Estimating Strategic Complementarity in a State-Dependent Pricing Model*

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December, 2010

Abstract

The predictions of a state-dependent pricing (SDP) model range from complete neutrality to results that are similar to those of a time-dependent approach. One way to generate relevant real effects is to incorporate strategic complementarities. This paper proposes a methodology to directly estimate the structural parameter related to strategic complementarity in pricing decisions in a SDP model. In addition, the method allows the estimation of some parameters of state-dependent pricing rules. The methodology uses the microfounded model to derive a structural, non-standard ordered probit. The identification strategy arises naturally from the parameter restrictions. We estimate the model using the microdata underlying the CPI-FGV in Brazil for the 1996-2006 period. The results indicate a substantial degree of strategic complementarity.

Keywords: State-dependent pricing, strategic complementarity, real rigidities
JEL Classification: E31, E32, E52

*This paper is based on the first chapter of the second author's Ph.D. thesis to be submitted to the Department of Economics of the Pontifical Catholic University of Rio de Janeiro (Puc-Rio). We would like to thank Ariel Burstein, Carlos Carvalho, Virgiliu Midrigan, Axel Weber and participants at the XII Annual Seminar of Inflation Targeting of the Central Bank of Brazil for helpful comments and discussions. We are grateful to IBRE-FGV (Brazilian Economics Institute of Getulio Vargas Foundation) for providing the dataset and to Solange Gouvea for helping us obtain access to the data. The views herein are those of the authors and do not necessarily reflect those of the Central Bank of Brazil.

1 Introduction

In recent years there has been an increasing interest in the study of price rigidity. It became well understood that not only the extent of price rigidity matters for monetary policy effects, but also the type of dynamic price setting policy. In particular, monetary policy shocks tend to have much less effect on models where price setters use state-dependent rules[1] than in time-dependent models[2]. The reason is that in the former models the selection effect leads to higher average price changes in response to shocks than in time-dependent models[3]. Recently, Gertler and Leahy (2008) have shown that when strategic complementarity in prices are large enough, even state-dependent models may generate aggregrate price-level stickness comparable to those in time-dependent model[4]. As a result, this research led us to conclude that the macroecomic effect of shocks depend on the details of the price-rigidity mechanism and the structural interactions among price-setters.

Additionally, the recent access to vast amounts of micro price data enabled the direct measurement of some those micro features, as the frequency of price adjustments and size of price adjustments[5], while estimulating research strategies to unveil the characteristics that are not directly observable. In this direction[6], Klenow and Willis (2006), Burstein and Hellwig (2007), Kryvtsov and Midrigan (2009), and Bils, Klenow and Malin (2009) pursue different strategies in relating aspects of the micro data with the degree of strategic complementarity (real rigidity). They all found indirect evidence against high degree of strategic complementarities in micro data, creating a huge gap regarding the degree of strategic complementarity necessary to explain macroeconomic effects.

In this paper we use individual price data from Brazilian CPI of Getulio Vargas Foundation (CPI-FGV) from 1996 to 2006 in order to directly estimate the degree of strategic complementarity in price-setting. Our approach has three distinctive features: i) we benefit from large amount of macroeconomic variation in Brazilian sample in those eleven years; ii) we estimate the degree of strategic complementarity by quasi-maximum likelihood, deriving the likelyhood function for price changes assuming that firms follow state-dependent pricing policies; iii) we allow for sector heterogeneity in pricing rules and in the structural parameters, estimating also the model for each sector separately and for the aggregate economy.

Barros et al. (2009) and Gouveia (2007) have used this same dataset to study the

[1] Some examples are Almeida and Bonomo (2002), Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2009), Nakamura and Steinsson (2009).

[2] This includes models where prices are fixed between adjustments, as Taylor (1979), Calvo (1983), and Bonomo and Carvalho (2004, 2010), as models where prices change continuously and information time is exogenous, as Mankiw and Reis (2004), and Reis (2006).

[3] Extreme cases are Caplin and Spulber (1987) model, for the state-dependent pricing rules and Calvo (1983) for time-dependent pricing rules. In the former case, the selection effect is so strong that it generates money neutrality. In Calvo (1983) there is no selection effect, since the firms that change their prices are randomly drawn.

[4] Since Ball and Romer (1990), it has been known that strategic complementarities amplify the impact of nominal rigidities.

[5] Bils and Klenow (2004), Nakamura and Steinson (2008), and Dhyne at al (2008).

[6] Dias, Marques, and Silva (2007), Klenow and Krivtsov (2008), and Midrigan (2008) investigate whether state or time dependent pricing rule are more in line with features of micro data.

relation between price setting and macroeconomic variables. In particular, Barros et al. (2009) have shown that the frequency of price change is significantly and positively related to inflation, output and exchange rate depreciation. If we assume that pricing rules are state-dependent, this should result from the relation of the frictionless optimal price with those same variables. Our identification strategy in this paper is then to infer the degree of strategic complementarity from the relation between the frictionless optimal price and macroeconomic variables which result from the microfounded model.

However, we observe price changes, and not the change in the frictionless optimal price. In order to estimate the model, we assumed that firms follow a two-sided asymmetric S,s rule and are also subject to aggregate and idiosyncratic shocks. The resulting econometric model for price-adjustments is a non-standard ordered probit model, where the emerging autocorrelation and heteroskedasticity in the residuals are treated. The parameter measuring strategic complementarity is obtained from parameter restrictions in the structural model. In order to account for heterogeneity in price-setting, we also estimated the model separately for each sector. In addition, by modeling the S,s pricing rules in an ordered probit model, our methodology is able to clean the selection effect and isolate strategic complementarity in a suitable way, providing a more straightforward way to measure real rigidities than those proposed in the literature.

Our results indicate that the parameter ζ measuring strategic complementarity ranges from 0.03 to 0.11 for the economy as a whole, implying a substantial degree of strategic complementarity. To give a sense of magnitude, the values for the deep parameters assumed in Gertler and Leahy (2008), whose S,s pricing model with strategic complementarities displays a significant real effect coming from a monetary shock, imply a degree of strategic complementarity of 0.08, inside the range of our estimations. Similar results of substantial strategic complementarities are found for the seven sectors of the CPI-FGV individually. Our results reconcile micro and macro effects, and are at odds with former indirect micro evidence against substantial degree of strategic complementarities.

Our methodology also allows estimating some parameters of the pricing rule as well as the variance of shocks affecting firms. We document some stylized facts about price rigidity in Brazil and relate them to the estimated variance of shocks and parameters of the pricing rules in each sector of the CPI-FGV and for the economy as a whole. In general, the results seem to explain the stylized facts.

The remaining parts of the paper ir organized as follows. In the next section we review the various sources of strategic complementarities emphasized in the literature and some papers that use indirect way to measure their relevance in actual data. In section 3 we derive the theoretical model and the equation for the frictionless optimal price. We also characterize the S,s pricing rule followed by the firms. In section 4 we derive the econometric methodology and present the identification strategy used to estimate the parameter of interest. Section 5 describes the dataset. Section 6 presents the estimation results and section 7 concludes.

2 Real rigidities and their measurement

Real rigidities (strategic complementarities) are mechanisms that dampen the price responses of firms when they have the opportunity of repricing, i.e., reduce the individual firm's willingness to respond to a shock when other firms do not respond right away.

Basically two types of real rigidities have received attention in the recent literature. The first type is real rigidities at the macro/industry level and emphasizes either input-output linkages across sectors or real factor price rigidities at the aggregate level that cause slow response of real marginal cost to output fluctuations. This slow responsiveness of real marginal cost to shocks is obtained theoretically with variations in the model that include the use of intermediate inputs, as in Basu (1995), segmented input markets as in Woodford (2003) and real wage rigidity as in Blanchard and Galí (2007).

The second type of real rigidities emphasizes the strategic interaction at the micro/firm level. This strategic complementarity arises from losses by an individual firm in having its price deviating from the prices of its competitors. The literature has stressed factors such as upward-sloping marginal cost (decreasing returns to scale) as in Burstein and Hellwig (2007) and Kryvtsov and Midrigan (2009) and non-CES (smoothed kinked) demand curves as in Kimball (1995) and Klenow and Willis (2006) as sources of concavity of the firm's profit function that may lead to strategic complementarities in pricing decisions.

Even though the concept of real rigidity is very appealing, it is hard to identify and measure its presence in the actual data. The difficulty mainly comes from two facts. First, strategic complementarity is linked to the firm's desired price (or frictionless optimal price), which is unobservable. In addition, real rigidities imply a softened response of the firm's prices, conditional on adjustment, to a marginal cost shock, but data on marginal costs is usually unavailable. That is why the literature has basically relied either on calibrations or indirect empirical tests to measure the presence of strategic complementarities in the data. Examples include the use of the reset price inflation concept in Bils, Klenow and Malin (2009) and in Gopinath and Itskhoki (2010), and calibrated models trying to reproduce some features of microdata, such as Klenow and Willis (2006), Burstein and Hellwig (2007), Kryvtsov and Midrigan (2009) among others.

In the present paper we depart from the recent literature by proposing a *direct* way to identify and measure strategic complementarities in pricing decisions. The metric we use to measure real rigidities is in fact very standard in the theoretical literature—it is given by the magnitude of the elasticity of the firm's frictionless optimal price with respect to the aggregate price—and has a direct interpretation—how much the firm's optimal price depends on the prices of its competitors. Although very straightforward, this measure has never been used before in papers trying to estimate real rigidities in the data, possibly because frictionless optimal price is unobserved. Our approach circumvent this fact by treating the frictionless optimal price as a latent variable in an ordered probit model that uses microdata at the firm level, and controlling for the aggregate shocks that affect all firms altogether, we propose a method to match the probit model parameters to the state-dependent pricing model structural parameters, including the one measuring real rigidities.

3 A model of state-dependent pricing

In this section we derive the model on which the econometric estimates are based and define the way we measure strategic complementarities. The model has three main elements. First, households obtain utility from consumption goods and disutility from supplying labor; and firms supply differentiated goods in a monopolistically competitive environment. In the (segmented) labor market, households and firms behave competitively. Second, we assume that firms must pay a fixed adjustment cost to change prices and, therefore, they will follow state-dependent pricing rules. Third, there are aggregate shocks and idiosyncratic productivity shocks.

3.1 Frictionless optimal price[7]

The representative household seeks to maximize the discounted sum of utilities

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t; \xi_t) - \int_0^1 v(L_{i,t}; \xi_t) di \right], \tag{1}$$

where C_t is a consumption index and $L_{i,t}$ is the quantity of labor of type i supplied. $v(L_{i,t}; \xi_t)$ represents the disutility of working and ξ_t is a vector of aggregate shocks. The consumption index over which utility is defined is given by

$$C_t = \left[\int_0^1 C_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/\theta-1}. \tag{2}$$

with $\theta > 1$, and $C_{i,t}$ is the consumption of variety i.

In this setting, the demand for an individual product has the familiar form:

$$C_{i,t} = C_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}, \tag{3}$$

where C_t is the aggregate consumption and the price index is

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}. \tag{4}$$

The optimal quantity of labor of type i supplied is implicitly given by

$$\frac{v_L(L_{i,t}; \xi_t)}{u_C(C_t; \xi_t)} = \frac{W_{i,t}}{P_t}, \tag{5}$$

where $W_{i,t}$ is the wage for labor of type i.

There is a continuum of monopolistically competitive firms supplying differentiated goods. We assume that each firm has the production function:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} M_t^{(1-\alpha)}, \tag{6}$$

[7] See Appendix A for a detailed derivation of the frictionless optimal price from fundamentals.

where $A_t(i)$ is a productivity factor and M_t is a foreign input used in the production process. The exogenous nominal exchange rate is given by $\tilde{E}_t$. We assume that each of the differentiated goods uses a specialized labor input in its production. Therefore, the suppliers do not hire labor from a single homogeneous competitive labor market.

If prices were perfectly flexible, i.e., if producer i could adjust prices continuously, it would choose $P^*_{i,t}$ to maximize profits according to the usual markup rule:

$$\frac{P^*_{i,t}}{P_t} = \mu\psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}), \tag{7}$$

where $\psi(.)$ is the real marginal cost, Y_t is the aggregated output, E_t is the real exchange rate and $\mu \equiv \theta/(\theta - 1)$ is the firm's desired markup.

