

Financing Constraints, Firm Dynamics, and International Trade*

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Abstract

This paper investigates the effects of financial frictions on the growth of firms and their decision to enter export markets. We propose a general equilibrium model economy in which monopolistically competitive firms face endogenous financing constraints. Financial constraints arise because of long-term lending contracts, which are optimal given an informational asymmetry between firms and investors about the firms performance over time. While there are no ex-ante productivity differences across firms, the history of revenue realizations and the optimal contract determine firms' access to credit, and thus their export decisions. Consistent with recent empirical regularities on exporters, the model predicts that (i) young exporters tend to be smaller and grow faster than established ones, (ii) the exit rate of new exporters is disproportionately higher than that of more established exporters (iii) investment by older exporters is more stable, and (iv) the share of export increases with firm age and size.

Keywords: private information, dynamic lending contracts, exporter dynamics, international trade

JEL classifications: F10, D82, L14

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1 Introduction

Research in both theoretical and empirical international trade increasingly focuses on firms and products in addition to the traditional analysis of countries and industries. Recent empirical studies of the financial decisions of firms documented important differences in the behavior of large and small firms, and are widely interpreted as indirect evidence of frictions in financial markets. Furthermore, recent studies from international trade have documented important differences in the behavior of new and established exporters, and suggested that access to credit may be an important factor in deciding to export. Our question is: Can financing constraints due to financial frictions account for empirical regularities on exporter dynamics?

To investigate how financial frictions affect firms' decisions to engage in international trade, we study an economy with monopolistic competition as in [Krugman \[1980\]](#), and in which financial frictions endogenously lead to firm heterogeneity in the spirit of [Clementi and Hopenhayn \[2006\]](#). Financial constraints arise because of long-term lending contracts, which are optimal given an informational asymmetry between firms and investors about the firms' performance over time. When financial intermediaries are able to offer and commit to long-term optimal financial contracts, firm heterogeneity arises because the history of revenue shocks and entrepreneurs reports to the lender determines the size of the loans, and the scale of firms' operation at any point in time. Thus, unlike heterogeneous-firms trade models based on [Melitz \[2003\]](#) in which ex-ante productivity differences across firms yield selection into export markets, the history of revenue realizations and the optimal contract determine firms' access to credit and thus their export decisions. We show that the model is at least qualitatively consistent with key empirical regularities about firms and exporter dynamics recently documented in both the industrial organization and

the international trade literature.

Cooley and Quadrini [2001] summarize empirical regularities on firm dynamics, which broadly include firm entry and exit, firm growth, and volatility of firm growth. They find that:

- conditional on age, the dynamics of firms are negatively related to the size of firms;
- conditional on size, the dynamics of firms are negatively related to the age of firms;
- smaller and younger firms pay fewer dividends, take on more debt and invest more; and
- the investments of small firms are more sensitive to cash flows, even after controlling for their future profitability

In the international trade literature, the surveys by Bernard, Jensen, Redding, and Schott [2007] and Wagner [2008] note that compared to non-exporters, exporters

- (1) are relatively rare;
- (2) are significantly larger;
- (3) are more productive;
- (4) pay higher wages;
- (5) are relatively more capital- and skill-intensive;
- (6) are more productive even before export begins; and
- (7) grow faster in employment and output after entering international markets.

Furthermore, using Colombian manufacturing data, [Eaton, Eslava, Kugler, and Tybout \[2007\]](#) and [Ruhl and Willis \[2008\]](#) find that¹

- (8) new exporters are smaller than older exporters;
- (9) the hazard rate of exit from international markets decreases with the time spent exporting; and
- (10) the share of exports in total sales increases with the time spent exporting.

[Krugman \[1980\]](#) recognizes the importance of fixed export cost to firms' decision to enter export markets, and shows that only the most profitable firms are willing to engage in trade. The seminal work by [Melitz \[2003\]](#) shows that firms of different productivity levels may co-exist in an industry when firms are uncertain about their own productivity, and must make an irreversible investment to enter the industry before the uncertainty is resolved and any export decision is made. When there are fixed cost of exports, [Melitz \[2003\]](#) shows that only the more productive firms engage in trade, thus accounting for regularities (1), (2), (3), and (6). A number of extensions to [Melitz \[2003\]](#) were since introduced to account for the other regularities listed above (see the next section for a brief overview). However, [Eaton, Eslava, Krizan, Kugler, and Tybout \[2011\]](#) argue that:

Models with sunk costs of exporting, which imply hysteresis in export behavior, help us understand patterns of foreign market entry and exit by individual firms [...]. But they provide little guidance as to why new exporters either exit or rapidly expand, while established exporters sales are much more stable. Nor do they convincingly reconcile the substantial market entry costs that they posit with the fact that many firms

¹ [Kohn, Leibovici, and Szkup \[2011\]](#) find similar empirical regularities using plant-level data on Chilean manufacturing.

export for short periods on a very small scale. Moreover, this analysis has not typically been integrated into general equilibrium or reconciled with aggregate export patterns.

As with other trade models, our model features fixed costs of exporting that cause hysteresis in export behavior. However, we show that, consistent with data, our model with financial frictions predicts that (i) the exit rate of new exporters is disproportionally higher than that of more established exporters, and declines the longer firms export, (ii) young exporters tend to be smaller and grow faster than established ones, (iii) investment by established exporters is more stable, and (iv) the share of export increases with firm age and size. Furthermore, our general equilibrium model lends itself well to (dynamic) policy analysis, as any change in the aggregate environment – for example due to a trade, a labor, or a financial reform – affects the dynamics of all firms in the economy and therefore firms’ decisions to hire labor and to export over time.

The rest of the paper is organized as follows: section 2 gives a brief overview of recent developments in heterogeneous firm trade models, and recent empirical studies investigating the link between financing constraints and exporter dynamics. Section 3 describes the economic environment. Section 4 discusses the source of the financial frictions and explains the mechanism generating firm heterogeneity, firm dynamics, and the pattern of trade. Section 5 defines the stationary equilibrium and section 6 analyzes the model numerically. Some extensions under consideration are discussed in section 7. Proofs of propositions and extensions are relegated to the Appendix.

2 Related literature

This paper proposes a general equilibrium model of international trade, in which firm dynamics and firms' decisions to engage in international trade are the outcome of long-term financial contracts that are constrained efficient under asymmetric information. This paper draws on, and relates to, several strands of literature from industrial organization, international trade, and macroeconomics which we briefly review in this section.

Firm dynamics are typically attributed to selection due to productivity differences between firms or to financing constraints. Two canonical models of selection are [Jovanovic \[1982\]](#) and [Hopenhayn \[1992\]](#). In [Jovanovic \[1982\]](#), entrepreneurs learn about their ability over time and decide to stay or exit depending on the signals they receive. In [Hopenhayn \[1992\]](#), persistent productivity shocks affect firm growth, and a long enough sequence of bad shocks may force a firm to exit the market.

[Melitz \[2003\]](#) incorporates the selection mechanism of [Hopenhayn \[1992\]](#) – albeit with permanent firm-level idiosyncratic productivity shocks – into a general equilibrium open economy environment. In this model, potential firms face uncertainty concerning their productivity, and have to make an irreversible investment before drawing a productivity level. Once in operation, a firm may export if it incurs an additional fixed cost and variable export costs. In equilibrium, the most productive firms export, moderately productive firms only serve domestic markets, and the least productive firms exit altogether. Since the uncertainty faced by firms in the Melitz model is resolved in the first period, a firm's status remains the same forever and there are no firm nor exporter dynamics.² Several extensions to [Melitz \[2003\]](#)

²[Bernard, Eaton, Jensen, and Kortum \[2003\]](#) propose another approach to generate firm heterogeneity building on [Eaton and Kortum \[2002\]](#) but assuming Bertrand competition between firms instead of perfect competition.

were proposed to account for the simultaneous growth and decline and entry and exit of firms in steady state. Arkolakis [2010] and Alessandria and Choi [2011] consider persistent idiosyncratic shocks to firms’ productivity, and Atkeson and Burstein [2010] and Burstein and Melitz [2011] introduce stochastic returns on firms’ R&D investment.³

The importance of financing constraints for firm dynamics has been the subject of extensive research dating back to Fazzari, Hubbard, and Petersen [1988], and there is now a consensus that financing frictions have an important impact on firm dynamics – see Hubbard [1998] and Stein [2003] for surveys.⁴ However, the possible relationship between financial constraints, firms’ decisions to engage in international trade, and exporter dynamics has only recently become the focal point of a small but rapidly growing empirical and theoretical literature.

Beck [2002] finds empirical support for the prediction of Kletzer and Bardhan [1987] that countries with better developed financial systems have a higher export share and trade balance. Campa and Shaver [2002] and Greenaway and Kneller [2007] find that exporters are less liquidity constrained than non-exporters. Bellone, Musso, Nesta, and Schiavo [2010] and Minetti and Zhu [2011] find that firms with greater access to external financing are more likely to export and to export more using French and Italian manufacturing data respectively. Suwantaradon [2008] and Wang [2010] use data from the World Bank Enterprise Survey and find that higher net-worth or less financially constrained firms are more likely to export; moreover, the likelihood of exporting and the volume of export increases with firm age. Last,

³Other papers that consider firm dynamics, but that are less related to this paper include Ruhl and Willis [2008], Eaton et al. [2011], and Chaney [2011].

⁴Other recent empirical contributions include Cabral and Mata [2003], Oliveira and Fortunato [2006], Fagiolo and Luzzi [2006], Lu and Wang [2010], Aghion, Fally, and Scarpetta [2007], and Beck, Demirg-Kunt, and Maksimovic [2008].

Manova, Wei, and Zhang [2011] find that credit constraints restrict international trade flows and affect the pattern of foreign direct investment in China.⁵

Several recent theoretical contributions relate financing constraints to firms' decisions to engage in international trade by extending Melitz [2003]. Chaney [2005] introduces collateral requirements in a model where firms must borrow to finance the export cost, and show that only the most productive firms are able to generate enough cash flows to meet the collateral requirement, thus exacerbating the selection effect of Melitz. Manova [2008] concentrates on cross-country differences in financial development, and shows that trade liberalization increases exports disproportionately in financially vulnerable sectors that require more outside finance, or employ fewer collateralizable assets. Feenstra, Li, and Yu [2011] motivate financing constraints with one-period financial contracts that are optimal given lenders have incomplete information about firms' productivity, and show that exporting firms are more severely credit constrained than non-exporting firms.

This paper contributes to this literature by studying the role of financial frictions and financial intermediation on firms decision to export and on exporter dynamics in general equilibrium. Our starting point are the models of Albuquerque and Hopenhayn [2004] and Clementi and Hopenhayn [2006] who show that long-term financial contracts that are constrained efficient under a particular type of market failure can help account for empirical regularities on the age-size dependence of firm dynamics. In Clementi and Hopenhayn [2006] borrowing constraints arise because borrowers

⁵Other studies of the impact of credit frictions on international trade followed in the aftermath of the financial crisis of 2008-2009. Chor and Manova [2011] suggest that credit conditions exerted a disproportionately disruptive effect on trade flows beyond their effect on domestic output. Paravisini, Rappoport, Schnabl, and Wolfenzon [2011] show that credit shortages explain 15% of the Peruvian exports decline during the crisis. Ahn, Khandelwal, and Wei [2011] argue that financial frictions are an important factor in explaining the collapse in world trade after the 2008-2009 financial crisis.

face limited liability, and the lender does not observe the firms' revenue realizations. In [Albuquerque and Hopenhayn \[2004\]](#) borrowing constraints arise because borrowers face limited liability, and debt repayments cannot be fully enforced. Our model adapts the long-term financing contract of [Clementi and Hopenhayn \[2006\]](#) to a general equilibrium multi-country environment with monopolistic competition and fixed export costs. This paper is related to [Wang \[2010\]](#) and [Brooks and DAVIS \[2011\]](#) who study the effects of optimal long-term financial contracts under limited contract enforcement as in [Albuquerque and Hopenhayn \[2004\]](#) on firms' decisions to export, with an emphasis on the welfare gains following a trade liberalization. A notable feature of this type of contractual arrangement however is that firm size never decreases in equilibrium, and thus a firm that starts exporting in these models never stops. This paper also relates in terms of focus to [Kohn et al. \[2011\]](#) who study the impact of collateral constraints on exporter dynamics in a partial equilibrium in which firms face persistent productivity shocks. Last, our model is also related to [Cooley, Marimon, and Quadrini \[2004\]](#), [Smith and Wang \[2006\]](#) and [Verani \[2011\]](#) who study the aggregate implications of optimal long-term lending contracts in a closed economy.

3 Environment

Time is discrete and infinite. There are two countries, each of which is populated by a mass of atomistic workers and entrepreneurs. Agents survive to the next period with a fixed probability. There is one all purpose good used for consumption and investment, which is costlessly assembled from a continuum of differentiated, and imperfectly substitutable domestic and foreign intermediate goods. Workers are risk averse, and are endowed with one unit of time each period, which they allocate

between labor and leisure. Workers supply labor in a competitive labor market. Entrepreneurs are risk neutral and have access to a technology to produce one of the differentiated goods. A project requires an initial fixed investment, and per-period resources to produce. An entrepreneur has the option to ship goods to the foreign market every period by paying a fixed cost. Entrepreneurs are born without assets, and do not make a labor-leisure decision. Once started, the project generates a stream of uncertain revenues subject to independent and identically distributed revenue shocks. Entrepreneurs finance their project by borrowing the initial capital and the period working resources from financial intermediaries. Death of an entrepreneur terminates the firm.

3.1 Workers

Workers are born with zero non-human wealth, survive into the next period with exogenous probability $(1-\gamma_w)$, and are instantly replaced by new ones when deceased. Workers discount the future at rate $\hat{\beta}$ and are endowed with one unit of time each period that they allocate between labor h_t and leisure. Labor is paid at wage w_t , and workers use their income to either buy the numéraire consumption good c_t , or to purchase contingent claims d_{t+1} at price p_t^a that pay $(1+r_t)$ units of consumption in the next period if the agent is alive, and zero otherwise. Agents do not value bequests, and will thus place all their savings in these claims. Workers assess their consumption-leisure decision according to

$$E_0 \sum_{t=0}^{\infty} (1-\gamma_w)^t \hat{\beta}^t u(c_t, 1-h_t), \quad (1)$$

which they maximize subject to the following budget constraint:

$$c_t + p_t^a d_{t+1} \leq d_t(1 + r_t) + w_t h_t. \quad (2)$$

The workers problem can be written recursively as

Problem 1 (Workers)

$$\begin{aligned} U(d) = \max_{d', c, h} \quad & u(c, h) + (1 - \gamma_w) \hat{\beta} \mathbb{E} U(d') \\ \text{s.t.} \quad & c + p^a d' = d(1 + r) + wh \\ & d' \geq 0 \end{aligned} \quad (3)$$

where $d' \geq 0$ is the no-Ponzi condition, and implies workers are not allowed to borrow. This extra condition simplifies the analysis, and is never binding given our parametrization.

