Unemployment Insurance in a Life Cycle General Equilibrium Model with Human Capital*

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Abstract

We evaluate the welfare gains of extending the duration and increasing the replacement ratio of the current unemployment insurance system in US. To this end, we build a general equilibrium overlapping generations model with on the job human capital accumulation. The model is able, among other things, to match the inequality of outstanding insurance in the economy (through the endogenous wealth distribution). The key features of the model are (i) agents are borrowing constraint, (ii) finding a job requires to exert (unobservable) effort, (iii) finite life span, and (iv) human capital accumulates *only* while the agent is employed. We find that the last two nobel features, in the class of models studying this problem, substantially increase the welfare gains of both: larger replacement ratios and long lasting unemployment benefits.

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1 Introduction

Several papers find that the welfare gains of switching from the current unemployment insurance system to the optimal one are negligible. This result hinges on the fact that under the current system enough insurance is provided. However, these papers fail to generate the observed high level of wealth inequality, and therefore the right level of insurance. We build a model that generates the right amount of inequality and we find that there are considerable welfare gains from increasing the replacement ratio from the outstanding 40% to 90%, and the duration from 6 months to 9 months.[1] This is true even with moral hazard concerns.

We build an incomplete markets overlapping generations general equilibrium model where agents, as they age, accumulate wealth and human capital through labor market participation. Employed agents are laid off exogenously but unemployed agents have to exert effort in order to find a job; as effort is unobservable, a generous unemployment insurance system generates moral hazard problems. However, the presence of a borrowing constraint makes a generous unemployment system desirable. A key feature in the design of any unemployment insurance system is the amount of insurance already available in the economy. This is given by the wealth distribution of the targeted population which are non-entrepreneurs [2]. The observed fat tails of the wealth distribution are an indication of the unequal distribution of insurance in the economy. In fact, the lack of insurance is most predominant among young generations. Our model generates fat tails in the wealth distribution by combining (i) unemployment risk, (ii) a borrowing constraint, (iii) a finite lifespan, and (iv) experience as the only source of human capital growth. Although not required to generate wealth dispersion, (iii) and (iv) are important in order to match the magnitude of wealth inequality observed in the data. On the one hand, the finite lifespan is important as it bounds above the discounted value of income, and thus, severely restricts the ability of agents to accumulate enough wealth to relax the borrowing constraint. On the other hand, the fact that human capital only grows

[1] Preliminary results.

[2] from now on when we refer to the wealth distribution we mean only among the non-entrepreneurial agents

with experience provides that both unemployment spells and job tenure have long lasting effects on labor income. These effects exacerbates the wealth inequality and give raise to a fat tails.

In addition, the returns to experience substantially mitigates the moral hazard problem among young agents. As Michelacci and Ruffo (2011) shows, there is a tight connection between age, moral hazard concerns and insurance. Young agents are more eager to accumulate human capital (through experience) than older agents, and thus, moral hazard concerns are almost absent at the beginning and increase with age. In this paper age and human capital play an important role in the sense that they imply a non trivial trade off between young generations with lack of insurance and low moral hazard concerns and old generations with a lot of insurance and serious moral hazard concerns. As far as we know, this has not been done in the literature.

When analyzing the welfare effects of alternative policies in economies with heterogeneous agents, an important issue arises: how to weight different agents? Based on the *Behind the veil of ignorance* approach, the standard solution to this problem is to put the same weight to all agents in the population. However, if there are losers and winners the answer ends up being somehow arbitrary. Different social weights could easily reverse the result. The life cycle approach provides a natural solution to this problem: in steady state, the present value of utility at birth encompasses all the necessary information and weights about all the possible states. It provides a clean answer to the question, if you could choose in which system to be born, which one would you choose? As we show later, this clean measure does not always agrees with the equally weighted preferences of all alive agents.

