Consumer default with complete markets* (Incomplete & in progress)

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Abstract

This paper studies properties of economies with complete markets where there is positive default on consumer debt. Households can default partially, at a punishment cost, and intermediaries price this risk competitively. This environment yields only partial insurance. The risk-based pricing of debt makes it too costly for the borrower to achieve full insurance and there is too little trade in securities. Consumption, as well as debt and the default rate, are positively correlated with idiosyncratic changes in income. This is in contrast with existing literature. Unlike the literature with default, there are no restrictions on the set of state contingent securities that are issued. Relative to the literature on lack of commitment, limited trade arises without debt constraints that rule default out. The approach in this paper may have novel implications for understanding consumption inequality.

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1 Introduction

The aim of this paper is to study the role of consumer credit default as a cause of financial frictions or incomplete trade. In spite of the evidence of consumer credit delinquency and bankruptcy, the existence of positive default in economies with conventional complete financial markets has received little attention. This paper takes up this issue and demonstrates that it is relevant, provides a natural story for financial frictions, and can be useful for understanding evidence on consumption inequality.

The specific objectives of the paper are three. The first is to propose and analyze a deliberately simple and tractable model that is suitable for the analysis of default with complete markets. The approach rests on the existence of partial default and risk-based pricing. The second objective is to identify, in that context, reasons why default matters for the allocation of resources, even in the absence of departures from a frictionless competitive environment. It will study the consequences of default for risk-sharing (i.e., insurance) and intertemporal consumption smoothing. The third objective is to start assessing the practical significance of this risk-pricing approach. We will compare its implications for consumption inequality with those from a model which – in the vein of much literature on endogenous incomplete markets – has debt constraints that rule out default.

Some motivation for entertaining partial default is in order. In the U.S., formal bankruptcy filings have attracted much analysis and discussion, yet default outside formal bankruptcy procedures is also prevalent. This is the theme of recent empirical and theoretical studies in Dawsey and Ausubel (2004), Dawsey, Hynes and Ausubel (2008) and Chatterjee (2010). Unlike the full discharge of unsecured debts in formal procedures like Chapter 7, informal bankruptcy is best seen as involving debtors failing to repay a chosen part of their liabilities. In the Survey of Consumer Finances 2007, about 1% of the U.S. population had filed under Chapter 7, whereas over 5% held delinquent loans.[1] This is suggestive that informal bankruptcy might account for the bulk of loan write-offs, a point already made by Dawsey and Ausubel (2004). The significance of informal bankruptcy supports the present focus on partial default.

The simple general equilibrium model consists of a two-period endowment economy populated by two types of households who are subject to idiosyncratic endowment risk in the second period. Regarding financial markets, agents can borrow and lend freely through perfectly competitive financial intermediaries, and have access to a full

[1] See charge-off rates and revolving debt from the Federal Reserve Board since 1988. See also Sullivan et al. (2000) and the Bankruptcy Data Project at Harvard `http://bdp.law.harvard.edu/filingsdb.cfm`.

set of Arrow securities. Borrowers can default fully or partially on their promised deliveries. Default carries a utility penalty to the household which depends on the scale of default. Because different levels of debt are associated with a different default rate, loans of different size commands a different price. In order to characterize the behavior of intermediaries and households, one needs to account for the price schedule associated with all possible contracts, including those that are not traded in equilibrium. We characterise the equilibrium and draw analytical results. The model will also be solved numerically in order to deal with situations where qualitative conclusions are elusive, as well as to provide some quantitative sense of the scale of the effects considered.

One key result is that bankruptcy on its own has real consequences for the allocation of consumption. The reason is that prices reflect the risk of different contract sizes in a way that affect the borrowing and lending decisions in equilibrium. Since a borrower's marginal gain in terms of current consumption from issuing debt is less than the price – a price wedge – an economy with endogenous bankruptcy will feature less trade in assets. Specifically, the possibility of bankruptcy implies imperfect consumption risk-sharing and suboptimal intertemporal consumption smoothing. One can show that, across states of nature in the second period, individual consumption varies positively with income, and so do the default rate and the level of liabilities. It is critical that the default decisions are endogenous. If default rates for each individual type and contingency were exogenous, then the price of claims would be given to households and allocations would be optimal, featuring full risk-sharing.

