Demand Shocks that Look Like Productivity Shocks*

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Tuesday 18$^{\text{th}}$ January, 2011

Abstract

The use of competitive search protocols in final goods allows for the specification of models where demand shocks generate propagations that look like productivity shocks. In such environments there is a role for expansion of public expenditures during recessions. We investigate the empirical and quantitative validity of model economies with these properties.

*We thank the comments of Ellen McGrattan and Larry Christiano. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, or the Federal Reserve System.

1 Introduction

In this paper we construct a model where measured productivity, the labor input are all strongly correlated with volatilities like those in the data. While real business cycle models deliver these features via persistent shocks to TFP, we generate such movements with shocks to demand. In our case these are preference shocks, but the exact nature is not so important. Moreover, our model economy is built over a growth model with capital and features a balanced growth path.

So how can we have simultaneously an increase in labor, decreasing returns to scale, no productivity shocks or externalities in production and procylical measured productivity? By explicitly posing the role of spenders in generating output. Restaurants need diners, hospitals need patients, in sum, firms need consumers and investors. For households and firms, the acquisition of goods is an active process that involves some costs that are not measured in NIPA. When shocks directly affect these costs households and firms change their spending, firms see increases in demand that allows them to deliver goods that would otherwise rot. In a sense, ours is a model of a modern economy, filled with services and perishable goods that if not sold, cannot be stored and do not show up in measured GDP. Consequently, our model displays notions of potential output, or full capacity.

Our model economy looks like a real business cycle economy with productivity shocks except that there is no such a thing. In fact, we estimate a process for the preference shocks to match the measured process for TFP. It has been almost 30 years since the seminal real business cycle paper **?** that brought to center stage productivity shocks, yet we have learned very little about such shocks, and in fact productivity movements remain pretty much like a black box. Moreover, casual discussions with firm managers does not seem to highlight the issue: business is good when customers show up and not when productivity has just increased.

Technically the paper poses a search shopping friction that is resolved via competitive search. The model poses shocks to the willingness to spend by consumers (and in extension also by investors). (in this version they are preference shocks and shopping technology shocks but it is easy and the stuff of future work to pose financial frictions as the mechanism responsible for shocks) that have processes that we estimate based on the observed properties of the time series of the Solow residual and the relative price of investment which is what in the real business cycle literature are take to be the underlying structural shocks (see **?** for instance).

In our model the friction is intrinsic to the process of production and is not arbitrarily imposed:

markets clear, and no agent has incentives to do things different. While some other models (of the new keynesian variety) have been able to generate demand induced shocks, they do that by making agents trade at prices that are not the equilibrium ones. In this paper there is no such a thing.

Literature. We have to cite various literatures.

1. Capacity utilization papers, **?**, **?** and others that we do not know yet.

2. Labor Hoarding **?**.

This paper has some additional contributions. First it shows how in models with production and competitive search, to achieve optimality it is necessary to index markets, not only by price and market tightness, but also by quantity. We provide an endogenous theory of aggregate adjustment costs of capital that comes from the additional search costs of acquiring investment goods which may not necessarily show up in the price. V: (Please verify whether these two statements are true.)

We start discussion a simple version of the model with Lucas trees in Section 2 and we go to a full version with the neoclassical growth model underlying the economy in Section 3. We then map the model to data in Section 4. In Section 5 we estimate processes for demand shocks that replicate the properties of the Solow residual while in Section 6 we explore the business cycle properties of economies with demand shocks and we asses the findings in detail. In Section 7 we study the performance of the Economy with various shocks simultaneously. We explore the robustness of our findings in Section 8. Section 9 concludes while an Appendix provides some proofs and computational and data details.

2 The simple Lucas-trees version of the model

We start by illustrating the workings of the model in a simple search model where output is produced by trees instead of capital and labor. We show that the sarch process impacts aggregate output in a way that appears as a level effect on the Solow residual. In an example we show how shocks to preferences are partly accommodated by increase in prices and interest rates and partly accommodated by increases in quantities, and demonstate that absent the search friction, the same shocks translate only to price increases.

2.1 Technology and preferences

There is a continuum of trees (i.e., suppliers), with measure $T = 1$. Each tree yields one piece of fruit every period. A standard search friction makes it difficult for consumers to find trees. To overcome this friction the consumer sends out a number of shoppers searching for fruit. The aggregate number of fruits found, Y, is given by the Cobb-Douglas matching function:

$$Y = A\, D^{\alpha}\ T^{1-\alpha}, \tag{1}$$

where D is the aggregate measure of shoppers searching for fruit (we sometimes call it aggregate demand) and A and α are parameters of the matching technology.

Following **?**, we assume a competitive search protocol where agents can choose to search in specific locations indexed by both the price and market tightness, defined as the ratio of trees per shopper, $Q = T/D$. The probability that a tree is found (i.e., matched with a shopper) is $\Psi_T(Q) = A\ Q^{-\alpha} = A\ D^{\alpha}/T^{\alpha}$. Once a match is formed, then the fruit is traded at the posted price p. By the end of the period, all fruit which is not found is lost. The trees pay out sales revenues as dividends and the expected dividend is $\varsigma = p\Psi_T(Q)$.

The economy has a continuum of identical, infinitely lived households of measure one. Their preferences are given by

$$E\left\{\sum_t\ \beta^t\ u\left(c_t, d_t, \theta_t\right)\right\}, \tag{2}$$

where c_t is consumption, d_t is the measure of shopping units (search effort) by the household, and θ is a preference shock that follows a stochastic Markov process. The probability that an individual shopping unit is successful is given by the ratio of matches to the aggregate number of shoppers. Given the matching technology (1), this can be expressed in terms of market tightness as $\Psi_d(Q) = A\ Q^{1-\alpha}$, so c_t is given by

$$c = d\ \Psi_d(Q) = d\ A\ Q^{1-\alpha}. \tag{3}$$

Trees are owned by households and are traded every period. We normalize the price of the tree to unity and use it as the numeraire good. Let s denote the number of shares owned by the household. The aggregate number of shares is unity. Consequently, with identical households the aggregate state of the economy is just θ, while the individual state includes also individual wealth s. The representative household problem cen then be expressed recursively as

$$
\begin{aligned}
v(\theta, s) &= \max_{c,d,s'} u(c, d, \theta) + \beta E\{v(\theta', s')|\theta\} && (4)\\
\text{s.t.} \quad & c = d\ \Psi_d[Q(\theta)] && (5)\\
& P(\theta)\ c + s' = s\ [1 + \varsigma(\theta)] && (6)
\end{aligned}
$$

2.1.1 Equilibrium

A competitive search equilibrium is defined by a set of individual decision rules, $c(\theta, s)$, $d(\theta, s)$, and $s'(\theta, s)$, the aggregate allocations $D(\theta)$, $Q(\theta)$, and $C(\theta)$, good prices $P(\theta)$, and the rate of return on trees $\varsigma(\theta)$ so that

1. The individual decision rules, $c(\theta, s)$, $d(\theta, s)$, and $s'(\theta, s)$ solve the household problem (4).

2. The individual decision rules are consistent with the aggregate functions

$$C(\theta) = c(\theta, 1) \qquad D(\theta) = d(\theta, 1) \qquad s'(\theta, 1) = 1$$

3. The aggregate consumption is consistent with aggregate search

$$C(\theta) = A\ D(\theta)^{\alpha} \tag{7}$$

4. Firms' dividends equal firm sales

$$\varsigma(\theta) = p(\theta)\ \ A\ D(\theta)^{\alpha}$$

5. The pair $(p(\theta), D(\theta))$ clears the goods market and aggregate tightness is $Q(\theta) = 1/D(\theta)$.

2.1.2 Competitive Search in the Market for Goods

We start by analyzing the equilibrium in the market for goods. There are differentiated markets indexed by the price, each of them having its own market tightness of number of trees per shopper. Let ς denote the outside value for firms of going to the most attractive market, yet to be determined.

Clearly a market can attract trees only if it offers trees at least ς. This puts a constraint on the feasible combinations of prices and market tightness that shoppers can offer:

$$\varsigma \leq \ p \ \Psi_T(Q) \tag{8}$$

The expected contribution to household utility of a shopper that chooses the best price–tightness pair is[1]

$$\Phi = \max_{Q,p} \left\{ u_d\left(\theta, s\right) \ + \ \Psi_d(Q) \ \left(u_c\left(\theta, s\right) \ - \ p \ m\right)\right\} \qquad \text{s.t.} \ \ \varsigma \leq \ p \ \Psi_T(Q), \tag{9}$$

where $u_d\left(\theta, s\right)$ and $u_c\left(\theta, s\right)$ are the marginal utility of increases in d and c, respectively. Moreover, $m = m(\theta, s'(\theta, s))$ is the expected discounted marginal utility of an additional unit of savings:

$$m\left(\theta, s'\right) = \beta \ E\left\{ \frac{P(\theta) \ [1 + \varsigma(\theta')]}{P(\theta')} \ \frac{\partial v\left(\theta', s'\right)}{\partial s'} | \theta \right\}$$

Solve (9) by substituting (8) at equality and take the first-order condition w.r.t. Q. This yields the unique equilibrium price p and value of the tree ς as functions of market tightness Q;

$$\begin{aligned} p &= (1 - \alpha) \ \frac{u_c\left(\theta, s\right)}{m}, \qquad (10) \\ \varsigma &= pA \ Q^{-\alpha}. \qquad (11) \end{aligned}$$

2.1.3 Characterizing the equilibrium

The competitive search equilibrium and its efficiency properties can then be characterized by the following proposition.

Proposition 1. *1. Aggregate search D is determined by the functional equation*

$$0 = \alpha \ A \ D\left(\theta\right)^{\alpha - 1} \ u_c\left(AD\left(\theta\right)^{\alpha}, D(\theta), \theta\right) + u_d\left(AD\left(\theta\right)^{\alpha}, D(\theta), \theta\right) \tag{12}$$

The functions C, Q and ς then follow directly from the equilibrium conditions $C\left(\theta\right) = A \ D\left(\theta\right)^{\alpha}$, $Q\left(\theta\right) = 1/D\left(\theta\right)$, and $\varsigma(\theta) = p\left(\theta\right) \ A \ D\left(\theta\right)^{\alpha}$.

[1] To derive this, take the first-order condition of (4) with respect to d and consider the price-tightness posting problem when the cost $\overline{u}_d$ of sending an additional shopper has been borne. The idea is that each shopper is equipped with a credit card and if no fruit is found, then utility is not affected over and above the sunk search cost.