A first-order log-linearization of this equation around the steady-state equilibrium in the case of flexible prices leads to the following equation for the frictionless optimal price[8]:

$$\begin{aligned} p^*_{i,t} &= \kappa + \zeta\mathcal{Y}_t + (1-\zeta)p_t + \frac{(1-\alpha)}{1+\alpha\omega\theta}e_t + \tilde{\xi}_t + \tilde{a}_{i,t} \\ &= \kappa + \boldsymbol{x}_t'\boldsymbol{\beta} + \tilde{\xi}_t + \tilde{a}_{i,t} \end{aligned} \tag{8}$$

where $\mathcal{Y}_t$ is the nominal expenditure, κ is a constant combination of primitive parameters, $\boldsymbol{\beta} = [\zeta, (1-\zeta), (1-\alpha)/(1+\alpha\omega\theta)]'$ and $\boldsymbol{x}_t = (\mathcal{Y}_t, p_t, e_t)'$. The variables $p^*_{i,t}$, $\mathcal{Y}_t$, p_t and e_t are in logarithm. $\tilde{\xi}_t$ is the aggregate shock and $\tilde{a}_{i,t}$ is the idiosyncratic shock.

Observe that when $\zeta < 1$, the optimal price the firm would like to charge depends positively on the prices set by its competitors (in our model represented by p_t), which we refer to as strategic complementarities in price setting. Therefore, the parameter ζ measures real rigidities—the smaller is ζ, the larger is the degree of strategic complementarities in pricing decisions—and a consistent estimation of it can provide an assessment of the intensity of real rigidities in the data.

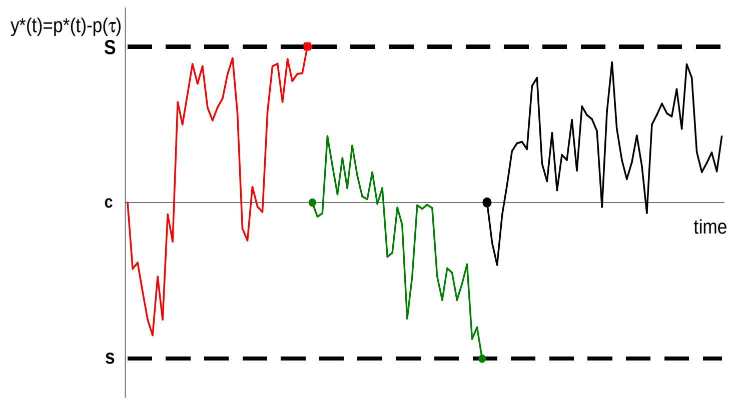
3.2 The pricing problem

We assume that the firm cannot adjust its price $p_{i,t}$ without paying a lump-sum cost F. In this case, it is not optimal for the firm to charge $p^*_{i,t}$ at all times. On the other hand, letting the price drift away from the optimal entails expected profit losses. Individual firms take $\mathcal{Y}_t$, p_t and e_t as given and solve for the optimal pricing rule. We do not derive the optimal policy here, but as will become clear ahead, for our purposes it is enough to know that in this case firms will follow S,s pricing policies, as illustrated in the figure below. The format of the pricing policy suffices to derive the econometric model to be estimated.

The graph shows a possible path for $y^*_{i,t} \equiv p^*_{i,t} - p_{i,\tau_t}$, where τ_t represents the time of the last price adjustment. Observe that τ_t has an index t because it refers to the last adjustment when we are considering the time t. The thresholds s and S

[8] Lowercase means that the variable is in logarithm.

Figure 1: A state-dependent pricing rule



determine the moments of price changes. While variable $y^*_{i,t}$ is inside the range (s, S), the firm maintains its price fixed. When $y^*_{i,t}$ reaches the threshold S, however, $p^*_{i,t}$ is sufficiently above the actual price charged and it is optimal for the firm to pay the cost F and increase its price. On the other hand, when the threshold s is reached, $p^*_{i,t}$ is sufficiently below $p_{i,t}$ and the firm decreases its price. In case of price change, the variable $y^*_{i,t}$ is set equal to c. The point c, however, may or may not be equal to zero. For instance, because the firm knows that it will maintain its actual price fixed for some time interval, when it adjusts it will be optimal to do so putting $p_{i,t}$ somewhat above $p^*_{i,t}$.

4 Econometric methodology

Using the equation (8) for the frictionless optimal price and the S,s pricing rule, we now derive the econometric methodology to estimate the parameter of strategic complementarity and some other parameters of the structural model, in particular those of the pricing rule.

In the dataset we only observe the price of each item charged by the firm over time, $p_{i,t}$, and the dates of price changes. The variable $y^*_{i,t} = p^*_{i,t} - p_{i,\tau_t}$ is unobservable. Define the observable variable $y_{i,t}$ as

$$y_{i,t} = \begin{cases} 1, & if \;\; p_{i,t} > p_{i,t-1} \\ 0, & if \;\; p_{i,t} = p_{i,t-1} \\ -1, & if \;\; p_{i,t} < p_{i,t-1} \end{cases} \tag{9}$$

By the above pricing rule, at the moment of a price change $p^*_{i,\tau_t} - p_{i,\tau_t} = c$. Then we can write

$$
\begin{aligned}
y^*_{i,t} \equiv p^*_{i,t} - p_{i,\tau_t} &= (\kappa + \boldsymbol{x}'_t\boldsymbol{\beta} + \tilde{\xi}_t + \tilde{a}_{i,t}) - (\kappa + \boldsymbol{x}'_{\tau_t}\boldsymbol{\beta} + \tilde{\xi}_{\tau_t} + \tilde{a}_{i,\tau_t} - c) \\
&= (\boldsymbol{x}_t - \boldsymbol{x}_{\tau_t})'\boldsymbol{\beta} + c + (\tilde{\xi}_t - \tilde{\xi}_{\tau_t}) + (\tilde{a}_{i,t} - \tilde{a}_{i,\tau_t}) \\
&= \boldsymbol{z}'_{i,t}\boldsymbol{\beta} + c + (\tilde{\xi}_t - \tilde{\xi}_{\tau_t}) + (\tilde{a}_{i,t} - \tilde{a}_{i,\tau_t}) \qquad (10)
\end{aligned}
$$

where $\boldsymbol{z}_{i,t} = \boldsymbol{x}_t - \boldsymbol{x}_{\tau_t}$. Note that although only macroeconomic variables are contained in vector $\boldsymbol{x}_t$, $\boldsymbol{z}_{i,t}$ is not the same for all firms. Once firms change their prices at different points in time, the cumulative variation will also be different, and at the end what matters for a firm's pricing decision is this cumulative change in the aggregate variables since the moment of its last price adjustment, τ_t. It is precisely this difference in each price spell which enables us to estimate the model with only aggregate variables as regressors, as will be clear ahead. For this reason we add the subscript i in $\boldsymbol{z}_{i,t}$.

Following the recent literature such as Boivin, Giannoni and Mihov (2009) that has provided evidence that the common component of inflation is very persistent, we approximate the aggregate shock by[9]

$$
\tilde{\xi}_t = \tilde{\xi}_{t-1} + v_t, \quad v_t \sim iid(0, \sigma_v^2). \qquad (11)
$$

In addition, to identify the parameters we assume that the idiosyncratic shock is given by $\tilde{a}_{i,t} = a_i + a_{i,t}$, where a_i is an individual fixed effect component and[10]

$$
a_{i,t} = \eta + a_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2). \qquad (12)
$$

Under these assumptions we can write equation (10) as[11]

$$
\begin{aligned}
y^*_{i,t} \equiv p^*_{i,t} - p_{i,\tau_t} &= \boldsymbol{z}'_{i,t}\boldsymbol{\beta} + c + (\tilde{\xi}_t - \tilde{\xi}_{\tau_t}) + (a_{i,t} - a_{i,\tau_t}) \\
&= \eta\delta_{i,t} + \boldsymbol{z}'_{i,t}\boldsymbol{\beta} + c + \sum_{j=1}^{T}\gamma_j d_{t,j} + u_{i,t}, \qquad (13)
\end{aligned}
$$

where $\delta_{i,t}$ is the time interval between t and τ_t,

$$
u_{i,t} = \varepsilon_{i,t} + ... + \varepsilon_{i,t-\delta_{i,t}+1} \sim \mathbb{N}(0, \delta_{i,t}\sigma^2)
$$

and

$$
d_{t,j} = \begin{cases} 1, & if \;\; j \in [t, t-\delta_{i,t}+1] \\ 0, & if \;\; otherwise \end{cases}, \qquad j = 1, ..., T. \qquad (14)
$$

[9]Maybe in the long run the process is stationary, but in the short run the process is persistent enough to be approximated by this assumption.

[10]In the main specification we assume that $a_{i,t}$ is a random walk with drift because we would like to take into account that productivity grows over time. But, given the recent evidence that idiosyncratic shock is short-lived, we also estimate the model assuming that $a_{i,t}$ is a white-noise, i.e., $a_{i,t} = \varepsilon_{i,t}$ and $\varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2)$. See the section with results.

[11]To obtain equation (13) we iterate backward $a_{i,t}$ and $\tilde{\xi}_t$ until the moment of the last price adjustment and use some algebra. Details are given in Appendix B.

Note that the dummy variables assume value one on each time between t and τ_t (the time of the last price adjustment). These terms are controlling for the aggregate shocks that hit the economy since the moment in which the firm has set its price for the last time. In addition, every firm i in our panel data that repriced on time τ_t and maintained its price fixed until time t has the same dummy variables with value 1. And more generally, every firm that kept its price unchanged during a period overlapping the time interval between τ_t and t has the respective dummy variables assuming value 1. By controlling these common shocks in our probit model (in addition to the other regressors), we want to make sure that two firms that changed prices at the same time repriced not because they were affected by the same shock, but because of some kind of strategic interaction between them.

Also observe that the residuals $u_{i,t}$ are naturally correlated. However, given our assumptions we know its autocorrelation structure—$u_{i,t}$ is a moving average $MA(\delta_{i,t} + 1)$ process—and we make use of this information in the estimation procedure.

Defining $\boldsymbol{w}_{i,t} = (\delta_{i,t}, \boldsymbol{z}'_{i,t}, \boldsymbol{d}'_t)'$, where $\boldsymbol{d}_t = (d_{t,1}, ..., d_{t,T})'$, and using the pricing rule, we can obtain the probability of observing a price increase:

$$
\begin{aligned}
Pr[y_{i,t} = 1|\boldsymbol{w}_{i,t}] &= Pr[y^*_{i,t} \geq S|\boldsymbol{w}_{i,t}] \\
&= Pr[\eta\delta_{i,t} + \boldsymbol{z}'_{i,t}\boldsymbol{\beta} + c + \sum_{j=1}^{T} \gamma_j d_{t,j} + u_{i,t} \geq S|\boldsymbol{w}_{i,t}] \\
&= Pr\left[\frac{u_{i,t}}{\sqrt{\delta_{i,t}}\sigma} \geq \frac{S - c - \eta\delta_{i,t} - \boldsymbol{z}'_{i,t}\boldsymbol{\beta} - \sum_{j=1}^{T}\gamma_j d_{t,j}}{\sqrt{\delta_{i,t}}\sigma}\right] \\
&= 1 - \Phi\left(\frac{S-c}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{t,j}}{\sqrt{\delta_{i,t}}}\right) \\
&= 1 - \Phi\left(\pi_1 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j \ddot{d}_{t,j}\right)
\end{aligned}
$$

where the variables with two dots represent the variables divided by $\sqrt{\delta_{i,t}}$, the parameters with tilde means that the parameters is scaled by σ, $\Phi(.)$ is the cumulative distribution function of a standard normal variable and $\pi_1 = (S - c)/\sigma$. In the third line we have used the fact that $u_{i,t}$ is independent of $\boldsymbol{w}_{i,t}$.

