions substantially. As already described above we also estimated the respective marginal contribution of each factor, not reported here, for the explanatory part in the variation of the data. The extracted factors are plotted in Figure (9) with the a probability band covering 95% highest posterior density. Furthermore we provide plots that show the extracted factors with their respective error bands.

5.3. *Convergence Diagnostics*

In order to assure that the results are based on converged simulations, that represent the respective target distribution completely and e.g. not only some local mode we have to apply further convergence diagnostics for the simulated parameters based on the Gibbs sampler. The respective diagnostics are not a guarantee for convergence but a battery of different diagnostics that can reduce the uncertainty researchers have regarding sampled parameters. Therefore we employed some diagnostics that are widespread in the MCMC literature which are only briefly explained here and reported in Appendix C.

All our results are based on 25000 simulation draws of the Gibbs sampler of which the first 5000 were discarded as a burn-in to avoid an initial transient of the starting values that initiated the simulation. Furthermore we chose a thinning parameter of 2 so that only every second draw was kept after the burn-in in order to reduce potential autocorrelation of the sequence. Furthermore we checked the precision of the sampler by plotting the associated error bands. An example for the extracted factors covering the 95% probability band is given in Figure (9). We furthermore checked the convergence by monitoring the sampler visually through trace plots which shows the evolvement of the sampler and helps to check whether there are jumps in the level of the respective parameter. Furthermore we plotted the first half of the kept draws against the second to check whether the sampler is still *"moving"* towards the target distribution or whether the whole density of the distribution is represented. Few deviation

FIGURE 1.— **Selected Impulse Responses for the Baseline Model.**



Results identified with sign restriction with the grey shaded area covering the 8 deciles (80% probability bands).

indicates convergence and vice versa (see Figure (10))[8].

5.4. *Impulse Response Analysis*

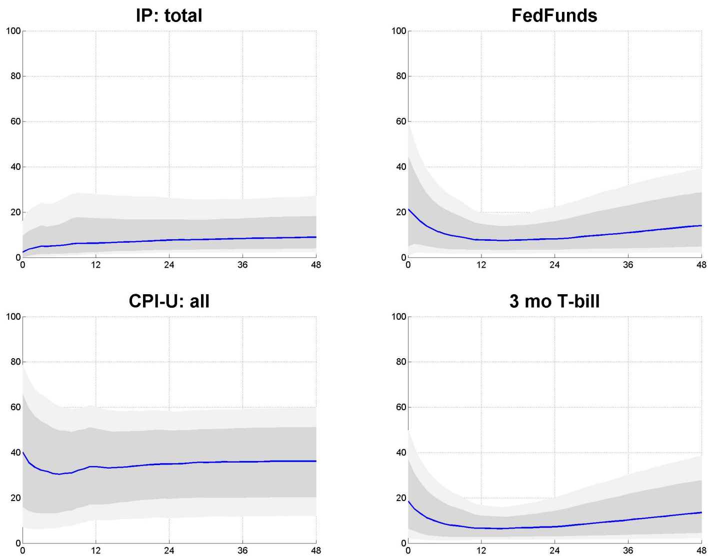
The key statistics suited to answer the question at hand are impulse response functions. Here we report the results for the different model versions as previously described and summarized in Table 1 in Appendix A. Starting with our benchmark specification **Model A** in Figure (1) we find a negative reaction for industrial production with a maximal impact after about a year before reverting back to pre shock levels. This is in contrast to BVAR results reporting an ambiguous effect[9].

For CPI we find a negative impact further decreasing for the entire horizon of 48 month considered. In contrast the commodity price index decreases less strong with a maximum impact after about a year before reverting back to the pre shock level a year later. Compared to CPI this is not surprising as the commodity price index reflects measures traded on the market with rather flexible prices. The federal funds rate and other short term interest rates increase

[8]Further statistics for the convergence such as the Rafetery-Lewis test, the I-statistic and more have been calculated to ensure convergence. Due to space limitations these results are available upon request.

[9]See Uhlig (2005).

FIGURE 2.— **Selected FEVD for the Baseline Model.**

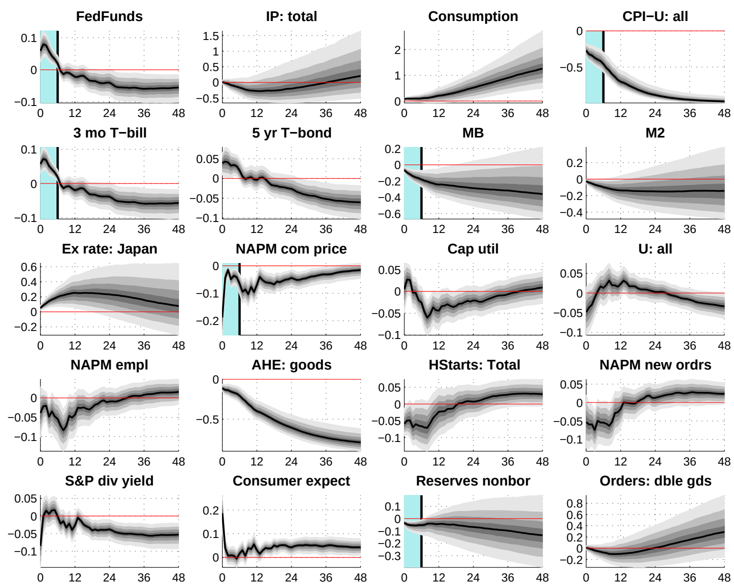


Forecast error variance decomposition for the baseline model identified with sign restriction. The grey shaded area cover the 84th and 68th percentile of the posterior distribution.

on impact but revert quite soon afterwards. After about a year it swings back to a slightly lower. Other interesting variables, such as capacity utilization rate, housing starts and new orders react rather similar by decreasing for about a year before reverting. In Figure (14) we report results for the same specification as the benchmark case except that we also restrict industrial production to react non positively. We find the results to be quite similar. This restriction doesn't seem to change much.

For comparing the differences in the results produced by the different identification schemes Figure (15) shows the impulse response functions of our baseline specification identified via a recursive Cholesky identification where the federal funds rate is ordered last. The most striking difference is the unreasonable high and long lasting "price puzzle". The CPI measure remains positive for about 18 month which is rather long. Even commodity prices increase on impact. Another anomaly is the increasing reaction of nonborrowed reserves which lasts for the whole horizon considered. A feature that has been analyzed in several studies because of its importance is changes in the monetary policy regime. We don't analyze that feature in greater detail as it is beyond the scope of the paper. But we provide results for the post Volcker disinflation period in order to check the fragility of the results. Figures (4) and (8) provide results of both identification schemes. The crucial difference regarding the full sample results and the post

FIGURE 3.— **Baseline Model: IRF**

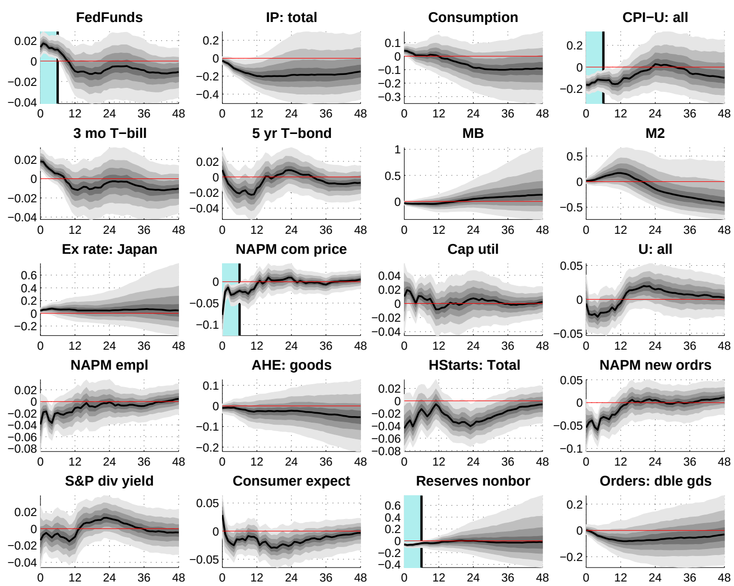


Volcker disinflation period for the sign restriction approach is that the response of industrial production is less ambiguous. The federal funds rate and other short term interest rates tend to have a positive reaction for a much longer period than in the full sample case. CPI reacts less stronger and reverts back after about 14 month opposed to the full sample results where CPI decrease for the whole horizon considered. The picture on the labor market is also somewhat different. Unemployment is affected stronger and for a longer period by a monetary policy shock. However employment reacts slightly after about 2 years. In comparison with results based on Cholesky identification we still find the "price puzzle". Nominal variables tend to have a stronger reaction comparing with results based on sign restriction.

