What Should I Be When I Grow Up? Occupations and Employment over the Life Cycle and Business Cycle*

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Abstract

Why is unemployment higher for young individuals than for the old? Why are business cycle fluctuations in employment more pronounced for the young than for the old? We address these questions in the context of a search-and-matching model of the labor market that features learning about occupational fit. Specifically, young workers enter the labor market not knowing the occupation they are most productive in, i.e., not knowing their “true calling” in life. In order to learn her true calling, a worker must sample occupational matches over her career. Matches are subject to both idiosyncratic and aggregate shocks, so that workers move in and out of unemployment. As such, our model generates heterogeneity in the labor force, where employed and unemployed workers can be categorized into types: lower type workers have little information about their true calling, while higher types are closer to discovering it. Hence, the model generates an endogenous mapping between ‘type’ and age, allowing us to address our unemployment facts. Young workers are more likely to be unemployed since they are more likely to be in transition between occupations. This generates age differences in unemployment levels. Young workers are more volatile over the business cycle since firms’ returns to posting vacancies for low types is more sensitive to fluctuations in aggregate productivity. This generates age differences in employment volatility.

1 Introduction

Labor market outcomes differ greatly for individuals at different points in the life cycle. Unemployment rates are much higher for the young than for all others. For example, in the U.S., individuals aged 20–24 years old are about 2.5 times more likely to be unemployed as compared to prime-aged individuals aged 45–54 years old; the unemployment rate of 25–34 year olds is about 50% greater than that of 45–54 year olds.[1] Moreover, employment of the young is much more sensitive to business cycle fluctuations. Over the business cycle, employment of 20–24 year olds is more than twice as volatile as that of the prime-aged.[2]

In our view, explaining these facts is interesting in their own right, given that the age differences are so stark. More importantly, we believe that developing such an explanation is potentially important for our understanding of business cycle and aggregate labor market dynamics. For example, Jaimovich and Siu (2009) show that while those under the age of 30 account for less than one third of aggregate employment, they account for over one half of the business cycle volatility in aggregate employment. Moreover, compositional change in the labor force due to the baby boom generated a substantial fraction of the rise and fall in U.S. macroeconomic volatility from the 1960s through to the 2000s. Similarly, the results of Shimer (1998) indicate that workforce compositional change generated a substantial fraction of the rise and fall in the aggregate unemployment rate.

In this paper, we study these fundamental age differences in employment and unemployment using a life-cycle search-and-matching model in the style of Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP, hereafter). The key life-cycle feature that we introduce is learning on the worker's part about her best occupational fit. Specifically, young workers enter the labor market not knowing the occupation that they are most productive in. We call this occupation the worker's "true calling". In order to learn her true calling, a worker must sample occupational matches over her career.

A worker enters the labor market unemployed, knowing only that her true calling

[1]See also Shimer (1998) and the references therein.

[2]See also Clark and Summers (1981), Gomme et al. (2004), Jaimovich and Siu (2009), and Jaimovich et al. (2010).

is one of a finite number of occupations. She searches for her first job in one of these occupations. Upon meeting a vacancy, an employment relationship is established. Over time, the firm and worker learn whether the current occupation is the worker's true calling. If not, the worker-firm pair can either maintain the employment relationship or choose to separate. Upon separation, the worker seeks a new occupation, knowing that one potential occupation has been ruled out. As the worker samples more occupational matches, the probability of finding her true calling rises, so that her expected productivity in a match rises.

Match formation and separation is stochastic in the DMP framework. As such, *ex ante* identical individuals experience different labor market histories over time, as they move in and out of unemployment and learn about their true calling. Despite this rich heterogeneity in the labor force, employed and unemployed workers can be summarized simply by their *type*: lower type workers have little information about their true calling, while higher types are closer to discovering it. Indeed, new workers are the lowest type, having no information about their best fit, while workers who have discovered their true calling are the highest type.

Hence, the model generates an endogenous mapping between type and age. This allows us to address our facts about the level differences in unemployment between young and old workers. In the presence of aggregate shocks to match productivity, workers experience cyclical movements between employment and unemployment. As such, the type and age distribution of employment varies over the business cycle. This allows us to address our facts about the volatility differences in employment between young and old workers.

A key element to our model analysis is a segmentation of labor markets that we discuss in detail below. This is what allows us to fully characterize worker heterogeneity by types.[3] Firms may operate in all markets by maintaining vacancies for each worker type. Differences in the behavior of job creation across types are key to our model results.

[3] Our market segmentation also delivers tractability—in the form of *block recursivity*—typically associated with models of directed search (see Shi (2009), Gonzalez and Shi (2010), and Menzio and Shi (2010) for the original contributions on block recursivity in the search-and-matching literature). Yet it allows for flexibility in the wage determination mechanism, facilitating comparison of our results to much of the work in the DMP literature (e.g. Andolfatto (1996), Shimer (2005a).

Matches with lower type workers generate lower expected productivity, inducing firms to maintain fewer vacancies per job searcher in those markets. Thus, lower types face lower job finding rates relative to higher types. Moreover, lower type matches are more likely to turn out to be bad matches. These matches are more likely to separate, as opposed to continuing in an employment relationship. Thus, lower type workers face higher job separation rates. Both of these effects imply higher unemployment rates for lower types (and lower types tend to be the young).

In response to productivity shocks, the model generates a greater impact on employment for lower types than for higher types. This stems from two forces. First, lower expected productivity of lower types implies that their match surplus is proportionately more sensitive to productivity shocks. Second, learning about occupational fit implies that worker types that have not found their true calling are willing to suffer a 'wage cut' in bilateral Nash bargaining. We refer to this as an *information value*, as it reflects the utility gain to a worker from working in the current occupational match, and advancing towards discovery of their true calling. This information value is procyclical, and acts as a source of endogenous wage rigidity. This mitigates the effect of productivity shocks on wages, strengthening firms' incentives for job creation for lower types. These two forces imply that cyclical shocks induce greater variation in job finding rates for lower types than for higher types, and therefore greater variation in employment of lower types.

Very little is known about the causes and consequences of age differences in labor market outcomes, and especially, the relative volatility of employment of young and old. This is due in part to the difficulty associated with handling heterogeneity and aggregate uncertainty in DMP models. Within the neoclassical framework, a small number of papers address age differences in the business cycle volatility of hours worked. Ríos-Rull (1996) and Gomme et al. (2004) study stochastic growth models in which households' preferences for home and market production as well as efficiency units of hours worked differ exogenously over the life-cycle. As these authors show, these models cannot explain the large differences in volatility between the young and old. A second approach is considered in Jaimovich et al. (2010). The focus in this paper is on age differences in labor demand over the business cycle, owing to a production technology that features complementarity between capital and experienced labor. While this model is quantitatively successful at matching the volatility of young and old hours, it shares a common failing with Ríos-Rull (1996) and Gomme et al. (2004): it lacks an explicit treatment of unemployment, so that all fluctuations in market work come from the intensive margin. This is problematic since in the data, the bulk of the cyclical variation is due to movements in and out of employment and unemployment.

In the labor market search-and-matching literature, our analysis is related to a number of papers. The notion that individuals learn about their types as they accumulate experience is reminiscent of the classic paper by Jovanovic (1979) where learning about the quality of a match is done by observing output over time. However, this quality is specific to the match, so that all knowledge learned on one job is lost upon separation. Instead, our emphasis on occupational fit implies that knowledge learned on one job is informative regarding the worker's productivity in all other jobs. Our model is also related to Neal (1999), who models employment relationships as having two components: 'career' quality and idiosyncratic match quality. In that work, the career quality is idiosyncratic and independent across matches, and hence, no learning occurs. Again, in our formulation the endogenous learning over the life cycle is crucial to our results.

Finally, our model is also related to recent work by Papageorgiou (2009), who studies a model of occupational mobility in which workers learn about occupational fit. Papageorgiou (2009) finds that an estimated version of his model does well in matching life-cycle wage dynamics, cross-sectional residual wage inequality, and the observation that occupational switching declines with experience. A key distinction is that in his work the job finding rate is taken parametrically, whereas in our analysis it is determined endogenously, for each worker type, through equilibrium free entry conditions. The exogenous imposition of the job finding rate in Papageorgiou (2009) makes it infeasible to study the central questions in our investigation—those of age differences in job creation and unemployment, as well as their cyclical fluctuations in response to aggregate shocks.

The rest of this paper is structured as follows. The next section documents the empirical observations of primary interest, as well as observations related to key maintained assumptions in our modeling approach. Section 3 introduces the economic environment, while Section 4 defines and characterizes equilibrium. The following section provides analytical results on key features of the model's steady state, and in particular, why unemployment rates vary by worker type. Section 6 discusses features of the model's dynamics and why the cyclicality of job creation varies by worer type. Simulation results will appear in section 8, and section 9 concludes the paper.

2 Empirical Motivation

In this section, we detail the empirical observations that motivate our work. First, we outline the age differences in labor market outcomes that are the principal objects of investigation. Then, we present empirical evidence that informs our theoretical approach.

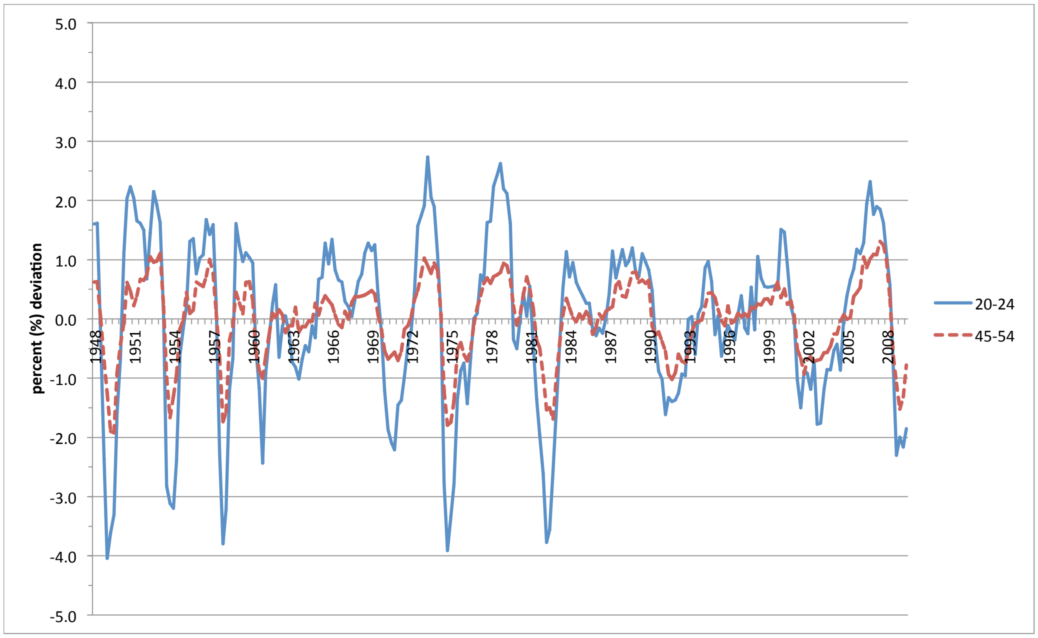
2.1 Age Differences in First and Second Moments

We begin by analyzing the responsiveness of employment to the business cycle for data disaggregated by age. We obtain labor force statistics for the U.S. from the Current Population Survey (CPS) for the period 1948:I–2010:III. From this we consider a measure of the *employment rate*—namely employed persons per labor force participant—for specific age groups, as well as an aggregate series for all individuals 16 years and up. We focus on the employment rate, as opposed to employed persons per capita, to emphasize that the age level differences that we study are not due to differences in the cyclicality of labor force participation.[4] To isolate fluctuations at the business cycle frequency (those greater than 32 quarters), we remove the trend from each series using the Hodrick-Prescott (HP) filter. Since the data are at a quarterly frequency, we use the standard smoothing parameter of $\lambda = 1600$.