Likewise, we can derive the probability of observing the other two possible outcomes for $y_{i,t}$. This results in the following ordered probit model for price changes:

$$
\begin{aligned}
Pr[y_{i,t} = 1|\boldsymbol{w}_{i,t}] &= 1 - \Phi\left(\frac{S-c}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{t,j}}{\sqrt{\delta_{i,t}}}\right) \\
&= 1 - \Phi\left(\pi_1 \ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j \ddot{d}_{t,j}\right)
\end{aligned}
$$

$$Pr[y_{i,t}=0|\boldsymbol{w}_{i,t}] = \Phi\left(\pi_1\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{t,j}\right) - $$

$$- \Phi\left(\pi_0\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{t,j}\right) \quad (15)$$

$$\begin{aligned} Pr[y_{i,t}=-1|\boldsymbol{w}_{i,t}] &= \Phi\left(\frac{s-c}{\sigma}\frac{1}{\sqrt{\delta_{i,t}}} - \frac{\eta}{\sigma}\sqrt{\delta_{i,t}} - \frac{\boldsymbol{z}'_{i,t}}{\sqrt{\delta_{i,t}}}\frac{\boldsymbol{\beta}}{\sigma} - \sum_{j=1}^{T}\frac{\gamma_j}{\sigma}\frac{d_{t,j}}{\sqrt{\delta_{i,t}}}\right) \\ &= \Phi\left(\pi_0\ddot{1}_{i,t} - \tilde{\eta}\ddot{\delta}_{i,t} - \ddot{\boldsymbol{z}}'_{i,t}\tilde{\boldsymbol{\beta}} - \sum_{j=1}^{T}\tilde{\gamma}_j\ddot{d}_{t,j}\right) \end{aligned}$$

The parameters are estimated by quasi-maximum likelihood method, which guarantees a consistent estimation. Inference is carried out using a robust variance-covariance matrix for autocorrelation and heteroskedasticity. Once we know the structure of the model in which our probit is based, given our assumptions we can derive the variance and covariance of $u_{i,t}$ and use this piece of information in the estimation of the parameters' robust variance-covariance matrix. Details are given in Appendix B.

4.1 Identification

The identification strategy to recover the structural parameter ζ measuring strategic complementarity in the model arises naturally from the restrictions on the frictionless optimal price parameters in equation (8) and the ordered probit model parameters in (15).

Observe that from equations (8) and (15), we have:

$$\tilde{\beta}_1 = \frac{\zeta}{\sigma}, \quad (16)$$

and

$$\tilde{\beta}_2 = \frac{1-\zeta}{\sigma}. \quad (17)$$

Then, by dividing equation (16) by (17) and isolating ζ we can obtain:

$$\zeta = \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}. \quad (18)$$

Similarly, we can estimate the variance of shocks σ^2 hitting the firms in each sector and in the economy as a whole. To obtain σ, substitute equation (18) in (16):

$$\sigma = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}. \tag{19}$$

Since we are able to write ζ and σ as a function of the probit model parameters, we can use the Delta method to construct confidence intervals for these parameters. Details are given in Appendix B.

Parameters π_0 and π_1 of the probit model are combinations of the pricing rule parameters and the variance of shocks. Even though we cannot obtain all parameters of the pricing rules, once we identify the variance of shocks we can estimate at least the widths of the top and bottom bands of the pricing rules. They are obtained respectively as

$$S - c = \pi_1 \sigma \quad \text{and} \quad s - c = \pi_0 \sigma. \tag{20}$$

5 Data

In this section we describe the dataset based on which the estimations are made and register some stylized facts that emerge from the data. These facts are not new. They are already documented in the literature[12]. However they will be useful as a guideline to our methodology—especially as a measure of the goodness of fit of the models we estimate—and we will relate them to our estimation results, in particular those of the pricing rules parameters.

5.1 Dataset description

The dataset used here consists of primary information of price quotes of individual products collected and used by the Brazilian Institute of Economics of the Getlio Vargas Foundation (FGV-IBRE) to compute the consumer price index (CPI-FGV). The CPI-FGV has wide coverage and it is the oldest Brazilian price index, being calculated since 1944.

The dataset contains information on prices at the firm level collected in the 12 largest Brazilian metropolitan regions, even though the coverage has changed during our sample period[13].

Currently, the CPI-FGV comprises 456 products and services grouped into seven different sectors: Food; Other Goods and Services; Education and Recreation; Housing; Medical and Personal Care; Transportation; and Apparel. The weight of each product or service used in computing the index mirrors the expenditure by households receiving income up to 33 times the minimum monthly wage, obtained from a household consumption survey—Pesquisa de Orcamento Familiar (POF)—also conducted by FGV.

[12] See, for example, Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008), Midrigan (2006), among others. A recent survey is Klenow and Malin (2010).

[13] Up to the end of 2000, the coverage comprised only the metropolitan regions of Rio de Janeiro and Sao Paulo. After January 2001, ten other cities were included in the survey: Belo Horizonte, Brasilia, Porto Alegre, Recife, Salvador, Belem, Curitiba, Florianopolis, Fortaleza and Goiania. At the beginning of 2005 the last five cities were dropped.

The data are systematically collected by FGV employees. Some data collections are made every ten days, while for other products prices are collected on a monthly basis. We refer to the most disaggregated level of data as an *item*. Each item is identified by a set of very specific characteristics, such as brand, model, packaging, type/variety that is sold in a particular outlet, in a specific city[14]. Each price also includes the exact date of collection. However, while the outlets are identified by codes, for confidential reasons no other feature of the firms, such as size, number of employees, revenues and costs amounts etc is available.

The number of items collected each month is not constant. There is variation due to item exclusions and inclusions over time. However, there is never substitution of an item by a similar one. Therefore, it is possible to follow the same item over time. When the price of a specific item is no longer collected, its price trajectory continues to be registered as missing. Thus, one can be assured that each change recorded is due to an actual price change.

5.2 Data sample and some stylized facts

In this subsection we describe the sample and register some stylized facts about price rigidity in Brazil. We have a very representative sample of the overall CPI-FGV (around 85%), containing 243 categories of products and services, whose prices are accompanied by a period of approximately 11 years, from 1996 to 2006. The seven sectors are very well represented. The sector with the lowest representation has products comprising 77.5% of its weight in the CPI-FGV (see Table 1).

Table 1: Original data sample

	Original Dataset		
Sector	**# of observations**	**# of trajectories**	**Representation**
Food	3,973,527	36,400	77.6%
Other Goods and Services	411,560	8,006	95.7%
Education and Recreation	346,095	13,613	88.4%
Housing	961,755	21,438	87.4%
Medical and Personal Care	1,087,647	17,922	91.0%
Transportation	149,185	5,026	85.4%
Apparel	501,202	19,919	77.5%
Total	7,430,971	122,324	84.8%

We start from the original full sample with approximately 7.4 million price quotes and 122 thousand quote lines, grouped in the seven sectors.

The original sample was treated in order to have a set of information more suited to our goals. First, we would like to have a dataset with monthly data. Thus, for products whose prices are collected every 10 days, in the aggregation we choose to keep always the first price quote in each month.

[14]For example, type I black beans of the Combrasil brand, sold in a 1kg package in the outlet number 16,352, in Belem.

Second, the very short price trajectories were excluded. Trajectories with less than 18 observations or with more than 30% of observations missing were dropped. We further refined our sample by treating specifically the remaining gaps. The gaps with only one observation missing were filled using the price collected in the immediately preceding month. Also, gaps with up to three observations missing, when preceded and succeeded by the same price, were filled with the last price available. In the case of four or more missing observations, we maintained the longest uninterrupted part of the price trajectory.

Third, retail prices are characterized by a significant number of temporary price decreases (sales or promotions). Following the recent literature, as Golosov and Lucas (2007), Midrigan (2006) and others, we treated observations identified as sales prices. Because sales are not explicitly classified in our database, we used the following algorithm to identify them. If a price decrease was large enough and reversed in the following months to levels near the one prevailing before the change, we classified that observation as a price sale[15]. Following Midrigan (2006), to deal with large price changes in a V shape, we repeated the algorithm three times. When a "sale" was identified, the price was replaced by the price collected in the immediately preceding period. Outlier were also treated. An observation was classified as an outlier when it was 10 times larger or 10 times smaller than the previous observation.

Finally, to avoid the problem of left-censored spells, for all price trajectories we dropped the observations before the first observed price change.

After this treatment were are left with a final data sample with approximately 3.2 million observations and 63.2 thousand price trajectories, very representative of the seven sectors of the CPI-FGV. The sector with the lowest number of observations (Transportation) still had more than 111 thousand price quotations.

Figure 2 below presents the histogram of the distribution of price changes for all sectors together, conditional on adjustment. The distribution is truncated, discarding the observations of 1st and 99th percentiles. For comparison, we add the density of a normal distribution with mean and variance equal to the distribution of price changes.

Table 2 presents some characteristics of the distribution with all sectors together and also of the distribution of each sector individually. We summarize these characteristics in the form of stylized facts listed below. Many of these facts are similar to those found in other studies using microdata of prices.

- *Fact 1: Prices change frequently, but the degree of price rigidity is quite different among sectors.*

  The mean aggregate price duration is 1.97 months, or approximately 59 days. This value is obtained by calculating the duration of each price spell on each item's price trajectory, and then computing a simple average of these durations for all sectors together, without considering their respective weights in the CPI-FGV. The duration obtained is very similar to the aggregate implied duration

[15]Formally, if $\frac{(p_{t-1}-p_t)}{p_{t-1}} > 25\%$ and $p_{t+1} \geq p_t \left(1 + \frac{p_{t-1}-p_t}{p_{t-1}}\right)$. One can always say that 25% is an arbitrary value, and/or that it is high for some sectors and/or low for some others. But we decided to keep a single rule to minimize arbitrariness. Changing this value does not significantly change our main results.

Figure 2: Distribution of price changes, conditional on adjustment

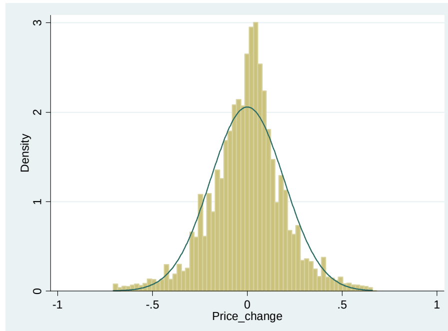


Table 2: Distribution of price changes, conditional on adjustment

Sector	Duration (days)	Mean of $\|\Delta p\|$ %	% of Decrease	Average decrease %	Average increase %
All sectors	59.27	15.99	0.44	-16.67	15.45
Food	50.53	16.27	0.45	-16.70	15.92
Apparel	53.89	25.32	0.48	-25.46	25.19
Housing	61.59	13.27	0.42	-14.04	12.72
Other Goods and Services	63.42	10.75	0.43	-11.05	10.53
Transportation	64.82	7.58	0.39	-7.13	7.86
Medical and Personal Care	75.90	12.35	0.41	-13.37	11.67
Education and Recreation	134.61	15.82	0.36	-17.66	14.78

Sector	mean of Δp %	Std deviation	Kurtosis	% of small Δp
All sectors	1.31	19.38	4.15	37.94
Food	1.15	19.41	3.81	37.27
Apparel	1.15	30.08	3.45	33.57
Housing	1.60	15.80	3.89	38.63
Other Goods and Services	1.31	12.48	3.16	33.69
Transportation	2.06	8.93	4.34	40.59
Medical and Personal Care	1.53	14.84	3.80	39.30
Education and Recreation	2.95	18.70	4.60	32.62

Note: 1) p is defined as the natural logarithm of the item price

2) All statistics are calculated based on unweighted price changes. Kurtosis is calculated excluding the top and bottom 1% of observations

3) Small Δp is defined as any price change whose absolute value is lower than 0.5 of the mean of $|\Delta p|$

of 1.8 months calculated in Barros et al. (2009)[16]. The table also shows the mean durations for each sector. The Food sector is the most flexible, while the Education and Recreation sector is the most rigid one, with durations of about 51 and 135 days, respectively. Therefore, there are considerable differences in the degree of price rigidity across sectors. Greater heterogeneity is found when sectors are disaggregated in more specific subsectors or products, as reported in Barros et al. (2009) and Gouveia (2007) for the case of Brazil and in Klenow and Malin (2010) for other countries. Because our main goal is not to study this heterogeneity, in the econometric estimations we maintain the aggregation level in the seven sectors.

