

Nash Bargaining, Credible Bargaining and Efficiency Wages in a Matching Model for the US

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This version: February 15, 2011

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Abstract

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This paper incorporates Nash bargaining, credible bargaining and efficiency wages as special cases of an over-arching model of wage determination in a matching model that is used to assess econometrically how well each fits US data. With Nash bargaining, estimates for worker bargaining power and the value of non-work activity are almost identical to those calibrated by Hagedorn and Manovskii (2008). However, the over-identifying restrictions are overwhelmingly rejected statistically, as they are for credible bargaining. Efficiency wages fit the data better, with the over-identifying restrictions not rejected statistically, and result in a lower, more plausible estimated value of non-work activity.

Keywords: Matching frictions, wage bargaining, efficiency wages, unemployment, shirking

JEL classification: E2, J3, J6

The matching model of Mortensen and Pissarides (1994), stemming from earlier models of matching frictions in Diamond (1982), Blanchard and Diamond (1989) and Pissarides (1985), has been seminal in recent discussions of unemployment. Tightly-specified aggregate formulations of the underlying model have been calibrated or estimated in many papers, including Cole and Rogerson (1999), Yashiv (2000), Hall (2005*a*), Shimer (2005) and Yashiv (2006).¹ These, however, have typically found it hard to match aspects of the US data, at least with wage determination based on the widely-used standard Nash bargain, see Shimer (2005). This has generated the search for alternative specifications, such as in Hagedorn and Manovskii (2008), and alternative wage determination mechanisms, such as in Hall (2005*b*), Hall and Milgrom (2008), Gertler and Trigari (2009), Rudanko (2009), Haefke et al. (2008) and Kudlyak (2009) to enable the matching approach to fit the data. This paper subjects the specifications in Hagedorn and Manovskii (2008) and Hall and Milgrom (2008) to rigorous econometric tests and compares them with yet another wage determination mechanism, efficiency wages, derived from the development in MacLeod and Malcomson (1998) of the efficiency wage model of Shapiro and Stiglitz (1984).

The paper uses formal econometric analysis, rather than calibration, for several reasons. First, calibrated models focus on fitting moments in the data that are thought to be of particular economic importance. That makes sense when the issue is how well a model can capture those particular moments. But for discriminating between models, a valuable test is how the models compare in fitting moments in the data, including autocorrelations, that they are not specifically designed to match. Econometric procedures provide an effective way of doing that. Second, calibration procedures rarely provide estimates of the uncertainty associated with point estimates of the parameters, such as confidence intervals, whereas this is a standard part of the econometric toolkit. Third, calibrated models typically require specific assumptions about the stochastic process underlying a model in order to obtain model-based moments for matching — in the models compared here this is the stochastic process driving productivity and separations. Moreover, specification of the data generating process often requires further assumptions to determine the equilibrium solution if it is not unique. These assumptions are not typically integral to the models. But imposing them makes tests of fit joint tests of the underlying model and of the assumptions about the stochastic process, not tests

¹There is also a growing literature applying disaggregated versions of the matching model with heterogeneous firms and employees to micro data. For a recent example, see Cahuc et al. (2006).

of the underlying model alone. This poses a serious threat to robustness that can be reduced by using appropriate econometric procedures. In this paper, we follow a limited information econometric approach that relies on much weaker assumptions about the underlying stochastic process, so tests of fit are focused more on the underlying models and less on the auxiliary assumptions used to implement them empirically. Finally, when, as here, models can be nested within a single over-arching framework, econometric procedures can use standard hypothesis tests to discriminate between them and the characteristics of these tests have been widely studied.

Set against these advantages are the disadvantages of econometric procedures. Most obvious is concern with identification. However, the models in Hagedorn and Manovskii (2008) and Hall and Milgrom (2008), and the efficiency wage model we analyze, are all forward-looking rational expectations models and this characteristic provides straightforward criteria for determining valid instruments. Moreover, we make use of recently-derived tests that are robust to weak instruments and thus reduce the risk of drawing inappropriate conclusions from statistical tests. Similarities between our estimation results and the calibration results in other papers also provide reassuring evidence that our results do not depend on differences in our methods.

The economic issues involved in identifying the differences between the models are the following. The standard Nash bargain has a worker and a firm share the gain from reaching agreement in fixed proportions that depend on bargaining power. Thus, graphically, the relationship between the worker's gain and the firm's gain is a straight line through the origin with slope equal to the ratio of their bargaining powers. Because it is a ratio of bargaining powers, the slope can be any non-negative number. In our formulation of the credible bargaining model of Hall and Milgrom (2008), the relationship between the worker's and the firm's gains is again a straight line but it can have a non-zero intercept, either positive or negative, that depends on the costs of making offers. The slope of the line, however, is the probability that the relationship breaks down between offers and is thus constrained to lie between zero and one. In the efficiency wage model, the relationship between the worker's and the firm's gains also has an intercept because the worker must receive some of the gain from the relationship to make it worthwhile not to shirk. In this case, the intercept has to be strictly positive but it differs subtly from the intercept in the credible bargaining model because the no-shirking condition requires that the worker's gain from the relationship *from the next period on* is sufficient to deter shirking, whereas the credible bargaining model is specified

in terms of gains *from the current period on*. Thus, the dynamics provide a way to discriminate between these two models.

Under the restrictions corresponding to the Nash bargaining model, our estimates of key parameters are very close to the calibrated values in Hagedorn and Manovskii (2008). That is reassuring evidence of the validity of our procedures. However, the over-identifying restrictions implied by that model are statistically rejected. The estimates of our formulation of the credible bargaining model cover the parameter values in the calibration of Hall and Milgrom (2008), though the parameter values are not precisely estimated. But again, the over-identifying restrictions implied by the model are rejected statistically. In contrast, the over-identifying restrictions implied by the efficiency wage model are not rejected statistically. Moreover, the estimated value of non-work activity is much lower, and more plausible, than with the Nash bargaining model. An important implication for future research is that the job creation equation implied by the canonical matching model is rejected by the data, so it seems further thought should be given to how the job creation equation is specified. We are, however, careful to check that this does not bias our comparison of wage determination mechanisms.

The paper is organized as follows. Section 1 describes the basic framework used in the canonical matching model. Section 2 sets out the different wage determination mechanisms. That is followed by sections on empirical specifications, the data and estimation results. These are, in turn, followed by a conclusion. An appendix contains derivations of equations in the main text.

1 The framework

The framework used in this paper is, apart from the wage determination mechanism, the canonical matching model used by Hagedorn and Manovskii (2008) with some minor generalization of inessential restrictions that may not be satisfied by the data and the addition of the possibility of shirking on the job to encompass the efficiency wage model. Denote by J_t the expected present value of current and future profits to a firm at t from having a filled job. This equals output p_t net of wage costs w_t , plus the expected present value of profits J_{t+1} from period $t + 1$ on, discounted by the discount factor δ_t and the probability $(1 - s_{t+1})$ that the relationship is not ended before production at $t + 1$ because the match is destroyed for exogenous reasons, plus the expected payoff

V_{t+1} of going back into the market for another employee if the match is destroyed. Thus

$$J_t = p_t - w_t + \delta_t E_t [(1 - s_{t+1})J_{t+1} + s_{t+1}V_{t+1}], \text{ for all } t, \quad (1)$$

where E_t is the expectation operator conditional on information available at t . (Hagedorn and Manovskii (2008) and Hall and Milgrom (2008) make use of the free entry condition (3) below to eliminate the term in V_{t+1} in this but for our present purpose it is convenient to retain that term. They also assume $s_{t+1} = s$ that is constant for all t and known to both parties but here we allow for separation shocks in view of the importance Mortensen and Nagypál (2007) attribute to these.) The probability of filling a vacancy at t is q_t . In Hagedorn and Manovskii (2008), a match at t results in employment starting at $t + 1$ and thus expected future profit $\delta_t E_t J_{t+1}$. For the empirical work, we are constrained to using quarterly data, for which a one-period delay between matching and employment starting may seem implausibly long. So we instead assume that employment resulting from a match at t starts at t with probability κ and at $t + 1$ with probability $1 - \kappa$. Thus $\kappa = 0$ corresponds to the assumption in Hagedorn and Manovskii (2008). Then, the present discounted value V_t of having a vacancy available for matching at t is

$$V_t = -c_t + q_t [\kappa J_t + (1 - \kappa) \delta_t E_t J_{t+1}] + (1 - q_t) \delta_t E_t V_{t+1}, \text{ for all } t, \quad (2)$$

where c_t is the vacancy posting cost incurred to have the vacancy available for matching in period t . With probability q_t , the vacancy is matched with a worker in period t and yields expected future profit as described above; with probability $1 - q_t$, it is not matched with a worker at t and remains available to be filled in period $t + 1$. Free entry of firms with vacancies implies

$$V_t = 0, \text{ for all } t. \quad (3)$$

For a worker in a match in period t , the expected present value of employment W_t is given by

$$W_t = w_t - e_t + \delta_t E_t [(1 - s_{t+1})W_{t+1} + s_{t+1}U_{t+1}], \text{ for all } t, \quad (4)$$

where e_t is the disutility of effort that can be avoided by shirking on the job, and U_{t+1} is the ex-

pected present value of starting period $t + 1$ unemployed, an event that happens with the probability s_{t+1} that the job comes to an end for exogenous reasons. In the models of Hagedorn and Manovskii (2008) and Hall and Milgrom (2008), $e_t = 0$. The probability that a worker unemployed at t finds a job in the matching process at t is denoted by f_t , the value of non-work activity (including any unemployment benefit) by z_t . Hence, the present discounted value U_t of seeking a match at t is

$$U_t = f_t [\kappa W_t + (1 - \kappa) (z_t + \delta_t E_t W_{t+1})] + (1 - f_t) (z_t + \delta_t E_t U_{t+1}), \text{ for all } t. \quad (5)$$

The right-hand side of (5) can be interpreted as follows. With probability f_t , the worker is hired at t and receives expected future utility $[\kappa W_t + (1 - \kappa) (z_t + \delta_t E_t W_{t+1})]$ from being matched. With probability $1 - f_t$ the worker is not hired at t and receives utility z_t for period t plus the expected utility from starting period $t + 1$ unmatched.