Figure 1 indicates the scale of the observed age differences. Specifically, the figure presents the HP detrended employment rates for the 20–24 and 45–54 year old age groups (expressed as percentage deviations about trend). While employment for both age groups clearly comove over the business cycle, the magnitude of the fluctuations is much greater for the young.

[4]This is relevant since the search-and-matching framework that we study focuses solely on transitions between employment and unemployment. Conclusions derived from looking at age-specific per capita employment are essentially the same; see the discussion below.

Figure 1: Cyclical Employment by Age Group



Notes: Data from the Current Population Survey (CPS) for the period 1948:I–2010:III. The cyclical component is extracted using the HP-filter with smoothing parameter $\lambda = 1600$.

Table 1 presents more detailed results for the volatility of employment of various age groups. The first row presents the percent standard deviation of the filtered age-specific series. Employment volatility is clearly decreasing with age. The second row presents the percent standard deviation for each age group, relative to that of the 45–54 year olds, i.e. the prime-aged.

Row 2 indicates clearly that the age differences in volatility are large. The standard deviation of employment fluctuations for 20–24 year olds is at least 2.1 times

Table 1: Volatility of Employment Rates by Age Group

	16+	20–24	25–34	35–44	45–54	55–64
% standard deviation	0.86	1.41	0.94	0.74	0.66	0.59
normalized % std dev	1.31	2.13	1.42	1.13	1.00	0.89
correlation with output	0.88	0.88	0.88	0.85	0.83	0.73

Notes: HP filtered data, 1948:III - 2010:I. Employment data from the CPS; real (chain-weighted) GDP data from the NIPA.

that of the prime-aged (and almost 2.5 times that of the 55–64 year olds). Relative to the prime-aged, employment is more than 40% more volatile for the 25–34 age group.

We are not interested in the high frequency fluctuations in these time series *per se*, but rather those that are due to the business cycle. The third row of Table 1 presents the correlation of detrended employment and detrended Real GDP. These correlations are all very high. Evidently, the preponderance of high frequency fluctuations in age-specific employment are attributable to the business cycle, as opposed to age-specific, non-cyclical shocks.[5] Hence, over the business cycle, the employment rate of the young varies much more than that of the prime-aged.

Finally, we point out that essentially the same results emerge when we consider per capita employment. As opposed to the employment rate, per capita employment also includes the labor force participation margin. That is:

$$\frac{employment}{population} = \frac{employment}{labor\ force\ participants} \times \frac{labor\ force\ participants}{population}.$$

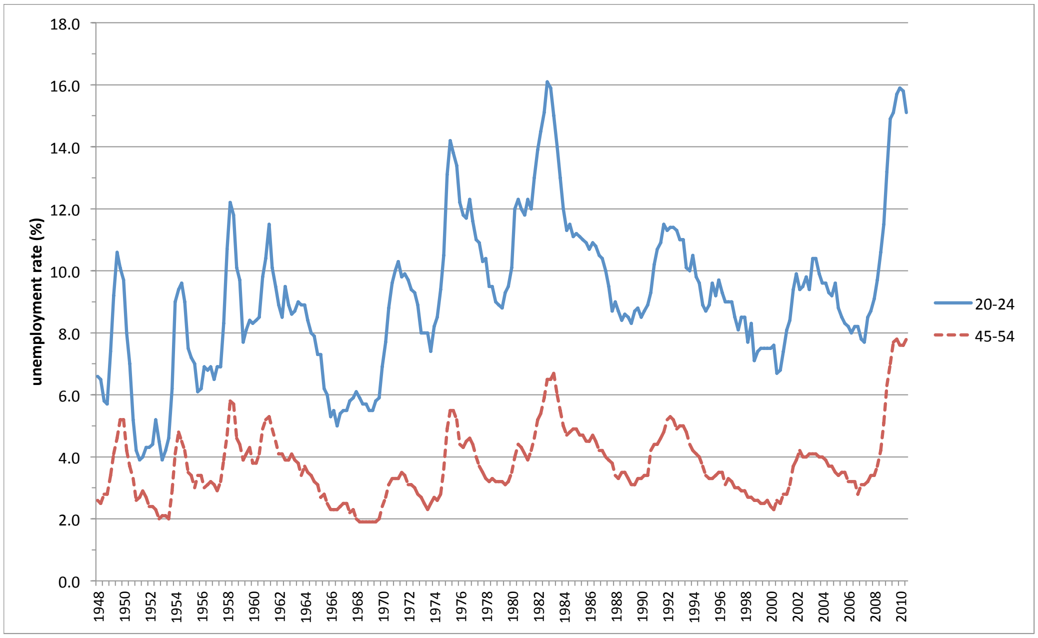
Hence, the variance in per capita employment factors in the variance of participation, as well as its covariance with the employment rate. For brevity, we do not present the results here. But as an example, the standard deviation of employment fluctuations for 20–24 year olds is 2.2 times that of the prime-aged when per capita employment is used. This compares to a relative volatility of 2.1 derived from employment rates. Hence, age differences in the volatility of per capita employment are due almost entirely to differences in the volatility of employment rates, as opposed to age differences in the volatility of participation.[6]

Our second observation is that unemployment among young individuals is much higher than for the prime-aged. To this end, we display CPS data on age-specific unemployment rates for the 20–24 and 45–54 age groups in Figure 2. At all points in time, the unemployment rate for 20–24 year olds is substantially higher than for the

[5]See Gomme et al. (2004) and Jaimovich and Siu (2009), who take similar analysis (for hours worked) one step further. They derive the "cyclical" component of age-specific market work series using projections on cyclical indicators. The results regarding magnitudes and age differences in age-specific volatility in the cyclical series are essentially unchanged relative to the results from the detrended series.

[6]These results echo those of Hall (2008). He finds that cyclicality in participation accounts for only 12% of the cyclical variation in aggregate hours worked, and that while the cyclicality of participation is greater for the young, the bulk of the variation in hours for this group stems from variation in employment.

Figure 2: Unemployment rates by Age Group



Notes: Data from the Current Population Survey (CPS) for the period 1948:I–2010:III.

prime-aged.

We summarize these differences for all age groups in Table 2. The first row presents the average unemployment rate observed in the sample. The second row presents the average for each age group, relative to that of the 45–54 year olds. Clearly, unemployment rates are decreasing with age. Moreover, the age differences are large. During the 1948:I–2010:III period, unemployment averaged 9.1% for 20–24 year olds, which is nearly 2.5 times that of 45–54 year olds. Similarly, the average unemployment rate of 5.4% for 25–34 year olds is about 45% greater than that of the prime-aged.

Table 2: Average Unemployment Rates by Age Group

	16+	20–24	25–34	35–44	45–54	55–64
average (%)	5.68	9.10	5.36	4.11	3.70	3.66
normalized average (%)	1.54	2.46	1.45	1.11	1.00	0.99

Notes: data from the CPS, 1948:III - 2010:I.

To summarize, we observe two stark age differences in labor market outcomes. First, the sensitivity of employment to the business cycle is much greater for the young than for other age groups. Second, the average unemployment rate is much higher for the young than for other age groups. These form our two principal observations of study.

2.2 Empirical Motivation of the Theory

In the remainder of this section, we present empirical evidence that motivate the maintained assumptions in our theoretical analysis. First, we present evidence on the cyclicality of job finding rates and job separation rates across age groups. We then present evidence on occupational mobility by age.

2.2.1 Job Finding and Job Separation Rates by Age

In Section 3, we consider a model in which job separation rates are essentially acyclical; all of the cyclicality in employment-unemployment transitions is due to cyclicality of job finding.[7] Here we show that this simplification represents an empirically relevant starting point. That is, our model's maintained assumption would be highly suspect if, for instance, the cyclicality of unemployment for the prime-aged was due primarily to variation in finding rates, whereas the cyclicality for the young was due primarily to variation in separation rates.

The empirical results indicate that this is not the case: the primary determinant of unemployment fluctuations over the business cycle—for all age groups—is the job finding rate. Our results are derived following the approach of Shimer (2005a,b, 2007). As such, we simply summarize here, and refer the reader to Shimer's work for details.[8]

[7] In principle, the model can generate countercyclical separation rates. However, in our numerical exercises, we have not found this to be quantitatively relevant. We discuss this in more detail below.

[8] In particular, the job finding and job separation rates are derived under two assumptions: that workers do not transit in and out of the labor force, and that workers within an age group are homogenous with respect to finding and separation probabilities. Of the two assumptions, the second is less stringent, as it simply alters the interpretation of the derived job finding rate; see Shimer (2005a, 2007). Note also that this assumption is more likely to be satisfied in our within-age-group analysis, as compared to Shimer's analysis of the aggregate labor force. The first assumption is stronger, and of course, applies equally to our results as it does to Shimer's. It is, however, informative to note that the cyclicality of labor force participation does not differ substantially

The approach uses monthly data on employment, unemployment, and short-term unemployment tabulated from the CPS. This data is available, disaggregated by age, only from June 1976 to August 2010.[9] The job finding rate, f_t, between month t and $t+1$ can be computed as:

$$f_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t}, \tag{1}$$

where u_t is the unemployment level in month t, and u^s_t is the number of workers unemployed for less than one month in month t. Next, the job separation rate, s_t, can be computed as:

$$s_t = \frac{u^s_{t+1}}{e_t\,(1 - 0.5 f_t)}, \tag{2}$$

where e_t is the employment level, and the 0.5 corrects for time-aggregation bias owing to individuals who lose their job and find a new one within the month.

With f_t and s_t, it is possible to determine the relative contribution of each to cyclical fluctuations in the unemployment rate. To do so, note that the unemployment rate, ur_t, can be approximated by:

$$ur_t \approx \frac{s_t}{s_t + f_t}. \tag{3}$$

Given this, we can construct two counterfactual series. The first is an unemployment rate that only allows for variation in the job finding rate. This is computed as:

$$cf^f_t = \frac{\overline{s}}{\overline{s} + f_t}, \tag{4}$$

where $\overline{s}$ is simply the time series average of the separation rate, s_t. The second is a counterfactual that only considers variation in the job separation rate:

$$cf^s_t = \frac{s_t}{s_t + \overline{f}}. \tag{5}$$

Which counterfactual series better accounts for the cyclical variation in the unemployment rate? Following Shimer (2007), we investigate this in two ways. The first is to simply compare the correlation of detrended ur_t with detrended cf^f_t and cf^s_t. To

across age groups; see Jaimovich et al. (2010), and our previous discussion regarding the cyclicality of per capita employment and the employment rate.

[9] Shimer (2005a, 2007) is able to study a longer time span since he is concerned only with deriving finding and separation rates in the aggregate.