2. *The equilibrium price is defined by the functional equation*

$$(1-\alpha)\ \theta_c\ u_c\left(C\left(\theta\right),D(\theta),\theta\right)=$$
$$p\left(\theta\right)\ \beta\ E\left\{\frac{P(\theta)\ [1+\varsigma(\theta')]}{P(\theta')}\ \left[u_c\left(C\left(\theta'\right),D(\theta'),\theta'\right)+\frac{u_d\left(C\left(\theta\right),D(\theta),\theta\right)}{\Psi_d[Q(\theta')]}\right]|\theta\right\} \quad (13)$$

3. *The competitive equilibrium is efficient.*

Proof. Substitute $d=c/\Psi_d\left(\hat{D}\right)$ into the period utility $U=\theta_c u(c)-\theta_d\ \overline{u}(c/\Psi_d(\hat{D}))$ and differentiate U w.r.t. c to get

$$\frac{dU}{dc}=\theta u_c+\frac{\partial U}{\partial d}\frac{\partial d}{\partial c}=\theta u_c-\frac{1}{\Psi_d(\hat{D})}\ \overline{u}_d\left(\frac{c}{\Psi_d(\hat{D})}\right)$$

The Euler equation can then be expressed as

$$\theta u_c-\frac{1}{\Psi_d(\hat{D})}\ \overline{u}_d\left(\frac{c}{\Psi_d(\hat{D}\left(\theta\right))}\right)=P(\theta)m.$$

Impose the representative-agent conditions $c=C=AD^{\alpha}$ and $d=D=\hat{D}$, and use (10) to substitute out the term $P(\theta)m$. This gives one functional equation:

$$\theta u_c\left(C(\theta)\right)-\frac{1}{\Psi_d(D\left(\theta\right))}\ \overline{u}_d\left(\frac{C(\theta)}{\Psi_d(D\left(\theta\right))}\right)=(1-\alpha)\ \theta\ u_c(C(\theta)).$$

Rearranging this equation and using (7) yields the functional equation (12). The functional equation (13) is derived from the equilibrium price equation (10) and the definition of m, where we exploit the envelope theorem and (5) to express $\partial v/\partial s$ as

$$\frac{\partial v\left(\theta,s\right)}{\partial s}=\theta u_c(c)-\bar{u}_d(d)\frac{c}{\Psi_d[D(\theta)]}$$

At the equilibrium, the agents' budget constraint (6) is satisfied. Given (12)-(13), the first-order conditions of (4) hold, which guarantees individual optimization.

Consider the planner problem $\max_{C,D}\left\{\theta u(C)-\overline{u}(D)\right\}$ subject to the aggregate resource constraint $C=A\ D^{\alpha}$. It is straightforward to verify that the solution to equation (12) solves this planner problem, which establishes efficiency. □

2.2 An example

Consider a version of this economy where

$$u(c,d,\theta) = \theta_c \, \log c \; - \; \theta_d \, d, \tag{14}$$

where the stochastic preference weights θ_c and θ_d are i.i.d. with $E\left\{(\theta_c') \; |\theta\right\} = E\left\{(\theta_d') \; |\theta\right\} = 1$ and $\theta_c > 0$ and $\theta_d > 0$. For notational convenience we define $\theta \equiv (\theta_c, \theta_d)$. Given these preferences, the equilibrium condition in equation (12) becomes

$$0 = \alpha \; A \; D(\theta)^{\alpha-1} \; \frac{\theta_c}{AD(\theta)^{\alpha}} - \theta_d.$$

Simplifying this expression and using (7), yields the equilibrium allocation:

$$\begin{aligned} D(\theta) &= \alpha \, \frac{\theta_c}{\theta_d} \\ C(\theta) &= A \; (D(\theta))^{\alpha} = A\left(\alpha \, \frac{\theta_c}{\theta_d}\right)^{\alpha} \end{aligned}$$

As is clear from this allocation, preference shocks generate aggregate fluctuations.

A central point of our paper is that in the presence of search frictions, the measured total factor productivity (TFP) is a function of the search effort and, hence, fluctuates in response to preference shocks. The only other production factor in this economy is the fixed stock of capital, $T = 1$. Therefore, if search effort is ignored, the contribution from search will show up as a Solow residual Z, defined as $C = Z \; T$;

$$Z(\theta) = A \; (D(\theta))^{\alpha} = A\left(\alpha \, \frac{\theta_c}{\theta_d}\right)^{\alpha}$$

Given the allocation, it is straightforward to verify that the equilibrium price satisfying (13) is given by

$$P(\theta) = \left(\frac{1}{\beta} - 1\right) \frac{1}{A\alpha^{\alpha}} (\theta_d)^{\alpha} \; (\theta_c)^{1-\alpha}.$$

The dividends (which is also the rate of return on the tree), is then

$$\varsigma(\theta) = p(\theta)\ A\left(\alpha\,\frac{\theta_c}{\theta_d}\right)^{\alpha} = \left(\frac{1}{\beta} - 1\right)\ \theta_c.$$

Thus, the rate of return is the discount rate times the preference shock. [2]

To get some intuition for the price expression, consider the price of the tree in terms of consumption units:

$$\frac{1}{p(\theta)} = \frac{\beta}{1-\beta} A\alpha^{\alpha}\ (\theta_d)^{-\alpha}\ (\theta_c)^{\alpha-1} = \frac{\beta}{1-\beta}\ \frac{C(\theta)}{\theta_c} = \frac{\beta}{1-\beta}\ \frac{1}{\theta_c\ u_c}.$$

Thus, the price of the tree in terms of consumption units is the discounted value of the current marginal utility of consumption for ever. Finally, note that the equilibrium interest rate in terms of the consumption good is given by

$$1 + r(\theta) = \frac{(\theta_d)^{\alpha}\ (\theta_c)^{1-\alpha}}{\beta\ E\left\{(\theta_d')^{\alpha}\ (\theta_c')^{1-\alpha}\right\}}$$

Note that this implies that in steady state, with $\theta' = \theta$, the interest rate would be $r(\theta) = \varsigma(\theta) = 1/\beta - 1$. In conclusion, the interest rate is increasing in θ_c and decreasing in θ_d. Note also that the elasticity of aggregate consumption (to changes in θ) is α.

It is instructive to compare the model to a version of the standard Lucas tree model without the search friction. As $\alpha \to 0$, the search economies converge to the standard Lucas tree model. In this case $Y = C = A$, $D = 0$, and

$$\begin{aligned} P(\theta) &= \left(\frac{1}{\beta} - 1\right)\frac{1}{A}\,\theta_c \\ 1 + r(\theta) &= \frac{\theta_c}{\beta\ E\{\theta_c'\}} = \frac{\theta_c}{\beta}. \end{aligned}$$

Clearly, in this case the aggregate consumption is invariant to the demand shock (so the elasticity

[2] It follows that when expressed in terms of consumption units, the dividends are given by

$$\frac{R(\theta)}{p(\theta)} = \left(\alpha\,\frac{\theta_c}{\theta_d}\right)^{\alpha} = C(\theta).$$

is zero), and all the adjustment to the shock takes place through the prices. Specifically, the elasticity of r and p to θ_c is unity, compared to $1-\alpha$ in the search case.

2.3 Fiscal policy in the Lucas-tree model

We now analyze a verison of the model where the government provides a public good financed with lump-sum taxation in terms of the consumption good. For simplicity, we assume that the government does not run deficits, so government expenditures G_t equals the lump-sum tax each period. Due to Ricardian equivalence, this assumption is without loss of generality. Since government consumption is in terms of the consumption good, the aggregate resource constraint is

$$Y_t = C_t + G_t = A\ D_t^{\alpha}\ T^{1-\alpha}.$$

Agents now search for fruit for both own consumption and tax purposes, so their budget constraint becomes

$$p_t\ (c_t + G_t) + s_{t+1} = s_t\ [1 + \varsigma_t],$$

and the search constraint becomes

$$c_t + G_t = d_t\ \Psi_d[Q_t].$$

Government expenditures G_t finances investment in government capital which is long-lived. We model this in a stark way by assuming that the marginal value of additional public-good spending is constant, so individual preferences are

$$E\left\{\sum_t \beta^t\ [\theta_{c,t}\log(c_t) - \theta_{d,t}\ d_t + G_t]\right\}.$$

2.3.1 Exogenous fiscal policy

Suppose first that the sequence of G_t is exogenous. As above, the competitive equilibrium allocations are constrained efficient, given the exogenous sequence of G_t. The equilibrium allocation is then a solution to the following static problem:

$$\begin{aligned} & \max_{C,D} \{\theta_c \log(C) - \theta_d \; D + G\} \\ \text{s.t.} \qquad C + G \;&=\; A \, D^{\alpha} \; T^{1-\alpha}, \end{aligned} \tag{15}$$

and the solution is the unique positive solution $D(\theta, G)$ to the following equation[3]

$$\frac{\theta_c}{\theta_d} \alpha A D^{\alpha-1} + G = A \; D^{\alpha}, \tag{16}$$

and equilibrium consumption is given by

$$C(\theta) = \frac{\theta_c}{\theta_d} \alpha A \; D(\theta)^{\alpha-1}. \tag{17}$$

The solution $D(\theta, G)$ is increasing in G and in the ratio θ_c/θ_d.[4] Intuitively, the aggregate search effort incrases if current consumption is more valuable ($\theta_c \uparrow$) or if search effort is less costly ($\theta_d \downarrow$). Increased government consumption will crowd out private consumption.[5] Therefore, a larger G will incrase the marginal utility of consumption and, hence, induce a larger search effort D. A larger D in turn increases aggregate output. Note, however, that the "fiscal multiplier" $\partial Y/\partial G$ is smaller than unity – an increase in G generates a less than one-for-one increase in output. Note also that an increase in the ratio θ_c/θ_d and an exogenous in G would generate lower aggegate output and, hence a fall in the Solow residual.

2.3.2 Endogenous fiscal policy

Consider now the Ramsey problem of public good provision in this economy. Since the search equilibrium is constrained efficient, the Ramsey planner problem amounts to maximize (15) over G, C, and D:

$$\max_{C,D,G} \Lambda = \{\theta_c \log(C) - \theta_d \; D + G - \lambda \left(A \; D^{\alpha} - C - G\right)\}$$

[3]To see that D is unique, note that the right-hand side is zero for $D = 0$ and monotone increasing. The left-hand side is monotone decreasing for $D > 0$ and converges to ∞ for $D \to 0$.

[4]To see this, note that both a larger G and a larger ratio θ_c/θ_d will increase the left-hand side of (16). Since the right-hand side of (16) is increasing in D, the equilibrium D must increase.

[5]To see this, use (17) to substitute D for C in equation (16) to obatin an expression of G as a decreasing function of C. Therefore, a larger G is associated with a lower C, *ceteris paribus*.

