Default risk and economic activity: A small open economy model with sovereign debt and default

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Abstract

Empirical evidence shows that sovereign defaults are associated with significant declines in economic activity in defaulting countries. However, the existing literature on sovereign debt and default mainly analyzes endowment economies and, therefore, does not speak for the interaction between default risk and macroeconomic dynamics. This paper develops a small open economy model with financial frictions, within which output and default risk are jointly determined, that allows to simultaneously examine the behavior of output, consumption and investment and their interaction with sovereign debt and default. When calibrated to match the business cycles properties of an average emerging market economy, the model is able to reproduce the countercyclicality of net exports, the countercyclicality of sovereign spreads and the negative correlation between investment and net exports observed in the data. Furthermore, when analyzing its dynamics around default episodes, the model captures the declines in output, consumption and investment that accompany sovereign defaults.

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1 Introduction

Empirical evidence shows that sovereign defaults are generally associated with significant downturns in economic activity in defaulting countries. Figure 1 shows the evolution of output, consumption and investment in Argentina, Ecuador, Mexico, Philippines and Russia around their default episodes of 2001, 1999, 1982, 1983 and 1998, respectively.[1] All cases are characterized by a decrease in output and consumption, interestingly, however, the largest declines are observed in investment dynamics. Table 1 further looks into this observation by summarizing these variables' deviations from trend around several default episodes within the last 30 years. While average output and consumption deviations from trend range roughly between -5 and -10 percent, respectively, their investment counterpart varies by as much as 22.5 percent below trend. This evidence, along with the fact that sovereign defaults are recurring events in world capital markets, emphasizes the importance of developing comprehensive frameworks aimed at jointly analyzing the behavior of sovereign default and economic activity in small open economies.[2] Yet, the existing literature on sovereign debt and default mainly analyzes endowment economies and, therefore, does not address the interaction between default risk and output and investment dynamics.

Motivated by these facts, in this paper we develop a general equilibrium small open economy model with financial frictions that features sovereign default in equilibrium. In doing so we follow the literature on sovereign debt and default in emerging economies which builds on the seminal work of Eaton and Gersovitz (1981), who provide a theoretical analysis of borrowing in international private capital markets with default penalties. Specifically, we embed investment dynamics into the standard economy model of sovereign debt and default to generate endogenous output fluctuations and costs of default. Unlike in other existing models of this nature but without investment, where the only means to smooth out consumption is by borrowing resources from the rest of the world, our set-up also allows the economy to self-insure against risk by changing the level of capital stock. This gives rise to two forces with potentially counteracting effects. On one hand, the need for external financing is reduced thus increasing incentives to default and, on the other hand, the lack of access to international credit markets (brought about by default) prevents the economy from accumulating as much capital, leading to a decrease in production capacities which increases default losses. This paper hence contributes to the literature by providing an RBC-like framework, within which output dynamics and default risk are jointly determined, that allows to simultaneously examine the behavior of investment and its interaction with external borrowing and sovereign default.

[1] Details on data and sources are provided in the Appendix.

[2] According to Tomz and Wright (2007), who use historical data on loans to sovereign governments from private foreign creditors, 106 countries defaulted a total of 250 times between 1820 and 2004. In the same respect, Reinhart and Rogoff (2010) document four default cylces in which a high share of countries in the world has been in the state of default or restructuring within the same period of time.

The model features negative correlations between the risk of default and the state of the world and the risk of default and the level of debt in equilibrium. This is consistent with empirical evidence showing that countries tend to default more in bad times and when they are more indebted.[3,4] It also suggests that countries that are richer, i.e. with a higher level of capital stock, tend to borrow more in absolute terms but that debt capacities relative to output remain fairly constant across capital levels. Moreover, when calibrated to match the business cycles dynamics of an average emerging market economy, the model is able to reproduce the countercyclical behavior of net exports and sovereign spreads observed in the data without having to rely on a specific stochastic process for shocks or on ad hoc output costs of default. Net exports are countercyclical because in bad times the economy is unable to borrow as much resources from the rest of the world as in good times. Sovereign spreads are countercyclical due to the negative correlation between default risk and output. One of the main reasons for the proposed framework is to account for investment dynamics both in normal times and around default episodes. Although the investment volatility in the model is still lower than in the data, the model successfully accounts for the observed negative correlation between investment and net exports and captures the declines in output, consumption and investment that accompany sovereign defaults.

The main anomalies of the model are twofold. First, it still displays a low rate of default; our country sample is characterized by a default rate of 1.8 percent while our economy model only produces a rate as high as 0.40 percent. This result is potentially driven by the increased ability of the economy to smooth out consumption: even though agents are impatient, in our framework the economy is able to adjust capital stock to avoid default and the costs associated with it. Second, the model fails to reproduce enough variability of country spreads. The latter is related to the tight relationship between default probabilities and the price of debt: the model produces little variability in default probabilities due to the same increased ability to smooth out consumption mentioned above. Nevertheless, low volatility of spreads is not an uncommon result in models of sovereign debt and default and introducing mechanisms that loosen this strong dependence may potentially alleviate the problem.

The paper is mainly related to two strands of the international macroeconomics literature; namely, the literature on sovereign debt and default represented by Aguiar and Gopinath (2006), Arellano (2008), Cuadra and Sapriza (2008) and Yue (2010), among many others, and the literature on financial frictions that introduces endogenous debt limits, based on Kehoe and Levine (1993), into fully-fledged RBC models such as the one studied in Kehoe and Perri (2002). We begin our analysis by introducing limited commitment into an otherwise standard bond economy model. The economy issues debt in the form

[3] When examining the relationship between economic activity and sovereign default, Tomz and Wright (2007) find that default is about twice as likely when output is 7 percent or more below trend, although the relationship between the two is not linear.

[4] D'Erasmo (2010) documents that average government debt to output ratio from 1970 to 2005 for countries that defaulted during the last decade is 65 percent.

of one-period, non-contingent discount bonds upon which she may default, if it is in her best interest to do so, at the expense of being temporarily excluded from international credit markets while facing direct output costs of default. The lack of contingency derived from the incomplete set of assets available for trade makes it more costly for the economy to fulfill her outstanding obligations when faced with negative productivity shocks because it is precisely then when she needs external resources the most. The fact that the economy is not constrained efficient makes the option of default a means to increase the degree of contingency, therefore, the economy becomes more prone to default in bad times.

Motivated by empirical evidence that suggests that emerging economies' distinguishing features are related to their access to international credit markets (see Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005), for instance), the models of sovereign debt and default, and hence this model, provide a framework that endogenizes interest rate dynamics. Sovereign debt is purchased by international risk neutral investors who are willing to lend or borrow resources at the expected international interest rate and need thus to be compensated for the likelihood of default. Since the probability of default, in turn, depends on the country's decision on how much debt to issue, the interest rates implied by debt prices are directly related to default risk.

Endogenous output dynamics are introduced by means of a capitalistic production function. In bad times output goes down increasing incentives to default while decreasing debt prices; however, given that investment decisions depend on the amount of resources available in the economy, future output levels are dependent on whether the economy has access to international credit markets at all and on the conditions of this access, i.e. on the price of debt. These two facts trigger a mechanism by which output dynamics feed back into default penalties and vice versa. In this respect, this paper relates to Mendoza and Yue (2010) who also develop a model of sovereign debt and default with endogenous output fluctuations. However, their model considers an economy with working capital loans where default triggers an efficiency loss due to the need to replace imported inputs with imperfect domestic substitutes, whereas our set-up considers investment dynamics as having a key role in the propagation mechanism of default risk throughout the economy. Bai and Zhang (2010) also study a model of sovereign debt and default with capitalistic production. In particular, they develop a model in which a continuum of small open production economies are able to walk away from their financial contracts. Nevertheless, the focus of their analysis is different from ours in that they aim at studying the effects of financial integration on international risk sharing within an environment where capital markets remain constrained by the existence of incomplete markets and limited commitment, whereas our goal is to assess the ability of this framework to reproduce both the real business cycles properties of an economy that suffers from default risk and the macroeconomic dynamics around default episodes.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents its quantitative analysis, which include the examination of the equilibrium and business cycles properties of the calibrated model, a sensitivity analysis and the study of the economy model's behavior around default episodes. Section 4 concludes.

