Parenting Style and the Development of Human Capital in Children

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Abstract

There is little consensus among social science researchers about the effectiveness of alternative parenting strategies in producing desirable child outcomes. Some argue that parents should set strict limits on the activities of their adolescent children, while others believe that adolescents should be given relatively wide discretion. In this paper, I develop and estimate a model of parent-child interaction in order to better understand the relationship between parenting styles and the development of human capital in children. Using data from the NLSY97, the estimates of the model indicate that the best parenting style depends on how much a child values human capital. Setting strict rules increases the study time of a child who places a low value on human capital, but decreases study time for a child who places a high value on human capital. According to the estimates, the impact of a public mandatory curfew, given these offsetting effects, is to increase slightly adolescent human capital.

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When questioned about who decides on how late they can stay out at night, about 67% of youths between the ages of 12 and 13 surveyed in the 1997 youth cohort of the *National Longitudinal Surveys* (NLSY97) declared that parents decide, 30% reported that the decision is jointly made with their parents and 3% that they alone decided. The responses are more heterogenous for questions about who decides what TV shows the youth can watch or about who the youth's friends can be. Although parents differ in the degree of self-regulation left to their children, there is no consensus on how effective alternative parenting strategies are in producing desirable child outcomes.[1] Some researchers argue that allowing children more discretion in making these kinds of choices is a better approach to parenting, while others believe that establishing strict rules is best at inducing good behavior.[2]

In this paper I develop and estimate a model of parent-child interaction to better understand the relationship between parenting styles and the development of human capital in children. In my model, parents optimally choose a parenting style in terms of the strictness of the limits they set for their children on their time allocation. By reducing the value of leisure time children have available, stricter limits induce greater effort of children in terms of the time they devote to study. However, stricter limits entail a higher monitoring cost for parents. Children value leisure and also human capital, which is produced by study time conditional on the stock of prior human capital. Parents and children are forward looking and solve a dynamic game in which child's effort is imperfectly monitored by parents and in which a child's valuation of knowledge is unknown. The model thus incorporates both moral hazard and adverse selection. Parents use the observed realization of a child's knowledge as a signal of the child's underlying valuation. A set of sufficient conditions on the primitives of the model are found that ensure the existence of a unique equilibrium in the class of cut-off strategies. The equilibrium has two features: i) parents punish low realizations of knowledge by setting a strict parenting style, and ii) the child plays a type-monotonic strategy (types that value knowledge more exert more effort).

As a way of understanding how the game-theoretic structure of the model affects the interpretation of data, consider the OLS regression of the number of hours per week a child spends doing homework on whether the curfew is decided only by the child or by the child together with the parents (controlling for the child's gender, race and human capital). The result of this exercise for a sample of children between the ages of 12 and 14 in 1997 is shown in table 1. Because the estimated parameters attached to the curfew variable are positive and statistically significant, one might conclude that not letting the child decide on the curfew is beneficial.

[1] In the words of Bornstein (1991):" Despite the fact that most people become parents and everyone who ever lived has had parents, parenting remains a mystifying subject about which everyone has opinions, but about few people agree. Freud once listed bringing up children as one of the three 'impossible profession'-the other two being governing nations and psychoanalysis.."

[2] Sells (2001) and Bernstein (2001) discuss different parenting attitudes toward limits. Edgette (2002) warns against a self-regulatory parenting style.

Table 1: OLS Regression: Time Study-Curfew

	time_study
jointly curf	0.963*
	(0.514)
parents curf	0.943*
	(0.501)
PIAT	0.029***
	(0.005)
sex	1.203***
	(0.166)
black	-0730***
	(0.242)
_cons	1.0374***
	(0.6502)
R^2	0.036
N	3227

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

According to this interpretation, the child studies, on a weekly basis, about one hour more when the parents decide than when the child and parents jointly decide or when the child alone decides. However, in the context of the behavioral model described above, this conclusion would likely be misleading for two reasons. First the OLS estimates are inconsistent because of the correlation between the curfew-limit variable and the child's unobserved valuation of human capital. Second, interpreting the parameters attached to the limits dummies as the "effect" of parenting style on the time the child spends doing homework is problematic because they are both jointly determined in equilibrium.[3]

The model is estimated by simulated maximum likelihood using two years of data (1997 and 1998) from the NLSY97. The sample consists of about 1500 youths between the ages of 12 and 13 in 1997. In the estimation, parents are assumed to have unbiased (heterogeneous) beliefs about their children's valuation of knowledge. There are assumed to be two types of children in terms of their valuation. The parent's prior about a child's type is assumed to drawn from a Beta distribution, which itself depends on the child's initial PIAT test score. The estimation procedure also allows for classification error in the limits variable and measurement error in the

[3] Section 2.5 clarifies more in detail this point.

time spent studying. Estimation requires the numerical solution for the equilibrium outcomes of the parent-child game (using a fixed point algorithm), that is, the time spent studying by the child and the limits set by the parents. The likelihood contribution for a given child-parents pair is the probability of observing the limits in 1997 and 1998, the time spent studying in 1997 (the only year it is available) and the PIAT test score in 1998 and 1999, conditional on the PIAT score in 1997. Estimation involves iterating between the solution of the fixed point algorithm and the calculation of the likelihood.

The model estimates indicate that a strict parenting style does not maximize child human capital for all children. Children who have an initial low level of human capital would study more and achieve a higher level of human capital with a strict parenting style. However, a strict parenting style would not maximize human capital for children with high initial levels of human capital. This result has important implications for evaluating the impact of a publicly enforced curfew. Establishing such a legal curfew, that is, taking the decision out of the hand of parents and thus eliminating their monitoring cost, would have offsetting effects, increasing the human capital of children with low level of initial skills and decreasing the human capital of children with high initial human capital. On net, given the distribution of initial human capital in the sample, the result would be a slight increase of the overall level of human capital.

This paper is organized as follows. Section 1 reviews some related literature, section 2 and 3 describe the model and the data. Section 4 talks about the estimation and describes the counterfactuals. Section 6 concludes. Section 7 contains the proof of the existence and the uniqueness of the equilibrium. Finally, in section 8 there are the details of the algorithm I used to solve the model.

1 Related Literature

One of the most famous theorems in the theory of incentives, *The Rotten Kid Theorem*, is concerned with the problem faced by the head of an household who needs to redistribute the resources of his family among the members of the household. Becker (1974) shows that "*each beneficiary, no matter how selfish, maximizes the family income of his benefactor and thereby internalizes all effects of his actions on other beneficiaries*". That is, with transferable utility, agency and free rider problems will be eliminated. As explained in Bergstrom (1989), the intuition behind this result is that the transfers made from the head of the household to each of his children are increasing in the total family income. This implies that "*any of the selfish children who has an opportunity to increase total family income, even if it is at the cost of reducing his or her pretransfer income, will choose to do so"*. Following these contributions, other papers, clarified the assumptions needed to guarantee that the Rotten Kid Theorem holds (see Cornes and Silva (1999) and Chiappori and Werning (2002)). Although they have in common the fact that they analyze parents-child interaction, these paper and mine address different issues. The

Rotten Kid regards the distribution of resources between the members of a family. In here I analyze how parents help their children's performances by setting limits. Two papers are more closely related to this work: Weinberg (2001) and Akabyashi (2006). In the first paper the author seeks to provide a new mechanism that generates correlation of outcomes between parents and their offsprings. He employs a static (one period) principal agent model with an altruistic parent who promises monetary reward to her child in case good outcomes are realized. Adding liquidity constraints the author generates intergenerational correlation of incomes. Also he shows that parents with less resources are willing to substitute monetary rewards with physical punishment, which is consistent with what observed in the data. The approach I take departs from Weinberg (2001) in a few aspects. First, my model is dynamic. Second I do not assume that parents and their offspring can sign binding contracts, as he does.[4] This is particularly important in a dynamic setting in which the behavior of both actors can change substantially if both do not have the possibility of credibly commit to certain actions. My model seems more closed to casual observation, as we don't observe courts enforcing agreements between parents and their children. Indeed parents are allowed to implement any measure they believe is beneficial for the well-being of their offspring, except those that that undermine physical and mental health.

Finally the nature of the link between parents and children in his model is different than mine. In his model the principal decides the the amount of transfers enjoyed by children and whether to use corporal punishment. These are argument of the child's utility function. In contrast in my model parents' choices change the child's preferences.

In the second paper a theoretical model that explains child maltreatment is built.Akabyashi (2006) shows that child maltreatment can arise when parents ignore the child's human capital. Parents with an initial high estimate of the human capital may tend to underestimate the child's effort. This " *results in persistently punitive abusive interactions*". There are several differences between my work and his one. First I want to give a quantitative assessment of the impact of different parenting methods on cognitive skills, whereas Akabayashi seeks to show that rational, altruistic and forward looking parents can engage into punitive behavior. Second, even though his model is dynamic from the point of view of the parents, the child behaves in a time-inconsistent fashion. In contrast I adopt the traditional rational expectation approach. Finally in his model parents choose the amount of spanking, hugs and so forth. These variables are conceptually different from the limits variables we use in the model. A recent contribution, Hotz et al. (2008), analyzes parenting in the context of birth order effects. Parents play twice, once with each of their children, starting with the oldest. If children ignore the type of parents they are playing with and they observe past history, parents have an incentive to be tougher with the older children to set the example and induce younger siblings not to misbehave. They show that the implication of this model is consistent with the observed birth order effects. Strictly

[4] A somehow more realistic approach might be represented by allowing the players to employ the so called "relational contracts". However these agreements are based on the assumption that the two players interact forever. The lack of data implies that such an approach is not empirically feasible

speaking their model is not fully dynamic because the problem solved by each of the child is static.

In sum, the main difference between my model and these contributions is that, although the equilibrium holds under specific assumptions, my model is dynamic. Second I use the model to address an empirical question, while these papers seek to provide an economic model which is consistent with some observed patterns of the parent-child interaction.

Finally a recent paper by Lizzeri and Siniscalchi (2008) studies parenting in the context of learning. In particular the authors analyze the problem of parents who need to supervise a child who is trying to learn how to perform an activity. They have superior information about how to perform the activity and face a trade off: *"sheltering the child from the consequences of mistakes and allowing him to learn from experience"*. The authors characterize the optimal parental intervention for a variety of loss functions. My work only shares with this paper the fact that parents choose actions to improve the well-being of their children, but we I not analyze a learning problem as they do.

Sociology. The greatest body of research on parenting practices has been produced by sociologists and researchers of child development. In particular parenting style is a term created by Baumrind (1968). She divides parents into three categories: authoritarian, permissive and authoritative. Authoritative parenting broadly refers to the method used by those parent who enforce their rules, set clear standards, provide consistent discipline that is not overly punitive. Authoritative parenting is also associated to the concept of "flexibility". Parents are flexible when they alter their behavior when appropriate. In contrast authoritarian parenting style is sometimes referred to as the military parenting style. This type of parents puts an emphasis on obedience, and usually impose very strict family rules. Authoritarian parenting stifles intellectual growth and creativity. It also encourages children to either rebel against their parents. Finally permissive parents give up most control to their children. Parents make few, if any, rules, and the rules that they make are usually not consistently enforced. They do not set clear boundaries or expectations for their children's behavior and tend to accept however the child behaves. A common result is that authoritative parenting is more likely to deliver good outcomes, see Dornbusch et al. (2007).
Sociologists documented that socio-economic status is correlated with parenting methods (see Bornstein (1991)). In particular middle-class parents are more likely to choose an authoritative parenting method while in low class households authoritarian parenting is the most common. However this result is obtained without accounting for the endogeneity problem discussed in the introduction.

My paper seeks to give economic content to the framework developed by Baumrind (1968), by allowing parents to optimally decide the strictness of limits in response to the child's per-

formances and characteristics. Further I want to extend the existing literature in economics by developing an economic model in which both parents and the child are forward looking players.

2 The model

My model describes the interaction between two players : *the parent* and the *child*.[5] For ease of exposition, in the context of a sentence, I refer to the child as "she" and to the parent as "he". Parents are treated as a single decision maker, that is from the prospective of the child it is *as if* she interacts with one single agent. [6] There are two periods, which are calendar years. Both players are forward looking and they discount the future using the same discount factor δ.[7]
In the next sections the primitives of the model will be describe; specific functional forms have been chosen to parametrize them, which is always required if one seeks to estimate the structural parameters. It is common practice to pick flexible functional forms, in order nest different specifications.[8] However, given my asymmetric information environment, I also needed to impose restrictions on the primitives (both on the functional forms and on the parameter's space), under which, in a specific class of equilibria, a unique equilibrium arises. Hence, although to a certain extent I sacrifice flexibility, the gain in terms of the feasibility of the estimation is substantial.
Not having a unique equilibrium is a serious issue if one wants to estimate the structural parameters of the model via maximum likelihood.[9] As explained in Moro (2003) and Tamer (2003), multiplicity of equilibria implies that the map from the parameter space to likelihood function is non unique, the so called problem of *incompleteness*. Incompleteness makes the estimation of the structural parameters problematic.[10] Although empirical strategies to deal with this problem have been developed, they are specific to the game at hand and they are not assumption-free. Methods able to deal with this issue have been developed both for static and dynamic games. Example of the first type are Moro (2003) and Sweeting (2008). Applications of the second type have mostly been developed in the IO literature and rely on the symmetry of players' (discrete) action space as well as on the stationarity of the environment, see Aguirregabiria and Mira (2007). Both these two elements are absent from my game. In light of this discussion, although a game that displays multiple equilibria can generate richer behavioral patterns, I believe that finding a set of (not unreasonable) sufficient conditions that deliver a

[5] This term is used to denote a young man or woman aged 12-13 at the beginning of the first period.

[6] I abstract from eventual disagreements that may arise between spouses/partners with respect to the strictness of the limits imposed on the child

[7] Akabyashi (2006) assumes that the child's discount factor is positively related to her stock of human capital.

[8] For example if one has to parametrize a production function, a CES production function would include a Cobb-Douglas and a Leontief production function.

[9] The maximum likelihood estimator is usually preferred to other estimators, such as GMM, because it is full information.

[10] In my model potential multiplicity could arise if both separating and pooling equilibria can be supported as equilibrium strategy.

unique equilibrium is a sensitive way of making my game empirically tractable.