3.2 Entrepreneurs

Entrepreneurs, like workers, are born without wealth, survive into the next period with probability $(1 - \gamma_e)$, and are instantly replaced upon death. Entrepreneurs are risk-neutral, and discount the future at the rate $(1 + r_t)^{-1}$. We assume entrepreneurs do not make a labor-leisure decision – an interpretation is that entrepreneurs devote a fixed fraction of their time to supervise the project operation. Entrepreneurs assess their consumption decision according to

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t, \quad (4)$$

where $\beta = \left(\frac{1-\gamma_e}{1+r_t}\right)$, so that $\beta^t c_t$ is the expected discounted utility at time t . Since entrepreneurs discount the future at the interest rate, they are indifferent between saving and consumption. For simplicity, we assume that they consume all their income every period, and do not take part in the annuity market introduced above.

3.3 Financial intermediation

Perfect competition in the financial market implies that we can focus on a representative financial intermediary. The representative financial intermediary is risk-neutral, discounts the future at $(1+r_t)^{-1}$ and offers one-period annuity contracts to workers. The assumptions on worker characteristics imply stationary demographics for workers so that annuities can be offered without risk by the financial intermediary. Zero-profit in the annuity market drive the price of annuities down to the survival rate $(1-\gamma_w)$.

Deposits from workers in period t are then invested into the entrepreneurs' project in period $t+1$. Repayments from the entrepreneurs to the intermediary are used to repay the deposits with interest. The financial contract, discussed in details below, specifies the optimal size of loans and repayments in each period conditional on an entrepreneur's report on the performance of his firm. Perfect competition in the financial market implies that the value of a new financial contract just offsets the sunk start-up investment.

3.4 Technology

3.4.1 Intermediate good

An entrepreneur has access to a project indexed by $\omega \in \tilde{\Omega}$, that requires an initial investment I_0 that is sunk, and per period working resources R_t . The ω -th firm produces the ω -th good according to a neoclassical production function $G(k, n)$, where k_t is capital input and n_t is labor input. An entrepreneur chooses the quantity q_t to sell to the domestic market, and the quantity q_t^* to sell abroad. In what follows, all variables with an asterisk denote exported goods, quantities and prices. A firm must pay a fixed export cost I_E before production begins if it chooses to export. That is, working resources R must be allocated such that

$$k + nw + \mathbf{1}(q^* > 0)I_E \leq R, \quad (5)$$

where $\mathbf{1}(q^* > 0)$ is an indicator function that is equal to 1 when $q^* > 0$. We assume that capital is fully depreciated at the end of the period. In order to sell one unit of its good abroad, a firm must ship $(1 + I_T)$ units of this good, which is an iceberg-style transport cost standard in the trade literature. Hence, output is allocated between domestic sales and exports such that

$$q + q^*(1 + I_T) \leq G(k, n). \quad (6)$$

We assume the ω -th firm is a monopolist for its differentiated product, but takes the inverse demand function for its product $p(q)$ – price as a function of quantity –

as given. It follows that the maximum revenues for a given level of resources are

$$\begin{aligned}
F(R) &= \max_{q, q^*, k, n} p(q)q + p^*(q^*)q^* \\
\text{s.t. } & q + q^*(1 + I_T) \leq G(k, n) \\
& k + nw + \mathbf{1}(q^* > 0)I_E \leq R .
\end{aligned} \tag{7}$$

Let us index firm status by $i \in \{D, E\}$, where D and E indicate whether the firm sells to the domestic market only, or to both the domestic and export market. Let $F_i(R)$ denote the maximum revenues a firm of type i can generate given it has access to a loan of size R . We assume there exists a unique level of resources R_{dx} past which a firm can only maximize its revenue by being an exporter: $F_D(R) > F_E(R)$ for all $R < R_{dx}$ and $F_D(R) < F_E(R)$ for all $R > R_{dx}$ – If there is no level for which it is optimal to export, there will be no trade.⁶

Project returns are subject to a sequence of independent and identically distributed idiosyncratic revenue shocks $(\theta_t)_{t \geq 0}$, where $Pr(\theta_t = \text{High}) = 1 - Pr(\theta_t = \text{Low}) = \pi$.⁷ We assume that all firms $\omega \in \tilde{\Omega}$ face the same type of idiosyncratic shocks. A firm is terminated if the entrepreneur dies, which is analogous to receiving a permanent zero-revenue shock. This assumption is convenient to capture other sources of exit not modeled explicitly, and allows to pin down the distribution of firms without having to keep track of individual firms.⁸

⁶In the appendix, we show that if such a level exists, then the crossing point of $F_D(R)$ and $F_E(R)$ is unique.

⁷One could think of the revenue shocks as productivity shocks. In that case, when productivity is zero, no goods can be produced and revenues will be zero. In the case of zero-one shocks the two definitions are identical; otherwise it is always possible to map one into the other.

⁸This assumption is common in the related literature. See for instance Cooley and Quadrini [2001], Cooley et al. [2004], and Smith and Wang [2006].

3.4.2 Final good

A large number of competitive firms costlessly assemble the final good Y from imported and domestically produced intermediate goods. Goods imported from foreign firms are denoted by a subscript f , as are the corresponding prices and quantities. The production function is a CES aggregator:

$$Y = \left(\int_{\Omega} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\Omega_f} y(\omega_f)^{\frac{\sigma-1}{\sigma}} d\omega_f \right)^{\frac{\sigma}{\sigma-1}}, \quad (8)$$

where $\sigma > 1$, and Ω is the set of goods available from domestic producers so that $\Omega \subseteq \tilde{\Omega}$, and $y(i)$ is the quantity of the i -th good used in production. A final goods producer takes prices as given and chooses a combination of $\omega \in \Omega$ and $\omega_f \in \Omega_f$ to maximize its profit function

$$Y - \int_{\Omega} y(\omega)p(\omega)d\omega - X \int_{\Omega_f} y(\omega_f)p(\omega_f)d\omega_f. \quad (9)$$

X denotes the exchange rate between the final domestic and foreign good. Constant returns to scale and perfect competition in the final goods market imply we can assume without loss of generality that there is one representative firm making zero profits.

4 Financial frictions and the optimal contract

Entrepreneurs are born with zero assets and must raise the initial sunk cost I_0 in order to start a project, plus per-period working resources R . Revenue shocks are only observed by the entrepreneurs, and determine the entrepreneurs' ability to pay back the loan. We assume there exists a perfectly competitive financial market

where financial intermediaries and new entrepreneurs are able to commit to a long-term contract. There is limited liability, so that entrepreneurs cannot be forced to make any payments beyond their current revenue.

A reporting strategy for an entrepreneur is a sequence of reports $\hat{\theta} = \{\hat{\theta}_t(\theta^t)\}$, where $\theta^t = (\theta_1, \dots, \theta_t)$ is the true history of realizations of revenue shocks received by an entrepreneur. The history of reports is denoted by $h^t = (\hat{\theta}_1, \dots, \hat{\theta}_t)$. A contract is a quintuple $\kappa_t = \{\ell_t(h^{t-1}), e_t(h^{t-1}), Q_t(h^{t-1}), R_t(h^{t-1}), \tau_t(h^{t-1}, \theta_t)\}$ that specifies a liquidation rule $\ell_t(h^{t-1})$, an export rule $e_t(h^{t-1})$, the payment $Q_t(h^{t-1}) \geq 0$ from the financial intermediary to the entrepreneur in case of liquidation, the period resource advancement $R_t(h^{t-1})$ to be allocated between capital k_t , labor n_t at cost w_t , and export costs I_E , and the period repayments τ_t to the financial intermediaries contingent on the entrepreneur's report on the revenue realization. Figure 1 summarizes the timing of events within one period.

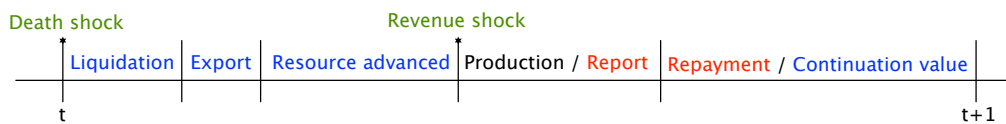


Figure 1: Timing

Conditional on surviving, a project is either liquidated, in which case the entrepreneur receives $Q_t(h^{t-1})$ and the financial intermediary receives $S - Q(h^{t-1})$, where $S \leq I_0$ is the project's salvage value, or it remains in operation. If a project is kept in operation, the contract specifies whether or not the firm should participate in international trade, and the size of the loan $R_t(h^{t-1})$. After production takes place and revenue is realized, an entrepreneur makes a repayment $\tau(h^{t-1}, \hat{\theta}_t)$ to the financial intermediary, that is conditional on his ex-post report $\hat{\theta}_t$.

After every history h^{t-1} , the pair of contract and report strategy $(\kappa, \hat{\theta})$ implies an

expected discounted cash flow $V_t(\kappa, \hat{\theta}, h^{t-1})$ and $B_t(\kappa, \hat{\theta}, h^{t-1})$ for the entrepreneur and the financial intermediary respectively. A feasible and incentive compatible contract is optimal if it maximizes $B_t(V_t)$ for every possible V_t . Following, [Clementi and Hopenhayn \[2006\]](#), we refer to V and B as equity and debt. It follows that an optimal contract will maximize the value of the firm $W(V) = B(V) + V$.

Let $\tau_t = \tau_t(h^{t-1}, \hat{\theta}_t = 1)$ since repayments are 0 when the entrepreneur reports a bad shock due to limited liability. Using the method of [Abreu, Pearce, and Stacchetti \[1990\]](#), the optimal contract can be written recursively by using V_t as a state variable and by defining V_t^H and V_t^L as promised continuation values. It follows equity must satisfy the following accounting identity

$$V = \pi(F_i(R) - \tau) + \beta[\pi V^H + (1 - \pi)V^L]. \quad (10)$$

That is, equity at the beginning of a period is equal to the expected cash flow minus repayments to the intermediary plus discounted expected equity in the following period. Incentive compatibility requires that truthful reporting is optimal in every period. Since the entrepreneur has no incentive to report a good shock when receiving a bad shock, the relevant incentive compatibility constraint is

$$F_i(R) - \tau + \beta V^H \geq F_i(R) + \beta V^L \Rightarrow \tau \leq \beta(V^H - V^L). \quad (11)$$

Limited liability requires the period repayment to never exceed the reported income, so that

$$\tau \leq F_i(R). \quad (12)$$

Conditional on surviving, a firm can choose to become an exporter. Index the export

status by $i \in \{D, E\}$. The value of a i -type firm is given by

$$\begin{aligned}\widehat{W}_i(V) &= \max_{\tau, R, V^H, V^L} \pi F_i(R) - (1+r)R + \beta \mathbb{E}W(V') \\ \text{s.t.} \quad & (10), (11), \text{ and } (12) \\ & R_E \geq I_E, \quad V^H, V^L \geq 0.\end{aligned}\tag{13}$$

Clementi and Hopenhayn [2006] show that for low values of equity, a greater value of the firm can be reached by allowing for a lottery on the liquidation decision such that

$$\begin{aligned}W(V) &= \max_{\alpha \in [0,1], Q, V_r} \alpha S + (1-\alpha)\widehat{W}(V_r) \\ \text{s.t.} \quad & \alpha Q + (1-\alpha)V_r = V \\ & Q, V_r \geq 0\end{aligned}\tag{14}$$

where α is the probability of liquidation, and V_r is the continuation value when the firm is not liquidated. Thus, whenever V falls below V_r , the financial intermediary offers the entrepreneur a lottery where the project is either liquidated with probability $\alpha = (V_r - V)/V_r$ in which case the entrepreneur receives Q from the intermediaries, or kept in operation with probability $1 - \alpha$ and is awarded V_r , so optimally $V_r = \sup\{V : \widehat{W}(V) - S = V\widehat{W}'(V)\}$ and $Q = 0$. Similarly, and by using an analogous argument, there is a region $[V_D, V_E]$ within which a greater value of the firm can be reached by allowing for a lottery on the export decision.⁹ We assume that this region does not overlap with the liquidation region, that is $V_D > V_r$. We can formally

⁹See proof in the appendix.

express this ‘export lottery’ as

$$\begin{aligned}
\widehat{W}(V) &= \max_{\delta \in [0,1], V_D, V_E} \delta \widehat{W}_E(V_E) + (1 - \delta) \widehat{W}_D(V_D) \\
\text{s.t. } &\delta V_E + (1 - \delta) V_D = V \\
&V_D, V_E \geq 0
\end{aligned} \tag{15}$$

where δ is the probability of becoming an exporter and V_D and V_E are continuation values of the firm if it becomes a domestic seller or an exporter respectively. Whenever a firm reaches a size $V \in [V_D, V_E]$, it is offered an ‘export lottery’ and becomes an exporter of size V_E with probability $\delta = (V - V_D)/(V_E - V_D)$, or a domestic seller of size V_D with probability $(1 - \delta)$. The boundaries of the ‘export region’ $[V_D, V_E]$ are determined such that the tangent of $\widehat{W}_D(V)$ at V_D is equal to the tangent of $\widehat{W}_E(V)$ at V_E . The firm’s problem can thus be written recursively as

Problem 2

$$\begin{aligned}
\widehat{W}_i(V) &= \max_{\tau, R, V^H, V^L} \pi F_i(R) - R(1+r) + \beta \mathbb{E}W(V') \\
s.t. \quad & V \geq \beta[\pi V^H + (1-\pi)V^L] \\
& \tau \leq \beta(V^H - V^L) \\
& \tau \leq F_i(R) \\
& R_E \geq I_E, \quad V^H, V^L \geq 0 \\
\\
\widehat{W}(V) &= \max_{\delta \in [0,1], V_D, V_E} \delta \widehat{W}_D(V_D) + (1-\delta) \widehat{W}_E(V_E) \\
s.t. \quad & \delta V_D + (1-\delta)V_E = V \\
& V_D, V_E \geq 0 \\
\\
W(V) &= \max_{\alpha \in [0,1], Q, V_c} \alpha S + (1-\alpha) \widehat{W}(V_c) \\
s.t. \quad & \alpha Q + (1-\alpha)V_c = V \\
& Q, V_c \geq 0
\end{aligned} \tag{16}$$

Figure 2 depicts the optimal contract.