The first-order conditions to that problem are

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial G} &= 1 - \lambda = 0 \\
\frac{\partial \Lambda}{\partial D} &= \theta_d - \lambda \alpha A D^{\alpha-1} = 0 \\
\frac{\partial \Lambda}{\partial C} &= \frac{\theta_c}{C} - \lambda = 0
\end{aligned}
$$

The implied optimal allocation is given by

$$
\begin{aligned}
D &= \left(\frac{\alpha A}{\theta_d}\right)^{\frac{1}{1-\alpha}} \\
C &= \theta_c \\
G &= \left(\frac{A}{\theta_d}\right)^{\frac{1}{1-\alpha}} (\alpha)^{\frac{\alpha}{1-\alpha}} - \theta_c
\end{aligned}
$$

It follows that optimal search is decreasing in θ_d, that consumption is increasing in θ_c, and that optimal fiscal policy reacts to these shocks. In particular, a fall in θ_c would, with exognous fiscal policy, generate a recession. The planner offsets this shock completely by increasing government expenditures one for one, so aggregate output is constant (countercyclical fiscal policy). However, when θ_d falls, fiscal policy exacerbates the aggregate fluctuations relative to an economy with exogenous fiscal policy. In the exogenous-fiscal-policy model above a fall in θ_d increases D and C (since G is constant by assumption in that model). This reduces the marginal utility of consumption which, in turn, mitigates the increase in search and, hence, mitigates the increase in aggregate output. However, when G is endogenous, it is optimal to let the whole increase in output go toward increasing G. The reason is that the marginal utility of G does not fall when G increases. Thus, fiscal policy stablizes private consumption but increases the volatility of aggregate output (procyclical fiscal policy).

3 Growth Model

We now turn to embody our ideas on a growth model suitable for business cycle analysis. There is capital, and increasing the stock requires both investment goods and professionals to shop for those goods. In addition, work generates a disutility. These two features generate substantial complications that we now analyize. We start with the problems faced by households and firms, we then look at the operation of competitive search for consumption and investment goods and we define equilibrium. Along the way we prove a few results that guarantee that all firms make the same choices of labor and investment. We also establish the Pareto Optimality of Equilibrium. We finish the section discussion some subtle issues of National Accounting that are involved in the determination of labor share and in the calculation of the Solow residual.

3.1 Households

There is a measure one of households that have preferences over consumption, shopping, and working and is affected by shocks that are perfectly correlated across households. All this is summarized by utility function $u(c, d, n, \theta)$.

We will proceed by assuming (which we will later verify) that the state of the economy is given by the pair (Θ, K), this is the aggregate preference shock and aggregate capital. The additional state variables for the household are the values of its own shock and the shares of the stock market that it owns or the vector (Θ, K, θ, s). We will denote with superindex c or i to indicate whether we refer to consumption or investment goods (which are identical from the point of view of production but not from the point of view of distribution).

The aggregate variables that are determined in equilibrium and that the households take as given include market tightness, price and quantity in the consumption good market which we denote by $\{Q^c, P^c, F^c\}$ and that are functions of (Θ, K); they also include the rate of return of the economy in terms of the numeraire (shares of the stock market), the wage, and the law of motion of capital denoted $G(\theta, K)$.

Household problem The representative household solves

$$\begin{aligned} v(\Theta, K, \theta, s) \quad &= \quad \max_{c,d,n,s'} \; u(c, d, n, \theta) + \beta \; E \left\{ v(\Theta', K', \theta', s') | \Theta, \theta \right\} && (18) \\ \text{s.t.} \qquad & \qquad c = d \; \Psi_d[Q^c(\Theta, K)] \; F^c(\Theta, K), && (19) \\ & \qquad P^c(\Theta, K) \; c + s' = s \; [1 + R(\Theta, K)] + n \; w(\Theta, K), && (20) \\ & \qquad K' = G(\Theta, K). && (21) \end{aligned}$$

Equation (19) shows that consumption requires the household's shopping effort, but it also depends on market tightness and the amount produced by consumption produced firms. Equation (20) is the budget constraint in terms of shares.

The solution to this problem is a set of functions $d(\Theta, K, \theta, s)$, $c(\Theta, K, \theta, s)$, $n(\Theta, K, \theta, s)$, and $s'(\Theta, K, \theta, s)$. Anticipating equilibrium conditions, we can bring the aggregate counterparts of these functions, yielding,

$$\begin{aligned} C(\Theta, K) &= c(\Theta, K, \Theta, 1) && (22) \\ D^c(\Theta, K) &= d(\Theta, K, \Theta, 1), && (23) \\ N(\Theta, K) &= n(\Theta, K, \Theta, 1) && (24) \\ s'(\Theta, K, \Theta, 1) &= 1. && (25) \end{aligned}$$

Notice that the use of stock market shares in lieu of assets as the state variable of the household accounts for the last condition.

Using these aggregate conditions, the intertemporal Euler is

$$u_c[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] = \beta \\ E\left\{\frac{P^c(\Theta, K)\,[1 + R(\Theta', K')]}{P^c(\Theta', K')} \;\; u_c\,[C(\Theta', K'), D(\Theta', K'), N(\Theta', K'), \Theta']\,|\Theta\right\}, \quad (26)$$

while the intratemporal is

$$u_c[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] = u_n\,[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] \;\; \frac{P^c(\Theta, K)}{w(\Theta, K)}. \quad (27)$$

Let $m(\Theta, K, \theta, s)$, denote the value in terms of marginal utility of an additional unit of savings and let $M(\Theta, K)$ be its aggregate counterpart

$$M(\Theta, K) = \beta\; E\left\{\frac{[1 + R(\Theta', K')]}{P^c(\Theta', K')} \;\; u_c\,[C(\Theta', K'), D(\Theta', K'), N(\Theta', K'), \Theta']\,|\Theta,\right\}. \quad (28)$$

The dynamic Euler equation becomes

$$u_c[C(\Theta, K), D(\Theta, K), N(\Theta, K), \Theta] = P(\Theta, K)\; M(\Theta, K). \quad (29)$$

3.2 Firms

There is a measure one of firms each one of them constituted by a plant or location or tree. The firm has certain amount of capital installed in that location. There is a technology that transforms capital services and labor into goods that is described by production function $f(k, n)$. To install new capital for the following period, the firm like the households has to shop. The firm has a technology than transforms one unit of labor for shopping (that cannot be used for production) into ζ shopping units.

Each firm has to choose three sets of things: whether to produce for investment or consumption, the specific submarket to which to go, and how much to invest.

Firm's problems The choices of the firm can be separated. First it chooses the type of good, then the submarket and then the investment choice. Firms choose to produce whichever good that gives higher value, i.e.

$$\Omega(\Theta, K, k) = \max\{\Omega^c(\Theta, K, k), \Omega^i(\Theta, K, k)\}, \tag{30}$$

where $\Omega^c(\Theta, K, k)$ is the best value for producing consumption goods. Firms obtain this value by choosing among the best available triplet of (Q^c, P^c, F^c) in the consumption submarket,

$$\Omega^c(\Theta, K, k) = \max \ \widetilde{\Omega}^c(\Theta, K, k, Q^c, P^c, F^c) \qquad \text{for all available } (Q^c, P^c, F^c).$$

imilarly, for investment. We denote by $T(\Theta, K)$ the share of firms that produce consumption goods.

A firm in a (Q^c, P^c, F^c) consumption goods submarket chooses labor for shopping n^k, investment i and capital to install tomorrow k' to solve the following problem,

$$\begin{aligned}
\widetilde{\Omega}^c(\Theta, K, k, Q^c, P^c, F^c) \ &= \ \max_{n^k, k', i} \ \frac{\Psi_d(Q^c)}{Q^c} \ P^c F^c - w(\Theta, K) \ [\widetilde{n}(k, F^c) + n^k] \\
&\qquad -P^i(\Theta, K) \ i + E\left\{ \frac{\Omega(\Theta', K', k')}{1 + R(\Theta', K')} \middle| \ \Theta \right\} && (31) \\
\text{s.t.} \qquad & i = n^k \ \zeta \ \ \Psi_d[Q^i(\Theta, K)] \ \ F[K, N^i(\Theta, K)] && (32) \\
& k' = i + (1-\delta)k && (33) \\
& K' = G(\Theta, K) && (34)
\end{aligned}$$

where $\widetilde{n}(k, y)$ is the inverse function of the production function $y = f(k, n)$ for a given k; $\zeta 1$ is the technological requirement for firms of how many shopping workers it needs to provide a unit of shopping service. Finally $N^i(\Theta, K)$ is the equilibrium amount of production workers in investment goods producing forms so $f(K, N^i(\Theta, K))$ is the amount of investment good produced by investment producing firms.

Similarly, a firm in the investment good market (Q^i, P^i, F^i) chooses labor for shopping n^k, investment i, and future capital stock k' to solve the following problem

$$\begin{aligned}\widetilde{\Omega}^i(\Theta, K, k, Q^i, P^i, F^i) &= \max_{n^k, k', i} \frac{\Psi_d(Q^i)}{Q^i} P^i\ F^i - w(\Theta, K)\ [\widetilde{n}(k, F^i) + n^k)] \\ &\quad -P^i(\Theta, K)\ i + E\left\{ \frac{\Omega(\Theta', K', k')}{1 + R(\Theta', K')} \middle| \Theta \right\} \end{aligned} \tag{35}$$

subject to (32), (33), and (34).

The FOC of this problem is given by

$$E\left\{ \frac{\Omega_k(\Theta', K', k')}{1 + R(\Theta', K')} \middle| \Theta \right\} = \frac{w(\Theta, K)}{\zeta\ \Psi_d[Q^i(\Theta, K)]\ f[K, N^i(\Theta, K)]} + P^i(\Theta, K). \tag{36}$$

Let $\varsigma(\Theta, K, k)$ denote the outside revenue for firms of going to the best market in units of shares. It is the best revenue obtained either by serving the consumption submarket or by serving the investment submarket, i.e.

$$\varsigma(\Theta, K, k) = \max\{\varsigma^c(\Theta, K, k), \varsigma^i(\Theta, K, k)\},$$

where $\varsigma^c(\Theta, K, k)$ is the maximum over the available submarkets of $\varsigma^c(\Theta, K, k, Q^c, P^c, F^c)$,

$$\varsigma^c(\Theta, K, k, Q^c, P^c, F^c) = P^c\ \frac{\Psi_d(Q^c)}{Q^c}\ F^c - w(\Theta, K)\ \ \widetilde{n}(k, F^c). \tag{37}$$

Similarly for investment goods.