[16] Even though they find a very different measure by calculating the frequency of price changes for each particular item and then aggregating the durations implied by these frequencies that were obtained in the more disaggregate level – 8.5 months. Gouvea (2007) estimated the aggregate duration between 2.7 and 3.8 months, depending on the calculation method.

- *Fact 2: On average, price changes (in absolute values) are large.*

  The average price change, considering both increases and decreases, is 1.31%. The small value is determined by the fact that increases and decreases cancel each other in the calculation. However, when considered in absolute values, price changes are in fact quite large: approximately 16%, on average. Although there are important differences between sectors, for all sectors the average change is large, which is consistent with the evidence presented by Klenow and Kryvtsov (2008) and Midrigan (2006). These results suggest that idiosyncratic factors are important for price setting, as modeled in a state-dependent pricing model, in which firms changing prices can be determined by idiosyncratic shocks[17].

- *Fact 3: Price decreases are frequent events.*

  Price decreases are not unusual events in our dataset. When considering all sectors, approximately 44% of price changes are price cuts. A similar result was found by Nakamura and Steinsson (2008). They found that in the U.S. economy, one-third of regular price changes are decreases.

- *Fact 4: On average price decreases are larger than price increases.*

  Note that except in the Transportation sector, when a firm changes its price, interestingly the percentage by which it decreases the price is larger than the percentage by which it increases. While the average price increase is 15.45%, the average price cut is 16.67%.

- *Fact 5: A large percentage of price changes is of small changes.*

  Consistent with evidence presented by Midrigan (2006), we found that a large fraction of price changes in Brazil are close to zero. Using the same criterion adopted by Midrigan to define "small", we conclude that in our dataset approximately 38% of price changes are small. This percentage is reinforced by the kurtosis coefficients. The value of 4.15 for the kurtosis of the distribution of all sectors indicates a higher concentration of observations around zero than that observed in a Gaussian distribution.

6 Estimation and results

In this section we use the dataset of individual prices just described combined with aggregate data to apply our methodology to estimate the parameter of strategic complementarity and some other parameters of interest of the structural model, in particular the variance of shocks and some parameters of the pricing policy, like the size of positive and negative price ajustments.

6.1 The estimated models

To have an idea of how important strategic complementarity is in different sectors as well as to have a single measure of real rigidities for the aggregate economy, we

[17] See Klenow and Malin (2009). Observe that we include idiosyncratic shocks in our model.

estimate the probit model in (15) for each one of the seven sectors of the CPI-FGV separately and also a model for the whole economy.

For both sectoral and aggregate model we estimate two specifications of the probit. In the first one, which we will call simple specification, we excluded the dummy variables that control for the aggregate shocks in (15). In the second, which we will name complete specification, we estimate the model exactly as it appears in (15). The objective of estimating the two specifications is to see the difference of controlling or not for the aggregate shocks, that is, the magnitude of the bias. For robustness, we also estimate the complete specification splitting the sample in two parts, using the data of each year separately and some other versions that we will present later.

Obviously the specification without the dummy variables is much simpler and computationally less intensive. In addition, as will become clear ahead, it is also easier to measure the model's ability to fit the data using the simple specification. But as we know that the absence of the dummy variables in the simple specification may cause some bias to the parameters estimation—once there may be some correlation between the residuals (which now include the aggregate shocks) and the aggregate variables included as regressors—our analysis will rely fundamentally on the complete specification results. After all, only after controlling for aggregate shocks we can argue that we manage to isolate strategic complementarities. In effect, in the complete specification, when we observe two firms jointly changing prices, we can argue that they are doing so because their pricing decisions are somehow complementaries and not because they were affected by a common shock. This means that we can consistently estimate the parameter ζ.

We combine the set of microdata described in the previous section with aggregate data for estimating both the simple and the complete specification. The dependent variable is the observed $y_{i,t}$ defined in (9). Following the theoretical model, there are five explanatory variables in the simple specification. The complete specification has the whole $\boldsymbol{w}_{i,t}$, i.e., it has one dummy variable for each month (constructed as given in (14) for each item) in addition to the five explanatory variables. The first two variables are respectively the square root of the time interval between t and τ_t and its inverse. These are obtained directly from the microdata for each item in the dataset. The last three variables are respectively the accumulated change in the aggregate price level $\Delta p_{i,t} = log(P_t/P_{\tau_t})$, in the nominal expenditure $\Delta \mathcal{Y}_{i,t} = log(\mathcal{Y}_t/\mathcal{Y}_{\tau_t})$ and in the exchange rate $\Delta e_{i,t} = log(E_t/E_{\tau_t})$ since the last price adjustment in time τ_t, divided by the square root of the time interval between t and τ_t, $\sqrt{\delta_{i,t}}$. Therefore, for each item in our database, in every time t, we calculate the changes in these aggregate variables between the moments t and τ_t. As previously pointed out, it is this difference in the length of price spells that allows us to estimate the model using basically aggregate variables as regressors—once firms reprice at different points in time, the accumulated changes of those variables since the last adjustment will be different. But we must divide the variables by $\sqrt{\delta_{i,t}}$ to account for the heteroskedasticity that appears in the residuals. It is worth noting, however, that two products with the same price adjustment history will have the same regressors.

As the measure of nominal expenditure we use the monthly nominal GDP series calculated by the Central Bank of Brazil using the quarterly national accounts of the Brazilian Institute of Geography and Statistics – IBGE. Our measure of aggregate price is the Consumer Price Index – IPC-FGV, which is calculated by the FGV from the set of microdata used here. The exchange rate is the monthly Real/US$ exchange rate (for sale) series provided by the Central Bank of Brazil.

To allow comparison among the sectoral estimations, we have ignored sectoral specificities and used the same regressors to estimate the model for each sector. However, at least some of these sectoral and firm specificities should appear in the form of a fixed effect. Observe, nonetheless, that we have a fixed effect a_i in our model, and once the variables in (10) are in differences, this individual effect disappears.

Before examining the estimation results, we will provide some information about the fit of the models. In this task we compare the implied price durations obtained from the estimated models with the durations calculated directly from the data. We also use the predicted probability of price change to assess the ability of the estimated model in inferring the actual degree of price rigidity in each sector. We carry out this exercise using the simple specification, since it is easier to predict the probabilities of price changes without worrying with the dummy variables. However, the use of the dummy variables in the complete specification should improve its fit, or at least should not worse it, which means that the exercise using the simple specification should give a good idea about the fit of the models.

Table (3) shows the estimated probabilities of price changes predicted by the simple specification of the model for each sector and also for the whole economy. They are obtained by calculating in the probit model the probability of a price increase, $Pr[y_t = 1|\bar{\boldsymbol{w}}_t]$, and the probability of a price decrease, $Pr[y_t = -1|\bar{\boldsymbol{w}}_t]$, using the average of the explanatory variables in the model. The sum of the two probabilities gives the probability of a price change. As we can see, the results in Table (3) provides the same picture (including the same ordering) about price rigidity in each sector that was previously obtained in the first stylized fact. The most flexible sector is Food, with a probability of price change of 0.63. The most rigid sector is Education and Recreation, with probability of only 0.25.

Table 3: Probabilities of price change and price duration, simple specification

Sector	**Pr[y_t= -1 or y_t=1|mean(w_t)]**	**Implied Duration (in days)**	**Duration (in days)**
All sectors	0.56	53.24	59.27
Food	0.63	47.35	50.53
Apparel	0.60	50.03	53.89
Housing	0.54	55.41	61.59
Other Goods and Services	0.53	56.82	63.42
Transportation	0.52	57.34	64.82
Medical and Personal Care	0.48	62.35	75.90
Education and Recreation	0.25	118.81	134.61

We also use these estimated probabilities to calculate a measure of induced duration of price spells to be compared to the actual duration. The implied duration is obtained by the inverse of the probability of price change[18]. Again, notwithstanding some differences, the estimated models make a good job. In general, the implied durations are very close to actual durations. For example, while the duration estimated from the data is about 59 days when considering all sectors together, the implied duration is approximately 53 days. For the Food and the Apparel sectors the difference is only about 3 days. These are remarkable results, if we consider that our theoretical model is very simple.

6.2 Strategic complementarity

After having provided some evidence that our estimated probit models make a good job in explaining the data, we now proceed to estimate the degree of strategic complementarity in firm's price decisions. Table (4) reports summarized results of the probit models parameters estimated for each sector and for the economy-wide case using the complete specification. Detailed results as well as results using the simple version are presented in Appendix C.

Before presenting the results regarding strategic complementarities, two comments are important. First, the parameters of the probit models for every sector and also for the aggregate economy are all significant at any of the usual significance levels. The parameters' signals are also in general as expected, except that we obtain negative parameters for the exchange rate variable in some sectors. Results for the other parameters—for p_t and $\mathcal{Y}_t$—seem fine. Of course those negative parameters may come from the fact that our theoretical model is too simple to capture every detail of each sector. Nonetheless, its structure is very standard in the literature and, as the previous exercise has shown, it can capture some salient facts about the firms' price setting behavior, which is our main purpose.

Second, the significance of the probit models parameters means that, at least in part, pricing decisions have a state-dependent component. As we have already pointed out in the introduction, Barros et al. (2009) have shown using the same dataset that the frequency of price change is significantly and positively related to inflation, output and exchange rate depreciation. In a state-dependent pricing model this should result from the relation of the frictionless optimal price with those same variables, and this is what our probit results are showing.

We infer the parameter of strategic complementarity from this relation between the frictionless optimal price and the macroeconomic variables which results from the microfounded model. The estimation of ζ comes from the probit parameters, through equation (18). Table (4) reports these estimations for each sector and for the aggregate economy, as well as a 95% confidence interval obtained using the Delta method.

The first important fact that Table (4) shows is that the estimated value for ζ in each sector and also in the economy-wide case is less than one. This result implies that firms' pricing decisions are strategic complementaries rather than substitutes. Actually, the upper bounds of the confidence intervals for all sectors are far from 1— the lower

[18]One can show that, for large samples, the inverse of the estimated probability is a consistent estimator of the durations of price spells. Other papers in the literature use the inverse of the frequency of adjustments. See, for example, Bils and Klenow (2004), Gouvea (2007) and works published by the ECB/IPN (`http://www.ecb.int/home/html/researcher_ipn.en.html`).

limit for strategic substitutability. The larger value for these estimated upper bounds is only 0.29, for the Education and Recreation sector.

Second, our results suggest that not only the pricing decisions are complementaries, but also that the degree of strategic complementarity is substantial in the data. The values obtained for ζ are close to zero in every estimation—for the aggregate economy, for example, the estimated value is 0.04, and for the Food sector its value is in fact negative, implying that the positive interaction between the frictionless optimal price of a given firm and the prices of its competitors is more than proportional. This negative value, however, is not obtained in the other estimations we made. What is robust in all estimations is a positive and close to zero value for ζ. The sector that has the largest estimated value is Education and Recreation.

Table 4: Probit parameters and strategic complementarity, complete specification

Sector	$\mathcal{Y}_t$	p_t	e_t	ζ	Conf. interval for ζ
All sectors	0.44	9.79	0.03	0.04	0.03 - 0.05
	(0.065)	(0.044)	(0.011)	(0.006)	
Food	-0.33	8.36	0.26	-0.04	-0.07 - -0.01
	(0.115)	(0.092)	(0.021)	(0.015)	
Apparel	1.16	6.22	-0.17	0.16	0.11 - 0.21
	(0.221)	(0.144)	(0.036)	(0.026)	
Housing	1.40	11.09	0.67	0.11	0.08 - 0.14
	(0.153)	(0.130)	(0.033)	(0.017)	
Other Goods and Services	2.44	12.69	-0.66	0.16	0.13 - 0.20
	(0.308)	(0.225)	(0.051)	(0.018)	
Transportation	2.32	11.20	0.92	0.17	0.13 - 0.22
	(0.359)	(0.236)	(0.059)	(0.024)	
Medical and Personal Care	1.36	11.93	-0.92	0.10	0.08 - 0.12
	(0.139)	(0.077)	(0.020)	(0.010)	
Education and Recreation	5.28	13.68	0.58	0.28	0.26 - 0.29
	(0.185)	(0.087)	(0.027)	(0.008)	

Note: These results are obtained using the complete specification
The robust standard deviations are in parenthesis
The confidence interval is 95% of confidence
The standard deviation of ζ was obtained by the Delta method

The estimation of the simple specification provides very similar results, as can be seen in Table (6) in Appendix C. Interestingly, the parameter of strategic complementarity obtained for the aggregate economy is somewhat larger than that estimated in the complete specification—0.09 compared to 0.04. Initially someone could imagine that controlling for aggregate shocks in the complete specification would lead to results indicating a lower degree of strategic complementarity, since joint repricing movements caused by common shocks could be mistakenly viewed as strategic complementarities in the simple specification. However, we have obtained just the opposite result. This may be puzzling at first, but in fact the correlation between the aggregate shocks and the aggregate variables included as regressors in the probit model makes difficult to anticipate the direction of the bias. In effect, for some sectors the estimated ζ is larger in the complete specification than in the simple one, and in some other sectors the opposite happens. In particular, the value obtained in the Food sector is not negative anymore. But despite the fact that there may be some bias in this simple specification, we should emphasize that its results are very close to those of the complete specification and they both support the evidence of strong real rigidities in the data.