2 The model economy

Consider a small open economy where output is produced using capital and labor and can be transformed one-to-one into consumption and investment goods. The economy is populated by infinitely-lived households, who are risk averse and have preferences over consumption goods and leisure, and by a benevolent government, that we will call the sovereign, who is in charge of maximizing households' utility at every point in time.

A shock affecting the economy's productivity is realized at the beginning of each period; however, the economy has access to international credit markets where she can borrow resources from the rest of the world as a way to insure against aggregate risk. Risk insurance in this case is partial though since financial markets are incomplete. On one hand, there is limited spanning in the set of financial assets available which limits trade to one-period, non-contingent bonds and, on the other hand, there is limited commitment arising from the lack of perfect enforceability of financial contracts which allows the sovereign to default upon the economy's outstanding debt whenever it is in her best interest to do so. If the sovereign decides to honor the economy's contract and pay the debt that comes due, the economy keeps her good credit standing and continues having access to international credit markets where she can issue more debt. If, instead, the sovereign decides not to pay the debt, the economy is said to be in default, falls in bad credit standing and faces some costs associated with this decision. Following the literature, we assume that the penalties associated to default are twofold: first, the economy is forced into financial autarky where she remains with certain probability until being redeemed, and second, the economy is penalized with a direct output cost while being out of the international credit markets.

After having described the general features of the economy, we now discuss the details of the economic problem. After this, we present the model's equilibrium and comment on the implications of introducing investment dynamics into the existing framework.

2.1 The sovereign

Let $V(a, k, z)$ be the economy's continuation value when initial assetholdings are given by a, the initial level of capital stock is k, and the productivity shock at the beginning of the period is represented by z. Then, the sovereign's problem can be stated recursively in the following way:

$$V(a, k, z) = \underset{d\{0,1\}}{max}\{(1-d)V^G(a, k, z) + dV^B(k, z)\} \tag{1}$$

where d is the default decision and $V^G(a,k,z)$ and $V^B(k,z)$ are the economy's continuation values of being in good and bad credit standing, respectively.[5]

If the economy is in good credit standing the sovereign chooses today's consumption and labor and tomorrow's capital stock and assetholdings in order to maximize the economy's present utility and expected discounted continuation value, given her budget constraint. This is, $V^G(a,k,z)$ is defined by:

$$\begin{aligned} V^G(a,k,z) &= \max_{c,l,k',a'} \{u(c,l) + \beta E\left[V(a',k',z')|z\right]\} \\ \text{s.t.} \quad & c + k' - (1-\delta)k + q(b',k',z)a' = e^z f(k,l) + a \end{aligned} \tag{2}$$

where $u(\cdot)$ and $f(\cdot)$ are the representative household's utility function and the economy's production function, respectively; E is the expectational operator conditional on today's information set and x' denotes x one period ahead; c and l stand for consumption and labor, respectively, δ is the depreciation rate and $q(a',k',z)$ represents the zero-coupon discount bond price.

If instead the economy is in bad credit standing, no debt is issued and the sovereign chooses today's consumption and labor and tomorrow's capital stock so as to maximize the economy's present utility and expected discounted continuation value, given available resources. Thus, $V^B(k,z)$ is defined by:

$$\begin{aligned} V^B(k,z) &= \max_{c,l,k'} \{u(c,l) + \beta[(1-\lambda)E[V^B(k',z')|z] + \lambda E[V(0,k',z')|z]]\} \\ \text{s.t.} \quad & c + k' - (1-\delta)k = y^{aut}(z, f(k,l)) \end{aligned} \tag{3}$$

where y^{aut} represents output in financial autarky, after considering the direct output cost penalties to which the economy is subject.[6] Notice that the second term in the objective function represents the fact that, in the following period, the economy may remain in autarky with probability $(1-\lambda)$ or be redeemed and regain access to credit markets with zero assetholdings with probability λ.[7]

2.2 International credit markets

International credit markets are perfectly competitive and populated by risk neutral investors. Bonds must hence be priced so that the zero-profit condition holds and investors' expected returns equal the

[5] a<0 implies the issuance of debt. The model will be calibrated so as to ensure that assetholding are non-positive in equilibirum.

[6] Assuming that the defaulting country is penalized with a direct output cost while in bad credit standing accounts for the output losses incurred when other trade relations are interrupted after sovereign default or as a result of the reduction of aggregate credit available in the economy while the economy is denied access to international credit markets. See Mendoza and Yue (2010) for a model of sovereign debt where the direct output cost of default is a result of the disruption of firms' private financing in a framework where default on public and private foreign obligations occurs simultaneously.

[7] Assuming an exogenous probability of redemption with a zero recovery rate after default is a simple alternative to considering a more complex environment where the defaulting country engages into a sovereign debt renegotiation process with international creditors. See Benjamin and Wright (2008) and Yue (2010) for examples of debt renegotiation processes in models with defaultable debt.

international risk-free interest rate. This implies that:

$$q(a', k', z) = \frac{1}{1 + r^w} E(1 - d') \tag{4}$$

where r^w denotes the world risk-free interest rate.

Two things are worth noting about the last expression. First, conditional on the current state of the world, the economy internalizes the fact that their actions affect international credit markets' perceptions regarding the degree of riskiness of her debt. In other words, the debt market price depends on a' and k'. Second, there is a direct and close relationship between the expected probability of default and the interest rate spread, defined as $q^{-1} - (1 + r^w)$; the higher the expected probability of default the higher the return investors would ask to purchase sovereign debt and, in consequence, the higher sovereign spreads.

2.3 Equilibrium

We now characterize the equilibrium of the model and some properties related to it. Section 3 further analyzes the debt and default equilibrium results once the model is calibrated.

Definition: *The recursive equilibrium for the economy is defined as a set of value functions* $V(a, k, z)$, $V^G(a, k, z)$ and $V^B(k, z)$, a set of *sovereign's decision rules for consumption, labor, capital accumulation, assetholdings and default, represented by* $c(a, k, z)$, $l(a, k, z)$, $k'(a, k, z)$, $a'(a, k, z)$ *and* $d(a, k, z)$, *respectively, and a bond price function,* $q(a', k', z)$*; such that, given initial levels for* a, k *and* z *and an exogenous law of motion for* z', $V(a, k, z)$, $V^G(a, k, z)$, $V^B(k, z)$, $c(a, k, z)$, $l(a, k, z)$, $k'(a, k, z)$, $a'(a, k, z)$ *and* $d(a, k, z)$ *solve the sovereign's optimization problem (1) with* $q(a', k', z) = q(a'(a, k, z), k'(a, k, z), z)$ *satisfying (4).*

Given the complexity of the problem at hand it is difficult to derive analytical results that serve as guidelines on the behavior of the economy. However, it can be proved that the model preserves one of the main features that characterizes the models of sovereign debt and default that examine endowment economies; namely, if at any given level of capital stock, k, and productivity shock, z, the economy is better off defaulting upon debt level a, she is also better off defaulting upon any other higher debt level. To formalize this statement notice that the sovereign's decision rule for default, $d(a, k, z)$, equals 1 whenever $V^G(a, k, z) < V^B(k, z)$. Let a default set, $D(a, k)$, be defined as the set of shock realizations for which $d(a, k, z) = 1$:

$$D(a, k) = \{z : V^G(a, k, z) < V^B(k, z)\}$$

then,

Proposition 1: For $0 \geq a_1 \geq a_2$, if default is optimal for a_1 in some combination of states (k, z) then default will be optimal for a_2 in the same combination of states (k, z). This is $D(a_2, k) \subseteq D(a_1, k)$.