We will proceed by establishing a series of lemmas that will allow us to verify our conjecture. We start by establishing that firms in all submarkets have the same expected revenue, V: (We should discuss the role of this lemma) so we write

Lemma 2. $\Omega^c(\Theta, K, k) = \Omega^i(\Theta, K, k)$ *implies* $\varsigma^c(\Theta, K, k) = \varsigma^i(\Theta, K, k)$.

3.3 Competitive Search in the Market for Consumption Good

In addition to the decisions made by the households with respect to the main elements of the allocation (how much to consume, shop, work and save) its shoppers choose how to implement the shopping, this is which market to go to. In our environment with competitive search this means choosing which market tightness, price, and output size, (Q^c, P^c, F^c). These choices will give us two conditions to be satisfied by these 3 variables.

To simplify notation let $\eta = u_d \ \frac{1}{\Psi_d(Q^c)F^c}$ denote the sunk utility cost in terms of current consumption of having an extra shopper and $u_c(\Theta, K) \ u_c[C(\Theta, K)]$. Recall that $m(\Theta, K, \theta, s')$ is the value in terms of marginal utility of an additional unit of savings. We can write the contribution to the utility of a household of a shopper that chooses the best price–tightness pair as

$$\Phi = \max_{Q^c, P^c, F^c} \quad -\eta + \Psi_d(Q^c)F^c \ (\theta \ u_c(\Theta, K) - P^c m)$$

subject to the constraint

$$\varsigma^c \leq P^c \frac{\Psi_d(Q^c)}{Q^c} F^c - w(\Theta, K)\widetilde{n}(k, F^c). \tag{38}$$

This constraint reflects the fact that the only relevant markets are those that guarantee certain expected revenue for the firms. Substituting (38) with $k = K$, and $\Psi_d(Q)$, we rewrite the problem as

$$\Phi = \max_{Q^c, F^c} \ -\eta + A \ (Q^c)^{1-\alpha} \ F^c \left(\theta u_c(\Theta, K) - m \ \frac{\varsigma^{*c} + w(\Theta, K) \ \widetilde{n}(k, F^c)}{A(Q^c)^{-\alpha} F^c} \right). \tag{39}$$

The FOC over Q^c is given by

$$\begin{aligned} 0 &= (1-\alpha)\frac{A\theta u_c(\Theta, K)}{(Q^c)^\alpha} F^c - m\left[\varsigma^{*c} + w(\Theta, K)\widetilde{n}(k, F^c)\right] \\ &= (1-\alpha)\ \frac{A\theta u_c(\Theta, K)}{(Q^c)^\alpha} F^c - m\frac{AP^c}{(Q^c)^\alpha} F^c \end{aligned}$$

which yields an equation for the unique price equilibrium P^c

$$P^c(\Theta, K) = (1-\alpha)\frac{\Theta \ u_c(\Theta, K)}{M(\Theta, K)}. \tag{40}$$

The FOC over F^c is given by

$$0 = A(Q^c)^{1-\alpha}\theta u_c(\Theta, K) - Q^c m \ w(\Theta, K)\frac{\partial \widetilde{n}}{\partial F^c}.$$

Substituting equilibrium price from equation (40) and the relation $\partial\widetilde{n}/\partial F^c = 1/f_n$ for the given

$k = K$, we have

$$\frac{w(\Theta, K)}{P^{c*}(\Theta, K)} = \frac{1}{1-\alpha} A(Q^c)^{-\alpha} f_n \left[K, n^c(\Theta, K)\right],$$

where $N^c(\Theta, K) = \widetilde{n}(K, F^c(\Theta, K))$ is the labor associated with $k = K$ and proposed F^c. The market tightness Q^c is a function of the equilibrium value for producing consumption goods $\varsigma^c(\Theta, K)$,

$$Q^c(\Theta, K) = \left[\frac{A\ P^c(\Theta, K)\ f(K, N^c(\Theta, K))}{\varsigma^c(\Theta, K) + w(\Theta, K)\ N^c(\Theta, K)}\right]^{\frac{1}{\alpha}}. \tag{41}$$

3.4 Competitive search in the market for investment goods

In the same way than household shoppers, firm shoppers choose how to implement the shopping, this is which market tightness, price, and output size, (Q^i, P^i, F^i), and these choices yield two conditions to be satisfied by these 3 variables.

A shopper for the firm chooses the best price-tightness pair $\{Q^i, P^i, F^i\}$ to solve

$$\begin{aligned} \Phi_F &= \max_{Q^i, P^i, F^i} -w(\Theta, K) + \zeta\ \Psi_d(Q^i) F^i \left[E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} - P^i\right] \\ \text{s.t.} \quad \varsigma^i &\leq P^i \frac{\Psi_d(Q^i)}{Q^i} F^i - w(\Theta, K)\widetilde{n}(k, F^i). \end{aligned}$$

Substituting P^i, we can rewrite the problem as

$$\max_{Q^i, F^i} -w(\Theta, K) + \zeta\ A(Q^i)^{1-\alpha} F^i \left[E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} - \frac{\varsigma^i + w(\Theta, K)\widetilde{n}(k, F^i)}{A(Q^i)^{-\alpha} F^i}\right]$$

The first order condition over Q^i is given by

$$0 = \frac{(1-\alpha)\zeta A}{(Q^i)^\alpha} E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} F^i - \zeta(\varsigma^i + w(\Theta, K)\widetilde{n}(k, F^i)),$$

which implies that the equilibrium price is

$$P^i(\Theta, K) = (1-\alpha)\ E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\}. \tag{42}$$

The first order condition over F^i is given by

$$0 = \zeta\, A\,(Q^i)^{1-\alpha}\, E\left\{\frac{\Omega_k(\Theta', K', k')}{1+R(\Theta', K')}\right\} - \zeta\, Q^i w(\Theta, K)\frac{\partial \widetilde{n}}{\partial F^i},$$

Substituting equation (42) for price and $\partial \widetilde{n}/\partial F^i = 1/f_n$ for given $k = K$, we have

$$\frac{w(\Theta, K)}{P^{i*}(\Theta, K)} = \frac{1}{1-\alpha} A(Q^i)^{-\alpha} f_n(K, N^i(K, \Theta))$$

where $n^i(\Theta, K) = \widetilde{n}(K, F^i(\Theta, K))$ is the labor associated with $k = K$ and the amount produced by investment firms proposed F^i. The equilibrium market tightness Q^i as a function of ς^i is given by

$$Q^i(\Theta, K) = \left[\frac{A\; P^i(\Theta, K)\; F[K, N^i(\Theta, K)]}{\varsigma^i(\Theta, K) + w(\Theta, K) n^i(\Theta, K)}\right]^{\frac{1}{\alpha}}. \tag{43}$$

3.5 Equilibrium

We have now established the necessary conditions for equilibrium that arise from the households' and the firm's problems and from the shoppers' problems under competitive search. Before formally defining equilibrium we provide a series of lemmas that show that our conjecture that a sufficient characterization of the state is the pair (Θ, K) is correct. They amount to show that if all firms are equal then the choice of sector does yield a different choice of either labor or output or capital for the following period and hence all firms the following period will have identical capital K'.

Lemma 3. *All firms with $k = K$ choose markets with the same output $F = F^c = F^i$ and also the same labor input for production.*

Lemma 4. *The expected revenue per unit of output is the same in both sectors:*

$$P^c(\Theta, K)\frac{\Psi_d[Q^c(\Theta, K)]}{Q^c(\Theta, K)} = P^i(\Theta, K)\frac{\Psi_d[Q^i(\Theta, K)]}{Q^i(\Theta, K)}. \tag{44}$$

Lemma 5. *Firms with the same k chooses the same k' as future capital stock.*

Thwo more lemmas that will prove useful later V: (Have to say what for) follow. The first two state properties of investment prices unveiling a relation between the direct and the indirect costs of installing capital. We use $N^y(\Theta, K)$ to denote the labor used in production.

Lemma 6. *The investment price is proportional to the ratio of the wage and the amount of shopping that a worker can carry out.*

$$\frac{w(\Theta, K)}{\zeta \Psi_d[Q^i(\Theta, K)] f(K, N^y(\Theta, K))} = \frac{\alpha}{1-\alpha} P^i(\Theta, K) \tag{45}$$

Lemma 7. *The Euler equation of a firm equals the price of investment to the value of capital tomorrow.*

$$E\left\{\frac{P^i(\Theta', K')}{1+R(\Theta', K')}\left[\frac{\Psi_d(Q^i)}{Q^i} f_k\left(K', N^y(K', \Theta')\right) + (1-\delta)\right]\right\} = P^i(Ta, K). \tag{46}$$

We are now ready to define equilibrium in this economy (it extends that of an environment in **?**). For simplicity and to reduce notation, we take advantage of some of the above lemmas.

Definition 8. *Equilibrium is a set of decision rules and values for the household* $\{c, d, n, s', v\}$ *as functions of its state* (Θ, K, θ, s)*, firms decisions and values* $\{n^y, n^i, i, k', \Omega\}$ *as functions of its state* (Θ, K, k)*, and aggregate functions for shopping for investment* D^i*, shopping for consumption* D^c *consumption* C*, labor* N*, labor for production* N^y*, labor for shopping* N^k*, investment* I*, aggregate capital* G*, the measure of consumption producing firms* T*, wages* w*, consumption good prices* P^c*, consumption market tightness* Q^c*, production of firms* F *expected revenues* ς*, investment good prices* P^i*, investment market tightness* Q^i*, and the rate of return of the economy* R *as functions of the aggregate state* (Θ, K) *that satisfy*

1. Households choices and values $d(\Theta, K, \theta, s)$, $c(\Theta, K, \theta, s)$, $n(\Theta, K, \theta, s)$, $s'(\Theta, K, \theta, s)$, and $v(\Theta, K, \theta, s)$ satisfy (18-20) and (26-27).

2. Firms choose $n^k(\Theta, K, k)$, $i(\Theta, K, k)$, $k'(\Theta, K, k)$ and $\Omega(\Theta, K, k)$ to solve their problem (30). They satisfy conditions (32-33) and (36).

3. Competitive Search Conditions. Shoppers and sellers go to the appropriate submarkets, i.e., equations (40– 41) and (42– 43).