4.1 Financing constraints and firm dynamics

Following Clementi and Hopenhayn [2006], there is a natural upper bound on equity $\tilde{V} = \pi F_E(\tilde{R})/(1-\beta)$, where $\tilde{R} = \arg\max_R \{\pi F_E(R) - R(1+r)\}$ corresponding to the unconstrained size of the firm – which must be an exporter. When the firm is borrowing constrained and not operating at full scale, the entrepreneur is awarded a

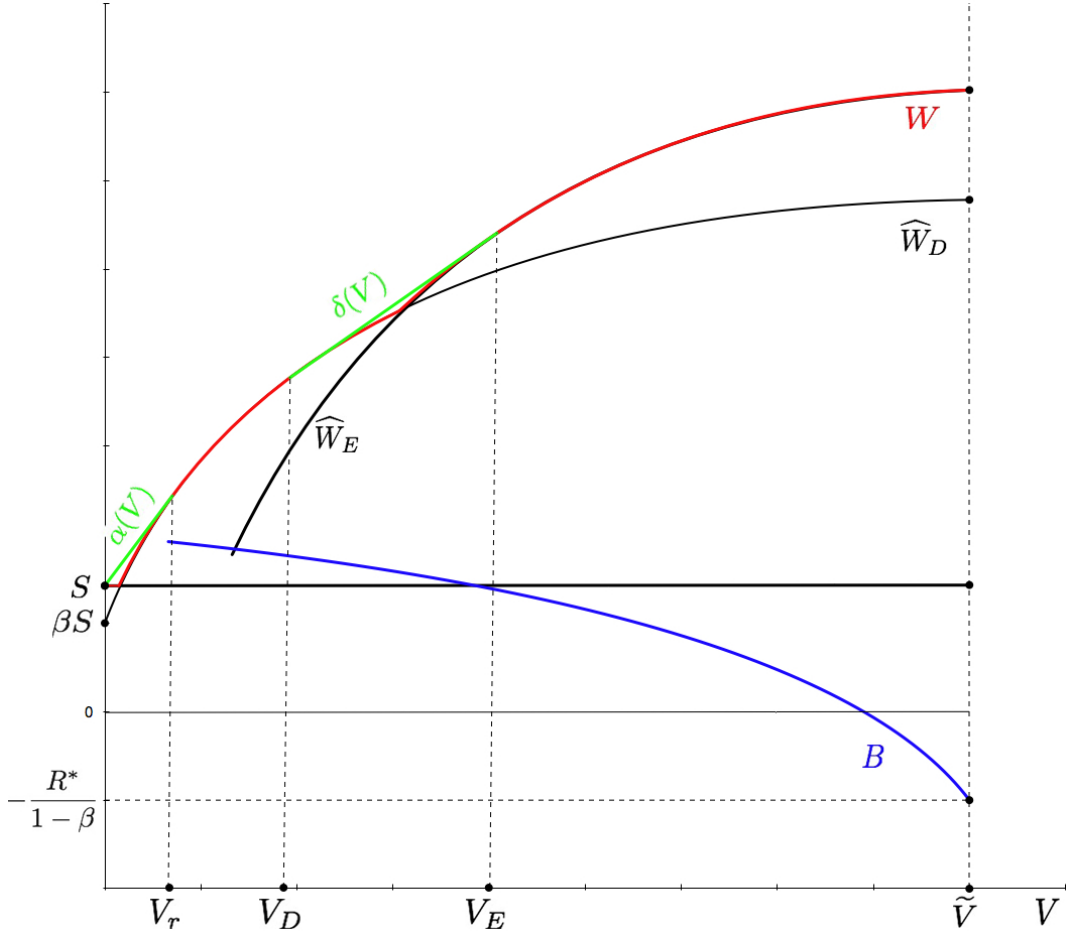


Figure 2: Value of the firm

continuation value V^L upon reporting a bad shock, where

$$V^L(V) = \begin{cases} \frac{V - \pi F_D(R(V))}{\beta} & \text{if } V \in [V_r, V_D] \\ \frac{V - \pi F_E(R(V))}{\beta} & \text{if } V \in [V_E, \tilde{V}) \end{cases}, \quad (17)$$

and is awarded V^H upon reporting a good shock, where

$$V^H(V) = \begin{cases} \frac{V+(1-\pi)F_D(R(V))}{\beta} & \text{if } V^H(V) \in [V_r, V_D) \\ \min \left\{ \tilde{V}, \frac{V+(1-\pi)F_E(R(V))}{\beta} \right\} & \text{if } V^H(V) \in [V_E, \tilde{V}) \end{cases} \quad (18)$$

Figure 3 summarizes the dynamics of a firm implied by the evolution of equity for a particular realization of shocks.

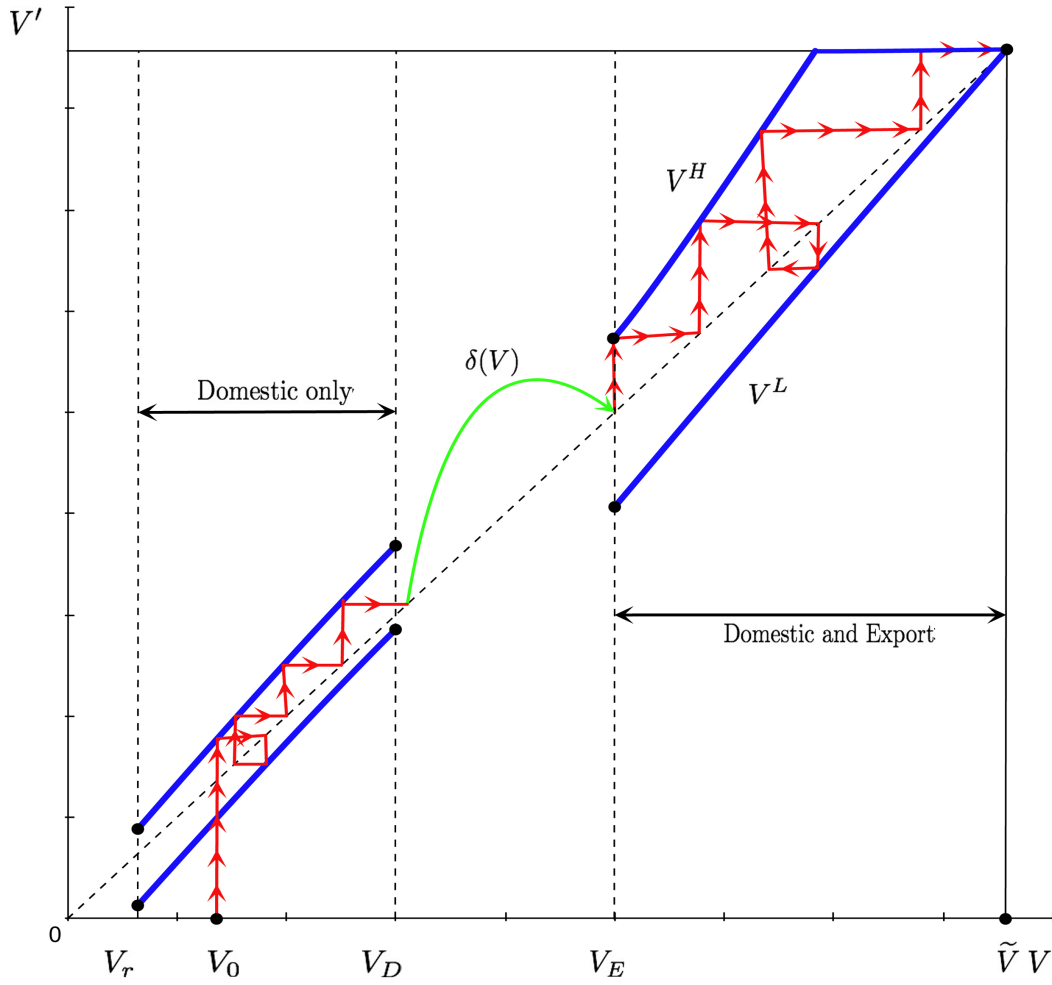


Figure 3: Firm dynamics

It is optimal for the financial intermediary to set the entrepreneur's repayments to $\tau(V) = F_i(R(V))$ for $i = \{D, E\}$ whenever $V^H(V) \leq \tilde{V}$ as it allows for the fastest accumulation of equity toward the unconstrained level. Furthermore, we assume the optimization problem takes place on the convex set $[0, \tilde{V}]$, which implies $V^H(V) = \tilde{V}$ whenever $(V + (1 - \pi)F_i(R(V)))/\beta > \tilde{V}$. Constraints (10) and (11) imply

$$\tau(V) = \begin{cases} F_i(R(V)) & \text{if } V^H(V) < \tilde{V} \\ \beta(\tilde{V} - V^L(V)) & \text{if } V^H(V) = \tilde{V} \end{cases} \quad (19)$$

so that conditional on a good shock resources advancement and repayments increase with the firms equity up until $V^H(V) = \tilde{V}$. Then repayments start declining until they eventually reach 0 when the firm becomes financially unconstrained. At this size, the firm's equity no longer changes, and the borrowing constraint ceases forever. The value of the firm is then

$$W(\tilde{V}) = \tilde{V} + B(\tilde{V}) = \frac{\pi F_E(\tilde{R})}{1 - \beta} - \frac{\tilde{R}(1 + r)}{1 - \beta}. \quad (20)$$

Thus, a firm becomes unconstrained when the financial intermediary has accumulated enough capital via the entrepreneur's repayment to finance the firm operation at full scale in every period and under all contingency.

Proposition 4.1 *The value function $W(V)$ is increasing and concave. There exist values $0 < V_r < V_D < V_E < \tilde{V}$ such that:*

1. *The firm gets liquidated with probability $\alpha(V) = (V_r - V)/V_r$ if $V \in [0, V_r]$;*
2. *The firm exports with probability one when $V \in [V_E, \tilde{V}]$, with probability $\delta(V) = (V_E - V)/(V_E - V_D)$ when $V \in (V_D, V_E)$, and else with probability zero;*

3. $W(V)$ is linear when $V \in [0, V_r) \cup (V_D, V_E)$, equal to $\widetilde{W}$ when $V = \widetilde{V}$ and strictly increasing when $V \in [V_r, V_D] \cup [V_E, \widetilde{V})$.

Proof In appendix.

Proposition 4.2 *Conditional on surviving, a firm grows on average. That is $\{V'\}_{t \geq 0}$ is a sub-martingale so that $E(V'|V) \geq V$.*

Proof In appendix.

5 Stationary equilibrium

All allocations and distributions are time-invariant in a stationary equilibrium. In what follows, we focus on the domestic economy, but analogous conditions must also hold in the foreign economy. The stochastic death assumption on workers implies a stationary distribution of workers since each period $(1 - \gamma_w)$ survive and γ_w are replaced by new ones, so aggregate deposits and the aggregate labor supply are constant. Setting the mass of workers to one, let d_j and h_j be the deposits and hours worked of a j -years old worker. In every period t , γ_w new workers are born with zero wealth and therefore contribute $\gamma_w d_0 = 0$ to aggregate deposits, and j -years old workers contribute $\gamma_w(1 - \gamma_w)^j d_j$ to aggregate deposits. It follows that aggregate deposits by workers each period are given by

$$D = \gamma_w \sum_{j=1}^{\infty} (1 - \gamma_w)^j d_j. \quad (21)$$

Similarly, the aggregate labor supply in each period is

$$H = \gamma_w \sum_{j=0}^{\infty} (1 - \gamma_w)^j h_j. \quad (22)$$

Let $\mathbf{M}$ be the state space for surviving firms so that $V \in \mathbf{M}$. Let $\mathcal{M}(V)$ be the Borel σ -algebra generated by $\mathbf{M}$, and μ the measure defined over $\mathcal{M}$.

Proposition 5.1 *There exists a unique stationary distribution of firms that is ergodic.*

Proof *In appendix* ■

The labor market clears when labor supply from workers equals the demand for labor by firms in the economy, so that

$$H = \int n d\mu. \quad (23)$$

Funds used to pay for labor cost, the various fixed costs, and to rent working capital every period are raised before production takes place. Recall that the intermediary receives deposits from workers who buy one-period annuity contracts, and payments from entrepreneurs in its portfolio every period. The net payment from each entrepreneur may be positive or negative depending on the size of the firm and the revenue shock realization each period. For instance, we showed that it is optimal for smaller firms to make repayments equal to their output, while larger firms that have accumulated enough equity no longer need to make any repayments.

It follows that total net payments from all the entrepreneurs in the intermediary's portfolio depend on the distribution of firms in the intermediary's portfolio, and can either be positive, negative or zero. Positive total net payments imply that the intermediary can use both workers and entrepreneurs deposits to fund projects while promising a rate of return r . Alternatively, negative total net deposits imply that the intermediaries have to raise more funds via workers' deposits to finance the firms in

their portfolio. In what follows, we refer to the difference between total net payments from entrepreneurs and total resources advanced as bank capital Z .

Therefore, the capital market clears when total resource advancements are equal to workers' deposits D plus banking capital Z ,

$$D + Z = \int R d\mu + \Gamma I_0. \quad (24)$$

Γ_b is the measure of liquidated (b for bankrupt) firms so that $\Gamma = \Gamma_b + \gamma_e$ is the measure of new born firms. The intermediary's budget must be balanced every period: intermediaries' receipts from the entrepreneurs' repayments plus the scrap value of liquidated firms and the returns from their own equity are large enough to finance the cost of borrowing funds on the capital market plus banking capital next period, Z' :

$$\pi \int \tau d\mu + \Gamma_b S + Z(1+r) = (1+r) \int R d\mu + (1+r)\Gamma I_0 + Z', \quad (25)$$

A stationary distribution of firms implies that $Z = Z'$ in equilibrium. Financial intermediaries break even on new contracts, i.e. $B_0(V_0) = I_0(1+r)$, which means that the initial loan size of a firm is equal to the discounted set-up cost.