Proof: See Appendix.

2.4 Adding investment into the picture

As mentioned above, this framework allows to jointly examine investment and sovereign debt dynamics while adding new mechanisms that affect the sovereign's borrowing and default decisions and the behavior of the economy in financial autarky. To make this statement clear, let us compare the implications of this production model with its endowment counterpart.

In the endowment economy model, the only way in which agents can smooth consumption is by borrowing resources from the rest of the world through the issuance of sovereign debt. In contrast, a production economy on its own is already able to self-insure by adjusting her investment level; doing so, however, implies affecting the pace at which capital stock is accumulated which, in turn, impacts the economy's size and ability to produce future output. Having the possibility to insure against aggregate risk by borrowing resources from international credit markets without affecting her investment level may indeed change the economy's incentives to accumulate debt. In particular, one may think that an economy with relatively low levels of capital would be more benefited by having the ability to access outside resources, while in the process of building up capital stock, than an economy whose level of capital stock is relatively high or close to its stationary level. Notice also that, for the case of an endowment economy, the value of being in bad credit standing depends exclusively on the current state of the world and on the future stream of shocks hitting the economy. This differs from the case of a production economy model where it is influenced by the capital policy rule, as well.

Another important implication of this model is that it features endogenous direct output costs after default. Typically, there are two ways to specify this loss in the existing literature: the first one, assumes that the cost is a constant share of endowment regardless its level, i.e. a constant productivity loss, while the second one assumes that output costs are asymmetric and increasing in endowment.[8] In any of these cases the output cost assumed, and its effect of V^B, is exogenous because endowment is exogenous. In our case, and regardless of the specification it takes, the direct output cost also depends on the level of capital stock of the economy.

All in all, the introduction of investment dynamics into the model modifies the way in which V^G and V^B are determined and their interaction. In the next section we proceed to analyze its quantitative implications for the model's performance.

[8] See Aguiar and Gopinath (2006) and Arellano (2008) for examples of these specifications.

3 Quantitative analysis

In order to test the numerical properties of the model, first, we describe the data used to contrast it with empirical evidence. Then, we present the calibration and the functional forms used and briefly comment on the solution method. We continue by examining the properties of the calibrated model. Specifically, we analyze its debt, default and debt prices implications and the business cycles properties that it delivers and present a sensitivity analysis of the results. Finally, we analyze the model's dynamics around a default episode.

3.1 The data

The literature on sovereign debt and default has historically focused on contrasting its models to the behavior of emerging market economies. This is mainly driven by two facts: first, because many of the recent episodes of sovereign default have occurred in emerging economies and, second, because empirical evidence suggests that emerging economies' distinguishing features are related to their access to international credit markets (see Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005)) which makes this type of models an ideal framework with which to study their behavior.[9] For this reason, our quantitative analysis focuses on contrasting the model' results with emerging economies' empirical evidence. In contrast with the common approach of using a specific country as a study case, we compare the model's results to the empirical properties of a representative emerging economy. In order to do so we collect quarterly data on national accounts and sovereign debt and defaults for a sample of 11 emerging market economies and assume that the representative economy's behavior is characterized by the sample average estimates. The emerging economies in our sample are Argentina, Brazil, Ecuador, Indonesia, Malaysia, Mexico, Peru, Philippines, Russia, South Africa and Turkey. As Table 2 shows, all of them, with the exception of Malaysia, went through at least one default episode during the last 30 years.

Table 3 summarizes these countries' main business cycles statistics and compares them with other representative estimates in the literature.[10] All in all, the business cycles properties of the representative emerging economy are in line with previous findings. On average, output standard deviation is 3.08 percent while consumption is 1.55 times more volatile than output. Although output is slightly more volatile in our sample, these estimates are not significantly different to the ones reported by Aguiar and Gopinath (2007) and by Neumeyer and Perri (2005), henceforth AG and NP, of 2.74 and 2.79 percent for output and 1.45 and 1.71 times for consumption, respectively. Investment is also more

[9] Reinhart and Rogoff (2010) document that the most recent default cycle is the one encompassing the emerging market debt crises of the 1980s and 1990s.

[10] Details on data and sources are provided in the Appendix.

volatile than output, 2.12 as much. However, notice that this value is small when compared to AG and NP's estimations; this may be due in part to the increased volatility of output observed in our sample. Conditional on being in good credit standing, sovereign spreads feature an average standard deviation of 2.67 percent which is similar to the one reported by NP.[11] Finally, the estimations are also consistent with two very well documented stylized facts of emerging economies, namely, the countercyclicality of net exports and of sovereign spreads; the average correlation between net exports and output and between sovereign spreads and output is -0.44 and -0.45, respectively.

3.2 Calibration and functional forms

The model is calibrated on a quarterly basis. The world risk-free interest rate, r^w, is set equal to 1%.

We assume that the period utility function takes the GHH form:

$$u(c_t, l_t) = \frac{\left(c_t - \frac{1}{\psi} l_t^{\omega}\right)^{1-\sigma}}{1-\sigma}$$

where σ, the coefficient of risk aversion, equals 2; ω, the parameter that regulates the curvature of labor supply, is set equal to 1.455 which implies a labor supply elasticity $(1/(\omega - 1))$ of around 2.2 and ψ is an adjustment factor set to target a labor participation rate of 1/3 in equilibrium. As emphasized by the literature (see, for instance, Mendoza (1991) and Neumeyer and Perri (2005)), abstracting the labor supply from wealth effects by using this functional form allows the model to better reproduce some business cycle facts that characterize emerging and small open economies.

The production function is assumed to be a constant returns to scale Cobb-Douglas:

$$f(k_t, l_t) = k_t^{\alpha} l_t^{(1-\alpha)}$$

where the capital share, α, is set equal to 0.36. In order to match an annual depreciation rate of capital of 10%, δ is set equal to 0.025.

We assume that productivity shocks follow an AR(1) process:

$$z_{t+1} = \mu_z + \rho_z z_t + \varepsilon_{t+1}$$

with $\varepsilon \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$. ρ_z and μ_z are set to 0.95 and $-\frac{1}{2}\sigma^2$, respectively, and are consistent with an unconditional mean of e^z of 1. σ_ε^2 is set so as to match the average output standard deviation of our country sample.

So far, all parameter values and functional forms discussed are in line with the RBC literature, now we comment on those whose values and forms are closely related to the literature on sovereign debt and

[11] NP's definition of interest rate is the expected 3-month real interest rate at which firms in a country can borrow. However, for reasons discussed in the paper, they use secondary market prices of emerging market bonds to construct this variable which makes it suitable for our comparison.

default: the discount factor, the probability of redemption after default and the direct output costs of default while in financial autarky. A high degree of impatience is needed in order to take the equilibrium of the economy to the region where default does occur, this is why the discount factor, β, is initially set equal to 0.91.[12] As we will see later, in the sensitivity analysis, the value of this parameter plays an important role in determining the frequency of default in the model. The probability of redemption after default, λ, is set equal to 10% which is consistent with an average stay in autarky of 2.5 years. This number is in line with the estimations of Gelos et al. (2004) and Reinhart and Rogoff (2010) who document an average time of re-entry into the international credit markets of 5.4 years in the 1980s and of roughly 1 year in the 1990s, and a median duration of default spells in the post World-War II period of 3 years, respectively. Finally, we refer to the direct output cost while the economy is in financial autarky, y^{aut}. As explained above, one of the novelties of our model has to do with the fact that this penalty affects the production capacities of the economy in contrast to other models that assume that the cost affects a set of income draws so, in this sense, there is no guideline to follow when it comes to model its form. We take a simple stance and model it as a constant, small share of output:

$$y^{aut}(z, k, l) = (1 - \gamma)e^z f(k, l)$$

with γ equal to 0.02. The sensitivity analysis will show that the value of this parameter has an important effect on the extension of credit in the model. Table 4 summarizes the values of the parameters used for the benchmark model.