5.5. *Forecast Error Variance Decomposition*

An important question after having identified monetary policy shocks is how much of the variation it explains. Results are provided by Figure (18) for the sign restriction case. For comparison purposes we also provide results for the case of Cholesky identification in Figure (19). According to the median estimates we find that a random shift in monetary policy explains about 12%-15% in the variation of industrial production and less than about 20% for most variables except for prices and short term interest rates. These results are amongst others

FIGURE 4.— **Post-Volcker: Sign Restriction Approach**



consistent with Sims [1999], Sims and Zha [2006], Uhlig [2005] and Amir-Ahmadi and Ritschl [2008].

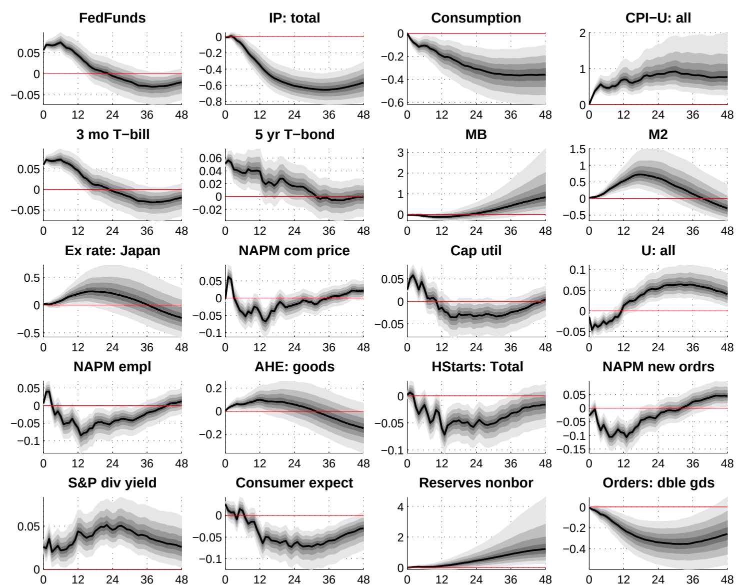
6. DISCUSSION

The impulse responses presented serve the indication that, for the researcher interested in measuring the effects of a shock to monetary policy, identifying restrictions that are consistent with the conventional wisdom, such as the proposed sign restrictions are decisive for the results to look reasonable. We have shown that our identification strategy is particularly well suited in a factor based model such as the employed BFAVAR. When comparing the results from the Gibbs sampling approach identified with the recursive identification scheme it is quite evident that the results seem more reasonable w.r.t. to unreasonable anomalies present such as the "price puzzle".

Advantage Towards BVAR.

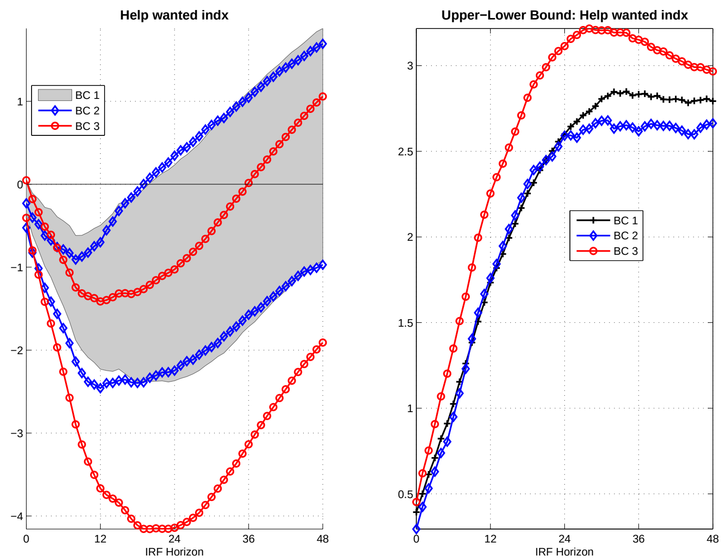
For Compactness we will briefly summarize the key arguments why factor based models are favorable compared to (B)VARs. The parsimoniously exploitation of a large set of disaggregated macroeconomic data through the dimension reduction methodology of BFAVARs avoids the omitted variable bias and

FIGURE 5.— **Recursive Cholesky identification for post-Volcker disinflation period**



overcomes the degree-of-freedom problem which are inherent drawbacks of the (B)VAR approach. Hence we do not have to restrict ourselves to a small scale VAR that apparently serves an unrealistic and fragmentary picture of the economy and the central bankers information set prior to taking decision. Hence small scale VARs might be a pitfall of biased inference. Furthermore the luxury position of large panels of macroeconomic data allows to check model validation and the robustness of the respective identification approaches applied due to the fact that a crucial amount of important data is considered. Hence we can have a much broader picture of the economy's reaction after structural shocks. On account of this we can have a thorough picture of the impact of shocks by considering and analyzing more variables of interest, other than the standard VAR indicators. We furthermore do not restrict ourselves to rely on single indicators that can impossibly reflect a whole economic concept. In such cases, inference might strongly depend the choice of such indicators. Through the methodology we let the data decide which weight to give to the single disaggregated series without excluding them a priori. Another important advantage is that the crucial dynamics of the data rich economy are captured more appropriate. Furthermore we find as an empirical result that the BFAVAR approach provides significant real effects whereas in the (B)VAR approach one comes up with insignificant

FIGURE 6.— **Comparison of different sets of sign restrictions: Help wanted Index**

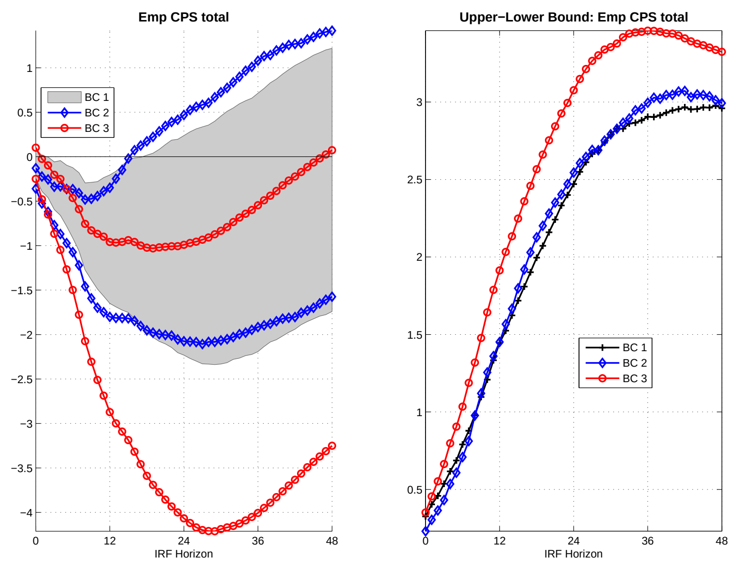


results where no statement is possible. The apparent reason might be the lack of information included in the (B)VAR framework.

Advantage towards Nonparametric FAVAR.

Admittedly one must say that the Bayesian approach by applying the Gibbs sampler in combination with the sign restriction is a cumbersome and time consuming procedure which is a disadvantage towards the nonparametric two step estimation procedure applied by BBE. The Two-Step estimation procedure delivers good results and is efficient regarding the computation procedure but cannot exploit the fully parametric factor structure with a lag polynomial which we evaluate of great relevance. Dynamic Principal component analysis and other estimation procedures for classical DFM's are convenient ways to reduce dimensions due to their computational efficiency and easy implementation. Nevertheless they lack on the possibility to exploit the factor structure. For that reason we prefer to take the Bayesian approach by estimating the model in the state space representation in order to impose an explicit parametrical model. The State space representation moreover serves the possibility to let the dynamics of

FIGURE 7.— **Comparison of different sets of Sign restrictions: Employment**



the state equation as unrestricted as possible and furthermore introduce sophisticated priors that can account for crucial interaction between the driving factors and forces of the economy. Hence we can be even more precise by capturing more realistically important comovements.

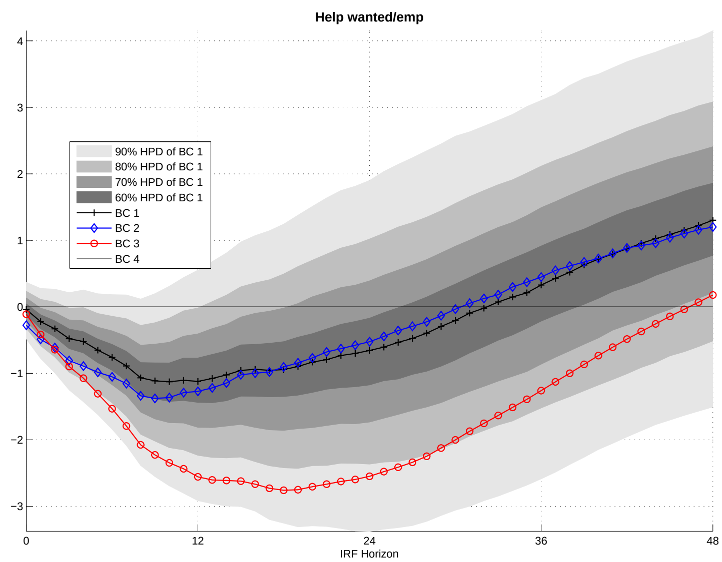
Sign Restriction vs. Recursive Identification in a BFAVAR approach.