We also carried out a number of estimations for robustness check, always using the complete specification. First, we split the sample in two time periods—from 1997 to 2000 and from 2001 to 2004. The first and second rows of Table (7) in Appendix C show the results for the economy-wide case. They are consistent with the previous estimations—the estimated ζ is 0.05 in the 1997-2000 period and 0.11 in the 2001-2004 period. We also estimated the model using data of each year separately during the whole sample period. Results are also consistent with those already presented.

Second, we estimated models without including the exchange rate variable in equation for $p^*_{i,t}$. This equation can be obtained from our theoretical model if we do not include the foreign input in the production function. This version is the same as that in Woodford (2002). Results for the economy-wide case are presented in the third row of Table (7). Even though the estimated ζ is larger than those obtained in the previous estimations, it is still well bellow 1 and also supports the evidence of significant real rigidities in the data.

Finally, taking into account the evidence in the literature showing that idiosyncratic shocks are typically short-lived, we changed the assumption in equation (12) and estimated the models assuming that the idiosyncratic shock is a white-noise, $a_{i,t} = \varepsilon_{i,t}$ and $\varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2)$. The last row of Table (7) presents the results again for the economy-wide case. The estimated value for ζ of only 0.03 shows that our results are very robust. Considering all models we have estimated, usually the point estimation of ζ for the aggregate economy is between 0.03 and 0.11.

6.2.1 Our results and those of the literature

Our main result indicating a substantial degree of strategic complementarities in the data is different from the usual result found in the literature. Here we will summarize some of those results and, by comparing our method to those in the literature and providing some other results, we argue why we believe our method may be better than (at least part of) those that have been proposed and used in recent papers.

First of all, we must emphasize that our procedure is very different from those in the literature, which rely basically on indirect methods to measure the presence of strategic complementarities in the data. For example, Klenow and Willis (2006) use a menu cost model with Kimball (1995) smoothed-kinked demand curve to investigate the compatibility of real rigidities with patterns of frequency and size of price adjustments observed in microdata of the U.S. They conclude that a significant degree of real rigidity would require implausibly large idiosyncratic shocks and menu cost.

Burstein and Hellwig (2007) propose an indirect procedure that uses a menu cost model with decreasing returns to scale at the variety level, calibrated to match the data of Dominick's database. They found moderate role for strategic complementarities. Kryvtsov and Midrigan (2009) emphasize real rigidities coming from sluggish adjustment of nominal factor prices and try to infer the degree of strategic complementarities from the behavior of inventories over the cycle in a sticky price model with motives for holding inventories calibrated to match U.S. microdata. They show that their model requires an elasticity of real marginal cost with respect to output only slightly below unity to reproduce the observed decline in inventory-sales ratio after a monetary policy shocks. This is compatible with absence of important real rigidities.

Bils, Klenow and Malin (2009) (hereafter BKM) develop a concept called "reset price inflation" to indirectly infer the role for strategic complementarities in the data. They use a general equilibrium price-setting model to simulate the behavior of reset price inflation and compare the simulated results to a measure of this variable constructed using the microdata underpinning the U.S. CPI. They report that a state-dependent model with strategic complementarity is fundamentally at odds with the data—the model displays unrealistically high persistence and low volatility of reset price inflation, once their empirical measure has low (in fact negative) serial correlation and high standard deviation[19]. Gopinath and Itskhoki (2010) use the same concept of reset price inflation, but applying it to import price microdata, arguing that incomplete pass-through from exchange rate to inflation can be seen as evidence of strategic complementarities. They find that reset price inflation for imports is less negatively autocorrelated as compared to that documented by BKM for retail prices.

Therefore, as can be seen, all these papers use indirect assessment. Since "desired" prices, marginal costs and markups (variables to which strategic complementarities are related) are not directly observable in practice, they mainly rely on calibrated models and/or statistical properties of specific observable variables to indirectly infer the presence of real rigidities in the data. In contrast, our method, by modeling the frictionless optimal price $p^*_{i,t}$ as a latent variable in an ordered probit model and by deriving the relationship between $p^*_{i,t}$ and p_t, which is captured by ζ, makes the measurement and interpretation of strategic complementarities clearer and more straightforward.

Compared to reset price inflation, developed by BKM and used by Gopinath and Itskhoki (2010), our method has an additional advantage. To make this point clearer, we have to review how they define and empirically measure reset price inflation. A *reset price* for an individual seller is defined as that price that the seller would choose if it implemented a price change in the current period. *Reset price inflation*, $\pi^*_{i,t}$, is the weighted average change of all reset prices, including those of current price changers and non-changers alike. Their empirical estimate of reset price for an item i at time t, $\hat{p}^*_{i,t}$, is given by:

$$\hat{p}^*_{i,t} = \begin{cases} p_{i,t}, & if\ p_{i,t} \neq p_{i,t-1} \\ \hat{p}^*_{i,t-1} + \hat{\pi}^*_t, & if\ p_{i,t} = p_{i,t-1} \end{cases}, \tag{21}$$

where $\hat{\pi}^*_t$ is the reset price inflation measured in the data by those changing prices only.

First, observe that their definition of $p^*_{i,t}$ is different from ours—in our notation, $p^*_{i,t}$ stands for frictionless optimal price, but their $p^*_{i,t}$ is reset price. Second, and this is of fundamental importance, because they cannot observe the reset price for those not changing prices, they update their empirical reset prices using the reset inflation calculated for those changing, i.e., the authors impute to all items, both those changing and those not, the reset price changes exhibited by price changers.

However, in a state-dependent pricing environment the decision to change prices reflects selection, which means that those changing prices are different from those not

[19]Actually, they found that TDP models with or without strategic complementarity, and especially SDP models with strategic complementarity, display these results.

changing. At this point we should emphasize that, when a shock hit the economy, two effects start to work. From one side, the *selection effect* means that the firms that change prices are not selected at random but are rather those firms whose prices are most out of line and, therefore, are those who want to change prices by a larger amount. By the other side, the existence of *strategic complementarities* acts softening these changes, because firms do not want their prices deviating a lot from those of their competitors.

If the world is state-dependent (and the strong significance of the probit parameters provides some evidence that there exists, at least in part, a state-dependent component in pricing decisions), Bils, Klenow and Malin (2009), by adjusting the reset price for those not repricing using the reset inflation estimated for those changing, mix these two effects together. Moreover, if selection is sufficiently strong, it can completely offset strategic complementarities[20]. In this case, if we use reset price inflation to measure the presence of strategic complementarities, we are possibly lead to conclude that they are not important in the data. This means that, even though the BKM's approach may be very interesting to the analysis of neutrality (or not) of monetary policy, the concept of reset price inflation as a measure of strategic complementarities should be viewed with caution.

On the other hand, by modeling the S,s pricing rule in an ordered probit model, our approach seems to take into account the selection effect—after all, firms whose prices are more out of line are those with larger probability of achieving the S or the s threshold—and isolate strategic complementarities in a more suitable way. That may be one reason why, unlike BKM, we find evidence of strong real rigidities in the data.

Finally, Bils, Klenow and Malin (2009) conclude that a state-dependent pricing model with strategic complementarities is in contradiction with the data because their empirical measure of reset price inflation does not have high persistence and stability, as predicted by their model. Given the problems in their estimation of reset price inflation, as already pointed out, we now analyze the statistical properties of our estimation of frictionless optimal inflation (which, by the above arguments, is free from the selection effect modeled in the probit model) to see whether it has the properties that BKM were looking for, after all, strategic complementarities should also make frictionless optimal inflation, like reset price inflation, persistent and stable.

We can take the first difference of equation (8) for the frictionless optimal price, take expectation conditional on the observable variables and use the estimated parameters of the probit model together with the observable regressors to construct a proxy for frictionless optimal inflation. We first estimate a proxy for each sector using this method (with the probit parameters estimated in the complete specification for each sector) and then aggregate them using their respective sectoral weights in the CPI-FGV to have an aggregate proxy. Figure (3) in Appendix C plots this measure of monthly frictionless optimal inflation and the actual inflation for the 1996-2006 period. Table (9) reports some basic statistics.

The dynamics of the estimated frictionless optimal inflation is very close to that of actual inflation. While the monthly average of actual inflation is 0.56%, the average frictionless optimal inflation is 0.47%. In addition, frictionless inflation is as volatile as actual inflation—with standard deviation of 0.57% in comparison to 0.59%—and has

[20] We thanks to Carlos Carvalho for this intuition.

a persistence only somewhat smaller than that of actual inflation, as measured by its first-order autocorrelation—0.42 compared to 0.48. The dynamics does not change if we use the seasonally adjusted series or not, as the last rows of Table (9) show.

Following the same procedure of BKM, to further investigate the persistence properties of the frictionless inflation we also estimate an univariate AR(6) regression and derived an impulse response function (in level), following a 1% impulse. As Figure (4) shows, our estimated frictionless optimal price[21] response has an upward sloping trajectory after a shock. This exercise once again seems to indicate some evidence of strategic complementarities in the data: the impact is initially small (as price setters wait for the average price to respond) but accumulates over time as more firms change prices. Even though the concepts are different, this impulse response for our frictionless optimal price is very similar to that of theoretical reset price predicted by Bils, Klenow and Malin (2009) using a state-dependent pricing model with strong strategic complementarity (in fact, their model has a strategic complementarity parameter of 0.05, very close to our point estimation and inside our 95% confidence interval in the main complete specification).

6.3 The estimated pricing rules

Finally, we explore the results for the estimated pricing rules for each sector and for the aggregate economy. Table (5) below shows the estimated parameters using the main complete specification. Results using the simple specification are presented in Appendix C. The results of the two specifications, however, are very close. In fact, in all models we estimated (splitting the sample in two parts, estimating the model year by year, excluding the exchange rate variable or even using the white noise assumption for the idiosyncratic shock) the parameters of the pricing rules never changed. In this sense the following results are very robust.

The intercepts[22]—π_1 and π_0—of the probit model for each sector and for the aggregate case are all statistically significant at 1%. Actually, their estimated standard deviations are very small in all models. Two facts may help to explain this large significance. First, we take into account the different durations of price spells—observe that all variables are divided by the square root of the time interval between t and τ_t, $\sqrt{\delta_{i,t}}$. In terms of π_1 and π_0 this means that in fact the intercepts are changing over time, which improves the fit of the model. Second, we are using a robust variance-covariance matrix estimator.

Since we are able to estimate the parameter σ for each sector and for the aggregate economy, from the intercepts of the probit models we can estimate the pairwise distances between S, s and c using equations in (20)—even though we cannot isolate the own parameters S, s and c. Some interesting evidences emerge from the results in Table (5).

The first interesting fact is that for each sector, and also for the aggregate economy, the estimated S,s band is larger at the bottom than the top, i.e., the estimated distance between c and s is larger than the distance between S and c. For example, for the

[21] Observe that the response function is in level.