3.3 Computation

We solve the model numerically using value function iteration. In contrast with most of the existing literature that uses the discrete state space technique approach, we follow Hatchondo et al. (2010), henceforth HMS, and use an interpolation method to solve for the equilibrium.

There are several reasons why we choose to use a continuos method rather than the discrete state space technique. One of the main challenges associated to the quantitative analysis of models of sovereign debt and default is its curse of dimensionality. Adding capital stock as a third state variable does increase the dimension of the problem specially as it is desirable that the grid defined for it is as fine as possible. In this respect, using a continuos method represents an advantage since, once the value functions are approximated, optimal policies can be chosen from continuos sets for a and k, $[\underline{a}, \overline{a}]\text{x}[\underline{k}, \overline{k}]$, in every iteration. Furthermore, using a continuos method enables us to construct a one-loop algorithm to iterate simultaneously on the value and the bond price functions, which significantly reduces the computation time. Lastly, in a recent study on the quantitative properties of sovereign debt and default models, HMS

[12] Values of β range from 0.72 to 0.95 for cases that match the business cycles properties of Argentina. See Aguiar and Gopinath (2006), Arellano (2008), Cuadra and Sapriza (2008) and Yue (2010), among others.

provides evidence showing that the simulated behavior of sovereign spreads can be very much affected by approximation errors which are more likely to occur when using coarse grids with the discrete state space technique: “We find that, to generate reliable results, the DSS [discrete state space] technique requires a significantly larger number of grid points than the ones used [in the literature] (...) [and show] that the results obtained using DSS converge towards the ones obtained using interpolation methods as the number of grid points increases.”[13]

Our algorithm follows HMS's strategy and extends it to include capital stock as an additional endogenous state variable. Value functions are approximated using Chebyshev collocation and the policy functions are solved using a derivative-free nonlinear optimization routine. Further details are provided in the Appendix.

3.4 Results

3.4.1 Equilibrium results: default, the extension of credit and debt prices

We begin by analyzing the sovereign's default decision in Figure 2 which depicts the repayment and default sets of the economy in the a-k space for the mean productivity shock. The colored region represents all combinations of assetholdings and capital stock levels for which the economy is better off defaulting (the default set), while its complement represents all combinations for which repaying outstanding debt is optimum (the repayment set) in equilibrium. Our results show that the model is consistent with the existing literature in that it displays a negative correlation between default risk and assetholdings. This is, for a given level of capital stock (y-axis), the economy is more likely to default the more indebted she is. The model also predicts a negative correlation between default risk and capital stock, in Bai and Zhang (2010). This is, for a given level of debt (x-axis), default is more likely to occur in economies with lower capital stock, i.e. in poor economies. Figure 2 also speaks for the amount of debt that the economy can borrow. In this respect, notice that the line dividing the repayment set from its default counterpart is negatively sloped meaning that the extension of credit increases the richer the economy is. Nevertheless, when further analyzing the model's implications on the extension of credit we find that, although absolute credit increases with the level of capital stock, credit limits relative to output remain fairly constant. Figure 3 displays the level of both absolute and relative debt capacities across capital levels.

An important implication derived from the simultaneous presence of limited spanning of assets and limited commitment is that default risk and the state of the world are negatively correlated. Paying non-contingent debt is more costly in bad times because it is precisely then when the economy needs more resources to finance consumption and investment; the option to default upon outstanding debt adds

[13] Hatchondo et al (2010) pg. 920.

some degree of contingency to sovereign decisions at the expense of increasing incentives to default, and hence default risk, at those difficult times. Figure 4 displays this property of the model by comparing the default and repayment sets across z. As it can be observed, default sets shrink as z increases which is also translated into larger extensions of credit.

The risk of default is tightly related to debt prices. In particular, from expression (4) we know that the issuance of debt is priced according to the expected probability of repayment which, in turn, depends on a' itself, on tomorrow's production possibilities, determined to some extent by k', and on z which provides relevant information as to what the state of the world will be when debt comes actually due. Figure 5 depicts several price schedules to examine the relationship between debt prices and their determinants. To begin with, notice that because of the negative correlation between risk of default and assetholdings, debt price schedules take their maximum value, $(1/(1 + r^w))$, for small levels of debt which are associated with a zero probability of default. Following the same reasoning, debt prices take positive but smaller values as debt becomes riskier until they eventually are zero for high levels of debt upon which the economy will default with certainty. The left panel of Figure 5 depicts debt prices across different states of the world showing that creditors are willing to pay a higher (lower) price for a given level of sovereign debt, and thus ask for lower (higher) returns, in good (bad) times. A similar pattern can be observed with respect to the investment decision of the economy. The right panel depicts debt prices across different levels for choices of k'; notice that, for a given level of capital stock, sovereign debt is more expensive (cheaper) the higher (lower) the level of k', this is, the higher (lower) the investment choice.

3.4.2 Real business cycles and debt and default properties

In order to compute the business cycles statistics of the model, we perform simulations of 500 periods of length with shocks drawn from the stationary distribution of z. The results reported correspond to the average over 100 simulations of the last 400 observations of every series, hence each set of these realizations may be interpreted as the evolution of a sample emerging economy over a hundred years. All statistics, with the exception of sovereign spreads standard deviation which is computed from the series levels, are computed based on the log-deviations of the Hodrick-Prescott filtered series with a smoothing parameter of 1600.

Table 4 reports the results of the model and contrasts them with the data. Several things are worth noting. To begin with, the model matches the countercyclicality of net exports and the countercyclicality of sovereign spreads. Net exports are countercyclical because in bad times the economy is unable to borrow as much resources from the rest of the world as in good times, and sovereign spreads are countercyclical due to the negative correlation between default risk and output. Importantly, our framework is able to generate the simultaneous countercyclicality without having to rely on a specific stochastic process for shocks or on ad hoc output costs of default like several papers in the literature. The model produces a volatility of consumption relative to output of 0.87, in contrast with the data which shows that consumption is actually more volatile than output. The smaller variability of consumption is brought about by the fact that in our framework adjusting investment serves as an additional means to smooth out consumption. This prevents it from moving as much as in the case of an economy where the only means to do so is by borrowing resources from the rest of the world. Notice also that, despite the above, the variability of investment is still smaller than in the data. This may be explained by the presence of financial frictions per se which tend to dampen investment dynamics; later on, in the sensitivity analysis, we will see that investment and net exports become more volatile the greater the penalties of default.[14] As for the relationship between investment and borrowing, the model is also able to reproduce their observed negative correlation. Finally, notice that the model fails to deliver sufficient variability in sovereign spreads which is common to other quantitative models of sovereign debt and default. This result is derived from the tight relationship between debt prices, and therefore interest rates, and the probability of default. Ways to loosen this relationship include modeling the debt renegotiation process after default which makes debt prices be also dependent on debt recovery rates as in Yue (2010), introducing longer-term debt which adds a short- vs long-term trade off in the issuance and hence riskiness of debt into the setup as in Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2010), and assuming that international creditors are also risk averse which makes them sensitive to changes in their valuation of consumption over time as in Arellano's (2008) example. Also related to the low volatility of sovereign spreads reported, Hatchondo et al. (2010) document that the discretization of the state variables introduces discontinuities in the bond price schedule that may overestimate the variability of interest rate spreads; when re-solving existing models with the discrete state space technique but with finer grids and with continuous methods, they find that the volatilities reported in other studies in the literature significantly decrease.