Equations (1)–(5) represent the canonical matching model. Pissarides (2009) has argued for an extension of that model to allow for fixed costs incurred after matching has occurred, such as training, negotiation, or one-off administrative costs. His proposed modification can be expressed in the notation used here by replacing (2) with

$$V_t = -c_t + q_t [-H_t + \kappa J_t + (1 - \kappa) \delta_t E_t J_{t+1}] + (1 - q_t) \delta_t E_t V_{t+1}, \text{ for all } t,$$

where H_t is a fixed cost incurred by the firm after meeting a new worker but before the wage is agreed (compare with Pissarides (2009, p. 1364)). This specification is obviously isomorphic to replacing the vacancy posting cost term c_t in (2) by the composite cost $c_t + q_t H_t$. It thus provides a simple generalization of the canonical model. To economize on notation, we retain the representation (2) and introduce the fixed cost by making c_t an affine function of q_t .

It is useful for later to note that (1) and (2) can be jointly solved forward to write

$$J_t - V_t = E_t \sum_{n=0}^{\infty} \bar{\delta}_{t,n}^{q,\kappa} [c_{t+n} + (1 - q_{t+n} \kappa) (p_{t+n} - w_{t+n})], \quad (6)$$

where $\bar{\delta}_{t,n}^{q,\kappa} = \prod_{i=1}^n \bar{\delta}_{t+i}^{q,\kappa}$, with $\bar{\delta}_{t,0}^{q,\kappa} = 1$, and $\bar{\delta}_t^{q,\kappa} = \delta_{t-1} (1 - s_t - q_{t-1} + \kappa s_t q_{t-1})$. Moreover, (4)

and (5) can be jointly solved forward to write

$$W_t - U_t = E_t \sum_{n=0}^{\infty} \bar{\delta}_{t,n}^{f,\kappa} [(1 - f_{t+n}\kappa)(w_{t+n} - e_{t+n} - z_{t+n})], \quad (7)$$

where $\bar{\delta}_{t,n}^{f,\kappa} = \prod_{i=1}^n \bar{\delta}_{t+i}^{f,\kappa}$, with $\bar{\delta}_{t,0}^{f,\kappa} = 1$, and $\bar{\delta}_t^{f,\kappa} = \delta_{t-1}(1 - s_t - f_{t-1} + \kappa s_t f_{t-1})$.

2 Wage determination

2.1 Nash bargaining

Hagedorn and Manovskii (2008) follow Mortensen and Pissarides (1994) in modelling wage determination by the Nash bargaining solution. Its generalized form satisfies the condition

$$W_t - U_t = \frac{\beta}{1 - \beta} (J_t - V_t), \text{ for } \beta \in [0, 1), \quad (8)$$

where $\beta/(1 - \beta)$ is the bargaining power of workers relative to that of firms. With the free entry condition $V_t = 0$, (8) reduces to the formulation in Hagedorn and Manovskii (2008)

$$W_t - U_t = \frac{\beta}{1 - \beta} J_t, \text{ for } \beta \in [0, 1). \quad (9)$$

2.2 Credible bargaining

Hall and Milgrom (2008) develop an alternative to the standard Nash bargain in which there is positive probability, here denoted α , that negotiations will break down irrevocably each time a new offer is made. They also include a cost to making an offer that we here denote by γ_t^f for each offer the firm makes in period t and γ_t^w for each offer the worker makes in period t . (Hall and Milgrom (2008) have $\gamma_t^w = 0$ but, for reasons that will become apparent, it is useful to generalize this to allow $\gamma_t^w > 0$.) If negotiations break down, the parties search for alternative matches.

In Hall and Milgrom (2008), the parties alternate in making offers, starting with the firm, with only one offer being made each period. Hall and Milgrom (2008), however, envisage each period as corresponding to a day. With the data available, we are constrained to having each period correspond to a quarter, so the assumption of one offer per period may well be implausible. For

this reason, we generalize the model to allow offers to be made at fixed intervals that may be less than a whole period. Consider an offer from the firm at time η ($0 \leq \eta < 1$) between t and $t + 1$ that would yield the worker present value payoff $W_{t+\eta}$. The worker will accept that offer if $W_{t+\eta}$ is at least as great as the payoff from rejecting the offer, having negotiations break down with probability α and receiving payoff $U_{t+\eta}$ of seeking an alternative match, but otherwise incurring the cost γ_t^w to make a counter-offer resulting in present value payoff denoted $\tilde{W}_{t+\eta}$. Recognizing this, the firm will make the lowest offer that satisfies this requirement, resulting in the indifference condition

$$W_{t+\eta} = \alpha U_{t+\eta} + (1 - \alpha) (-\gamma_t^w + \tilde{W}_{t+\eta}), \quad \eta \in [0, 1]. \quad (10)$$

Symmetrically, the firm will accept an offer with present value payoff $J'_{t+\eta}$ made by the worker at time η ($0 \leq \eta < 1$) between t and $t + 1$ if $J'_{t+\eta}$ is at least as great as the payoff from rejecting the offer, having negotiations break down with probability α and receiving payoff $V_{t+\eta}$ of seeking an alternative match, but otherwise incurring the cost γ_t^f to make a counter-offer resulting in present value payoff denoted $\tilde{J}_{t+\eta}$. Recognizing this, the worker will make the lowest offer that satisfies this requirement, resulting in the indifference condition

$$J'_{t+\eta} = \alpha V_{t+\eta} + (1 - \alpha) (-\gamma_t + \tilde{J}_{t+\eta}), \quad \eta \in [0, 1]. \quad (11)$$

In Hall and Milgrom (2008), the firm makes the first offer and in equilibrium that offer is always accepted, so the bargained outcome corresponds to $W_{t+\eta}$ for $\eta = 0$.

In the specification in Hall and Milgrom (2008) with only one offer per period,

$$\tilde{J}_t = \delta_t E_t J_{t+1}; \quad \tilde{W}_t = z_t + \delta_t E_t W'_{t+1}, \quad (12)$$

where W'_{t+1} is the payoff to the worker from making an offer at $t + 1$. In that case, the indifference conditions (10) and (11) with $\eta = 0$ can be solved to give the following sharing rule as an

alternative to (8):

$$\begin{aligned}
W_t - U_t = (1 - \alpha) & \left[(1 - \alpha) \delta_t E_t \gamma_{t+1}^f - \gamma_t^w \right] - (1 - \alpha) \kappa f_t (w_t - e_t - z_t) \\
& + (1 - \alpha) \delta_t E_t \left\{ [(1 - f_t (1 - \kappa s_{t+1})) (W_{t+1} - U_{t+1})] + \alpha (J_{t+1} - V_{t+1}) \right. \\
& \left. + (1 - \alpha) (p_{t+1} - w_{t+1}) - (1 - \alpha) \delta_{t+1} [s_{t+2} (J_{t+2} - V_{t+2})] \right\}. \quad (13)
\end{aligned}$$

The alternative we consider here is to let the time interval between offers go to zero. Then

$$\tilde{J}_t = J_t; \quad \tilde{W}_t = W_t'. \quad (14)$$

In that case, the indifference conditions (10) and (11) with $\eta = 0$ can be solved to give the following sharing rule as an alternative to (8):

$$W_t - U_t = (1 - \alpha) (J_t - V_t) + \frac{(1 - \alpha)}{\alpha} \gamma_t, \quad (15)$$

where $\gamma_t = (1 - \alpha) \gamma_t^f - \gamma_t^w$. Note that γ_t^f and γ_t^w cannot be separately identified from (15). But permitting $\gamma_t^w > 0$ allows the model to be consistent with an estimated $\gamma_t < 0$.

2.3 Efficiency wages

In the development in MacLeod and Malcomson (1998) of the efficiency wage model of Shapiro and Stiglitz (1984), the *no-shirking condition* that must hold for the worker not to shirk on the job at $t - 1$ is

$$\delta_{t-1} E_{t-1} [(1 - s_t) (W_t - U_t)] \geq e_{t-1}, \text{ for all } t. \quad (16)$$

MacLeod and Malcomson (1998) show that any wage sequence that satisfies this condition and also ensures non-negative expected profits for the firm can be supported as an equilibrium. Thus, any sharing rule that divides the surplus in excess of the no-shirking condition specifies an equilibrium. Such a sharing rule acts as an equilibrium selection mechanism in the way described in Hall (2005b). In particular, the sharing rules derived from Nash bargaining and from credible bargaining satisfy this requirement as long as the worker's share satisfies the no-shirking condition (16)

and the firm's share of the surplus is non-negative. These rules can, therefore, be used to nest all the models considered here within a single, more general formulation.

2.4 Nesting wage determination mechanisms

The Nash and credible bargaining outcomes (8) and (15) are special cases of the more general formulation

$$W_t - U_t = \lambda (J_t - V_t) + \frac{\lambda}{1 - \lambda} \gamma_t, \quad (17)$$

with the models satisfying the restrictions

$$\begin{aligned} \text{Nash bargaining (8)} : \lambda &= \frac{\beta}{1 - \beta} \in [0, \infty); \gamma_t = 0; \\ \text{Credible bargaining (15)} : \lambda &= 1 - \alpha \in [0, 1). \end{aligned} \quad (18)$$

The efficiency wage model can also be nested with these in the following way. Multiply both sides of (17) by $\delta_{t-1} (1 - s_t)$ and take expectations at $t - 1$ to get

$$\delta_{t-1} E_{t-1} [(1 - s_t) (W_t - U_t)] = \delta_{t-1} E_{t-1} \left\{ (1 - s_t) \left[\lambda (J_t - V_t) + \frac{\lambda}{1 - \lambda} \gamma_t \right] \right\}.$$

This is nested with the no-shirking condition (16) in the formulation

$$\begin{aligned} \delta_{t-1} E_{t-1} [(1 - s_t) (W_t - U_t)] \\ = e_{t-1} + \delta_{t-1} E_{t-1} \left[\lambda (1 - s_t) (J_t - V_t) + \frac{\lambda}{1 - \lambda} (1 - s_t) \gamma_t \right]. \end{aligned} \quad (19)$$

Restrictions on (19) necessary for the different models are as follows:

$$\begin{aligned} \text{Nash bargaining (8)} : e_{t-1} &= \gamma_t = 0; \lambda = \frac{\beta}{1 - \beta} \in [0, \infty); \\ \text{Credible bargaining (15)} : e_{t-1} &= 0; \lambda = 1 - \alpha \in [0, 1); \\ \text{Efficiency wages (16)} : e_{t-1} &> 0. \end{aligned} \quad (20)$$

Note that the restrictions in (20) for the Nash and credible bargaining models are necessary but not sufficient because those models imply that (17) holds for each t , not just in expectation at $t - 1$ as in (19) with $e_{t-1} = 0$. In addition, for the credible bargaining model, it is also necessary to restrict $\frac{\lambda}{1-\lambda} (1 - s_t) \gamma_t$ to ensure $W_t - U_t$ and $J_t - V_t$ are both non-negative.