2.1 Preferences

The parent cares about the final level of human capital achieved by the child at the end of the second period. Human capital achieved at the end of period t is denoted by G_t and should be thought as the set of cognitive skills the child has accumulated. Time spent by the child producing human capital (henceforth referred as "effort") is one of the inputs of its production function, which is described in section 3.2. The parent can influence the effort level selected by the child by choosing one of three mutually exclusive parenting styles. I denote by $R_t \in \{s, p, n\}$ parents choice set; whenever they choose s parents enforce *strict limits*, a *'tough'* parenting style; whereas $R_t = p$ and $R_t = n$ imply that parents set *permissive limits* and *no limits*, respectively. *Strict* limits are set when the parent imposes (possibly) binding constraints on the leisure activities enjoyed by the child when not at school (TV shows, parties etc.); under this parenting style the child has no role in deciding the limits. The limits are *permissive* when the parent decides about the rules together with the child; as a consequence the amount of monitoring tends to be low and more autonomy is left to the child. Finally if the child has no limit to comply to, she has complete freedom in her leisure time and not subject to any parental control. Because enforcing stricter limits implies a higher degree of (costly) monitoring, a stricter parenting style entails a higher psychic cost.[11]

The parent's preferences is given by:

$$\mathbf{v}_t = \begin{cases} -\mathbb{I}[R_t = n]c_n - \mathbb{I}[R_t = p]c_p - \mathbb{I}[R_t = s]c_s & \text{if } t = 1, 2 \\ \log(G_2) & \text{when the game is over.} \end{cases}$$

where $\mathbb{I}[\cdot]$ denotes the indicator function which takes value 1 if the expression in parenthesis is true. Consistently with the idea that stricter limits imply a higher monitoring cost I assume that $c_s > c_p > c_n = 0$. Thus the *no-limits* alternative is a costless option.

The child cares about her own level of human capital at the end of period 2 and leisure. To cumulate knowledge, she decides how much of the available time (normalized to 1) to dedicate to its production. Time devoted to this activity is denoted by e_t. The child's valuation of her own knowledge is denoted by ω_i, which is heterogenous across children. In particular I assume the existence of N finite number of types, that is $\omega_i \in \{\omega_1, \omega_2, \ldots, \omega_N\}$. Further ω_i is known to child but not to the parent (adverse selection).
Expressing the child's utility in terms of effort we have:

$$\mathbf{u}_t = \begin{cases} (1 - e_t)w(R_t) & \text{if } t = 1, 2 \\ \omega_i G_2 & \text{when the game is over} \end{cases}$$

[11] An alternative way of rationalizing the observation that the parent dislikes to set strict limits would be to assume that he is altruistic. This would pose additional problems in terms of tractability of the theoretical model.

In the above specification $w(R_t) \equiv \mathbb{I}[R_t = s]R_s + \mathbb{I}[R_t = p]R_p + \mathbb{I}[R_t = n]R_n$.

Moreover $R_n > R_p > R_s = 1$. This restriction captures the idea that the higher degree of monitoring implied by stricter limits diminishes the value of recreational activities.[12] This mechanism -stricter the limits, lower the value of leisure time- represents the link between the parent and the child. The model simply takes it as given and is silent about where it comes about.

In essence the model postulates an incongruence in preferences between the parents and the child. That is, the parent cares less about the leisure of his child less than she does. In particular, as often observed in reality, the parent is worried that not enough time is dedicated to the production of human capital. This concern may induce him to take actions-strict limits- which are undesirable from the perspective of the child. This is the source of conflict between my actors.[13] In the literature, the conflict between parents and children is typically generated by assuming that children are myopic whereas parents are forward looking agents. The idea behind this class of model is that parents care about their children's "future"[14] more than they do. On the one hand, my model is consistent with this idea by letting ω_i to be arbitrarily closed to zero. On the other, I depart from the traditional literature by allowing the child to anticipate that the parent will respond to her performance. In other words, the child may not care about her own human capital (low ω), but the desire to avoid stricter limits in the future induces her to exert effort. I feel that the implications of my model are more consistent with children's observed behavior than the traditional myopic-child model. In fact assuming that parents and children can sign binding contracts, is the only way the latter model can generate a positive effort level, as done in Weinberg (2001).

2.2 Human Capital Production Function

The human capital production function is given by:

$$G_{t+1} = F(e_t, G_t)\epsilon_t = \left[\gamma G_t^{\beta}\left(e_t^{\alpha} - \alpha e_t\right)\right]\epsilon_t$$

with $\alpha \in (0,1)$, $\beta > 0$ and $\gamma > 0$. Therefore both higher effort and current human capital increase future human; the production function is increasing and concave in e_t because $\alpha \in (0,1)$. Human capital is always non-negative because $e_t^{\alpha} - \alpha e_t > 0 \quad \forall e \in (0,1)$ and $\epsilon_t \in [0,\infty)$. Moreover $F_e(0, G_t) = +\infty$ and $F_e(1, G_t) = 0$, hence choosing $e_t = 0$ or $e_t = 1$ is

[12] As an example, suppose that the child is not allowed to play with video-games. In the model she could still decide to do so, but the fear of being caught and incurring into some punishment diminishes the value of the spent playing with respect to the scenario in which this activity is not forbidden.

[13] In reality, there are many other issues (use of drugs, involvement in criminal activities, etc.) children and parents disagree about. Incorporating those, while keeping my asymmetric environment, would make my model not empirically tractable.

[14] In the model the "future" of the child is represented by G_2

never optimal. The idiosyncratic shock ϵ_t represents a random event that affects the production of human capital and is unknown to the child before she decides her effort level. I assume that $\epsilon_t \sim \exp(\lambda)$. I denote the initial stock of human capital by G_0 which is a sufficient statistic for all of the permanent factors that affects human capital such as school quality and the initial ability of the child. As it will be explained in section 4, the initial stock of human capital is treated as observable. Given the above assumptions, the *monotone likelihood ratio property* holds:

$$\frac{\partial}{\partial G_{t+1}}\Big(\frac{f_e(G_{t+1}|e,G_t)}{f(G_{t+1}|e,G_t)}\Big) > 0 \quad (MLRP)$$

In words, the MLRP property implies that high realizations of human capital are more likely to have been generated by a high effort level. Thus the realization of human capital at the end of the first period is used by parents as a noisy signal of the child's underlying valuation (ω_i). Also note that because ϵ_t is multiplicative, for any given effort level (e_t) and initial human (G_0) there is a positive probability of observing any $G_1 \in [0,\infty)$. Thus G_1 together with Bayes' rule and the equilibrium play is used by parents to update their prior p_0 over the vector of types $(\omega_1, \omega_2, \ldots, \omega_N)$.[15]

2.3 Information structure, timing

Parents are uncertain about the child's underlying valuation of her human capital (ω_i) (adverse selection). Moreover they do not observe the effort level exerted by the child (moral hazard) and the realization of the idiosyncratic shock ϵ_t. However the realization of the human capital G_t is public and that is everything they observe. All the other primitives of the model, as well as the initial stock of human capital, are also known by parents.
The stage game is sequential and the order of moves is as follows:

1. The parent chooses a parenting style R_t
2. The child chooses an effort level e_t
3. A shock ϵ is drawn
4. The stock of human capital G_t becomes public
5. The parent updates his beliefs using Bayes rule

The stage game is played twice.

2.4 Equilibrium

The solution concept I adopt is the standard Perfect Bayesian Equilibrium. I focus my attention on the class of cut-off equilibria. Lemma 2.1 establishes my uniqueness result, that allows us to estimate the theoretical model.

[15] This specification implies that I do not need to worry about off the equilibrium path beliefs.

Lemma 2.1. *In the class of cut-off equilibria, under the parametric assumptions made on the primitives in sections 2.1 and 3.2, there exists a unique pair of equilibrium cut-offs* $(\underline{G}, \overline{G})$*, with* $\overline{G} \geq \underline{G} \geq 0$ *such that, in the second period, the parent chooses the following optimal strategy:*

$$R_2^{eq} = \begin{cases} \textit{no limits} & \textit{if } G_1 = 0 \\ \textit{strict limits} & \textit{if } 0 < G_1 < \underline{G} \\ \textit{permissive limits} & \textit{if } \underline{G} \leq G_1 < \overline{G} \\ \textit{no limits} & \textit{if } G_1 \geq \overline{G} \end{cases}$$

Moreover the child adopts a type-monotonic effort strategy in both periods and there is a unique optimal strategy R_1^{eq} *in the first period.*

Proof. See appendix □

As stated in the lemma my equilibrium is consistent with intuitive idea that a good performance is rewarded by a less strict parenting style. Since the stricter the parenting style adopted, the worse off the child is (lower is her value function), the child has an incentive to exert effort not to fall below $\overline{G}$. Further, the lemma establishes that the higher is the valuation of her human capital, the higher the effort level will be (type-monotonicity). Cut-off equilibria have been used in the political economy literature to model voters' behavior (see Ferejohn (1986) and Fiorina (1977)). In this approach it is assumed that voters adopt a retrospective voting strategy. That is, if the performance of a politician in office exceeds a given threshold, she gets re-elected, otherwise the incumbent prevails. In these two papers cut-off strategies are not derived as an equilibrium strategy, differently from Banks and Sundaram (1998). Their game has in common with mine the fact that type monotonic strategies and cut-off equilibria are analyzed, as well as the information structure. However the games are not strategically equivalent. In their model the principal lives forever; this, together with the supermodularity of the agent's payoff function and few other assumptions, allows them to derive cut-off strategies as the equilibrium strategy in presence of moral hazard and adverse selection. It is also worth mentioning that their result rests on much weaker assumptions than mine.

2.5 Interpretation of OLS Regression and My Model

Because I described my model, the interpretation of the OLS regression shown in the introduction in table 1 can be better discussed. As a special case of the model suppose the parents know the type ω_i of the child. According to the model, linearized versions of parents' and child's decision rules are given by the following expressions:

$$t = \alpha_1 G_0 + \alpha_2 l + \alpha_3 \omega_i \tag{1}$$

$$l = \beta_1 G_0 + \beta_2 t + \beta_3 \omega_i \tag{2}$$

where t stands for time spent studying and l for limits. G_0 denotes the reported PIAT test scores which measures the child's human capital when t and l are chosen. In the equations

the α's and β's represent function of structural parameters of the problem, preferences' and technology's parameters. The observed t and l are then the solution to this system of best responses. By obtaining ω_i from the second equation and plugging it into the first I obtain the following relationship:

$$t = \gamma_1 G_0 + \gamma_2 l$$

where $\gamma_1 = \frac{\alpha_1\beta_3 - \alpha_3\beta_1}{\beta_3 + \alpha_3\beta_2}$ and $\gamma_2 = \frac{\alpha_2\beta_3 + \alpha_3}{\beta_3 + \alpha_3\beta_2}$. If time spent studying is measured with error, I can then run the OLS regression described in the introduction. The estimated parameters attached to the limits variables γ_2 is a function of functions of the structural parameters of the problem: the α's and β's. Hence it cannot be interpreted as the effect of limits on time spent studying. In light of this discussion, the estimation of a behavioral model of parent-child interaction seems a reasonable way of addressing my question while obtaining interpretable results.

3 Data, Descriptive Statistics and Patterns

In this section I describe some important aspect of the data I use for the empirical analysis. I highlight some important patterns and I report some descriptive statistics of my three endogenous variables in sections 3.1,3.3 and 3.2. Section 3.1.1 describes the the correlation between the variables I use to proxy for parenting styles and children's family background. Finally section 3.4 contains information about my sample.

The *National Longitudinal Study of Youth 1997* (NLSY97) provides information about youths living in the Unites States and born during the years 1984 through 1980. The original sample contains 8,984 individuals living in 6,819 households. The interviews were conducted annually and there are 9 waves currently available, ranging from 1997 to 2005. Blacks and Hispanics are over represented in the sample, which consists of roughly 50 percent of boys.
In each survey round, the *Youth Questionnaire* is administered. The topics covered range from child's schooling to employment activities and also information about the respondent's family background are available. In the first round the *Parent questionnaire* collected information from one of the child's biological parent if present, or from the child's guardian/parent figure.[16] In the next three sections we introduce the variables available in the NLSY97 that could be used to measure our endogenous variables. Moreover we describe the advantages and disadvantages of each possible choice.

3.1 Limits variables

The *Autonomy/Parental control* section of the NLSY97 *Youth Questionnaire* asks in the first three survey years to children born in 1983 and 1984 about the person or people who make

[16]For detailed information about the topics covered in the *Parent's questionnaire* and the *Youth Questionnaire* see the NLSY97 guide

decisions concerning three of their activities: i) how late they may stay out at night ii) the kind of TV shows or movies they may watch, and iii) who they are allowed to "hang out" with. Three alternative responses are available:

1. Parent or Parents Set Limits
2. My Parents and I Decide Jointly
3. Parents Let Me Decide

That is for each limit it is reported if the decision is made only by the respondent, by the the child jointly with her parents or only by the parents.[17] A follow up question asks about the number of times the respondent broke the rules in the last month, what kind of consequence (if any) there would be if parents find out that the rules have been violated, and who would implement the (eventual) punishment. In the *Parent questionnaire*, responding parents of children born in 1983 and 1984 are asked the same set of questions. Because in the first round we have the responses from both parents and the child we can compare the responses. By doing so, we can learn the extent to which the parenting style adopted by the parent is perceived differently by the child. As for the curfew limit, we can see from table 13 that in roughly 66% of cases (sum of numbers on the diagonal) the parents and the child agree on the person(s) sets the limit. Most of the disagreement (25%) is when parents declare that they set the limits whereas the child declares that she does. Inspecting tables 14 and 15 for the TV and friends limits, we see that answers do not match in 43.52% and 41.32% of the cases (sum of elements off the diagonal). Overall the mismatch between responses that conceptually differ more-one actor declaring that she sets the limits or that the other does, while the other reporting the opposite-is fairly limited. This happens in 16%, 13% and 2% of the cases for the curfew, TV and friends limits respectively.

It is interesting to explore to what extent parents enforce those limits and make sure that rules are not systematically violated. As we can see from table 16 the majority of children do not violate the curfew in the first round survey, but the propensity to do so increases with age. Moreover children are more likely to break friends and TV limits. It is still the case that the majority of them violates those limits only once in the month prior to the interview date. What this statistics tell us is that most of the time parents make sure that agreed or imposed rules are respected.
In order to have a better understanding of how rules, if any, are enforced we can examine responses to the following questions:

- 'Which of the following would your parent or parents do if they found out you had come home an hour late for no good reason?'