As for the debt and default properties, notice first that the mean debt to output ratio delivered by the model is 17.47 per cent. Although this number is still small when compared to the data, it is significantly larger than the ones produced by other existing models. This has to do with the fact that our framework introduces endogenous penalties of default by modeling output dynamics in and out financial autarky. In the sensitivity analysis we will show that debt limits increase the larger the penalties of default. Results also show that default is still a rare event. The maximum number of defaults in our simulated economies is 2 per 400 quarters which occurs in 3.2 percent of the cases while 25 percent of the cases present a single default occurrence. This translates into a rate of default of 0.28 percent which is much lower than the rate of default of our country sample of 1.18 percent.[15] As commented above, default occurrences in

[14] Kehoe and Perri (2002) document the fact that limited comitmment leads to more severe restrictions on risk sharing and investment flows.

[15] The default rate is computed based on all default occurrences in the span of the data in our sample. When computing

the model are related to the degree of agents' impatience, our sensitivity analysis will also show how the model's results change when the value of β changes.

3.5 Sensitivity analysis

In this section we analyze the effects of changes in the values of β, λ and γ on the business cycles and debt and default properties of the model. These parameters are closely related to sovereign debt and default dynamics, therefore, studying their impact on the model's behavior may shed light on the direction towards which we should go to improve upon our results.

Changing the degree of agents' impatience through changes in β has a clear impact on the rate of default of the economy, as columns 2 and 3 in Table 6 show. In particular, when agents are more impatient, i.e. when β equals 0.87, the rate of default increases by more than 40 percent, from 0.28 to 0.40 percent. Lower levels of β reduce the value of future consumption. This decreases the losses of being outside the international credit markets and having to face direct output costs thus increasing incentives to default. As a result of this, sovereign spreads observe an increased variability and a higher correlation with output.

Regarding the effects of changing the average time that the economy stays in financial autarky after default, we analyze the cases where λ is equal to 0.044 and 0.25 which imply average stays in autarky of 5.4 and 1 years, respectively. As columns 4 and 5 in Table 6 show, changes in λ have a great impact in both the amount of debt that the model is able to sustain and borrowing and investment dynamics. In particular, when the economy is outside the international credit markets for a longer period of time after default, the debt to output ratio goes up by around 90 percent, from 17.47 to 33.22 percent. Increasing default penalties has two main consequences: first, it reduces default incentives which make international investors willing to lend more resources and ,second, it forces the economy to adjust borrowing and, in turn, investment levels more in order to avoid the increased penalties of default. Notice also that, as a consequence of the higher levels of debt, the probability of default becomes slightly more volatile and thus translate into more volatile sovereign spreads. Interestingly, the negative relationship between sovereign spreads and output loosens.

Finally, columns 6 and 7 in Table 6 show the effects of changing the proportion of output that is lost while the economy remains in financial autarky once she defaults. As it can be noticed, increasing the proportion of loss from 0.02 to 0.03, has a very similar outcome than increasing the average stay in financial autarky. This is, the extent of credit increases (this time by more than 50 percent, to 26.51 percent) and investment, borrowing and spreads dynamics are more volatile. In contrast to the last case, the negative relationship between sovereign spreads and output becomes stronger. This discrepancy suggests that the two penalties have different effects on default probabilities and that further analysis

the default rate for the same set of countries for the last 30 years (1980-2010) the default rate increases to 1.73 percent.

needs to be carried out to understand the underlying mechanisms at work in each case.

3.6 The model around default episodes

We conclude the study of our results by examining the model's dynamics around a default episode. As commented above, sovereign defaults are generally associated with significant declines in output, consumption and investment dynamics. Figure 6 shows the deviations of these three variables from their respective trend around a realization of the benchmark model where a default episode occur. As it can be noticed, the model is able to endogenously produce declines in output, consumption and investment around sovereign default. This points out that these episodes indeed contribute to the downturn in economic activity and that further analysis needs to be done to assess the quantitative importance of their effects.

4 Conclusions

This paper develops a small open economy model with financial frictions, within which output and default risk are jointly determined, that allows to simultaneously examine the behavior of output, consumption and investment and their interaction with sovereign debt and default. When calibrated to match the business cycles properties of an average emerging market economy, the model is able to reproduce the countercyclicality of net exports, the countercyclicality of sovereign spreads and the negative correlation between investment and net exports observed in the data. However, the benchmark calibrated model fails to deliver default rates of the magnitude observed in the data and enough variability in sovereign spreads. Our sensitivity analysis, shows that changing the values of the discount factor, the probability of staying in financial autarky before redemption and the direct output costs of default, which are closely related to debt and default dynamics, could potentially improve upon the results obtained so far. Furthermore, the model is able to endogenously produce declines in output, consumption and investment around sovereign defaults suggesting that these episodes contribute to the downturn in economic activity and that further analysis needs to be done to assess the quantitative importance of their effects on macroeconomic dynamics.

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6 Appendix

6.1 Proofs

Proof of Proposition 1:

First, notice that $V^G(a,k,z)$ increases monotonically in assetholdings:

Let $u(c,l)$ be such that, given (1), optimal labor, l^*, is independent of c, i.e. $l^*(k,z)$. Consider any pair of assetholdings levels a_1 and a_2 such that $0 \geq a_1 \geq a_2$, then for combinations $(a_1'^*, k_1'^*)$ and $(a_2'^*, k_2'^*)$ were $V^G(a_1,k,z)$ and $V^G(a_2,k,z)$ are attained, respectively:

$$\begin{aligned}
V^G(a_1,k,z) &\equiv u(e^z f(k,l(k,z)) + (1-\delta)k + a_1 - q(a_1'^*, k_1'^*, z)a_1'^* - k_1'^*, l(k,z)) + \beta E\left[V(a_1'^*, k_1'^*, z')\right] \\
&\geq u(e^z f(k,l(k,z)) + (1-\delta)k + a_1 - q(a_2'^*, k_2'^*, z)a_2'^* - k_2'^*, l(k,z)) + \beta E\left[V(a_2'^*, k_2'^*, z')\right] \\
&\geq u(e^z f(k,l(k,z)) + (1-\delta)k + a_2 - q(a_2'^*, k_2'^*, z)a_2'^* - k_2'^*, l(k,z)) + \beta E\left[V(a_2'^*, k_2'^*, z')\right] \\
&\equiv V^G(a_2,k,z)
\end{aligned}$$

The proof of Proposition 1 is now straightforward:

Given that $V^G(a, k, z)$ increases monotonically in assetholdings and $V^B(k, z)$ is independent of assetholdings, if $V^B(k, z) \geq V^G(a_1, k, z)$, then for $0 \geq a_1 \geq a_2$, $V^B(k, z) \geq V^G(a_1, k, z) \geq V^G(a_2, k, z)$.

QED.

6.2 The data

The representative emerging economy is characterized by the average statistics of a country sample consisting of 11 emerging economies: Argentina, Brazil, Ecuador, Indonesia, Malaysia, Mexico, Peru, Philippines, Russia, South Africa and Turkey. All of these countries belong to the set included in the literature documenting emerging economies' stylized facts (see Neumeyer and Perri 2005 and Aguiar and Gopinath (2007)), are emerging economies that defaulted in the past 15 years (see Beers and Chambers (2006) and Beers, Chambers and Ontko (2010)), or both.