3 Empirical wage equations

We derive testable implications of the above models that can be used to compare them and identify the relevant structural parameters. Our objective is to make as few auxiliary assumptions as possible in order to increase the scope of the results. For this reason, we adopt a limited-information perspective that requires minimal assumptions about the underlying data generating process, e.g., stationarity, ergodicity and existence of eighth moments, see Newey and McFadden (1994). The testable implications of the models can be expressed naturally in terms of orthogonality restrictions of the form $E[Z_t' \phi_t(\theta)] = 0$, where $\phi_t(\theta)$ is a parametric function whose expectation conditional on information at time t or $t - 1$ (as appropriate for the particular model) vanishes, and Z_t is an appropriate set of instruments. These over-identifying restrictions are obtained from the assumptions of forward-looking behavior and rational expectations. This approach has a long history in economics since the seminal work of Hansen (1982) and has been used successfully in monetary economics to study forward-looking inflation dynamics and monetary policy, see Galí and Gertler (1999) and Clarida et al. (2000).

3.1 General specification of wage equation

The Nash bargaining, credible bargaining and efficiency wage models can be converted into empirical wage equations by using the other equations of the models to substitute for $W_t - U_t$ and $J_t - V_t$ in (13), (17) and (19) with terms that are empirically observable and by providing empirical specifications for e_t , z_t , c_t and γ_t , which are unobserved. For $W_t - U_t$, (7) is the relevant condition. To account for trends in the data stemming from secular productivity growth, we make the standard assumption that all relevant trending variables grow at the same rate as productivity. For z_t , e_t and

γ_t , natural assumptions are that they are proportional to productivity p_t , so

$$z_t = zp_t; \quad e_t = ep_t; \quad \gamma_t = \gamma p_t, \quad (21)$$

where z , e and γ are non-negative constants. The first of these implies that the value of non-work activity is proportional to productivity. Provided c_t grows at the same rate as productivity, the models are then consistent with the unemployment rate being untrended in the long run. To account for fixed costs H_t of the type introduced by Pissarides (2009), we use

$$c_t = (c + Hq_t) p_t. \quad (22)$$

Thus, we can obtain stationary representations by normalizing all equations by p_t .

There are several ways to substitute for $J_t - V_t$, each of which involve different assumptions. A general specification can be obtained using (6). For (17), this results in the specification:

$$\begin{aligned} E_t \sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{f,\kappa} \left[(1 - f_{t+n}\kappa) \left(\frac{w_{t+n}}{p_{t+n}} - e - z \right) \right] \\ = \lambda E_t \sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{q,\kappa} \left[c + Hq_{t+n} + (1 - q_{t+n}\kappa) \left(1 - \frac{w_{t+n}}{p_{t+n}} \right) \right] + \frac{\lambda}{1 - \lambda} \gamma, \end{aligned} \quad (23)$$

where $\tilde{\delta}_{t,n}^{y,\kappa} = \prod_{i=1}^n \tilde{\delta}_{t+i}^{y,\kappa}$, with $\tilde{\delta}_{t,0}^{y,\kappa} = 1$, $\tilde{\delta}_t^{y,\kappa} = \delta_{t-1} \theta_t (1 - s_t - y_{t-1} + \kappa s_t y_{t-1})$ for $y = f, q$ and $\theta_t \equiv p_t / p_{t-1}$. For (19), it results in

$$\begin{aligned} E_{t-1} \left[\delta_{t-1} (1 - s_t) \theta_t \sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{f,\kappa} (1 - f_{t+n}\kappa) \left(\frac{w_{t+n}}{p_{t+n}} - e - z \right) \right] \\ = e + E_{t-1} \left\{ \lambda \delta_{t-1} (1 - s_t) \theta_t \left[\sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{q,\kappa} \left[c + Hq_{t+n} + (1 - q_{t+n}\kappa) \left(1 - \frac{w_{t+n}}{p_{t+n}} \right) \right] \right. \right. \\ \left. \left. + \frac{\gamma}{1 - \lambda} \right] \right\}. \end{aligned} \quad (24)$$

Note that neither (23) nor (24) makes use of the free entry condition (3). They therefore remain valid even if that condition is misspecified.

For empirical purposes, equations (23) and (24) require truncating the infinite sum of wage

terms on the left-hand side, and of profit terms on the right-hand side, at some horizon k (though, because of the discounting of future terms, the approximation error from truncation can be made arbitrarily small for k sufficiently large).² In our empirical work we use $k = 12$, mainly to keep the sample comparable to previous studies since results are virtually indistinguishable for values of $k > 4$.

3.2 Wage equation with predetermined separation rate

There is an alternative way to substitute for $J_t - V_t$ if we assume, as in both Hagedorn and Manovskii (2008) and Hall and Milgrom (2008), that s_t is known at $t - 1$ and make use of the free entry condition (3). We then obtain the alternative empirical specification of the wage equation (17)

$$(1 - f_t \kappa) \frac{w_t}{p_t} = \frac{z + e + \lambda}{1 + \lambda} (1 - f_t \kappa) + \frac{\lambda}{1 + \lambda} \frac{f_t}{q_t} (c + H q_t) + \frac{\lambda \gamma}{1 - \lambda^2} E_t \left(1 - \tilde{\delta}_{t+1}^{f, \kappa} \right). \quad (25)$$

For the Nash bargaining model, $\lambda = \beta / (1 - \beta)$ and $e = \gamma = 0$, so the above equation can be written

$$\frac{w_t}{p_t} = \beta + (1 - \beta) \frac{z_t}{p_t} + \beta \frac{c_t}{p_t q_t} \frac{f_t}{(1 - f_t \kappa)}. \quad (26)$$

With $\kappa = 0$, this reduces to exactly the wage equation in Hagedorn and Manovskii (2008). Note that under our assumptions for z_t , and c_t in (21) and (22), equation (26) is under-identified because the number of unknown structural parameters exceeds the number of estimable coefficients by one. This occurs because the free entry condition causes some terms that appear in the general formulation (23) to be perfectly collinear. Therefore, we need at least one more moment restriction to achieve identification. This can be obtained from the job creation equation implied by (2) and the free entry condition (3). When s_t is known at $t - 1$, the job creation equation can be written

$$E_t \left[\theta_{t+1} \frac{\frac{(1-s_{t+2})}{q_{t+1}} \frac{c_{t+1}}{p_{t+1}} + (1 - \kappa) \left(1 - \frac{w_{t+1}}{p_{t+1}} \right)}{1 - \kappa s_{t+2}} - \frac{\frac{c_t}{p_t q_t} - \kappa \left(1 - \frac{w_t}{p_t} \right)}{\delta_t (1 - \kappa s_{t+1})} \right] = 0. \quad (27)$$

²This approach has been used, for example, by Rudd and Whelan (2006) for studying the new Keynesian Phillips curve.

3.3 Wage specification

The wage w_t in the analysis above is that for an individual worker-firm match. The macroeconomic data used in empirical matching models for the US is the average wage for the nonfarm business sector, denoted here by w_t^* . There is thus an issue of how to match the theoretical specification to the data.

One assumption used in the literature is that the wage is bargained in every match in every period. Then $w_t = w_t^*$ for all t . This is the specification used by Hagedorn and Manovskii (2008) in their calibration of (26) and (27).

The Nash and credible bargaining models do not, however, require bargaining in every match in every period. Equations (8) and (15) to which they give rise determine only the present discounted values of the payoffs W_t and J_t . There has been considerable discussion in the matching literature about wage rigidity over time within matches — see, for example, Shimer (2004), Hall (2005b), Hall and Milgrom (2008), Gertler and Trigari (2009), Rudanko (2009), Haefke et al. (2008) and Kudlyak (2009). The specifications in (23) and (24) with $w_t = w_t^*$ for all t are actually robust to different time profiles of wages over the duration of a match as long as the present discounted value of payoffs W_t and J_t at the start of a match are the same as if the wage were equal to the average wage over the duration of the match.

One of the concerns in the literature has, however, been that there may be persistent cohort effects in wages. As a result, in matches that start with a low wage because, at the time they are formed, the labour market is slack and the average wage thus low, the wage is below the average wage in some subsequent periods that have a higher average wage but at no time rises above the average wage. The converse applies in matches that start with a high wage because the labour market is tight, and the average wage thus high, at the time they are formed. Such cohort effects can result from the use of contracts that result in staggered wage negotiations, as in Gertler and Trigari (2009). They can also result from contracts to share risk with a risk-averse worker when there is limited commitment as in Thomas and Worrall (1988) and contracts to protect general investments in the presence of turnover frictions as in MacLeod and Malcomson (1993). In Thomas and Worrall (1988), the wage remains constant in real terms until one party would do better by ending the match at that wage, at which point (provided it is efficient to continue the match) the wage is renegotiated

by just enough to prevent a separation. That approach is used in Rudanko (2009). But it depends on workers being risk averse, which is not the assumption in the canonical matching model nor that used here. In MacLeod and Malcomson (1993), the underlying motivation for a contract is not to share risk but to ensure that each party receives the return on its general investments in order to provide efficient incentives for investment. In that model, the wage also remains constant until one party would do better by ending the match at that wage, at which point (again, provided it is efficient to continue the match) the wage is renegotiated by just enough to prevent separation occurring. But, for the purpose of protecting general investments, the constant wage can be in either real or nominal terms. The point is to prevent the wage being renegotiated, which might allow one party to capture part of the return on the other's general investment, except to an outside market level which, as noted by Becker (1975), reflects all the returns to general investments. Wages constant in nominal or real terms are equally good for this purpose and both are consistent with risk-neutral parties. Both, moreover, satisfy the property emphasized by Hall (2005b) that wage rigidity should not, in itself, result in inefficient separations.