The markets for domestically sold and exported intermediary goods must clear, so that demand from the final good producer must be equal to the supply of intermediate goods. That is,

$$y(\omega) = q(\omega) \quad \forall \omega \in \Omega, \quad (26)$$

and

$$y(\omega_f) = q(\omega_f) \quad \forall \omega_f \in \Omega_f. \quad (27)$$

where Ω and Ω_f are the set of all domestically produced and imported good respectively available in an economy to be converted into final good. Similar conditions must also hold abroad and thus

$$y(\omega^*) = q(\omega^*) \quad \forall \omega^* \in \Omega^* , \quad (28)$$

where Ω^* is the set of exported good. In a stationary equilibrium, trade has to balance in order to satisfy agents' maximization problems:

$$\int_{\Omega_f} y(\omega_f) p(\omega_f) d\omega_f = X \int_{\Omega^*} y(\omega^*) p(\omega^*) d\omega^* , \quad (29)$$

where X is the exchange rate. The left and right-hand side are the aggregate value of imports and exports, respectively. The exchange rate is one when countries are symmetric. It can be viewed as the relative price of the final good between the two countries. No condition concerning arbitrage between the home and foreign final good is necessary because the final good cannot be traded.

By Walras's law, if the above equilibrium conditions hold, the final goods market also clears so that total production equals aggregate consumption of workers and entrepreneurs plus capital expenditures. That is,

$$Y = C_w + C_e + K, \quad (30)$$

where

$$K = \int k d\mu + \Gamma_e I_E + \Gamma I_0 - \Gamma_b S , \quad (31)$$

which is the total capital expenditure – total investment in capital plus export fixed costs plus initial sunk investment in new projects minus salvaged capital from liqui-

dated firms – and Γ_e denotes the measure of firms that export. Aggregate consumption by workers C_w is defined by

$$C_w = \gamma_w \sum_{j=0}^{\infty} (1 - \gamma_w)^j c_{wj} , \quad (32)$$

and aggregate consumption by entrepreneurs is given by

$$C_e = \pi \int F(R) d\mu - \pi \int \tau d\mu , \quad (33)$$

which is equal to total revenues accruing to entrepreneurs minus repayments made to the financial intermediary.

To show this condition holds, we start from the zero profit condition for final goods producers and invoke the market clearing condition for intermediate goods (Equation (26) and equation (27)):

$$Y = \int p y d\omega + X \int p_f y_f d\omega_f = \int p q d\omega + X \int p_f q_f d\omega_f \quad (34)$$

where we omit the argument ω in $p(\omega)$ for notational simplicity. Given the balanced trade condition equation (29), the market clearing for exported goods equation (28) and the definition of revenues, it follows that

$$Y = \int p q d\omega + X \int p_f q_f d\omega_f = \int p q d\omega + \int p^* q^* d\omega^* = \pi \int F(R) d\mu . \quad (35)$$

Using the definition of entrepreneurial consumption equation (33) and the intermediaries' balanced budget constraint equation (25) yields

$$Y = \pi \int F(R) d\mu = C_e + \pi \int \tau d\mu = C_e + (1+r) \int R d\mu + (1+r) \Gamma I_0 - r Z - \Gamma_b S . \quad (36)$$

The clearing of the capital market, equation (24), allows to substitute for rZ :

$$Y = C_e + (1+r) \int R d\mu + (1+r) \Gamma I_0 - rZ - \Gamma_b S = C_e + \int R d\mu + \Gamma I_0 - \Gamma_b S + rD . \quad (37)$$

Plugging in for the use of resource advancements yields

$$Y = C_e + \int R d\mu + \Gamma I_0 - \Gamma_b S + rD = C_e + \int k d\mu + \Gamma_E I_E + \Gamma I_0 - \Gamma_b S + \int nwd\mu + rD . \quad (38)$$

The definition of total capital expenditures allows us to rewrite this as

$$Y = C_e + \int k d\mu + \Gamma_E I_E + \Gamma I_0 - \Gamma_b S + \int nwd\mu + rD = C_e + K + \int nwd\mu + Dr . \quad (39)$$

Finally, the labor market clearing condition equation (23), and the aggregate budget constraint for workers $C_w + D = wH + D(1+r)$, complete the proof:

$$Y = C_e + K + \int nwd\mu + Dr = C_e + K + wH + Dr = C_e + C_w + K . \quad (40)$$

The definition of the stationary equilibrium follows:

Definition 5.2 (Stationary Equilibrium) *A stationary equilibrium consists of*

1. *decision rules for labor supply h , consumption c_w , and deposits d' for workers in each country;*
2. *a contract policy in each country, consisting of: promised values $V^H(V)$ and $V^L(V)$, period resource advancements $R(V)$, liquidation lottery $\alpha(V)$, export lottery $\delta(V)$, and repayments $\tau(V)$;*
3. *an initial contract state V_0 in each country;*

4. wages $\{w, w^*\}$ and interest rates $\{r, r^*\}$;
5. prices $\{p(\omega), p_f(\omega)\}$ and $\{p(\omega^*), p_f(\omega^*)\}$ for intermediate goods;
6. and an exchange rate X ,

such that

1. the labor and consumption function maximize the workers' value function $U(d)$ in each country;
2. the contract policy maximize the value of the firm $W(V)$ in each country;
3. the initial state V_0 is such that the intermediary in each country breaks even on a new contract – i.e. $V_0 = \sup\{V : W(V) - V = I_0\}$;
4. the intermediary's budget in each country is balanced every period (equation (25));
5. the labor (equation (23)) and capital (equation (24)) markets clear in each country;
6. the domestic (equation (26)) and imported (equation (27)) intermediary goods market clear in each country;
7. trade between the two countries is balanced (equation (29)).

Proposition 5.3 *There exists a stationary equilibrium.*

Proof *In appendix* ■

6 Numerical analysis

In order to complete the analysis of the model, we parametrize the instantaneous utility function for workers, the production function for firms, the stochastic process for the idiosyncratic revenue shocks, the various fixed costs, and solve numerically for the loan size $R(V)$. While the model can be calibrated to answer a specific question, the following parametrization serves to illustrate that the model is able to account for, at least qualitatively, the empirical regularities discussed in the introduction. In this example, we simplify the analysis by considering the case of two identical countries for which trade costs are low enough to engage in bilateral trade. Throughout the section, firm size refers to firm equity V .

6.1 Parametrization

Let the instantaneous utility function for the workers be:¹⁰

$$u(c, h) = \ln(c) + \lambda \ln(1 - h) , \quad (41)$$

and let the production function be:

$$G(k, n) = k^{\eta_k} n^{\eta_n} . \quad (42)$$

Given the above parametrization, it remains to assign values on workers' intertemporal discount rate β , the elasticity of leisure λ , the probability of death for workers γ_w , the probability of high revenue shocks π , the production parameters η_l

¹⁰Smith and Wang [2006] use this functional form and show it implies a close-form solutions for the aggregate supply of labor and aggregate deposits given the workers' demographic assumption. This simplifies the numerical implementation and reduces the computational burden.

and η_k , the exogenous exit rate for firms γ_e , the salvage value S , the setup investment I_0 , the fixed export cost I_E , the iceberg cost I_T , and the elasticity of substitution between intermediate goods.

A period in this model is 1 year. The death rate γ_w for workers is set to 0.02, which implies an average working life of 50 years. Workers' discount rate β is set to 0.9633, which implies an interest around 4% at steady state in the two economies. Last, the elasticity of leisure of leisure λ is set to 1.7, which implies aggregate hour worked of 1/3.

Table 1: Parameter values

Worker's discount rate	$\hat{\beta}$	0.9633
Elasticity of leisure	λ	1.7
Workers' death shock	γ_w	0.02
Production technology	η_l	0.33
	η_k	0.67
Salvage value	S	0.06
Setup investment	I_0	0.06
Iceberg cost	I_T	0.0001
Fixed export cost	I_E	0.009
Firm revenue shock	π	0.5
Exogenous' death shock	γ_e	0.06
Elasticity of substitution for intermediates	σ	1.5

The exogenous probability of firm exit is set to $\gamma_e = 0.06$ which is consistent with studies on industry dynamics such as [Lee and Mukoyama \[2008\]](#). We set the capital η_k and labor η_l share are set to 0.67 and 0.33 respectively which implies constant

return to scale technology, and σ to 1.5, which implies a relatively high monopoly power and large profit for firms producing intermediates good.¹¹ The salvage value S , setup investment I_0 , and probability of high revenue shock π affect the value of the contract and the distribution of firms in the economy, and can be calibrated to moments from studies on firm-level investment dynamics. Last, we set the fixed export cost I_E to 0.009 and the iceberg cost I_T to 0.0001 to make trade between the two economy profitable. Table 1 summarizes the parametrization of the model.

6.2 The age-size distribution of firms and the exporter cutoff

After solving the model, we simulate the life of 6,000 firms and compute the statistics discussed in the following two sub-sections. Note that while we use a firm's equity V as a measure of firm size, the results discussed below are robust to other definitions of firm size such as employment n , quantities produced q or $q + q^*$ or sales F , which are all highly positively correlated with V in this model. Figure 4 plots the age-size stationary distribution of firms using a 2-dimensional histogram with hexagonal bins and log firm counts.

Table 2 shows that, given this parametrization, new-born firms begin their life by selling their production in domestic markets only. New firms are about 20 percent smaller than the average domestic firm, and 4.5 times smaller than the average exporter. Exporters are significantly larger than domestic firms – about 4 times larger than the average domestic firm – and Figure 4 shows that a new firm must double its size in order to have access to a loan large enough to pay the fixed cost of exporting and ship some of its production overseas. Table 2 also shows that it takes on average 9 years, and no less than 3 years for a firm to begin exporting, and that

¹¹All the qualitative results discussed in this section hold for $1 < \sigma < \infty$.

Table 2: Firm characteristics in steady state

Initial firm size	0.047	
Unconstrained firm size	0.319	
Share of unconstrained firms	0.241	
Share of exporting firms	0.523	
	Mean	Std. dev.
Firm size	0.151	0.112
non-exporting firms	0.057	0.022
exporting firms	0.236	0.092
Firm age	13.94	13.55
non-exporting firms	6.470	5.905
exporting firms	20.74	13.55
Age of new exporters	9.190	6.058

exporters are on average 3 times older than domestic firms. Last, about 50 percent of the firms are exporters, while the other sell their goods to the domestic market only.

It follows that one of the key differences between this model and other models based on Melitz [2003] is that a cutoff level of exporters endogenously arises not because of an ex-ante productivity differential, but because of different financial conditions faced by firms of different size. Furthermore, while the aggregate quantities are fixed in steady state, the evolution of individual firms in the economy every period is not. For instance, in every period, new firms are born and older ones die, a fraction of the surviving firms expand while the others shrink, and thus some firms enter the export market while others exit it even if the fraction of exporters remains constant. In other words, financial frictions have an important effect on the intensive

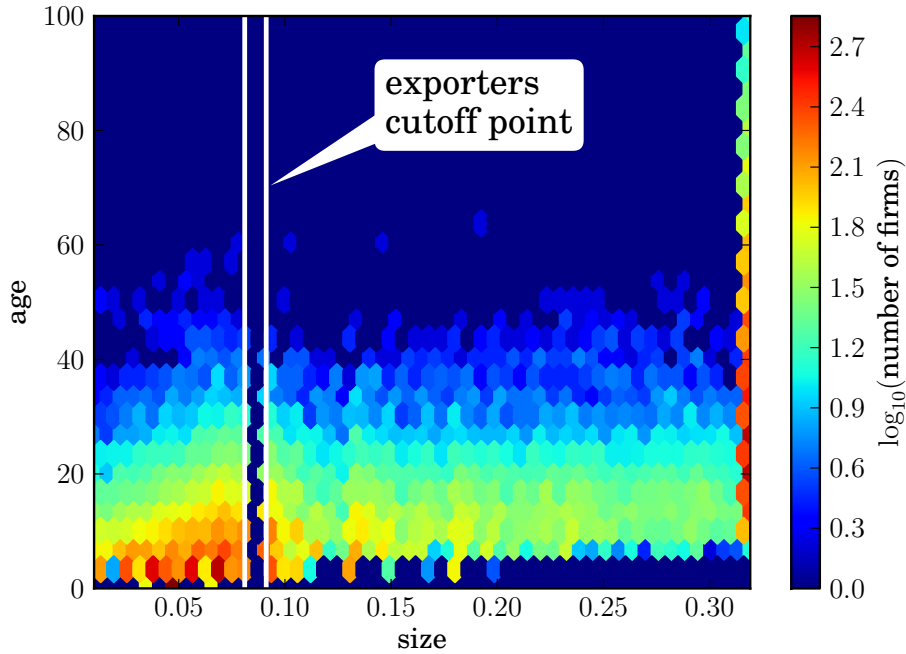


Figure 4: Age-size density of firms

as well as the extensive margin of trade. It is to the characterization of the steady state firm dynamics that we turn next.

6.3 Firm dynamics in general equilibrium

Figure 5 plots the life-cycle of three firms from our simulated sample. Since there is no ex-ante technological differences, all three firms start at the same size V_0 . Firm 1, in blue, never succeeded entering the export market. Firm 2, in red, received a long enough sequence of good shocks to enter the export market, and could accumulate enough equity to reach $\tilde{V}$ to self-finance its operation under all contingencies. Firm 3, in green, grew enough to begin exporting, but exited the export market soon after entering it. Figure 5 shows that even though revenue shocks are identically

and independently distributed, the effect of an individual shock on firms' equity is long lasting. The asymmetric response of equity to revenue shocks for firms of different size, together with the fact that the endogenous process for firms' equity is a sub-martingale, yields several key implications.