Collecting and standarizing data on national accounts and sovereign debt for emerging economies is not an easy task; the process of gathering and organizing aggregate data within a country is costly and, being the economies in our sample still in a developing stage, it is common that not many resources have been devoted to build up large historical information sets, specially on a quarterly basis. Despite this, we choose to keep this time frequency because one of our interests is to study the RBC properties of the model. Most of the data thus covers the 1990s and 2000s; however, for each country and variable, our time series go as far back in time as possible. The specifics of the data set are the following.

National accounts data:

Real GDP, consumption (defined as the sum of private and government consumption, change in inventories and a statistical discrepancy), investment (defined as gross fixed capital formation) and net exports are seasonally adjusted and expressed in domestic currency. For most of the cases the data is from the International Financial Statistics service of the International Monetary Fund (IFS); the exceptions are Argentina and Ecuador whose data was obtained from the Ministry of Finance (Ministerio de Economía) and the Central Bank (Banco Central del Ecuador), respectively. Available periods vary across countries as follows: Argentina 1993Q1-2010Q2, Brazil 1995Q1-2010Q1, Ecuador 1993Q1-2010Q3, Indonesia 1997Q1-2010Q2, Malaysia 1991Q1-2010Q2, Mexico 1981Q1-2010Q2, Peru 1980Q1-2010Q2, Philippines 1981Q1-2010Q1, Russia 1995Q1-2009Q3, South Africa 1980Q1-2010Q2 and Turkey 1987Q1-2010Q1.

Sovereign spreads:

Sovereign spreads are measured by the EMBI+ Government Bond Spread taken from the Global Financial Data database. Available starting dates vary across countries as follows: Argentina 1994Q1, Brazil 1994Q2, Ecuador and South Africa 1995Q1, Turkey 1996Q3, Malaysia 1996Q4, Mexico, Peru, Philippines and Russia 1998Q1 and Indonesia 2004Q1. All series end in 2010Q4.

Sovereign debt:

XXX

6.3 Optimization algorithm

The optimization algorithm follows HMS and extends it to allow for capital stock as an additional endogenous state variable. The algorithm consists of the following steps:

1. Create grids for assetholdings, capital stock and productivity shocks within the approximation set $[\underline{a}, \overline{a}]x[\underline{k}, \overline{k}]x[\underline{z}, \overline{z}]$ using Chebyshev nodes.

2. Guess initial values for $V^{G(0)}$ and $V^{B(0)}$ and use them to determine $V^{(0)}$ for every combination of (a, k, z).

3. Use $V^{G(0)}$, $V^{B(0)}$ and $V^{(0)}$ to solve for (2) and (3) for every combination of (a, k, z).

Value functions are approximated using Chebyshev collocation for all (a, k, z) that belong to the approximation set $[\underline{a}, \overline{a}]x[\underline{k}, \overline{k}]x[\underline{z}, \overline{z}]$. To get optimal values of a' and k' a derivative-free nonlinear optimization routine is used. The initial guesses $a'^{(0)}$ and $k'^{(0)}$ used to feed the optimization routine are set equal to the solution derived from solving the same problem with a discrete state space technique. Notice that whenever the sovereign's objective functions are evaluated within the optimization routine $E\left[V(a', k', z')|z\right]$, $E[V^B(k', z')|z]$ and $E[V(0, k', z')|z]$, as well as $q(a', k', z)$, need to be determined. The expectational objects are calculated as numerical integrals using Gauss-Legendre quadrature while $V(a', k', z')$, $V^B(k', z')$ and $V(0, k', z')$ are approximated with $V^{G(0)}$, $V^{B(0)}$ and $V^{(0)}$. Knowing the expected values of next-period's V^G and V^B for any given (a', k', z) allows us to determine the probability of default and hence to evaluate $q(a', k', z)$ using (4). Notice also that, in the search for the optimal level of a' and k' by the optimization routine or while determining the possible values of z' given z to evaluate the numerical integrals, the algorithm may want to evaluate the value functions on values of a, k or z lying outside the approximation set. Whenever this is the case, a first-order Taylor approximation is performed around the closest point in the approximation set to calculate the value functions.

4. Use the results in step 3 to update the value functions $V^{G(1)}$ and $V^{B(1)}$ and to determine $V^{(1)}$.

5. Use $V^{G(i)}$ and $V^{B(i)}$ to repeat step 3 and step 4 until $\left|V^{(i)} - V^{(-i)}\right| < \varepsilon$, where i is the number of iteration and $\varepsilon = 5e^{-4}$.

7 Graphs and tables

Table 1. Output, consumption and investment dynamics in selected default episodes

Country	Default episode (starting date)	Deviation from trend: Output	Consumption	Investment
Argentina	2001Q4	-11.2	-11.4	-44.2
Ecuador	1999Q3	-6.0	-15.1	-25.1
Indonesia	1998Q2	-7.6	-18.1	-16.3
Mexico	1982Q3	-3.1	-4.9	-21.8
Peru	1980Q1	-3.7	-8.7	-25.7
Peru	1983Q1	-8.4	-10.0	-12.9
Philippines	1983Q4	-7.7	-9.6	-39.1
Russia	1998Q4	-10.8	-10.6	-38.3
South Africa	1985Q3	-2.1	-8.1	-8.2
South Africa	1989Q4	1.5	-1.3	6.4
Average		-5.9	-9.8	-22.5

Notes: The selection of default episodes is guided by data availability. Starting dates of default episodes are from Levy-Yeyati and Panizza (2006). Variables are in logs and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Numbers reported correspond to maximum deviation within the quarter of default and the next four quarters. All deviations are in percentages.

Table 2. Foreign currency debt default episodes

Country	Bond debt	Bank debt	Num. default episodes
Argentina	1989; 2001-2005	1982-1993; 2001-2005	3
Brazil		1983-1994	1
Ecuador	1999-2000	1982-1995; 2008-2009	3
Indonesia		1998-1999; 2000; 2002	2
Malaysia			0
Mexico		1982-1990	1
Peru		1980; 1984-1997	2
Philippines		1983-1992	1
Russia	1998-2000	1991-1997	2
South Africa		1985-1987; 1989; 1993	3
Turkey		1982	1

Sources: Beers and Chambers (2006) and Beers, Chambers and Ontko (2010)

Table 3. Business cycles and sovereign spreads in emerging economies

Country	(1) $\sigma(y)$	(2) $\frac{\sigma(c)}{\sigma(y)}$	(3) $\frac{\sigma(i)}{\sigma(y)}$	(5) $\sigma(\frac{nx}{y})$	(6) $\sigma(spr)$	(7) $\rho(\frac{nx}{y}, y)$	(8) $\rho(spr, y)$
Argentina	4.15	1.00	3.42	1.60	6.81	-0.91	-0.63
Brazil	1.97	1.02	2.61	0.86	3.92	-0.05	-0.32
Ecuador	2.18	2.13	1.64	3.70	4.23	-0.47	-0.48
Indonesia	3.12	1.73	1.37	2.29	1.62	-0.49	-0.29
Malaysia	2.82	2.07	2.05	4.40	1.36	-0.60	-0.80
Mexico	3.03	1.28	2.08	2.05	1.71	-0.38	-0.80
Peru	5.03	1.17	1.80	2.36	2.07	-0.41	-0.19
Philippines	2.80	1.51	2.62	2.92	1.39	-0.43	-0.21
Russia	3.46	1.50	1.99	3.83	3.14	0.02	-0.42
South Africa	1.66	2.31	1.60	2.39	0.90	-0.55	-0.18
Turkey	3.64	1.37	2.14	2.33	2.22	-0.61	-0.68
Average	3.08	1.55	2.12	2.61	2.67	-0.44	-0.45
AG	2.74	1.45	3.91	3.22	N.A.	-0.51	N.A.
NP	2.79	1.71	3.29	2.40	2.32	-0.61	-0.55

Notes: Sovereign spreads standard deviation, in column 6, is conditional on being in good credit standing. For all cases but the latter, variables are in logs and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. All standard deviations are in percentages. Rows AG and NP are taken from Table 1 in Aguiar and Gopinath (2007) and Table 1 in Neumeyer and Perri (2005), respectively.