The spirit of these forms of cohort effect can be captured using the aggregate data available by the following formulation for a match started at t

$$w_t = w_t^* + \rho \left(w_t^* - \frac{w_{t-1}^*}{p_{t-1}} p_t \right), \quad \rho \geq 0, \quad (28)$$

$$w_{t+n} = (1 - \psi^n) w_{t+n}^* + \psi^n \pi_{t+n}^{-\mu} w_t, \quad \psi, \mu \in [0, 1], n \geq 1, \quad (29)$$

where π_{t+n} is the ratio of prices at $t+n$ to prices at t . The specification in (28) allows the starting wage w_t for new matches at t to differ from the average wage w_t^* because of cohort effects in wages in continuing matches. For $\rho > 0$, the wage in new matches is below the average wage if the average wage is falling relative to productivity and above the average wage if the average wage is rising relative to productivity. In (29), ψ captures the strength of the cohort effect. It can be interpreted as the per period probability that the match wage is not renegotiated to the current market wage. For $\psi = 0$, the wage within each match always equals the current average wage in the market (so $\rho = \psi = 0$ nests the formulation $w_t = w_t^*$ for all t) whereas, for $\psi = 1$, the match wage is unaffected by changes in market conditions once the initial wage has been agreed. For $\mu = 1$, the unrenegotiated wage is set in nominal terms, for $\mu = 0$ in real terms, with $\mu \in (0, 1)$

being interpreted as the proportion of the cohort effect in nominal terms. We use the specifications (28) and (29) in (23) for estimating the bargaining models.

4 Data

We use data on the nonfarm business sector of the USA, obtained mainly from the Bureau of Labor Statistics (BLS) and the OECD. The data are quarterly and cover the postwar period. The estimation sample is 1951q1 to 2004q4.

To construct model-consistent data series, we make use of the following model equations. Employment in period t is the number of jobs in the previous period j_{t-1} that are not destroyed, $(1 - s_t)j_{t-1}$, plus newly matched vacancies m_t , so

$$j_t = (1 - s_t)j_{t-1} + m_t. \quad (30)$$

The stock of vacancies available at the end of period t , after matching takes place, is v_t . Hence, the total number of vacancies available to be filled in period t is $v_t + m_t$. The stock of unemployed workers seeking matches in period t consists of workers who were unemployed in the previous period, $l_{t-1} - j_{t-1}$ (where l_t is the labor force at t), workers who were employed in the previous period but have lost their job, $s_t j_{t-1}$, and new workers, $\Delta l_t = l_t - l_{t-1}$, making $l_t - (1 - s_t)j_{t-1}$ in total. Equivalently, this is given by the stock of unemployed workers at the end of the period plus the total matches during the period, $u_t + m_t$. Thus, the job-filling probability for firms in period t is given by

$$q_t = \frac{m_t}{v_t + m_t} \quad (31)$$

and the job-finding probability for workers by

$$f_t = \frac{m_t}{u_t + m_t}. \quad (32)$$

Employment j_t and unemployment u_t are constructed and seasonally adjusted by the BLS from the CPS, and they correspond to the last month in the quarter in accordance with the model used here. Employment consists of total nonfarm dependent employment (excluding the self-

employed). The labour force is measured as the sum of unemployment and employment.

We adopt the time-honoured practice discussed by Blanchard and Diamond (1990) of constructing a series for job destructions from the number of short-term unemployed, u_t^s , in our case (because we are using quarterly data) those with spells shorter than 14 weeks. Moreover, if the increase in the labour force all goes through the unemployment pool first, this increase should be subtracted from the short-term unemployed before calculating the job destruction rate. We adjusted the data for this, though the effect on the calculated series for the separation rate s_t is very small. We also adjusted for direct job-to-job flows using the procedure suggested in Shimer (2005) based on the idea that, on average, a worker losing a job has half a period to find a new one before being recorded as unemployed. Thus, short-term unemployment satisfies

$$u_t^s = \left(1 - \frac{1}{2}f_t\right) (\Delta l_t + s_t j_{t-1}).$$

Use of (30) and (32) to express f_t as $[j_t - (1 - s_t)j_{t-1}] / [l_t - (1 - s_t)j_{t-1}]$ enables us to solve for a series for s_t that is consistent with the model.³ The resulting series is plotted in Figure 1. This series is higher than the monthly separation rate series reported elsewhere (e.g., Shimer (2005, Figure 7)), but it matches the cyclical pattern of the (monthly) series exactly. It illustrates the importance of allowing for separation shocks, as emphasized by Mortensen and Nagypál (2007).

Vacancy stocks v_t are measured using the Conference Board Help-Wanted Index (HWI), which is available in quarterly frequency since 1951. The index is converted to total units using the job-openings series from the Job Openings and Labor Turnover Survey (JOLTS), which is available only since December 2000. The HWI series is known to contain low frequency fluctuations, such as those resulting from newspaper consolidation in the 1960s and the internet revolution recently, that are unrelated to labor market trends, see Shimer (2005). Following Shimer (2005), we remove the effect of those trends using a low frequency filter. The job-filling probability q_t is then calculated using data on employment and vacancies via equations (30) and (31). The resulting series is plotted in Figure 2. We also plot on the same graph the corresponding series for v_t derived using the JOLTS data over the period (2001 on) for which it is available. This shows that the two series match very closely (their correlation is 0.9).

³Even with the adjustment suggested by Shimer (2005), the measure of separations does not include workers moving directly from jobs to self-employment or leaving the labour force but it is not clear how to allow for that.

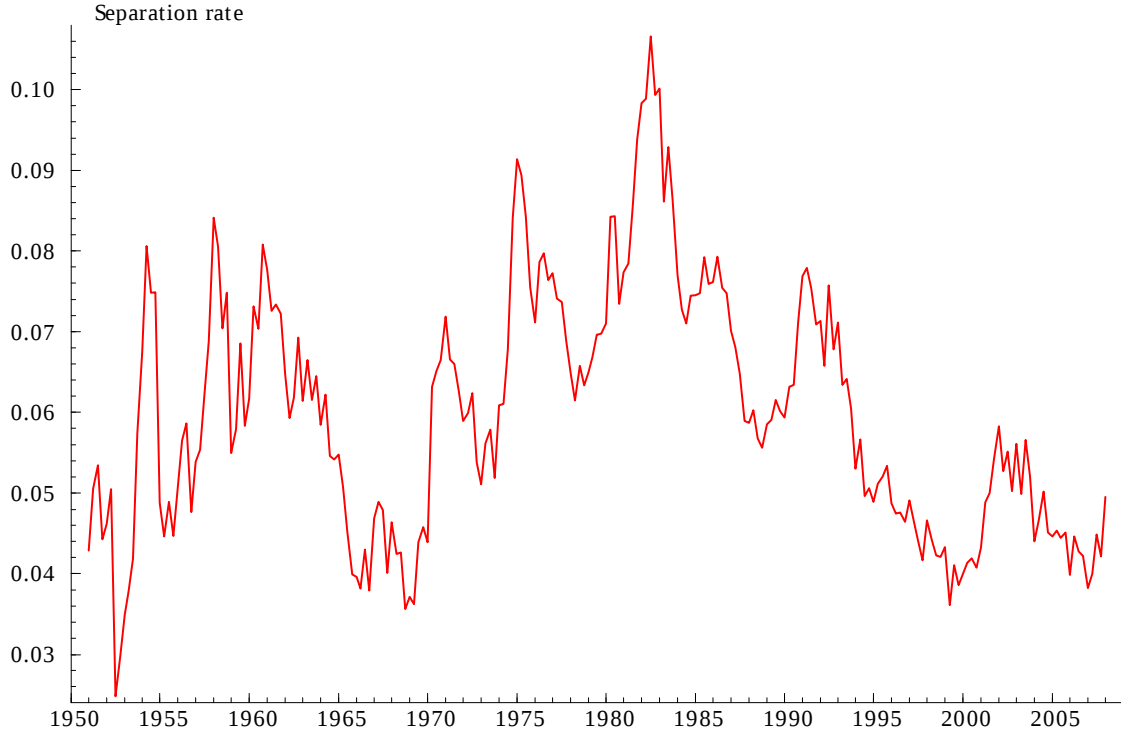


Figure 1: The separation rate s_t computed from $u_t^s = \left(1 - \frac{1}{2} \frac{j_t - (1-s_t)j_{t-1}}{l_t - (1-s_t)l_{t-1}}\right) (\Delta l_t + s_t l_{t-1})$ using employment, unemployment and short-term unemployment data from the BLS.

Productivity and wages are obtained from the BLS, which provides a measure of the labor share and output per person in the nonfarm business sector in index form. We scale the former using data from the OECD to obtain the wage share in levels. We also adjust for the difference between average and marginal productivity, as well as for sales taxes, using the scaling computed by Hagedorn and Manovskii (2008).⁴ Because we use quarterly data, we specify the discount factor as $\delta_t = \frac{1}{1+r_t/4}$, where r_t is the *annualized gross* real interest rate, deflated using the implicit price deflator for nonfarm business obtained from the BLS.

⁴The ratio of marginal to average productivity is 0.679, and effective average sales taxes are 5.1%, yielding an extra normalization of 1/0.949.

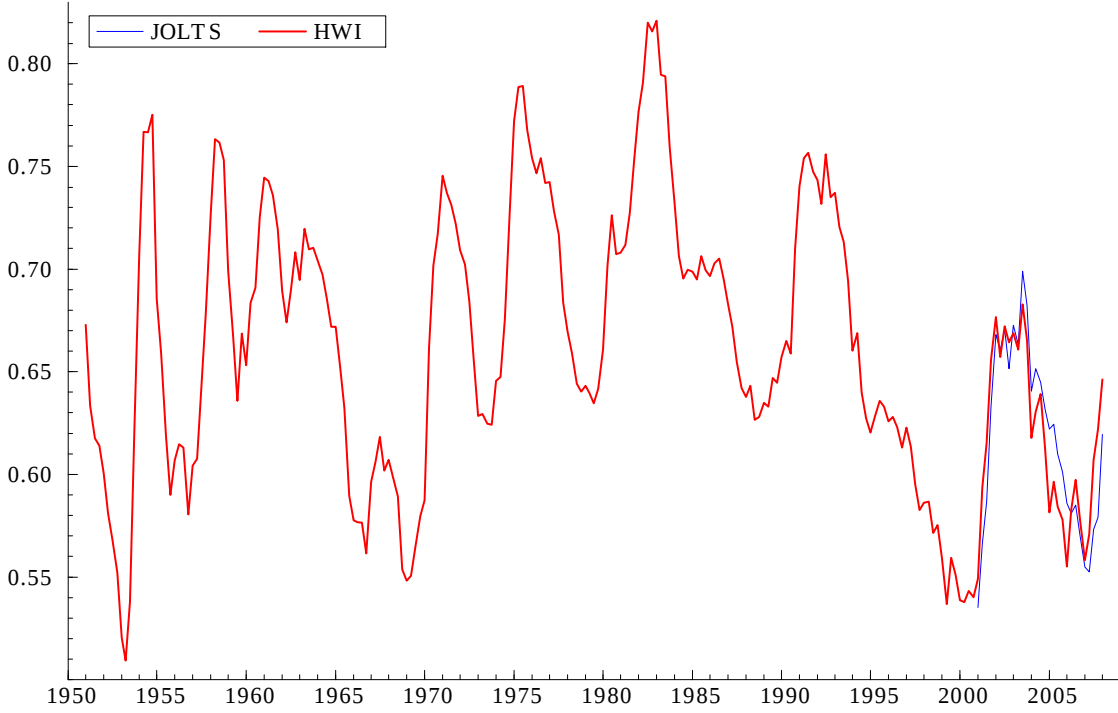


Figure 2: The job-filling probability q_t using employment data from the BLS and vacancy data from the Conference Board HWI and from JOLTS.