4. Representative Agent and Equilibrium Conditions:

$$
\begin{aligned}
C(\Theta, K) &= c(\Theta, K, \Theta, 1) && (47)\\
D^c(\Theta, K) &= d(\Theta, K, \Theta, 1) && (48)\\
D^i(\Theta, K) &= \zeta N^k(\Theta, K) && (49)\\
N(\Theta, K) &= n(\Theta, K, \Theta, 1) && (50)\\
N^y(\Theta, K) &= n^y(\Theta, K, K) && (51)\\
N^k(\Theta, K) &= n^k(\Theta, K, K) && (52)\\
I(\Theta, K) &= i(\Theta, K, K) && (53)\\
K'(\Theta, K) &= G(\Theta, K) = k'(\Theta, K, K) && (54)
\end{aligned}
$$

5. Market Clearing Conditions:

$$
\begin{aligned}
s'(\Theta, K, \Theta, 1) &= 1. && (55)\\
I(\Theta, K) &= D^i(\Theta, K)\, \Psi_d(Q^i(\Theta, K))\, f(K, N^y(\Theta, K)) && (56)\\
C(\Theta, K) &= D^c(\Theta, K)\, \Psi_d(Q^c(\Theta, K))\, f(K, N^y(\Theta, K)) && (57)\\
N(\Theta, K) &= N^y(\Theta, K) + N^k(\Theta, K) && (58)\\
Q^c(\Theta, K) &= \frac{T(\Theta, K)}{D^c(\Theta, K)} && (59)\\
Q^i(\Theta, K) &= \frac{1 - T(\Theta, K)}{D^i(\Theta, K)} && (60)\\
& && (61)
\end{aligned}
$$

Comments Note that $\Omega(\Theta, K, K) = 1 + R(\Theta, K)$, a direct implication from the fact that the numeraire is the stock market pre-dividends. The financial wealth of the household is $s(1+R)$ and is equal to the stock market today including the dividends. Note also that the share of consumption expenditure equals the share of firms T, i.e. $\frac{P^c C}{P^c C + P^i I} = T$.

3.6 Equilibrium are Optimal

We now turn to establish optimality of equilibrium. V: (Kjetil, Can you write this?)

3.7 Understanding Solow Residual and Labor Share

To analyze movements in the measured Solow residual in this economy, we need to define real GDP and labor share. In the appendix we provide a detailed analysis of various definitions of output in the data and we show that the historical measure of GDP based on base year prices is very similar to the current measure based on a Tornqvist index. This is important because of the changing relative prices of consumption and investment. We measure GDP using base year prices, which is given by

$$Y = P_0^c C + P_0^i I \tag{62}$$

where P_0^c is the base year consumption price and P_0^i is the base year investment price. Replacing C and I with the aggregate production function the matching terms, we can write GDP as

$$Y = \left[P_0^c\, A(D^c)^\alpha (T^c)^{1-\alpha} + AP_0^i\, (D^i)^\alpha (T^i)^{1-\alpha}\right] zK^{\gamma_k}(N^y)^{\gamma_n}.$$

Denote labor labor share by γ, the Solow residual $\overline{Z}$ is defined as

$$\overline{Z} = \frac{Y}{K^{1-\gamma}N^\gamma}.$$

Substituting Y in the Solow residual, we have

$$\overline{Z} = z\left[P_0^c\, A(D^c)^\alpha (T^c)^{1-\alpha} + AP_0^i (D^i)^\alpha (T^i)^{1-\alpha}\right] K^{\gamma_k+\gamma-1}(N^c)^{\gamma_n} N^{-\gamma},$$

or

$$\overline{Z} = A\, z \underbrace{\left[P_0^c\, (D^c)^\alpha (T^c)^{1-\alpha} + P_0^i (D^i)^\alpha (T^i)^{1-\alpha}\right]}_{\text{Demand Effect 1.04}} \overbrace{\left(\frac{N^y}{N}\right)^{\gamma_n}}^{\text{Effective Work -.00}} \underbrace{K^{\gamma_k-(1-\overline{\gamma})} N^{\gamma_n-\overline{\gamma}}}_{\text{Share's Error -.00}}. \tag{63}$$

The Solow residual depends on the demand effects, increases in demand without changes in technology increase the Solow residual. The two additional terms are due to the mismeasure of productive labor and to the imputation of CRS with a measure of labor share.

Labor share is defined as the ratio of total labor compensation over GDP, $\gamma = wN/Y$. Some algebra yields the following awful expression for labor share.

$$\gamma \;=\; \frac{\frac{1}{1-\alpha}\gamma_n P_0^c A(D^c)^{\alpha}(T^c)^{-\alpha}K^{\gamma_k}(N^y)^{\gamma_n - 1}N}{P_0^c A(D^c)^{\alpha}(T^c)^{1-\alpha}K^{\gamma_k}(N^y)^{\gamma_n} + P_0^i \gamma_n A(D^i)^{\alpha}(T^i)^{1-\alpha}K^{\gamma_k}(N^y)^{\gamma_n}}.$$

Labor share will be time varying but we can use steady states to both define base year prices and to compute labor share which yields a much more palatable expression.

$$\gamma = \frac{1}{1-\alpha}\frac{\gamma_n (N^y)^{-1}N}{T^c + \frac{P_0^i}{P_0^c}\frac{A(D^i)^{\alpha}(T^i)^{-\alpha}}{A(D^c)^{\alpha}(T^c)^{-\alpha}}T^i} = \frac{1}{1-\alpha}\frac{\gamma_n (N^y)^{-1}N}{T^c + T^i} = \frac{1}{1-\alpha}\gamma_n \frac{N}{N^y}$$

Further manipulations allow us to get the ratio of N and N^y from the parameters of the economy yielding an expression for steady state labor share that depends only on the parameters of the economy and that is given by

$$\gamma = \frac{1}{1-\alpha}\gamma_n + \frac{\alpha\delta}{(1-\alpha)(1/\beta - 1 + \delta)}\gamma_k, \tag{64}$$

which reflects the fact that there are workers in production in shopping. When $\alpha = 0$, we get the same labor share of γ_n as the standard model.

In empirical applications $N - N^y$ is small and γ_n is close to γ. This implies that the last two terms in equation (63) are almost constant over the business cycle and that all movements in the Solow residual are due to movements in demand.

4 Mapping the model to data

We now turn to map the model to data describing which properties of the steady state of the model we are seeking. We also make the case for a few functional forms that we find appropriate.

Preferences We pose separable preferences to be able to clearly discuss the role of the Frisch labor elasticity and also isolate the role of shopping. The per period utility function or felicity is given by (65)

$$u(c, n, d) = \frac{c^{1-\sigma}}{1-\sigma} - \chi\frac{n^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - d \tag{65}$$

where we omit the shocks to preferences for simplicity. This choice of utility function requires that we specify four parameters: the discount rate β, the coefficient of risk aversion, σ, Frisch elasticity of labor, ν, and the parameter that determines average hours worked, χ. That the disutility of shopping is linear is not important as the shocks that affect it will shape its properties.

Production Technology Firms have a Cobb-Douglas production function with decreasing returns.

$$f(k, n) = z \;\; k^{\gamma_k} \, (n^y)^{1-\gamma_n}$$

There is no need to impose constant returns to scale given the fact that there are a limited number of locations that are valuable.[6] Note also that parameter z is just a parameter that determines units, not a shock as in the standard model. In addition a worker devoted to shop for investment goods produces ζ units of shopping services, allowing for the possibility that firms could be a lot more efficient shoppers than people. Capital depreciates at rate δ. This adds 5 more parameters.

Matching Technology As we have explicitly imposed throughout the paper matching in goods markets occurs via competitive search where they are index by tightness, prices, and quantities via a Cobb-Douglas matching technology

$$A \, D^{\alpha} \; T^{1-\alpha} \tag{66}$$

which adds two more parameters.

4.1 Calibration

We now turn to describe the calibration process. We first describe the steady state targets for the model economy and then we estimate the process for the shocks. Some of the targets are standard macroeconomic variables, while others are specific to this economy. Some of those are not exactly steady state targets as much as values of parameters themselves (the intertemporal elasticity of substitution and the Frisch elasticity of labor). Table 1 reports the targets and, following tradition, reports also the parameter values most associated with those targets.

The targets are defined in yearly terms even though the model period is a quarter. The value

[6] The model could be easily extended to accommodate the costly creation of such locations.

Table 1: Calibration Targets, Implied Aggregates and (quarterly) Parameter values

Targets	Value	Parameter	Value
First Group: Parameters are set autonomously			
Risk aversion	2.	σ	2.
Real interest rate	4.%	β	0.99
Frisch elasticity	0.72	$\frac{1}{\psi}$	0.72
Second Group: Standard Targets			
Fraction of time spent working	30%	χ	16.81
Physical Capital to Output Ratio	2.75	δ	0.07
Consumption Share of Output	0.80	γ_k	0.23
Labor Share of income	0.67	γ_n	0.59
Units of output	1.	z	2.03
Third Group: Targets specific to this economy			
Share of production workers	97.%	ζ	3.16
Capacity Utilization of Consumption Sector	0.81	A	0.97
Capacity Utilization of Investment Sector	0.81	α	0.09
Fourth Group: Aggregate Variables Implied			
Percentage of GDP payble to Shoppers	2.%		
Percentage of Cost of New Capital that is internal	9%		
Wealth to output ratio	3.33	From hhold budget constraint	
Relative price of investment in terms of consumption	1.	Equal capacity utilizations	

of 2 for the intertemporal elasticity of substitution is quite standard it does not merit discussion as does the value of the rate of return of 4.%. The Frisch elasticity is much argued among economists. We will provide below V: (In section) alternative assessments of the performance of the model under different values of this elasticity. The specific value of .72 that we have chosen is based on the measurements of **?** who take into account the response of hours worked by he hole household (and not only its head). V: (Is this true?)

The targets in the second group are what we label standard in the sense that they are targets commonly used in the literature. Perhaps, the only one that deserves some discussion is the consumption to output ratio that we set at 0.8 rather than at 0.75 percent which is more usual. We do this because we take a stand that investment is business investment and not household investment given that there are separate shopping technologies.

The third group of targets is not standard because they relate to variables that are not present in standard real business cycle models. We set the share of workers that are not productive workers but are in charge of purchasing to 3%. While there is no obvious statistic to compare this variable with, we think that it deems a low value. Note that, as the fourth group of variables reports, 2 percent of GDP is used to pay for these workers and the the cost of installing new capital involves internal costs (a form of adjustment costs) as well as the cost of the investment itself and the internal costs are 9% of the total cost of new capital. In the same fashion we target capacity utilization to be 81 percent in both sectors. While our notion of capacity utilization is not the same as that collected by the Federal Reserve System, it is clearly related.[7] Our choices for steady state capacity in both sectors are close to the postwar average of 0.81 (see **?**) of the official data series.