Table 3: Correlation of Counterfactuals with Unemployment by Age Group

	16+	20–24	25–34	35–44	45–54	55–64
job finding	0.97	0.90	0.93	0.91	0.86	0.77
job separation	0.58	0.49	0.63	0.66	0.57	0.49

Notes: data from the CPS, 1976:III - 2010:I; job finding refers to counterfactual where only the job finding rate varies (and the job separation rate is fixed); job separation refers to counterfactual where only the job separation rate varies.

maintain comparability with Shimer's results, we time-aggregate the monthly data to a quarterly frequency, and apply an HP filter with $\lambda = 10^5$. This is an unusually large value for the smoothing parameter for quarterly data; as such, we have also derived results using $\lambda = 1600$; for brevity, we do not present the results here, but simply note that the results are very similar.

The first column of Table 3 presents the correlations for the counterfactuals with the unemployment rate for all individuals, aged 16 years and older. The correlation of cf_t^f with ur_t is near unity, and is clearly much higher than that of cf_t^s with ur_t. Hence, in the aggregate, fluctuations in unemployment are much more closely associated with fluctuations in the job finding rate.

The remaining columns of Table 3 present the same statistics for each of the age groups. As is obvious, the same conclusions derived in the aggregate apply when disaggregated by age: variation in the job finding rate is driving variation in the unemployment rate over the cycle, and crucially, this is true for young, prime-aged and old individuals alike.

To quantify this further, we perform a simple decomposition exercise as in Shimer (2007), by comparing the covariance of detrended cf_t^f and cf_t^s with ur_t. This is obviously very similar to the comparison of correlations, but also accounts for the fact that the job finding rate is much more volatile over the business cycle than the job separation rate.

The first column of Table 4 presents the decomposition for aggregate unemployment. Since this is not an exact decomposition, the two counterfactual series need not capture all of the variation in unemployment. As such, we have normalized the results, so that each row displays the fraction of explained variation accounted for

Table 4: Relative Contribution to Unemployment Volatility by Age Group

	16+	20–24	25–34	35–44	45–54	55–64
job finding	0.77	0.75	0.69	0.68	0.70	0.70
job separation	0.23	0.25	0.31	0.32	0.30	0.30

Notes: data from the CPS, 1976:III - 2010:I; job finding refers to counterfactual where only the job finding rate varies (and the job separation rate is fixed); job separation refers to counterfactual where only the job separation rate varies.

by each counterfactual.[10] In our sample period, variation in the job finding rate accounts for 77% of the variation in the unemployment rate for all workers aged 16 years and older; variation in the job separation rate accounts for the remaining 23%. This corresponds very closely to the results of Shimer (2007).[11]

The remaining columns of Table 4 present the same decomposition for each age group. Again the same conclusion arrived at in the aggregate holds for all age groups: variation in unemployment over the cycle is due primarily to variation in job finding rates. When disaggregated by age, the decomposition attributes between 68% (for 35–44 year olds) and 75% (20–24 year olds) of the volatility in age-specific unemployment rates to the job finding margin.

This finding regarding the relative importance of the cyclicality of job finding rates in the aggregate has had a substantial impact in the literature. Indeed, recent research investigating the business cycle volatility of aggregate unemployment has focused almost exclusively on representative worker models with exogenous—and hence, acyclical—match destruction rates.[12] The motivation for our analysis is similar. Across all age groups, variation in the job finding rate accounts for the bulk of the variation in employment-unemployment transitions over the business cycle. As such, in our model of heterogeneous workers, we maintain the simplifying assumption that match destruction rates are acyclical for all match types.

[10]See Fujita and Ramey (2009) for further discussion.

[11]See also Fujita and Ramey (2009) and Elsby et al. (2009); they arrive at similar conclusions, though with a slightly larger contribution to separation rates, and more nuanced findings regarding timing within downturns.

[12]See for instance, Shimer (2007), Hagedorn and Manovskii (2008), and Hall and Milgrom (2008).

2.2.2 Age-Specific Occupational Mobility

The mechanism underlying our theory is that individuals learn about their true calling in life by experimenting various occupations as they age. Following Kambourov and Manovskii (2008, 2009), who argue that human capital is largely occupation specific, we use data from the Panel Study of Income Dynamics (PSID) to document the age-profile of occupational mobility.

As is well-known, measures of occupational mobility are subject to sizable measurement error. In order to mitigate this measurement error, the PSID released Retrospective Occupation-Industry Supplemental Data Files (Retrospective Files hereafter) in 1999. The goal of this release was to make available retrospectively assigned 3-digit 1970 census codes to the reported occupations and industries of household heads and wives for the period 1968–80. Kambourov and Manovskii (2008, 2009) argue that the methodology used to construct the Retrospective Files minimizes the error in identifying true occupation switches.[13]

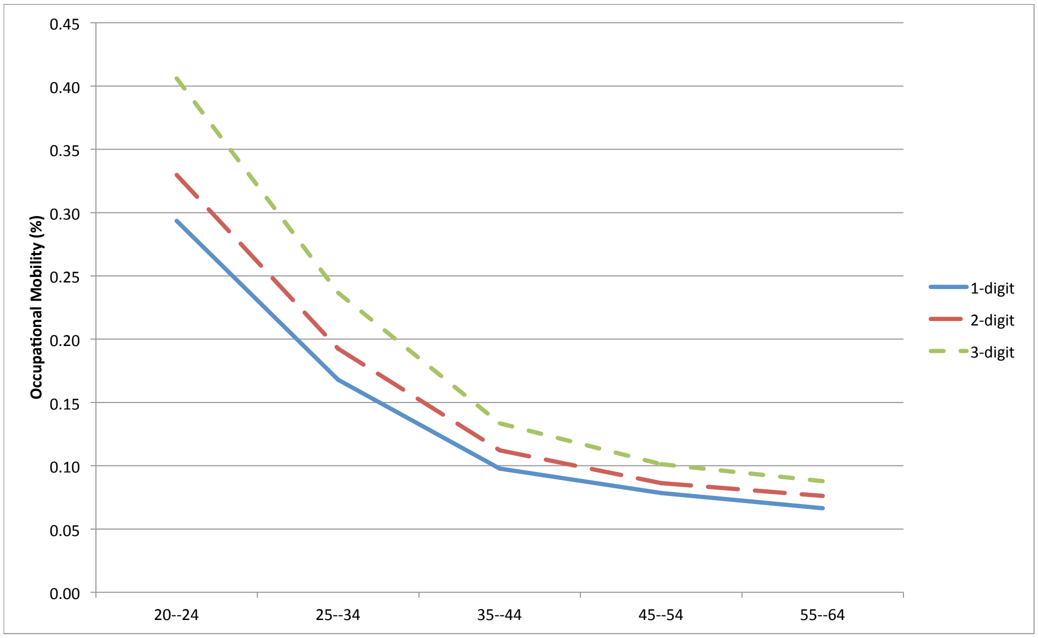
Figure 3 shows occupational mobility rates by age group for occupations at the 1, 2, and 3-digit level.[14] While the data and methodology underlying Figure 3 comes from Kambourov and Manovskii (2008), we adapt their exercise for the purpose of this study. First, we only use data from 1968 to 1980, that is, the period covered by the Retrospective files. Restricting the sample in this way makes our estimates conservative, in the sense that Kambourov and Manovskii (2008) document a rise in occupational mobility over the 1969–1997 period. Second, we use a broader sample, consisting of male heads of households who aren't self-employed and are either employed or temporarily laid off.[15] The main difference with their sample is our inclusion

[13] For example, approximately half of occupation switches in the original data are identified as legitimate occupation switches in the Retrospective Files. A key aspect of the Retrospective files is that a single individual was responsible for coding the occupation of a particular individual over the entire sample period, thereby minimizing changes in interpretation of the occupation reported by a given individual over time.

[14] See Appendix B of Kambourov and Manovskii (2008) for details of the occupation classification system.

[15] The latter restriction implies that we do not include individuals who leave the labor force in the calculation of the occupational mobility rate. The mobility rate is the ratio of the number of individuals who switch occupations, divided by the total number of workers (i.e. the sum of 'switchers' and 'non-switchers'). Assume an individual who leaves the labor force (retires or goes to school). If we count this individual as a 'non-switcher' we generate a downward bias in the occupational mobility measure (because the denominator is larger). Excluding those who leave the

Figure 3: Occupational Mobility by Age Group



Notes: Data from Kambourov and Manovskii (2008). Original data from the PSID and the Retrospective files from 1968 to 1980. The sample consists of male heads who aren't self-employed and are either employed or temporarily laid off.

of government workers and farmers, who tend to switch occupation less frequently than the rest of the sample. As such, our estimates are again conservative.

Figure 3 reveals that occupational mobility declines sharply as people age. For the young, occupational mobility rates are very high: nearly one in three individuals in the 20–24 age group switches occupation every year, even at the 1-digit level. For prime-aged workers, fewer than one in 12 changes 1-digit occupation on a yearly basis. The information contained in Figure 3 will be used to guide our calibration in Section 7.

These mobility rates can be used to calculate the number of occupations an individual experiences over her working life. For example, assume that an individual enters the labor force and experiences her first occupation at age 20. On average, the probability that she switches to a different one-digit occupation during each of the first five years of her career is 29.3%, which implies that she switches occupations

labor force from the sample avoids this bias.

1.47 times during the first five years. Similarly, over the next 10 years the average occupation switching probability is 16.8%, implying 1.68 switches. Repeating this calculation for all the age groups yields that the average worker switches about 5.6 one-digit occupations, hence working in 6.6 one-digit occupations over her career. A similar procedure indicates that the average person works in 7.3 two-digit occupations and 8.6 three-digit occupations over her career.

3 Economic Environment

3.1 Description

There are M occupations or "jobs" in the economy that are identical, except in name (think bus driver, cook, economist, rockstar). Each worker is best-suited for one occupation; that is, only one job, $m \in \{1, 2, ..., M\}$, is a good match, or the worker's "true calling". When a worker is employed in her true calling occupation m, she produce f_G units of output. In all other $M-1$ occupations, the worker-firm pair produces f_B units of output, with $f_B < f_G$. This is true for all occupations $m \in \{1, 2, ..., M\}$.[16]

In each period, a mass of newly born workers enter the economy not knowing their true calling. A new worker has her true calling randomly assigned into one of the M occupations with probability $1/M$. This assignment is distributed independently across all new workers. In order to learn whether job m is a good fit, the worker must search, be matched, and work in this job.

Given stochastic match creation and destruction, workers are heterogeneous with respect to their labor market history. Worker heterogeneity can be summarized as *types*, $i \in \{1, \ldots, M\}$, indicating the number of ill-suited jobs the worker has identified (plus one). A worker of type $i < M$ has previously sampled $i-1$ ill-suited jobs, and therefore has $M-i+1$ jobs left to try; a worker of type M knows her true calling (either having sampled $M-1$ ill-suited jobs, or having sampled fewer than that but got "lucky").

[16]We abstract from aggregate uncertainty at this point to maintain simplicity. Later on, we introduce aggregate productivity shocks keeping the ratio of productivity the same.

Workers understand that their true calling is uniformly distributed across all occupations, and use this to form their beliefs. Hence, an unemployed worker of type $i = 1$ has a flat prior over occupations, and randomly selects an occupation to search for employment. If that occupation turns out to be the wrong fit, the worker becomes type $i = 2$ and updates her prior according to Bayes' rule. She now understands that her true calling is uniformly distributed over the remaining $M - 1$ jobs, and randomly picks one of those occupations to search in the next time she becomes unemployed. We can now define p_i, the probability conditional on being matched in the i^{th} job that the worker has found her true calling. Given our assumptions, this is given by:

$$p_i = \frac{1}{M - i + 1}, \quad i = 1, \ldots, M.$$

We assume that each of the M job markets can be divided into "submarkets" that are distinguished by the unemployed workers' type, i. That is, in each job market $m \in \{1, \ldots, M\}$, a firm can maintain a vacancy for workers who are looking for their first job (we call this submarket 1 in job m), for workers who have already sampled one job (that was a bad fit) and are now looking for a second one (submarket 2 in job m), and so on. There is, obviously, a submarket for workers who know that job m is their true calling; we call this submarket M for the obvious reason. We assume that a worker's type (i.e. history of jobs) is perfectly observable. The cost of maintaining a vacancy in any submarket of any job market is k. There is free entry into vacancy posting on the part of firms.