5 Estimation results

Combined with rational expectations, the empirical models in Section 3 imply over-identifying restrictions on moments of the data that can be used for estimation and testing. Together with the bargaining models, for example, rational expectations imply that the realized values of variables in the data should satisfy equation (23) with errors that are uncorrelated with any vector Z_t of variables that are in the information set at t . This restricts the (vector) covariance terms $cov\left\{\sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{f,\kappa} \left[(1 - f_{t+n}\kappa) \frac{w_{t+n}}{p_{t+n}}\right], Z_t\right\}$, $cov\left\{\sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{f,\kappa} (1 - f_{t+n}\kappa), Z_t\right\}$, $cov\left\{\sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{q,\kappa}, Z_t\right\}$, $cov\left\{\sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{q,\kappa} q_{t+n}, Z_t\right\}$ and $cov\left\{\sum_{n=0}^{\infty} \tilde{\delta}_{t,n}^{q,\kappa} \left[(1 - q_{t+n}\kappa) \left(1 - \frac{w_{t+n}}{p_{t+n}}\right)\right], Z_t\right\}$ to be (nonlinearly) related. A natural choice of instruments Z_t consists of lags of the variables in the model, w_t/p_t , q_t , f_t and s_t . This procedure yields restrictions on the high-order autocovariances of the underlying stochastic process. This identification strategy does not, unlike calibration, require further restrictions on the underlying stochastic process (the law of motion of the exogenous variables p_t, s_t and

δ_t) and so is robust to possible misspecification of such additional restrictions.⁵

Estimation is performed in the framework of the Generalized Method of Moments (GMM) using methods robust to weak instruments. These are the continuously updated estimator (CUE) of Hansen et al. (1996), which is median-unbiased when instruments are moderately weak, see Newey and Smith (2004), and the GMM-AR test statistic of Stock and Wright (2000), which is fully robust to weak instruments.⁶ Given the quarterly frequency of the sample, we use four lags of the data as instruments, starting at t for equation (23) and $t - 1$ for (24). For the estimation, parameter values are constrained to be consistent with theory, specifically $c, \kappa, \psi, \mu \in [0, 1]$ and $\lambda, H, \rho \geq 0$.

5.1 Nash bargaining, credible bargaining and free entry

We start by presenting estimates for the Nash bargaining formulation (26) derived using the free entry condition (3) with $w_t = w_t^*$ for all t , as in Hagedorn and Manovskii (2008). Table 1 reports the results of estimation based on equation (26), together with the job creation equation (27) that, as discussed above, is used to identify the parameters for this formulation.

Column (1) of Table 1 reports estimates for the specification closest to that in Hagedorn and Manovskii (2008) with κ , the fraction of newly matched jobs that become active within the quarter, set to 0. The parameter estimates are not significantly different from the calibrated values in Hagedorn and Manovskii (2008), $\beta = 0.05$ and $z = 0.95$, but the fit of the model is very poor — the over-identifying restrictions are overwhelmingly rejected by the Hansen test. Column (2) reports estimates with κ restricted only to $[0, 1]$. The point estimate of κ is 1, indicating that our modification of the timing in their formulation is appropriate for application to quarterly data, though the improvement in fit, as measured by the value of the GMM objective function, is actually small. Indeed, in this specification the point estimates of β and z are almost identical to the calibrated values in Hagedorn and Manovskii (2008) — reassuring evidence that, when applied to the same formulation, our estimation procedures yield essentially the same results as their calibration procedures. But the over-identifying restrictions for this formulation are also overwhelmingly rejected

⁵One assumption used in calibration of matching models that seems particularly problematic in view of the recent literature on the “great moderation” is that volatilities are constant throughout the sample period.

⁶Stock et al. (2002), Dufour (2003) and Andrews and Stock (2005) are excellent surveys of methods robust to weak instruments.

Parameter	(1)	(2)	(3)
β	0.122 (0.033)	0.050 (0.016)	0.304 (0.120)
z	0.933 (0.012)	0.944 (0.016)	0.812 (0.084)
c	0.327 (0.026)	0.321 (0.030)	0.130 (0.094)
H	—	—	0.175 (0.160)
κ	—	1.000 (0.136)	1.000 (0.180)
GMM objective	63.094	60.948	47.356
Hansen test p value	0.000	0.000	0.000

Table 1: Estimates of the Hagedorn and Manovskii (2008) model, eq. (26) with $w_t = w_t^*$ for all t . β is the workers’ bargaining weight, z the value of non-work activity, c the vacancy posting cost, H the Pissarides (2009) fixed cost parameter and κ the fraction of matched jobs that become active within the quarter. Estimation method is CUE-GMM with Newey-West weight matrix with prewhitening over the sample 1951q1-2004q4, with a constant and four lags of w/p , f , q and s as instruments. Standard errors in parentheses.

by the Hansen test.

Column (3) of Table 1 reports estimates allowing for the fixed cost H suggested by Pissarides (2009). The fit of the model improves substantially and the parameter estimates change in a direction consistent with the predictions of Pissarides (2009). But the standard errors are also much larger and, most importantly, the over-identifying restrictions of the model are still overwhelmingly rejected by the Hansen test.

An alternative to Nash bargaining is the credible bargaining developed by Hall and Milgrom (2008). Equation (25) with $e = 0$, γ unrestricted and $\lambda \in [0, 1]$ corresponds to a plausible formulation of credible bargaining applied to quarterly data when, as in Hall and Milgrom (2008), the free entry condition (3) is imposed. We estimated this specification jointly with the job creation equation (27) under the assumption $w_t = w_t^*$ for all t . Allowing γ to be non-zero, however, makes little difference to the fit, as measured by the value of the GMM objective function, and leaves the over-identifying restrictions rejected just as overwhelmingly by the Hansen test.

All in all, the parameter estimates in these results are consistent with the values reported in the literature using calibration, which is reassuring evidence of the validity of our procedures, but the rejection of the over-identifying restrictions by the Hansen test is strong evidence of misspecification. Because, however, the estimates rely on the job creation equation (27) implied by the free

Parameter	(1)	(2)	(3)
c	0.359 (0.039)	0.097 (0.028)	0.655 (0.184)
H	-	0.307 (0.053)	0.000 (0.195)
κ	0.000 (0.240)	0.000 (0.196)	1.000 (0.120)
ψ	-	-	1.000 (1.550)
μ	-	-	1.000 (5.530)
ρ	-	-	0.000 (0.186)
GMM objective	55.676	52.154	44.803
Hansen test p value	0.000	0.000	0.000

Table 2: Estimates of the job creation equation implied by the free entry condition. Columns (1) and (2) correspond to equation (27) with $w_t = w_t^*$ for all t , column (3) corresponds to equation (33) with w_t specified by (29). c is the vacancy posting cost, H the Pissarides (2009) fixed cost parameter, κ the fraction of matched jobs that become active within the quarter, ψ measures the strength of cohort effects, μ the proportion of cohort effects in nominal terms and ρ the deviation of the starting wage from the average wage. Estimation method is CUE-GMM with Newey-West weight matrix with prewhitening over the sample 1951q1-2004q4, with a constant and lags 0 to 3 of w/p , f , q and s as instruments. Standard errors in parentheses.

entry condition (3), the misspecification might be in that condition rather than because the Nash and credible bargaining models of wage determination are themselves inadequate. We investigate that next.

5.2 The job creation equation

We test the specification of the job creation equation through the testable implications of (1) and (2) combined with (3). Under the assumption used in Hagedorn and Manovskii (2008) that s_t is known at $t - 1$ and $w_t = w_t^*$ for all t , these equations imply the job-creation equation (27). Columns (1) and (2) in Table 2 report estimates of this specification, without and with the fixed cost parameter H , respectively. Although $H > 0$ improves fit, both specifications are overwhelmingly rejected by the Hansen test.

In addition to the free entry condition, equation (27) also requires the assumption that s_t is known in period $t - 1$ for all t and the specification does not allow for cohort effects in wages. Thus, the tests reported in columns (1) and (2) of Table 2 are joint tests of the validity of these assumptions. To check whether the additional assumptions are the cause of misspecification, we

estimate an alternative version of the job creation equation that does not impose these restrictions. With s_t not predetermined, the orthogonality conditions can be written

$$E_t \sum_{n=1}^{\infty} \left[\kappa \tilde{\delta}_{t,n}^{q,\kappa} + (1 - \kappa) \delta_t \theta_{t+1} \tilde{\delta}_{t+1,n-1}^{q,\kappa} \right] \left[\frac{c_{t+n}}{p_{t+n}} + (1 - q_{t+n} \kappa) \left(1 - \frac{w_{t+n}}{p_{t+n}} \right) \right] + (1 - q_t \kappa) \left[\kappa \left(1 - \frac{w_t}{p_t} \right) - \frac{1}{q_t} \frac{c_t}{p_t} \right] = 0. \quad (33)$$

Estimates of the specification in (33) with w_t specified by (28) and (29) in order to allow for cohort effects are reported in column (3) of Table 2. The fit of the model is somewhat better than before but the Hansen test still gives strong evidence of misspecification.

These results indicate that the rejection of the over-identifying restrictions in the estimates of the wage equation (26) may result from the job creation equation (on which they depend) being misspecified, rather than from the Nash and/or credible bargaining models themselves being misspecified. For this reason, the subsequent analysis investigates versions of the wage equation that do not depend on the specification of the job creation equation. But the results of this subsection point to an important implication for future research: the job creation equation implied by the canonical version of the matching model appears to be misspecified when applied to US data. It is not difficult to think of alternative specifications that are fully consistent with the underlying matching framework, so this conclusion should not be seen as a rejection of the matching approach. But further thought needs to be given to how the job creation equation is specified in matching models.