Among the papers that evaluate the welfare gains from changing the current system are Abdulkadiroglu et al. (2002) and Wang and Williamson (2002). These papers, using calibrated models similar to Hansen and Imrohoroglu (1992), analyze the gains of extending or decreasing current unemployment insurance duration.[3] They find that allowing the policy

[3]In both papers there is no endogenous interest rate on savings.

to be dependent on the unemployment spell marginally increases welfare; and that even the best combination of replacement ratio and unemployment spell generates minimal welfare gains. Further, Young (2004) adds capital accumulation to Wang and Williamson (2002) and finds that the optimal replacement ratios, for each possible duration, are zero. But still generating negligible welfare gains. More recently Mukoyama (2012), evaluates alternative unemployment insurance policies in a general equilibrium model with endogenous interest rate. He finds limited gains from changing the current unemployment insurance system even though there is a substantial feedback general equilibrium effect through changes in the interest rate. In addition, he generates fat tails for the wealth distribution by introducing heterogeneous discount factor.

This paper differs from the above in that models the capital markets, does not relies on preferences for saving (discount factor) to generate fat tails, and that explicitly considers the intergenerational trade off implicit in unemployment insurance systems. First, as Mukoyama (2012), we show that there is an important feedback effect through the interest rate. Second, the rich structure on age heterogeneity and human capital accumulation allow us to generate high levels of wealth inequality without relying on preference parameters. As a result, our model has a more involved trade off between insurance need and inequality which generates important departures from the standard results: newborns, independently of their initial wealth level, enjoy important welfare gains from changes in the unemployment insurance system.

From the partial equilibrium point of view, the trade off between insurance needs and moral hazard concerns has been the focus of many studies. For example, Hopenhayn and Nicolini (1997) notes that this trade off should restrict the shape of the unemployment insurance system. In particular, it is shown that the unemployment benefits should depend on the length of the unemployment spell. In a related matter, Shimer and Werning (2008) show in a partial equilibrium search model where agents are allowed to save, that unemployment benefits should be constant if agents are homogeneous and face a stationary problem,

while the result does not hold any more if heterogeneity or human capital depreciation are introduced (see Shimer and Werning (2006)).

From a general equilibrium perspective the opposing forces between insurance and moral hazard generate a trade off that, as noted by Hansen and Imrohoroglu (1992) in a calibrated incomplete markets economy were the interest rate on savings is zero, greatly affect the design of the unemployment insurance system. In an environment where the planner is constraint to a replacement ratio policy, Hansen and Imrohoroglu (1992) show that absent moral hazard concerns the level of unemployment benefits should be high but when agents are allowed to turn down offers the optimal level of unemployment benefits should fall sharply. Similar result can be found in Alvarez and Veracierto (2001) analysis of a general equilibrium economy, where the optimal level of unemployment benefits is shown to be low. Related to this issues Ljungqvist and Sargent (2008) show the importance of the interaction between on the job human capital accumulation and unemployment insurance.

2 Model

The economy is populated by a measure of heterogenous agents and a continuum of competitive firms which hire workers and rent capital to produce final consumption goods.

Agents differ in their age j, wealth a, human capital h, and their activity status $i \in \{e, u, r\}$ (employed, unemployed, retired). We assume that there is compulsory retirement after T periods in the labor force. While still in working age, an employed worker is fired with exogenous probability $\pi(j)$. An unemployed worker has to exert effort $\gamma(s)$ to find employment with probability s, where $\gamma(s)$ is increasing and convex, $\gamma(0) = 0$, and $\lim_{s\uparrow 1} \gamma(s) = +\infty$. Human capital accumulates as workers gain experience in the labor force. Let $\tilde{\alpha}(h, i)$ denote the rate at which human capital accumulates,

$$\tilde{\alpha}(h,i) = \begin{cases} 1 & \text{if } i \neq e; \\ \alpha(h) & \text{if } i = e. \end{cases}$$

with $\frac{\partial \alpha(h)}{\partial h} > 0$, $\frac{\partial^2 \alpha(h)}{\partial h^2} \leq 0$, and $\alpha(1) > 1$. That is, an agent accumulates human capital only while employed. We also assume that agents die stochastically with exogenous probability δ_j with

$$\delta_j = \begin{cases} \delta(j) & \text{for } j \leq T; \\ \delta_r & \text{for } j > T. \end{cases}$$

where $\frac{\partial \delta(j)}{\partial j} \geq 0$, $\frac{\partial^2 \delta(j)}{\partial j^2} \geq 0$, and $\delta_r > \delta(T)$. That is, the death probability increases with age. We restrict the parameter space so that, independently of their age, retired agents die with the same exogenous probability.[4] Furthermore, at every point in time, a new cohort is born.

