Competitive Search and Firm Growth

Erzo G.J. Luttmer
University of Minnesota
Federal Reserve Bank of Minneapolis

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Abstract

This paper describes a model of search unemployment in an economy with multi-worker firms. It combines competitive labor market search with the model of firm growth of Luttmer [2011]. In a baseline version, Gibrat's law holds approximately and aggregate shocks that destroy blueprint capital —for example, because certain activities have become obsolete— lead to an extremely slow adjustment to the steady state. This can serve as a benchmark explanation for protracted unemployment. Slow rates of convergence arise as well in an economy in which Gibrat's law is replaced with more realistic accounts of firm growth. Some suggestive evidence on the limited impact of recessions on fast-growing firms is provided.

JEL Classification: J6, L1.

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Note: For SED 2011 evaluation only. Please do not distribute.

1. Introduction

Many models of labor market search consider firms with only one worker. At the opposite extreme, search models with multi-worker firms tend to consider firms that have a continuum of employees. These extreme assumptions make it hard, if not downright impossible, to make serious contact with micro evidence on firm employment dynamics and the labor market. This paper presents a benchmark model of firm employment dynamics in which a search friction generates unemployment. The implied firm employment size distribution is discrete and closely matches the one observed in US data.

In the model, there are two types of organization capital: blueprint capital and match capital. Blueprints are job descriptions, and blueprints need to be matched with suitable workers. A match produces a flow of consumption goods and new blueprints. Firms are agglomerations of blueprints that can all be traced to the same initial start-up blueprint.

Recessions can result from the destruction of match capital alone or match and blueprint capital combined. Traditional models of labor market search suggest that match capital quickly mean reverts. But the calibrations reported in Luttmer [2011] suggest that the same is far from true for blueprint capital. Recessions triggered by a significant destruction of blueprint capital can lead to periods of protracted unemployment.

2. The Economy

Firms are collections of blueprints produced from initial start-up blueprints. These start-up blueprints are created by entrepreneurs. Blueprints are specific job descriptions that can be matched with workers. Firms and workers sign binding long-term contracts. The terms of these contracts are such that the competitive search equilibrium implements the efficient allocation for this economy.

2.1 Technology and Preferences

The economy is populated by a continuum of entrepreneurs and a continuum of workers. Entrepreneurs create a flow α of new blueprints. A blueprint is a description for a specific job that can be done by a single worker. It must be matched with a worker and a newly created blueprint is initially unmatched.

There is a unit measure of workers. Employed workers are matched with blueprints and unemployed workers are not. Employed workers can allocate effort $a_t \in [0,1]$ to producing $(1-a_t)y$ units of output and a flow $F(a_t,1)$ of new blueprints, where F is a standard production function that exhibits constant returns to scale. Unemployed workers can home-produce a flow $h < y$ of output. Blueprints are destroyed at a rate $\sigma > \alpha$. The aggregate stock of employed workers is N_t and the aggregate stock K_t of blueprints evolves according to

$$\mathrm{D}K_t = -\sigma K_t + \alpha + F(A_t, N_t), \tag{1}$$

where $A_t = a_t N_t \in [0, N_t]$ is the aggregate effort allocated by employed workers to creating new blueprints.

Employed workers lose the ability to do their assigned job at a rate θ, and the blueprint for the job they are assigned to is destroyed at an independent rate σ. Thus employed workers become unemployed at the rate $\sigma + \theta$. A vacancy is an unmatched blueprint. The measures of unemployed workers u_t and vacancies v_t are therefore

$$u_t = 1 - N_t, \quad v_t = K_t - N_t, \tag{2}$$

respectively. These unemployed workers and vacancies produce a flow of new matches $M(u_t, v_t)$. As a result, the stock employed workers evolves according to

$$\mathrm{D}N_t = -(\sigma + \theta)N_t + M(u_t, v_t). \tag{3}$$

The matching function M exhibits constant returns to scale and satisfies $M(0,1) = M(1,0) = 0$. This ensures that $N_t \in [0, \min\{1, K_t\}]$ starting from any initial condition $N_0 \in [0, \min\{1, K_0\}]$.