By comparing the results of the recursive identification scheme and the sign restriction approach it turns out that the results of the latter are associated with less uncertainty as can be seen from the error bands. This is an enticing promise insofar that an economically desirable identification scheme that is "weak" with respect to the restrictions and structure imposed delivers reasonable results.

7. CONCLUSION

In this paper we propose to estimate the effects of monetary policy shocks in a BFAVAR framework identified via sign restriction. We estimate the model from a Bayesian perspective relying on MCMC methods. To infer the effects of a contractionary shock to monetary policy we impose sign restriction on the impulses

FIGURE 8.— **Comparison of posterior IRF uncertainty of different sets of Sign restrictions**



responses on many prices, monetary aggregates, spreads and short term interest rates. More restrictions can be set in factor based models which is promising as the robustness of identification can be monitored and identifying assumption, tight to economic theory can be imposed broadly. Our identification strategy outperforms the recursive identification in the BFAVAR framework hence we conclude that the combination of the BFAVAR model and a reasonable identification scheme are crucial for a successful identification. Furthermore, sign restrictions hold independent from subsamples considered unlike the Cholesky case where results and inference are sensitive sometimes contradicting economic theory. We find that after a contractionary monetary policy shock a negative response of output but modest in size. This effect vanishes when we consider the post Volcker disinflation period. Monetary policy shocks account for a small fraction of the variance for the forecast revision of output.

APPENDIX A: ESTIMATION AND INFERENCE

For the estimation of FAVAR models Bernanke, Boivin and Eliasz [2005] present two competing approaches. The first one, which they prefer due to their results and the computational simplicity is the two-step estimation based on a dynamic principal component approach. This classical approach goes back to Stock and Watson [2003]. A detailed survey with several identification schemes in the classical estimation approach can be found in Stock and Watson [2005]. The second estimation approach described in their paper is the one that we employ in this paper because the likelihood based one-step estimation approach employing MCMC methods explicitly exploits the factor structure.

We pursue the multi-move Gibbs sampler for which we have to cast the model into the state space representation. Let (2.3) be extended by f_t^y which results in

$$\begin{bmatrix} X_t^c \\ f_t^y \end{bmatrix} = \begin{bmatrix} \lambda^c & \lambda^y \\ 0 & I_{N_y} \end{bmatrix} \begin{bmatrix} f_t^c \\ f_t^y \end{bmatrix} + \begin{bmatrix} e_t^c \\ 0 \end{bmatrix} \tag{A.1}$$

Let $X_t \equiv (X_t^{c\prime}, f_t^{y\prime})'$, $e_t \equiv (e_t^{c\prime}, 0_{[N_y \times 1]}{}')'$ and $f_t \equiv (f_t^{c\prime}, f_t^{y\prime})'$, then the model can be rewritten as

$$X_t = \lambda f_t + e_t \tag{A.2}$$

$$f_t = \sum_{p=1}^{P} \phi_p f_{t-p} + u_t^f \tag{A.3}$$

where X_t has dimension $[N \times 1]$ with $N = N_c + K_y$. In most empirical applications and also in our specification the lag order P exceeds one hence we have to rewrite the state space in a stacked first order Markov process. This requires the following straight forward definitions for the companion form of the model:

$$\begin{aligned} \lambda &\equiv \begin{bmatrix} \lambda^c & \lambda^y \\ 0_{N_y \times K_c} & I_{N_y} \end{bmatrix} \\ \phi &\equiv [\phi_1, \phi_2, \ldots, \phi_P]' \\ F_t &\equiv (f_t, f_{t-1}, ..., f_{t-p+1}) \\ u_t &\equiv (u_t^f, 0, ..., 0)' \end{aligned}$$

The lag polynomial of the FAVAR equation in the first-order representation changes to:

$$\Phi = \begin{bmatrix} \phi_1 & \cdots & \phi_P \\ I_{K(P-1)} & & 0_{K(P-1) \times K} \end{bmatrix}.$$

Now we have to transform the VCV of the FAVAR disturbances with 0's in a straightforward way to adjust the dimensions of the state equation which results in the following matrix:

$$Q = \begin{bmatrix} Q_u & 0 \\ 0 & 0 \end{bmatrix}$$

where Q is of dimension $[PK \times PK]$ extended by zero matrices to match the companion form. We define $\Lambda \equiv [\lambda \quad 0 \quad \cdots \quad 0]$. Then

$$F_t = \Phi F_{t-p} + u_t \tag{A.4}$$

$$X_t = \Lambda F_t + e_t \tag{A.5}$$

$$u_t \sim \mathcal{N}(0, Q) \tag{A.6}$$

$$e_t \sim \mathcal{N}(0, R) \tag{A.7}$$

is the final state-space representation prepared to fit the estimation procedure. Note again that R is diagonal and that e_t and u_t are mutually independent.

A.1. *Factor Identification*

The factors are only identified up to an invertible rotation. Any rotation of the factors results in the same likelihood for the factors though the models are different. Identifying restrictions have to be set, in order to distinguish the idiosyncratic from the common component. Additionally one can set further identifying assumptions in order to identify the factors and the loadings, separately. We follow the standard identification restrictions either on the factor loading matrix employed by Bernanke, Boivin and Eliasz [2005] for unique identification against rotational indeterminacy. Since factors are estimated up to a rotation, the normalization should not affect the space spanned by the estimated factors. In the joint estimation case the specified identification against rotation requires that the factors are uniquely identified in the following form

(A.8) $$f_t^* = Af_t^c - Bf_t^y$$

where A and B are nonsingular. Restrictions are only imposed on the observation equation. Here we substitute F_t^* into (2.1) due to the fact that restrictions should not be imposed on the VAR dynamics we obtain

(A.9) $$X_t^c = \lambda^c A^{-1} f_t^* + (\lambda^y + \lambda^c A^{-1} B) f_t^y + e_t^c.$$

For unique identification of the factors and the loadings it is required that $\lambda^c A^{-1} = \lambda^c$ and $\lambda^y + \lambda^c A^{-1} B = \lambda^y$. As discussed in Bernanke, Boivin and Eliasz [2005] sufficient conditions are to set the upper $K_c \times K_c$ block of λ^c to identity and the upper $K_c \times N_y$ block of λ^y to a zero matrix[10].

A.2. *Inference*

Bayesian analysis treats the parameters of the model as random variables. We are interested in inference on the parameter space $\theta = \left(\lambda^f, \lambda^y, R_e, \phi, Q_u\right)$ and the factors $\{f_t\}_{t=1}^T$. Multi move Gibbs Sampling alternately samples the parameters θ and the factors f_t, given the data. We use the multi move version of the Gibbs sampler because this approach allows us, as a first step, to estimate the unobserved common components, namely the factors via the Kalman filtering technique conditional on the given hyperparameters and data, and as a second step calculate the hyperparameters of the model given the factors and data via the Gibbs sampler in the respective blocking. Let $X^T = (X_t, \ldots, X_T)$ and $F^T = (F_1, \ldots, F_T)$ define the respective histories. For the estimation of the model we want to derive the posterior densities which requires to empirically approximating the marginal posterior densities of F^T and θ:

$$\begin{aligned} p(F^T) &= \int p(F^T, \theta) d\theta \\ p(\theta) &= \int p(F^T, \theta) dF^T \end{aligned}$$

where

$$p(F^T, \theta)$$

is the joint posterior density and the integrals are taken with respect to the supports of θ and F^T respectively. The procedure applied to obtain the empirical approximation of the posterior distribution is the previously mentioned multi move version of the Gibbs sampling technique by Carter and Kohn [1994] and Frühwirth-Schnatter [1994][11].

A.3. *Choosing the Starting Values*

In general one can start the iteration cycle with any arbitrary randomly drawn set of parameters, as the joint and marginal empirical distributions of the generated parameters will converge at an exponential rate to its joint and marginal target distributions as $S \to \infty$. This has been shown by Geman and Geman [1984]. Since Gelman and Rubin [1992] have shown that a single chain of the Gibbs sampler might give a "false sense of security ", it has become common practice to try out different starting values. We check our results based on four different strategies regarding the set of starting values. One out of many convergence diagnostics involves testing the fragility of the results with respect to the starting values. For the results to be reliable, estimates based on different stating values should not differ. Strictly speaking,

[10] Note that this identification strategy is over-identified. However, for comparison purposes we follow closely the approach of Bernanke, Boivin and Eliasz [2005].