[17] From now on I will refer to the limits of type i) as the curfew-limit, of type ii) as the TV limits and to limits in iii) as to the friends limits.

- 'Which of the following would your parent or parents do if they found out you had hung out with someone you weren't supposed to be with?'
- 'Which of the following would your parent or parents do if they found out you had watched something you weren't supposed to watch?'

Seven mutually exclusive answers are available:

1. Discuss it calmly with you;
2. Ignore it, pretend that it didn't happen or let you get away with it;
3. Sulk, pout, or give you the silent treatment;
4. Take away a privilege, ground you, or give you a chore;
5. Make threats that won't be kept;
6. Yell, shout, or scream at you;
7. Use physical punishment;

If we inspect table 18, we can see that there are two most common consequences: 'Discuss it calmly' and 'Take away a privilege, ground you, or give you a chore'. For each of limit these two alternatives are chosen in more than 80% of the cases. It thus seem, that when rules are broken roughly half of children expect their parents to approach the situation using confrontation and discussion, while the other half expect to suffer from some kind of tangible punishment.[18] We are now ready to learn about the extent to which limits vary across households and outline some patterns related to their dynamic behavior. First as can be seen from table 17, parents adopt different approaches on how to decide the limits, namely different parenting styles. The magnitude of this variation however differs depending on the type of limit we consider. For example, if we pick the TV limits when the youth is 12-13 years old we can see that roughly the same proportion of parents chooses each of the three options. Moreover, at every age of the child, the type of limit parents are more likely to set are those on how late he/she can stay out at night (curfew), followed by the TV limits and friends limits. The biggest difference, when the child is aged 12-13, occurs between the proportion of parents setting the curfew (67.89%)

[18] Although presumably the propensity to set limits and to break them are substantially affected by the consequences of the latter, we will not use these information in my analysis. Indeed including these variables in the empirical analysis would be possible. However we feel that it would go beyond the idea that justifies the use of the limits variable to proxy for parenting style choices. In our model adopting a *tough* parenting method is conceptualized by a significant degree of regulation/monitoring toward child's leisure activities. With this respect the underlying assumption is that whenever it is reported that parents set the limit about some activity it is more likely the this situation is in place. To include the punishment variables it would be necessary to model the choice of breaking limits and and to punish the teenager. Beside the fact that this would make our game much more complicated, it would be unfeasible because no information about whether the child was caught breaking the rules is available

and the proportion of parents choosing the friends (23.89%).[19] Second, there is an age effect: for each of the three activities the child is more likely to decide as she gets older. Interestingly though, this is not true for the friends limits. As table 17 reveals, more parents decide to set limits when the youth is aged 16-7 with respect to when the youth is 14-15.

Along these lines we are interested in learning to what extent, for all the three type of limits, parents switch parenting style, i.e. in the transitions of the limits variables. Table 19 reflects the age effect documented by table 17. In addition we observe parents that did not set limits or decide them together with the youth at a certain age find it optimal to set limits when she is one year older. For example with respect to the curfew limits almost 35% of the youths that were allowed to decide and 41% of those that decided together with parents on how late to stay out at night, have no longer a role in deciding about this matter when they age one year. Older children also experience these transitions: overall about 63% percent of youths who have some or all the power in deciding on how late to stay out at night when 13-14 years old, face parents that set the curfew when they are 14-15.
These cases transitions also happen in the case of the friends and the TV limits. The 'loss of power situation' happens in about 18% and 19% of the times for the TV and for the friends limits when the child is 13-14 and 15% 16% of the times when she is 14-15 years old. These transitions highlight the importance of having a dynamic model able to accommodate both types of switches in parenting styles observed in the data, as well the age pattern showed in table 17.

3.1.1 Limits and Family Characteristics

Table 20 presents a correlation between parents' propensity to set curfew-friends and limits and the race of the child.[20] In particular parents of black children set limits 75% of the times parents set limits versus 67%and 65% for Hispanic and White children[21]. This difference persists for the friends limits but it is not there for the TV limits. One explanation for this fact might be that, because they are more likely to live in dangerous neighborhoods, Black and Hispanic parents have more incentives to impose those type of limits that prevent the interaction of their offspring with those environments. Such an explanation would be consistent with the fact that parents belonging to these minorities are roughly equally likely as White parents to decide which TV shows the youth is allowed to watch. For all of these racial/ethnic groups we observe the age pattern discussed before.

As for the correlation limits-parental income we can examine table 22. We see that the av-

[19]This presumably indicates that parents feel that setting an upper bound on the amount of leisure (curfew) is more effective than avoiding the interaction with dangerous peers and/or that setting the curfew entails a lower cost.

[20]Most likely parents are of the same race of the respondent. Difference in race in the NLSY97 is limited to roughly 4 per cent of the cases

[21]I define White whoever does not belong to Black or Hispanic ethnicity

erage yearly gross income of parents that decide on the curfew and on the TV shows together with the child ($37859 and $37664) is higher than those who let the child decide ($25900 and $30175) or decide themselves on this issues ($31139 and $33942). This is not true for the friends limits in which the highest average income is earned by parents that let the child decide($40330 versus $31509 for parents who decide and $24544 for parents that decide jointly). In sum richer parents are more likely to decide the curfew together with their child and they are more likely to let her decide with whom to spend time and which TV shows to watch. One reasonable explanation to rationalize this fact is that parents with lower resources try to make up for the inability to purchase home and school inputs-e.g. private lessons, books, high quality school-by imposing a strict discipline. Overall the message that we get from tables 20-22 is consistent with the well known correlation between socio-economic class and child rearing practices: the lower the former, the stricter the latter.

Further there is no substantial difference in parents behavior conditional on the sex of the child, as Table 21 documents. This indicates that outcomes that are gender specific, e.g. unwanted pregnancies, are not of first order in shaping parents' choices about limits.
We can then learn about the correlation between limits and family structure. In particular we are interested in learning if children whose guardian(s) are not both her biological parents have a different probability of having to comply with limits.Table 23 shows that single mothers of children aged 12-15 have an higher probability of setting both the curfew limit and the friends limit. This not true however for the TV shows, in which case single mother are less likely to impose limit on. The same kind of explanation for the correlation race-limits can be advocated to interpret this fact: single mothers may respond to more dangerous environments by restricting more recreational activities the child is involved outside the household. There is not a significant difference in parenting style by both biological parents and by her biological mother and a step-father; while when the biological father and a step mother discipline the child there is an higher probability that limits are set.

3.2 Human Capital

In the context of the model human capital is interpreted as the set of cognitive skills the child is endowed with. I do not consider non cognitive skills both for tractability purposes and for the lack of appropriate data.[22] I use the Peabody Individual Achievement Test (PIAT)test scores to measure cognitive skills.[23] This test measures academic achievement of children ages five and over and it is among the most widely used brief assessments of academic achievement. One of the PIAT subtests, the Mathematics Assessment, was given in round 1 respondents not yet enrolled in the 10th grade and in rounds 2 through 6 to respondents who were age 12 as

[22] Although presumably parents care about both for tractability purposes we focus only on cognitive skills. Further the type of tests typically used to proxy for the non cognitive skills, such as the ones used in Cunha and Heckman (2006) are not available in the NLSY97

[23] See Todd and Wolpin (2005) to learn about the impact of school and home inputs on the PIAT

of December 31, 1996, and who were in the 9th grade or lower in round 1. Consistently with well known facts regarding racial-ethnic test score gaps-see Todd and Wolpin (2007) table 24 documents this gap across races-ethnic groups also for the NLSY97 respondents born in 1984. In particular, for children aged 12-13 there is a difference of roughly 13 points in the PIAT math scores between White children and Hispanic-Blacks. With respect to our choice of using the PIAT as measure of human capital, it might be questionable to postulate that parents are aware of the PIAT math score of their offspring. Grades achieved at the end of each academic year are an alternative variable which is less likely to violate this kind of assumption. Indeed, schools inform parents about the academic performances of their offspring. The NLSY97 data contain self reported grades achieved in eight grade in the form of a discrete variable taking eight values ranging from 'Mostly below Ds' to 'Mostly As'. Moreover, for grades 9-12, the transcript data survey provides the academic GPA achieved at the end of each academic year. However using the data on grades presents two drawbacks. First, the theoretical model employs a continuous variable, and the grades achieved in eight grade are discrete. Second, because they are affected by school specific factors, they are not comparable across students, as opposed to the PIAT math scores which are standardized.[24] Using the PIAT math as the state variable on which parents condition their parenting style is, however, less problematic if we take into account the correlation between grades and PIAT math. Table 26 and table 27 show a significant and positive correlation between the PIAT MATH test scores reported during an academic year and the GPA achieved at the end of the same year. Such a correlation still holds when we control for the GPA the child achieved at the end of the previous academic year. (see table 27). Table 28 shows the existence of a monotonic relationship between PIAT and parents propensity set TV-friends limits for children of age 12-15. However, if we look at the curfew-limit, the relationship between PIAT and *toughness* of parenting style is non monotonic.

3.3 Allocation of Time

The *Youth questionnaire* in the first survey round asks children of any age how many hours they spend doing homework in the typical week. This variable, among the ones available, seem the most appropriate to proxy for effort.[25] Table 29 shows the difference in allocation of time conditional on ethnic/racial groups. Interestingly, on average, in a typical week Hispanic children spend more time doing homework then White and Black youths.[26] Table 30 shows that time spent studying is positively correlated with children's educational attainment. Finally table 10 documents a positive effect of past test scores and time spent doing homeworks on

[24] For example if it is the case that parents are aware that their son/daughter can easily get an A because of the (unobserved) characteristic of the school he/she is attending, their response to academic performances will take this into account and inference will be contaminated

[25] Obviously if schools differ systematically in the amount of home-works assigned time spent doing homeworks poorly reflects the actual time invested in the production of skills. It is worth mentioning that no information regarding other activities clearly beneficial to the production of skills is available

[26] As the same table reveals, it is also remarkable the average number of hours that 12-13 years old American children spend watching TV in the typical week

current PIAT test scores.[27]

3.4 Sample

The sample is populated by 1512 children who were born in 1984. I abstract from the grade attended because grade progression at this stage is nearly universal. I restrict attention to this cohort for two reasons. First the limits-variables are available only for those born in 1983 and 1984. Second, the PIAT test scores in later rounds are available only for those who were age 12 as of December 31, 1996, and who were in the 9^{th} grade or lower in round 1.[28] Children that fall into this category were born in 1984. Further, in the 1984 cohort I retain children whose scores are non missing in 1997 and 1998. By doing, so I avoid to have a missing-state-variable problem for individuals in the sample. This case deletion approach leaves my sample size relatively large. As common, the underlying assumption is that this procedure does not generate any selection bias problem. However, this should be the case because the only criterion which determines whether a child takes the PIAT test is her age.[29]

4 Estimation

This section is structured as follows: section 4.1 clarifies the criteria I use to measure the three parenting styles, the interpretations which justify my choices and the measurement error structure I adopted. Sections 4.2 and 4.3 describe the sources of heterogeneity in estimation and how the likelihood function is computed. Sections 4.4 and 5.1 describe my estimates and the fit of the model. Finally section 5.2 discusses the results of the counterfactuals and their economic interpretation.

4.1 Measurement and Interpretations

Because the NSLY97 contains three type of limit variables, I need to choose which variable to use to proxy for parenting style.[30] Becuase I want to use the estimated model to perform a counterfactual public policy experiment in which a mandatory curfew is introduced my measure will be the curfew variable. The map between the responses and the three possible parenting styles in the model is the following. Whenever it is reported that the parent decides the curfew I

[27] The specification is an the added value type. Measures of home and school inputs are not available in the NLSY97

[28] In the first round every child not enrolled in the 10^{th} grade took the PIAT

[29] Every child born in 1984 is not enrolled yet in 10^{th} grade. I also made sure that the difference in the distributions of the endogenous variables between children born in 1984 that belong to my sample and those who were excluded are not statistically significant

[30] A better approach to the problem would be to use all of the three limits-variables in a latent factor model, as done in Cunha and Heckman (2006). The difficulty of this approach is that both our latent and observable variables are discrete. Unlike the continuous case the conditions for the identification of the factor loadings of the measurements equations have not been established for this case. When this econometric issue is resolved it will be possible for us to incorporate all of the three limits variables in estimation.

assume the parent is choosing *strict limits*. If it is reported that the decision is taken jointly limits are *permissive*. If the child declares to decide herself the curfew there are *no limits*. Two main assumptions underlie this approximation. First, whenever the parent has some or all the role in deciding the curfew, this implies that the child has to come back home at night at an earlier hour than she would if she were the only person deciding the curfew. In other words, the curfew limit is more "binding". This interpretation is consistent with the mechanism that links parenting-style choices and children's behavior described in section 2.1. Second the responses to the question about the person that decides on the curfew are considered a truthful statement about the role the child has in the decision. In other words the responses only reflect whether the parent sets a curfew, or participate in the decision, and not the effect of the curfew-limit on the respondent's behavior.[31]

I denote by $R_t^o \in \{s, p, n\}$ parent's observed choice at time t. Consistent with the notation adopted in the previous sections, I denote by e_t^o and G_t^o the self-reported average weekly time spent doing homework and the PIAT test scores. It is reasonable to assume that both e_t^o and R_t^o are measured and classified with error. Denoting by e_t the true effort level I adopt the following specification for the measurement error: $e_t^o = e_t + \epsilon_t^e$, with $\epsilon_t^e \sim N(0, \sigma_e^2)$.[32] Further I assume that there is a positive probability that responses to the question on the curfew are classified with error. For parsimony, I assume that there is a positive probability that the response is correctly reported. I denote by π_s the probability that $R_t^o = s$ when the true choice is in fact s; π_p and π_n are defined similarly and referred to when it is reported that the curfew is decided jointly ($R_t^o = p$) and by the child ($R_t^o = s$), respectively.

4.2 Heterogeneity

There are two sources of heterogeneity. First children observably differ in their initial human capital G_0, which is measured by the PIAT test scores reported in the 1997 survey round. To ease identification, I assume that there is no unobserved heterogeneity in ability across children: children's ability-motivation at the age of 12 is incorporated in G_0. The second source of heterogeneity is the value children attach on G_2, described in section 2. I allow for the existence of two types: low and high. That is, $\omega_i \in \{\omega_L, \omega_H\}$ The proportion of high types is ϕ. Finally, parents are heterogenous in their initial belief p_0 about the child being an high type. To incorporate the fact that the parent received a number of signals about the child's type from birth to the age of 12, I allow his belief to be correlated with the child's initial stock of

[31] For example if the parent establish the curfew and the child systematically violates the limit, I assume that she reports that the parent sets the curfew. As described in the 3 section this situation does not occur often.