[22] We are calling them intercepts, but observe that our probit is not standard and the variable $1/\sqrt{\delta_{i,t}}$ changes over time.

aggregate economy, while $S - c$ is 0.06, $-(s - c)$ is equal to 0.09. This means that if c were zero, we could say that on average a firm waits for its frictionless optimal price $p^*_{i,t}$ deviating 6% from its actual price until it increases and deviating 9% until it reduces its price. Unfortunately, we are not able to estimate the isolated value of c. Nonetheless, these results seems to indicate that a typical firm is a little more tolerant with negative than with positive shocks. This evidence is compatible with the fourth stylized fact we raised from the data—that on average price decreases are larger than price increases—and may possibly be explained by the positive average inflation during the period. When inflation is positive, if a firm is hit by a small negative shock and want to decrease its price, by maintaining its nominal price unchained it will have actually a lower price in real terms. But when the distance from its optimal price is sufficiently large it adjusts and the size of adjustment will possibly be larger.

Table 5: Estimated parameters of the pricing rules, complete model

Sector	π_0=(s-c)/σ	π_1=(S-c)/σ	σ	s-c	S-c	S-s	(S-s)/σ
All sectors	-0.93	0.63	0.10	-0.09	0.06	0.15	1.55
Food	-0.71	0.48	0.12	-0.09	0.06	0.15	1.19
Apparel	-0.72	0.62	0.14	-0.10	0.08	0.18	1.35
Housing	-1.05	0.60	0.08	-0.08	0.05	0.13	1.66
Other Goods and Services	-1.05	0.72	0.07	-0.07	0.05	0.12	1.77
Transportation	-1.25	0.63	0.07	-0.09	0.05	0.14	1.88
Medical and Personal Care	-1.29	0.83	0.08	-0.10	0.06	0.16	2.12
Education and Recreation	-2.11	1.48	0.05	-0.11	0.08	0.19	3.59

Note: These results are obtained using the complete specification
All parameters are significant at 1%. See detailed results in Appendix C.

Second, as it is well known (for example, Bonomo (1992)), the higher the adjustment cost and the variance of shocks, the wider the S,s band is. Here we do not estimate the adjustment cost, but only the variance of shocks. Then, to obtain a comparable measure of the bands' sizes among the different sectors, we divide the estimated width of the band by the estimated variance of shocks—$(S - s)/\sigma$—for each sector. Using this measure, our estimations confirm the theoretical intuition in the data: the more flexible the sector, the smaller the size of the band is (controlling for the variance of shocks). In addition, our estimations tell the same story as that of the stylized facts previously discussed. Even the ranking of sectors according to the degree of price flexibility is preserved by our measure of the bands' sizes. In particular, after controlling for the variance of shocks the Food sector has the lowest band and the Education and Recreation sector has the largest one. These are, respectively, the most flexible and the most rigid sectors. Observe, however, that the two most flexible sectors—Food and Apparel— do not have the narrowest bands measured by the distance between S and s. But the variances of shocks in these two sectors are larger than those in the other sectors. In contrast, for the Education and Recreation sector σ is only 0.05, beside the fact that this sector has the largest band measured by $S - s$.

7 Conclusions

In this paper we have developed an econometric methodology to directly estimate the structural parameter measuring the degree of strategic complementarities in pricing decisions in a state-dependent pricing model. At the micro level, firms face idiosyncratic shocks and fixed costs of adjusting prices. We obtain the firm's frictionless optimal price from fundamentals and based on the S,s pricing rule we derive a structural, non-standard ordered probit model. We use a quasi-maximum likelihood method that takes into account the autocorrelation and the heteroskedasticity that emerge from the micro-founded model to estimate the probit using the microdata underpinning the CPI-FGV in Brazil for the 1996-2006 period.

Unlike the results in the literature, we find a substantial degree of strategic complementarity in firms' pricing decisions, for both the aggregate economy and all sectors of the CPI-FGV individually. For the aggregate economy we estimate the parameter ζ measuring strategic complementarities ranging from 0.03 to 0.11. We also obtained a highly persistent frictionless optimal inflation and an impulse response for the frictionless optimal price that has a small initial impact after a shock, but that accumulates over time. Therefore, we do not find that a state-dependent pricing model with strategic complementarities is fundamentally in contradiction with the data, as suggested by Bills, Klenow and Malin (2009). Our results are more in line with Gertler and Leahy (2008). They developed a state-dependent pricing model that has both strategic complementarity and idiosyncratic shocks. The calibration they use produces a strategic complementarity parameter of 0.08 for the aggregate economy, which lies inside the range of our estimations.

Our results are obtained assuming a state-dependent pricing rule, however. The next step that should be taken is to develop a similar methodology to estimate the degree of strategic complementarities in a time-dependent pricing model and verify whether the same results are obtained.

The methodology also allows us to estimate some characteristics of the pricing rules. Even though we cannot separately identify the parameters S, s and c, we can easily estimate some characteristics of the pricing rules, like their widths, the length of the top and the bottom bands etc. The variance of shocks is also estimated. All the results are quite consistent with the stylized facts documented from the data. In particular, after we control for the magnitude of the variance of shocks, the sectors in which prices are most flexible have narrower S,s bands. Additionally, for all sectors and for the aggregate economy the distance between c and s seems to be larger than the distance between S and c, which would explain, at least partially, why price decreases are on average larger than price increases.

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Appendix A

Here we derive the frictionless optimal price in a general equilibrium framework.

Households. The representative household seeks to maximize the discounted sum of utilities:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t;\xi_t) - \int_0^1 v(L_{i,t};\xi_t) di \right], \tag{22}$$

subject to its budget constraint. C_t is a consumption index and $L_{i,t}$ is the quantity of labor of type i supplied. The term $v(L_{i,t};\xi_t)$ represents the disutility of supplying labor and ξ_t is a vector of aggregate shocks.

There is a continuum of differentiated goods $C_{i,t}, i \in [0,1]$. Thus, following Dixit and Stiglitz (1977), C_t is an index

$$C_t = \left[\int_0^1 C_{i,t}^{(\theta-1)/\theta} di \right]^{\theta/\theta-1}, \tag{23}$$

with $\theta > 1$. The expenditure minimization problem of households implies that the optimal demand for an individual product i has the following familiar relation with the aggregated demand:

$$C_{i,t} = C_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}, \tag{24}$$

where,

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di \right]^{1/(1-\theta)}. \tag{25}$$

In addition, the household must choose an optimal quantity of each labor type to supply, given the wages that it faces. The optimal quantity $L_{i,t}$ is implicitly given by the first-order condition:

$$\frac{v_L(L_{i,t};\xi_t)}{u_C(C_t;\xi_t)} = \frac{W_{i,t}}{P_t}, \tag{26}$$

where $W_{i,t}$ is the wage for labor type i at time t.

Firms. There is a continuum of monopolistically competitive firms indexed by $i \in [0,1]$, supplying differentiated goods. We assume that each good i has the following production function:

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} M_t^{(1-\alpha)}, \tag{27}$$

where $A_{i,t}$ is a productivity factor and M_t is a foreign input used in the production process. The nominal exchange rate used to import M_t is given by $\tilde{E}_t$, which is exogenously determined. We may think of capital as being allocated to each firm in a fixed amount, and never depreciating. Moreover, observe the presence of heterogeneity, once $A_{i,t}$ is an idiosyncratic shock affecting only firm i.

Marginal Cost. The optimal quantities of labor and foreign input required to produce the quantity $Y_{i,t}$ of good i is given by

$$L_{i,t} = \left(\frac{\alpha}{(1-\alpha)} \frac{\tilde{E}_t}{W_{it}} \right)^{1-\alpha} \frac{Y_{i,t}}{A_{i,t}}, \tag{28}$$

$$M_t = \left(\frac{(1-\alpha)}{\alpha} \frac{W_{i,t}}{\tilde{E}_t} \right)^{\alpha} \frac{Y_{i,t}}{A_{i,t}}. \tag{29}$$

Then, the nominal cost of supplying the quantity $Y_{i,t}$ is

$$\begin{aligned} W_{i,t}L_{i,t} + \tilde{E}_t m_t &= \alpha^{-\alpha}(1-\alpha)^{(\alpha-1)} \frac{W_{i,t}^{\alpha} \tilde{E}_t^{(1-\alpha)} Y_{i,t}}{A_{i,t}} \\ &= \lambda \frac{W_{i,t}^{\alpha} \tilde{E}_t^{(1-\alpha)} Y_{i,t}}{A_{i,t}}, \end{aligned} \tag{30}$$

where $\lambda = \alpha^{-\alpha}(1-\alpha)^{(\alpha-1)}$. And the nominal marginal cost can be written as

$$\Psi_{i,t} = \lambda \frac{W_{i,t}^{\alpha} \tilde{E}_t^{(1-\alpha)}}{A_{i,t}}. \tag{31}$$

By equation (26), we can see that the quantity of labor is positively related to the firm's product. Therefore, we will rewrite equation (26) as

$$W_{i,t} = \frac{v_L(Y_{i,t};\xi_t)P_t}{u_C(C_t;\xi_t)} \tag{32}$$

Substituting equation (32) into (31), and considering that $C_t = Y_t$ in equilibrium, leads to the following real marginal cost function:

$$\begin{aligned} \psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}) \equiv \frac{\Psi_{i,t}}{P_t} &= \frac{\lambda}{A_{i,t}} \left\{ \frac{v_L(Y_{i,t};\xi_t)}{u_C(Y_t;\xi_t)} \right\}^{\alpha} \left\{ \frac{\tilde{E}_t}{P_t} \right\}^{1-\alpha} \\ &= \frac{\lambda}{A_{i,t}} \left\{ \frac{v_L(Y_{i,t};\xi_t)}{u_C(Y_t;\xi_t)} \right\}^{\alpha} E_t^{1-\alpha} \end{aligned} \tag{33}$$

where $E_t = \tilde{E}_t / P_t$ is the real exchange rate.

Marginal Revenue. In a model of monopolistic competition, each supplier chooses the price that will maximize its profit, taking into account the demand function given in (24):

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\theta}. \tag{34}$$

Using this equation, we can write the real revenue as

$$Y_{i,t}\frac{P_{i,t}}{P_t} = Y_{i,t}\left(\frac{Y_{i,t}}{Y_t}\right)^{-1/\theta} = Y_{i,t}^{(\theta-1)/\theta}Y_t^{1/\theta}, \tag{35}$$

which gives us the following equation for the real marginal revenue:

$$\frac{\theta-1}{\theta}\left(\frac{Y_t}{Y_{i,t}}\right)^{1/\theta}. \tag{36}$$

Optimality. Because the good i only contributes with an infinitesimal part to the aggregate output, the supplier chooses its optimal price taking Y_t and P_t as given, i.e., he equals marginal revenue to marginal cost

$$\frac{\theta-1}{\theta}\left(\frac{Y_t}{Y_{i,t}}\right)^{1/\theta} = \psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}). \tag{37}$$

Once equation (28) shows that $\psi(.)$ is increasing in $Y_{i,t}$, this equation must have a unique solution for $Y_{i,t}$ given Y_t.

Substituting equation (34) into (37), we have the following equation for the frictionless optimal price:

$$\frac{P_{i,t}^*}{P_t} = \mu\psi(Y_{i,t}, Y_t, E_t; \xi_t, A_{i,t}), \tag{38}$$

where $\mu = \theta/(\theta-1)$ is the seller's desired markup.