The output produced at home and by employed workers consists of tradable consumption goods. Aggregate consumption is given by

$$C_t = (1 - N_t)h + (N_t - A_t)y. \tag{4}$$

Entrepreneurs and consumers have additively separable preferences over consumption flows. Everyone discounts at the rate ρ and the flow utility function is logarithmic.

2.2 Competitive Search

There are enough securities markets to share risk perfectly. The resulting interest rates are

$$r_t = \rho + \frac{\mathrm{D}C_t}{C_t}. \tag{5}$$

More general homothetic preferences can be considered without much difficulty.

Firms and workers in a newly created match sign long-term contracts that allow the firm to tell the worker what to do, and that require the firm to pay the worker the going wage w_t. The value of a typical firm is linear in the number of matched and unmatched blueprints it has. The price of a blueprint matched with a worker is p_t, and the price of an unmatched blueprint is q_t. Finding a worker for an unmatched blueprint generates a capital gain $p_t - q_t$. As in Moen [1997], matches between vacancies and unemployed workers are created by competitive matchmakers. These matchmakers pay s_t per unit of time for a vacancy, and a matching fee $w_t - h$ for unemployed workers who make themselves available for employment. In exchange, firms pay the matchmaker the capital gain $p_t - q_t$ when a successful match is created. Anyone can become a matchmaker. The profits from matchmaking must therefore be zero,

$$s_t = \max_x \{(p_t - q_t)M(x, 1) - (w_t - h)x\}.$$

Assuming an interior solution, equilibrium requires that

$$(p_t - q_t)\mathrm{D}M(u_t, v_t) = [w_t - h, s_t]. \tag{6}$$

The unemployment-vacancy ratio u_t/v_t is implied by the state $[K_t, N_t]$ of the economy via (2). Given a capital gain $p_t - q_t$ of a newly created match, (6) then determines the factor prices $[w_t - h, s_t]$. Note that workers earn w_t per unit of time no matter what: from home production and matching fees when unemployed, or as a wage when employed.

An unmatched blueprint is valuable only because it can be rented out to matchmakers as a vacancy. Although firms enjoy capital gains when their blueprints are matched with workers, they owe those capital gains to the matchmakers who created the matches. Therefore

$$(r_t + \sigma)q_t = s_t + \mathrm{D}q_t. \tag{7}$$

A matched blueprint generates revenues from the output of consumption goods and the creation of new blueprints. The cost is the wage w_t and the anticipated capital loss $\theta(p_t - q_t)$ when the matched worker is no longer able to do the job. The value of a blueprint matched to a worker therefore satisfies

$$(r_t + \sigma)p_t = \max_{a \in [0,1]} \{q_t F(a, 1) + y(1 - a)\} - w_t - \theta(p_t - q_t) + \mathrm{D}p_t. \tag{8}$$

Again assuming an interior solution, this gives rise to the first-order condition

$$q_t \mathrm{D}_1 F(A_t, N_t) = y. \tag{9}$$

Given the state N_t and an unmatched-blueprint price q_t, this condition pins down the effort $a_t = A_t/N_t$ assigned to the creation of new blueprints by the typical employed worker.

The value of the aggregate capital stock in this economy is $q_t K_t + p_t N_t$. The value of an individual firm is simply the market value of its unmatched and matched blueprints. If the firm is integrated with the matchmakers that help fill its vacancies, then the stock price of the firm will see a capital gain when a new match is formed. If not, then the firm can issue securities to pay for a new match.

2.3 Equilibrium

The equilibrium is determined by (1)-(9) together with initial conditions for K_0 and N_0, and a transversality condition that requires $e^{-\rho t}(p_t K_t + q_t N_t)/C_t \to 0$ for large t.

To see how the equilibrium is determined, note that (2) together with (6) pins down (w_t, s_t) given (p_t, q_t). The first-order condition (9) pins down A_t/N_t and therefore the profits that accrue to the owner of a worker-matched blueprint, again given (p_t, q_t). With this, the equilibrium conditions (1), (3) and (7)-(8) become a first-order differential equation in (p_t, q_t, K_t, N_t). There are initial conditions for (K_t, N_t) and the transversality condition for $p_t K_t + q_t N_t$ implies analogous transversality conditions for p_t and q_t.