[11] For a survey and more details see Kim and Nelson [1999], Eliasz [2005] and Bernanke, Boivin and Eliasz [2005]

the different chains should represent the same target distribution. In order to verify we start our Gibbs sampler with the following summarized starting values respectively.

(i) Randomly draw θ_0 from (over)dispersed distribution

(ii) Set θ_0 to rather "agnostic values" which involves setting 0's for coefficients and 1's for variances[12]

(iii) Set θ_0 to results from principal component analysis.[13] In such a way the number of draws required for convergence can be reduced considerably.

(iv) Set θ_0 to parameters of the last iteration of the previous run.

Despite the strategies above convergence is never guaranteed, particularly in large models. Hence it is recommended to restart a chain many times applying the strategy 4.

A.4. *Conditional density of the factors F^T given X^T and θ*

In this subsection we want to sample from $p(F^T \mid X^T, \theta)$ assuming that the data and the hyperparameters of the parameter space θ are given, hence we describe Bayesian inference on the dynamic evolution of the factors f_t conditional on X_t^c for $t = 1, \ldots, T$ and conditional on θ. The transformations that are required to draw the factors have been done in the previous section. The conditional distribution, from which the state vector is generated, can be expressed as the product of conditional distributions by exploiting the Markov property of state space models in the following way

$$p(F^T \mid X^T, \theta) = p(F_T \mid X^T, \theta) \prod_{t=1}^{T-1} p(F_t \mid F_{t+1}, X^T, \theta)$$

The state space model is linear and Gaussian, hence we have:

$$F_T \mid X^T, \theta \quad \sim \quad N(F_{T|T}, P_{T|T}) \tag{A.10}$$

$$F_{t|T} \mid F_{t+1|T}, X^T, \theta \quad \sim \quad N(F_{t|t,F_{t+1|T}}, P_{t|t,F_{t+1|T}}) \tag{A.11}$$

with

$$F_{T|T} \quad = \quad E(F_T \mid X^T, \theta) \tag{A.12}$$

$$P_{T|T} \quad = \quad Cov(f_T \mid X^T, \theta) \tag{A.13}$$

$$F_{t|t,F_{t+1|T}} \quad = \quad E(F_t \mid F_{t|t}, F_{t+1}, \theta) \tag{A.14}$$

$$P_{t|t,F_{t+1|T}} \quad = \quad Cov(F_t \mid F_{t|t}, F_{t+1}, \theta). \tag{A.15}$$

We first run the Kalman filter generating $F_{t|t}$ and $P_{t|}$ for $t = 1, \ldots, T$. For the initialization we set $F_{1|0} = 0_{KP\times 1}$ and $P_{1|0} = I_{KP}$ and iterate through the Kalman filter according to

$$F_{t|t} \quad = \quad F_{t|t-1} + P_{t|t-1}\Lambda' H^{-1}\eta_{t|t-1} \tag{A.16}$$

$$P_{t|t} \quad = \quad P_{t|t-1} - P_{t|t-1}\Lambda' H^{-1}\Lambda P_{t|t-1} \tag{A.17}$$

where $\eta_{t|t-1} = (X_t - \Lambda F_{t|t-1})$ is the conditional forecast error and its covariance is denoted by $H_{t|t-1} = (\Lambda P_{t|t-1}\Lambda' + R_e)$. Furthermore let

$$F_{t|t-1} \quad = \quad \Phi F_{t-1|t-1} \tag{A.18}$$

$$P_{t|t-1} \quad = \quad \Phi P_{t-1|t-1}\Phi' + Q_u. \tag{A.19}$$

The last iteration of the Kalman filter yields $F_{T|T}$ and $P_{T|T}$ required for (A.10) to draw the last observation and start the Kalman smoother according to (A.11) going backwards through the sample for F_t, $t = T-2, T-3, \ldots, 1$ updating the filtered estimates with the sampled factors one period up subject to

$$F^*_{t|t,F_{t+1|T}} \quad = \quad F_{t|t} + P_{t|t}\Phi^{*\prime} J^{-1}_{t+1|t}\xi_{t+1|t} \tag{A.20}$$

$$P^*_{t|t,F_{t+1|T}} \quad = \quad P_{t|t} - P_{t|t}\Phi^{*\prime} J^{-1}_{t+1|t}\Phi^* P_{t|t}. \tag{A.21}$$

[12] This strategy has been applied by Belviso and Milani [2007].

[13] This strategy is particularly suited for large models as the ones studied here and has been proposed by Eliasz [2005].

where $\xi_{t+1|t} = F^*_{t+1} - \Phi^* F_{t|t}$ and $J_{t+1|t} = \Phi^* P_{t|t} \Phi^* + Q^*$. Note that Q^* refers to the upper $K \times K$ block of Q and Φ^* and F_t^* denote the first K rows of Φ and F_t respectively. This is required when Q is singular which is the case for the companion form when there is more than one lag in (A.3). Here we closely follow Kim and Nelson [1999] where a detailed explanation and derivation can be found.

A.5. *Conditional density of the parameters θ given X^T and F^T*

Sampling from the conditional distribution of the parameters $p(\theta \mid X^T, F^T)$ requires the blocking of the parameters into the two parts that refer to the observation equation and to the state equation respectively. The blocks can be sampled independently from each other conditional on the extracted factors and the data.

A.5.1. *Conditional density of Λ and R_e*

This part refers to observation equation of the state space model which, conditional on the estimated factors and the data, specifies the distribution of Λ and R_e. The errors of the observation equation are mutually orthogonal with diagonal R_e. Hence we can apply equation by equation OLS in order to obtain the ols estimates $\hat{\Lambda}_n$ and $\hat{e}^c$ as the observation equation amounts to a set of independent regressions. Note that the subscript n refers to the n-th equation and all hat variables refer to the respective ols estimates. We assume conjugate priors

$$\begin{aligned} p(R_{nn}) &= \mathcal{IG}(\delta_0/2, \eta_0/2) \\ p(\Lambda_n \mid R_{nn}) &= \mathcal{N}(\Lambda_{n0}, R_{nn} M_{n0}^{-1}) \end{aligned}$$

which according to Bayesian results[14] conform to the following conditional posterior distribution

$$\begin{aligned} p(R_{nn} \mid \tilde{X}_T, \tilde{F}_T) &= \mathcal{IG}(\delta_i/2, \eta_i/2) \\ p(\bar{\Lambda}_{nn} \mid \tilde{X}_T, \tilde{F}_T, R_{nn}) &= \mathcal{N}(\bar{\Lambda}_n, R_{nn} M_n^{-1}). \end{aligned}$$

with

$$\begin{aligned} \eta_n &= \eta_0 + T \\ \delta_n &= \delta_0 + (\hat{e}_n^c)'(\hat{e}_n^c) + (\hat{\Lambda}_n - \Lambda_{n0})' \left[M_{n0}^{-1} + (F_T^{n\prime} F_T^n)^{-1} \right]^{-1} (\hat{\Lambda}_n - \Lambda_{n0}) \\ \bar{M}_n &= M_{n0} + \left(F_T^{n\prime} F_T^n \right) \\ \Lambda_n &= \bar{M}_n (M_{n0}^{-1} \Lambda_{n0} + \left(F_T^{n\prime} F_T^n \right) \hat{\Lambda}_n) \end{aligned}$$

where we set the same prior specification ($\delta_0 = 6, \eta_0 = 10^{-3}, M_{n0} = I_{K_c}, \Lambda_{n0} = 0_{K_c \times 1}$) as in Bernanke, Boivin and Eliasz [2005] in order to allow an adequate comparison. M_0 denotes the matrix in the prior on the coefficients of the n-th equation of Λ_n. The factor normalization discussed earlier requires to set $M_0 = I$. The regressors of the n-th equation are represented by F_T^n and the fitted errors of the n-th equation are represented by $\hat{e^c_{tn}}$.