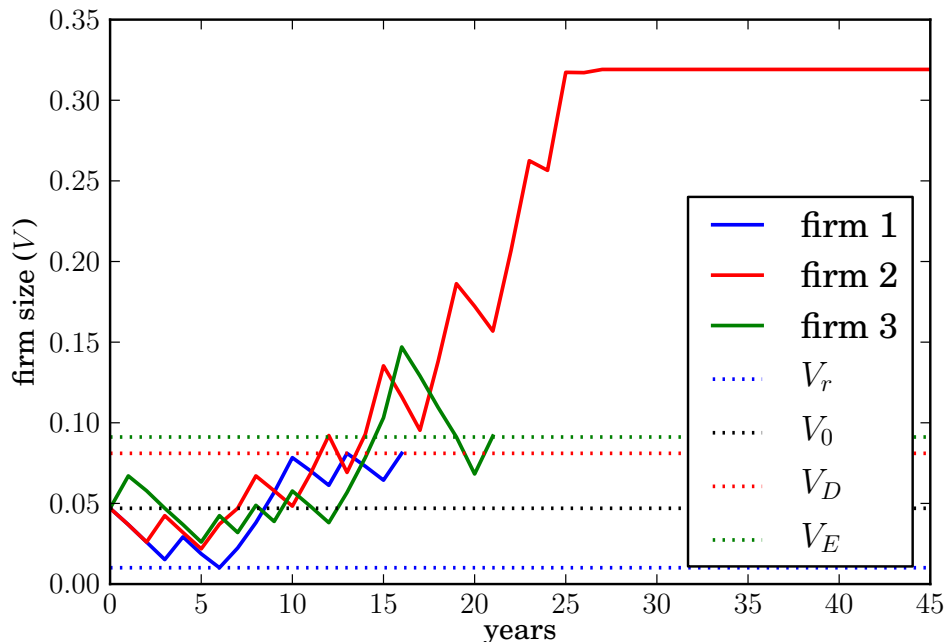


Figure 5: Life-cycle of three firms

First, the hazard rate of liquidation decreases with firm age since firms grow on average, and thus require a longer sequence of low shocks to be brought down to the liquidation region. Table 3 shows that about 0.8 percent of firms are liquidated every period, and Figure 6 shows that young firms face a high hazard rate of liquidation that is decreasing in age.

Second, new exporters re-enter the export lottery if they receive a low shock during their first year exporting. Table 3 shows that about 28 percent of domestic

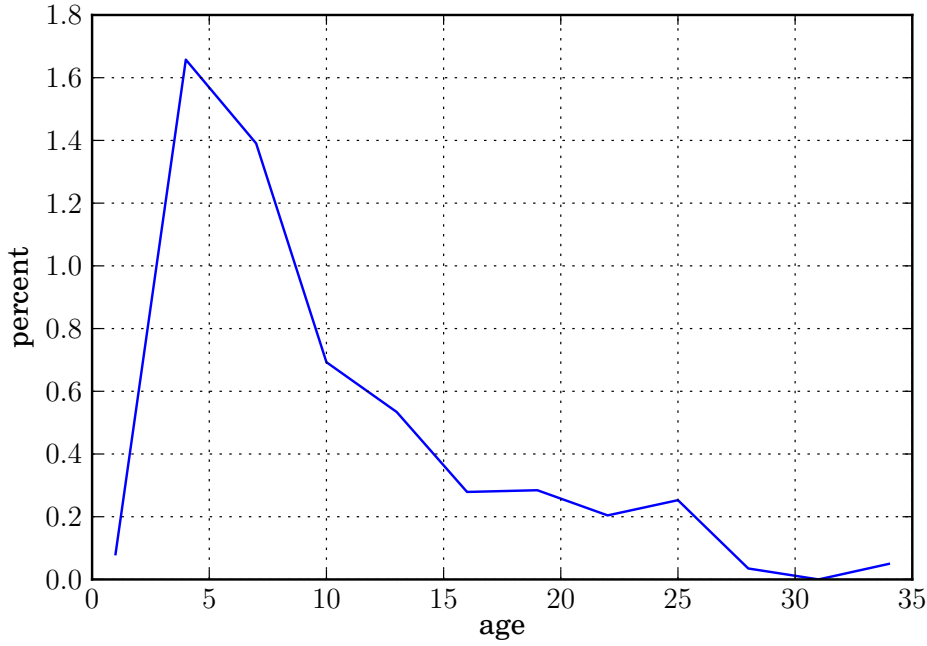


Figure 6: Hazard rate of liquidation by age

firms enter the export market every year, but only 69 percent of them continue exporting after the first year. This effect is consistently found in data, but is typically difficult to account for with current trade models. Furthermore, Figure 7 shows that since constrained firms grow on average, the probability of exit declines the longer a firm has been exporting.

Third, exporting firms are more productive than non-exporters even before they export. Inspection of Figure 5 reveals that Firm 1 (blue) and Firm 2 (red) started at the same size, and are both alive after 15 years of operation. However, Firm 2 is exporting while Firm 1 is not, which implies that Firm 2 has performed better than Firm 1 in its early life to raise its equity to a level where exporting becomes possible – i.e., Firm 1 was on average less productive than Firm 2.

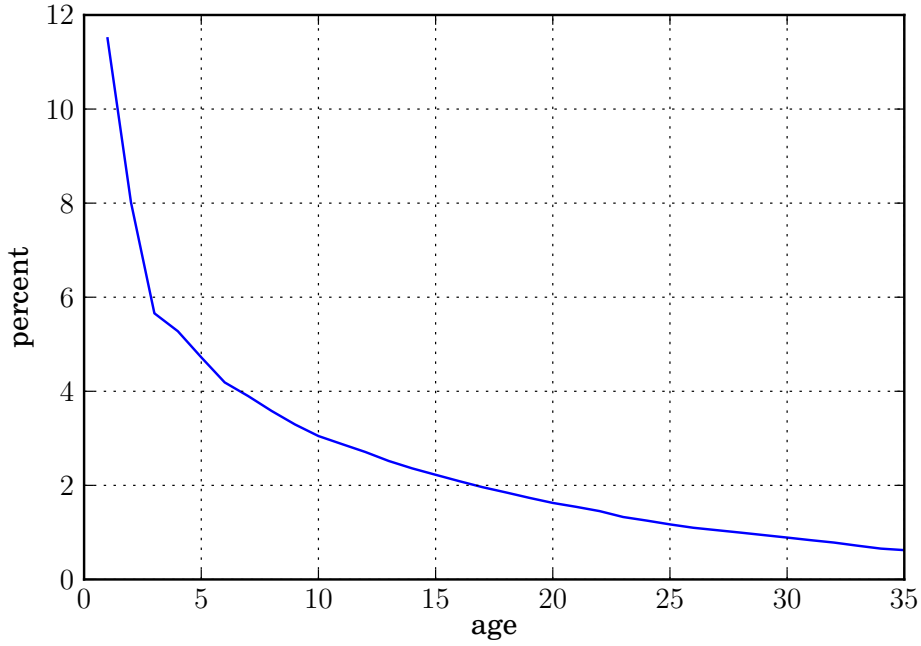


Figure 7: Distribution of exporters by age

Table 3: Firm entry and exit

Entry/exit rate	0.068
Entry rate in export market	0.141
Exit rate from export market after 1 year	0.308

Fourth, exporters grow faster after entering the export market. Note that new exporters are more likely to have received a high shock if they remain in the export market, which implies growth. Furthermore, for $V \geq V_E$ close to the linear part of $W(V)$, the cost of decreasing next period's equity after a low shock V^L is lower compared to the strictly concave region, so the working capital in that region can

be higher than elsewhere – and, similarly for $V \leq V_D$ and $V > V_r$ close to the linear region. For 5 periods after entering an export market, we find that new exporters have grown faster on average than other firms.

Last, new exporters increase their volume of exports gradually over time. Figure 8 plots the domestic and export production of intermediates as a percentage of the unconstrained firm level by firm size. New exporters are not financially unconstrained, and sell their products at home and abroad such that marginal revenues equal marginal cost. Note however, that the ratio of goods exported to goods sold domestically is constant since the marginal extra cost of exporting I_T is constant. This last observation and the property that firms grow on average imply that older exporters have access to more working capital, allowing them to expand their production, and thereby to sell more in both markets.

6.4 Firms' investment dynamics

Central to this model are financial frictions that limit firms' access to credit, and in turn their decision to produce and ship some of their production overseas. In this section, we discuss the extent to which the model predicts investment dynamics that are consistent with the empirical regularities on exporting and non-exporting firms.

Figure 9 plots the decision rule for loan size as a function of firm equity. Recall that $E(V'|V) \geq V$ so that younger firms are smaller on average, and vice versa. We showed above that it is optimal for financially constrained firms with $V^H < \tilde{V}$ to delay all dividend payments, as this leads to the fastest accumulation of equity toward the unconstrained level. We also showed that only older and larger firms with $V^H = \tilde{V}$ pay dividends. Furthermore, young and small firms take on more debt since the initial debt is $B_0 = I_0(1+r)$, and the debt level decreases on average to the

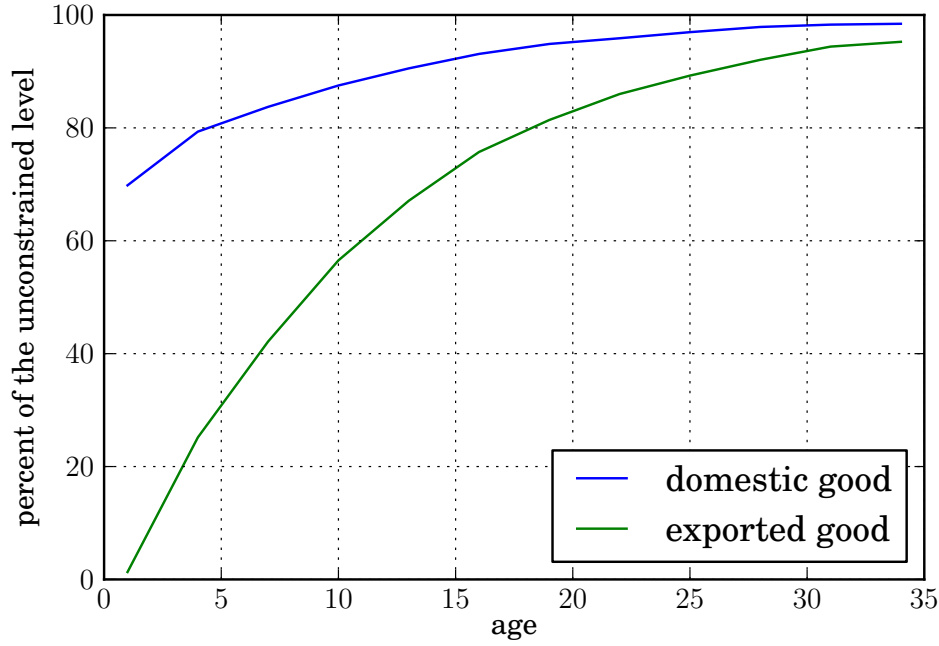


Figure 8: Domestic and export production of intermediates as a percentage of the unconstrained firm level

lower bound $\tilde{B} = -\tilde{R}(1+r)/(1-\beta)$ with a sufficiently long sequence of high shocks – and conditional on remaining in operation. Figure 10 summarizes the above by plotting debt, repayments, and dividends schedule as a function of equity.

Define investment as R_t/R_{t-1} , which is the change in loan size from one period to the next.¹² Figure 11 plots the log-change in loan size conditional on receiving a high and low shock as a function of equity. Consistently with data, smaller non-exporting firms' investment are most sensitive to cash flows, while the investment of large unconstrained firms does not depend on cash-flow at all. For constrained firms, investment is always positive after receiving a high shock, and always negative

¹²One could alternatively use capital expenditure k , which is a constant fraction of total resources R .

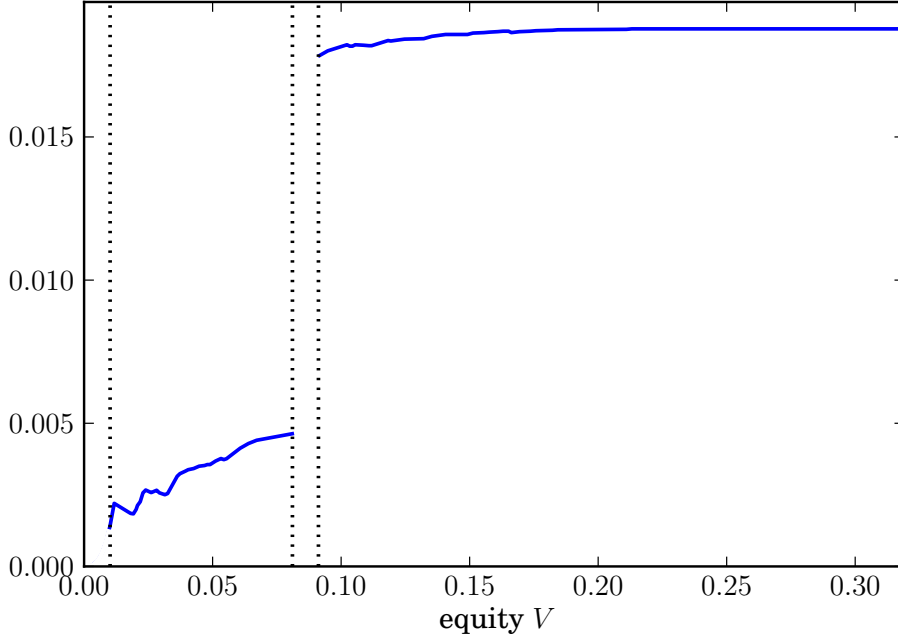


Figure 9: Loan size as a function of equity

after a low shock. There is a large increase in investment once the firm becomes an exporter, with subsequent very high possible disinvestment should the firm receive a low shock and exit the export markets.

These results also hold along the age dimension. Figures 12 and 13 plot the mean and standard deviation of firms' investment conditional on age, and shows that younger firms invest more, and their investment is more volatile. For instance, 25 years old firms invest on average 5 times less than 3 years old firms or younger, and their investment is about 50 percent less volatile.