For simplicity, we assume that all workers, regardless of the specific names of their past occupations, produce the same output in ill-suited jobs, f_B, and the same output in their true calling, f_G. As well, recall that true-callings are uniformly and independently distributed across workers. Hence, from the perspective of a firm posting a vacancy, all unemployed workers searching in a submarket are identical. This validates our summarization of worker types and submarkets by $i \in \{1, \ldots, M\}$, the number of ill-suited jobs a worker has identified.

Finally, recall that all occupations are identical (except in name). In addition, since each individual from the mass of new workers independently draw their true calling from a uniform distribution over occupations each period, each occupation is the true calling of exactly $1/M$ times the mass of new workers. And since the law of large numbers also applies to all other individuals with no work experience, all occupation-specific job markets for type 1 workers are symmetric. This allows us to consider a "representative" occupation for type 1 workers. A similar argument allows us to focus on the "representative" occupation for all types of individuals, which we refer to as submarkets $i \in \{1, \ldots, M\}$.[17]

To close the description of the environment, we need to take a stand on "death/exit from the economy". For simplicity, we assume that each worker faces the same, constant probability of surviving from one period to the next, σ. In this case, we interpret β in what follows as the "true" discount factor times the survival probability.

3.2 Market Tightness

Since each submarket is completely segmented, it is natural to define market tightness in submarket i, θ_i, as the ratio of the number of vacancies created by firms to the number of workers looking for jobs in submarket i. While the tightness of each submarket is an equilibrium object, they are taken parametrically by both firms and workers.

We denote the probability that a worker will meet a vacant job in submarket i by $\mu(\theta_i)$, where $\mu : \mathcal{R}_+ \rightarrow [0, 1]$ is a strictly increasing function with $\mu(0) = 0$. Similarly, we let $q(\theta_i)$ denote the probability that a firm with a vacancy meets a worker in submarket i, where $q : \mathcal{R}_+ \rightarrow [0, 1]$ is a strictly decreasing function with $q(\theta) \rightarrow 1$ as $\theta \rightarrow 0$. Naturally, $\mu(\theta) = \theta q(\theta)$.

3.3 Contractual Arrangement and Timing

The wage in a match is determined via Nash bargaining at the beginning of the period. When an unemployed worker and a firm match, they become productive in the following period. In the first productive period of a match, the output of the match does not become revealed until the end of the period, after the wage has been determined. In all subsequent periods that a worker and firm are matched, the wage is bargained with complete knowledge of the occupational fit, as is standard in the literature.

[17] Given the symmetric equilibrium we consider we cannot consider sectoral or occupation specific shocks.

Hence, three types of matches potentially exist at the beginning of a period: (i) good matches, for workers employed with a firm at their true calling; (ii) bad matches of type $i \in \{1, \ldots, M-1\}$, for workers employed with a firm at an occupation that is not their true calling; and (iii) matches of unknown quality of type $i \in \{1, \ldots, M-1\}$, for workers who are newly employed and do not know whether this is their true calling or not. We denote ω_M the wage of a worker in a good match, $\omega_{B,i}$ the wage of a worker of type i in a bad match (that is, a worker who is working in her i^{th} occupation, where $i < M$), and $\omega_{f,i}$ the wage of a newly hired worker who is just starting her i^{th} occupation.

3.4 Worker's Problem

Workers can either be unemployed or employed. Employed workers can either be in a good match, a bad match, or a match of yet unknown quality. Accordingly, we define U_i as the value of being unemployed for a worker of type i (recall, a type $i \in \{1, \ldots, M-1\}$ worker has experienced $i-1$ occupations without finding her true calling, whereas a type $i = M$ knows her true calling), W_M the value of a worker employed in a good match, $W_{B,i}$ the value of a type i worker in a bad match (for $i \in \{1, \ldots, M-1\}$), and $W_{f,i}$ the value of a worker who is experiencing her i^{th} occupation (for $i \in \{1, \ldots, M-1\}$). More formally,

$$U_i = z + \beta E\left[\mu(\theta_i) W'_{f,i} + (1 - \mu(\theta_i)) U'_i\right], \tag{6}$$

where the expectation is taken over the state of aggregate productivity which follows a Markov process. The value of being unemployed of type i equals the contemporaneous return from being unemployed, z, plus the continuation value, where the prime symbol denotes next period values. This continuation value equals the probability of finding a match for the first time in the i^{th} occupation times its expected return plus the probability of not finding a match and remaining unemployed. Notice that since the worker does not learn anything if no match is formed, the worker retains her type.

This formulation assumes that workers will never sample an occupation which they already know is not their true calling. We show later in Proposition 1 that this is indeed a feature of the equilibrium: workers who have sampled i occupations that are not their best fit never sample one of these again upon separation.

The value of being employed in a good match is given by

$$W_M = \omega_M + \beta E\left[(1-\delta)W_M' + \delta U_M'\right], \tag{7}$$

where δ denotes the (exogenous) probability that a good match will be destroyed at the end of the period. Notice that upon separation, the worker retains her type M as she knows her true calling.[18]

The value of being employed in a bad match for a type-i worker is given by

$$W_{B,i} = \max\left\{\omega_{B,i} + \beta E\left[(1-\delta_B)W_{B,i}' + \delta_B U_{i+1}'\right], U_{i+1}\right\}, \tag{8}$$

where $\delta_B \geq \delta$ denotes the probability that a bad match will be destroyed at the end of the period. The worker's value in a bad match either consists of today's wage plus the expected continuation payoff, or, if it is higher, the value of being unemployed today. In the event of separation—whether exogenous or endogenous—the worker becomes unemployed of type $i+1$ as she now knows that her last occupation was not her true calling.

Finally,

$$W_{f,i} = \omega_{f,i} + \beta E\left[p_i\left((1-\delta)W_M' + \delta U_M'\right) + (1-p_i)\left((1-\delta_B)W_{B,i}' + \delta_B U_{i+1}'\right)\right]. \tag{9}$$

Recall that when the wage is determined, neither the worker nor the firm know whether this will turn out to be a good or a bad match. Similarly, the right-hand-side of this last value function takes into account the fact that the match will turn out to be of good quality with probability p_i, or of bad quality with complementary probability. If the latter occurs, the match might dissolve exogenously with probability δ_B.

3.5 Firm's Problem

We assume that there is a large number of inactive firms who can potentially post vacancies in any of the submarkets they so desire, as long as they pay the cost of posting a vacancy, k. The value of posting a vacancy in submarket i is defined as

$$V_i = -k + \beta E\left[q(\theta_i)J_{f,i}' + (1-q(\theta_i))\max_j\left(V_j', 0\right)\right]. \tag{10}$$

[18]Again, Proposition 1 indicates that type M workers who separate only search in this market.

The second term in the square bracket expresses the idea that if a firm does not match with a worker, it can either become inactive or post a vacancy in any of the submarkets next period. With free-entry, this term will of course be zero in equilibrium. The term $J_{f,i}$ denotes the value of being in a match of unknown quality, given by[19]

$$\begin{aligned} J_{f,i} &= p_i f_G + (1-p_i) f_B - \omega_{f,i} \\ &+ \beta E\left[p_i(1-\delta)J'_M + (1-p_i)(1-\delta_B)J'_{B,i} + (p_i\delta + (1-p_i)\delta_B)\max_j\left(V'_j, 0\right)\right]. \end{aligned} \tag{11}$$

This value is composed of the contemporaneous profits—expected productivity minus the wage—plus the discounted value from next period on. This latter part consists of the value of being in a good match, which we denote J_M and occurs with probability p_i if the match survives, and the value of being in a bad match, $J_{B,i}$, also if it survives. The last term simply allows the firm to open a new vacancy or to remain inactive next period in case of separation. Notice that opening a vacancy in market M eliminates uncertainty about match quality since these workers already know their true calling ($p_M = 1$). In this case, only the value of being in a good match appears inside the square bracket, and so $J_{f,M}$ collapses to the value of being in a good match, defined as

$$J_M = f_G - \omega_M + \beta E\left[(1-\delta)J'_M + \delta \max_j\left(V'_j, 0\right)\right]. \tag{12}$$

Finally, the value of being in a bad match is given by

$$J_{B,i} = \max\left\{f_B - \omega_{B,i} + \beta E\left[(1-\delta_B)J'_{B,i} + \delta_B \max_j\left(V'_j, 0\right)\right], \max_j\left(V_j, 0\right)\right\}. \tag{13}$$

Similarly to the discussion that followed equation 8, firms in bad matches may choose to separate if it is in their best interest to do so.

4 Equilibrium

4.1 Definition of Equilibrium

An equilibrium with Nash bargaining is a collection of value functions $\Big(\{V_i, J_{B,i}, J_{f,i}\}_{i=1}^{M-1}$, V_M, J_M; $\{U_i, W_{B,i}, W_{f,i}\}_{i=1}^{M-1}$, U_M, $W_M\Big)$, wages $\Big(\{\omega_{f,i}, \omega_{B,i}\}_{i=1}^{M-1}, \omega_M\Big)$, and tightness

[19] It is worth noting again that this formulation assumes that workers searching in submarket i have never experienced that occupation in the past.

ratios, $\{\theta_i\}_{i=1}^{M}$ such that:

1. workers are optimizing. That is, given wages and tightness ratios, workers that are matched must prefer to remain matched rather than to be unemployed (with a weak preference in bad matches), $W_{f,i} > U_i, W_{B,i} \geq U_{i+1}, i = 1, \ldots, M-1$, and $W_M > U_M$;

2. firms are optimizing. That is, given wages and tightness ratios, the value of posting a vacancy in submarket i is greater than the value of not posting, $V_i \geq 0$, $i = 1, \ldots, M$, and the value of posting a vacancy is equal in all submarkets (since firms are free to choose where to post), $V_i = V_j \equiv V$, $\forall\, i, j$. As well, firms that are matched must prefer to remain matched as opposed to maintaining a vacancy (with a weak preference in the case of poorly matched firms), $J_{f,i}, J_M > V$, $J_{B,i} \geq V$, $i = 1, \ldots, M$;

3. wages solve the generalized Nash bargaining problems displayed below;

4. the free entry condition is satisfied. That is, since there is free entry on the part of firms posting vacancies, $V = 0$.

4.2 Equilibrium Wages

As is standard in the labor search literature, we assume that wages are determined via generalized Nash bargaining. We discuss this in detail, since the behavior of wages in response to productivity shocks is critical to our model results.