In order to assess the implications for consumption inequality in this risk-pricing model, we consider a debt-constrained version of the economy as a comparison benchmark. This obtains naturally if the penalty for defaulting is fixed regardless of its scale. Default, which becomes in effect a zero-or-one choice, is ruled out and liabilities cannot exceed a certain endogenous debt limit. While the source of imperfect risk sharing is the same lack of commitment in the two models, the mechanisms are quite different. Illustrative numerical examples support the conjecture that the risk-pricing model encompasses less risk sharing and more consumption inequality than the debt-constrained model.

This paper is related to an emerging recent literature on consumer credit and bankruptcy. The works that undertake a quantitative general equilibrium analysis of incomplete markets include Chatterjee et al. (2007) and Livshits et al. (2007), Mateos-Planas (2008), and Mateos-Planas and Ríos-Rull (2010). The present paper deals in a similar way with the pricing of default risk but does not rule out trade opportunities arbitrarily. In theoretical literature, default has also been typically studied in models with incomplete financial markets. The emphasis has been on the role of default as providing some degree of insurance against individual risk. This is the case, for example, in Zame (1993) and Dubey et al. (2005). This paper studies instead situations where there is a complete set of securities and insurance considerations are abstracted from. The role of default in this context must be a distinct one.

In considering complete markets, this paper is also related to a large and influential literature on limited enforcement with complete markets. This includes Alvarez and Jerman (2000), Krueger and Perri (2006), Kehoe and Levine (1993, 2001), and Kehoe and Perri (2002). Like those works, we also derive endogenous incomplete insurance given a full set of available securities, but this is done without imposing an exogenous participation constraint or not-too-tight borrowing limits. Furthermore, there is positive default in equilibrium in the present paper, whereas these other papers rule out positive bankruptcy as an outcome. In The case of other papers could be regarded as a special case of ours were we to assume default is a discrete all-or-nothing decision. The existing literature has used that framework to address important practical questions regarding consumption and wealth inequality. The approach we propose should also be relevant for understanding that evidence.

2 A basic model

The economy has two periods. The possible states are $s = 0, 1, , , , S$. In the first period the state is 0; in the second period the state of nature can be $s = 1, ..., S$. The probability of a state s in the second period is denoted as π_s. There are two types $i = A, B$ of individuals. The two groups are of the same size. These agents receive endowments of goods that depend on their type i and the state of nature $s = 0, 1, ..., S$, and are denoted by y_s^i. In the first period, agents can borrow or save against second-period states $s = 1, ..., S$ using securities traded through banks. Let a_s^i denote the risk-free delivery values of the assets that pay in state s held by an individual of type i. Let l_s^i denote the promised delivery values of the loans held by an individual of type i that pay in state s. Let p_s denote the price of assets at 0. As for debts, since different levels of debt may carry a different risk of failure, the price at 0 of debt held by agent i with promised delivery value l in state s is a function of its value $Q_s^i(l)$. A borrower can default on a fraction of their debt. Let d_s^i denote the fraction of promised repayments l_s^i that borrower i fails to deliver in state s.[2] Individuals consume in each period their net available resources, and c_s^i denotes consumption in state s by an individual of type i.

[2]Think of a household as pooling the resources of a large number of identical members. The correct interpretation of the default rate is the fraction of them who default.

Individual preferences are represented as a function consumption and default. A borrower who defaults experiences a utility stigma loss η that depends on the rate of default incurred d_s^i. More specifically, the utility of an individual of type i is represented by

$$u(c_0^i) + \beta \sum_{s=1,\dots,S} \pi_s[u(c_s^i) - \eta(d_s^i)],$$

where $u(.)$ is the period utility functions and $\eta(.)$ describes the penalty for default. We will often use specific functional form for preferences, with

$$u(c) = \log c \tag{0a}$$

and

$$\eta(d) = \eta d^\gamma. \tag{0b}$$

Banks intermediate the trade in assets and liabilities. Their lending activity carries a risk of default. Let $D_s^i(l)$ denote the default rate associated with a value l of promised deliveries in state s by agent i. The revenues to the bank for lending l claims against state s to agent i are then $(1 - D_s^i(l))l$. The costs to the bank consist of the delivery value of the deposits taken a and an intermediation cost, a proportion ν of the debt value. The bank's book balancing requires the value of deposits taken to equalise the value of loans made, that is $Q_s^i(l)l = p_s a$. Therefore the bank's cash flow is:

$$\left[1 - D_s^i(l) - \frac{Q_s^i(l)}{p_s} - \nu\right] l.$$

There is an open market for loans of every possible size. All markets clear under perfect price-taking competition and free entry in intermediation.