3. Convergence Rates and the Firm Size Distribution

Consider an aggregate economy that is in a steady state $[K, N]$. The state of an individual firm at time t is then defined by the numbers n_t of unmatched and k_t of matched blueprints it has. Its matched blueprints generate new unmatched blueprints at the rate $\mu = F(a, 1)$, become unmatched at the rate θ, and are destroyed at the rate σ. Its unmatched blueprints are also destroyed at the rate σ and become matched at the rate $M(u/v, 1)$. The event that a blueprint becomes unmatched is assumed to be independent across blueprints. To complete the description of the stochastic process for the state (n_t, k_t), suppose that $\sigma = \delta + \lambda$, where δ is the rate at which all of a firm's blueprints are destroyed at once, and λ is the idiosyncratic rate at which its individual blueprints are destroyed. Firms are assumed to exit when they no longer have any blueprints, either because of a firm-wide destruction of blueprints, or because a firm loses its last remaining blueprint. Firms enter when entrepreneurs find workers for their unmatched blueprints.

The state of a firm follows a two-dimensional birth-death process. The state of an individual blueprint cycles back and forth between being unmatched and matched with a worker, as long as it survives. When it is in the matched state, it generates new blueprints. To gain some insight into the properties of this type of process, consider an extreme scenario in which the matching function is infinitely productive. That is, new blueprints are immediately matched with workers and workers who leave are immediately replaced. This extreme scenario cannot be a good approximation for the aggregate economy because it implies no unemployment. But if matching blueprints with workers happens much quicker than creating new blueprints, then it can shed some light on the relation between the technology for creating new blueprints and the observed firm size distribution.

In this scenario, $K = N$ and (1) requires $N = \alpha/[\delta - (\mu - \lambda)]$, and hence $\delta > \mu - \lambda$. An individual firm grows at the rate $\mu - \lambda$ as long as it is not hit by the firm-wide shock, and aggregate employment at incumbent firms grows at the rate $\mu - \lambda - \delta < 0$. The entrepreneurial creation of new blueprints ensures stable aggregate employment.

Let P_n be the measure of firms with $k = n$ employees in a steady state. Then

$$\delta P_1 = \lambda 2 P_2 - (\mu + \lambda) P_1 + \alpha$$

and

$$\delta P_n = \mu(n-1) P_{n-1} + \lambda(n+1) P_{n+1} - (\mu + \lambda) n P_n$$

for all $n - 1 \in \mathbb{N}$. This is a second-order difference equation that can be solved explicitly. Luttmer [2011] provides the solution and gives the following characterization of the size distribution.

Proposition 1 *The employment size distribution of firms has a right tail that behaves like $n^{-\zeta}$, where $\zeta = \delta/(\mu - \lambda)$.*

Note that $\delta - (\mu - \lambda) > 0$ and so $\zeta > 1$. Estimates for US data give a ζ is very close to 1, somewhere in between 1 and 1.1. Luttmer [2011] estimates $\zeta \approx 1.06$.

To see what this implies, write $\mu_t = F(a_t, 1)$ outside the steady state. Then (1) and $K_t = N_t$ gives

$$\mathrm{D}N_t = -(\delta - [\mu_t - \lambda]) N_t + \alpha.$$

That is, the speed with which employment converges to its steady state is $\delta - (\mu_t - \lambda)$, and this is approximately $\delta - (\mu - \lambda) = (\zeta - 1)(\mu - \lambda)$ if $F(\cdot, 1)$ exhibits enough curvature. As noted, $\zeta - 1$ is close to zero. A calibration reported in Luttmer [2011] suggest that $\mu - \lambda \approx .02$ on an annual basis, and so $(\zeta - 1)(\mu - 1) < .1 \times .02 = .002$, or .2% per annum. This back-of-the envelope calculation suggests that employment can approach the steady state very slowly when the underlying stock of blueprints is away from its steady state.

As many have observed, basic calibrations of the matching function suggest that unemployment should quickly return to its steady state. But here that steady state is itself driven by a very slowly moving stock of blueprints. Recessions that are triggered by a destruction of match capital lead to quick recoveries in employment, but recessions triggered by a destruction of blueprint capital can lead to protracted periods of low employment.

References

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