Table 4: The benchmark model: parameter values

Risk aversion	σ	2
Curvature of labor	ω	1.455
Adj. factor for labor	ψ	targets $l = 1/3$
Capital share	α	0.36
Depreciation	δ	0.025
Mean of transitory shock	μ_z	$-\frac{1}{2}\sigma_\varepsilon^2$
Autorregresive coeff. of transitory shock	ρ_z	0.95
Std. deviation of transitory shock	σ_ε	0.007*1.8
World rate interest rate	r^w	1%
Discount factor	β	0.91
Direct loss of output in autarky	γ	2%
Probability of redemption	λ	10%

Table 5. Business cycles and sovereign debt and default properties: the data vs the model

Business cycles properties:			Sovereign debt properties:		
	Data	Benchmark		Data	Benchmark
$\sigma(y)$	3.08	3.10	Mean$(debt/y)$		17.47
$\sigma(c)/\sigma(y)$	1.55	0.87	Default rate	1.18	0.28
$\sigma(l)/\sigma(y)$	N.A.	0.69			
$\sigma(i/y)$	1.56	0.92			
$\sigma(nx/y)$	2.61	0.60			
$\rho(nx/y, y)$	-0.44	-0.28			
$\rho(nx/y, i/y)$	-0.59	-0.84			
$\sigma(spr)$	2.67	0.012			
$\rho(spr, y)$	-0.45	-0.26			

Notes: All standard deviations, the mean debt to output ratio and the default rate are in percentages.

Table 6. Business cycles and sovereign debt and default properties: sensitivity analysis

	(1) Benchmark	(2) $\beta = 0.87$	(3) $\beta = 0.93$	(4) $\lambda = 0.044$	(5) $\lambda = 0.25$	(6) $\gamma = 0.01$	(7) $\gamma = 0.03$
$\sigma(y)$	3.10	3.20	3.04	3.17	3.04	3.04	3.16
$\sigma(c)/\sigma(y)$	0.87	0.92	0.84	0.93	0.84	0.85	0.89
$\sigma(l)/\sigma(y)$	0.69	0.68	0.69	0.69	0.69	0.69	0.68
$\sigma(i/y)$	0.92	0.92	0.93	1.42	0.71	0.73	1.18
$\sigma(nx/y)$	0.60	0.73	0.55	1.34	0.25	0.30	0.96
$\rho(nx/y, y)$	-0.28	-0.24	-0.29	-0.21	-0.29	-0.29	-0.26
$\rho(nx/y, i/y)$	-0.84	-0.89	-0.80	-0.95	-0.66	-0.70	-0.91
$\sigma(spr)$	0.012	0.021	0.014	0.019	0.008	0.011	0.015
$\rho(spr, y)$	-0.26	-0.58	-0.05	-0.10	0.14	-0.06	-0.33
Mean$(debt/y)$	17.47	17.70	17.45	33.22	7.65	8.70	26.51
Default rate	0.28	0.40	0.17	0.24	0.24	0.27	0.28

Notes: All standard deviations, the mean debt to output ratio and the default rate are in percentages.