2.1 Financing the unemployment insurance system

Agents in working age transit from employment to unemployment status stochastically. During their unemployment spells they receive a transfer $B_u(j,\nu)\ h\ w$ from the government in the form of unemployment benefits, where $B_u(j,\nu)$ is the replacement ratio, h is the accumulated human capital of the laid-off worker, and w is the wage per unit of effective labor. The replacement ratio $B_u(j,\nu)$ (potentially) depends on the age j of the agent and the length of her unemployment spell ν. To finance the unemployment insurance scheme the government levies a tax on labor earnings τ. The government also uses part of the tax revenues to finance a pension system with payments P_p to retired agents per unit of time.[5]

We restrict our attention to stationary equilibrium. The government keeps a balanced budget,

$$\tau w \int\int\int hX^e(j,h,a)\ dh\ da\ dj = \int\int\int\int B_u(j,\nu)X^u(j,h,a,\nu)\ d\nu\ dh\ da\ dj + \frac{P_p}{1-\delta_r}$$

where $X^e(j,h,a)$ is the stationary measure of employed agents with age j, human capital h, and wealth a, and $X^u(j,h,a,\nu)$ is the stationary measure of unemployed agents with age j,

[4] This can be easily relaxed but it has almost no effect on the result.

[5] The pension system is second order to our analysis. It only exists to guarantee interior solutions on the search intensity s while unemployed.

human capital h, wealth a, and ν periods of unemployment. Furthermore, $(1-\delta_r)^{-1}$ is the amount of retired agents in the economy. The government budget constraint provides that the revenue raised by a tax on every unit of employed human capital is used to finance the unemployment and pension system schemes.

Note that, because the probability of finding a job once unemployed s is under the span of control of the agent, the measures of employed and unemployed agents are endogenous.

2.2 Agents

In this section we describe the problem faced by agents at different moments of their life cycle.

2.2.1 The problem of an agent in working age ($j \leq T$)

Let $V_j^e(a,h)$ denote the value for an employed (e) agent with age j, wealth level a, and accumulated human capital h. Analogously, let $V_j^u(a,h,\nu)$ denote the value for an unemployed (u) agent with age j, wealth level a, accumulated human capital h, and length of unemployment spell ν.

While $j < T$, the value function for a worker solves

$$V_j^e(a,h) = \max_{c,a'} u(c) + \beta(1-\delta(j)) \left[(1-\pi(j))V_{j+1}^u(a',h',0) + \pi(j)V_{j+1}^e(a',h')\right]$$

subject to the constraints,

$$\begin{aligned} c + a' &= (1+r)a + wh(1-\tau) \\ h' &= \alpha(j)h \\ a' \geq 0 \quad &, \quad c \geq 0 \end{aligned}$$

and when $j = T$ the value function for a worker solves

$$V_j^e(a,h) = \max_{c,a'} u(c) + \beta(1-\delta(T))V^r(a)$$

subject to the constraints,

$$\begin{aligned} c + a' &= (1+r)a + wh(1-\tau) \\ h' &= \alpha(T)h \\ a' \geq 0 \quad &, \quad c \geq 0 \end{aligned}$$

The optimal policy function is $a' \equiv a^{*e}(j,h,a)$.

When unemployed and $j < T$ the value function solves

$$V_j^u(a,h,\nu) = \max_{c,a',s} u(c) - \gamma(s) + \beta(1-\delta(j)) \left[(1-s)V_{j+1}^u(a',h',\nu+1) + sV_{j+1}^e(a',h')\right]$$

subject to the constraints,

$$\begin{aligned} c + a' &= (1+r)a + B_u(j,\nu)\ h\ w \\ h' &= h \\ a' \geq 0 \quad &, \quad c \geq 0 \end{aligned}$$

and when $j = T$ it solves

$$V_j^u(a,h,\nu) = \max_{c,a'} u(c) + \beta(1-\delta(T))V^r(a')$$

subject to the constraints,

$$\begin{aligned} c + a' &= (1+r)a + B_u(T,\nu)\ h\ w \\ h' &= h \\ a' \geq 0 \quad &, \quad c \geq 0 \end{aligned}$$

where we already imposed that when $j = T$, the optimal search intensity is $s = 0$.