[32] Although according to this specification I may get a negative observed effort, the alternative classic specification $\log(e_t^o) = \log(e_t) + \epsilon_t^e$ is problematic because e_t^o might be 0. Empirically, because only 7 per cent of time it happens that $e_t^o = 0$, working with my specification does not creates substantial problems. An alternative heuristic approach is to assume that the observed zeros are the result of rounding. That is a child that studies a very small amount of time tends to declare that she does not study at all. Hence the probability of reporting a 0 is a function of the "true" effort level implied by the model.

human capital. I assume that p_0 is distributed in the population according to a Beta distribution, that is $p_0 \sim \text{beta}(\theta_0, \exp(\theta_1 G_0))$. I restrict θ_0 to be non-negative. Given my parametrization if $\theta_1 < 0$, an higher G_0 implies an higher mean for the prior distribution. For identification purposes, I assume that parent has unbiased (rational) expectations:

$$\phi = \frac{\theta_0}{\theta_0 + \exp(\theta_1 G_0)} \tag{3}$$

This restriction implies that the mean of parent's prior about the child being a high type is the true proportion of high type in the population.

4.3 Likelihood

The model is parsimoniously specified in terms of 16 parameters :

$$\Theta = \left\{ R_p, R_n, c_s, c_p, \alpha, \beta, \gamma, \lambda, \omega_L, \omega_H, \sigma_e^2, \pi_1, \pi_2, \pi_3, \theta_0, \theta_1 \right\}$$

I estimate the model by simulated maximum likelihood. The estimation procedure iterates between the solution of the model, obtained through the fixed point algorithm described in section 8, and the computation of the simulated likelihood function, given a set of parameters. To briefly clarify how the the estimation works, once I solve for the equilibrium of the model, for a given initial level of human capital G_0 and a prior p_0, I obtain a set of decision rules for both the parent and the child. The decision rules establish a deterministic map between G_t, the state variable, and players' actions. The decision rule for the parent is an optimal $R_t \in \{s, p, n\}$, given G_t. For the child, it provides an optimal effort level e_t as a function of G_t and R_t. Assuming that e_t^o and R_t^o are measured and classified with error, respectively, allows us to place a probability on the the observed events.

The vector containing the observed endogenous variables in period 1 for a parent-child pair i is given by: $O_1^i = \{e_1^o, d_1^o, G_1\}$. Given that the measurement and classification errors are uncorrelated among themselves, the probability of observing O_1^i conditional on G_0, $P(O_1|G_0)$, is computed as the joint probability of observing each of the element in O_1. Conditional the prior belief hold by the parent and the type of the child, we can write down the joint probability of observing the events in O_1 as the product of conditional probabilities:

$$\Pr(O_1|G_0) = \Pr(G_1, e_1^o, d_1^o|G_0) = \Pr(G_1, e_1^o|G_0)\Pr(d_1^o|G_0) = \Pr(G_1|e_1, G_0)\Pr(e_1^o|G_0)\Pr(d_1^o|G_0)$$

The first and the second equality come from the fact that parental choices and the effort level e_1 are deterministically determined by the state G_0, once I solve for the equilibrium of the model. These conditional probabilities are given by:

1. $\Pr(e_1^o|G_0) = \Pr(e_1^o|e_1(G_0)) = \Pr\left(\epsilon_t^e = e_1^o - e_1\right)$;
2. $\Pr(d_1^o = i\,|d_1 = j) = \pi_i \mathbb{I}[j = i] + (1 - \pi_i)\mathbb{I}[j \neq i] \;\; i \in \{\text{s,p,n}\}$;

3. $\Pr(G_1|G_0) = \Pr\left(\epsilon = \dfrac{G_1}{F(e_1, G_0)}\right)$;

Conditional on G_1, the probability statements for the observed vector of outcomes $O_2 = \{d_2^o, G_2\}$ are computed according to the expressions 2 and 3.

In computing the contribution to the likelihood for a pair i, I integrate out the unobserved prior p_0 and weight by the type probability ϕ. Because the measurement errors are serially uncorrelated the joint probability of observing O_1^i and O_2^i is given by the product of the conditional probabilities. Denoting by $g(p_0)$, the distribution of the prior, the contribution to the likelihood of a parent-child pair i for a given set of parameters $\theta \in \Theta$ is given by:

$$L^i(\theta) = \int_{p_0} \Big[(1-\phi)\mathbb{I}\big[\omega_i = \omega_L\big] + \phi\mathbb{I}\big[\omega_i = \omega_H\big]\Big] P(O_1^i|G_0) P(O_2^i|G_1) g(p_0)$$

where $P(O_1^i|G_0)$ and $P(O_2^i|G_0)$ are meant dependent on p_0 and ω_i. The above integral is computed through Monte Carlo simulation. The likelihood for a sample of N pairs is given by $L(\theta) = \prod_{i=1}^{N} L^i(\theta)$.

4.4 Estimates

Table 2 shows preliminary estimates of the structural parameters. The marginal utility of leisure

Table 2: Parameter Estimates

R_p	R_n	c_s	c_p	α	β	γ	λ	ω_L	ω_H
2.07	4.44	0.06	0.03	0.08	0.64	2.08	1.54	0.14	0.78
(1.03)	(1.25)	(0.01)	(0.02)	(0.02)	(0.15)	(1.48)	(1.01)	(0.06)	(0.03)
σ_e^2	π_1	π_2	π_3	θ_0	θ_1				
1.23	0.55	0.08	0.41	0.32	0.03				
(0.03)	(0.15)	(0.02)	(0.11)	(0.08)	(0.01)				
log-likelihood=-2190.3		s.e. in parenthesis							

when parents choose permissive limits is less than half of when they choose no limits. Also recall that, for identification purposes, the marginal utility of leisure when parents choose strict limits is normalized to 1. The valuation of human capital of the high type (ω_L) is about six times as big as the valuation of the low type (ω_H). The estimated psychic cost of choosing a strict limits (c_s) is three times as big as the one attached permissive parenting parenting (c_p). The psychic cost of permissive parenting is normalized to 0.

Turning to the production function parameters, it is possible to see that my estimates are not far from the ones that one would obtain by taking the log of my human capital produc-

tion function and performing an OLS regresssion.[33] The estimates from an OLS regression are displayed in table 10 in column 3. Under the assumptions of model and because of the heterogeneity structure I adopted, such an OLS regression delivers consistent estimates of the production function parameters. Specifically I assumed that the there is no unobserved heterogeneity in ability and the shock is drawn after the effort decision is taken, hence the error term is uncorrelated with the reported time spent studying. The estimated marginal productivity of effort (α) and past human capital (β) in my model are 0.08 and 0.64, while in the OLS regression they are 0.41(0.252). The constant ($\gamma + \lambda$) in my model is estimated at 3.62 while the OLS regression delivers 3.19. Overall, these estimates of the structural parameters are in the 95% confidence interval of the OLS estimates.

Finally, the mean of the prior distribution about the child being an high type is positively correlated with the initial stock of human capital ($\theta_1 < 0$). By taking draws from the empirical distribution of the initial stock of human capital (PIAT test scores in 1997), I found that the rational expectations restrictions (equation 3), together with my estimates of θ_0 and θ_1, imply that the proportion of high type children is 56.2%.

5 Identification

In this section I seek to provide some intuitions about the sources of identification of the structural parameters. As we saw in lemma 2.1, in the unique equilibrium of the game, children play a type-monotonic strategy, that is children with high valuation of human capital spend more time doing homework, in both the periods, than children with low valuation. Suppose now the parent has the same prior about the type of the child. In this case, given that their preferences' parameters were not assumed to differ, conditional on an observed choice they made in the first period, all children would face the same cut-offs in the second period. Thus, conditional on the initial test score G_0 which is observed by the econometrician, the child's underlying valuation is identified by the observed time she spends doing homework in the first period. That is, higher the latter, higher the probability the child is of high type. Conditional on the type of the child, the marginal utilities of leisure conditional on parental choices, R_p and R_n, are identified by the difference in the average time spent doing homework between children whose parents choose R_p and children whose parents choose R_n.
Turning to parents preference parameters, the main issue regards the separate identification of the parameters of parents' prior, θ_0 and θ_1, from the cost of choosing *strict* and *permissive* limits, c_s and c_p. Holding fixed c_s and c_p , the degree to which parents' choices in the first period differ as the initial (observed) test score G_0 changes, is informative of θ_1. Further, the distribution of parents' choices in the first period *conditional* on a given G_0, is indicative of θ_0 because of the rational expectation restriction I imposed. In order to identify c_s and c_p I

[33] For practical purposes I imputed a small number whenever it is declared that the time spent studying is zero. A different specification for the measurement error of the time spent studying is under examination.

can use the observed choices in the second period, for a given realization of G_1. Thus being able to observe parents' choices in both periods allows us to separately identify θ_0 and θ_1 from c_s and c_p. Notice that the normalization that was made regarding parent's terminal value as well as the other restriction imposed (no unobserved heterogeneity in abilities or in the costs of choosing strict limits) also play a crucial role in these arguments.
Finally the identification of the parameters of the production function is ensured by the assumption I made on the timing: the shock ϵ_t is drawn after the choice regarding the optimal effort has been made, hence the regressors, e_1 and G_0, and the random shock in the human capital production function are uncorrelated.

5.1 Model Fit

To see how well the model is able to reproduce some main patterns observed in the data, we investigate its within sample fit by generating simulated data using the estimates and comparing some statistics of the simulated data with the actual data. Table 3 shows the fit of the model with respect to the means of the endogenous variables. The model matches well the average PIAT test scores as well as the increase in average test score as the child becomes older. The average time spent studying in a typical week is also matched quite closely. Notice that because time spent studying is available only for the first survey round, I have no information about the average effort level exerted by the child in the second period of the model. Thus, as indicative of this statistics, I report its value by using the cohort of the 1983; that is, by using time spent doing homework by children who are one year older. Under the assumption that the distribution of the unobservable is the same across cohorts, this approximation should be fairly good. The model predicts a smaller increase in the average weekly time spent studying the one observed in the data. Table 4 shows that the model matches the limits-choices in the first period very

Table 3: PIAT Test Scores

	Av. PIAT Test Scores		Av. Weekly Study Time	
	Period 1	Period 2	Period 1	Period 2
Data	72.5	75.8	5.21	5.34
Model	74.21	76.88	4.92	4.87

The test score's range is [1,100]

well. The fit for the second period is not as good: the proportion of parents deciding the curfew alone is lower than the one observed in the data. Moreover the proportion of parents who let the child decide is overstated. However the qualitative behavior generated by the model is still consistent with the data: parents decide alone in the majority of the cases while letting the child decide is the least often adopted parenting option.

Table 4: Limits

	Period 1			Period 2		
	Strict	Permissive	No Limits	Strict	Permissive	No Limits
Data	67.79%	29.32%	2.89%	55.45%	39.35%	5.2%
Model	62.31%	24.15%	13.54%	48.15%	31.11%	19.74%

I examine how well the model does in reproducing changes from one parenting style to the other in table 5. The table reveals that the model exhibits less persistence than the data. This pattern is in part related to our earlier observation that the distribution of parenting styles predicted by the model in the second period is a less accurate. Too high a frequency of changes may also be due to the multiplier effect given by the Bayesian updating rule used by the parent to revise their beliefs about the type of the child. Hence, with respect to the perfect information case, a bad realization of human capital at the end of the second period are punished to a greater extent. Symmetrically, a good performance is more likely to be rewarded by switching to a more lenient parenting style. Although, it would be desirable to generate transitions regarding the person who decides the curfew closer to the one observed in the data, the model is better able to accommodate the transitions-pattern similar to the ones observed for the other two limits variables: the friends and TV limit, as seen in table 19.[34]

Table 5: Transitions

		Period 2		
Period 1		Strict	Permissive	No Limits
Data	Strict	63.07%	32.47%	4.46%
Model	Strict	58.24%	36.18%	5.58%
Data	Permissive	39.05%	55.08%	4.96%
Model	Permissive	42.24%	50.21%	7.55%
Data	No Limits	33.33%	41.03%	25.64%
Model	No Limits	37.61%	29.87%	35.52%

The last measure of fit is given by the contemporaneous correlation between parenting styles and average time spent doing homework. Table 6 shows that the model is consistent with the data. However the difference in children's behavior conditional on different parenting styles predicted by the model is higher than the one we observe in the data. For example in the first period, the difference between the average weekly time spent studying conditional on parents choosing strict limits and parents choosing no limits is one hour and eighteen minutes, while the difference predicted by the model is three hours and 43 minutes. Table 6 shows that in the first period there is a monotonic relationship between limits and time spent doing homework: the stricter the former, the higher the latter.

[34]This is particularly important for the future development of this project in which I plan to use all of the limits variables.

Table 6: Av. Weekly Time Spent Studying and Parenting Style

	Period 1			Period 2		
	Strict	Permissive	No Limits	Strict	Permissive	No Limits
Data	5.32	5.18	4.14	5.13	5.54	3.23
Model	6.71	4.45	3.38	5.09	5.22	3.19

In the second period this relationship becomes non monotonic: children who are subject to a permissive limits study slightly more than children whose parents set strict limits. Further, when limits are strict, children study 1.9 hours more, on average, then when they are not subject to any limits, which is what the model predicts. Table 6 shows that the model closely matches the data in replicating these two patterns. [35]

5.2 Counterfactuals

The estimates of the model allow to perform a public policy counterfactual experiment and a thought experiment. In the first we use the model to assess the impact of a mandatory curfew. In the second we remove the asymmetric information, that is we assume that parents know the type of the child. There are two parameters of interest. First, we look at the percentage change in children's average test score at the end of the second period ΔG, where:

$$\Delta G = \frac{\mathbb{E}[G_2|c] - \mathbb{E}[G_2|d]}{\mathbb{E}[G_2|d]}$$

In this notation $\mathbb{E}[G_2|d]$ stands for the average test score observed in the data and $\mathbb{E}[G_2|c]$ denotes the one predicted by the model in the counterfactual scenario. This parameter captures the effect on the *level* of children's human capital. Second we investigate the degree to which the human capital development of children would change under the two scenarios. That is, we compare ΔG_0^c with its data counterpart, ΔG_0^d, where

$$\Delta G_0^i = \frac{\mathbb{E}[G_2|i] - \mathbb{E}[G_0|i]}{\mathbb{E}[G_0|i]} \quad i \in \{c, d\}$$

Hence, this parameter of interest is concerned with the effect on the *growth* of children's human capital.