A.5.2. *Conditional density of $vec(\phi)$ and Q_u*

The next Gibbs block requires to draw $vec(\phi)$ and Q_u conditional on the most current draws of the factors and the data. We employ the Normal-Inverse Wishart prior according to Uhlig [1994]

$$\begin{aligned} p(Q_u) &= \mathcal{IW}(S_0, \nu_0) \\ p(vec(\phi) \mid Q_u) &= \mathcal{N}(\bar{\phi}_0, Q_u \otimes N_0^{-1}) \end{aligned}$$

which results in the following posterior:

$$\begin{aligned} p(Q_u \mid X^T, F^T) &= \mathcal{IW}(S_T, \nu_T) \\ P(vec(\phi) \mid X^T, F^T, Q_u) &= \mathcal{N}(vec(\bar{\phi}_T), Q_u \otimes N_T^{-1}) \end{aligned}$$

[14] For a derivation see Gelman, Carlin, Stern and Rubin [1995],

with

$$\begin{aligned}
\nu_T &= T + \nu_0 \\
N_T &= N_0 + (F'_{T-1}F_{T-1}) \\
\bar{\phi}_T &= N_T^{-1}(N_0\bar{\phi}_0 + F'_{T-1}F_{T-1}\hat{\phi}) \\
S_T &= \frac{\nu_0}{\nu_T}S_0 + \frac{T}{\nu_T}\hat{Q}_u + \frac{1}{\nu_T}(\hat{\phi} - \bar{\phi}_0)' N_0 (N_T)^{-1}(F'_{T-1}F_{T-1})(\hat{\phi} - \bar{\phi}_0)
\end{aligned}$$

This prior and has the following specification

$$\begin{aligned}
\nu_0 &= 0 \\
N_0 &= 0_{K \times K}
\end{aligned}$$

where the choice of S_0 and $\bar{\phi}_0$ are arbitrary as they cancel out in the posterior. We alternatively also implemented the Normal-Wishart prior for according to Kadiyala and Karlsson [1997] where the diagonal elements of Q_0 are set to the corresponding p-lag univariate autoregressions, σ_i^2. The diagonal elements of Ω_0 are constructed such that the prior variances of the parameter of the k lagged j'th variable in the i'th equation equals $\sigma_i^2/k\sigma_j^2$. Hence $S_0 = Q_0$ and $\bar{\phi}_0 = \mathbf{0}$. Results were virtually the same. To ensure stationarity, we truncate the draws by discarding the draws of ϕ with the larges eigenvalue greater than 1 in absolute value.

APPENDIX B: TABLES AND FIGURES

B.1. *Data*

Table (III) is taken from Stock and Watson [2005] and lists the short name of each series, its mnemonic (the series label used in the source database), the transformation applied to the series, and a brief data description. All series are from the Global Insights Basic Economics Database, unless the source is listed (in parentheses) as TCB (The Conference Boards Indicators Database) or AC (authors calculation based on Global Insights or TCB data). In the transformation column, 1 denotes the level of the series, 2 denotes the first difference of the series, 4 denotes logarithm, 5 and 6 denote the first and second difference of the logarithm respectively.

Pos.	Shortname	Mnemonics	TC	Description
1	PI	a0m052	5	Personal income (AR, bil. chain 2000 $)
2	PI less transfers	A0M051	5	Personal income less transfer payments (AR, bil. chain 2000 $)
3	Consumption	A0M224R	5	Real Consumption (AC) A0m224/gmdc
4	M&T sales	A0M057	5	Manufacturing and trade sales (mil. Chain 1996 $)
5	Retail sales	A0M059	5	Sales of retail stores (mil. Chain 2000 $)
6	IP: total	IPS10	5	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
7	IP: products	IPS11	5	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
8	IP: final prod	IPS299	5	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
9	IP: cons gds	IPS12	5	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
10	IP: cons dble	IPS13	5	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
11	iIP:cons nondble	IPS18	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
12	IP:bus eqpt	IPS25	5	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
13	IP: matls	IPS32	5	INDUSTRIAL PRODUCTION INDEX - MATERIALS
14	IP: dble mats	IPS34	5	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
15	IP:nondble mats	IPS38	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
16	IP: mfg	IPS43	5	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
17	IP: res util	IPS307	5	INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES
18	IP: fuels	IPS306	5	INDUSTRIAL PRODUCTION INDEX - FUELS
19	NAPM prodn	PMP	1	NAPM PRODUCTION INDEX (PERCENT)
20	Cap util	A0m082	2	Capacity Utilization (Mfg)
21	Help wanted indx	LHEL	2	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
22	Help wanted/emp	LHELX	2	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
23	Emp CPS total	LHEM	5	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
24	Emp CPS nonag	LHNAG	5	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
25	U: all	LHUR	2	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)

Pos.	Shortname	Mnemonics	TC	Description
26	U: mean duration	LHU680	2	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
27	U < 5 wks	LHU5	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
28	U 5-14 wks	LHU14	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
29	U 15+ wks	LHU15	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
30	U 15-26 wks	LHU26	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
31	U 27+ wks	LHU27	5	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
32	UI claims	A0M005	5	Average weekly initial claims, unemploy. insurance (thous.)
33	Emp: total	CES002	5	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE
34	Emp: gds prod	CES003	5	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING
35	Emp: mining	CES006	5	EMPLOYEES ON NONFARM PAYROLLS - MINING
36	Emp: const	CES011	5	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION
37	Emp: mfg	CES015	5	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING
38	Emp: dble gds	CES017	5	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS
39	Emp: nondbles	CES033	5	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS
40	Emp: services	CES046	5	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING
41	Emp: TTU	CES048	5	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES
42	Emp: wholesale	CES049	5	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE
43	Emp: retail	CES053	5	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE
44	Emp: FIRE	CES088	5	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES
45	Emp: Govt	CES140	5	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT
46	Emp-hrs nonag	A0M048	5	Employee hours in nonag. establishments (AR, bil. hours)
47	Avg hrs	CES151	1	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR
48	Overtime: mfg	CES155	2	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR
49	Avg hrs: mfg	aom001	1	Average weekly hours, mfg. (hours)
50	NAPM empl	PMEMP	1	NAPM EMPLOYMENT INDEX (PERCENT)
51	HStarts: Total	HSFR	4	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA
52	HStarts: NE	HSNE	4	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
53	HStarts: MW	HSMW	4	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
54	HStarts: South	HSSOU	4	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
55	HStarts: West	HSWST	4	HOUSING STARTS:WEST (THOUS.U.)S.A.
56	BP: total	HSBR	4	HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
57	BP: NE	HSBNE	4	HOUSES AUTHORIZED BY BUILD. PERMITS:NORTHEAST(THOU.U.)S.A
58	BP: MW	HSBMW	4	HOUSES AUTHORIZED BY BUILD. PERMITS:MIDWEST(THOU.U.)S.A.
59	BP: South	HSBSOU	4	HOUSES AUTHORIZED BY BUILD. PERMITS:SOUTH(THOU.U.)S.A.
60	BP: West	HSBWST	4	HOUSES AUTHORIZED BY BUILD. PERMITS:WEST(THOU.U.)S.A.
61	PMI	PMI	1	PURCHASING MANAGERS' INDEX (SA)
62	NAPM new ordrs	PMNO	1	NAPM NEW ORDERS INDEX (PERCENT)
63	NAPM vendor del	PMDEL	1	NAPM VENDOR DELIVERIES INDEX (PERCENT)
64	NAPM Invent	PMNV	1	NAPM INVENTORIES INDEX (PERCENT)
65	Orders: cons gds	A0M008	5	Mfrs' new orders, consumer goods and materials (bil. chain 1982 $)
66	Orders: dble gds	A0M007	5	Mfrs' new orders, durable goods industries (bil. chain 2000 $)
67	Orders: cap gds	A0M027	5	Mfrs' new orders, nondefense capital goods (mil. chain 1982 $)
68	Unf orders: dble	A1M092	5	Mfrs' unfilled orders, durable goods indus. (bil. chain 2000 $)
69	M&T invent	A0M070	5	Manufacturing and trade inventories (bil. chain 2000 $)
70	M&T invent/sales	A0M077	2	Ratio, mfg. and trade inventories to sales (based on chain 2000 $)
71	M1	FM1	5	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
72	M2	FM2	5	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,
73	M3	FM3	5	MONEY STOCK: M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL$,SA)
74	M2 (real)	FM2DQ	5	MONEY SUPPLY - M2 IN 1996 DOLLARS (BCI)
75	MB	FMFBA	5	MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)
76	Reserves tot	FMRRA	5	DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)
77	Reserves nonbor	FMRNBA	5	DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)
78	C&I loans	FCLNQ	5	COMMERCIAL & INDUSTRIAL LOANS OUSTANDING IN 1996 DOLLARS (BCI)
79	C&I loans	FCLBMC	1	WKLY RP LG COM'L BANKS:NET CHANGE COM'L & INDUS LOANS(BIL$,SAAR)
80	Cons credit	CCINRV	5	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
81	Inst cred/PI	A0M095	2	Ratio, consumer installment credit to personal income (pct.)
82	S&P 500	FSPCOM	5	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)