The optimal policy function is $a' \equiv a^{*u}(j, h, a, \nu)$ and $s = s(j, h, a, \nu)$.

2.2.2 The problem of a retired agent

As long as $j > T$, pension transfers P_p are independent of an individual's age j, and human capital h. Therefore, once retired, an agent's value function only depends on her wealth level a,

$$V^r(a) = \max_{c,a'} u(c) + \beta(1 - \delta_r)V^r(a')$$

subject to the constraints

$$\begin{aligned} c + a' &= (1 + r)a + P_p \\ a' \geq 0 \quad &, \quad c \geq 0 \end{aligned}$$

The optimal policy function is $a' \equiv a^{*r}(a)$.

2.2.3 Bequests

Newborns receive part of the wealth of the dying population. Let ϕ denote the tax on bequests and $\tilde{a}$ the effective bequest received by a newborn,

$$\tilde{a} \equiv (1 - \phi)a$$

Because $\tilde{a}$ is increasing in a there is persistence in the wealth level across generations.

2.3 Firms

There is a continuum of competitive firms which chooses the aggregate demand of capital K and aggregate demand of units of human capital H. The standard maximization problem of

a neoclassical firm provides,

$$\begin{aligned} r &= f_K(K,H) - d \\ w &= f_H(K,H) \ . \end{aligned}$$

2.4 Measures

Recall that $x^e(j,h,a)$ denotes the measure of employed agents with age j, human capital h, and wealth level a. In the same fashion, $x^u(j,h,a,\nu)$ denotes the measure of unemployed agents with age j, human capital h, wealth level a, and duration of unemployment spell ν. Also, let $X^r(a)$ denote the measure of retired agents with wealth level a. In this section we produce the Kolmogorov forward equations that can be used to compute these objects.

Let $g(a)$ denote the (endogenous) distribution of wealth of agents that die at a given period,

$$g(a) \equiv \int \delta(j) \int X^e(j,h,a) \ dh \ dj + \int \delta(j) \int \int X^u(j,h,a,\nu) \ d\nu \ dh \ dj + \delta_r X^r(a)$$

We use $g(a)$ to construct the measures for newborns,

$$\begin{aligned} X^e(1,1,\tilde{a}) &= g(a)(1-\phi)^{-1}\pi_0 \\ X^u(1,1,\tilde{a}) &= g(a)(1-\phi)^{-1}(1-\pi_0) \end{aligned}$$

for $\tilde{a} \in [0,(1-\phi)\bar{a}]$.

For the population in working age (i.e. $j \leq T$) we have that

$$\begin{aligned} X^e(j+1,h',a') &= (1-\delta(j))\pi(j)\int X^e\left(j,\frac{h}{\alpha(j)},a\right)\ \mathbb{I}\left(a'=a^{*e}(j,h,a)\right)\ da \\ &\quad +(1-\delta(j))\int\int s(j,h',a,\nu)\ X^u\left(j,h',a,\nu\right)\ \mathbb{I}\left(a'=a^{*u}(j,h',a,\nu)\right)\ d\nu\ da \\ X^u(j+1,h',a',\nu+1) &= (1-\delta(j))(1-\pi(j))\int X^e\left(j,\frac{h}{\alpha(j)},a\right)\ \mathbb{I}\left(a'=a^{*e}(j,h,a)\right)\ da \\ &\quad +(1-\delta(j))\int\int(1-s(j,h',a,\nu))\ X^u\left(j,h',a,\nu\right)\ \mathbb{I}\left(a'=a^{*u}(j,h',a,\nu)\right)\ d\nu\ da \end{aligned}$$