Figure 1. Output, consumption and investment dynamics around default episodes

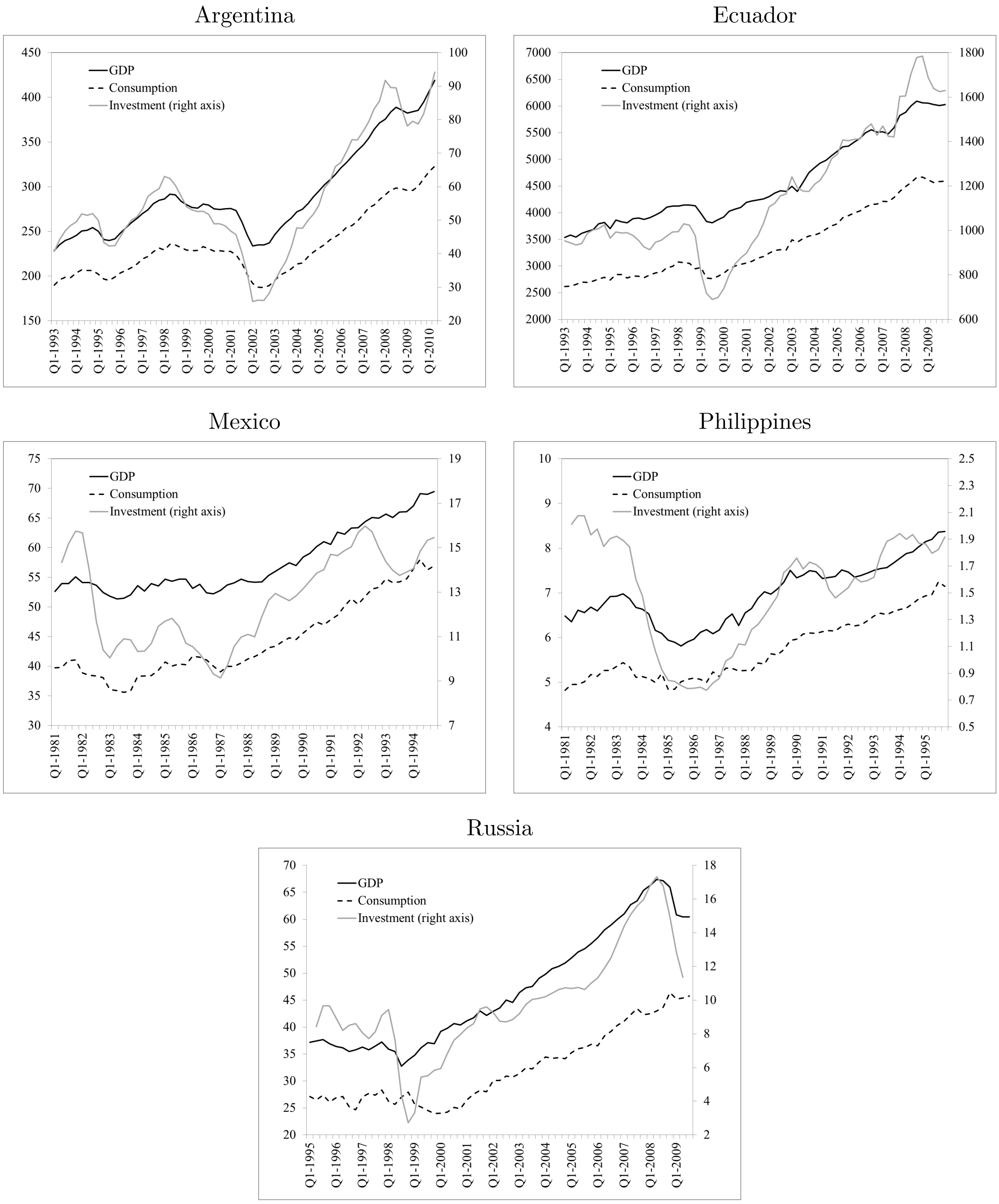


Figure 2. Default and repayment sets for $z = E(z)$

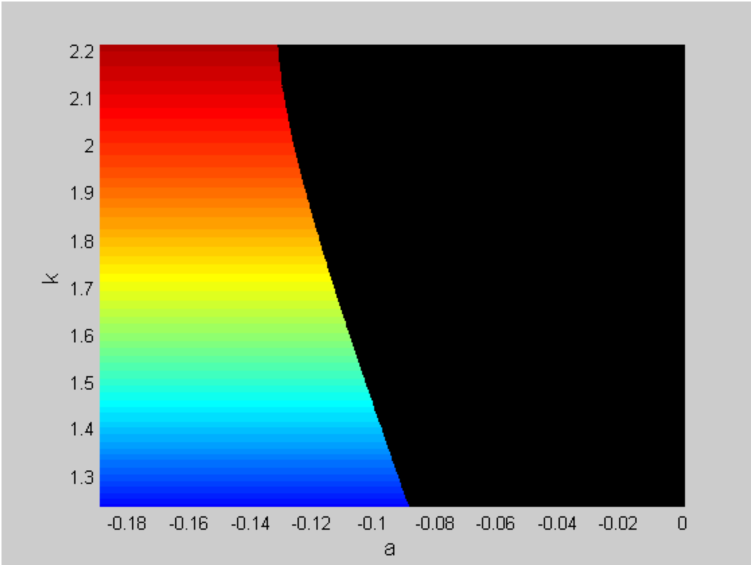


Figure 3. Absolute and relative credit limits

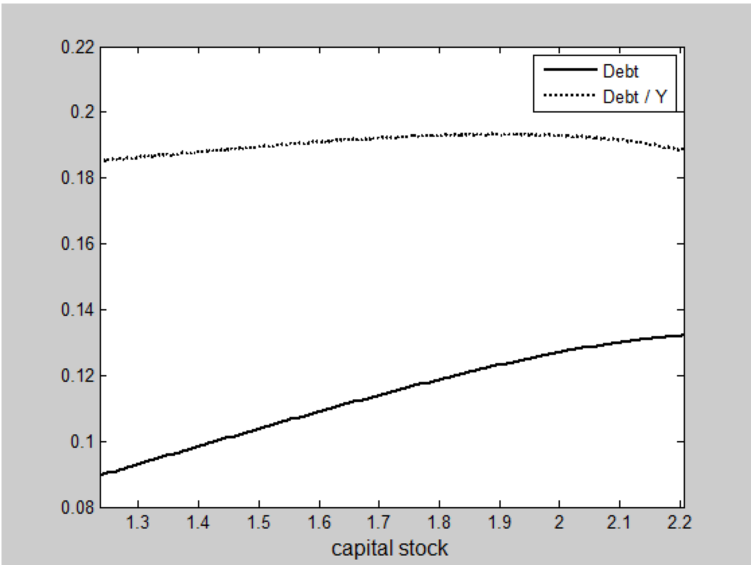
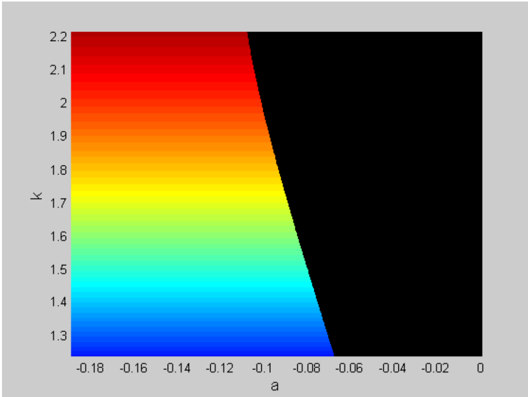
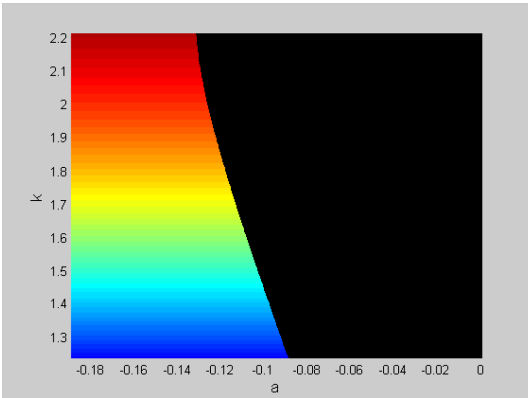


Figure 4. Default and repayment sets across z

Low productivity shock



Mean productivity shock



High productivity shock

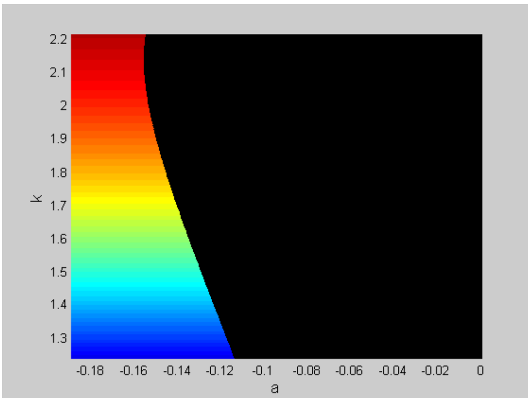
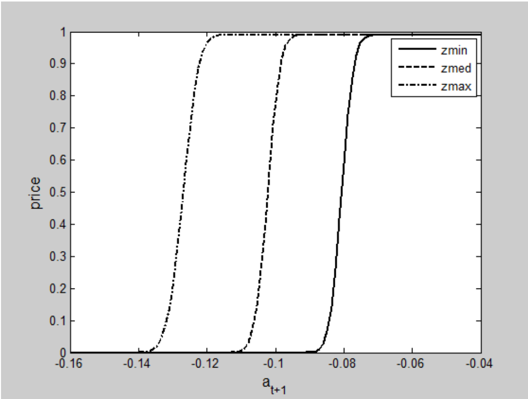
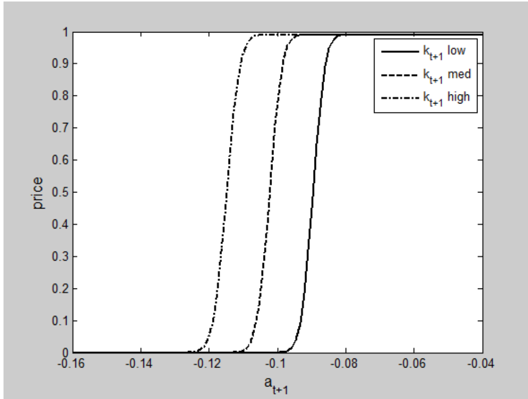


Figure 5. Debt price schedules

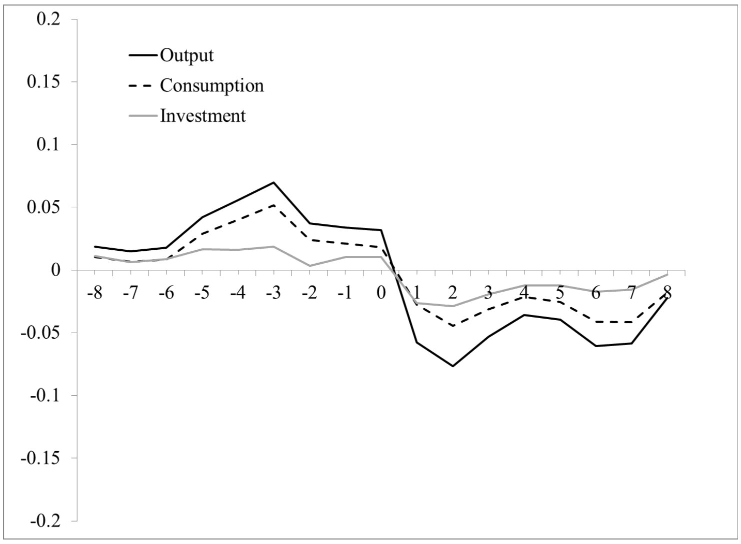
Price function across z | Price function for different levels of k'





Notes: The left panel is calculated for an intermediate level of k_{t+1} while the right one is calculated for the mean level of z.

Figure 6. Output, consumption and investment deviations from trend around a default episode



Notes: The date of default is normalized to zero.