Pos	Shortname	Mnemonics	TC	Description
83	S&P: indust	FSPIN	5	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
84	S&P div yield	FSDXP	2	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
85	S&P PE ratio	FSPXE	5	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
86	FedFunds	FYFF	2	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
87	Commpaper	CP90	2	Cmmercial Paper Rate (AC)
88	3 mo T-bill	FYGM3	2	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
89	6 mo T-bill	FYGM6	2	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
90	1 yr T-bond	FYGT1	2	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
91	5 yr T-bond	FYGT5	2	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
92	10 yr T-bond	FYGT10	2	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
93	Aaabond	FYAAAC	2	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
94	Baa bond	FYBAAC	2	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
95	CP-FF spread	scp90	1	cp90-fyff
96	3 mo-FF spread	sfygm3	1	fygm3-fyff
97	6 mo-FF spread	sFYGM6	1	fygm6-fyff
98	1 yr-FF spread	sFYGT1	1	fygt1-fyff
99	5 yr-FFspread	sFYGT5	1	fygt5-fyff
100	10yr-FF spread	sFYGT10	1	fygt10-fyff
101	Aaa-FF spread	sFYAAAC	1	fyaaac-fyff
102	Baa-FF spread	sFYBAAC	1	fybaac-fyff
103	Ex rate: avg	EXRUS	5	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
104	Ex rate: Switz	EXRSW	5	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)
105	Ex rate: Japan	EXRJAN	5	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)
106	Ex rate: UK	EXRUK	5	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
107	EX rate: Canada	EXRCAN	5	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
108	PPI: fin gds	PWFSA	5	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
109	PPI: cons gds	PWFCSA	5	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
110	PPI: int matls	PWIMSA	5	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
111	PPI: crude matls	PWCMSA	5	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
112	Commod: spot price	PSCCOM	5	SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)
113	Sens matls price	PSM99Q	5	INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)
114	NAPM com price	PMCP	1	NAPM COMMODITY PRICES INDEX (PERCENT)
115	CPI-U: all	PUNEW	5	CPI-U: ALL ITEMS (82-84=100,SA)
116	CPI-U: apparel	PU83	5	CPI-U: APPAREL & UPKEEP (82-84=100,SA)
117	CPI-U: transp	PU84	5	CPI-U: TRANSPORTATION (82-84=100,SA)
118	CPI-U: medical	PU85	5	CPI-U: MEDICAL CARE (82-84=100,SA)
119	CPI-U: comm.	PUC	5	CPI-U: COMMODITIES (82-84=100,SA)
120	CPI-U: dbles	PUCD	5	CPI-U: DURABLES (82-84=100,SA)
121	CPI-U: services	PUS	5	CPI-U: SERVICES (82-84=100,SA)
122	CPI-U: ex food	PUXF	5	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
123	CPI-U: ex shelter	PUXHS	5	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
124	CPI-U: ex med	PUXM	5	CPI-U: ALL ITEMS LESS MEDICAL CARE (82-84=100,SA)
125	PCE defl	GMDC	5	PCE,IMPL PR DEFL:PCE (1987=100)
126	PCE defl: dlbes	GMDCD	5	PCE,IMPL PR DEFL:PCE; DURABLES (1987=100)
127	PCE defl: nondble	GMDCN	5	PCE,IMPL PR DEFL:PCE; NONDURABLES (1996=100)
128	PCE defl: services	GMDCS	5	PCE,IMPL PR DEFL:PCE; SERVICES (1987=100)
129	AHE: goods	CES275	5	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
130	AHE: const	CES277	5	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
131	AHE: mfg	CES278	5	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
132	Consumer expect	HHSNTN	2	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)

B.2. *Tables*

TABLE IV

Share of Variance Explained by Common Factors.

This table provides the share of variance explained by the extracted factors, CPI and FFR. Note that the series were transformed to induce stationarity.

Variable	R^2	**Variable**	R^2	**Variable**	R^2
IPE	0,31	LPHRM	0,38	SFYGM	0,83
IPI	0,19	LPMOSA	0,36	SFYGT	0,92
IPM	0,23	Employment	0,91	SFYGT	0,93
IPMD	0,22	Pers Cons	0,07	SFYGT	1,00
IPMND	0,10	Dur Cons	0,03	SFYAAAC	0,99
IPMFG	0,30	Non Dor Cons	0,04	SFYBAAC	1,00
IPD	0,27	GMCSQ	0,04	M	0,99
IPN	0,17	GMCANQ	0,01	M	0,13
IPMIN	0,03	Housing Starts	0,56	M	0,05
IPUT	0,03	HSNE	0,45	FM DQ	0,07
IP	0,70	HSMW	0,62	M Base	0,31
CapUtilRate	0,71	HSSOU	0,52	FMRRA	0,17
PMI	0,96	HSWST	0,36	NBR	0,04
PMP	0,88	HSBR	0,58	FCLNQ	0,07
GMPYQ	0,09	HMOB	0,51	FCLBMC	0,17
GMYXPQ	0,27	PMNV	0,67	CCINRV	0,07
LHEL	0,28	New Order	0,87	CommPI	0,21
LHELX	0,81	PMDEL	0,67	PWSFA	0,65
LHEM	0,13	MOCMQ	0,08	PWFCSA	0,45
LHNAG	0,16	MSONDQ	0,02	PWIMSA	0,41
Uneployment	0,81	FSNCOM	0,06	PWCMSA	0,37
LHU	0,89	FSPCOM	0,06	PSMQ	0,11
LHU	0,90	FSPIN	0,05	CPI	1,00
LHU	0,95	FSPCAP	0,04	PU	0,31
LHU	0,98	FSPUT	0,06	PU	0,19
LHU	0,97	Dividends	0,66	PU	0,46
LPNAG	0,46	FSPXE	0,58	PUC	0,39
LP	0,46	EXRSW	0,03	PUCD	0,78
LPGD	0,47	XR YEN	0,06	PUS	0,41
LPMI	0,04	EXRUK	0,03	PUXF	0,58
LPCC	0,11	EXRCAN	0,03	PUXHS	0,77
LPEM	0,52	FFR	1,00	PUXM	0,80
LPED	0,45	m TreasBills	0,98	Earnings (avg/h)	0,89
LPEN	0,37	FYGM	0,98	LEHM	0,06
LPSP	0,27	FYGT	0,99	Cons Expec	0,12
LPTU	0,02	y TreasBond	0,99	Real FFR	0,60
LPT	0,29	FYGT	1,00		
LPFR	0,13	FYAAAC	1,00		
LPS	0,17	FYBAAC	1,00		
LPGOV	0,13	SFYGM	1,00		

B.3. *Figures*

Figure 9.— **Estimated Macro Factors and 95%-Confidence Intervals**

This figure provides plots of the estimated macro factors together with their 95%-confidence intervals. The estimates have been obtained as the median of the factor draws kept after the initial burn-in period of the Metropolis-within-Gibbs sampler.

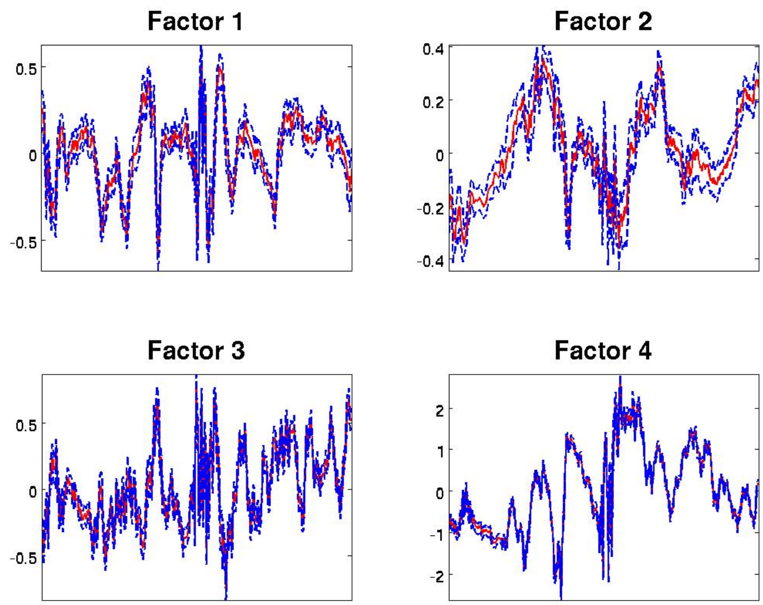


FIGURE 10.— **Convergence of Macro Factors (First Half vs. Second Half)**

The plot the first half of the extracted factors against the second half in order to asses the convergence of the sampler. Strong deviation would indicate that the samplers still moves towards its target distribution. No to small deviation indicates that the simulated chain went through the whole target distribution and has converged.