5.3 Wage bargains independent of the job creation equation

The wage equation in (23) was derived without the use of the free entry condition (3). Moreover, as long as the free entry condition is not imposed, this model does not suffer from the identification problem of (25).⁷ It can, therefore, be used to test the Nash and credible bargaining assumptions independently of the specification of the job creation equation. Nash bargaining corresponds to the special case of (23) with $\gamma = e = 0$ and $\lambda = \beta / (1 - \beta)$. The results from estimating that specification, with w_t specified by (28) and (29) to allow for cohort effects, are given in columns

⁷Formally, the free entry condition requires some terms that appear in the general form of the wage equation to be precisely zero, thus preventing them from playing a role in identifying parameters.

Parameter	(1)	(2)	(3)
λ	0.122 (0.572)	0.098 (1.588)	0.134 (1.838)
z	0.792 (0.029)	0.798 (0.040)	0.804 (0.042)
c	0.812 (3.951)	0.895 (16.66)	0.884 (14.19)
H	—	0.050 (0.365)	0.000 (1.684)
κ	1.000 (0.218)	1.000 (0.995)	1.000 (1.129)
ψ	0.000 (1.621)	0.000 (1.942)	0.000 (2.484)
μ	— (∞)	— (∞)	— (∞)
ρ	0.183 (2.076)	0.265 (3.327)	0.000 (1.876)
γ	—	—	-0.277 (5.269)
GMM objective	47.932	47.843	47.589
Hansen test p value	0.000	0.000	0.000

Table 3: Estimates of the general sharing rule that nests both Nash bargaining and credible bargaining, eq. (23) with w_t specified by (29). λ determines surplus sharing proportions, z is the value of non-work activity, c is the vacancy posting cost, H is the Pissarides (2009) fixed cost parameter, κ is the fraction of matched jobs that become active within the quarter, ψ measures the strength of cohort effects, μ the proportion of cohort effects in nominal terms and ρ the deviation of the starting wage from the average wage. Estimation method is CUE-GMM with Newey-West weight matrix with prewhitening over the sample 1951q1-2004q4, with a constant and four lags of w/p , f , q and s as instruments. Standard errors in parentheses.

(1) and (2) of Table 3, with the fixed cost H being zero and allowed to be different from zero, respectively. The estimate of the value of non-work activity z is now around 0.8, significantly below the calibration of 0.95 in Hagedorn and Manovskii (2008) but still higher than Mortensen and Nagypál (2007) and Hall and Milgrom (2008) regard as plausible. With ψ estimated at zero and ρ not significantly different from zero, cohort effects do not appear to play an important role. (Note that μ is not identified when $\psi = 0$.) But it remains the case that the over-identifying restrictions are overwhelmingly rejected by the Hansen test. Since these results do not depend on the validity of the free entry condition, this is strong evidence that the Nash bargaining specification is not appropriate for US data. It really does seem that the Nash bargaining approach itself, even with a high value of non-work activity, is not a satisfactory specification for application to US data.

Equation (23) with γ unrestricted, $e = 0$ and $\lambda \in [0, 1]$ corresponds to one formulation of credible bargaining applied to quarterly data. Estimates of this specification with w_t given by (29) are reported in column (3) of Table 3. It is not, however, statistically satisfactory either. The coeffi-

cient γ is not significantly different from zero and the fit of the model (as shown by the value of the objective function) improves only marginally relative to the Nash bargaining specification. Moreover, the over-identifying restrictions are still overwhelmingly rejected by the Hansen test. The parameter estimates are somewhat different from the values reported for the calibration in Hall and Milgrom (2008) but the standard errors are sufficiently large to cover those reported values. Again, given the overwhelming evidence of misspecification, one should not read too much into these estimates.

To check whether this misspecification results purely from the specific formulation of the credible bargaining model in (15), we also estimated the alternative formulation in (13). That, however, fared no better empirically. It fitted the data somewhat less well in that the value of the GMM objective function was higher. Moreover, the over-identifying restrictions were just as overwhelmingly rejected by the Hansen test. On this evidence, the credible bargaining does not look a promising alternative to Nash bargaining for application to US data.

5.4 Efficiency wages

The alternative to Nash and credible bargaining we consider is based on the development in MacLeod and Malcomson (1998) of the efficiency wage model of Shapiro and Stiglitz (1984). This is nested with the Nash and credible bargaining models in equation (24). The results of estimating (24) with $w_t = w_t^*$ for all t are reported in Table 4. Estimates with unrestricted H are omitted from the table because in all specifications this parameter is estimated at zero. Column (1) reports estimates of the model that are unrestricted in other respects. The fit of this model is dramatically better than in all the previous cases considered. It is the first model considered here for which the over-identifying restrictions are not rejected by the Hansen test at the 5% level of significance. Moreover, the parameter e (the disutility of effort avoided by shirking) that crucially distinguishes the efficiency wage model from the Nash and credible bargaining models is significantly different from zero at the 5% level by the conventional t test. It is also significantly different from zero at the 5% level using the GMM-AR test that is robust to weak identification. As a final test of the importance of this parameter for a satisfactory specification, we estimated (24) under the restriction $e = 0$ and under the restrictions $e = \gamma = 0$. These can be thought of as weak tests of the credible

Parameter	(1)	(2)	(3)	(4)
λ	0.123 (0.379)	0.238 (0.349)	0.162 (0.857)	-
z	0.289 (0.254)	0.642 (0.086)	0.830 (0.025)	0.437 (0.186)
c	0.000 (0.267)	0.000 (0.137)	0.474 (2.673)	-
κ	0.000 (0.405)	0.000 (0.373)	1.000 (0.403)	0.428 (0.262)
γ	1.129 (4.278)	1.850 (3.459)	-	-
e	0.369 (0.176)	-	-	0.305 (0.125)
GMM objective	19.078	24.572	46.421	19.658
Hansen test p value	0.089	0.039	0.000	0.141

Table 4: Estimates of a specification nesting Nash and credible bargaining with efficiency wages. eq. (24). λ determines surplus sharing proportions, z is the value of non-work activity, e is the disutility of non-shirking effort, c is the vacancy posting cost and κ is the fraction of matched jobs that become active within the quarter. Estimation method is CUE-GMM with Newey-West weight matrix with prewhitening over the sample 1951q1-2004q4, with a constant and four lags of w/p , f , q and s as instruments. Standard errors in parentheses.

and Nash bargaining specifications, as discussed in Section 2.4. Columns (2) and (3) of Table 4 report the results for $w_t = w_t^*$ for all t . The credible bargaining specification (with γ unrestricted) fits the data better than the Nash bargaining specification. But in both cases, setting $e = 0$ results in the over-identifying restrictions being rejected by the Hansen test. We also estimated (24) with $e = 0$ and w_t specified by (28) and (29) but the estimate of ψ is zero, so the results are the same as in columns (2) and (3) of Table 4. We conclude that $e > 0$, and thus the efficiency wage element, really is essential to enable the over-arching formulation (24) to satisfactorily fit US data.

Apart from e , the parameters in column (1) of Table 4 are not precisely determined. Moreover c , the vacancy posting cost incurred to have a vacancy available for matching, has a point estimate of zero, a value inconsistent with the assumption in the canonical matching model that it is this cost which limits the number of vacancies. However, it is apparent from (24) that neither c nor γ are identified if $\lambda = 0$ and the estimate of λ in column (1) of Table 4 is certainly not significantly different from zero. Moreover, when estimated with γ restricted to zero, the point estimate of λ is precisely zero. Having $\lambda = 0$ with $e > 0$ corresponds to (16) holding with equality at all dates, that is, having the wage actually on, rather than above, the no-shirking condition. The final column of Table 4 reports estimates of (24) with the restriction $\lambda = 0$. The fit of the model deteriorates only slightly relative to the specification in column (1), which is consistent with the

finding that λ is not significantly different from zero. Moreover, the p value of the Hansen test is now higher because of the degrees of freedom adjustment. With this specification, the parameters z and e are both significantly different from zero according to the individual t tests. Indeed, using the identification-robust GMM-AR test, they are both significantly different from zero at the 1% level. The point estimates imply that the disutility of effort avoided by shirking amounts to about 30% of a worker's output, whereas the value of non-work activity amounts to only about 44%, far lower than the 95% in the calibrated model of Hagedorn and Manovskii (2008) and the 71% in the calibrated model of Hall and Milgrom (2008).

In the efficiency wage model, the opportunity cost to taking a job and not shirking is $z + e$, not just z as in the Nash and credible bargaining models. Thus, to get an appropriate comparison with the bargaining models, it is instructive to look at a joint confidence set on the parameters z and e . Figure 3 reports a joint identification-robust confidence set for these parameters at the 90%-level (because there are two parameters), obtained by inverting a joint GMM-AR test on (e, z) .⁸ Interestingly, even though the confidence intervals for z and e are both fairly wide, the joint confidence set is very tight in the direction $z + e$. Moreover, $z + e$ is not significantly different from the calibrated value of 0.71 for non-work activity in Hall and Milgrom (2008), although it is still significantly lower than the calibrated value of 0.95 reported by Hagedorn and Manovskii (2008). This suggests that the high calibrated value for non-work activity in the bargaining models may arise because fitting the data requires a high opportunity cost to not working and in those models all of that opportunity cost is attributed to the value of non-work activity. In contrast, the efficiency wage model does not attribute all the opportunity cost of working to the value of non-work activity because part of the opportunity cost is accounted for by the disutility that can be avoided by shirking on the job.

Taken together, the results show clear evidence to support the hypothesis that $e > 0$. When $e > 0$ is included in the specification, the over-identifying restrictions are not rejected by the Hansen test whereas we have found no specification with $e = 0$ for which that is the case. In addition, it seems that the data are consistent with an equilibrium path of wages that satisfies the no shirking condition with equality at all dates. However, given the uncertainty associated with the estimates of λ , the

⁸Inverting a test means collecting all parameter values that are not rejected by the test at the specified level of significance.