Let τ denote the bargaining weight of workers. The wage earned by workers in good matches solves the following problem:

$$\omega_M = \arg\max\, (W_M - U_M)^{\tau} (J_M - V_M)^{1-\tau}.$$

The solution takes the form:[20]

$$\omega_M = \tau f_G + (1-\tau)\Big\{ z + \mu(\theta_M)\, \beta E\left[W_G' - U_M'\right] \Big\}.$$

The interpretation of this is standard. In a good match, the worker is paid: a fraction, τ of the match output; a fraction, $1-\tau$, of the flow benefit of unemployment; and a

[20]See the Appendix for details on all wage derivations presented in this subsection.

fraction, $1-\tau$, of the *option value* of being unemployed. This last term represents a compensation to the worker to remaining in the match, which requires foregoing the opportunity to be unemployed this period (namely, the discounted worker surplus of a future match, adjusted by the probability of matching).

Using the fact that Nash bargaining results in a proportional split of total surplus, and the zero profit condition on vacancy posting in the M^{th} submarket, we obtain the more familiar representation:

$$\omega_M = \tau(f_G + \theta_M k) + (1-\tau)z. \tag{14}$$

This makes clear that a rise in market tightness, θ_M, increases the option value, and thus increases the worker's wage.

The wage for workers who are in their i^{th} bad match solves:

$$\omega_{B,i} = \arg\max\ (W_{B,i} - U_{i+1})^{\tau}(J_{B,i} - V_i)^{1-\tau}.$$

For these workers, the threat point is the value of unemployment in the $(i+1)^{th}$ market, U_{i+1} (as opposed to U_i): absent a wage agreement in the current match, the worker enters unemployment as type $i+1$. As such, the wage is given by:

$$\begin{aligned}\omega_{B,i} &= \tau f_B + (1-\tau)\Big\{z + \mu\left(\theta_{i+1}\right)\beta E\left[W'_{f,i+1} - U'_{i+1}\right]\Big\}, \\ &= \tau(f_B + \theta_{i+1}k) + (1-\tau)z. \end{aligned} \tag{15}$$

In a bad match of type i, the option value stems from the foregone opportunity of searching for the next job as type $i+1$.

Finally, the wage in the first phase of an i^{th} match solves:

$$\omega_{f,i} = \arg\max\ (W_{f,i} - U_i)^{\tau}(J_{f,i} - V_i)^{1-\tau}.$$

For these workers, the unemployment threat value is U_i; if agreement is not reached in the bargaining stage, they return to unemployment having not learned anything about their type. The solution for $\omega_{f,i}$ is given by:

$$\begin{aligned}\omega_{f,i} = \tau\left[p_i f_G + (1-p_i)f_B\right] + (1-\tau)\Big\{z + \mu\left(\theta_i\right)\beta E\left[W'_{f,i} - U'_i\right]\Big\} \\ - (1-\tau)\beta E\left[p_i U'_M + (1-p_i)U'_{i+1} - U'_i\right].\end{aligned}$$

Note that relative to ω_M and $\omega_{B,i}$, the first phase wage contains a new term. We refer to:

$$\beta E\left[p_i U'_M + (1-p_i)U'_{i+1} - U'_i\right]$$

as the *information value*. By working in the match, the worker learns about her competency in the current occupation. With probability p_i she learns that it is her true calling, and with complementary probability she learns that her type (when next unemployed) is $i+1$. Hence, the information value is the discounted, expected gain to learning and, therefore, augmenting her threat point in future wage bargaining.

Moreover, the information value is *positive*. This follows as an immediate corollary of Proposition 1, which we establish in Section 5. Hence, the information value term represents a 'pay cut' to the worker relative to a standard DMP model. Because the worker has this additional incentive to work—in order to advance toward her true calling—she is willing to accept this pay cut in her negotiated wage.

Finally, as we show in the Appendix, $\omega_{f,i}$ can also be expressed as a function tightness ratios (and exogenous parameters), as is the case for ω_M and $\omega_{B,i}$; specifically:

$$\omega_{f,i} = \tau\left[p_i f_G + (1-p_i) f_B + \theta_i k\right] + (1-\tau)z \\ - \tau k E \sum_{s=1}^{\infty} \beta^s \left[p_i \theta_M^{+s} + (1-p_i)\theta_{i+1}^{+s} - \theta_i^{+s}\right], \quad (16)$$

where the '$+s$' superscript indicates an 's-period ahead' variable. Obviously, the information value term represents a 'pay back' to the firm, and augments the firm's profitability of being in a first phase match. We discuss the implications of this for job creation in Sections 5 and 6.

4.3 Recursivity of Equilibrium

In this subsection we highlight two key sources of recursivity in the model's equilibrium. These features are important, in that they make model analysis tractable, despite the rich degree of heterogeneity in worker types. Moreover, they are key to understanding the mechanisms that deliver our results on differences in unemployment and employment volatility.

First, equilibrium in our model is *block recursive* in the sense that agents' value functions and decision rules are independent of the distribution of workers across employment/unemployment status and labor market histories. This is particularly useful in our analysis of the model's business cycle implications in response to aggregate shocks. This block recursivity is manifested in the fact that solving for model equilibrium can be summarized as finding a vector of M labor market tightness ratios, $\{\theta_i\}_{i=1}^{M}$ (specifically, one vector for each state of aggregate productivity). As such, all remaining equilibrium objects (wages, value functions, and measures of employed and unemployed workers) by type, i, as well as the mapping from type to age, can be backed out given $\{\theta_i\}_{i=1}^{M}$. [21]

Second, our equilibrium exhibits *recursivity in markets*. In a technical sense, this implies that the vector of equilibrium tightness ratios, $\{\theta_i\}_{i=1}^{M}$, need not be solved for simultaneously. Instead, they are found recursively: θ_M is independent of all lower markets, $i = 1, \ldots, M-1$; θ_{M-1} is independent of all lower markets, $i = 1, \ldots, M-2$; and so on.

This recursivity is seen most easily in the stationary version of the model (with no aggregate uncertainty). In this case, θ_M can be characterized by imposing equilibrium free entry in the M^{th} market, by using the stationary versions of equations (10) and (12):

$$k = \beta q\left(\theta_M\right) \frac{f_G - \omega_M}{1-\beta(1-\delta)}, \tag{17}$$

where ω_M is given by equation (14). Clearly, the M^{th} market is independent of all other markets. Moreover, this market operates *identically* to the single, representative market of the standard DMP model. This feature is especially useful in facilitating comparison of our results to those of the literature.

Next, consider the free entry condition in market $M-1$. The tightness ratio, θ_{M-1}, must be such that:

$$\begin{aligned} k = \beta q\left(\theta_{M-1}\right) \Big\{ & p_i f_G + (1-p_i) f_B - \omega_{f,M-1} \\ & + \beta \left[p_i (1-\delta) \frac{f_G - \omega_M}{1-\beta(1-\delta)} + (1-p_i) \frac{f_B - \omega_{B,M-1}}{1-\beta(1-\delta)} \right] \Big\}. \end{aligned}$$

From equations (14) and (15), ω_M and $\omega_{B,M-1}$ depend on θ_M; from (16), $\omega_{f,M-1}$ depends on θ_{M-1} and θ_M (the latter through the information value). Having solved for market M, we are able to solve for market $M-1$ independently of markets

[21] See the Appendix for details on the construction of worker flows.

$1, \ldots, M-2$. This, of course, continues to hold as we move down from market $M-2$ through to the first market.

The recursivity of markets has a couple of useful implications. The first is that the model is especially easy to solve (we present the formal algorithm in the Appendix). More importantly, it indicates how equilibrium in markets $1, \ldots, M-1$ differ from market M. In particular, the level and volatility differences in employment that the model generates is due to the dependence of θ_i on $\theta_{i+1}, \ldots, \theta_M$. We turn to this in the next two sections.

5 Characterizing Steady State Equilibria

In this section we establish that in any steady state equilibrium, the value of unemployment increases with workers' type, that is, $U_i < U_{i+1}$, $i = 1, \ldots, M-1$. This result is useful not only as it leads to a sharp characterization of steady state equilibria, but also because unemployed workers in such equilibria have an incentive to search in the submarket of their type, which was assumed throughout Section 3. Before showing that result, we first establish a useful relationship between the value of unemployment in a particular submarket and the market tightness in that market.

Lemma 1 *In any steady state equilibrium, $U_i < (>) U_{i+1}$ implies that $\theta_i < (>) \theta_{i+1}$.*

Proof. In steady state, the value of being unemployed for a type-i worker is

$$U_i = z + \beta \left[\mu(\theta_i) \left(W_{f,i} - U_i \right) + U_i \right].$$

Since Nash bargaining implies that $\tau J_{f,i} = (1-\tau)(W_{f,i} - U_i)$, and free entree implies that $\beta q(\theta_i) J_{f,i} = k$ this can be re-written as

$$U_i = \frac{z + \tau k \theta_i / (1-\tau)}{1-\beta}.$$

■

A number of results follow immediately from this Lemma, collected in the following Corollary:

Corollary 1 *If $U_i < U_{i+1}$ for $i = 1, \ldots, M-1$, then*

1. $q(\theta_{i+1}) < q(\theta_i)$;
2. $J_{f,i+1} > J_{f,i}$;
3. $W_{f,i+1} - U_{i+1} > W_{f,i} - U_i$;
4. $W_{f,i+1} - W_{f,i} > U_{i+1} - U_i > 0$, *so* $W_{f,i+1} > W_{f,i}$;
5. $\omega_{B,i+1} = \tau(f_B + \theta_{i+2}) + (1-\tau)z > \tau(f_B + \theta_{i+1}) + (1-\tau)z = \omega_{B,i}$
6. $J_{B,i+1} = \frac{f_B - \omega_{B,i+1}}{1-\beta(1-\delta_B)} < \frac{f_B - \omega_{B,i}}{1-\beta(1-\delta_B)} = J_{B,i}$;[22]
7. $\omega_{f,i} < p_i \omega_M + (1-p_i)\omega_{B,i}$.

All the inequalities are reversed if $U_i > U_{i+1}$.

Proof. All these results are obvious, except perhaps for the last one, that $\omega_{f,i} < p_i\omega_M + (1-p_i)\omega_{B,i}$. To see this, consider the steady state version of the wage first obtained in market i, equation (9):

$$\omega_{f,i} = (1-\tau)z + \tau\left[p_i f_G + (1-p_i) f_B\right] + \beta\left(\omega_{f,i} - p_i\omega_M - (1-p_i)\omega_{B,i}\right) + \tau k\theta_i.$$

Using the wage equations in good and bad matches (7) and (8) and rearranging, we have

$$(1-\beta)\left[\omega_{f,i} - p_i\omega_M - (1-p_i)\omega_{B,i}\right] = \tau k\left(\theta_i - p_i\theta_M - (1-p_i)\theta_{i+1}\right).$$

The result follows if $\theta_i < \theta_{i+1}$ for $i = 1, \ldots, M$, which by Lemma 1 must be the case if U_i is increasing in i. ■

This last result states that the wage of an individual experimenting in submarket i is smaller than the expected value of the future wage once productivity has been revealed, much as in Jovanovic (1979). As mentioned earlier, this result emanates from the notion that workers gains information by experimenting an occupation,

[22]Note that this result does not hold for two subsequent submarkets in which bad matches are endogenously destroyed, as the firm's value of being in a bad match is zero for both submarkets in this case. This result also implies that bad matches for higher types are more likely to lead to endogenous separation than bad matches for lower types.

which firms exploit through Nash bargaining. In addition, it implies that from the perspective of the firm,

$$[p_i f_G + (1 - p_i) f_B] - \omega_{f,i} > p_i \left(f_G - \omega_M\right) + (1 - p_i) \left(f_B - \omega_{B,i}\right), \tag{18}$$

which states that the "first period" momentary profit is greater than the expected value of momentary profit in any subsequent period.