[35] As we did for the unconditional time spent studying in table 3, we use the data of 1983 cohort to compute the average time spent studying conditional on parenting style

5.2.1 Mandatory Curfew

The role of government in helping parents to discipline their children is subject to debate. Coleman and Roker (2001) illustrates some aspects of this discussion in UK :

> *"In the early days of the Labour government there was much discussion in the media about where the boundary lay between interferences and appropriate involvement. There are some who view the family as a private institutions and believe that, except in the extreme cases, it should be largely free from state interferences. On the other hand there are those who take the position that government does have a role in enhancing support for the family and in modifying attitudes about the importance of parenting."*

The paper contributes to this debate by simulating the introduction of publicly enforced curfew. Curfew laws restricts the right of children to be outdoors or in public places during certain hours of the day. If a teenager breaks curfew, he or she can be temporarily detained by police and returned home. Currently, there is no state curfew. Such laws or ordinances are typically passed and enforced by local municipalities, cities and townships. Courts in California have generally upheld such laws as long as the local ordinance seeks to discourage loitering or remaining in certain places after certain hours.[36] From the prospective of the children, the restriction subsequent to the implementation of the curfew law, is interpreted as having the same effect as when parents decide themselves the curfew. Thus the child solves her problem with her marginal utility of leisure being equal to one in both periods.[37]

The estimates of my model indicate that the average effect of the policy is small. In fact, the effect on the level of children's human capital, ΔG, is 2.74%. The growth effect is also small. In fact the growth in children's test scores if a curfew law is in place would be equal to 11.2%, whereas in the data it equals 9.41%. However the effect of the policy, both in terms of level and growth, is different, qualitatively and quantitatively depending on the initial stock of human capital. Given that the test scores are continuous, I discretize the PIAT test score at the age of 12 by using the deciles of the empirical distribution.

Table 7: Conditional Means

$G1$	$G2$	$G3$	$G4$	$G5$	$G6$	$G7$	$G8$	$G9$	$G10$
35.18	51.67	58.37	63.36	66.95	71.5	75.64	81.62	86.5	92.43

[36] Typically curfew laws have been passed in communities where the juvenile crime rates became too high. Kline (2006) finds that curfew laws are effective in reducing the juvenile crime rate. He also includes a detailed description of the historical application of curfew laws in the US.

[37] In the model, this is the normalized value of the marginal utility of leisure when parents decide the curfew alone

To ease notation, a child belongs to group $G1$ if her initial test score falls in the first decile, she belongs to group G_2 if her test score falls in the second decile and so forth. The average test scores of children belonging to each group are shown in table 5.2.1. Table 8 shows the distribution of level effect, ΔG, conditional test scores at the age of 12/13. It is shown that for children with low level of human capital, a curfew law would have fairly large positive effects. The PIAT scores at the age of 14 would increase by as much as 28.31% (for children belonging to the $G2$ group). However, the human capital of children with the highest test scores would be decreased by the curfew law by as much as about 14%. Further the distribution of the level

Table 8: Distribution of Level Effect

$G1$	$G2$	$G3$	$G4$	$G5$	$G6$	$G7$	$G8$	$G9$	$G10$
25.11%	28.31%	15.16%	2.35%	8.91%	4.12%	−5.32%	−7.44%	−4.12%	−13.67%

effect is non-monotonic in the initial level of human capital. Table 9, shows how the growth in human capital in children, ΔG_0, would change after the implementation of the curfew law and exhibits the same pattern for the level of effect.

Table 9: Distribution of the Growth Effect

	$G1$	$G2$	$G3$	$G4$	$G5$	$G6$	$G7$	$G8$	$G9$	$G10$
Data	50.2%	21.1%	16.00%	14.01%	13.00%	10.3%	6.88%	5.6%	3.6%	−1.6%
Model	78.61%	54.81%	37.13%	16.32%	25.28%	17.45%	0.54%	−0.28%	−0.31%	12.36%

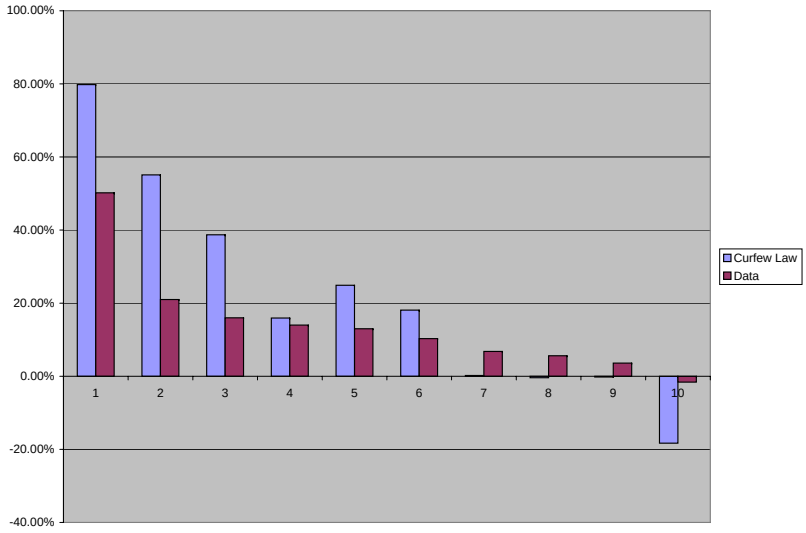
In the data we see high growth in test scores for children with initial low test scores. Whereas the growth in test scores is small for children with low initial test scores. The effect of the introduction of the curfew law is to exacerbate this pattern: children with low initial test scores would accumulate human capital faster, while the human capital development is slowed down or reverted for children with high level of skills. Figure 1 compares children's development of human capital in the data and under the curfew law. According to our estimates, a curfew law would be detrimental for children with the highest level of skills: their human capital would decrease by as much as 12%.[38]

Thus my main empirical result is that setting strict limits independently of performance does not maximize the human capital of all children. In light of the model and the equilibrium, the reason that such a parenting strategy is not always optimal seems intuitive. On the one hand, in the both periods, children respond to strict limits by dedicating more time to study. On the other, in the first period, they have an incentive to exert effort to obtain permissive limits in the

[38]This peak in the upper tail of the policy distribution is due to the fact that children belonging to group $G10$ are almost totally high types.

future. By passing a curfew law, such an incentive is removed. Thus, in general, the net effect of a curfew law on the effort level in the first period is ambiguous; consequently, the effect on G_2 cannot be established ex-ante. Our results indicate that the overall effect is quite different depending on the level of initial human capital. Everything else being the same, for children with low level of human capital the contemporaneous effect prevails; for children with high level of skills the dynamic incentives dominate. This empirical result clarifies an hidden, or not so obvious, undesirable consequence that has to be paid if curfew laws are passed: because both parents and children prefer not having limits, not setting limits can be used to reward good performance. A publicly enforced curfew, while it reduces the cost of setting limits for parents who would otherwise have done so, restricts the instruments available to parents to induce effort.

Figure 1: Policy's Distribution



5.3 No Asymmetric Information

In order to understand to what extent the uncertainty about children's type is affecting parents' ability to induce effort, we assume that the type of the child is known. Thus, we simply solve for the subgame perfect Nash equilibrium of the game and generate simulated data using our estimates. In this case the level effect is 2.48%. Under this hypothetical scenario the growth of children's human capital would be equal to 11.3% versus the 9.6 observed in the data. Hence, the inefficiency subsequent the existence of private information does not seem to be big.

6 Conclusions

This paper is a first attempt to understand the extent to which parental monitoring, in the form of limits on leisure activities, contributes to the development of human capital in children. I addressed this question by providing a framework able to empirically analyze parent-child interaction. My main result is consistent with some of the child development literature in confirming that a strict ("authoritarian") parenting style, which does not reward children's good performances, is not the optimal policy for all type of children. Passing a curfew law, aimed at helping parents to discipline their offspring, would have different effects depending on the type of adolescent children. For children with high initial level of human capital this public policy would be counterproductive. This result has important implications for public policies that restrict children's leisure activities, such as bans and prohibitions.

My results have been obtained by means of a quite stylized model: no school-attendance decision are made by children and there are only two periods. Given that school attendance is nearly universal between the ages of 12 and 14, this is not a major issue for the validity of our empirical results. Extending the model to more periods-until the child reaches the age of 17 or 18-is the object of my current research. Incorporating a more detailed description of the school environment, i.e. grade progression and credits accumulation as done in Eckstein and Wolpin (1999), while allowing for both moral hazard and adverse selection, is in my view a legitimate but hardly feasible extension. Applications of our model to other kind of children's behavior seem to be more easily implementable. Examples are criminal behavior, the use of alcohol and(or) drugs and other risky behaviors. Future research should also try to incorporate, in a dynamic framework, the use of monetary incentives as done in Weinberg (2001). All these extensions constitute a rather full and challenging research agenda, which I hope to pursue in the near future. In my view this work is a useful first step toward the development of richer game-theoretic models of parent-child interaction.

7 Proof

The proof is organized as follows. In section (7.1) and (7.2) I show that, given parent's conjectured strategy R_2^{eq}, the child plays a type monotonic strategy as best response. In sections (7.3) I show R_2^{eq} is parent's best response whenever the child plays a type monotonic strategy.

7.1 Child's problem in period $T = 2$

In this section we characterize the problem of the child in the last decision period for a given $R_T \in \{s, p, n\}$ and $G_{T-1} > 0$, conditional on the type ω_i:

$$V_T(G_{T-1}, R_T; \omega_i) = \max_{e \in (0,1)} (1 - e_T) w(R_T) + \delta \omega_i F(e_T, G_{T-1}) \mathbb{E}[\epsilon_T]$$

The problem is concave in e and continuously differentiable in e. Moreover the optimal solution will be interior because $F'(1, G) = 0$ and $F_e(0, G) = \infty$. We can use the $F.O.C$ condition to characterize the optimal solution. Because the production function is linear in ϵ_T we have:

$$T(G_{T-1}, e_T, R_T) = -w(R_T) + \delta \omega_i F_e(e_T, G_{T-1}) \mathbb{E}[\epsilon_T] = 0 \tag{4}$$

7.1.1 Characterization of the optimal effort in T

Using the implicit function theorem and equation (4), omitting time indexes to ease notation, and denoting by $e^\star$ the effort level that solves (4), we can work out:

$$\frac{\partial e^\star}{\partial G} = -\frac{\delta \omega_i \alpha F_{eG}(e^\star, G) \mathbb{E}[\epsilon_T]}{\delta \omega_i F_{ee}(e^\star, G) \mathbb{E}[\epsilon_T]} = \frac{\beta}{G(1-\alpha)} \frac{e^{\alpha-1} - 1}{e^{\alpha-2}} \equiv \frac{\beta}{G(1-\alpha)} H(e) > 0$$

In words, because of the complementarity between effort and knowledge, the above inequality says that child'effort is increasing in knowledge.
Finally, analogously to the previous step, we can see that type monotonicity holds:

$$\frac{\partial e^\star}{\partial \omega_i} = -\frac{\frac{\partial T}{\partial \omega_i}}{\frac{\partial T}{\partial e}} = -\frac{\delta F_e(e^\star, G) \mathbb{E}[\epsilon_T]}{\delta \omega_i F_{ee}(e^\star, G) \mathbb{E}[\epsilon_T]} = -\frac{\alpha(e^{\alpha-1} - 1)}{\alpha(\alpha - 1) e^{\alpha-2}} = \frac{1}{(1-\alpha)\omega_i} \frac{e^{\alpha-1} - 1}{e^{\alpha-2}} \equiv \mu_i G^\beta H(e) > 0 \tag{5}$$

In the case in which $G_{T-1} = 0$ the optimal choice of the child will be $e^\star = 0$ because effort is not productive, yet costly. Moreover $V_T(0, R_t; \omega_i) = 0$

7.2 Child's problem in period $T - 1$

We now proceed to characterize the optimal effort level when the parent plays the described cut-off strategy in state (G_{T-2}, R_{T-1}), with $G_{T-2} > 0$. The $T - 1$ problem for a child of type

ω_i is given by:

$$\max_{e\in(0,\infty)} V_t(G_{T-1}, R_{T-1}, e_{T-1}; \omega_i) \equiv \Big\{u(e_{T-1})w(R_{T-1}) + \delta\Big[\int_{0}^{\underline{G}} V_T(G_{T-1}, s; \omega_i)h(G_{T-1}|e_{T-1}) + \quad (6)$$
$$\int_{\underline{G}}^{\overline{G}} V_T(G_{T-1}, p; \omega_i)h(G_{T-1}|e_{T-1}) + \int_{\overline{G}}^{\infty} V_T(G_{T-1}, n; \omega_i)h(G_{T-1}|e_{T-1})\Big]\Big\}$$

Lemma 7.1. *In period $T-1$ the child plays a type monotonic strategy as a best response to R_T^{eq}*

Proof. We proceed by contradiction. In particular *without loss of generality* suppose that there exists only two type of children with $\omega_L < \omega_H$. Let's now suppose that the ω_L-type chooses an effort level e_h while ω_H-type chooses an effort level e_h, with $e_l < e_h$. It then follows that the following inequalities hold:

$$u(e_h)w(R) + \mathbb{E}[V_T(G_{T-1}, R_T|\omega_L; e_h)] > u(e_l)w(R) + \mathbb{E}[V_T(G_{T-1}, R_T|\omega_L; e_l)] \quad (7)$$
$$u(e_l)w(R) + \mathbb{E}[V_T(G_{T-1}, R_T|\omega_H; e_l)] > u(e_h)w(R) + \mathbb{E}[V_T(G_{T-1}, R_T|\omega_H; e_h)] \quad (8)$$

If we sum (8) and (7) we get the following inequalities:

$$\mathbb{E}[V_T(G_{T-1}, R_T|\omega_L; e_h)] - \mathbb{E}[V_T(G_{T-1}, R_T|\omega_H; e_h)] > \mathbb{E}[V_T(G_{T-1}, R_T|\omega_L; e_l)] - \mathbb{E}[V_T(G_{T-1}, R_T|\omega_H; e_l)] \quad (9)$$