For retired agents with age $j = T+1$,

$$\begin{aligned} X^r(T+1,a') &= (1-\delta_T)\int\int X^e(T,h,a)\ \mathbb{I}\left(a'=a^{*e}(T,h,a)\right)\ da\ dh \\ &\quad +(1-\delta_T)\int\int\int X^u(T,h,a,\nu)\ \mathbb{I}\left(a'=a^{*u}(T,h,a,\nu)\right)\ d\nu\ da\ dh \end{aligned}$$

and for retired agents with age $j > T+1$,

$$X^r(a') = (1-\delta_r)\int X^r(a)\ \mathbb{I}\left(a'=a^{*r}(a)\right)\ da$$

2.5 Stationary Equilibrium

Given a policy rule $(\tau, B_u(j,\nu), P_p)$, a stationary equilibrium is a wage w and an interest rate r, and densities $X^e(j,h,a),\ X^u(j,h,a,\nu),\ X^r(j,a)\ \forall\ j,h,a,\nu$ such that (i) agents maximize their expected utility, (ii) labor markets clear

$$H = \int\int\int hX^e(j,h,a)djdhda\ \ ,$$

(iii) capital markets clear

$$K = \int\int\int\int a\left(X^e(j,h,a) + X^u(j,h,a,\nu) + X^r(j,a)\right) djdhdad\nu + K^e \quad ,$$

where K^e is entrepreneurial capital[6], (iv) the government keeps a balanced budget, and (v) feasibility constraint

$$f(K,L) = \int\int\int\int \left(c^e_{j,h,a} X^e(j,h,a) + c^u_{j,h,a,\nu} X^u(j,h,a,\nu) + c^r_{h,a} X^r(j,a)\right) djdhdad\nu$$

is satisfied, where, for example, $c^e_{j,h,a}$ denote the consumption of an employed agent of age j, human capital h, and asset level a.

3 Calibration

The utility function and the search cost functions are,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

$$\gamma(s) = -\gamma_0 \log(1-s)$$

As standard in the literature we set $\sigma = 2$. Since search effort is unobservable there is not reliable estimates of γ_0. Later we discuss how we calibrate the search parameter.

We first discuss how we quantify the exogenous processes: the human capital accumulation function $\alpha(h)$, the death probability $\delta(j)$, and the job-keeping probability $\pi(j)$. These functions are thought to be independently determined from our model and are constructed using micro data as we describe next.

To parameterize the human capital accumulation function $\alpha(h)$ we use results from Heck-

[6]For the moment we are assuming that there is some wealth held by agents outside of our model to match the aggregate wealth distribution. We are working on improving this assumption by letting the elasticity of wealthy entrepreneurs outside the model to be the same as the elasticity of rich workers. This will amplify the general equilibrium effects.

man et al. (2006). Note that human capital in the model is only accumulated when an agent is working and that $\alpha(h)$ is assumed to be monotonic. This implies there is a monotonic mapping between human capital h and experience in the labor market, say η. Heckman et al. (2006) uses the following Mincerian expanded regression to account for the return to experience,[7]

$$\ln(w_\eta) = a + b\eta + c\eta^2$$

where w_η is the wage for a worker with experience in the labor market η (which is measured in a yearly basis) and a, b, c are the estimated coefficients. Our theory provides that $w_\eta = h_\eta w$, where w is the wage per unit of effective labor. Using this relationship we get that

$$\ln \frac{h'}{h} = b + c + 2c\eta$$

In our calibration we use the estimated coefficients in Table 2 of Heckman et al. (2006) for white men where $b = 0.13$ and $c = -0.0023$.

We quantify the death probability using 2007 United States survival probability actuarial data collected by the United States Social Security Administration.[8] The empirical death probability by age is plotted in Figure 1, which we use to parameterize the function $\delta(j)$. Until middle-aged, agents die with a probability below 0.002, and later grows exponentially up to 0.014 when they retire, around age 62. Note that in the model we are assuming that retired agents share the same exogenous death probability, which in this case is $\delta^r = 0.11$.

To parameterize the job-keeping probability $\pi(j)$ we fit a fifth order polynomial to a series of the average quarterly job-keeping probability by age[9]. In Figure 2 we plot the empirical series and the fitted polynomial. The probability of remaining employed increases with age.