Log-linearization. When $A_{i,t} = 1, \forall i$, it follows that in equilibrium each good must be supplied in the same quantity, and such common quantity must equal Y_t. We take a first-order log-linearization of the cost function around the steady-state equilibrium in the case of flexible prices and $A_{i,t} = 1, \xi_t = 0$. Let $\bar{Y}$ be the output level in this steady state. Then,

$$\begin{aligned}
\log P_{i,t}^* - \log P_t &= \log\left(\frac{P_{i,t}^*}{P_t}\right) - \log(1) \\
&= \widehat{\psi}_{it} \\
&= \alpha\omega\hat{y}_{i,t} + \alpha\sigma^{-1}\hat{y}_t + (1-\alpha)\hat{e}_t + \frac{\partial log\,\psi_{i,t}}{\partial\xi_t}\xi_t - \log A_{i,t},
\end{aligned}$$

where ω represents the elasticity of real marginal cost with respect to the firm's output; the elasticity of marginal cost with respect to the aggregate demand corresponds to the inverse of the intertemporal elasticity of substitution of private expenditure, $\frac{\partial\log\psi}{\partial\log Y_t} = -\frac{\partial\log u_C}{\partial\log Y_t} = -\frac{u_{CC}\bar{Y}}{u_C} = \sigma^{-1}$ and $\overline{x} = \log\left(\frac{X_t}{\bar{X}}\right)$ for all variables. Therefore,

$$\log P_{i,t}^{*}-\log P_t=\alpha\omega\left(\log Y_{i,t}-\log\overline{Y}\right)+\alpha\sigma^{-1}\left(\log Y_t-\log\overline{Y}\right)+$$
$$+(1-\alpha)\left(\log E_t-\log\overline{E}\right)+\frac{\partial\log\,\psi_{i,t}}{\partial\xi_t}\xi_t-\log A_{i,t}$$

Now, substituting the demand equation (34):

$$\log P_{i,t}^{*}-\log P_t=\alpha\omega\left(\log\left(\frac{P_{i,t}^{*}}{P_t}\right)^{-\theta}Y_t-\log\overline{Y}\right)+\alpha\sigma^{-1}\left(\log Y_t-\log\overline{Y}\right)+$$
$$+(1-\alpha)\left(\log E_t-\log\overline{E}\right)+\frac{\partial log\,\psi_{i,t}}{\partial\xi_t}\xi_t-\log A_{i,t}$$

And finally, substituting $\mathcal{Y}_t=P_tY_t$, we have:

$$\log P_{i,t}^{*}-\log P_t=-\theta\alpha\omega\left(\log P_{i,t}^{*}-\log P_t\right)+\alpha\left(\omega+\sigma^{-1}\right)\log\,\mathcal{Y}_t-$$
$$-\alpha\left(\omega+\sigma^{-1}\right)\log P_t-\alpha\left(\omega+\sigma^{-1}\right)\log\overline{Y}+$$
$$+(1-\alpha)\left(\log E_t-\log\overline{E}\right)+\frac{\partial\log\,\psi_{i,t}}{\partial\xi_t}\xi_t-\log A_{i,t}$$

$$\log P_{i,t}^{*}=-\frac{\alpha\left(\omega+\sigma^{-1}\right)}{(1+\theta\omega\alpha)}\log\overline{Y}-\frac{(1-\alpha)}{(1+\theta\omega\alpha)}\log\overline{E}+$$
$$+\frac{\left(1+\theta\omega\alpha-\alpha\left(\omega+\sigma^{-1}\right)\right)}{(1+\theta\omega\alpha)}\log P_t+\frac{\alpha\left(\omega+\sigma^{-1}\right)}{(1+\theta\omega\alpha)}\log\,\mathcal{Y}_t+\frac{(1-\alpha)}{(1+\theta\omega\alpha)}\log E_t+$$
$$+\frac{1}{(1+\theta\omega\alpha)}\frac{\partial\log\,\psi_{i,t}}{\partial\xi_t}\xi_t-\frac{\log A_{i,t}}{(1+\theta\omega\alpha)}$$

Letting lowercase variables representing variables in logs, we obtain equation (8):

$$p_{i,t}^{*}=\kappa+(1-\zeta)p_t+\zeta\mathcal{Y}_t+\frac{(1-\alpha)}{(1+\theta\omega\alpha)}e_t+\tilde{\xi}_t+\tilde{a}_{i,t},$$

where $\zeta=\frac{\alpha\left(\omega+\sigma^{-1}\right)}{(1+\theta\omega\alpha)}$, $\kappa=-\frac{\alpha\left(\omega+\sigma^{-1}\right)}{(1+\theta\omega\alpha)}\log\overline{Y}-\frac{(1-\alpha)}{(1+\theta\omega\alpha)}\log\overline{E}$, $\tilde{\xi}_t=\frac{1}{(1+\theta\omega\alpha)}\frac{\partial\log\,\psi_{i,t}}{\partial\xi_t}\xi_t$, and

$$\tilde{a}_{i,t}=-\frac{\log A_{i,t}}{(1+\theta\omega\alpha)}$$

Appendix B

Obtaining the equation for $y^*_{i,t}$

From equation (10) we have:

$$y^*_{i,t} \equiv p^*_{i,t} - p_{i,\tau_t} = \boldsymbol{z}'_{i,t}\boldsymbol{\beta} + c + (\tilde{\xi}_t - \tilde{\xi}_{\tau_t}) + (\tilde{a}_{i,t} - \tilde{a}_{i,\tau_t}).$$

First, we assume that the aggregate shock is given by

$$\tilde{\xi}_t = \tilde{\xi}_{t-1} + v_t, \quad \text{and} \quad v_t \sim iid(0, \sigma_v^2).$$

Iterating $\tilde{\xi}_t$ backward gives

$$\tilde{\xi}_t - \tilde{\xi}_{\tau_t} = v_t + ... + v_{t-\delta_{i,t}+1}.$$

We also assume that

$$v_t = \sum_{j=1}^{T} \gamma_j \hat{d}_{t,j} \quad \text{and} \quad \hat{d}_{t,j} = \begin{cases} 1, & if \;\; t = j \\ 0, & if \;\; t \neq j \end{cases} .$$

Then,

$$\begin{aligned} \tilde{\xi}_t - \tilde{\xi}_{\tau_t} &= v_t + v_{t-1} ... + v_{t-\delta_{i,t}+1} \\ &= \sum_{j=1}^{T} \gamma_j \hat{d}_{t,j} + \sum_{j=1}^{T} \gamma_j \hat{d}_{t-1,j} + ... + \sum_{j=1}^{T} \gamma_j \hat{d}_{t-\delta_{i,t}+1,j} \\ &= \sum_{j=1}^{T} \gamma_j d_{t,j}, \end{aligned}$$

where $d_{t,j} = \begin{cases} 1, & if \;\; j \in [t, t-\delta_{i,t}+1] \\ 0, & if \;\; otherwise \end{cases}$.

Second, regarding the idiosyncratic shock we assume that $\tilde{a}_{i,t} = a_i + a_{i,t}$, where a_i is an individual fixed effect component and

$$a_{i,t} = \eta + a_{i,t-1} + \varepsilon_{i,t} \quad \text{and} \quad \varepsilon_{i,t} \sim \mathbb{N}(0, \sigma^2).$$

Iterating $a_{i,t}$ backward, this assumption implies that

$$\begin{aligned} \tilde{a}_{i,t} - \tilde{a}_{i,\tau_t} &= (\eta + ... + \eta) + (\varepsilon_{i,t} + ... + \varepsilon_{i,t-\delta_{i,t}+1}) \\ &= \eta\delta_{i,t} + u_{i,t}, \end{aligned}$$

where $\delta_{i,t}$ is the time interval between t and τ, and $u_{i,t} = \varepsilon_{i,t} + ... + \varepsilon_{i,t-\delta_{i,t}+1}$ is a moving average MA($\delta_{i,t} - 1$) process. Therefore, $u_{i,t} \sim \mathbb{N}(0, \delta_{i,t}\sigma^2)$.

Then, putting all parts together we obtain equation (13) in the text:

$$y_{i,t}^* \equiv p_{i,t}^* - p_{i,\tau_t} = \eta\delta_{i,t} + \boldsymbol{z}_{i,t}'\boldsymbol{\beta} + c + \sum_{j=1}^{T} \gamma_j d_{t,j} + u_{i,t}$$

7.1 Likelihoood and robust variance-covariance matrix

For notational reason, write equations in (15) in the following simpler way:

$$Pr[y_{i,t} = 1|\boldsymbol{w}_{i,t}] = 1 - \Phi\left(\pi_1 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right)$$

$$Pr[y_{i,t} = 0|\boldsymbol{w}_{i,t}] = \Phi\left(\pi_1 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right) - \Phi\left(\pi_0 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right)$$

$$Pr[y_{i,t} = -1|\boldsymbol{w}_{i,t}] = \Phi\left(\pi_0 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right)$$

where $\tilde{\boldsymbol{w}}_{i,t} = (\ddot{\delta}_{i,t}, \ddot{\boldsymbol{z}}_{i,t}', \ddot{\boldsymbol{d}}_t')'$ and $\boldsymbol{\psi} = (\tilde{\eta}, \tilde{\boldsymbol{\beta}}', \tilde{\gamma}_1..., \tilde{\gamma}_T)'$.
Then, the partial log-likelihood of an observation i at time t is given by:

$$\begin{aligned} l_{i,t}(\boldsymbol{\Psi}) = {} & \mathbb{I}\{y_{i,t} = 1\} \log\left[1 - \Phi\left(\pi_1 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right)\right] + \\ & + \mathbb{I}\{y_{i,t} = 0\} \log\left[\Phi\left(\pi_1 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right) - \Phi\left(\pi_0 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right)\right] + \\ & + \mathbb{I}\{y_{i,t} = -1\} \log\left[\Phi\left(\pi_0 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right)\right], \end{aligned}$$

where $\boldsymbol{\Psi} = (\pi_1, \pi_0, \boldsymbol{\psi}')'$ and $\mathbb{I}\{.\}$ is an indicator function.
Now define the following notation

$$\begin{aligned} \Phi_1(.) &\equiv \Phi\left(\pi_1 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right) \\ \Phi_0(.) &\equiv \Phi\left(\pi_0 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right) \\ \phi_1(.) &\equiv \phi\left(\pi_1 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right) \\ \phi_0(.) &\equiv \phi\left(\pi_0 \ddot{\imath}_{i,t} - \tilde{\boldsymbol{w}}_{i,t}'\boldsymbol{\psi}\right) \end{aligned}$$

Then, the score of an observation i at time t is given by:

$$\boldsymbol{s}_{i,t}(\boldsymbol{\Psi}) = \begin{bmatrix} \frac{\partial l_{i,t}}{\partial \pi_1} \\ \frac{\partial l_{i,t}}{\partial \pi_0} \\ \frac{\partial l_{i,t}}{\partial \boldsymbol{\psi}} \end{bmatrix} = \begin{bmatrix} s_{1,i,t} \\ s_{2,i,t} \\ \boldsymbol{s}_{3,i,t} \end{bmatrix},$$

where,

$$s_{1,i,t} = \left\{\frac{\mathbb{I}\{y_{i,t} = 0\}}{\Phi_1(.) - \Phi_0(.)} - \frac{\mathbb{I}\{y_{i,t} = 1\}}{1 - \Phi_1(.)}\right\} \phi_1(.)\ddot{\imath}_{i,t}$$

$$s_{2,i,t} = \left\{\frac{\mathbb{I}\{y_{i,t} = -1\}}{\Phi_0(.)} - \frac{\mathbb{I}\{y_{i,t} = 0\}}{\Phi_1(.) - \Phi_0(.)}\right\} \phi_0(.)\ddot{\imath}_{i,t}$$

$$\boldsymbol{s}_{3,i,t} = \left\{ \frac{\mathbb{I}\{y_{i,t} = 1\}\phi_1(.)}{1 - \Phi_1(.)} + \frac{\mathbb{I}\{y_{i,t} = 0\}}{\Phi_1(.) - \Phi_0(.)} [-\phi_1(.) + \phi_0(.)] - \frac{\mathbb{I}\{y_{i,t} = -1\}\phi_0(.)}{\Phi_0(.)} \right\} \tilde{\boldsymbol{w}}_{i,t}$$

A variance-covariance matrix of parameters $\boldsymbol{\Psi}$ robust for autocorrelation and heteroskedasticity can be estimated by

$$\frac{1}{N}\hat{\boldsymbol{A}}^{-1}\hat{\boldsymbol{B}}\hat{\boldsymbol{A}}^{-1},$$

where matrix $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ are defined as

$$\hat{\boldsymbol{A}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \boldsymbol{s}_{i,t}(\hat{\boldsymbol{\Psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\Psi}})'$$

and

$$\hat{\boldsymbol{B}} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T} \boldsymbol{s}_{i,t}(\hat{\boldsymbol{\Psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\Psi}})' + \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{r\neq t} \boldsymbol{s}_{i,r}(\hat{\boldsymbol{\Psi}})\boldsymbol{s}_{i,t}(\hat{\boldsymbol{\Psi}})',$$

where the second term of $\hat{\boldsymbol{B}}$ accounts for the autocorrelation in the scores.