Because of the cut-off strategy adopted by the parent we have that for the generic type ω_i and effort level e:

$$\mathbb{E}\big[V_T(G_{T-1}, R_T|\omega_i; e)\big] = \int_{0}^{\underline{G}} V_T(G_{T-1}, s; \omega_i)h(G_{T-1}|e_{T-1}) + \quad (10)$$
$$\int_{\underline{G}}^{\overline{G}} V_T(G_{T-1}, p; \omega_i)h(G_{T-1}|e_{T-1}) + \int_{\overline{G}}^{\infty} V_T(G_{T-1}, n; \omega_i)h(G_{T-1}|e_{T-1})$$

In order to ease notation I will rewrite (9) considering only the first integral in (10):

$$\int_{0}^{\underline{G}} V_T(G_{T-1}, s; \omega_L)h(G_{T-1}|e_h) - \int_{0}^{\underline{G}} V_T(G_{T-1}, s; \omega_H)h(G_{T-1}|e_h) > \quad (11)$$
$$\int_{0}^{\underline{G}} V_T(G_{T-1}, s; \omega_L)h(G_{T-1}|e_l) - \int_{0}^{\underline{G}} V_T(G_{T-1}, s; \omega_H)h(G_{T-1}|e_l)$$

We can re-write (11) as follows:

$$\int_{0}^{\underline{G}} \big[V_T(G_{T-1}, s; \omega_L) - V_T(G_{T-1}, s; \omega_H)\big]h(G_{T-1}|e_h) > \quad (12)$$
$$\int_{0}^{\underline{G}} \big[V_T(G_{T-1}, s; \omega_L) - V_T(G_{T-1}, s; \omega_H)\big]h(G_{T-1}|e_l)$$

We can now define $y(G_{T-1}, 1) = \left[V_T(G_{T-1}, s; \omega_H) - V_T(G_{T-1}, s; \omega_L)\right]$ and re-write (12) as follows:

$$\int_0^{\underline{G}} y(G_{T-1}, s) h(G_{T-1}|e_h) < \int_0^{\underline{G}} y(G_{T-1}, s) h(G_{T-1}|e_l)$$

Because in (10) we also have other two integrals, in re-writing (9) we also need to include other two integrals involving terms $y(G_{T-1}, p)$ and $y(G_{t-1}, n)$. We get:

$$\int_0^{\underline{G}} y(G_{T-1}, s) h(G_{T-1}|e_h) + \int_{\underline{G}}^{\overline{G}} y(G_{T-1}, p) h(G_{T-1}|e_h) + \int_{\overline{G}}^{\infty} y(G_{T-1}, n) h(G_{T-1}|e_h) \quad (13)$$

$$< \int_0^{\underline{G}} y(G_{T-1}, s) h(G_{T-1}|e_l) + \int_{\underline{G}}^{\overline{G}} y(G_{T-1}, p) h(G_{T-1}|e_l) + \int_{\overline{G}}^{\infty} y(G_{T-1}, n) h(G_{T-1}|e_l)$$

Now notice that $y(G_{T-1}, R_{T-1}) > 0, \ \forall R$ because $V_T(G_{T-1}, R; \omega_L) < V_T(G_{T-1}, R; \omega_H), \ \forall R$. We can now define the random variable $x(G) \in [0, \infty)$ as follows:

$$x(G_{T-1}) = \begin{cases} y(G_{T-1}, s) & \text{if } G_{T-1} < \underline{G} \\ y(G_{T-1}, p) & \text{if } \underline{G} < G_{T-1} < \overline{G} \\ y(G_{T-1}, n) & \text{if } G_{T-1} > \overline{G}. \end{cases}$$

and re-write (13) as follows:

$$\int_0^{\infty} x(G_{T-1}) h(G_{T-1}|e_h) < \int_0^{\infty} x(G_{T-1}) h(G_{T-1}|e_l)$$

The above inequality cannot hold because $h(G_{T-1}|e_h)$ FOSD $h(G_{T-1}|e_l)$. Hence, the contradiction. □

7.3 Parent's best response

Given the above strategies we want to check that the best response adopted by the parent in the last period takes indeed the form of a cut-off one. Let G_{T-1} be the state variable at the beginning of the last. After having selected a certain R_{T-1} in the previous period, in the last period the parent's value function is given by:

$$W_T(G_{T-1}) = \max_{R_T \in \{s,p,n\}} \left\{ \mathbb{E}[v(G_T)|s] - c_s, \mathbb{E}[v(G_T)|p] - c_p, \mathbb{E}[v(G_T)|n] - c_n \right\}$$

Let $p^i_{T-1}(G_{T-1})$ denote the posterior probability of the child being type i at the beginning of period T after the generic action R_{T-1} was selected in the previous period. We denote by $\Pi^{sj}_T(G_{T-1})$ with $j \in \{p, n\}$ the following function:

$$\Pi^{1j}_T(G_{T-1}) = \sum_i p^i_{T-1}(G_{T-1}) \left[\mathbb{E}[v(G_T)|s; \omega_i)] - \mathbb{E}[v(G_T)|j; \omega_i] \right]$$

Therefore $\Pi_T^{sj}(G_{T-1})$ is a weighted average of the expected gain from choosing alternative s (*strict* limits)versus the *permissive* or *no limits* alternative, conditional on being type ω_i. $(\Gamma(G_{T-1}^{sj}|\omega_i))$ with weights given by the beliefs on the child being of type ω_i. Holding fixed the type ω_i, the parent will choose alternative s versus alternative j in state G_{T-1} if and only if:

$$\mathbb{E}[v(G_T)|s;\omega_i] - c_s > \mathbb{E}[v(G_T)|j;\omega_i] - c_j \Leftrightarrow \mathbb{E}[v(G_T)|s;\omega_i] - \mathbb{E}[v(G_T)|j;\omega_i] > c(s) - c(j) > 0$$

Lemma 7.2. *Let* $\Gamma_{sj}(G_{T-1}|\omega_i) \equiv \mathbb{E}[v(G_T)|s;\omega_i] - \mathbb{E}[v(G_T)|j;\omega_i]$ *denote the gain from choosing alternative s versus alternative* j, $\forall j \in \{p,n\}$. *It is the case that* $\Gamma_{sj}(G_{T-1}|\omega_i)$ *is strictly decreasing in* $G_{T-1} \in (0,\infty)$.

Proof. Omitting the dependence of the optimal level e of effort on ω_i we have that the derivative of the expected value of knowledge in the last period with respect to the current state G, when the parenting option i is selected is given by:

$$\frac{\partial \mathbb{E}[v(G_T)|\omega_i,i]}{\partial G} = \mathbb{E}[\epsilon]\kappa(G)\frac{1}{e^\alpha - \alpha e}\Big[(e^\alpha - \alpha e) + \Big(\frac{\alpha}{1-\alpha}(e^{\alpha-1}-1)\frac{e^{\alpha-1}-1}{e^{\alpha-2}}\Big)\Big] =$$

$$\kappa(G)\mathbb{E}[\epsilon]\Big[1 + \frac{\alpha}{1-\alpha}\frac{e^{\alpha-1}-1}{e^\alpha - \alpha e}(e^{\alpha-1}-1)e^{2-\alpha}\Big] = \kappa(G)\mathbb{E}[\epsilon]\Big[1 + \frac{\alpha}{1-\alpha}\frac{e^{\alpha-1}-1}{e^\alpha - \alpha e}(e - e^{2-\alpha})\Big] =$$

$$\kappa(G)\mathbb{E}[\epsilon]\Big[1 + \frac{e^\alpha - 2e + e^{2-\alpha}}{e^\alpha - \alpha e}\Big] = \kappa(G)[\epsilon]\psi(e)$$

where $\kappa(G) = \dfrac{\beta}{G}$. Hence we have:

$$\frac{\partial \Gamma_{sj}(G|\omega_i)}{\partial G} = \kappa(G)\big[\psi(e_s) - \psi(e_j)\big] < 0 \Leftrightarrow \psi'(e) < 0$$

where:

$$\psi(e) = 1 + \frac{e^\alpha - 2e + e^{2-\alpha}}{e^\alpha - \alpha e}$$

It is easy to check that $\psi'(e) < 0\ \forall e \in (0,1),\ \alpha \in (0,1)$. The claim follows

□

Lemma 7.3. *The (expected) gain form choosing option s versus* $j \in \{p,n\}$ $\Gamma_{sj}(G_{T-1}|\omega_i)$, *is strictly decreasing in* ω_i

Proof. Let $\chi(e) = \dfrac{e^\alpha - e}{e^\alpha - \alpha e}\dfrac{e^{\alpha-1}-1}{e^{\alpha-1}}$. It is easy to see that

$$\frac{\partial \Gamma_{sj}(G|\omega_i)}{\partial \omega_i} = \mu_i\big[\chi(e_s) - \chi(e_j)\big]$$

Because $\chi(e)$ is the product of two decreasing functions (in e), our result follows.

□

Lemma 7.4. *Suppose that there are only two types, $\omega_L < \omega_H$. Then, the equilibrium gain from choosing alternative s versus alternative j, $\Pi_T^{sj}(G_{T-1})$, is strictly decreasing in G_{T-1} $\forall j \in \{p, n\}$.*

Proof. The result is obvious when we refer to its geometrical interpretation. We are taking the convex combination of two curves $\Gamma_{sj}(G_{T-1}|\omega_L)$ and $\Gamma_{sj}(G_{T-1}|\omega_H)$, decreasing in G_{T-1} because of lemma 7.2, with the latter lying below the former(because of lemma 7.3). Further we weight (using posterior) the lower curve more and more as G_{T-1} increases because of type monotonicity. This operation will yield a decreasing curve. Formally:

$$\frac{\partial \Pi_T^{sj}(G_{T-1})}{\partial G_{T-1}} = \frac{\partial}{\partial G_{T-1}}\big[p_{T-1}(G_{T-1})\Gamma_{sj}(G_{T-1}|\omega_H) + (1 - p_{T-1}(G_{T-1}))\Gamma_{sj}(G_{T-1}|\omega_L)\big] > 0$$

We can now see that:

$$\frac{\partial \Pi_T(G_{T-1})}{\partial G_{T-1}} = \underbrace{p'_{T-1}(G_{T-1})}_{X}\underbrace{\big[\Gamma_{sj}(G_{T-1}|\omega_H) - \Gamma_{sj}(G_{T-1}|\omega_L)\big]}_{Y}$$
$$+ p_{T-1}(G_{T-1})\Gamma'_{sj}(G_{T-1}|\omega_H) + \big(1 - p_{T-1}(G_{T-1})\big)\Gamma'_{sj}(G_{T-1}|\omega_L) > 0$$

Now because of lemma type monotonicity $X > 0$. Moreover because of lemma 7.3 $Y < 0$. Finally because of lemma 7.2 the second line is strictly negative.

□

Lemma 7.5. *Suppose there exists N types with $\omega_1 < \omega_2 < \cdots < \omega_N$. The gain from choosing alternative i versus alternative j $\Pi_{ij}(G)$, where limits i are stricter than j , is strictly decreasing in G.*

Proof. **Step 1**

The first step is borrowed from to Banks and Sundaram (1998). For any $G_{T-1} > \overline{G}_{T-1}$ let $p(i)$ and $p(\bar{i})$ the beliefs placed on the kid being of type i after witnessing G_{T-1} and $\overline{G}_{T-1}$, respectively. Then it follows that p stochastically dominates $\overline{p}$, in the sense that for any $l \in \{1, \dots N\}$ it is the case that:

$$\sum_{k=l}^{N} p(\bar{k}) > \sum_{k=l}^{N} p(k) \tag{14}$$

This means that when higher level of knowledge are realized, beliefs shift toward higher types. To see why this result holds we need to show that, for any given l:

$$\frac{\sum_{j=l}^{N} \pi_j \varphi(G|e_j)}{\sum_{i=1}^{N} \pi_i \varphi(G|e_i)} \geq \frac{\sum_{j=l}^{N} \pi_j \varphi(\overline{G}|e_j)}{\sum_{i=1}^{N} \pi_i \varphi(\overline{G}|e_i)}$$

where π_i, $\forall i = 1, \dots, N$ denotes the prior on the kid being of type i and $\varphi(G|e_i)$ denotes the probability of observing G conditional on the the effort level chosen by type i. We can now

rewrite the following expression as follows:

$$\sum_{j=l}^{N}\pi_j\varphi(G|e_j)[\sum_{i=1}^{l-1}\pi_i\varphi(\overline{G}|e_i)+\sum_{i=l-1}^{N}\pi_i\varphi(\overline{G}|e_i)]\geq\sum_{j=l}^{N}\pi_j\varphi(\overline{G}|e_j)[\sum_{i=1}^{l-1}\pi_i\varphi(G|e_i)+\sum_{i=l-1}^{N}\pi_i\varphi(G|e_i)]\Leftrightarrow$$

$$\sum_{i=1}^{l-1}\sum_{j=l}^{N}\pi_i\pi_j\varphi(G|e_j)\varphi(\overline{G}|e_i)+\sum_{i=l}^{N}\sum_{j=l}^{N}\pi_i\pi_j\varphi(\overline{G}|e_i)\varphi(G|e_j)\geq$$

$$\sum_{i=1}^{l-1}\sum_{j=l}^{N}\pi_i\pi_j\varphi(\overline{G}|e_j)\varphi(G|e_i)+\sum_{i=l}^{N}\sum_{j=l}^{N}\pi_i\pi_j\varphi(G|e_i)\varphi(\overline{G}|e_j)$$

Canceling common terms the inequality becomes:

$$\sum_{i=1}^{l-1}\sum_{j=l}^{N}\pi_i\pi_j\varphi(G|e_j)\varphi(\overline{G}|e_i)\geq\sum_{i=1}^{l-1}\sum_{j=l}^{N}\pi_i\pi_j\varphi(\overline{G}|e_j)\varphi(G|e_i)$$

We can now know that:
i) because of type-monotonicity $e_j \geq e_i, \forall\, j > i$
ii) because of the monotone likelihood ratio property $\varphi(\overline{G}|e_j)\varphi(G|e_i) \geq \varphi(\overline{G}|e_i)\varphi(G|e_j)$ It follows that (14) holds. It is now easy to see that $\Pi_T^{sj}(G_{T-1})$ is increasing in G_{T-1} because of i)