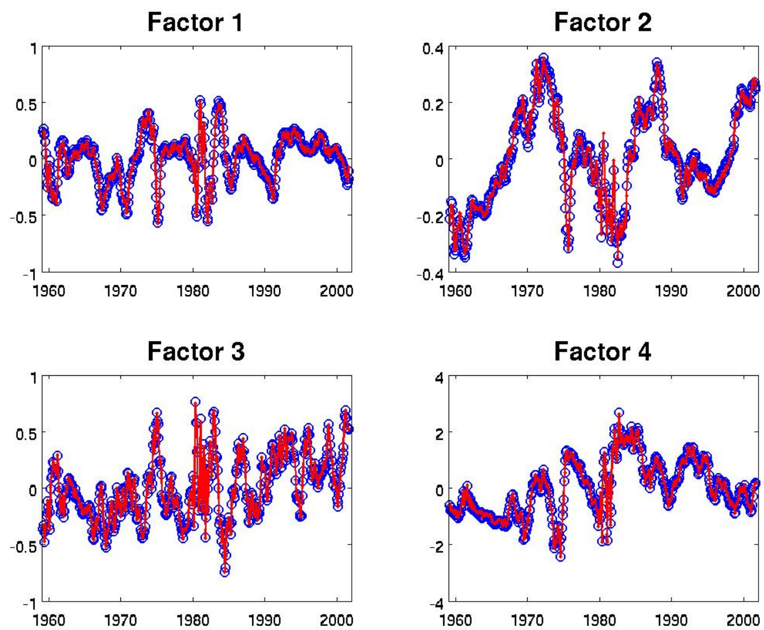


FIGURE 11.— **Impulse response functions of different industrial production indicators.**

IRF are based on the baseline model.

Model A: Baseline FAVAR

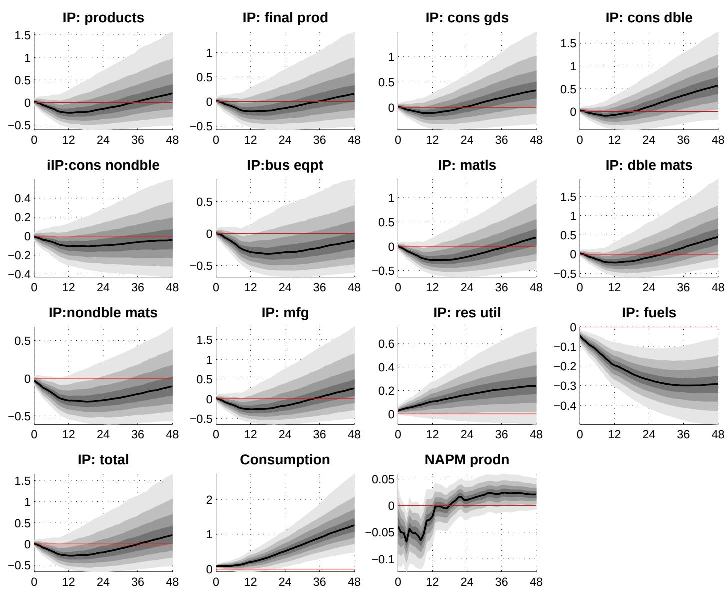


FIGURE 12.— **Impulse response functions of II. Pure FAVAR**

This amounts to only consider FFR as a Factor and excluding CPI out of the core VAR. Those series in the plot that were restricted are visualized in black and a horizontal line is added to the horizon where the restriction was active.

Model B: Pure FAVAR

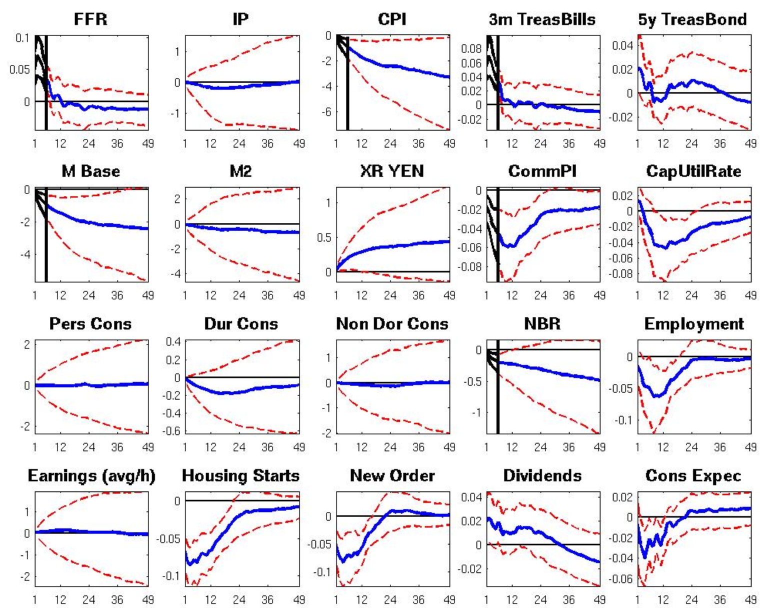


FIGURE 13.— **Impulse Responses for *MODEL C: Maximal sign restrictions***

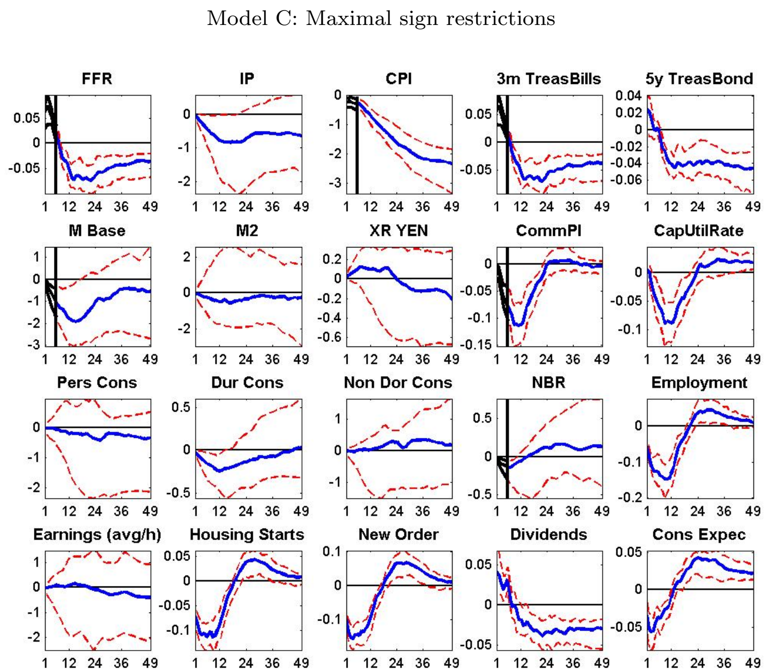


FIGURE 14.— **Impulse Responses for *MODEL D: Output Model***

In this model version we additionally impose the restriction on the Industrial production that was left agnostically unrestricted to analyze its impact. The difference to the baseline model is that the reaction of output itself is slightly stronger negative with less uncertainty for a longer period of time. The remaining indicators are unaffected.

Model D: Output Model

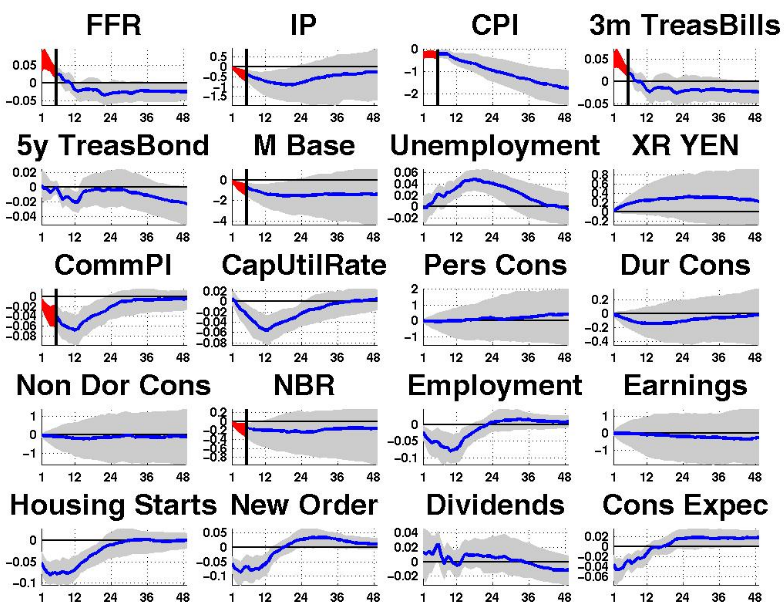


FIGURE 15.— **Impulse Responses for *Model E: Baseline Cholesky***

Here the set up is exactly the same as in the baseline case of Model A however the identification is employed with the recursive Cholesky identification. The key differences in the results are the clear price puzzles in CPI and commodity price index. The latter shows how flawed this identification schemes is. The commodity price index was originally included to overcome the price puzzle which is obviously not the case here. For CPI the price puzzle holds up to two years.