3 Equilibrium

The equilibrium determines the price of assets p_s and, for all tradeable debt sizes l, the price schedule $Q_s^i(l)$ and default risk risk schedule $D_s^i(l)$. There will be a single type of borrower i under each state s. All traded loans are of a particular size l_s^i and carry a single effective default rate $d_s^i = D_s^i(l_s^i)$ and price $q_s^i = Q_s^i(l_s^i)$, if $l_s^i > 0$.

In order to determine these specific realizations, one will need to understand the values associated with off equilibrium allocations for contracts that are not traded, as described by the mappings $Q_s^i(.)$ and $D_s^i(.)$. One one hand, consumers make their borrowing and default plans bearing in mind how the interest charged changes with the liabilities through $Q^i_s(.)$. On the other hand, banks bidding entry into the industry take into account the variation in the risk of default associated with loans of different size through $D^i_s(.)$. In this model, these off equilibrium expectations will be consistent with the assumption made that markets for non traded contracts are open.[3] We turn now to defining the equilibrium more precisely.

One remark about the equilibrium is that borrowing or savings can only be positive for one agent in any state $s = 1, ..., S$. That is, if $l^i_s > 0$ then $a^i_s = 0$, $l^j_s = 0$, and $a^j_s > 0$.

3.1 Definition

An equilibrium for the above economy consists of the following objects for each state s, and for all agents i and $j \in \{A, B\}$: Traded prices q^i_s and p_s, price menus for debt $Q^i_s(.)$, default functions $D^i_s(.)$, portfolios a^j_s and l^i_s, and consumption allocations c^i_s and c^j_s, and default rates d^i_s. They satisfy the following conditions:

(i) Consumer maximisation: For each i, given p_s and $Q^i_s(.)$, the choices $a^i_s \geq 0$, $l^i_s \geq 0$ and d^i_s maximise utility for i subject to

$$c^i_0 = y^i_0 - \sum_{s\geq 1} p_s a^i_s + \sum_{s\geq 1} Q^i_s(l^i_s) l^i_s$$

and, if $l^i_s > 0$,

$$c^i_s = y^i_s - (1 - d^i_s) l^i_s, \ \ s = 1, ..., S$$

or, if $a^i_s > 0$,

$$c^i_s = y^i_s + a^i_s, \ \ s = 1, ..., S$$

(ii) Off-the-equilibrium default: For a given loan size $l > 0$, the value of the default function schedule $D^i_s(l)$ is the $\tilde{d}$ that maximises agent i's utility in state s, $u(c^i_s) - \eta(\tilde{d})$, subject to

$$c^i_s = \ \ y^i_s - (1 - \tilde{d}) l, \ \ s = 1, ..., S \ .$$

(iii) Bank competition: Given the default schedule $D^i_s(.)$, the price menu $Q^i_s(.)$ satisfies zero-profit on any potential credit contract size l.

[3]This is not unlike Dubey et al. (2005)'s refinement to prevent the existence of markets being ruled out by excessive pessimistic expectations.

(iv) Consistency: the traded price corresponds to the traded contracts

$$q_s^i = Q_s^i(l_s^i), \;\; l_s^i > 0$$

and the default rate corresponds to the traded contract

$$d_s^i = D_s^i(l_s^i), \;\; l_s^i > 0.$$

(v) Market clearing:

$$\sum_i c_0^i = \sum y_0^i,$$

$$\sum_i c_s^i + \sum_i \nu l_s^i = \sum_i y_s^i.$$

In point(i) the budget constraint (2b) shows that the borrower i bears in mind the effect of the level of debt on the price through $Q_s^i(.)$. More specifically, if $l_s^i > 0$, for the borrower the optimality conditions for debt is

$$u'(c_0^i)\left[q_s^i + Q_s^{i\prime}(l_s^i)l_s^i\right] = \beta u'(c_s^i)(1-d_s^i)\pi_s, \;\; l_s^i > 0 \tag{1}$$