[7] The Federal Reserve Statistical Release in its oficial documentation of Industrial Production and Capacity Utilization has some Capacity Utilization Explanatory notes that states *"Federal Reserve Board constructs estimates of capacity and capacity utilization for industries in manufacturing, mining, and electric and gas utilities. For a given industry, the capacity utilization rate is equal to an output index (seasonally adjusted) divided by a capacity index. The Federal Reserve Board's capacity indexes attempt to capture the concept of sustainable maximum output-the greatest level of output a plant can maintain within the framework of a realistic work schedule, after factoring in normal downtime and assuming sufficient availability of inputs to operate the capital in place.*

Coverage. Capacity indexes are constructed for 89 detailed industries (71 in manufacturing, 16 in mining, and 2 in utilities), which mostly correspond to industries at the three- and four-digit NAICS level. Estimates of capacity and utilization are available for a variety of groups, including durable and nondurable manufacturing, total manufacturing, mining, utilities, and total industry. Manufacturing consists of those industries included in the North American Industry Classification System, or NAICS, definition of manufacturing plus those industries-logging and newspaper, periodical, book and directory publishing-that have traditionally been considered to be manufacturing and included in the industrial sector. Also, special aggregates are available, such as high-technology industries and manufacturing excluding high-technology industries."

The fourth group of variables in Table 1 report the implied variables of some interesting aggregates. The one that we have not talked about yet is the wealth to output ratio.[8]

5 Estimating Demand Shocks off the Solow Residuals

We now turn to estimate a process for preference shocks that yield the measured Solow residual. We proceed by estimating by Maximum Likelihood a process for various shocks so that the implied Solow residual generated by the model resembles that of the U.S. economy.

We now proceed to estimate various versions of our model using Solow residual data, linearly detrended, from 1960.Q1–2009.Q4 with a labor share of XXX. We proceed by estimating by Maximum Likelihood various processes, one for each of the four natural sources of demand shocks in our model, three of them to the utility function:

$$\theta_c \, \frac{c^{1-\sigma}}{1-\sigma} - \theta_n \, \frac{n^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \theta_d \; d \tag{67}$$

and one to the shopping technology ζ. In all cases the best specification turned out to be an AR(1).

The estimates are in Table 2. The last column of the table reports the estimates and its properties of the Solow residual as an exogenous process as is done in the RBC literature. In all cases, except perhaps for investment shopping shocks, we can estimate a process that yields observable Solow residuals like those in the U.S. economy. The estimates are quite precise and they are as good as those of a technology shock taken directly of the Solow residual without the need of a model.

The addition of the shopping process to the mechanism that transforms produced goods into used goods is capable to generate productivity movements that look like technology shocks without the technology moving at all.

We now turn to explore the business cycle statistics of economies with demand shocks and compare them with the data and the statistics generated by standard RBC models.

[8]This statistic is implied by the steady state budget constraint of the household given the rate of return, consumption share, and labor share.

Table 2: Estimates of the Processes for the Shocks look like TFP shocks

	Our Model with Shopping					Standard RBC
Shocks to	Shop. Disut	Cons Ut	Labor Disut	Firm's Shop.	Tech	Tech
Shocks to	θ_d	θ_c	θ_n	ζ	z	z
ρ	0.942	0.788	0.720	0.990	0.954	0.940
st. dev.	0.022	0.072	0.060	0.007	0.018	0.023
t-stat	43.80	10.98	12.06	135.99	52.08	40.97
σ	0.086	0.164	0.170	0.333	0.006	0.006
st. dev.	0.004	0.008	0.009	0.017	0.0003	0.0003
t-stat	20.00	19.96	20.00	19.99	20.03	20.04
Likelihood	738.75	739.90	739.96	737.21	738.54	738.78
Var of $\bar{Z}$	3.08	3.05	3.01	2.54	3.02	3.06
Autocorr of $\bar{Z}$	0.94	0.94	0.94	0.93	0.94	0.94

6 Business cycle properties of economies with demand shocks

We start reporting in Table 3 the values of the business cycle moments for the U.S. data and for a standard RBC model that shares the steady state targets with our model. Appendix B reports how the standard RBC model is calibrated and gives some details of how this Table is constructed. V: (Yan can you fill this?). Notice that we report variances and not standard deviations and that all variables move together. Note also that the variance of hours is quite small. This is due to the relatively low Frisch elasticity of substitution that we use in our calibration (that we are using variances gives an additional optical illusion of being small).

Table 4 reports the main business cycle statistics of the model economies with various shock processes that affect the willingness to spend of agents. Each one of these shocks generates a process for the measured Solow residual that resembles that of the U.S. Economy.

The business cycles properties of these economies are very different from each other and have various important differences with those of the data and the standard RBC model. Perhaps, their only feature in common is that output and the Solow residual move together, something that should be implied almost automatically by the way that the Solow residual is constructed.

1. *The Shopping disutility shock*. Panel (a) of the Table reports the statistics of this economy.

Table 3: Main Business Cycle Moments: U.S. Data and Standard RBC Model

	(a) Data			(b) Standard RBC		
	Variance	Cor w Y	Autocor	Variance	Cor w Y	Autocor
Z	3.19	0.43	0.94	3.06	0.99	0.94
Y	**2.38**	1.00	0.86	**0.96**	1.00	0.71
N	**2.50**	0.87	0.91	**0.10**	0.97	0.71
C	1.55	0.87	0.87	0.05	0.93	0.76
I	34.15	0.92	0.80	17.07	0.97	0.71
cor(C, I)	**0.74**			**0.91**		

All variables except the Solow residual are HP-filtered. The variances of Labor, Consumption and Investment are relative to that of Output.

The first thing that jumps to the eye is that labor is negatively correlated with output. This is clearly counterfactual as the trigger of fluctuations. It seems that shocks to the shopping disutility act pretty much like a wealth effect. Consumption goes up and work go down. Consumer shoppers are more effective and consumption producing firms operate at higher capacity allowing for lower work effort. However, firms do not improve in their shopping ameliorating the ability of transferring the gains to the future.

2. *The Consumption utility shock*. There is now a tremendous expansion associated to this shock partly due to its huge variance. The volatiliy of output is more than twice that of the data. Hours go up even more than output and consumption goes up tremendously. The reasons for these are two. First, there is only one shock which makes all variables be strongly correlated which means that if the measured Solow residual and hours move as much as in the data, then their high correlation imply that output goes up a lot more than in the data. The second reason is that now investment is negatively correlated with the other variables. The demand for consumption increases the price of investment enough to deter firms from increasing their capital despite the persistence of the shock. This is very counterfactual.

3. *The Working disutility shock*. Panel (c) of the Table reports the statistics of this economy. This shock has also a very large variance and it generates a tremendous volatility of output, six times that of the data. The volatility of hours is even larger, about 15 times that of the data. Clearly, to induce a movement in the Solow residual as large as in the data, the movement in hours has to be tremendous. So much, in fact, as to make the Solow reisudual countercyclical. V: (Clearly this is weird given that there is only one shock. This has to be

Table 4: Main Business Cycle Moments: Various Economies with Demand Shocks

(a) Shop Disut θ_d

	Variance	Cor w Y	Autocor
Z	3.08	1.00	0.94
Y	**0.38**	1.00	0.71
N	**0.06**	**-1.00**	0.72
C	0.50	1.00	0.71
I	0.06	0.99	0.69
cor(C, I)	**0.99**		

(b) Cons Ut θ_c

	Variance	Cor w Y	Autocor
Z	3.05	0.71	0.94
Y	**5.87**	1.00	0.60
N	**7.62**	0.93	0.69
C	58.03	1.00	0.60
I	338.15	**-1.00**	0.60
cor(C, I)	**-1.00**		

(c) Labor Disut θ_n

	Variance	Cor w Y	Autocor
Z	3.01	**-0.85**	0.94
Y	**35.84**	1.00	0.58
N	**98.13**	0.99	0.55
C	2.89	0.76	0.84
I	634.89	0.98	0.55
cor(C, I)	**0.63**		

(d) Firms' Shopping Tech ζ

	Variance	Cor w Y	Autocor
Z	2.53	0.78	0.93
Y	**1.68**	1.00	0.75
N	**0.50**	0.78	0.69
C	0.42	**-0.52**	0.75
I	66.43	0.96	0.70
cor(C, I)	**-0.73**		

(e) Technology Shock z

	Variance	Cor w Y	Autocor
Z	3.02	0.98	0.94
Y	**0.62**	1.00	0.73
N	**0.01**	**-0.38**	0.96
C	0.22	0.98	0.78
I	4.56	0.98	0.69
cor(C, I)	**0.91**		

All variables except the Solow residual are HP-filtered. The variances of Labor, Consumption and Investment are relative to that of Output.

a result of the transmission mechanism (the variables are not simultaneous) or perhaps due to the two other components of the Solow residual that in this case may be very large.

4. *The shock to the firm's shopping technology*. The last panel of Table 4 reports the findings when the demand shocks are due to a reduction in the costs of shopping for investment goods. Clearly, investment goes up a sizeable amount. Interestingly, this also seems to act like a wealth effect for the household as it increases its consumption (the price of consumption goes down given the higher efficiency of the investment goods market) and reduces its leisure. Clearly, this shock alone cannot be the trigger of the business cycle.

To summarize, the model economies with demand shocks do not display the business cycle properties of the data in terms of the comovements of the major variables. In these economies either investment or labor are countercyclical which is dramatically counterfactual. The RBC economies, display the right comovements of variables but a low induce variability of labor. Perhaps the main shortcoming of these demand shocks is that they either generate gains in market efficiency that act as a type of wealth effect that makes labor countercyclical (shocks to the shopping disutility and the the firm's shopping technology) or they generate a negative correlation of consumption and investment (shocks to consumption utility) while requiring enormously volatile preference shocks.

We now turn to investigate whether a certain combination of demand shocks generates simultaneously, in addition to the movements in the Solow residual, the right amount of output volatility and the same comovements of variables as in the data.

7 Multiple Shocks to the Economy

We now turn to explore the performance of the model economies when we have various shocks. We start with various combinations of demand shocks designed to replicate specific statistics of the data in Section 7.1 where we use exactly identified GMM. We then use Maximum Likelyhood to attempt to discriminate the role played by various types of shocks including technology shocks in Section 7.2.

7.1 Combination of Demand Shocks

We have seen that while the demand shocks are capable of generating the properties of the observed Solow residuals, they do not, by themselves, generate comovements like those in the data or even in the standard RBC model. This is partly due to the particular type of demand shocks that hit disproportionaly a particular subset of the relevant variables. We know turn to estimate various combinations of demands shocks targeting some of the comovements. We estimate the parameters of the shock process by exactly identified GMM.