Our next proposition shows that only equilibria in which the value of unemployment increases with types can emerge.

Proposition 1 *In any steady state equilibrium, $U_i < U_{i+1}$ for $i = 1, 2, \ldots, M-1$.*

The proof is provided in the Appendix. The intuition behind Proposition 1 is that because expected productivity is increasing in submarkets, so is the total surplus from a match. Under Nash bargaining, firms receive a constant fraction of that surplus, which means that conditional on forming a match, firms' profits are increasing in submarkets. But in order for potential firms to be indifferent between opening vacancies in any of the M submarkets, the probability of forming a match must be lower in high submarkets. Accordingly, the tightness ratio must increase with submarkets.

We close this section by stating formally a result regarding the value of information which follows directly from Proposition 1. This result, which will prove useful in the next section, simply states that the value of information must be positive.

Corollary 2 *In any steady state equilibrium, the information value is positive. That is, $[p_i U_M + (1 - p_i) U_{i+1} - U_i] > 0$, or, equivalently, $[p_i \theta_M + (1 - p_i)\theta_{i+1} - \theta_i] > 0$.*

6 Characterizing Cyclical Variation

In the model, differences in employment volatility across worker types are due to differences in the volatility of the job finding rate.[23] As the job finding rate is an

[23]Note that the model features differences in the separation rate of matches across types/submarkets. This is due to the fact that workers and firms may choose to separate once they discover that the match productivity is f_B. Hence, the model can feature (in principle), cycli-

increasing function of the vacancy-unemployment ratio, θ_i, we focus on characterizing the volatility of θ_i across markets. Differences in volatility comes from three sources of heterogeneity across markets: (1) the cyclicality of the information value, (2) the expected steady state level of productivity, and (3) the information value in steady state.

To illustrate this, we analyze deviations in equilibrium variables from steady state arising from a shock to aggregate productivity. Consider markets $M-1$ and M.[24] As discussed in Section (4.3), the M^{th} market in our model functions as a 'stand-alone' DMP model: θ_M is pinned down by the free entry condition, (17), with the wage given by (14). The effect of a positive shock is well understood. The direct effect is to raise match output, and hence, profitability. Free entry implies that firms create more vacancies per unemployed worker, raising θ_M. This raises the option value of unemployment, and thus raises wages. This feedback effect dampens the rise in profits, acting to mute the required increase in θ_M in equilibrium.

In steady state, the free entry condition in market $M-1$ can be written as:

$$\frac{k}{\beta}\theta_{M-1}^{ss1-\alpha} = (1-\tau)[p_{M-1}f_G + (1-p_{M-1})f_B - z] \\ - \tau k\theta_{M-1}^{ss} + \beta p_{M-1}(1-\delta)J_M^{ss} + VI_{M-1}^{ss}, \qquad (19)$$

where J_M^{ss} is the steady state value of a firm in a good match, and:

$$VI_{M-1}^{ss} = \tau k\left(\frac{\beta}{1-\beta}\right)\left[\theta_M^{ss} - \theta_{M-1}^{ss}\right]$$

is the steady state information value in market $M-1$ (multiplied by constant terms). In this expression, we have assumed that $J_{B,M-1} = 0$. This is for expositional simplicity, and our results hold even when $J_{B,M-1} > 0$.[25]

Log linearizing equation (19) and denoting by a hat the percentage deviations

cality of the separation rate if, for instance, bad matches are retained in booms, but separated in recessions. However, in our numerical simulations, we find no such cyclicality in response to aggregate productivity shocks for plausible calibrations. As a result, we find no quantitatively plausible difference in the cyclicality of job separation across match types in our analysis.

[24] The analysis is identical in nature in all other markets.

[25] It is also worth noting that in our calibrated examples, we find that $J_{B,M-1} = 0$ is the quantitatively relevant case.

from the steady state, we can express $\hat{\theta}_{M-1}$ as:

$$\hat{\theta}_{M-1} = \frac{\beta p_{M-1}(1-\delta)J_M^{ss}\hat{J}_M + (1-\tau)\left[p_{M-1}\hat{f}_G + (1-p_{M-1})\hat{f}_B\right] + VI_{M-1}^{ss}\widehat{VI}_{M-1}}{\left[\frac{k}{\beta}\theta_{M-1}^{ss1-\alpha}(1-\alpha) + \tau k\theta_{M-1}^{ss}\right]}.$$

We now show that the endogenous response in the information value induces greater volatility in $\hat{\theta}_{M-1}$ than absent this movement. Consider a positive shock to productivity (the argument is identical for the case of a negative shock). The first two terms in the numerator are independent of market $M-1$ and are, therefore, independent of $\widehat{VI}_{M-1}$. The denominator is a function only of the steady state, and hence, does not react to the shock. Since Corollary 2 shows that the information value is always positive, the effect of $\widehat{VI}_{M-1}$ on $\hat{\theta}_{M-1}$ depends on the cyclical nature of the information value. The following proposition speaks directly to this cyclicality:

Proposition 2 *The information value is procyclical. That is,* $\widehat{VI}_i \gtreqless 0$ *for* $\hat{f}_G, \hat{f}_B \gtreqless 0$ *for all* $i = 1, \ldots, M-1$.

The proof is provided in the Appendix. It follows immediately that the procyclicality of the information value induces a stronger response of θ_{M-1} relative to a model absent fluctuations in $\widehat{VI}_{M-1}$.

The intuition for is as follows. Following a positive shock, $\hat{\theta}_M > 0$, so that the job finding rate is higher for workers who know their true calling. Since the worker's value is increasing in type, and since it is now easier to find a good job, it is more valuable for workers to work and learn about their true calling.

Therefore, a worker of any type $i < M$ is willing to take a bigger pay cut when productivity rises. This effectively weakens the worker's bargaining position, and mitigates the wage increase that is associated with higher productivity. The weaker feedback effect of market tightness on wages strengthens firms' incentives to create vacancies for types $i < M$. Hence, the procyclicality of the information value acts as a form of *endogenous wage rigidity*, generating greater cyclicality in vacancies and the job finding rate. It is worth noting that a number of recent papers have emphasized the ability of sticky wage models to amplify job creation in the DMP model.[26] Relative to these models, our model displays wage rigidity as a consequence

[26]See, for instance, Shimer (2004) and Hall and Milgrom (2008).

of procyclicality in the information value, without deviating from the convention of Nash bargaining.

Aside from the effect of fluctuations of the information value, the remaining two mechanisms operate through their effect on steady state match profitability. Hagedorn and Manovskii (2008) show that in a DMP model, reducing the profit of matches in steady state results in productivity shocks having a larger effect on match profitability in percentage terms, and as a result, a larger effect on the response of vacancies.

In our model, two forces operate in opposite directions to affect steady state profitability. On the one hand, expected match productivity is lower for lower types. This reduces steady state profit and, therefore, increases the volatility of vacancies. On the other hand, the existence of a positive information value works to increase profit. This reduces the volatility of vacancies. We are not able to analytically sign the overall effect of the two opposing forces since it depends in a nonlinear way on the steady state values of θ_M and θ_{M-1}. Nonetheless, these two forces operate through the same channel, and generate differences in the volatility of job creation across markets.

7 Calibration

TBW

8 Simulation Results

TBW

9 Conclusion

TBW

A Deriving Wages

A.1 Deriving ω_M :

From the Nash bargaining we get that the solution can be characterized as

$$(1-\tau)(W_M - U_M) = \tau J_M$$

Starting with the left hand side we have

$$\begin{aligned} W_M &= \omega_M + \beta E\left[(1-\delta)W'_M + \delta U'_M\right] \\ U_M &= z + \beta E\left[\mu(\theta_M)W'_M + (1-\mu(\theta_M))U'_M\right] \end{aligned}$$

Hence

$$\begin{aligned} (W_M - U_M) &= (\omega_M - z) + \beta E\left[((1-\delta) - \mu(\theta_M))(W'_M - U'_M)\right] \\ (W_M - U_M) &= (\omega_M - z) + \beta E\left[((1-\delta) - \mu(\theta_M))\frac{\tau}{1-\tau}(J_M)'\right] \end{aligned}$$

Now substitute this last equation to the wage function

$$(1-\tau)(\omega_M - z) + \tau\beta E\left[((1-\delta) - \mu(\theta_M))(J_M)'\right] = \tau J_M$$

From

$$J_M = (f_G - \omega_M) + \beta E\left[\delta \max_j \left((V_j)', 0\right) + (1-\delta)(J_M)'\right]$$

we get

$$\begin{aligned} (1-\tau)(\omega_M - z) + \tau\beta E\left[((1-\delta) - \mu(\theta_M))(J_M)'\right] &= \tau\left[(f_G - \omega_M) + \beta E\left[(1-\delta)(J_M)'\right]\right] \\ (1-\tau)(\omega_M - z) - \tau\beta E\left[\mu(\theta_M)(J_M)'\right] &= \tau\left[(f_G - \omega_M)\right] \\ (1-\tau)(-z) + \omega_M - \tau\beta E\left[\mu(\theta_M)(J_M)'\right] &= \tau f_G \\ \omega_M &= \tau f_G + \tau\mu(\theta_M)\beta E\left[(J_M)'\right] + z(1-\tau) \end{aligned}$$

Using the free entry (zero profit) condition we can sub for $\beta E q(\theta_M)(J_M)'$ and thus get

$$\omega_M = \tau\left(f_G + \theta_M k\right) + z(1-\tau)$$

A.2 Deriving $\omega_{B,i}$:

From the Nash bargaining we get that the solution can be characterized as

$$(1-\tau)(W_{B,i} - U_{i+1}) = \tau J_{B,i}$$

Using the defnitions of the value function we get that $W_{B,i} - U_{i+1}$ equals

$$\begin{aligned} W_{B,i} - U_{i+1} &= \left\{ \begin{array}{c} \omega_{B,i} + \beta E\left[(1-\delta_B)W'_{B,i} + \delta_B U'_{i+1}\right] \\ - \\ \left(z + \beta E\left[\mu(\theta_{i+1})W'_{f,i+1} + (1-\mu(\theta_{i+1}))U'_{i+1}\right]\right) \end{array} \right\} \\ &= (\omega_{B,i} - z) + \beta E(1-\delta_B)E\left(W'_{B,i} - U'_{i+1}\right) - \beta\mu(\theta_{i+1})E\left(W'_{f,i+1} - U'_{i+1}\right) \end{aligned}$$

Note that

$$\begin{aligned} W'_{B,i} - U'_{i+1} &= \frac{\tau}{1-\tau}J'_{B,i} \\ W'_{f,i+1} - U'_{i+1} &= \frac{\tau}{1-\tau}J'_{f,i+1} \end{aligned}$$

Use this to get that

$$(1-\tau)(W_{B,i} - U_{i+1}) = (1-\tau)\left(\omega_{B,i} - z\right) + \tau\beta E(1-\delta_B)E\left(J'_{B,i}\right) - \tau\beta\mu(\theta_{i+1})E\left(J'_{f,i+1}\right)$$

Now, from the zero profit conditions we know that for any i

$$k = q(\theta_i)\beta E\left[J'_{f,i}\right]$$

thus, since $\mu(\theta_{i+1}) = \theta_{i+1}q(\theta_{i+1})$,

$$(1-\tau)(W_{B,i} - U_{i+1}) = (1-\tau)\left(\omega_{B,i} - z\right) + \tau\beta E(1-\delta_B)E\left(J'_{B,i}\right) - \tau k\theta_{i+1}$$