Model E: Baseline Cholesky

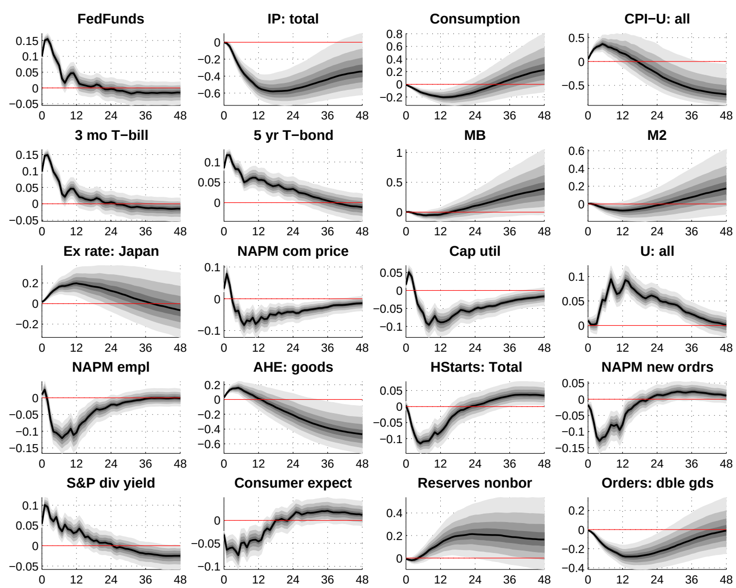


FIGURE 16.— **Impulse Responses for *Model F: Baseline Minimal SR***

This figure shows the same results as in the baseline case except that we impose the minimal restrictions as proposed in Uhlig [2005]. Results are similar to the baseline case except for a slightly higher degree of uncertainty around the reaction of industrial production.

Model F: Baseline Minimal SR

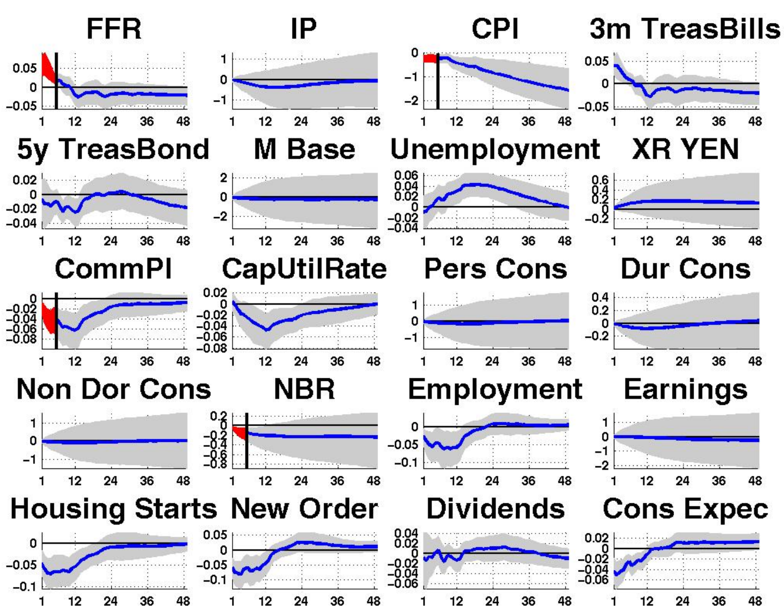


FIGURE 17.— **Impulse Responses for *Model G: Baseline w/o CPI Cholesky***

This plot reports results based on the Cholesky identification where in the core VAR the CPI indicator is not included. Interestingly the strong price puzzle vanishes to the extend that a positive response is as likely as a negative. This might be due to the fact that it is less well explained by the factors. However the price puzzle in the commodity price index is still present.

Model G: Baseline w/o CPI Cholesky

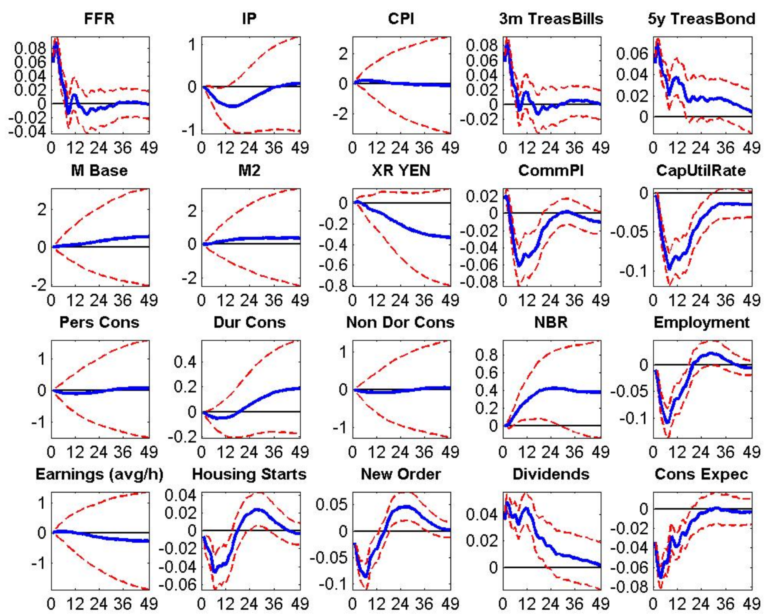


FIGURE 18.— **Forecast Error Variance Decomposition *Model A: Baseline Model* identified via sign restriction.**

This plot shows the forecast error variance decomposition of a contractionary monetary policy shock in the baseline case of model A. As expected a contractionary monetary policy mostly contributes to the variation in interest rates at the short horizon of around 25 % quickly going down after 3-4 month to around 10 %. This is much lower than in the case of Cholesky identification as can be seen in the next plot. The contractionary monetary policy shocks account on average only for a low fraction in the variation of industrial production and other output indicators of around 10%-12% over the entire horizon.

Forecast Error Variance Decomposition: Baseline Model

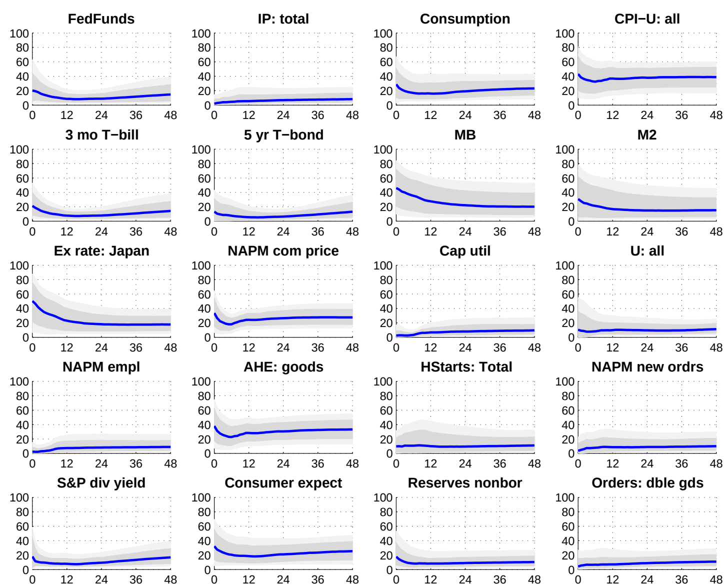
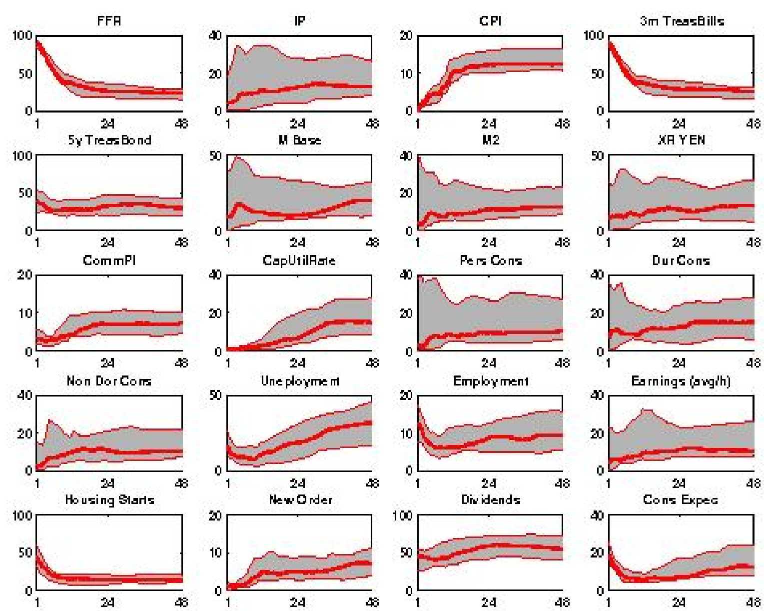


FIGURE 19.— **Forecast Error Variance Decomposition *Model A: Baseline Model* identified via Cholesky.**

This plot shows the forecast error variance decomposition of baseline model with Cholesky identification.

Forecast Error Variance Decomposition: Baseline Model



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