where use has been made of the definition in (iv). Regarding the default choice,

$$u'(c_s^i)l_s^i = \eta'(d_s^i), \;\; l_s^i > 0 \tag{2}$$

For the lender with $l_s^i = 0$, the standard conditions for optimal savings holds, so

$$u'(c_0^i)p_s = \beta u'(c_s^i)\pi_s, \;\; l_s^i = 0 \tag{3}$$

Point (ii) describes the default behavior for arbitrary levels of debt. The optimality condition in this case is analogous to the condition for the equilibrium allocation (2), so $D_s^i(l)$ solves

$$u'(y_s^i - (1 - D_s^i(l))l)l = \eta'(D_s^i(l)) \quad \text{all} \;\; l > 0. \tag{4}$$

Note that, because of separability of the utility function, the default menu has the convenient property that it does depend on the price schedule $Q_s^i(.)$.

Point (iii) describes the zero-profit condition for all loan sizes. Given the expression for the bank's cash-flow presented earlier, this can be written more explicitly as:

$$1 - D_s^i(l) - \frac{Q_s^i(l)}{p_s} - \nu = 0 \quad \text{all} \;\; l > 0. \tag{5}$$

The definition of the price of debt traded in equilibrium in point (iv) along with the fact just discussed that the realised default $d_s^i = D_s^i(l_s^i)$ if $l_s^i > 0$, permit to write an analogous expression specifically for the active intermediaries:

$$1 - d_s^i - \frac{q_s^i}{p_s} - \nu = 0 \tag{6}$$

Point (v) implies clearing in the financial market $q_s^i l_s^i = p_s a_s^j$ when $l_s^i > 0$. Combining these market clearing conditions with the budget constraints in point (i) and the zero-profit condition (6), one can write the consumption allocation as follows:

$$\begin{array}{rcl} c_0^i & = & y_0^i - \sum_s q_s^j l_s^j + \sum_s q_s^i l_s^i \\ c_s^i & = & y_s^i - (1 - d_s^i) l_s^i, \quad l_s^i > 0 \\ c_s^j & = & y_s^j + (q_s^i / p_s) l_s^i, \quad l_s^j = 0 \end{array} \tag{7}$$

In the special case where the intermediation cost is zero, (6) implies that the last equality can be written as $c_s^j = y_s^j + (1 - d_s^i) l_s^i$.

3.2 Characterisation

The set of equations (1)-(3) and (6)-(7) forms a system in the endogenous variables consisting of prices p_s and q_s^i, debt promised deliveries l_s^i, default d_s^i, and consumption allocations c_0^i and c_s^i, for $i = A, B$ and $s = 1, ..., S$. However this system is not fully determined because it says nothing about the sensitivity of the price on debt to changes in the borrowing decision expressed in derivative of the price schedule $Q_s^{i\prime}$ in (1). The two remaining equations (4) and (5) precisely describe the pattern for default and pricing off the equilibrium which is needed to pin down this derivative. Therefore, in order to characterise the equilibrium, we will first study the properties of the price menu implied by (4) and (5).

To be specific, we will consider the functional forms for utility and stigma costs in (0). The default mapping $D_s^i(l)$ is determined from (4) as the solution to

$$\frac{l}{y_s^i - (1 - D_s^i(l) l)} = \gamma \eta D_s^i(l)^{\gamma - 1}.$$

Computing its derivative $D_s^{i\prime}(l)$ and, using condition (2) with (7), evaluating it at the equilibrium l_s^i, one obtains:

$$D_s^{i\prime}(l_s^i) = \frac{1 - d_s^i + \frac{1}{\gamma\eta} {d_s^i}^{1-\gamma}}{\left(\frac{1}{\gamma\eta} {d_s^i}^{-\gamma} - 1\right)(\gamma - 1) + \gamma} \frac{1}{l_s^i}$$