Table 7 reports the business cycle statistics of various economies with multiple demand shocks. The first column of the Table shows the performance of an economy with shocks to the shopping disutility of households and to the shopping efficiency of firms designed to target the properties of the Solow residual and the relative variances and correlation of Consumption and Investment. The reason is to get the relative size of these consumption and demand shocks *"right"*. We can see immediately that these shocks gnerate a tremendous volatility of output with only limited volatility of hours, which in turn are countercyclical. These comovements are not good in the sense that it seems we are overshooting the possible contributions of these shocks in shaping productivity.

7.2 Disentangling technology shocks from demand shocks

8 Robustness of Findings

We now turn to explore other features of the class of models that we have developed that go beyond the standard business cycle analysis. After that we look at the properties of the model economies when some targets are changed.

8.1 Implications for other variables

There are some macroeconomics aggregates of interest that are present in our model economies and are absent in standard business cycle models.

1. *The relative price of investment.* The role of the relative price of investment in shaping economic performance has been studied in various contexts, from its role in shaping the skill premia (**?**) to its role as a direct source of business cycles (**?**). In most of these cases such relative price is taking to be an exogenous object that depends purely on technological considerations (an exception is **?** that uses a non-linear production possibility frontier). In our economy the relative price of consumption and investment is purely an economic object since consumption and investment are perfect substitutes in production. The reason why this price oscillates is due to changes in the willingness to pay relative to shop by consumers and firms. When consumers get a reduction in the disutility of shopping, they send more shoppers relative to firms which makes market tightness higher V: (lower?) than that of

Table 5: Combination of Demand Shocks

Shocks Involved	Data	θ_d, ζ	θ_c, ζ	θ_n, ζ	θ_d, θ_c	θ_d, θ_n	θ_c, θ_n
Parameters		$\rho_d, \rho_\zeta, \sigma_d, \sigma_\zeta$ $cor(\epsilon_c, \epsilon_\zeta)$	$\rho_c, \rho_\zeta, \sigma_c, \sigma_\zeta$ $cor(\epsilon_c, \epsilon_\zeta)$	$\rho_n, \rho_\zeta, \sigma_n, \sigma_\zeta$ $cor(\epsilon_n, \epsilon_\zeta)$	$\rho_d, \rho_c, \sigma_d, \sigma_c$ $cor(\epsilon_d, \epsilon_c)$	$\rho_d, \rho_n, \sigma_d, \sigma_n$ $cor(\epsilon_d, \epsilon_n)$	$\rho_c, \rho_n, \sigma_c, \sigma_n$ $cor(\epsilon_c, \epsilon_n)$
Restrictions Imposed		$\rho_d = \rho_\zeta$	$\rho_c = \rho_\zeta$	$\rho_n = \rho_\zeta$	$\rho_d = \rho_c$	$\rho_d = \rho_n$	$\rho_c = \rho_n$
Moments Targeted		$Var(Z)$ $Atcor(Z)$ $\frac{Var(\widehat{C})}{var(\widehat{I})}$ $cor(C, I)$	$Var(Z)$ $Atcor(Z)$ $\frac{Var(\widehat{C})}{var(\widehat{I})}$ $cor(C, I)$	$Var(Z)$ $Atcor(Z)$ $var(\widehat{Y})$ $cor(C, I)$	$Var(Z)$ $Atcor(Z)$ $var(\widehat{Y})$ $cor(C, I)$	$Var(Z)$ $Atcor(Z)$ $var(\widehat{Y})$ $cor(C, I)$	$Var(Z)$ $Atcor(Z)$ $var(\widehat{Y})$ $cor(C, I)$
Var Z	3.19	3.19	3.20	3.19	3.19	3.19	3.15
Var $\widehat{Y}$	2.38	0.65	6.74	2.47	2.31	2.38	2.41
Var $\widehat{N}$	2.50	0.02	6.98	6.42	3.28	5.96	5.84
Var $\widehat{C}$	1.55	0.24	2.55	0.94	3.60	0.91	1.60
Var $\widehat{I}$	34.15	5.47	56.68	20.67	0.00	19.91	9.99
cor($\widehat{Z}$, $\widehat{Y}$)	.63	.99	.92	-0.60	0.67	-0.06	-0.40
cor($\widehat{N}$, $\widehat{Y}$)	.87	-.21	.98	0.98	0.93	0.91	0.99
cor($\widehat{C}$, $\widehat{I}$)	.74	.74	.74	0.74	0.92	0.74	0.77

Table 6: Combination of Demand Shocks and/or Technology Shocks

Shocks Involved	Data	$\theta_d, \theta_c, \zeta$	θ_d, θ_c, z
Parameters		$\rho_d, \rho_c, \rho_\zeta, \sigma_d, \sigma_c, \sigma_\zeta$	$\rho_d, \rho_c, \rho_z, \sigma_d, \sigma_c, \sigma_z$
		$cor(\epsilon_d, \epsilon_c)$	$cor(\epsilon_d, \epsilon_c)$
		$cor(\epsilon_d, \epsilon_\zeta)$	$cor(\epsilon_d, \epsilon_z)$
		$cor(\epsilon_c, \epsilon_\zeta)$	$cor(\epsilon_c, \epsilon_z)$
Restrictions Imposed		$\rho_d = \rho_c = \rho_\zeta$	$\rho_d = \rho_c$
Moments Targeted		$Var(Z)$, $Atcor(Z)$	$Var(Z)$, $Atcor(Z)$
		$Var(\widehat{Y})$, $Atcor(\hat{Y})$	$Var(\widehat{Y})$, $Atcor(\hat{Y})$
		$Var(\hat{N})$, $\frac{Var(\widehat{C})}{var(\widehat{I})}$	$Var(\hat{N})$, $\frac{Var(\widehat{C})}{var(\widehat{I})}$
		$cor(\widehat{Y}, \widehat{N})$, $cor(\widehat{C}, \widehat{I})$	$cor(\widehat{Y}, \widehat{N})$, $cor(\widehat{C}, \widehat{I})$
Var Z	3.19	3.19	3.26
Var $\widehat{Y}$	2.38	2.47	2.56
Var $\widehat{N}$	2.50	2.52	2.07
Var $\widehat{C}$	1.55	1.01	1.15
Var $\widehat{I}$	34.15	24.23	21.89
cor($\widehat{Z}$, $\widehat{Y}$)	.63	.74	0.80
cor($\widehat{N}$, $\widehat{Y}$)	.87	.84	0.78
cor($\widehat{C}$, $\widehat{I}$)	.74	.55	0.59
Var $\widehat{p}^i/\widehat{p}^c$	1.08	2.69	0.04
cor $\widehat{p}^i/\widehat{p}^c$, $\widehat{Y}$	-0.23	-0.98	0.96

Table 7: Technology Shocks

Shocks Involved	Data	Standard RBC	Our Model				
			Benchmark	Low σ_d	Low σ_d	Low persistence	High Frisch
Shock		$\rho_z = 0.94$	$\rho_z = 0.954$	$\rho_z = 0.954$	$\rho_z = 0.954$	$\rho_z = 0.80$	$\rho_z = 0.954$
		$\sigma_z = 0.006$	$\sigma_z = 0.0062$	$\sigma_z = 0.0062$	$\sigma_z = 0.0062$	$\sigma_z = 0.0062$	$\sigma_z = 0.0062$
Parameters			$\sigma_d = \infty$	$\sigma_d = 0.72$	$\sigma_d = 0.01$	$\sigma_d = \infty$	$\sigma_d = \infty$
		$\nu = 0.72$	$\nu = 0.72$	$\nu = 0.72$	$\nu = 0.72$	$\nu = 0.72$	$\nu = 1.1$
Var Z	3.19	3.06	3.02	3.11	3.21	0.83	3.07
Var $\widehat{Y}$	2.38	0.96	0.62	0.59	0.58	0.73	0.62
Var $\widehat{N}$	2.50	0.10	0.01	0.01	0.01	0.04	0.02
Var $\widehat{C}$	1.55	0.05	0.22	0.24	0.25	0.10	0.21
Var $\widehat{I}$	34.15	17.07	4.56	4.51	4.47	10.34	4.80
cor$(\widehat{Z}, \widehat{Y})$	.63	.99	.99	.99	.99	.99	.99
cor$(\widehat{N}, \widehat{Y})$	.87	.97	-.38	-.46	-.52	.76	-.28
cor$(\widehat{C}, \widehat{I})$	.74	.71	.91	.91	.92	.78	.90
var $\widehat{D}^c$			0.22	0.05	0.00	0.10	0.21
var $\widehat{D}^i$			1.76	1.73	1.70	6.33	1.95
var $\widehat{T}^c$			0.12	0.12	0.12	0.36	0.13
var $\widehat{N}^c$			0.01	0.01	0.02	0.02	0.02
var $\widehat{R}$			0.049	0.047	0.046	0.81	0.06
cor $(\widehat{D}^c, \widehat{Y})$			-.98	-.98	-.98	-.88	-.97
cor $(\widehat{D}^i, \widehat{Y})$			.92	.92	.91	.96	.92
cor $(\widehat{T}^c, \widehat{Y})$			-.95	-.95	-.94	-.97	-.95
cor $(\widehat{N}^c, \widehat{Y})$			-.70	-.74	-.77	.60	-.57
cor $(\widehat{R}, \widehat{Y})$			-.969	-.966	-.964	-.992	-.967
var DF			0.014	0.006	0.002	0.003	0.014
var SF			0.002	0.002	0.002	0.003	0.002
var FF			0.072	0.070	0.069	0.027	0.061

Table 8: Business Cycle Moments of Other Variables: U.S. Data and model with shocks to

(a) Data

	Variance	Cor w Y	Autocor
Y	**2.38**	1.00	0.86
p_i/p_c	0.47	-0.23	0.92
S&P 500	101.48	0.41	0.82
Cap Ut	8.97	-0.45	0.89

(b) Model with Shocks to

	Variance	Cor w Y	Autocor
Y			
p_i/p_c			
Stock Market			
Cap Ut			

All variables are HP-filtered. The variances are relative to that of Output.

investment but the price of consumption lower V: (higher?) than that of investment.

2. *The stock market.* Note that we have normalized the stock market to 1. Consequently the inverse of the price of consumption p^c is the value of the stock market in terms of current consumption. In this economy the value of firms changes not only because of changes in the costs of shopping for capital, but also because the value of the location that matches households or firms' shoppers changes.

3. *Capacity Utilization.* In the model economies the ratio of output to potential output is constantly changing, with higher utilization resulting in higher measured productivity. In the U.S. economies, the Federal Reserve Board has constructed a series of capacity utilization that while does not coincide with our notion, is clearly related.

Table 8 shows the business cycle behavior of these three variables in the model and in the data. V: (Yan: Need to complete table and discuss it.)