Figures 14 and 15 plot the mean and standard deviation, respectively, of domestic firms' investment conditional on age. Figure 16 and 17 are analog pictures for exporters. These pictures illustrate the property that exporters are always larger

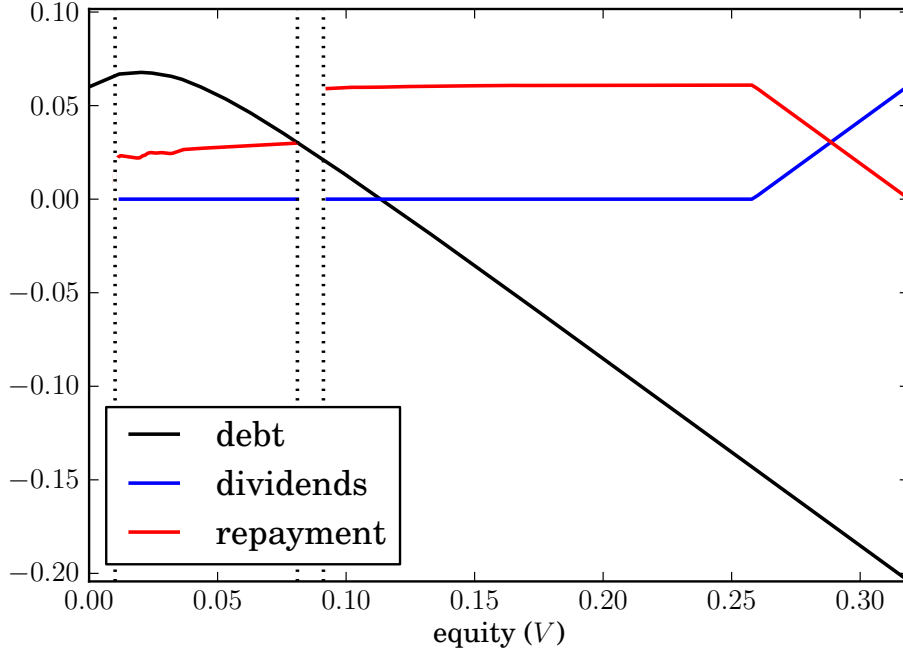


Figure 10: Firm's debt, dividend and repayment as a function of its equity

and less leveraged than domestic firms, have greater access to financing, and their investment is less sensitive to cash flows.

It follows that our model is able to account for all the empirical regularities discussed in the introduction except that exporters are more productive, pay higher wages, and are relatively more capital- and skill-intensive. Nevertheless, we show in the Appendix that the model can be extended with a production function that exhibits capital-skill complementarity. With this assumption, exporters choose to hire relatively more skilled labor and capital, than domestic firms. When wages are set competitively, high-skilled workers are paid a higher wage than low-skilled workers, which implies that the average wage paid by exporters is higher. Lastly, if the production function is allowed to have increasing returns to scale (which is

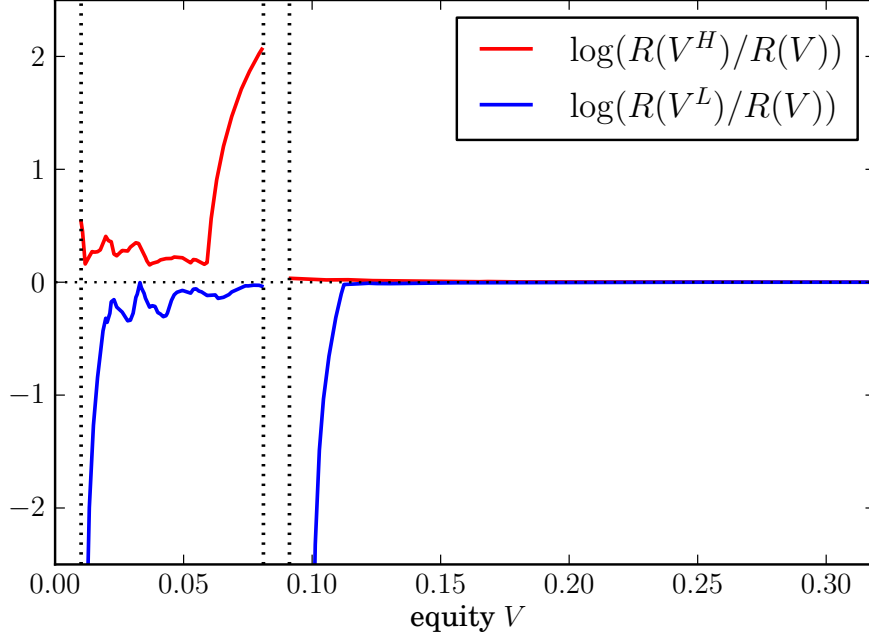


Figure 11: Firm investment conditional on revenue shock realization

possible in this model), exporters become more productive than non-exporters, and that without an ex-ante differential in productive technology.

7 Concluding comments

We proposed a model of international trade with monopolistic competition, in which financial frictions lead to firm heterogeneity. The key difference between this model and other trade models in the spirit of Melitz [2003] is that financing constraints arise endogenously because firms have uncertainty about their future revenue realization in each period, and the realization of one period revenue shocks is private information. When a financial intermediary is able to offer and commit to a long-term optimal

financial contract, firm heterogeneity arises because the history of revenue shocks the firms receive, and the entrepreneurs' reporting decision determine the size of the loans and the scale of the firm's operation. In turn, this mechanism determines the growth path for firms and which firms participate in international trade at any point in time. There are no inherent productivity differences and all firms have the same expected productivity at any point in time, irrespective of their history, size, and export status.

We showed that in its simplest form, the model can account for most of the key empirical regularities documented in the industrial organization and the international trade literature, and that with a simple extension of the production function the model can account for all of them. We argued that while this model must be solved numerically, its qualitative predictions are robust to a wide range of parameter values and different production technologies. Current work in progress focuses on calibrating the model to the US economy and study quantitatively the welfare effect of trade liberalization.

The dynamic general equilibrium nature implies that any change in the economic environment has an impact on the optimal financial contract between firms and financial intermediaries, and thus the growth path of all firms, thereby affecting employment, trade and welfare. For example, one could use the model to study the equilibrium impact of a labor reform such as the imposition of a minimum wage, a banking reform such as the imposition of a capital reserve requirement for lending institutions, or the imposition of trade barriers such as tariffs or quotas. One could then use the method proposed in [Verani \[2011\]](#) to compute the transitional dynamics between the equilibria associated with each policy change.

Our longer term research agenda includes an aggregate shocks version of this model to study firm dynamics, and the pattern of trade over the business cycle. We

are particularly interested in the implications of financing frictions on the propagation of aggregate shocks from one country to its trading partners. Furthermore, a version of the model with open capital markets would allow to study how investment shifts across countries following a productivity shock, and compare the effects of exchange rate perturbations to longer-term reductions in trade costs.

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A Demand for Intermediates goods

The first-order condition for the maximization of equation (9) with respect to variety $\hat{\omega} \in \Omega$ yields

$$y(\hat{\omega})^{-1/\sigma} Y^{1/\sigma} = p(\hat{\omega}). \quad (43)$$

Dividing by the equivalent expression of another variety $\omega \in \Omega$ leads to

$$\frac{y(\hat{\omega})}{y(\omega)} = \left(\frac{p(\hat{\omega})}{p(\omega)} \right)^{-\sigma} \quad (44)$$

and similarly for imported goods to

$$\frac{y(\hat{\omega}_f)}{y(\omega)} = \left(\frac{Xp(\hat{\omega}_f)}{p(\omega)} \right)^{-\sigma}. \quad (45)$$

Multiplying by $p(\hat{\omega})$ (or $Xp(\hat{\omega}_f)$ for imported goods) and $y(\omega)$ and summing over all varieties $\hat{\omega} \in \Omega + \Omega_f$, we obtain

$$\begin{aligned} & \int_{\Omega} p(\hat{\omega}) y(\hat{\omega}) d\hat{\omega} + X \int_{\Omega_f} p(\hat{\omega}_f) y(\hat{\omega}_f) d\hat{\omega}_f = \\ & y(\omega) p(\omega)^{\sigma} \left(\int_{\Omega} p(\hat{\omega})^{(1-\sigma)} d\hat{\omega} + X^{1-\sigma} \int_{\Omega_f} p(\hat{\omega}_f)^{(1-\sigma)} d\hat{\omega}_f \right). \end{aligned} \quad (46)$$

Since the final goods producer generates zero profits, the left-hand side is simply equal to total output Y :

$$Y = y(\omega) p(\omega)^{\sigma} \left(\int_{\Omega} p(\hat{\omega})^{(1-\sigma)} d\hat{\omega} + X^{1-\sigma} \int_{\Omega_f} p(\hat{\omega}_f)^{(1-\sigma)} d\hat{\omega}_f \right). \quad (47)$$

Let $P = \left(\int_{\Omega} p(\hat{\omega})^{1-\sigma} d\hat{\omega} \right)^{\frac{1}{1-\sigma}}$ be the price index of domestic intermediate goods, and P_f the equivalent price index of imported goods. Then we can define the inverse demand function for domestic intermediate inputs as

$$p(\omega) = y(\omega)^{-1/\sigma} Y^{1/\sigma} (P^{1-\sigma} + X^{1-\sigma} P_f^{1-\sigma})^{-1/\sigma}, \quad (48)$$

and equivalently $p(\omega_f)$. From analogue functions abroad, we can infer the inverse demand functions for exported goods $p(\omega^*)$:

$$p(\omega^*) = y(\omega^*)^{-1/\sigma} X Y^{1/\sigma} (P_f^{*1-\sigma} + X^{\sigma-1} P^{*1-\sigma})^{-1/\sigma}. \quad (49)$$

These inverse demand functions are used in the intermediate goods producers' maximization problems. Since each entrepreneur is atomistic, he cannot affect the demand functions and thus takes them as given. Note that Ω and Ω^* denote the set of firms producing and selling their goods, so that $y(\omega) > 0$ and $y(\omega^*) > 0$.

B Optimal revenue function for intermediate goods producers

Let us redefine the inverse demand functions as $p = q^{-1/\sigma} A$ and similarly $p^* = q^{*-1/\sigma} A^*$, where

$$A = Y^{1/\sigma} (P^{1-\sigma} + X^{1-\sigma} P_f^{1-\sigma})^{-1/\sigma} . \quad (50)$$

and

$$A^* = Y^{*1/\sigma} X (P_f^{*1-\sigma} + X^{\sigma-1} P^{*1-\sigma})^{-1/\sigma} . \quad (51)$$

Our analysis thus far has made no assumption on the form of the production function, and the results of this paper are robust to a wide choice of production technology. Consider now the case of a Cobb-Douglas production function such that $G(k, n) = k^{\eta_k} n^{\eta_n}$, which is what we use to solve the model numerically. The cost minimization then implies that the quantity sold by a domestic firm is

$$q_D = R^{\eta_k + \eta_n} (1 + \eta_n/\eta_k)^{-\eta_k} [w(1 + \eta_k/\eta_n)]^{-\eta_n} = R^\nu x. \quad (52)$$

where $\nu = \eta_k + \eta_n$ is the returns to scale parameter of the production function $G(k, n)$ and $x = (1 + \eta_n/\eta_k)^{-\eta_k} [w(1 + (1 + \eta_k/\eta_n))]^{-\eta_n}$ reflects the impact of wages and the shares of capital and labor on production. When a firm maximizes its cash-flow $F_i(R) - R(1 + r)$, the optimal amount of resources used by a non-exporting firm is then

$$\tilde{R}_D = \left[\pi \frac{A x^{(1-1/\sigma)} \nu (1 - 1/\sigma)}{1 + r} \right]^{\frac{1}{1-\nu(1-1/\sigma)}} , \quad (53)$$

From the underlying first-order condition one can deduce that $\nu(1 - 1/\sigma) < 1$ must hold, or otherwise there would not be a finite optimum. This means that if the production $G(k, n)$ exhibits increasing returns to scale, the reduction in marginal cost by expanding capacity must not outpace the reduction in marginal revenue. Analogously, we can derive the expressions for goods sold domestically and abroad

by an exporter

$$q_E = \frac{(R - I_E)^\nu x}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} = (R - I_E)^\nu B^{\sigma/(\sigma-1)}, \text{ and} \quad (54)$$

$$q_E^* = \frac{(R - I_E)^\nu x (A^*/A)^\sigma (1 + I_T)^{-1/\sigma}}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} = (R - I_E)^\nu B^{*\sigma/(\sigma-1)}. \quad (55)$$

where

$$B = \left(\frac{x}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} \right)^{1-1/\sigma}, \text{ and} \quad (56)$$

$$B^* = \left(\frac{x (A^*/A)^\sigma (1 + I_T)^{-1/\sigma}}{1 + (A^*/A)^\sigma (1 + I_T)^{1-\sigma}} \right)^{1-1/\sigma}. \quad (57)$$

The quantity sold domestically depends positively on the (endogenous) domestic demand parameter A and negatively on the foreign demand parameter A^* . The higher the transportation cost, the more goods an exporter sells at home. The reverse applies to goods sold abroad. A firm operating at full scale when it is profitable to trade requires period resources

$$\tilde{R}_E = \left[\pi \frac{(AB + A^*B^*)\nu(1 - 1/\sigma)}{1 + r} \right]^{\frac{1}{1-\nu(1-1/\sigma)}} + I_E. \quad (58)$$

In this case, the markup is constant, as is common in the literature with monopolistic competition. Finally, we can infer the amount of resources from where on it pays off to incur the fixed export cost, i.e. the R_{dx} such that $F_D(R) = F_E(R)$:

$$R_{dx} = \frac{I_E}{1 - \left(\frac{x^{(1-1/\sigma)} A}{AB + A^*B^*} \right)^{\frac{1}{\nu(1-1/\sigma)}}} = \frac{I_E}{1 - \phi}. \quad (59)$$

where

$$\phi = \left(\frac{x^{1-1/\sigma} A}{AB + A^*B^*} \right)^{\frac{1}{\nu(1-1/\sigma)}} \quad (60)$$

It follows that the necessary condition for exports to be profitable is $\phi > 1$. When $\phi \leq 1$ the fixed costs of exporting cannot be compensated by the access to a new market. For given other parameters, we can also interpret this as the implicit lower bound on returns to scale ν , so that exports are still profitable. That is, if there are

decreasing returns to scale of the production function $G(k, n)$, then the increase in marginal cost must not be too steep to prevent accessing the export market of being profitable.

C Labour Supply and Deposits

Following [Smith and Wang \[2006\]](#), we assume the instantaneous utility function

$$u(c_w, 1 - h) = \log(c_w) + \lambda \log(1 - h) \quad (61)$$

for workers, where $\lambda > 0$ is a parameter governing the relative weight of leisure as compared to consumption. The use of this specific utility function, along with allowing negative working hours, permits to find a closed-form solution for worker deposits and labor supply, which greatly facilitates computation. One could interpret the negative working hours (which will only arise for the very small fraction of the population that survives a very long time) as purchasing services from others, such as hiring a cook, personal secretaries etc.