As standard in the literature we calibrate $d = 0.0125$[10] and we set $\alpha = 0.3$, which replicates

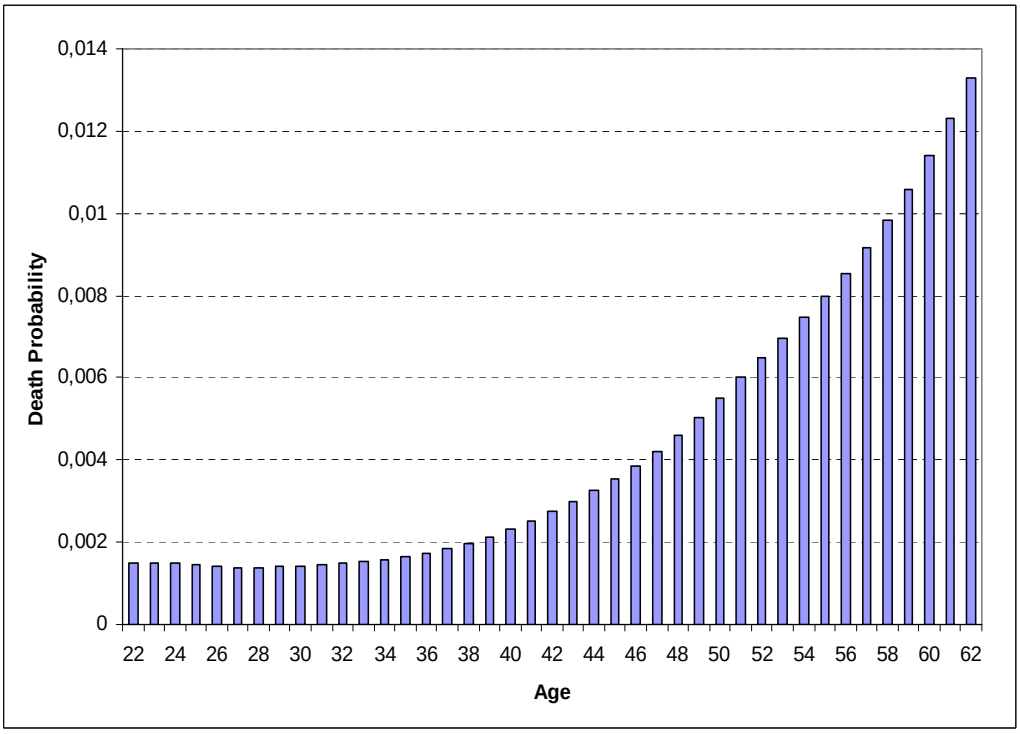
[7] Here we are abstracting from education and other controls also included in their regressions.

[8] The data is available at www.ssa.gov/oact/STATS/table4c6.html.

[9] This data was constructed by Robert Shimer using CPS monthly microdata from 1976 to 2005. The original series corresponds to average monthly statistics. Since the time period in our model correspond to a quarter we use the implied quarterly frequency.

[10] That is assume an annual depreciation rate of 0.05.

Figure 1: Death probability $\delta(j)$



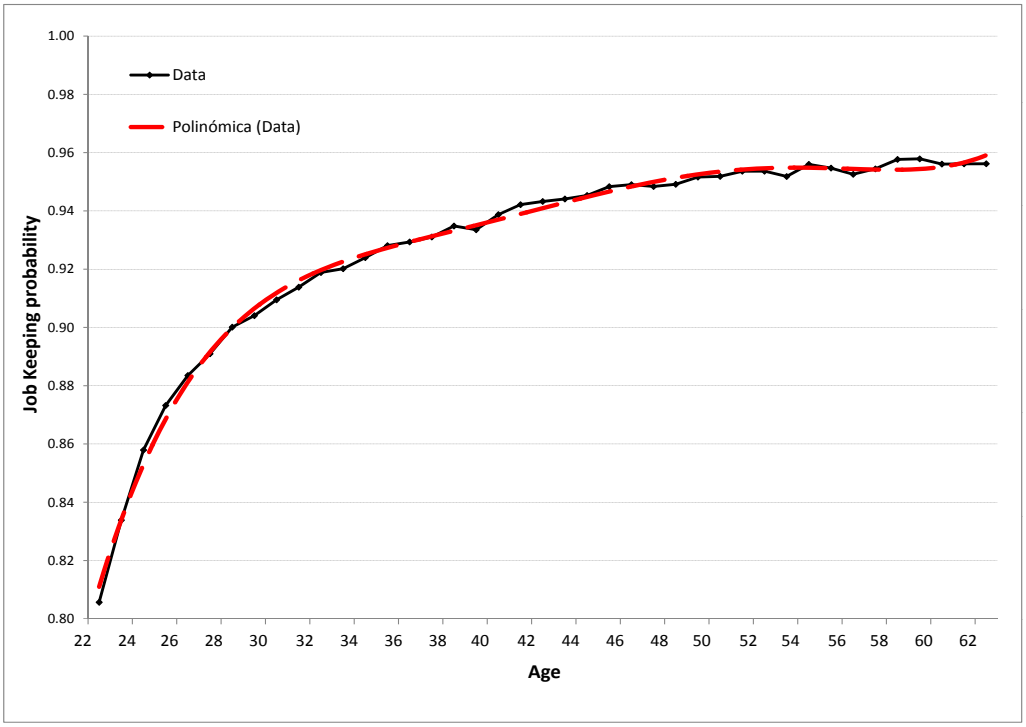
CHANGE TO QUARTELY Own calculations using data from 2007 US survival probability actuarial data from the United States Social Security Administration (www.ssa.gov/STATS/table4c6.html).