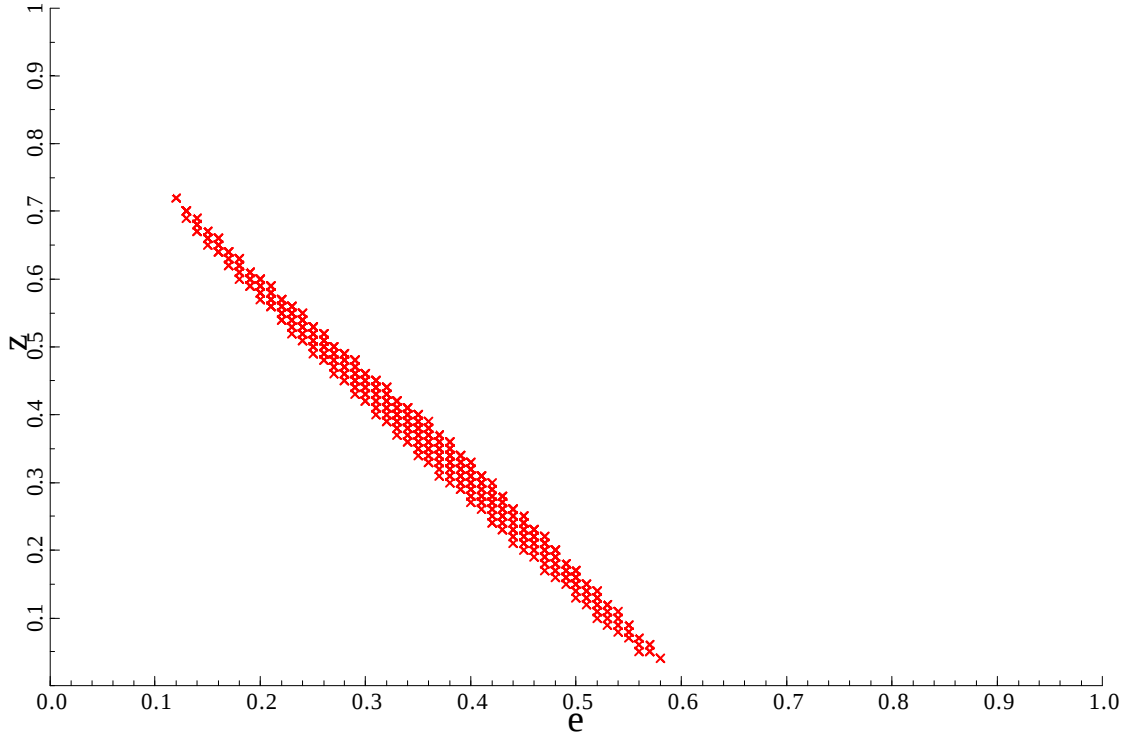


Figure 3: Identification robust 90% confidence set for the parameters e and z in column (4) of Table 4, obtained by inverting the GMM-AR test of Stock and Wright (2000).

results do not rule out wages considerably higher than the minimum necessary to deter shirking.

6 Conclusion

In this paper, we have investigated econometrically appropriate wage determination mechanisms for a matching model of the US by nesting the Nash bargaining model, the credible bargaining model of Hall and Milgrom (2008), and the development in MacLeod and Malcomson (1998) of the efficiency wage model of Shapiro and Stiglitz (1984) within a common over-arching framework of which each is a special case. That has enabled us to apply standard statistical tests to investigate which models are statistically acceptable restrictions of the over-arching framework. Our conclusion from these tests is that only for the efficiency wage model are the restrictions statistically acceptable. Moreover, the data are not inconsistent with the wage being on the no-shirking condition, that is, at the lowest level necessary to deter shirking, though it is not possible to reject wages

higher than this. Furthermore, the results indicate the importance of shocks to the separation rate emphasized by Mortensen and Nagypál (2007).

The efficiency wage formulation addresses a puzzle in the literature. One common finding is that the value of non-work activity needs to be implausibly high to enable the Nash bargaining model to fit the data. In the efficiency wage model, the opportunity cost to taking a job consists of not only the value of non-work activity but also the disutility of incurring the effort not to shirk. The efficiency wage formulation is thus able to generate the high opportunity cost to working needed to match wages with a lower value of non-work activity. Consistent with this, the sum of our estimates of the value of non-work activity and the disutility of effort is actually remarkably close to the calibrated value of non-work activity in Hall and Milgrom (2008). But the efficiency wage formulation changes more than just that — if it did not, it would affect only the interpretation of the results, not fit the data better. An essential characteristic of the efficiency wage formulation is that, to satisfy the no-shirking condition (16) at $t - 1$, wages from t on must give the worker sufficient gain from continued employment. Thus the wage at t must satisfy conditions that involve variables at $t - 1$. That is apparent in the wage equation (24) from the term in δ_{t-1} and the fact that $\theta_t \equiv p_t/p_{t-1}$. This dynamic element is not captured by the Nash and credible bargaining models but seems crucial for fitting US data.

The Nash and credible bargaining models are, of course, only two of the potentially many possible bargaining models. A natural question is whether there might be some alternative bargaining model, perhaps with varying relative bargaining powers, that would result in a bargaining outcome capturing this characteristic of the efficiency wage model and thus be able to fit the US data satisfactorily without an efficiency wage element. We do not have a definitive answer to that. However, we rather doubt it for the following reason. The outcome of a bargaining game to determine the wage at t would typically be expected to depend only on variables dated t and later, as in the wage equation (23). Thus it is hard to see how a bargaining game could capture the dynamic element introduced by the efficiency wage formulation any better than the Nash and credible bargaining models.

Since our estimate of the opportunity cost to working in the efficiency wage formulation is similar to that in Hall and Milgrom (2008), one might expect many of the non-wage characteristics of the calibration in Hall and Milgrom (2008) to apply to that formulation. It would, however, be

unwise to jump to a conclusion on this, for two reasons. First, the additional dynamic element introduced by the efficiency wage formulation may have an impact on non-wage characteristics. Second, a by-product of our analysis has been to show that the job creation equation implied by the canonical version of the matching model appears to be misspecified for application to US data. In our analysis, we have been careful to base our conclusions only on estimates that do not rely on the specification of the job creation equation, so this issue does not bias our statistical comparisons of wage determination mechanisms — these should hold whatever the specification of the job creation equation. Moreover, as noted above, it is not difficult to think of alternative specifications of the job creation equation that are fully consistent with the underlying matching framework, so misspecification of the canonical formulation should certainly not be interpreted as a rejection of the matching approach. But it does indicate that further thought needs to be given to how the job creation equation is specified in matching models to be applied to US data before the implications for non-wage characteristics can be determined.

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A Mathematical Appendix

A.1 Derivation of equation (6)

From (1) and (2),

$$\begin{aligned}
& J_t - V_t \\
&= (1 - q_t \kappa) J_t + c_t - [q_t (1 - \kappa) \delta_t E_t J_{t+1} + (1 - q_t) E_t (\delta_t V_{t+1})] \\
&= (1 - q_t \kappa) [p_t - w_t + \delta_t E_t ((1 - s_{t+1}) J_{t+1} + s_{t+1} V_{t+1})] \\
&\quad + c_t - [q_t (1 - \kappa) \delta_t E_t J_{t+1} + (1 - q_t) E_t (\delta_t V_{t+1})] \\
&= (1 - q_t \kappa) (p_t - w_t) + c_t \\
&\quad + \delta_t E_t \{ (1 - q_t \kappa) [(1 - s_{t+1}) J_{t+1} + s_{t+1} V_{t+1}] - [q_t (1 - \kappa) J_{t+1} + (1 - q_t) V_{t+1}] \} \\
&= (1 - q_t \kappa) (p_t - w_t) + c_t \\
&\quad + \delta_t E_t \{ [(1 - q_t \kappa) (1 - s_{t+1}) - q_t (1 - \kappa)] J_{t+1} + [(1 - q_t \kappa) s_{t+1} - (1 - q_t)] V_{t+1} \} \\
&= (1 - q_t \kappa) (p_t - w_t) + c_t \\
&\quad + \delta_t E_t \{ [1 - s_{t+1} - q_t \kappa (1 - s_{t+1}) - q_t + q_t \kappa] J_{t+1} \\
&\quad + [s_{t+1} - q_t \kappa + q_t \kappa (1 - s_{t+1}) - 1 + q_t] V_{t+1} \} \\
&= (1 - q_t \kappa) (p_t - w_t) + c_t + \delta_t E_t [(1 - s_{t+1} - q_t + \kappa q_t s_{t+1}) (J_{t+1} - V_{t+1})]. \tag{34}
\end{aligned}$$

This can be solved forward to give (6), defining $\delta_t^{q, \kappa} = \delta_{t-1} (1 - s_t - q_{t-1} + \kappa s_t q_{t-1})$.

A.2 Derivation of equation (7)

From (4) and (5),

$$\begin{aligned}
& W_t - U_t \\
&= (1 - f_t \kappa) W_t - [f_t (1 - \kappa) (z_t + \delta_t E_t W_{t+1}) + (1 - f_t) (z_t + \delta_t E_t U_{t+1})] \\
&= (1 - f_t \kappa) \{w_t - e_t + \delta_t E_t [(1 - s_{t+1}) W_{t+1} + s_{t+1} U_{t+1}]\} \\
&\quad - [f_t (1 - \kappa) (z_t + \delta_t E_t W_{t+1}) + (1 - f_t) (z_t + \delta_t E_t U_{t+1})] \\
&= (1 - f_t \kappa) (w_t - e_t - z_t) \\
&\quad + \delta_t E_t \{(1 - f_t \kappa) [(1 - s_{t+1}) W_{t+1} + s_{t+1} U_{t+1}] - [f_t (1 - \kappa) W_{t+1} + (1 - f_t) U_{t+1}]\} \\
&= (1 - f_t \kappa) (w_t - e_t - z_t) \\
&\quad + \delta_t E_t \{[(1 - f_t \kappa) (1 - s_{t+1}) - f_t (1 - \kappa)] W_{t+1} + [(1 - f_t \kappa) s_{t+1} - (1 - f_t)] U_{t+1}\} \\
&= (1 - f_t \kappa) (w_t - e_t - z_t) \\
&\quad + \delta_t E_t \{[1 - s_{t+1} - f_t \kappa (1 - s_{t+1}) - f_t + f_t \kappa] W_{t+1} \\
&\quad + [s_{t+1} - f_t \kappa + f_t \kappa (1 - s_{t+1}) - 1 + f_t] U_{t+1}\} \\
&= (1 - f_t \kappa) (w_t - e_t - z_t) + \delta_t E_t [(1 - s_{t+1} - f_t + \kappa f_t s_{t+1}) (W_{t+1} - U_{t+1})]. \tag{35}
\end{aligned}$$

This can be solved forward to give (7), defining $\delta_t^{f, \kappa} = \delta_{t-1} (1 - s_t - f_{t-1} + \kappa s_t f_{t-1})$.