The worker's value function can be determined by the guess-and-verify method:

$$U(d) = a_0 + a_1 \log(d + a_2) \quad (62)$$

The optimal choice functions are

$$d' = \frac{w[(1+r)\hat{\beta} - 1]}{(1+r) - (1-\gamma_w)} + (1+r)\hat{\beta}d \quad (63)$$

$$h = \frac{(1+r) + \lambda(1-\gamma_w)\hat{\beta}(1+r) - (1-\gamma_w)(1+\lambda)}{(1+\lambda)[(1+r) - (1-\gamma_w)]} - \frac{\lambda(1+r)(1-\hat{\beta}(1-\gamma_w))}{w(1+\lambda)}d. \quad (64)$$

As long as the interest rate plus principal is bigger than the inverse of the workers' discount factor, i.e. $(1+r)\hat{\beta} > 1$, workers will accumulate deposits as they grow older. Deposits depend positively on the interest rate, that is $\partial d' / \partial r > 0$, and on wages. Labor supply decreases linearly in deposits, which means that ultimately workers will accumulate enough deposits to have negative working hours. Labor supply depends positively on the interest rate for younger workers with few deposits and negatively when $d > w(1+r - (1-\gamma_w))^{-2}$. It always depends positively on wages. Knowing the optimal choice function, it is easy to infer deposits and hours

worked for a j -old worker and thus aggregate deposits and labor supply. These are

$$D = \frac{w(1 - \gamma_w)((1 + r)\hat{\beta} - 1)}{(1 - (1 - \gamma_w)(1 + r) * \hat{\beta})((1 + r) - (1 - \gamma_w))} \quad (65)$$

$$H = \frac{(1 + r) + \lambda(1 - \gamma_w)\hat{\beta}(1 + r) - (1 - \gamma_w)(1 + \lambda)}{(1 + \lambda)[(1 + r) - (1 - \gamma_w)]} \quad (66)$$

$$- \frac{\lambda(1 + r)(1 - \hat{\beta}(1 - \gamma_w))(1 - \gamma_w)((1 + r)\hat{\beta} - 1)}{(1 + \lambda)(1 - (1 - \gamma_w)(1 + r)\hat{\beta})((1 + r) - (1 - \gamma_w))}.$$

D Capital-Skill Complementarity

In order to account for differences in skill and capital intensities between exporters and non-exporters, we have to extend the model to include high and low- skilled workers. It also enables us to make predictions on which groups of society benefit or lose out due to trade liberalization (in progress). The empirical trade literature documents that exporters are more capital intensive, hire more high-skilled labor and pay higher wages than non-exporting firms. In order to account for this, we introduce a production function which features capital-skill complementarity which is increasing in the size of the firm. As before, the production function can have decreasing, constant or increasing returns to scale (within the limits outlined above).

Consider an environment with two types of workers, high and low-skilled, which are otherwise identical. Let m_h and m_l denote the mass of workers of each type. The agents' optimization problem stays exactly the same as before, except that the two worker types face different wages, w_h and w_l . A firm's production function could then be defined as $G(k, n_h, n_l)$, where n_h and n_l stand for the amount of labor of each typed hired by a firm:

$$G(k, n_h, n_l) = \left[(k^{\eta_k} n_h^{\eta_h})^{1/\rho} + \eta_l n_l^{1/\rho} \right]^\nu, \quad 1 < \eta_h + \eta_k < \rho. \quad (67)$$

Capital and high-skilled labor are complements in this function and imperfect substitutes for unskilled labor, since $\eta_h + \eta_k < \rho$. The assumption of $1 < \eta_h + \eta_k$ implies that larger firms will employ relatively more capital and high-skilled labor. The parameter η_l determines the relative productivity of unskilled compared to skilled labor, while returns to scale are governed by ν . As before, the efficient allocation of resources R (net of the potential fixed exporting cost) to the factors of production

solves

$$\begin{aligned} \max_{k, n_h, n_l} \quad & \left[(k^{\eta_k} n_h^{\eta_h})^{1/\rho} + \eta_l n_l^{1/\rho} \right]^\nu \\ \text{s.t.} \quad & w_l n_l + w_h n_h + k \leq R. \end{aligned} \quad (68)$$

Factor demands are then (implicitly) given by

$$w_h n_h (1 + \eta_k / \eta_h) + w_l \zeta n_h^{(\eta_k + \eta_h - \rho)/(1-\rho)} = R \quad (69)$$

$$k = n_h \frac{\eta_k w_h}{\eta_h} \quad (70)$$

$$n_l = \zeta n_h^{(\eta_k + \eta_h - \rho)/(1-\rho)} \quad (71)$$

where

$$\zeta = \left[\left(\frac{\eta_h w_l}{\eta_l w_h} \right)^\rho \left(\frac{\eta_k w_h}{\eta_h} \right)^{\eta_k} \right]^{1/(1-\rho)}. \quad (72)$$

Taking total derivatives of the demand for skilled labor (69) yields

$$\frac{dn_h}{dR} = \left(w_h + w_h \frac{\eta_k}{\eta_h} + \frac{\eta_k + \eta_h - \rho}{1 - \rho} w_l \zeta n_h^{\frac{\eta_k + \eta_h - 1}{1-\rho}} \right)^{-1} > 0. \quad (73)$$

We can now determine how the ratio of unskilled to skilled labor changes with the amount of resources R , which is generally tied to firm size:

$$\frac{d(n_l/n_h)}{dR} = \zeta \frac{\eta_k + \eta_h - 1}{1 - \rho} n_h^{\frac{\eta_k + \eta_h - 1}{1-\rho} - 1} \frac{dn_h}{dR} < 0. \quad (74)$$

As pointed out before, the inequality holds because $1 < \eta_h + \eta_k$. The ratio of unskilled to skilled labor thus decreases as firm size increases. Under the innocuous assumption that high-skilled workers earn higher wages, larger firms hence also pay higher wages on average. The ratio of skilled labor to capital stays constant, therefore larger firms have a higher capital to labor ratio.

The derivations in the section before also hold for the case of capital-skill complementarity, except that the value of x , which reflects the impact of wages and the shares of capital and labour on production, is different and itself a function of working capital R . For example, the higher R , the more important are the wages of the high-skilled. The parameter ν that governs returns to scale of the production function $G(\cdot)$ is also not identical. While derivations are different (and do not have a closed-form solution), the same logic as before applies: production for a given amount of resources R is maximized, and then goods are optimally allocated between the domestic and the foreign market in the case of an exporter.

E Other proofs

Proposition E.1 *There exists a point V_{dx} such that $\widehat{W}_D(V) < \widehat{W}_E(V)$ for all $V \in [0, V_{dx})$, and $\widehat{W}_D(V) > \widehat{W}_E(V)$ for all $V \in (V_{dx}, \tilde{V}]$.*

Proof It is optimal to reach the unconstrained value $\tilde{V}$ in the shortest time possible because the joint surplus is maximized there and both the entrepreneur and the financial intermediary are risk-neutral and share the same discount factor. Hence, repayments should be set equal to revenues $\tau = \pi F_i(R)$ as long as $V^H < \tilde{V}$. This follows the argument set forth in [Clementi and Hopenhayn \[2006\]](#). Let us thus rewrite the value of a firm with a given export status $i \in \{D, E\}$ as

$$\begin{aligned} \widehat{W}_i(V) = & \max_{R, V^H, V^L} \{ \pi F_i(R) - R(1+r) + \beta[\pi W(V^H) + (1-\pi)W(V^L)] \} \\ \text{s.t. } & V = \beta(\pi V^H + (1-\pi)V^L) \\ & F_i(R) \leq \beta(V^H - V^L) \\ & V^H, V^L \geq 0 \\ & R_E \geq I_X. \end{aligned} \tag{75}$$

Since $F_i(R)$ is a strictly increasing, strictly concave function, then the expected cash flows $\pi F_i(R) - R(1+r)$ for $R_i \leq \tilde{R}_i$ are too. Let $\widehat{W}_D(V) = \widehat{W}_D(\tilde{V}) \forall V > \tilde{V}$. Therefore, the function $\widehat{W}_i(V)$ for $V \in [0, \tilde{V}]$ inherits the same properties, where $\tilde{V}_i$ is the equity of an unconstrained firm of type i . We have to show that the function $\widehat{W}_E(V)$ is lower than $\widehat{W}_D(V)$ for all $V < V_{dx}$ and vice versa – i.e. that there is a unique crossing point V_{dx} . It is easy to see that the unconstrained value of an exporting firm is higher than that of a purely domestic firm, $\max \widehat{W}_E(V) > \max \widehat{W}_D(V)$, since $\max\{F_E(R) - R(1+r)\} > \max\{F_D(R) - R(1+r)\}$.

When the equity of a domestic firm goes to zero, the value of it goes to βS : the first constraint, together with the fact that continuation values have to be non-negative, forces continuation values to go to zero, as equity V approaches zero. It follows that the spread between V^H and V^L goes to zero, so $F_i(R)$ has to go to zero to maintain incentive compatibility. Therefore, the optimal resource advancement of a domestic firm will approach zero and thus its value $\widehat{W}_D(V)$ will go to the discounted scrap value. In the case of an exporting firm, the logic is very similar, except that the resource advancement approaches the fixed cost of exporting I_X , which cannot be seized by the entrepreneur. The value of an exporting firm with equity zero will hence be the discounted scrap value minus the cost of paying the export cost, $\widehat{W}_E(0) = -I_X + \beta S$. As both firm value functions are increasing and concave, and strictly so for $\widehat{W}_E(V), V < \tilde{V}$, the fact that $\widehat{W}_E(0) < \widehat{W}_D(0)$ and $\widehat{W}_E(\tilde{V}_E) > \widehat{W}_D(\tilde{V}_D)$

implies a unique crossing. ■

Proposition E.2 *The function $\max\{\widehat{W}_D, \widehat{W}_E\}$ contains an interval $[V_D, V_E] \subset (0, \tilde{V})$ on which it is not concave. This implies together with risk neutrality that it is optimal to use an export lottery.*

Proof As shown in the previous proposition, there exists a unique equity value V_{dx} where the two value functions cross. For any given $V < \tilde{V}$, the slope of the exporting firm's value function is steeper than the slope of the non-exporting firm, i.e. $W'_D(V) < W'_E(V) \quad \forall V < \tilde{V}$. This follows from the fact that the same is true for the underlying revenue functions. Therefore, by continuity of the value functions, $\widehat{W}'_D(V_{DX} - \epsilon) < \widehat{W}'_E(V_{DX} + \epsilon)$. This implies that the function $\max\{\widehat{W}_D, \widehat{W}_E\}$ is not concave on some interval $[V_D, V_E]$. Since the marginal profit of an extra unit of resources goes to infinity as R approaches zero, $V_D > 0$, and as the slope of the exporting value function is zero at the unconstrained level, $V_E < \tilde{V}$. ■

Proof of Proposition 4.1 As outlined above, $\widehat{W}_i(V)$ is increasing and concave for $i \in \{D, E\}$, so any convex combination of the two functions and S is too. Since $\widehat{W}(0) < S$ and $\widehat{W}(V) > S$ for some $V < \tilde{V}$ (otherwise it would not pay to finance firms at all), $V_r > 0$ and by assumption, $V_D > V_r$. We know that $\widehat{W}'_E(V) > \widehat{W}'_D(V) \forall V < \tilde{V}$, so by concavity of the functions $\widehat{W}_i(V)$, it follows that $V_E > V_D$. Finally, because $\widehat{W}'_E(\tilde{V}) = 0$, it has to be that $V_E < \tilde{V}$.

The first point follows immediately from the fact that the expected value of the liquidation lottery is equal to the equity with which an entrepreneur enters it, thus pinning down the probabilities of liquidation and survival. To prove the second point, we have to show that $V_{dx} \in (V_D, V_E)$. From the above, it follows that the interval is non-empty. By definition, $\widehat{W}_E(V_{dx}) = \widehat{W}_D(V_{dx})$, which implies together with concavity that $V_D < V_{dx} < V_E$. This means that no company with $V \leq V_D$ finds it profitable to export, and all companies with $V \geq V_E$ do. A company with equity $V \in (V_D, V_E)$ is offered a lottery with expected value equal to the equity the entrepreneur had before. The probabilities of getting V_D and V_E are determined thereby. An entrepreneur wins the lottery with probability $\delta(V)$, receiving V_E and thus exporting; with probability $(1 - \delta(V))$, he gets V_D and will hence not export.

Concerning the third point, it is clear that $W(V)$ is linear in the two lottery regions. When $V = \tilde{V}$, the value of the firm does not change anymore, so it will stay constant at $\tilde{W}$. The functions $\widehat{W}_i(V)$ are strictly increasing as long as $V < \tilde{V}$, since $F_E(R) - (1 + r)R$ is strictly increasing in that region. Therefore the firm value function $W(V)$ is strictly increasing for all $V < \tilde{V}$. ■

Proof of Proposition 4.2 Partition the domain of the contract $[0, \tilde{V}]$ in five parts $[0, V_r) \cup [V_r, V_D] \cup (V_D, V_E) \cup [V_E, \tilde{V}) \cup \{\tilde{V}\}$. From the above $E(V'|V) = V/\beta$ when $V \in [V_r, V_D]$ and when $V \in [V_E, \tilde{V})$. When $V = \tilde{V}$, the firm is unconstrained and there is no need to provide any incentives to report the truth (as all revenues will go to the entrepreneur), and hence $V^L = V^H = \tilde{V}$, so $E(V'|\tilde{V}) = \tilde{V}$. Whenever $V^H \in (V_D, V_E)$ or $V^L \in (V_D, V_E)$, the firm enters a lottery and will either end up with V_D or V_E , with the expected value of the lottery being exactly equal to the promised value V^L or V^H . The expected next period equity for each of these is V^L/β and V^H/β , so that $E(V'|V \in (V_D, V_E)) = V/\beta$. Similarly, when $V^L \in [0, V_r)$, the lottery for liquidation yields the expected payoff V^L , and the expected equity for next period is then just V^L/β . ■

F Stationary equilibrium

In the stationary steady state, given interest rate r and wage w , perfect competition in the financial market implies financial intermediaries earn zero-profit on a new contract. This implies new entrepreneurs receive an initial equity $V_0 = \sup_V \{V|W(V) - V - I_0 = 0\}$, for which the lender earns just enough to break even. The initial equity V_0 is endogenous in the general equilibrium.