Now the zero-profit condition (5), with (6), shows that the change in the price of debt with liabilities has the opposite sign, $Q_s^{i\prime}(l) = -q_s^i(1-d_s^i-\nu)^{-1}D_s^{i\prime}(l)$. With this information, one can find an explicit expression for the change in the amount that one can borrow by incurring extra debt:

$$q_s^i + Q_s^{i\prime}(l_s^i)l_s^i = q_s^i(1-d_s^i-\nu)^{-1}\left[\frac{d_s^{i\,-\gamma}}{(\gamma-1)d_s^{i\,-\gamma}+\gamma\eta}((1-d_s^i)(\gamma-1)-d_s^i)-\nu\right] \quad (8)$$

This term (8) has to be positive in an equilibrium with positive borrowing and lending where (1) holds. By incurring more liabilities for tomorrow, the borrower must be able to raise more consumption today. Note that the default penalty being steep enough, or $\gamma > 1$, is a necessary condition for this to happen. On the other hand, this condition guarantees that, off the equilibrium, the default risk increases and hence the debt price decreases with the value of debt promised repayments.

Proposition 1 *Assume the functional forms in (0). Then $q_s^i + Q_s^{i\prime}(l_s^i)l_s^i$ is given by (8). In an equilibrium with positive borrowing its sign must be positive and so $\gamma > 1$. In such an equilibrium, $D_s^{i\prime}(l_s^i) > 0$ and $Q_s^{i\prime}(l_s^i) < 0$.*

4 Risk-sharing and smoothing

In this section, we discuss the the properties of the equilibrium regarding risk-sharing and the consequences of default for allocations of consumption. For each of the two issues, we proceed in two steps. We first consider the case where default is exogenous as the standard benchmark where there is full risk-sharing and default does not matter. Then we study the model with endogenous default in order to establish and explain that these properties do not hold.

To facilitate the discussion we will assume away intermediation costs and set $\nu = 0$. Also, we assume there is no aggregate risk so $y_s^i + y_s^j$ is a constant y for all s.

4.1 Risk sharing

Consider first the situation where the default rates d_s^i are exogenous. An equilibrium is still characterized by the above conditions except (2),(4) and (5), and with the property that risk does not depend on loan size or $Q_s^{i\prime}(l) = 0$.

In this setting with exogenous default, the first-order conditions (1) and (3), alongside the no-arbitrage (6) that $p_s = q_s^i/(1-d_s^i)$, imply the equalization of the intertemporal marginal rates of substitution of the two agents at each state, that is $u'(c_s^i)/u'(c_0^i) = u'(c_s^j)/u'(c_0^j)$. On the other hand, the market clearing condition (7) implies that $c_s^i + c_s^j = y$. Combining these two properties,

$$\frac{u'(c_s^i)}{u'(y - c_s^i)} = \frac{u'(c_0^i)}{u'(c_0^j)}.$$

The implication is that consumption of any agent is invariant to the state s. There is, in other words, complete risk-sharing. The intuition for this result is standard. With default rates given exogenously, no-arbitrage prices fully account for default risk. Technically, no-arbitrage prices lead to the equalization of marginal rates of substitution across agents.

We turn now to the case with endogenous default. The equilibrium if described by equations (1)-(7) with (8). Complete risk sharing fails to hold in this case. No-arbitrage does not lead to the equalization of marginal rates of substitution across agents. The reason is that the borrower household takes into account the endogenous response of the price to their loan choice. More formally, let i the agent that borrows against a particular state s. First, from (1) and (3) we obtain, respectively,

$$\frac{u'(c_s^i)}{u'(c_0^i)} = \frac{q_s^i + Q_s^{i\prime}(l_s^i) l_s^i}{\beta(1 - d_s^i)\pi_s},$$

and

$$\frac{u'(c_s^j)}{u'(c_0^j)} = \frac{p_s}{\beta \pi_s}.$$

Now the no-arbitrage condition (6) that $p_s = q_s^i/(1 - d_s^i)$ does not imply equalization of marginal rates of substitution as long as $Q_s^{i\prime} \neq 0$. Combining these expressions with market clearing (7), one obtains:

$$\frac{u'(c_s^i)}{u'(y - c_s^i)} = \frac{q_s^i + Q_s^{i\prime}(l_s^i) l_s^i}{q_s^i} \frac{u'(c_0^i)}{u'(c_0^j)}.$$

This shows that in general full-risk sharing will fail to hold. Specifically, using (8), we write this more explicitly as

$$\frac{u'(c_s^i)}{u'(y - c_s^i)} = \frac{1}{(\gamma - 1) + \gamma \eta {d_s^i}^{\gamma}} \left[\gamma - 1 - \frac{d_s^i}{1 - d_s^i} \right] \frac{u'(c_0^i)}{u'(c_0^j)}. \tag{9}$$

So individual consumption varies with the state s as long as the default rate d_s^i does so.