the observed proportion of capital incomes to GDP in the United States. We assume that the pension payments parameter, P_p, is 10% of the minimum wage.

We calibrate the remaining parameters, namely, the discount factor β, search cost parameter γ_0 and the capital held by entrepreneurs K^e, to match a set of moments. To this end we consider a benchmark model with an unemployment insurance system with a replacement ratio of 0.36 and two quarters of duration. This is approximately the current unemployment system in the United States.

Since our model includes the possibility of death, which modifies the effective discounting factor, we calibrate the discount factor to generate a net worth to GDP ratio of 4, which is the approximate ratio for the US economy in the last 10 years. This implies $\beta = 0.985$ (equivalent to a 0.94 annually).

Figure 2: Job-keeping probability $\pi(j)$



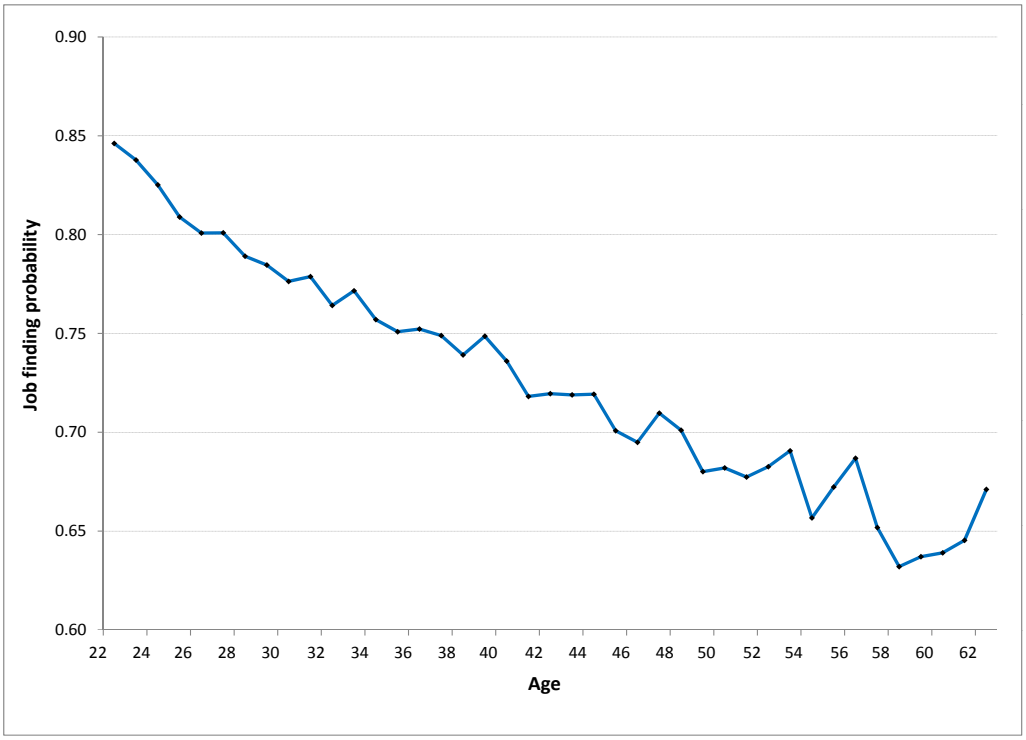
The data was constructed by Robert Shimer using CPS monthly microdata from 1976 to 2005. We fitted a fifth order polynomial to this data to obtain parameterize $\pi(j)$