Consider the sequence $(X_t)_{t \geq 0}$ of equity levels from a single firm indefinitely replaced by a new one when it dies. It is clear $(X_t)_{t \geq 0}$ is a sequence of random variables, and its evolution depends on the properties of the contracts and on the sequence of shocks – productivity, death, export, and liquidation. In what follows, we show that $\mathbf{X} = (X_t)_{t \geq 0}$ is a time-homogeneous Markov chain such that

$$X_{t+1} = T_\omega(X_t, \epsilon_t), (\epsilon_t)_{t \geq 0} \sim \phi_\omega \in \mathcal{P}(Z), X_0 = V_0 \in S \quad (76)$$

where $T_\omega : S \times Z \rightarrow S$ is a collection of measurable functions indexed by $(r, w) = \omega \in \Omega$ the parameter space, $(\epsilon_t)_{t=1}^\infty$ is a sequence of independent random shocks with (joint) distribution ϕ_ω , and S and Z are the state space and the probability space respectively. The existence of an invariant distribution of firms follows if $\mathbf{X}$ has a unique and ergodic invariant distribution. Existence of stationary equilibrium follows if the stationary distribution is continuous in prices.

Proposition F.1 (Stationary distribution) *$\mathbf{X}$ is a time-homogeneous Markov chain on a general state space and is globally stable.*

Equip the state space S with a boundedly compact, separable, metrizable topology $\mathcal{B}(S)$. Let $(Z, \mathcal{Z})$ be the measure space for the shocks. Let A be any subset of

$\mathcal{B}(S)$. It follows for any $x \in \{x : V_r < x < V_D \text{ and } V^L(x) < V_r\}$

$$P(x, A) = \begin{cases} (1 - \gamma)(1 - \pi)\alpha(V^L(x)) + \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma)(1 - \pi)(1 - \alpha(V^L(x))) & \text{if } A = \{V_r\} \\ (1 - \gamma)\pi & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \quad (77)$$

For any $x \in \{x : V_r < x < V_D \text{ and } V_r \leq V^L(x) \leq V_D \text{ and } V^H(x) \leq V_D\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma)(1 - \pi) & \text{if } A = \{V^L(x)\} \\ (1 - \gamma)\pi & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \quad (78)$$

For any $x \in \{x : V_r < x < V_D \text{ and } V_r \leq V^L(x) \leq V_D \text{ and } V^H(x) \geq V_D\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma)(1 - \pi) & \text{if } A = \{V^L(x)\} \\ (1 - \gamma)\pi\delta(V^H(x)) & \text{if } A = \{V_E\} \\ (1 - \gamma)\pi(1 - \delta(V^H(x))) & \text{if } A = \{V_D\} \\ 0 & \text{otherwise} \end{cases} \quad (79)$$

For any $x \in \{x : V_E < x < \tilde{V} \text{ and } V^L(x) \leq V_E \text{ and } V_E \leq V^H(x) < \tilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma)(1 - \pi)\delta(V^L(x)) & \text{if } A = \{V_E\} \\ (1 - \gamma)(1 - \pi)(1 - \delta(V^L(x))) & \text{if } A = \{V_D\} \\ (1 - \gamma)\pi & \text{if } A = \{V^H(x)\} \\ 0 & \text{otherwise} \end{cases} \quad (80)$$

For any $x \in \{x : V_E < x < \tilde{V} \text{ and } V_E \leq V^L(x) \leq \tilde{V} \text{ and } V^H(x) \geq \tilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma)(1 - \pi) & \text{if } A = \{V^L(x)\} \\ (1 - \gamma)\pi & \text{if } A = \{\tilde{V}\} \\ 0 & \text{otherwise} \end{cases} \quad (81)$$

And for $x = \{\tilde{V}\}$

$$P(x, A) = \begin{cases} \gamma & \text{if } A = \{V_0\} \\ (1 - \gamma) & \text{if } A = \{\tilde{V}\} \\ 0 & \text{otherwise} \end{cases} \quad (82)$$

For each $A \in \mathcal{B}(S)$, $P(\cdot, A)$ is a non-negative function on $\mathcal{B}(S)$, and for each $x \in S$, $P(x, \cdot)$ is a probability measure on $\mathcal{B}(S)$. Therefore, for any initial distribution ψ , the stochastic process $\mathbf{X}$ defined on S^∞ is a time-homogeneous Markov chain. Let $\mathbf{M}$ denote the corresponding Markov operator, and let $\mathcal{P}(S)$ denote the collection of firms distribution generated by $\mathbf{M}$ for a given initial distribution.¹³

Write the stochastic kernel P with the density representation p so that $P(x, dy) = p(x, y)dy$ for all $x \in S$. The Dobrushin coefficient $\alpha(p)$ of a stochastic kernel p is defined by

$$\alpha(p) := \min \left\{ \int p(x, y) \wedge p(x', y) dy : (x, x') \in S \times S \right\}$$

$(\mathcal{P}(S), \mathbf{M})$ is globally stable if $(\psi \mathbf{M}^t)_{t \geq 0} \rightarrow \psi^* \mathbf{M}$ where $\psi^* \in \mathcal{P}(S)$ is the unique fixed point of $(\mathcal{P}(S), \mathbf{M})$. This occurs if the Markov operator is a uniform contraction of modulus $1 - \alpha(p)$ on $\mathcal{P}(S)$ whenever $\alpha(p) > 0$. A firm dies with a fixed, exogenous and independent probability γ each period, and is instantaneously replaced by a new one of size V_0 . Therefore,

$$P(x, \{V_0\}) \geq 0 \quad \forall x \in S. \quad (83)$$

Equation (11.15) and Exercise (11.2.24) in [Stachurski \[2009\]](#) yield $\alpha(p) > \gamma$. By [Stachurski \[2009, Th. 11.2.21\]](#), this implies

$$\|\psi \mathbf{M} - \psi' \mathbf{M}\|_{TV} \leq (1 - \gamma) \|\psi - \psi'\| \quad (84)$$

for every pair ψ, ψ' in $\mathcal{P}(Z)$, and where $_{TV}$ indicate the total variation norm. ■

Proposition F.2 (Existence of a stationary equilibrium) *The unique and ergodic invariant distribution of $\mathbf{X}$ is continuous in prices.*

The result follows if the conditions of [LeVan and Stachurski \[2007, Proposition2\]](#) are satisfied and the proof is similar to the one in ?.

¹³Note that [Stokey, Lucas, and Prescott \[1989, Theorem 12.12\]](#) fails to apply in this case because the stochastic kernel is not monotone near the randomization region. See ? for more details.

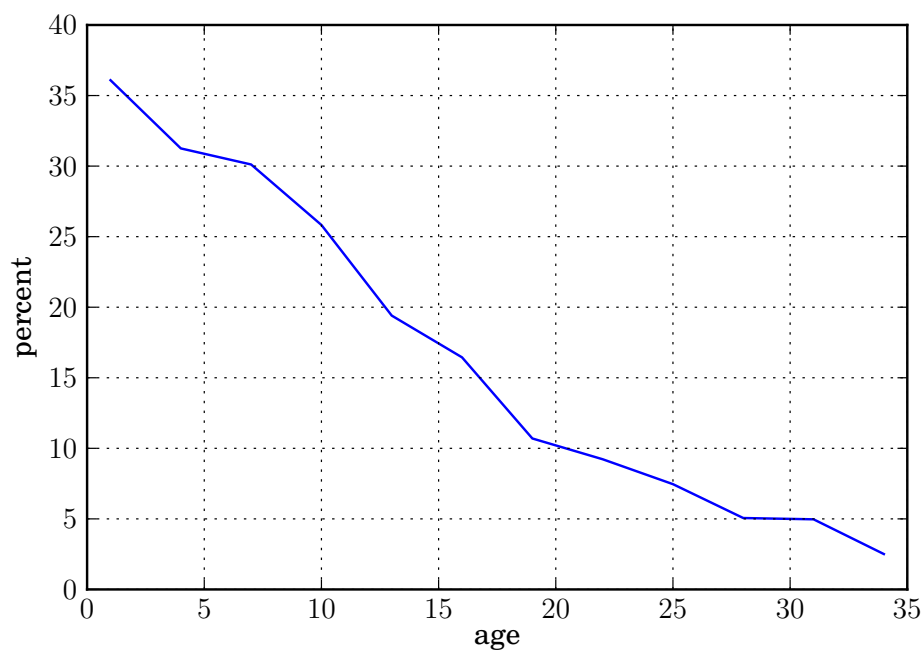


Figure 12: Firms' investment growth rate by age

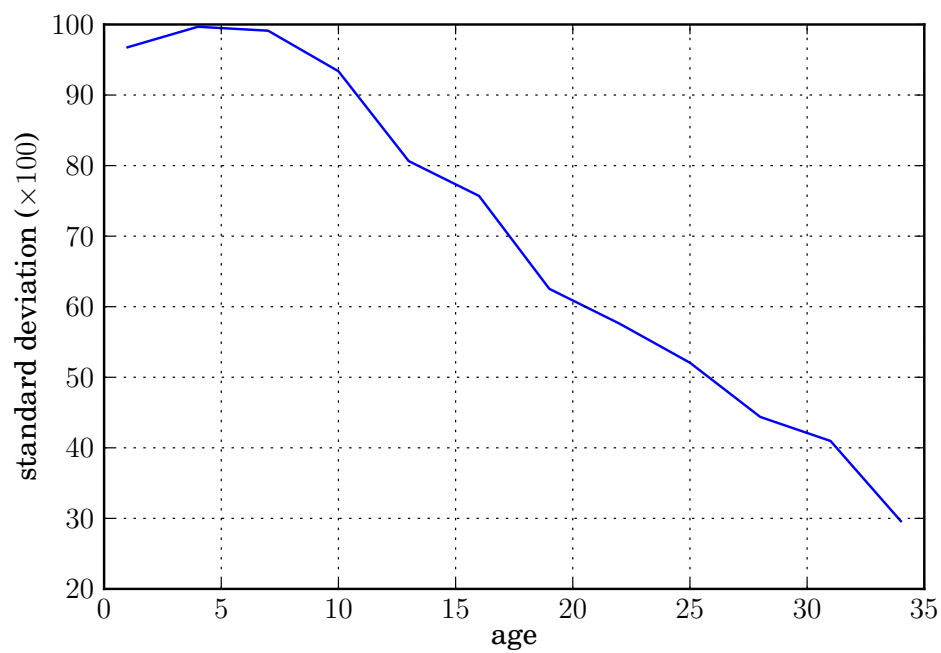


Figure 13: Volatility of firms' investment growth rate by age



Figure 14: Domestic firms' investment growth rate by age



Figure 15: Volatility of domestic firms' investment growth rate by age

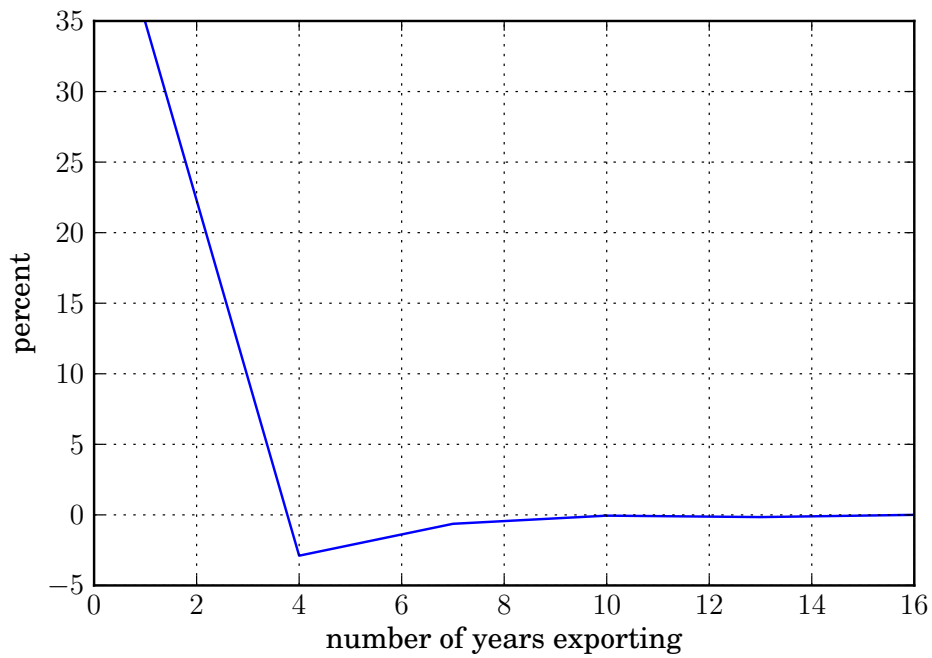


Figure 16: Exporters' investment growth rate by age

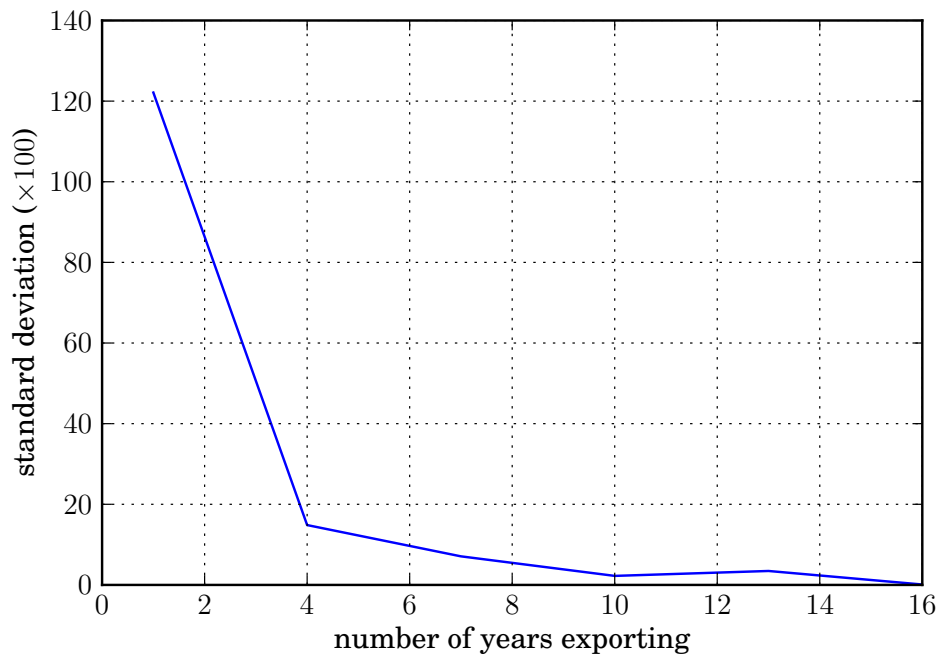


Figure 17: Volatility of exporters' investment growth rate by age