We discuss now how individual variables change across states s. To simplify, we start by considering changes across states where the household i remains the debtor across these states. The previous expression (9) implies that the borrower i's default rate d^i_s is higher in states where its consumption c^i_s is higher – and, hence, the lender j's consumption is lower. On the other hand, the default optimality condition (2) implies that this must be accompanied by a higher level of debt l^i_s. We have the following proposition.

Proposition 2 *(Default, debt and consumption with constant roles) Across states s where type i remains the debtor, its consumption c^i_s, default rate d^i_s, and debt level l^i_s are positively correlated.*

We now consider the central association between consumption and the income endowment. The following background will facilitate the discussion. Expression (9) – derived form the optimality condition (1) and (3), no-arbitrage (6), and clearing (7) – involves default and consumption in a particular state. On the other hand, the optimality condition for default (2), using the functional assumptions made and (7), can be written more explicitly as:

$$\frac{y^i_s - c^i_s}{c^i_s} = \gamma\eta(1 - d^i_s){d^i_s}^{\gamma-1}. \tag{10}$$

This expression relates consumption, default and the endowment in a particular state. It will be important to know how the right-hand side of this expression changes with the default rate. Its derivative has the same sign as $(\gamma-1)/\gamma - d^i_s$ so the RHS of (10) is hump shaped. Now, the fact discussed that the RHS of (8) is positive implies that, given $\gamma > 1$, the sign of the expression $\gamma - 1 - d^i_s/(1 - d^i_s)$ must be positive. But then this implies that $(\gamma - 1)/\gamma - d^i_s > 0$ so the RHS of (10) is increasing in the default rate d^i_s.

Suppose now that consumption c^i_s increases across two states. From (9) we already know that default d^i_s must increase. From (10), income y^i_s would have to be higher if the right-hand side in increasing in d^i_s, a property we have just established. Conversely, a larger income must also imply higher consumption.

Proposition 3 *(Imperfect income risk sharing with constant roles) Across states s where type i is the debtor, its consumption c^i_s and income endowment y^i_s are positively correlated.*

Consider now comparisons across states such that a particular household changes role. That is, suppose an equilibrium where agent i holds debt in state s and holds assets in another state s'. Again, optimal borrowing against state s is characterized using (1) by

$$\frac{u'(c_s^i)}{u'(c_0^i)} = \frac{q_s^i + {Q_s^i}'(l_s^i)l_s^i}{\beta(1-d_s^i)\pi_s},$$

and optimal saving against state s' reads from (3) as

$$\frac{u'(c_{s'}^i)}{u'(c_0^i)} = \frac{p_{s'}}{\beta\pi_{s'}} = \frac{q_{s'}^j}{\beta(1-d_{s'}^j)\pi_{s'}},$$

where the last equality follows from the zero-profit condition (6). Now perfect risk sharing amounts to having an equality between the two RHS terms in these two expressions. We show this cannot be true by way of contradiction. Assuming this equality, the property that ${Q_s^i}'(.) < 0$ implies that $q_s^i/(\beta(1-d_s^i)\pi_s) > q_{s'}^j/(\beta(1-d_{s'}^j)\pi_{s'})$. From market clearing in (7), there must also be risk sharing for agent j, which, by an analogous argument, means that the contrary inequality must hold. Exactly the same argument can be used to establish that risk-sharing fails to hold precisely in the sense that

$$\frac{q_s^i + {Q_s^i}'(l_s^i)l_s^i}{\beta(1-d_s^i)\pi_s} < \frac{q_{s'}^j}{\beta(1-d_{s'}^j)\pi_{s'}}. \tag{11}$$

This property together with the preceding two expressions implies that $c_s^i > c_{s'}^i$. That is, there is imperfect risk sharing in the sense that a household's consumption is higher in the states where they hold debt than in the states where they hold savings. Given this we can also establish how income correlates with consumption. Consumption levels for household i are determined from (7) as $c_s^i = y_s^i - (1-d_s^i)l_s^i$ and $c_{s'}^i = y_{s'}^i + (1-d_{s'}^j)l_{s'}^j$. The fact that $c_s^i > c_{s'}^i$ clearly requires that $y_s^i > y_{s'}^i$ so consumption and income a are positively related.