Step 2

We now prove the claim by induction. Consider the three-types, with $\omega_1 < \omega_2 < \omega_3$. Let' denote by $\Gamma_i(G)$ the gain from choosing alternative i versus alternative j, if the parent knew the kid is of type i. In the previous step we showed that the function $\Delta(G) = p_2(G) + p_3(G)$ is increasing in G, where $p_j(G)$ denotes the posterior placed by the parent on the kid being of type j. Because the density of G is continuously differentiable in G, it follows that $\Delta'(G) > 0$. We now seek to show that $\Pi(G) = (1-p_1(G)-p_2(G))\Gamma_1(G)+p_2(G)\Gamma_2(G)+p_3(G)\Gamma_3(G)$ is strictly decreasing in G, provided that as shown in lemmas 7.3 and 7.2 it is the case that $\Gamma_j'(G) < 0$ and $\Gamma_3(G) > \Gamma_2(G) > \Gamma_1(G)$. Suppose now that, without loss of generality, $p_2'(G) > 0$. We can calculate:

$$\Pi'(G) = p_2'(G)\Gamma_2'(G) + p_3'(G)\Gamma_3'(G) - (p_2'(G) + p_3'(G))\Gamma_1'(G) +$$
$$(1 - p_1(G) - p_2(G))\Gamma_1'(G) + p_2(G)\Gamma_2'(G) + p_3(G)\Gamma_3'(G)$$

The second line of the above expression is strictly negative because of lemma 7.2. We can now add and subtract to the first line (FL) $p_2'(G)\Gamma_1(G)$. Rearranging terms we get:

$$FL = \underbrace{p_2'(G)}_{>0}\underbrace{\Big[\Gamma_2(G) - \Gamma_1(G)\Big]}_{=A<0} + \underbrace{\big[p_2'(G) + p_2'(G)\big]}_{>0}\underbrace{\Big[\Gamma_3(G) - \Gamma_1(G)\Big]}_{=B<0}$$

Where A and B are negative because of lemma 7.2. Hence our claim holds if $N = 3$. The argument is analogous if we assume that $p_3'(G) > 0$ and is trivial if $p_3'(G) < 0$ and $p_2'(G) < 0$. We can easily extend the above argument to the case of $N = 4$ and so forth.

□

Corollary 7.6. *The gain from choosing alternative p versus alternative n in period $T-1$ $\Gamma_{pn}(G_{T-1}|\omega_i)$ is strictly decreasing in G_{T-1}*

Proof. The claim follows . The argument used in the lemma does not rely on the fact that we were analyzing behavior of the gain from choosing alternative 1 over alternative j and they go through if we re-label $\Gamma_{sj}(G)$ by $\Gamma_{sp}(G)$ □

Lemma 7.7. *In the last period the parent adopts the following strategy:*

$$R_2^{eq} = \begin{cases} n & \text{if } G_{T-1} = 0 \\ s & \text{if } G_{T-1} < \underline{G} \\ p & \text{if } \underline{G} \leq G_{T-1} < \overline{G} \\ n & \text{if } G_{T-1} \geq \overline{G} \end{cases}$$

with $\underline{G} \leq \overline{G}$

Proof. Let $\Pi_{ij}(G_{T-1}) = \sum_{k=1}^{N} \Gamma_{ij}(G_{T-1}|\omega_k) p_{T-1}^k(G_{T-1})$ be defined as the equilibrium expected gain of option i versus j, with $i < j$, i.e. the average of each curve $\Gamma_{ij}(G_{T-1}|\omega_k)$ using the beliefs $p_{T-1}^k(G_{T-1})$. As mentioned before the parent prefers alternative s over alternative p if $\Pi_{sp}(G_{T-1}) > c_s - c_p$ and alternative s over alternative n if $\Pi_{sn}(G_{T-1}) > (c_s - c_n)$. Because in lemma 7.3 we showed that Π_{ij} is strictly decreasing there exists a unique $\overline{G}_{sp} \in [0, \infty)$ such that $\Pi_{sp}(\overline{G}_{sp}) = c_s - c_p$ and a unique $\overline{G}_{sn} \in [0, \infty)$ such that $\Pi_{sn}(\overline{G}_{sn}) = c_s - c_n$. Let $\underline{G} \equiv \min\left\{\overline{G}_{sp}, \overline{G}_{sn}\right\}$. It then follows that the parent will choose option s (*strict* limits) whenever $G_{T-1} < \underline{G}$. [39] Similarly in the interval $[\underline{G}, \infty)$ the parent will choose p (*permissive* limits) if $G_{T-1} < \overline{G}$, where $\overline{G}$ is such that $\Pi_{pn}(\overline{G}) = c_p - c_n$. If $G_{T-1} = 0$, no effort will be exerted, regardless of the chosen parenting method, hence the parent will choose the option less costly: *no limits*. □

[39] I assume that whenever the parent is indifferent between two options $i < j$ he chooses j, i.e. the more lenient parenting option. Although arbitrary this assumption is necessary to ensure that there exists a unique equilibrium in pure strategy. Indeed the potential multiplicity coming from multiple mixed strategy in $(\underline{G}, \overline{G})$ is a measure zero event

8 Solution of the Model

We start by fixing an initial level of human capital G_0 and a prior p_0 about the child being of an high type. The equilibrium can be described as follows: i) parents choose an initial action R_1 and ii) announce two cut-offs $(\underline{G}, \overline{G})$. The child believes the announcement and plays her best response. From the point of view of the parents the chosen action R_1 and the announcements are reflected in the way beliefs are updated in the second period. Let $\Gamma^j_{tn}(G_1) = \mathbb{E}[\log(G_2)|\omega_j, G_1, t] - \mathbb{E}[\log(G_2)|\omega_j, G_1, n]$ denote the expected gain of choosing strict versus no-limits conditional on the child being an ω_j-type and after the realization of G_1. Let $\Gamma^j_{tj}(G_1)$ and $\Gamma^j_{np}(G_1)$ be defined similarly. Now, to any candidate strategy $s_p(R_1) = \big(R_1, (\underline{G}, \overline{G})\big)$, it is associated a unique function $p(G_1; s_p(R_1))$, which represents the posterior about the child being an ω_H type, given the realization of G_1. Hence a unique set of equilibrium-gain curves is associated to any candidate strategy $s_p(R_1)$. The curves are obtained by weighting each of the type-specific curve using the posterior beliefs $p(G_1; s_p(R_1))$. By definition the equilibrium cut-offs are such that:

1. $\Gamma^{j\star}_{np}(\overline{G}) = c_p$
2. $\Gamma^{j\star}_{sn}(\underline{G}) = c_s - c_p$ or $\Gamma^{j\star}_{sp}(\underline{G}) = c_s$

Essentially finding the cut-offs requires we solve a system of two simultaneous equations in two unknowns (the cut-offs). The content of lemma 2.1 ensures us the the solution is unique. The solution is found through an iterative algorithm. That is:

1. Fix an action $R_1 \in \{s, p, n\}$ and guess a pair of cut-offs $(\underline{G}, \overline{G})$
2. Find the best response of the child by solving in both periods her problem as a best response to parents conjectured strategy $s_p(R_1)$
3. Check that $(\underline{G}, \overline{G})$ are the equilibrium cut-offs for the initial action R_1. If this is the case compute the value of the strategy s_p. If not guess another set of cut-offs $(\underline{G}, \overline{G})$ until the equilibrium cut-offs associated to R_1 are found. Let $s^\star_p(R_1) = \Big(R_1, (\underline{G}, \overline{G})\Big)$ denote the equilibrium strategy if R_1 is selected and $V(R_1)$ the associated value
4. Pick the strategy that delivers the higher value. That is $s^\star_p(R_1) \in argmax\big\{V(s), V(p), V(n)\big\}$

We perform steps 1-4 for both of the systems. This procedure will deliver two candidate strategy: $s^p_1 = \Big(R_1, (\underline{G}_1, \overline{G}_1)\Big)$ and $s^p_2 = \Big(R_2, (\underline{G}_2, \overline{G}_2)\Big)$ with associated values V_1 and V_2. By picking the strategy that delivers the highest value we find parents' equilibrium strategy and the child's best response.

9 Tables

Table 10: Human Capital Production Function

	(1)	(2)	(3)
	piat_m_raw_98	piat_m_raw_98	log_piat_98
piat_m_raw_97	0.702*** (36.96)	0.699*** (36.67)	
time_study	0.150* (2.29)	0.150* (2.31)	
p_curf		-0.210 (-0.11)	
j_curf		1.748 (0.90)	
log_piat_97			0.252*** (15.11)
log_study			0.0101** (3.26)
_cons	24.28*** (17.90)	24.23*** (11.10)	3.190*** (45.69)
R^2	0.482	0.485	0.147
N	1482	1465	1480

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 11: Available data for high school graduates born in 83 or 84 with normal grade progression

	attending 6^{th} grade in in ac year 96/97						
	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}	12^{th}
TV Limits	YES	YES	YES	NO	NO	NO	NO
Curfew limits	YES	YES	YES	NO	NO	NO	NO
Friends limits	YES	YES	YES	NO	YES	NO	NO
Study time	YES	NO	NO	NO	NO	NO	NO
Self Decl. Grades	NO	NO	YES	NO	NO	NO	NO
Transcript GPA	NO	NO	NO	YES	YES	YES	YES
PIAT MATH	YES	YES	YES	YES	YES	YES	YES
CAT-ASVAB	YES	NO	NO	NO	NO	NO	NO

	attending 7^{th} grade in in ac year 96/97						
	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}	12^{th}
TV Limits	NO	YES	YES	YES	NO	NO	NO
Curfew limits	NO	YES	YES	YES	NO	NO	NO
Friends limits	NO	YES	YES	YES	NO	YES	NO
Study time	NO	YES	NO	NO	NO	NO	NO
Self Decl. Grades	NO	NO	YES	NO	NO	NO	NO
Transcript GPA	NO	NO	NO	YES	YES	YES	YES
PIAT MATH	NO	YES	YES	YES	YES	YES	YES
CAT-ASVAB	NO	YES	NO	NO	NO	NO	NO

	attending 8^{th} grade in in ac year 96/97						
	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}	11^{th}	12^{th}
TV Limits	NO	NO	YES	YES	YES	NO	NO
Curfew limits	NO	NO	YES	YES	YES	NO	NO
Friends limits	NO	NO	YES	YES	YES	NO	YES
Study time	NO	NO	YES	NO	NO	NO	NO
Self Decl. Grades	NO	NO	YES	NO	NO	NO	NO
Transcript GPA	NO	NO	NO	YES	YES	YES	YES
PIAT MATH	NO	NO	YES	YES	YES	YES	YES
CAT-ASVAB	NO	NO	YES	NO	NO	NO	NO

Table 12: Distribution of Grade Attended in ac. Year 96/97

grade attended	born in 84	born in 83
4	0.06	0
5	0.35	0.06
6	5.85	0.9
7	36.44	5.88
8	56.26	39.54
9	0.87	52.6
10	0.12	0.85
11	0.06	0.11
12	0	0.06
	N=1726	N=1768

Table 13: Agreement in Response: Curfew Limits

N=1532	Parents' Responses		
	Who sets the curfew?		
	Parents	Kid	Joint
Parents	61.88	0.39	5.35
Kid	2.15	0.07	0.52
Joint	25.33	0.2	4.11

Table 14: Agreement in Response: Friends Limits

N=1532	Parents' Responses		
	Who sets the curfew?		
	Parents	Kid	Joint
Parents	15.27	2.15	6.46
Kid	14.23	12.99	19.58
Joint	12.01	4.24	13.05

Table 15: Agreement in Response: TV Limits

N=1532	Parents' Responses		
	Who sets the curfew?		
	Parents	Kid	Joint
Parents	20.65	2.87	13.39
Kid	11.47	6.78	12.38
Joint	13.29	3.19	16.09

Table 16: Breaking Limits in the Last Month

	Youths aged 12-13 in Survey Year 1997		
N=1771	Curfew Limits	Friends Limits	TV Limits
Never	66.46	42.63	48.95
Once	8.2	3.11	5.87
More than once	25.24	54.26	46.08
	Youths aged 13-14 in Survey Year 1998		
N=1771	Curfew Limits	Friends Limits	TV Limits
Never	58.84	28.4	32.69
Once	8.13	2.6	2.99
More than once	33.03	69	64.31
	Youths aged 14-15 in Survey Year 1999		
N=1771	Curfew Limits	Friends Limits	TV Limits
Never	50.14	8.58	41.28
Once	28.83	1.98	74.2
More than once	25.07	2.94	71.99

Table 17: Limits by Age

	Youths aged 12-13 in Survey Year 1997		
	Curfew Limits	Friends Limits	TV Limits
Parents	67.89	23.89	36.63
Kid	2.88	46	31.34
Jointly	29.23	30.11	32.03
N	1738	1737	1739
	Youths aged 13-14 in Survey Year 1998		
	Curfew Limits	Friends Limits	TV Limits
Parents	56.24	12.49	20.11
Kid	5.4	58.79	50.47
Jointly	38.36	28.71	29.42
N	1611	1609	1611
	Youths aged 14-15 in Survey Year 1999		
	Curfew Limits	Friends Limits	TV Limits
Parents	46.3	9.8	14.38
Kid	7.65	64.5	59.92
Jointly	46.05	25.7	25.7
N	1568	1572	1572
	Youths aged 16-17 in Survey Year 2001		
	Curfew Limits	Friends Limits	TV Limits
Parents	-	13.81	-
Kid	-	51.59	-
Jointly	-	34.6	-
N	-	1477	-