A.3 Derivation of equation (13)

With the specification in (12), (10) and (11) with $\eta = 0$ become

$$W_t = \alpha U_t + (1 - \alpha) (-\gamma_t^w + z_t + \delta_t E_t W'_{t+1}) \tag{36}$$

$$J'_t = \alpha V_t + (1 - \alpha) (-\gamma_t^f + \delta_t E_t J_{t+1}). \tag{37}$$

The terms in W'_{t+1} and J'_t in (36) and (37) can be eliminated in the following way. Forward (37) one period, take expectations at t , multiply it by $(1 - \alpha) \delta_t$ and subtract from (36) to get

$$W_t - (1 - \alpha) \delta_t E_t J'_{t+1} = \alpha U_t + (1 - \alpha) (-\gamma_t^w + z_t + \delta_t E_t W'_{t+1}) \\ - (1 - \alpha) \delta_t E_t \left[\alpha V_{t+1} + (1 - \alpha) \left(-\gamma_{t+1}^f + \delta_{t+1} E_{t+1} J_{t+2} \right) \right]$$

or

$$W_t = \alpha U_t + (1 - \alpha) [-\gamma_t^w + z_t + \delta_t E_t (W'_{t+1} + J'_{t+1})] \\ - (1 - \alpha) \delta_t E_t \left[\alpha V_{t+1} + (1 - \alpha) \left(-\gamma_{t+1}^f + \delta_{t+1} E_{t+1} J_{t+2} \right) \right].$$

Since $W'_t + J'_t = W_t + J_t$ necessarily, this can be written

$$W_t = \alpha U_t + (1 - \alpha) (-\gamma_t^w + z_t) + (1 - \alpha) \delta_t E_t (W_{t+1} + J_{t+1}) \\ - (1 - \alpha) \delta_t E_t \left[\alpha V_{t+1} + (1 - \alpha) \left(-\gamma_{t+1}^f + \delta_{t+1} E_{t+1} J_{t+2} \right) \right]. \quad (38)$$

Equation (5) can be rewritten as:

$$U_t = z_t + \delta_t E_t U_{t+1} + f_t \delta_t E_t (W_{t+1} - U_{t+1}) + \kappa f_t [W_t - (z_t + \delta_t E_t W_{t+1})] \\ = z_t + \delta_t E_t U_{t+1} + f_t \delta_t E_t (W_{t+1} - U_{t+1}) + \kappa f_t \{w_t - e_t - z_t - \delta_t E_t [s_{t+1} (W_{t+1} - U_{t+1})]\} \\ = z_t + \kappa f_t (w_t - e_t - z_t) + \delta_t E_t U_{t+1} + f_t \delta_t E_t [(1 - \kappa s_{t+1}) (W_{t+1} - U_{t+1})],$$

since $W_t - \delta_t E_t (W_{t+1}) = w_t - e_t - \delta_t E_t [s_{t+1} (W_{t+1} - U_{t+1})]$, from (4). Multiply this expression for U_t by $1 - \alpha$ and subtract from (38) to get

$$W_t - (1 - \alpha) U_t = \alpha U_t + (1 - \alpha) [-\gamma_t^w + z_t] + (1 - \alpha) \delta_t E_t (W_{t+1} + J_{t+1}) \\ - (1 - \alpha) \delta_t E_t \left[\alpha V_{t+1} + (1 - \alpha) \left(-\gamma_{t+1}^f + \delta_{t+1} E_{t+1} J_{t+2} \right) \right] \\ - (1 - \alpha) \{ (z_t + \kappa f_t (w_t - e_t - z_t) + \delta_t E_t U_{t+1}) + f_t \delta_t E_t [(1 - \kappa s_{t+1}) (W_{t+1} - U_{t+1})] \}$$

or

$$\begin{aligned}
W_t - U_t &= (1 - \alpha) \left[(1 - \alpha) \delta_t E_t \gamma_{t+1}^f - \gamma_t^w \right] - (1 - \alpha) \kappa f_t (w_t - e_t - z_t) \\
&\quad + (1 - \alpha) \delta_t E_t [(1 - f_t (1 - \kappa s_{t+1})) (W_{t+1} - U_{t+1})] \\
&\quad + (1 - \alpha) \delta_t E_t [(J_{t+1} - \alpha V_{t+1}) - (1 - \alpha) (\delta_{t+1} E_{t+1} J_{t+2})]. \tag{39}
\end{aligned}$$

From (30) we have

$$\delta_t E_t J_{t+1} = J_t - (p_t - w_t) + \delta_t E_t [s_{t+1} (J_{t+1} - V_{t+1})].$$

Forward this one period and use it to substitute for $\delta_{t+1} E_{t+1} J_{t+2}$ in equation (39) to obtain (13).

With free entry the last term in (13) becomes

$$\begin{aligned}
\alpha J_{t+1} + (1 - \alpha) (p_{t+1} - w_{t+1}) - (1 - \alpha) \delta_{t+1} E_{t+1} (s_{t+2} J_{t+2}) \\
= (p_{t+1} - w_{t+1}) + \delta_{t+1} E_{t+1} [(\alpha - s_{t+2}) J_{t+2}]
\end{aligned}$$

Also, (1) can be solved for $E_t (\delta_t J_{t+1})$ to give

$$E_t (\delta_t J_{t+1}) = \frac{J_t - (p_t - w_t)}{(1 - s_{t+1})}. \tag{40}$$

With s_t predetermined, use of (40) forwarded one period to substitute for $E_{t+1} \delta_{t+1} J_{t+2}$ yields

$$\begin{aligned}
(p_{t+1} - w_{t+1}) + (\alpha - s_{t+2}) \frac{\frac{c_{t+1}}{q_{t+1}} - \kappa (p_{t+1} - w_{t+1})}{1 - \kappa s_{t+2}} \\
= \frac{\alpha - s_{t+2}}{1 - \kappa s_{t+2}} \frac{c_{t+1}}{q_{t+1}} + \frac{1 - \kappa \alpha}{1 - \kappa s_{t+2}} (p_{t+1} - w_{t+1}).
\end{aligned}$$

So (13) becomes

$$\begin{aligned}
W_t - U_t = (1 - \alpha) & \left[(1 - \alpha) \delta_t E_t \gamma_{t+1}^f - \gamma_t^w \right] - (1 - \alpha) \kappa f_t (w_t - e_t - z_t) \\
& + (1 - \alpha) \delta_t E_t \left[(1 - f_t (1 - \kappa s_{t+1})) (W_{t+1} - U_{t+1}) + \frac{\alpha - s_{t+2}}{1 - \kappa s_{t+2}} \frac{c_{t+1}}{q_{t+1}} \right. \\
& \left. + \frac{1 - \kappa \alpha}{1 - \kappa s_{t+2}} (p_{t+1} - w_{t+1}) \right].
\end{aligned}$$

A.4 Derivation of equation (15)

With the specification in (14), (10) and (11) with $\eta = 0$ become

$$W_t = \alpha U_t + (1 - \alpha) (-\gamma_t^w + W_t') \quad (41)$$

$$J_t' = \alpha V_t + (1 - \alpha) (-\gamma_t^f + J_t). \quad (42)$$

The terms in W_t' and J_t' in (41) and (42) can be eliminated in the following way. Multiply (42) by $1 - \alpha$ and subtract it from (41) to get

$$W_t - (1 - \alpha) J_t' = \alpha U_t + (1 - \alpha) (-\gamma_t^w + W_t') - \alpha (1 - \alpha) V_t + (1 - \alpha)^2 (-\gamma_t^f + J_t).$$

Next note that $W_t' + J_t' = W_t + J_t$ necessarily, so this can be written

$$W_t = \alpha U_t - (1 - \alpha) \gamma_t^w + (1 - \alpha) (W_t + J_t) - \alpha (1 - \alpha) V_t + (1 - \alpha)^2 (-\gamma_t^f + J_t)$$

or, subtracting W_t from both sides

$$0 = \alpha U_t - (1 - \alpha) \gamma_t^w + \alpha W_t + (1 - \alpha) (1 - (1 - \alpha)) J_t - \alpha (1 - \alpha) V_t - (1 - \alpha)^2 \gamma_t^f.$$

This can be rewritten as (15).

A.5 Derivation of equation (27)

With $V_{t+1} = 0$ and s_{t+1} predetermined, (1) can be written

$$\delta_t E_t J_{t+1} = \frac{J_t - (p_t - w_t)}{1 - s_{t+1}}, \text{ for all } t.$$

Use of this in (2) with $V_t = 0$ gives

$$0 = -c_t + q_t \left[\kappa J_t + (1 - \kappa) \frac{J_t - (p_t - w_t)}{1 - s_{t+1}} \right], \text{ for all } t,$$

or

$$J_t = \frac{(1 - s_{t+1}) \frac{c_t}{q_t} + (1 - \kappa) (p_t - w_t)}{1 - \kappa s_{t+1}}. \quad (43)$$

Now combine (2) with the free entry condition (3) to get

$$\kappa J_t + (1 - \kappa) \delta_t E_t J_{t+1} = \frac{c_t}{q_t}$$

with, from (1) with $V_{t+1} = 0$ and s_{t+1} predetermined,

$$J_t = p_t - w_t + \delta_t (1 - s_{t+1}) E_t J_{t+1}$$

to get

$$\kappa (p_t - w_t - \delta_t s_{t+1} E_t J_{t+1}) + \delta_t E_t J_{t+1} = \frac{c_t}{q_t}$$

or

$$E_t J_{t+1} = \frac{\frac{c_t}{q_t} - \kappa (p_t - w_t)}{\delta_t (1 - \kappa s_{t+1})} \quad (44)$$

Substituting for J_{t+1} using (43) and dividing by p_t yields (27).

A.6 Derivation of equation (33)

Substituting for J_t and J_{t+1} in (2) using (6) with $V_t = 0$ yields

$$0 = -c_t + q_t \kappa E_t \sum_{n=0}^{\infty} \bar{\delta}_{t,n}^{q,\kappa} [c_{t+n} + (1 - q_{t+n} \kappa) (p_{t+n} - w_{t+n})] \\ + q_t (1 - \kappa) \delta_t E_t \sum_{n=1}^{\infty} \bar{\delta}_{t+1,n-1}^{q,\kappa} [c_{t+n} + (1 - q_{t+n} \kappa) (p_{t+n} - w_{t+n})]$$

or

$$0 = -(1 - q_t \kappa) c_t + q_t \kappa (1 - q_t \kappa) (p_t - w_t) \\ + E_t \sum_{n=1}^{\infty} \left[q_t \kappa \bar{\delta}_{t,n}^{q,\kappa} + q_t (1 - \kappa) \delta_t \bar{\delta}_{t+1,n-1}^{q,\kappa} \right] [c_{t+n} + (1 - q_{t+n} \kappa) (p_{t+n} - w_{t+n})].$$

Division by $q_t p_t$ yields equation (33).