Proposition 4 *(Default, debt/saving and consumption with changing roles) Across states s where household i switches between debtor and saver, its consumption c_s^i, debt level l_s^i and income endowment y_s^i are positively correlated.*

Having established that risk sharing fails, some intuition is in order. One way to gain this intuition is by comparison the the outcomes for an economy where default rates are exogenous but coincide with the equilibrium default rates of our economy. Consider the constraints that determine consumption in (7). In both economies, consumption changes less than income as the household holds more debt liabilities in states where income is higher. In the exogenous-default case the portfolio composition varies across states to an extent that neutralizes the effect of income on consumption. In our endogenous-default economy, portfolios also adjust but to a lesser extent thus making consumption responsive to income. So the key point behind imperfect risk sharing is that households stop short of arranging their portfolios as needed to achieve complete consumption smoothing. Why is that? The intuition is that the interest rate faced by the household rises with the level of the debt issued. This makes them more cautious about taking large debt positions needed to fully insure consumption. This is a reflection that financial markets price the fact that risk depends on individual (borrowing) decisions.

With a complete set of assets, default is not insurance. This is in contrast with the role ascribed to default in the incomplete markets literature.

4.2 Consumption smoothing

We also would like to discuss how the two economies differ in terms of aggregate allocations and prices. By construction, the risk premium is the same in the two economies. Therefore, one needs to figure out how the level of interest rates and hence consumption differ between the two economies.

The simplest way to make this point is to consider a deterministic version of the model, with only one state in the second period. Suppose that the borrower is of type i. One can now compare this economy with the analogous case where default is exogenous, and the default rate is the same. The main difference between these two economies is found in the intertemporal optimality condition for the borrower in equation (1). For given prices of debt, with endogenous default, the borrower faces a steeper interest rate, or a lower marginal benefit to issuing debt. She will issue less debt and consume less in the first period compared to the exogenous case. In equilibrium, a higher price of debt brings about the corresponding reduction in lending from the lender according to (3). (Formal details to be added.)

5 Consumption inequality with risk-pricing or debt-constraints

This section discusses the differences between the present risk-pricing model and the debt-constrained model regarding the implications for consumption inequality. The ability of different models for understanding the evidence on risk-sharing has been the subject of an important body of quantitative literature (e.g., Krueger and Perri (2006), Kaplan and Violante (2009)). The conclusion that seems to emerge is that the debt-constrained model in the Kehoe-Levine (1993) tradition underestimates inequality. It is thus important to understand how the risk-pricing model could comparatively perform on this dimension.

The debt-constrained model obtains as a special case of the setting used here. It suffices to assume that the utility penalty is a fixed amount η, independently of the scale of default.[4] This assumption naturally leads defaulters to default in all their debts. This being so will lead intermediaries not to offer loans that induce any default (i.e., to command a price of zero). This result in a situation where loans traded carry no default risk, so lending rates are risk free, but the size of loans is constrained by the maximum debt that would trigger default. Formally, individuals of type i face a debt limit against state s that we denote $\bar{l}_s^i$. This is determined as the value that makes the individual indifferent between defaulting or not:

$$u(y_s^i) - \eta = u(y_s^i - \bar{l}_s^i).$$

One can argue that, at least for some level of the default penalty, the risk-pricing model brings about more inequality than the debt-constrained economy. We have discussed that there is imperfect risk-sharing in the risk-pricing economy as long as there is positive default in equilibrium. For any finite penalty, there will always be some default in that setting. On the other hand, in the debt-constrained economy one can surely find a large enough punishment that makes consumption inequality zero. By a continuity argument, there will also be a region where the debt-constrained economy has binding debt limits and incomplete insurance but still displays less inequality than the risk-pricing economy. While some especial assumption on the punishment technology made in this paper might play a part, it betrays the sense that the risk-pricing economy might have less risk-sharing more generally. If default can be partial, there will be more states where default and hence the distortion that follows is positive.