8.2 Alternative calibration targets

To find out how our findings change when we change the calibration targets, we have performed a large number of experiments, much larger than is worth reporting. Perhaps, the most controversial of calibration targets is the Frisch elasticity of labor. For this reason we have repeated the experiments reported in Tables 2 and 4 with a higher Frisch elasticity. In particular the value that we have looked at is 1.1. This value comes from considering in additional to the two–adult typical household the response of other less typical individuals such as teenagers and people in the thresholds of retirement. The findings are in Table V: (Yan: We have to add this)

9 Conclusion

APPENDIX

A Proof of some results

Lemma 2. $\Omega^c(\Theta, K, k) = \Omega^i(\Theta, K, k)$ *implies* $\varsigma^c(\Theta, K, k) = \varsigma^i(\Theta, K, k)$.

Proof. From the firms' first order condition over k' (36) it is clear that both marginal return and marginal cost of capital are independent of firm's current choice over which goods to produce and choice of labor for production. This implies that firms simply search for the markets that give them the best current revenue. Thus $\Omega^c(\Theta, K, k) = \Omega^i(\Theta, K, k)$ implies $\varsigma^{c*}(\Theta, K, k) = \varsigma^{i*}(\Theta, K, k)$. $\square$

Lemma 3. *All firms with $k = K$ choose markets with the same output $F = F^c = F^i$ and also the same labor input for production.*

Proof. Let's define $n^c(K, \Theta)$ as the necessary labor for a consumption-producing firm with capital $k = K$ to produce output $y^c(\Theta, K)$, namely $n^c(K, \Theta) = \widetilde{n}(K, y^c(\Theta, K))$. Similarly, we define $n^i(K, \Theta) = \widetilde{n}(K, y^i(\Theta, K))$ for investment-producing labor. In equilibrium, firms are indifferent of producing consumption goods or investment goods, i.e. $\varsigma^{c*} = \varsigma^{i*}$. By definition, $\varsigma^{c*} = P^c(\Theta, K)A[Q^c(\Theta, K)]^{-\alpha}y^c(\Theta, K) - w(\Theta, K)n^c(\Theta, K)$. We can further rewrite ς^{c*} using equilibrium conditions from the competitive search,

$$
\begin{aligned}
\varsigma^{c*} &= P^c(\Theta, K)A[Q^c(\Theta, K)]^{-\alpha}y^c(\Theta, K) - w(\Theta, K)n^c(\Theta, K) \\
&= w(\Theta, K)\left[\frac{P^c(\Theta, K)A[Q^c(\Theta, K)]^{-\alpha}y^c(\Theta, K)}{w(\Theta, K)} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{A[Q^c(\Theta, K)]^{-\alpha}y^c(\Theta, K)}{A[Q^c(\Theta, K)]^{-\alpha}f_n[K, n^c(\Theta, K)]} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{y^c(\Theta, K)}{f_n[K, n^c(\Theta, K)]} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{zK^{\gamma_k}(n^c(\Theta, K))^{\gamma_n}}{\gamma_n zK^{\gamma_k}(n^c(\Theta, K))^{\gamma_n - 1}} - n^c(\Theta, K)\right] \\
&= w(\Theta, K)\left[(1-\alpha)\frac{n^c(\Theta, K)}{\gamma_n} - n^c(\Theta, K)\right] \\
&= \frac{1-\alpha-\gamma_n}{\gamma_n}w(\Theta, K)n^c(\Theta, K).
\end{aligned}
$$

The third equality uses the following equilibrium condition from the competitive search problem of consumption goods

$$\frac{w(\Theta, K)}{P^c(\Theta, K)} = \frac{1}{1-\alpha} A(Q^c)^{-\alpha} f_n[K, n^c(\Theta, K)].$$

The fifth equality uses the definition of the production function $f(k, n) = zk^{\gamma_k} n^{\gamma_n}$. Similarly, we have

$$\varsigma^{i*} = \frac{1-\alpha-\gamma_n}{\gamma_n} w(\Theta, K) n^i(\Theta, K).$$

Equalizing ς^{i*} and ς^{c*} implies that $n^c(\Theta, K) = n^i(\Theta, K)$, the labor inputs for production are the same for the firms with the same capital $k = K$. Their outputs must be the same too. □

Lemma 4. *In equilibrium, the following holds*

$$P^c(\Theta, K) \frac{\Psi_d[Q^c(\Theta, K)]}{Q^c(\Theta, K)} = P^i(\Theta, K) \frac{\Psi_d[Q^i(\Theta, K)]}{Q^i(\Theta, K)}. \tag{68}$$

Proof. Under Lemma 3, firms with the same $k = K$ have the same labor input for production, the same output, and the same ς^*. This implies equation (44). □

Lemma 5. *Firms with the same k chooses the same k' as future capital stock.*

Proof. According to Lemma 3, firms with the same k choose the same labor input for both consumption producing and investment producing. For a firm that considers to produce consumption tomorrow, the first order condition over k' is given by

$$E\left\{ \frac{-w(\Theta', K')\widetilde{n}_k(k', y(\Theta', K')) + (1-\delta)\left(\frac{w(\Theta', K')}{\zeta\, \Psi_d[Q^i(\Theta', K')]\, f[(\Theta', K')]} + P^i(\Theta', K')\right)}{1 + R(\Theta', K')} \middle| \Theta \right\}$$
$$= \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^y(\Theta, K)]} + P^i(\Theta, K). \tag{69}$$

For a firm that considers to produce investment tomorrow, the first order condition over k' is

$$E\left\{ \left. \frac{-w(\Theta', K')\widetilde{n}_k(k', y(\Theta', K')) + (1-\delta)\left(\frac{w(\Theta',K')}{\zeta\, \Psi_d[Q^i(\Theta',K')]\, f[K'^i(\Theta',K')]} + P^i(\Theta', K')\right)}{1 + R(\Theta', K')} \right| \Theta \right\}$$
$$= \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^y(\Theta, K)]} + P^i(\Theta, K),$$

With Lemma 3, it is easy to see that the first order conditions for k' of future consumption producing firms and investment producing firms are identical. Thus, all the firms with the same current capital choose the same future capital. □

Lemma 6. *In equilibrium, the following holds*

$$\frac{w(\Theta, K)}{\zeta\Psi_d[Q^i(\Theta, K)]f(K, N^y(\Theta, K))} = \frac{\alpha}{1-\alpha}P^i(\Theta, K) \tag{70}$$

Proof. From investment-producing firms' first order condition over k', we have

$$E\left\{ \left. \frac{\Omega_k(\Theta', K', k')}{1 + R(\Theta', K')} \right| \Theta \right\} = \frac{w(\Theta, K)}{\zeta\, \Psi_d[Q^i(\Theta, K)]\, f[K, N^i(\Theta, K)]} + P^i(\Theta, K) \tag{71}$$

The equilibrium search in the investment goods market implies

$$P^i(\Theta, K) = (1-\alpha)\, E\left\{ \frac{\Omega_k(\Theta', K', k')}{1 + R(\Theta', K')} \right\}. \tag{72}$$

Putting together the above two equations proves the Lemma. □

Lemma 7. *In equilibrium, investment-producing firm has the Euler equation with the usual form*

$$E\left\{ \frac{P^i(\Theta', K')}{1 + R(\Theta', K')} \left[\frac{\Psi_d(Q^i)}{Q^i} f_k\left(K', N^y(K', \Theta')\right) + (1-\delta) \right] \right\} = P^i. \tag{73}$$

Proof. The FOC for future capital stock evaluated at K' for an investment-producing firm is given by

$$E\left\{ \frac{-w(\Theta', K')\widetilde{n}_k(K', y(\Theta', K')) + (1-\delta)\left(\frac{w(\Theta',K')}{\zeta\,\Psi_d[Q^i(\Theta',K')]\,f[K',N^y(\Theta',K')]} + P^i(\Theta', K')\right)}{1+R(\Theta', K')} \right| \Theta \Bigg\}$$

$$= \frac{w(\Theta, K)}{\zeta\,\Psi_d[Q^i(\Theta, K)]\,f[K, N^y(\Theta, K)]} + P^i(\Theta, K),$$

According to Lemma 6, we can replace $\frac{w(\Theta,K)}{\zeta\Psi_d[Q^i(\Theta,K)]f(K,N^y(\Theta,K))}$ with $\frac{\alpha}{1-\alpha}P^i(\Theta, K)$. Similarly for the future variables. The Euler becomes

$$E\left\{ \frac{-w(\Theta', K')\widetilde{n}_k(K', y(\Theta', K')) + (1-\delta)\frac{P^i(\Theta',K')}{1-\alpha}}{1+R(\Theta', K')} \right| \Theta \Bigg\} = \frac{P^i(\Theta, K)}{1-\alpha}$$

Multiplying $1-\alpha$ on both hand side and reorganizing the equation, we have

$$E\left\{ \frac{P^i(\Theta', K')}{1+R(\Theta', K')}\left[-(1-\alpha)\frac{w(\Theta', K')\widetilde{n}_k(K', y(\Theta', K'))}{P^i(\Theta', K')} + (1-\delta)\right] \right| \Theta \Bigg\} = P^i(\Theta, K).$$

Substituting $w(\Theta', K')/P^i(\Theta', K')$ with $\frac{1}{1-\alpha}\frac{\Psi_d(Q^i)}{Q^i}f_n(K, N^y(\Theta', K'))$ from the competitive search problem, we can rewrite the Euler as

$$E\left\{ \frac{P^i(\Theta', K')}{1+R(\Theta', K')}\left[-\frac{\Psi_d(Q^i)}{Q^i}f_n(K, N^y(\Theta', K'))\widetilde{n}_k(K', y(\Theta', K')) + (1-\delta)\right] \right| \Theta \Bigg\} = P^i(\Theta, K).$$

Under the production function $f(k,n) = zk^{\gamma_k}n^{\gamma_n}$, $\widetilde{n}_k(k,y) = -\gamma_k n/(\gamma_n k)$ and $f_n = \gamma_n z k^{\gamma_k} n^{\gamma_n - 1}$. Thus $-f_n\widetilde{n}_k = \gamma_k z k^{\gamma_k - 1}n^{\gamma_n} = f_k$. Substituting $-f_n\widetilde{n}$ with f_k in the Euler equation, we have

$$E\left\{ \frac{P^i(\Theta', K')}{1+R(\Theta', K')}\left[\frac{\Psi_d(Q^i)}{Q^i}f_k(K, N^y(\Theta', K')) + (1-\delta)\right] \right| \Theta \Bigg\} = P^i(\Theta, K).$$

□

B Calibration of the standard RBC Model