Now let's use the right hand side

$$\tau J_{B,i} = \tau\left(f_B - \omega_{B,i}\right) + (1-\delta_B)\tau\beta E\left[J'_{B,i}\right],$$

we thus get

$$\begin{aligned} (1-\tau)\left(\omega_{B,i} - z\right) + \tau\beta E(1-\delta_B)E\left(J'_{B,i}\right) - \tau k\theta_{i+1} &= \tau\left(f_B - \omega_{B,i}\right) + (1-\delta_B)\tau\beta E\left[J'_{B,i}\right] \\ (1-\tau)\left(\omega_{B,i} - z\right) - \tau k\theta_{i+1} &= \tau\left(f_B - \omega_{B,i}\right) \end{aligned}$$

and rearranging terms we get

$$\omega_{B,i} = \tau(f_B + \theta_{i+1}k) + (1-\tau)z$$

A.3 Deriving $\omega_{f,i}$:

From the Nash bargaining we get that the solution can be characterized as

$$(1-\tau)\left(W_{f,i} - U_i\right) = \tau J_{f,i}$$

Using the definitions of the value function we get that $W_{f,i} - U_i$ equals

$$
\begin{aligned}
(W_{f,i} - U_i) &= \left\{ \begin{array}{c} \omega_{f,i} + \beta E \left[p_i \left\{ \begin{array}{c} (1-\delta)W'_M \\ + \\ \delta U'_M \end{array} \right\} + (1-p_i) \left\{ \begin{array}{c} (1-\delta_B)W'_{B,i} \\ + \\ \delta_B U'_{i+1} \end{array} \right\} \right] \\ - \\ \left(z + \beta E\left[\mu(\theta_i)W'_{f,i} + (1-\mu(\theta_i))U'_i\right]\right) \end{array} \right\} \\
&= \left\{ (\omega_{f,i} - z) + \beta E \left[\begin{array}{c} (p_i(1-\delta)(W'_M - U'_M) + p_i U'_M \\ +(1-p_i)(1-\delta_B)(W'_{B,i} - U'_{i+1}) + (1-p_i)U'_{i+1} \\ -\mu(\theta_i)\left(W'_{f,i} - U'_i\right) - U'_i \end{array} \right] \right\}
\end{aligned}
$$

From the Nash bargaining solutions we have

$$
\begin{aligned}
(W_M - U_M) &= \frac{\tau}{(1-\tau)} J_M \\
(W_{B,i} - U_{i+1}) &= \frac{\tau}{(1-\tau)} J_{B,i} \\
(W_{f,i} - U_i) &= \frac{\tau}{(1-\tau)} J_{f,i}
\end{aligned}
$$

and thus

$$
(1-\tau)\left(W_{f,i} - U_i\right) = (1-\tau)\left(\omega_{f,i} - z\right) + \beta E \left[\begin{array}{c} p_i(1-\delta)\tau J'_M + (1-\tau)p_i U'_M \\ +(1-p_i)(1-\delta_B)\tau J'_{B,i} + (1-\tau)(1-p_i)U'_{i+1} \\ -\mu(\theta_i)\tau J'_{f,i} - (1-\tau)U'_i \end{array} \right]
$$

Now, the right hand side is

$$
\begin{aligned}
\tau J_{f,i} &= \tau\left([p_i f_G + (1-p_i)f_B] - \omega_{f,i} + \beta E\left[p_i(1-\delta)J'_M + (1-p_i)(1-\delta_B)J'_B\right]\right) \\
&= \tau\left[p_i f_G + (1-p_i)f_B\right] - \tau\omega_{f,i} + \tau\beta E\left(p_i(1-\delta)J'_M\right) + \tau\beta E\left[(1-p_i)(1-\delta_B)J'_B\right]
\end{aligned}
$$

Note that $\tau\beta E\left(p_i(1-\delta)J'_M\right) + \tau\beta E(1-p_i)(1-\delta_B)J'_B$ cancels from both sides and thus we get

$$
\begin{aligned}
(1-\tau)\left(\omega_{f,i} - z\right) + (1-\tau)\beta E\left[p_i U'_M + (1-p_i)U'_{i+1} - U'_i\right] - \beta E\mu(\theta_i)\tau J'_{f,i} \\
= \tau\left[p_i f_G + (1-p_i)f_B\right] - \tau\omega_{f,i},
\end{aligned}
$$

or

$$
\omega_{f,i} = (1-\tau)z + \tau\left[p_i f_G + (1-p_i)f_B\right] + (1-\tau)\beta E\left[U'_i - p_i U'_M - (1-p_i)U'_{i+1}\right] + \beta E\left(\theta_i q(\theta_i)\tau J'_{f,i}\right).
$$

From the zero profit condition we know that $k = \beta E\left[q(\theta_i)J'_{f,i}\right]$ and thus

$$
\omega_{f,i} = (1-\tau)z + \tau\left[p_i f_G + (1-p_i)f_B\right] + (1-\tau)\beta E\left[U'_i - p_i U'_M - (1-p_i)U'_{i+1}\right] + \tau k\theta_i
$$

Now, what do we know about $\left[U_i' - p_i U_M' - (1-p_i) U_{i+1}'\right]$?
First note that that we can write $J_{f,i}$ and $W_{f,i}$ as

$$\begin{aligned} J_{f,i} &= p_i \left(J_M + \omega_M\right) + (1-p_i)\left(J_{B,i} + \omega_{B,i}\right) - \omega_{f,i} \\ W_{f,i} &= p_i \left(W_M - \omega_M\right) + (1-p_i)\left(W_{B,i} - \omega_{B,i}\right) + \omega_{f,i} \end{aligned}$$

Let's use these last two equations in the Nash bargaining condition $(1-\tau)\left(W_{f,i} - U_i\right) = \tau J_{f,i}$

$$(1-\tau)\left[\begin{array}{c} p_i \left(W_M - \omega_M\right) + (1-p_i)\left(W_{B,i} - \omega_{B,i}\right) + \omega_{f,i} \\ - \\ U_i \end{array}\right] \\ = \tau \left[p_i \left(J_M + \omega_M\right) + (1-p_i)\left(J_{B,i} + \omega_{B,i}\right) - \omega_{f,i}\right]$$

We can rewrite this condition as

$$\left\{\begin{array}{c} p_i \left[(1-\tau)\left(W_M - \omega_M\right) - \tau J_M\right] \\ + \\ (1-p_i)\left[(1-\tau)\left(W_{B,i} - \omega_{B,i}\right) - \tau J_{B,i}\right] \\ - \\ (1-\tau)\left[U_i - \omega_{f,i}\right] \end{array}\right\} = \tau \left[p_i \omega_M + (1-p_i)\omega_{B,i} - \omega_{f,i}\right]$$

Now $\pm U_M$ and $\pm U_{i+1}$ to get

$$\left\{\begin{array}{c} p_i \left[(1-\tau)\left(W_M - \omega_M \pm U_M\right) - \tau J_M\right] \\ + \\ (1-p_i)\left[(1-\tau)\left(W_{B,i} - \omega_{B,i} \pm U_{i+1}\right) - \tau J_{B,i}\right] \\ - \\ (1-\tau)\left[U_i - \omega_{f,i}\right] \end{array}\right\} = \tau \left[p_i \omega_M + (1-p_i)\omega_{B,i} - \omega_{f,i}\right]$$

Using the two Nash conditions

$$\begin{aligned} \left(W_M - U_M\right) &= \frac{\tau}{(1-\tau)} J_M \\ \left(W_{B,i} - U_{i+1}\right) &= \frac{\tau}{(1-\tau)} J_{B,i} \end{aligned}$$

we get

$$
\begin{aligned}
\left\{\begin{array}{c} p_i\left[(1-\tau)\left(U_M-\omega_M\right)\right] \\ + \\ (1-p_i)\left[(1-\tau)(U_{i+1}-\omega_{B,i})\right] \\ - \\ (1-\tau)\left[U_i-\omega_{f,i}\right] \end{array}\right\} &= \tau\left[p_i\omega_M+(1-p_i)\omega_{B,i}-\omega_{f,i}\right] \\
\left\{\begin{array}{c} p_i(1-\tau)U_M \\ + \\ (1-p_i)(1-\tau)U_{i+1} \\ - \\ (1-\tau)U_i \end{array}\right\} &= \omega_M p_i(1-\tau)+(1-\tau)\omega_{B,i}(1-p_i) \\
-(1-\tau)\omega_{f,i}+\tau\left[p_i\omega_M+(1-p_i)\omega_B-\omega_{f,i}\right] & \\
(1-\tau)\left[p_iU_M+(1-p_i)U_{i+1}-U_i\right] &= p_i\omega_M+(1-p_i)\omega_B-\omega_{f,i}.
\end{aligned}
$$

Thus we get that

$$
\begin{aligned}
\omega_{f,i} &= (1-\tau)z+\tau\left[p_if_G+(1-p_i)f_B\right]+(1-\tau)\beta E\left[U_i'-p_iU_M'-(1-p_i)U_{i+1}'\right]+\tau k\theta_i \\
&= (1-\tau)z+\tau\left[p_if_G+(1-p_i)f_B\right]+\beta E\left[\omega_{f,i}'-p_i\omega_M'-(1-p_i)\omega_{B,i}'\right]+\tau k\theta_i \qquad (20)
\end{aligned}
$$

where we already solved for $\omega_{B,i}$ and ω_M

$$
\begin{aligned}
\omega_{B,i} &= (1-\tau)z+\tau f_B+\tau k\theta_{i+1} \\
\omega_M &= (1-\tau)z+\tau f_G+\tau k\theta_M
\end{aligned}
$$

B Algorithm

Below is a simple algorithm that can be used to compute our equilibrium objects in a stationary environment. A straightforward extension of this procedure is used in for our stochastic environment featuring fluctuations in the state of aggregate productivity.

Step 1 Compute ω_M and θ_M by solving

$$
\omega_M=\tau(f_G+\theta_Mk)+(1-\tau)z
$$
$$
k=\frac{\beta q(\theta_M)(f_G-\omega_M)}{1-\beta(1-\delta)}
$$

Step 2 Compute $\omega_{B,i}$ for $i=M-1$ as

$$
\omega_{B,i}=\tau(f_B+\theta_{i+1}k)+(1-\tau)z
$$

Step 3 Compute $\omega_{f,i}$ and θ_i for $i = M-1$ by solving

$$\omega_{f,i} = \frac{(1-\tau)z + \tau[p_i f_G + (1-p_i)f_B] - \beta[p_i\omega_M + (1-p_i)\omega_{B,i}] + \tau k\theta_i}{1-\beta}$$
$$k = \beta q(\theta_i) J_{f,i}$$

where ω_M and $\omega_{B,i}$ were computed in Steps 1 and 2, and

$$J_{f,i} = [p_i f_G + (1-p_i)f_B] - \omega_{f,i} + \beta[p_i(1-\delta)J_M + (1-p_i)(1-\delta_B)J_{B,i}]$$
$$J_M = \frac{f_G - \omega_M}{1-\beta(1-\delta)}$$
$$J_{B,i} = \frac{f_B - \omega_{B,i}}{1-\beta(1-\delta_B)}$$

Step 4 Repeat Steps 2 and 3 for $i = M-2, \ldots, 1$.