Table 18: Punishment by Age

	Youths aged 12-13 in Survey Year 1997		
	Curfew Limit Punishment	TV Limit Punishment	Friends Limit Punishment
Discuss it calmly with you	41.8	53.62	54.28
Ignore it, pretend that it didn't happen or let you get away with it	2.02	3.37	1.82
Sulk, pout, or give you the silent treatment	0.59	0.76	0.86
Take away a privilege, ground you, or give you a chore	45.12	32.91	30.41
Make threats that won't be kept	1.72	1.85	2.03
Yell, shout, or scream at you	6.36	5.13	6.53
Use physical punishment	2.38	2.36	4.07
N	1682	1188	934
	Youths aged 13-14 in Survey Year 1998		
	Curfew Limit Punishment	TV Limit Punishment	TV Limit Punishment
Discuss it calmly with you	39.22	54.22	56.38
Ignore it, pretend that it didn't happen or let you get away with it	3.89	4.92	3.04
Sulk, pout, or give you the silent treatment	0.73	1.13	1.06
Take away a privilege, ground you, or give you a chore	43.18	28.75	28.27
Make threats that won't be kept	2.97	2.52	1.22
Yell, shout, or scream at you	8.64	7.19	8.21
Use physical punishment	1.38	1.26	1.82
N	1517	793	658
	Youths aged 14-15 in Survey Year 1999		
	Curfew Limit Punishment	TV Limit Punishment	TV Limit Punishment
Discuss it calmly with you	41	55.41	53.07
Ignore it, pretend that it didn't happen or let you get away with it	3.25	6.37	3.97
Sulk, pout, or give you the silent treatment	0.9	0.8	1.26
Take away a privilege, ground you, or give you a chore	41.41	27.87	29.24
Make threats that won't be kept	4.57	2.55	3.79
Yell, shout, or scream at you	7.96	5.57	7.22
Use physical punishment	0.9	1.43	1.44
N	1444	628	554

Table 19: Transitions

Transitions of the Curfew Limits

	Age 13-14					Age 14-15					Age 14-15			
Age 12-13	Parents	Jointly	Kid	*N*	Age 13-14	Parents	Jointly	Kid	*N*	Age 12-13	Parents	Jointly	Kid	*N*
Parents	63.48	31.97	4.55	1076	Parents	60.61	5.42	33.96	848	Parents	52.35	6.81	40.84	1043
Jointly	41.36	53.09	5.54	469	Jointly	27.09	8.86	64.05	587	Jointly	33.41	8.30	58.3	458
Kid	34.78	41.30	23.91	46	Kid	35.00	22.50	42.50	80	Kid	41.86	18.60	39.53	43

Transitions of the Friends Limits

	Age 13-14					Age 14-15					Age 14-15			
Age 12-13	Parents	Jointly	Kid	*N*	Age 13-14	Parents	Jointly	Kid	*N*	Age 12-13	Parents	Jointly	Kid	*N*
Parents	26.70	31.06	42.23	367	Parents	30.65	38.71	30.65	186	Parents	19.06	48.07	32.87	362
Jointly	11.98	39.46	48.55	484	Jointly	10.38	49.44	40.18	443	Jointly	10.09	57.68	32.24	456
Kid	5.56	20.33	74.12	738	Kid	4.50	78.04	17.45	888	Kid	4.66	77.23	18.11	729

Transitions of the TV Limits

	Age 13-14					Age 14-15					Age 14-15			
Age 12-13	Parents	Jointly	Kid	*N*	Age 13-14	Parents	Jointly	Kid	*N*	Age 12-13	Parents	Jointly	Kid	*N*
Parents	32.14	32.33	35.53	1030	Parents	38.92	30.14	30.94	501	Parents	24.16	45.35	30.5	1010
Jointly	12.8	33.8	53.40	1000	Jointly	12.92	45.55	41.53	797	Jointly	9.17	62.95	28.24	981
Kid	5.74	14.13	80.12	1132	Kid	4.27	83.57	12.16	1710	Kid	4.56	84.49	10.95	1096

Table 20: Limits and Race

	Age 12-13				Age 13-14				Age 14-15			
Curfew Limits												
	Parents	Kid	Jointly	N	Parents	Kid	Jointly	N	Parents	Kid	Jointly	N
Hispanic	67.45	3.94	28.61	381	60.06	5.38	34.56	353	52.19	5.83	41.98	343
Black	75.52	3	21.48	433	63.06	7.65	29.29	379	56.87	5.49	37.64	364
White	64.55	2.37	33.08	928	51.76	4.42	43.38	883	39.51	9.5	50.98	863
Friends' Limits												
Hispanic	28.16	43.36	28.16	380	13.31	55.81	30.88	353	10.17	62.5	27.33	344
Black	34.87	36.49	28.64	433	17.32	52.23	30.45	381	14.25	56.99	28.77	365
White	17.35	51.08	31.57	928	10.24	62.8	26.96	879	7.86	68.21	23.93	865
TV Limits												
Hispanic	35.96	35.17	28.87	381	18.64	48.59	32.77	354	14.53	55.81	29.65	344
Black	35.33	35.33	29.33	433	20.63	52.65	26.72	378	10.68	62.74	26.58	365
White	37.5	27.8	34.7	928	20.39	50.4	29.22	883	15.95	60.12	23.93	865

Table 21: Limits and Gender

	Age 12-13				Age 13-14				Age 14-15			
	Curfew Limits											
	Parents	Kid	Jointly	N	Parents	Kid	Jointly	N	Parents	Kid	Jointly	N
Boys	67.15	3.82	29.02	889	54.32	7.43	38.25	834	44.84	10.2	44.96	814
Girls	68.67	1.88	29.45	849	58.3	3.22	38.48	777	47.88	4.91	47.21	754
	Friends' Limits											
Boys	27.3	43.03	29.66	890	12.76	60.05	27.2	831	10.43	65.4	24.17	815
Girls	20.31	49.11	30.58	847	12.21	57.46	30.33	778	9.11	63.54	27.34	757
	TV Limits											
Boys	38.58	30.71	30.71	889	22.38	49.7	29.72	831	15.95	57.91	26.13	815
Girls	34.59	32.00	33.41	850	17.69	51.28	31.03	780	12.68	62.09	25.23	757

Table 22: Limits and Family income

Survey Year: 1997			Survey Year: 1998			Survey Year: 1999		
Curfew Limits								
Parents	Kid	Jointly	Parents	Kid	Jointly	Parents	Kid	Jointly
32139	25900	37859	31993	30348	40681	33919	30574	38406
Friends' Limits								
24544	40330	31509	25308	38248	31338	26500	37964	33830
TV Limits								
33942	30175	37664	33136	35604	34162	32282	35767	34010

Table 23: Limits and Family Structure

	Age 12-13				Age 13-14				Age 14-15			
Curfew Limits												
	Parents	Jointly	Kid	N	Parents	Jointly	Kid	N	Parents	Jointly	Kid	N
Both Biological Parents	65.82	30.98	3.2	907	55.15	40.28	4.57	854	44.65	6.62	48.74	831
Two Parents, Biological Mother	64.74	31.4	3.86	207	56.13	37.74	6.13	212	46	8	46	200
Two Parents, Biological Father	85	15	0	40	66.67	33.33	0	42	63.41	2.44	34.15	41
Biological Mother Only	71.55	26.75	1.7	471	57.54	36.19	6.26	431	48.56	9.57	41.87	418
Biological Father Only	64.29	36.71	0	42	52.27	38.64	9.09	44	40.82	14.29	44.9	49
Adoptive Parents	70.59	23.53	5.88	17	70	30	0	10	63.64	0	36.36	11
Friends Limits												
	Parents	Jointly	Kid	N	Parents	Jointly	Kid	N	Parents	Jointly	Kid	N
Both Biological Parents	22.96	30.79	46.25	906	11.59	28.81	59.6	854	8.29	65.26	26.44	832
Two Parents, Biological Mother	24.27	33.5	42.23	206	13.88	25.84	60.29	209	9.5	65.5	25	200
Two Parents, Biological Father	20	37.5	42.5	40	14.29	19.05	66.67	42	19.51	58.54	21.95	41
Biological Mother Only	24.63	28.66	46.71	471	13.66	30.56	55.79	432	12.62	62.86	24.52	420
Biological Father Only	25.58	25.58	48.84	43	11.36	27.27	61.36	44	8.16	65.31	26.53	49
Adoptive Parents	29.41	23.53	47.06	17	10	20	70	10	9.09	72.73	18.18	11
TV Limits												
	Parents	Jointly	Kid	N	Parents	Jointly	Kid	N	Parents	Jointly	Kid	N
Both Biological Parents	40.79	32.75	26.46	907	22.55	32.24	45.21	856	16.35	55.77	27.88	832
Two Parents, Biological Mother	37.2	31.88	30.92	207	15.57	26.42	58.02	212	13	64	23	200
Two Parents, Biological Father	32.5	35	32.5	40	24.39	14.63	60.98	41	17.07	58.54	24.39	41
Biological Mother Only	29.09	31.63	39.28	471	17.21	26.74	56.05	430	10	66.67	23.33	420
Biological Father Only	30.95	33.33	35.71	42	9.09	29.55	61.36	44	22.45	57.14	20.41	49
Adoptive Parents	55.56	16.67	27.78	18	50	20	30	10	25	50	25	8

Table 24: PIAT MATH Raw Average Score By Race/Ethnic Group

Age	Hispanic	Black	White
12-13	60.5	61.2	72.9
13-14	68.1	63.2	78.1
14-15	70.6	67.6	80.7
15-16	73.5	69.4	83.7
16-17	74.9	70.9	86.1

Table 25: PIAT MATH Raw Average Score and 1997 Limits

1997	AV.PIAT M 1997			AV. PIAT M 1998		
	Kid	Parents	Jointly	Kid	Parents	Jointly
curfew limit	67.2	60.8	68.9	71.8	67.1	74.6
friends limit	62.7	69.2	68.7	66.6	74.4	74
TV limit	66.4	66.6	69.6	71.6	70.7	75.1
1998				Kid	Parents	Jointly
curfew limit				71.5	68.2	75.5
friends limit				64.9	74.4	73
TV limit				70.5	73.5	73.4

Table 26: PIAT MATH RAW and GPA

	(1)	(2)	(3)	(4)
	GPA_98_99	GPA_99_00	GPA_00_01	GPA_01_02
GPA_97_98	0.198*			
	(0.0902)			
piat_m_raw_98	1.091*			
	(0.408)			
GPA_98_99		0.722***		
		(0.0301)		
piat_m_raw_99		0.340*		
		(0.135)		
GPA_99_00			0.696***	
			(0.0324)	
piat_m_raw_00			0.572***	
			(0.142)	
GPA_00_01				0.683***
				(0.0350)
piat_m_raw_01				0.271
				(0.156)
_cons	161.9***	52.24***	38.11**	77.29***
	(35.40)	(11.22)	(12.15)	(13.61)
N	45	571	555	510
F	8.005	367.0	316.2	237.1

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 27: PIAT MATH RAW and GPA

	(1)	(2)	(3)	(4)
	GPA_98_99	GPA_99_00	GPA_00_01	GPA_01_02
piat_m_raw_98	1.517***			
	(0.156)			
piat_m_raw_99		1.577***		
		(0.171)		
piat_m_raw_00			1.696***	
			(0.178)	
piat_m_raw_01				1.412***
				(0.189)
_cons	175.1***	164.7***	148.4***	179.7***
	(12.24)	(13.92)	(14.99)	(16.36)
N	599	582	559	514
F	94.92	84.61	90.53	55.91

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 28: Limits and Average PIAT MATH

	Age 12-13			Age 13-14			Age 14-15		
Curfew Limits									
	Parents	Jointly	Kid	Parents	Jointly	Kid	Parents	Jointly	Kid
	67.27	68.94	60.8	71.52	75.11	68.23	74.16	77.68	76.34
N	1118	475	45	849	594	82	652	657	106
Friends Limits									
	Parents	Jointly	Kid	Parents	Jointly	Kid	Parents	Jointly	Kid
	62.73	68.71	69.26	64.96	73.02	74.42	69.42	75.65	77.04
N	388	490	759	183	436	905	133	358	928
TV Limits									
	Parents	Jointly	Kid	Parents	Jointly	Kid	Parents	Jointly	Kid
	66.43	69.68	66.69	70.51	73.46	73.55	74.21	76.45	76.2
N	604	535	500	300	449	776	204	467	848

Table 29: Allocation of Time By Race/Ethnic Group 12-13 Years Old Youths

Average Weekly Time Spent Doing Homework			Average Weekly Time Spent Watching TV		
White	Black	Hispanic	White	Black	Hispanic
5.31	4.62	5.82	16.25	23.4	18.73
N=925	N=432	N=376	N=930	N=430	N=380

Table 30: Allocation of Time and Eventual Educational Attainment

Average Hours per Week Devoted to Study at 12-13 by Educational Attainment

Dropout	High School Graduate	Enrolled in Some College
4.51	5.42	5.99

Average Hours per Week Devoted to Watch TV at 12-13 by Educational Attainment

Dropout	High School Graduate	Enrolled in Some College
21.11	19.94	16.6

Table 31: Limits and Average Hours per week Spent Studying

	curfew limits	friends limits	TV limits
parents	5.33	5.13	5.41
joint	5.17	5.19	5.48
kid	5.01	5.42	4.89

Table 32: Correlation Limits-Study Time Conditional On Family Structure

	Curfew Limit							
	12-13 years old				13-14 years old			
	Parent(s)	N	Kid	N	Parent(s)	N	Kid	N
Both Biological Parents	5.63	587	5.44	305	5.772	556	5.8	318
Two Parents, Biological Mother	5.33	133	5.41	73	4.49	133	5.827	57
Two Parents, Biological Father	4.27	33	3.708	6	5.07	26	4.29	17
Biological Mother Only	5.25	330	4.76	132	4.64	315	4.48	158
Biological Father Only	4.98	26	4.266	15	3.94	31	5.55	27
	Friends Limit							
	12-13 years old				13-14 years old			
	Parent(s)	N	Kid	N	Parent(s)	N	Kid	N
Both Biological Parents	5.422	205	5.622	686	5.97	157	5.78	720
Two Parents, Biological Mother	3.75	5	4.73	50	5.633	35	4.72	155
Two Parents, Biological Father	6.28	7	3.72	32	9.23	7	3.89	36
Biological Mother Only	4.87	114	5.199	348	4.377	113	4.653	259
Biological Father Only	4.55	10	4.72	32	2.559	7	4.99	51
	TV Limit							
	12-13 years old				13-14 years old			
	Parent(s)	N	Kid	N	Parent(s)	N	Kid	N
Both Biological Parents	5.346	366	5.7299	526	5.52	267	5.94	610
Two Parents, Biological Mother	5.981	77	4.993	129	5.957	59	4.417	131
Two Parents, Biological Father	4.448	13	4.051	26	4.47	13	4.88	30
Biological Mother Only	5.618	132	4.919	330	5.064	98	4.47	375
Biological Father Only	5.638	12	4.344	29	3.87	13	4.933	45

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