At this point, no formal result is available though and we resort to illustrate this conjecture using numerical examples. Only two random states s are assumed. The first example has symmetric consumers where the only difference is the state when they get the bad shock, but the probability is the same. The parameters are $\beta = 1$, $y_0^i = 0.50$ all i, equal probabilities $\pi_s = 0.50$, symmetric income processes y_s^i, $(0.60, 0.40)$, and punishment $\eta = 0.12$ and $\gamma = 3.0$. It suffices the consider outcomes for just one agent. Table ** shows the results for the two economies and, as a

[4] This cost can be made individual and state dependent.

benchmark reference, those under full risk-sharing for an economy with commitment. The risk-pricing model delivers far more consumption volatility and less trade in assets. This is reflected in a lower risk-free lending rate and higher borrowing rates. This is a conservative example in that it sets a considerably low default penalty. Mild punishment parameters, needed that generate some inequality in the debt-constrained economy, are associated with very large default rates in the risk-pricing economy.

Example 1: symmetric case

	RISK-SHARING	RISK-PRICING	DEBT-CONSTRAINTS
c_0^i	0.500	0.500	0.500
$\{c_s^i\}$	(0.500, 0.500)	(0.583, 0.417)	(0.532, 0.468)
l_s^i	0.100	0.0268	0.0678
p_s	0.500	0.599	0.534
d_s	–	0.357	0.000
q_s	0.500	0.385	0.534
$\bar{l}$	–	–	0.0678

The previous example focuses on within period insurance only. The second example involves effects on smoothing over time as well. This is an asymmetric economy in that household A, having a back-loaded endowment profile, borrows against all future states. The parameters are as follows: $\beta = 1$, $\pi_s = 0.50$ all s, $y_0^A = 0.40$, $\{y_s^A\} = (0.54, 0.48)$, $y_0^B = 0.60$, $\{y_s^B\} = (0.46, 0.52)$. The punishment parameters are the curvature $\gamma = 3.0$, and the fixed penalty η, of 0.20 for the risk-pricing model and a lower cost 0.10 for the debt-constrained model. This different choice works against out contention that the debt-constrained model features less inequality. The results are displayed in the next Table. The debt-constrained model has the limit binding only in one of the states and individuals can achieve perfect intertemporal smoothing in the other state. We see that the risk-pricing economy generates less smoothing both over time and across states since there is less trade.

Example 2: asymmetric case

	RISK-SHARING	RISK-PRICING	DEBT-CONSTRAINTS
c_0^A	0.4550	0.4227	0.4452
$\{c_s^A\}$	(0.4550, 0.4550)	(0.5133, 0.4673)	(0.4887, 0.4452)
c_0^B	0.5450	0.5773	0.5548
$\{c_s^B\}$	(0.5450, 0.5450)	(0.4867, 0.5327)	(0.5113, 0.5548)
l_s^A	(0.085, 0.025)	(0.0424, 0.0168)	(0.0513, 0.0348)
p_s	(0.500, 0.500)	(0.5931, 0.5419)	(0.5420, 0.5000)
q_s^A	(0.500, 0.500)	(0.3729, 0.4094)	(0.5420, 0.5000)
d_s	–	(0.3712, 0.2445)	–
$\bar{l}_s^A$	–	–	(0.0513, 0.0457)
$\bar{l}_s^B$	–	–	(0.0438, 0.0494)

6 Concluding remarks

Partial default in consumer credit can help account for limited trade even when there is a complete set securities. The specific mechanism differs from the existing literature on debt-contrained economies. The risk-pricing model in this paper brings about price wedges that distort the borrowing decisions. This model seems to imply more consumption inequality than the debt-constrained model typical of much applied literature on the subject. This property might prove significant for understanding the evidence.

This analysis can be extended fruitfully in various directions. First, one can investigate with this novel approach the ability of different factors to account for the observation of rising bankruptcy and the expansion of credit and levels of debt. This type of questions have only being pursued in incomplete markets economies. Second, bankruptcy matters even without the frictions introduced by market incompleteness. This insight might prove important for a better understanding of financial frictions and welfare policy in actual economies.

The model in this paper is deliberately simple and, in the interest on analytical tractability, exploits some specific functional assumption. In order to draw firm practical conclusions, a quantitative model of risk-pricing with complete markets is called for. This is being pursued in ongoing research.

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