C Worker Flows

The model features a rich set of flows of workers across jobs and unemployment over time. To describe these flows, we first need to introduce some notation for the measure of each type of individual in the model.

- $\{\upsilon_{i,t}\}_{i=1}^{M}$ denotes the total number of unemployed workers of each type at the beginning of date t. Note that by symmetry, the number of unemployed of type i searching in a specific job market m would be $u_{i,t}^m = \upsilon_{i,t}/M$. This symmetry argument holds for all remaining definitions.

- $\{N_{f,i,t}\}_{i=1}^{M-1}$ denotes the total number of workers, in the very first period of the match, of each type i at the beginning of date t. Note that at the end of date t these workers will learn whether they are good or bad in their current match. (Again, to get the number of in each specific job, divide by M.)

- $\{N_{B,i,t}\}_{i=1}^{M-1}$ denotes the total number of badly matched workers of each type at the beginning of date t.

- $N_{M,t}$ denotes the total number of well matched workers at the beginning of date t.

Unemployed The law of motion for the unemployed who know their true calling is given by:

$$u_{M,t+1} = \sigma\Big[(1-\mu(\theta_{M,t}))u_{M,t} + \delta N_{M,t} + \delta_B N_{B,M-1,t} + \delta\sum_{i=1}^{M-1} p_i N_{f,i,t} + (1-p_{M-1})\delta_B N_{f,M-1,t}\Big].$$

That is, the number of searchers who know their true-calling is composed of surviving individuals who:

1. were unemployed and already knew their true calling last period, but did not find a job;
2. were employed in their true calling last period, $N_{M,t}$, but were exogenously separated;
3. were employed in a bad match last period, were exogenously destroyed, but because it was their $(M-1)^{th}$ bad match, learn their true calling;
4. were employed in the first period of their i^{th} match last period, $N_{f,i,t}$ for $i = 1,\ldots,M-1$, learned at the end of last period that they had found their true calling (with probability p_i), but were separated;
5. were employed in the first period of their $(M-1)^{th}$ match last period, $N_{f,M-1,t}$, learned they were again in a bad match, and were separated (implying that they enter unemployment knowing their true calling is their last remaining job).

The law of motion for unemployed who have not found their true calling and are sampling their i^{th} job for $i = 2,\ldots,M-1$ is:

$$u_{i,t+1} = \sigma\left[(1-\mu(\theta_{i,t}))u_{i,t} + \delta_B N_{B,i-1,t} + (1-p_{i-1})\delta_B N_{f,i-1,t}\right].$$

The number of searchers for their i^{th} job is composed of surviving individuals who:

1. were unemployed of this type last period, but did not find a job;
2. were employed in a bad match last period, were exogenously destroyed, but because it was their $(i-1)^{th}$ bad match, move up to searching for their i^{th} job;
3. were employed in the first period of their $(i-1)^{th}$ match last period, learned at the end of last period they were again in a bad match, and were separated.

Finally, the law of motion for unemployed who are sampling their first job is:

$$u_{1,t+1} = \sigma(1 - \mu(\theta_{1,t}))u_{1,t} + X_{t+1}.$$

These are composed of:

1. surviving individuals who were unemployed of type 1 last period, but did not find a job;

2. an inflow of "newly born" entrants to the labor market searching for their first job; since we normalize the population to unitiy, there is no population growth, and all individuals face a constant probability of death in between periods:

$$X_t = 1 - \sigma \quad \forall t.$$

Employed The law of motion for the the total number of well matched workers is:

$$N_{M,t+1} = \sigma \left[(1 - \delta)N_{M,t} + \sum_{i=1}^{M-1} p_i(1 - \delta)N_{f,i,t} + \mu(\theta_{M,t})u_{M,t} \right].$$

So the number of workers who have found their true calling is composed of surviving individuals who:

1. were workers of this type last period, and did not get separated;

2. were employed in the first period of their i^{th} match last period, learned at the end of last period that they had found their true calling, and were not separated;

3. were unemployed of type M last period (already knew what their true calling is), and got matched.

The law of motion for the the total number of poorly matched workers in their i^{th} job, for $i = 1, \dots, M - 1$, is:

$$N_{B,i,t+1} = \sigma \left[(1 - \delta_B)N_{B,i,t} + (1 - p_i)(1 - \delta_B)N_{f,i,t} \right].$$

These are surviving individuals who:

1. were workers of this type last period, and did not get separated;

2. were employed in the first period of their i^{th} match last period, learned at the end of last period that it is a bad match, and were not separated.

Finally, the law of motion for the the total number of workers in the first period of their i^{th} job, for $i = 1, \dots, M - 1$, is:

$$N_{f,i,t+1} = \sigma\mu(\theta_{i,t})u_{i,t}.$$

These are surviving individuals who were unemployed of type i last period and got matched.

D Proofs

Proposition 1. The proof is by induction. Assume, by contradiction, that $U_{M-1} > U_M$. Then Lemma 1 implies $\theta_{M-1} > \theta_M$, and so $J_M < J_{f,M-1}$. Since the value of these matches is proportional to total surplus of the respective match, it follows that $TS_M < TS_{f,M-1}$. But by definition

$$\begin{aligned}
TS_M &= J_M + W_M - U_M \\
&= f_G - \omega_M + \beta E\left[(1-\delta)J'_M\right] + \omega_M + \beta E\left[(1-\delta)W'_M + \delta U'_M\right] - U_M \\
&= f_G + \beta E\left[(1-\delta)TS'_M + U'_M\right] - U_M \\
&= f_G + \beta E\left[p_{M-1}(1-\delta)TS'_M + (1-p_{M-1})(1-\delta)TS'_M + U'_M\right] - U_M,
\end{aligned}$$

and

$$\begin{aligned}
TS_{f,M-1} = E_{M-1}f &+ \beta E\left[p_{M-1}(1-\delta)TS'_M + (1-p_{M-1})(1-\delta_B)TS'_{B,M-1}\right] \\
&+ \beta E\left[p_{M-1}(1-\delta)U'_M + (1-p_{M-1})(1-\delta_B)U'_M\right] - U_{M-1}.
\end{aligned}$$

The difference between the last two expressions is

$$\begin{aligned}
TS_M - TS_{f,M-1} = f_G &- E_{M-1}f \\
&+ (1-p_{M-1})\beta E\left[(1-\delta)TS'_M - (1-\delta_B)TS'_{B,M-1}\right] \\
&+ \beta E\left[U'_M - p_{M-1}(1-\delta)U'_M - (1-p_{M-1})(1-\delta_B)U'_M\right] \\
&+ U_{M-1} - U_M.
\end{aligned}$$

The first term is clearly positive, the third term is also positive, and under the assumption that $U_{M-1} > U_M$, the last term is also positive. We still need to show that $TS_M > TS_{B,M-1}$. By definition,

$$\begin{aligned}
TS_{B,M-1} &= J_{B,M-1} + W_{B,M-1} - U_M \\
&= f_B - \omega_{B,M-1} + \beta E\left[(1-\delta_B)J'_{B,M-1}\right] + \omega_{B,M-1} + \beta E\left[(1-\delta_B)W'_{B,M-1} + \delta_B U'_M\right] - U_M \\
&= f_B + \beta E\left[(1-\delta_B)TS'_{B,M-1} + U'_M\right] - U_M.
\end{aligned}$$

It follows that

$$TS_M - TS_{B,M-1} = f_G - f_B + \beta E\left[(1-\delta)TS'_M - (1-\delta_B)TS'_{B,M-1}\right],$$

which by recursion is positive since $f_G - f_B > 0$, thus contradicting that $TS_M < TS_{f,M-1}$. It follows that $U_{M-1} < U_M$.

Now assume that $U_{i+1} < U_{i+2}$ but, by contradiction, that $U_i > U_{i+1}$ for some $i \leq M-2$. As before, Lemma 1 implies that $\theta_{i+1} < \theta_i$, which implies that $J_{B,i+1} > J_{B,i}$, and, since $J_{B,i}$ is proportional to total surplus in bad matches of type i, $TS_{B,i+1} > TS_{B,i}$. But

$$\begin{aligned} TS_{B,i+1} - TS_{B,i} =& \beta E\left[(1-\delta_B)\big(TS'_{B,i+1} - TS'_{B,i}\big)\right] \\ &+ \beta E\left[U'_{i+2} - U'_{i+1}\right] + U_{i+1} - U_{i+2} \\ =& \beta E\left[(1-\delta_B)\big(TS'_{B,i+1} - TS'_{B,i}\big)\right] \\ &+ \Big(U_{i+1} - \beta E\left[U'_{i+1}\right]\Big) - \Big(U_{i+2} - \beta E\left[U'_{i+2}\right]\Big). \end{aligned}$$

The term $\big(U_{i+1} - \beta E\left[U'_{i+1}\right]\big)$ can be simplifies using the value of unemployment, which can be expressed as

$$U_{i+1} = z + \beta E\left[U'_{i+1}\right] + \mu(\theta_{i+1})\beta E\left[W'_{first,i+1} - U'_{i+1}\right].$$

Using the Nash bargaining condition, $(W_{first,i+1} - U_{i+1}) = \tau J_{first,i+1}/(1-\tau)$, we have

$$U_{i+1} - \beta E\left[U'_{i+1}\right] = z + \frac{\tau}{1-\tau}\mu(\theta_{i+1})\beta E\left[J'_{first,i+1}\right]$$

which, using the free entry condition, implies that $\big(U_{i+1} - \beta E\left[U'_{i+1}\right]\big) = z + \tau\theta_{i+1}k/(1-\tau)$. Using this in the expression for the difference in total surplus above, we have

$$\begin{aligned} TS_{B,i+1} - TS_{B,i} =& \beta E\left[(1-\delta_B)\big(TS'_{B,i+1} - TS'_{B,i}\big)\right] \\ &+ \tau\theta_{i+1}k/(1-\tau) - \tau\theta_{i+2}k/(1-\tau) \\ =& \beta E\left[(1-\delta_B)\big(TS'_{B,i+1} - TS'_{B,i}\big)\right] \\ &+ \frac{\tau k}{(1-\tau)}(\theta_{i+1} - \theta_{i+2}), \end{aligned}$$

which by recursion is negative, contradicting that $TS_{B,i+1} > TS(B,i)$. Thus it must be that $U_i < U_{i+1}$ for all i. ■

Remark 1 *If some bad matches are immediately destroyed, a similar argument can be used. In this case, assume that $U_i > U_{i+1}$ for some i in which $TS_{B,i} \leq 0$. Lemma 1 implies that $TS_{f,i+1} < TS_{f,i}$, which will lead to a contradiction. To see this,*

$$\begin{aligned} TS_{f,i} &= J_{f,i} + W_{f,i} - U_i \\ &= E_i f - \omega_{f,i} + \beta E\left[p_i(1-\delta)J'_M\right] \\ &+ \omega_{f,i} + \beta\left[p_i\big((1-\delta)(W'_M - U'_M) + U'_M\big) + (1-p_i)U'_{i+1}\right] - U_i \\ &= E_i f + \beta p_i(1-\delta)E\left[TS'_M - U'_M\right] - U_i. \end{aligned}$$

It follows that

$$TS_{f,i+1} - TS_{f,i} = E_{i+1}f - E_i f + \beta(p_{i+1} - p_i)(1-\delta)E\left[TS'_M - U'_M\right] + (U_i - U_{i+1}).$$

Under the assumption that $U_i > U_{i+1}$, all three terms are positive, contradicting that $TS_{f,i+1} < TS_{f,i}$.

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