Regarding the search parameter γ_0, we choose it to minimize the distance between the search effort by age generated by the model with the observed job finding probabilities. We obtain $\gamma_0 = 0.625$. Figure 3 shows the job finding probabilities [11].

Finally, we assume that the capital held by entrepreneurs is constant across simulations. We set $K^e = 700$, which implies that 40% of the net worth are held by entrepreneurs. This is consistent with the Survey of Consumer Finances database.

[11] Again this data was provided by Robert Shimer using CPS monthly microdata from 1976 to 2005

Figure 3: Job-finding probability by age $s(j)$



The data was constructed by Robert Shimer using CPS monthly microdata from 1976 to 2005.

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Appendix

3.1 Numerical Algorithms

Given any policy rule $B_u(j,\nu)$ first, fix a equally spaced grid $A = [a_1, a_2, ..., a_{Na}]$ of points for assets. Here we set $a_1 = 0$, $a_{Na} = 115$ and $Na = 250$. Fix a grid for human capital $H = [h_1, h_2, ..., h_{Nh}]$. With $h_1 = 1$, $Nh = 160$ (one point for each possible model age) and each h_i for $i = 2, ..., Nh$ is generated using the Mincerian equation. Finally fix a tolerance level $\epsilon > 0$ sufficiently small. These are the parameters of the algorithm and are kept fixed throughout. Then, choose a capital-labor ration R_0 and total government expenses on unemployment insurance Ψ_0. Then.

Step 1 Given R_0 compute the implied wage, w, and interest rate, r, using the firm's first order conditions. Then, given prices we can solve the problem of the retired agent. This is done using the standard value function iteration method. The solution to this problem generates a value function $V^r(a)$ and a policy function $a'^r(a)$.

Step 2 Given prices and Ψ_0 compute the tax, τ, that makes the government budget constraint hold with equality.

Step 3 Given τ, r, w and $V^r(a)$ we solve the employed and unemployed problem by backward induction. In this step is important to notice that the optimal search effort depends only on the continuation utilities. That is, taking first order conditions we obtain

$$\hat{s}(j,h,a',\nu) = 1 - \frac{\gamma_0}{\beta(1-\delta_j)[V^e_{j+1}(a',h) - V^u_{j+1}(a',h,\nu+1)]}$$

Since the solution to this equation does not guaranty that $s \in [0,1]$ we choose

$$s(j,h,a',\nu) = \min\{\max\{\hat{s}(j,h,a',\nu), 0\}, 1\}$$

Notice that this is not the optimal search effort yet, since it depends on a' and not on a. It only says how much effort the agent would exert contingent on saving a'. However, we can replace the above equation in the value function of the unemployed agent reducing the dimensionality of the maximization problem. Once we performed the maximization we obtain $a'^u(j,h,a,\nu)$ and therefore the optimal search effort is given by,

$$s^*(j,h,a,\nu) = s(j,h,a'^u(j,h,a,\nu),\nu)$$

Finally, the employed agent problem generates $a'^e(j,h,a,\nu)$

Step 4 Given $a'^e(j,h,a,\nu)$, $a'^u(j,h,a,\nu)$, $a'^r(a)$ and $s^*(j,h,a,\nu)$ we compute the measures using the laws of motions of Section 2.4. Once the measures has been computed we calculate aggregate workers capital, K', aggregate labor, L and total expenses in unemployment insurance Ψ_1.

Step 5 Given K' and L compute $R_1 = \frac{K'+K^{ent}}{L}$ and check distances. If $|R_0 - R_1| < \epsilon$ stop: solution found. Otherwise set $R_0 = \phi R_1 + (1-\phi)R_0$, for some $\phi \in (0,1)$ and $\Psi_0 = \Psi_1$ go to Step 1.