Confidence interval for $\hat{\zeta}$ and for $\hat{\sigma}$

Theorem (Delta Method): Suppose that $\hat{\boldsymbol{\theta}}$ is an estimator of the Px1 vector $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ and that

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{d} \mathbb{N}(\boldsymbol{0}, \boldsymbol{V}),$$

where $\boldsymbol{V}$ is a PxP positive definite matrix. Let $\boldsymbol{c} : \boldsymbol{\Theta} \rightarrow \mathbb{R}^Q$ be a continuous differentiable function on the parameter space $\boldsymbol{\Theta} \subset \mathbb{R}^P$, where $Q \leq P$, and assume that $\boldsymbol{\theta}$ is in the interior of the parameter space. Define $\boldsymbol{C}(\boldsymbol{\theta}) \equiv \nabla_\theta \boldsymbol{c}(\boldsymbol{\theta})$ and the QxP Jacobian of $\boldsymbol{c}$. Then

$$\sqrt{N}[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})] \xrightarrow{d} \mathbb{N}[\boldsymbol{0}, \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{V}\boldsymbol{C}(\boldsymbol{\theta})']. \tag{39}$$

Define $\hat{\boldsymbol{C}} \equiv \boldsymbol{C}(\hat{\boldsymbol{\theta}})$. Then plim $\hat{\boldsymbol{C}} = \boldsymbol{C}(\boldsymbol{\theta})$. If plim $\hat{\boldsymbol{V}} = \boldsymbol{V}$, then

$$N[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})]'[\hat{\boldsymbol{C}}\hat{\boldsymbol{V}}\hat{\boldsymbol{C}}']^{-1}[\boldsymbol{c}(\hat{\boldsymbol{\theta}}) - \boldsymbol{c}(\boldsymbol{\theta})] \xrightarrow{d} \chi^2_Q. \tag{40}$$

Proof: See Wooldridge (2002), pp. 44-45 ■

Once ζ and σ are given respectively by $\zeta = \frac{\tilde{\beta}_1}{\tilde{\beta}_1 + \tilde{\beta}_2}$ and $\sigma = \frac{1}{\tilde{\beta}_1 + \tilde{\beta}_2}$, we can use the theorem above to obtain

$$\sqrt{N}[\hat{\zeta} - \zeta] \xrightarrow{d} \mathbb{N}[0, \boldsymbol{C}_\zeta \boldsymbol{V}_1 \boldsymbol{C}_\zeta'] \tag{41}$$

and

$$\sqrt{N}[\hat{\sigma} - \sigma] \xrightarrow{d} \mathbb{N}[0, \boldsymbol{C}_\sigma \boldsymbol{V}_1 \boldsymbol{C}'_\sigma], \tag{42}$$

where $\boldsymbol{C}_\zeta = [C_\zeta^1 \;\; C_\zeta^2]'$, $\boldsymbol{C}_\sigma = [C_\sigma^1 \;\; C_\sigma^2]'$ and $C_\zeta^1 = \frac{1}{\tilde{\beta}_1+\tilde{\beta}_2} - \frac{\tilde{\beta}_1}{\tilde{\beta}_1+\tilde{\beta}_2}$, $C_\zeta^2 = -\frac{\tilde{\beta}_1}{(\tilde{\beta}_1+\tilde{\beta}_2)^2}$, $C_\sigma^1 = -\frac{1}{\tilde{\beta}_1+\tilde{\beta}_2}$, $C_\sigma^2 = -\frac{1}{\tilde{\beta}_1+\tilde{\beta}_2}$. $\boldsymbol{V}_1$ is the respective variance-covariance matrix of $\tilde{\beta}_1$ and $\tilde{\beta}_2$.

Now the confidence interval for the desired significance level can be constructed as usual.

Appendix C

Table 6: Probit parameters and strategic complementarity, simple specification

Sector	y_t	p_t	e_t	ζ	Conf. interval for ζ
All sectors	0.92	9.15	-0.09	0.09	0.08 - 0.10
	(0.037)	(0.036)	(0.007)	(0.003)	
Food	0.20	8.79	0.07	0.02	0.01 - 0.03
	(0.058)	(0.072)	(0.013)	(0.006)	
Apparel	1.09	4.91	-0.17	0.18	0.15 - 0.22
	(0.122)	(0.119)	(0.025)	(0.018)	
Housing	1.36	11.02	0.11	0.11	0.09 - 0.13
	(0.106)	(0.111)	(0.022)	(0.008)	
Other Goods and Services	1.43	10.45	-0.36	0.12	0.09 - 0.15
	(0.173)	(0.183)	(0.034)	(0.013)	
Transportation	1.36	14.29	0.42	0.09	0.06 - 0.11
	(0.211)	(0.189)	(0.039)	(0.013)	
Medical and Personal Care	1.24	10.06	-0.70	0.11	0.10 - 0.12
	(0.082)	(0.064)	(0.014)	(0.007)	
Education and Recreation	6.09	11.80	0.32	0.34	0.33 - 0.35
	(0.123)	(0.076)	(0.020)	(0.005)	

Notes: These results are obtained using the simple specification
The robust standard deviations are in parenthesis
The confidence interval is 95% of confidence
The standard deviation of ζ was obtained by the Delta method

Table 7: Probit parameters and strategic complementarity (Robustness)

Model	y_t	p_t	e_t	ζ	Conf. interval for ζ
Period 1997 - 2000	0.28	5.06	0.62	0.05	0.00 - 0.11
	(0.154)	(0.142)	(0.023)	(0.028)	
Period 2001 - 2004	1.24	10.25	-0.24	0.11	0.09 - 0.12
	(0.087)	(0.051)	(0.014)	(0.007)	
Without Exchange Rate	2.61	3.48	-	0.43	0.42 - 0.44
	(0.053)	(0.023)	-	(0.006)	
Idiosyncratic White Noise	0.44	12.82	0.01	0.03	0.02 - 0.04
	(0.064)	(0.064)	(0.016)	(0.005)	

Note: These results are obtained using the complete specification, i.e., including the dummy variables for the aggregate shocks.
These results are for the economy-wide case.
The robust standard deviations are in parenthesis.
The confidence interval is 95% of confidence and the standard deviation of ζ was obtained by the Delta method.

Table 8: Estimated parameters of the pricing rules, complete specification

Sector	π_0**=(s-c)/**σ	π_1**=(S-c)/**σ	σ	**s-c**	**S-c**	**S-s**	**(S-s)/**σ
All sectors	-0.93	0.63	0.10	-0.09	0.06	0.15	1.55
Food	-0.70	0.48	0.11	-0.08	0.05	0.13	1.18
Apparel	-0.72	0.62	0.17	-0.12	0.10	0.22	1.34
Housing	-1.05	0.61	0.08	-0.08	0.05	0.13	1.65
Other Goods and Services	-1.06	0.70	0.08	-0.09	0.06	0.15	1.76
Transportation	-1.21	0.60	0.06	-0.08	0.04	0.12	1.81
Medical and Personal Care	-1.30	0.81	0.09	-0.11	0.07	0.19	2.10
Education and Recreation	-2.12	1.45	0.06	-0.12	0.08	0.20	3.58

Note: These results are obtained using the simple specification

All parameters are significant at 1%. See detailed results in Appendix C.

Table 9: Summary statistics for frictionless and actual inflation

	Average	**Std Deviation**	**Persistence**
Actual Inflation IPC-FGV	0.55%	0.56%	0.47
Frictionless Inflation	0.46%	0.55%	0.40

Notes: The statistics are calculated using the period Jan/1996-Dec/2006.

Frictionless inflation is obtained by $\pi^*_t = \eta + (\mathbf{x}_t - \mathbf{x}_{t-1})'\boldsymbol{\beta}$ in each sector, where $\boldsymbol{\beta}$ are the estimated parameters of the probit model using the complete specification, and then aggregating using the sectoral weight in IPC-FGV.

Persistence is measured by the first-order autocorrelation.

Table 10: Detailed results of probit models, complete specification

Sector: All sectors					
Log-likelihood:	-3863826.87			**Number of obs:** 2851318	
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	0.63	0.008	76.81	0.61	0.64
(s-c)/σ	-0.93	0.008	-113.99	-0.94	-0.91
$(\delta_{i,t})$^(0.5)	-0.35	0.059	-5.90	-0.47	-0.23
$\mathcal{Y}_t$	0.44	0.065	6.79	0.31	0.57
$\mathbf{p_t}$	9.79	0.044	224.11	9.70	9.88
$\mathbf{e_t}$	0.03	0.011	3.07	0.01	0.05
ζ	0.04	0.006	-	0.03	0.05
$\boldsymbol{\sigma}$	0.10	0.006	-	0.09	0.11
Sector: Food					
Log-likelihood:	-1639878.65			**Number of obs:** 1229052	
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	0.48	0.013	38.33	0.46	0.51
(s-c)/σ	-0.71	0.013	-56.37	-0.73	-0.68
$(\delta_{i,t})$^(0.5)	-0.61	1.017	-0.60	-2.60	1.39
$\mathcal{Y}_t$	-0.33	0.115	-2.84	-0.55	-0.10
$\mathbf{p_t}$	8.36	0.092	91.27	8.18	8.54
$\mathbf{e_t}$	0.26	0.021	12.53	0.22	0.30
ζ	-0.04	0.015	-	-0.07	-0.01
$\boldsymbol{\sigma}$	0.12	0.015	-	0.10	0.15
Sector: Other Goods and Services					
Log-likelihood:	-175303.29			**Number of obs:** 131933	
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	0.72	0.025	29.16	0.67	0.77
(s-c)/σ	-1.05	0.025	-42.96	-1.10	-1.00
$(\delta_{i,t})$^(0.5)	0.63	1.041	0.60	-1.41	2.67
$\mathcal{Y}_t$	2.44	0.308	7.94	1.84	3.05
$\mathbf{p_t}$	12.69	0.225	56.54	12.25	13.14
$\mathbf{e_t}$	-0.66	0.051	-12.92	-0.76	-0.56
ζ	0.16	0.018	-	0.13	0.20
$\boldsymbol{\sigma}$	0.07	0.021	-	0.03	0.11
Sector: Education and Recreation					
Log-likelihood:	-211577.62			**Number of obs:** 196576	
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	1.48	0.018	80.32	1.45	1.52
(s-c)/σ	-2.11	0.018	-115.67	-2.15	-2.07
$(\delta_{i,t})$^(0.5)	-1.08	1.672	-0.65	-4.36	2.19
$\mathcal{Y}_t$	5.28	0.185	28.62	4.92	5.64
$\mathbf{p_t}$	13.68	0.087	156.73	13.51	13.85
$\mathbf{e_t}$	0.58	0.027	21.80	0.53	0.63
ζ	0.28	0.008	-	0.26	0.29
$\boldsymbol{\sigma}$	0.05	0.008	-	0.04	0.07

Table 10: Detailed results of probit models, complete specification (cont.)

Sector: Housing

Log-likelihood: -479116.74 **Number of obs:** 364110

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.60	0.019	31.55	0.57	0.64
(s-c)/σ	-1.05	0.019	-55.49	-1.09	-1.01
$(\delta_{i,t})$^(0.5)	-2.08	0.110	-18.89	-2.30	-1.87
$\mathcal{Y}_t$	1.40	0.153	9.15	1.10	1.69
p_t	11.09	0.130	85.32	10.84	11.35
e_t	0.67	0.033	20.03	0.60	0.73
ζ	0.11	0.017	-	0.08	0.14
σ	0.08	0.013	-	0.06	0.10

Sector: Medical and Personal Care

Log-likelihood: -674949.22 **Number of obs:** 515849

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.83	0.010	79.58	0.80	0.85
(s-c)/σ	-1.29	0.010	-126.76	-1.31	-1.27
$(\delta_{i,t})$^(0.5)	1.48	0.279	5.32	0.94	2.03
$\mathcal{Y}_t$	1.36	0.139	9.76	1.08	1.63
p_t	11.93	0.077	154.31	11.78	12.09
e_t	-0.92	0.020	-46.45	-0.96	-0.88
ζ	0.10	0.010	-	0.08	0.12
σ	0.08	0.009	-	0.06	0.09

Sector: Transportation

Log-likelihood: -123726.40 **Number of obs:** 95908

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.63	0.013	48.68	0.61	0.66
(s-c)/σ	-1.25	0.013	-100.01	-1.28	-1.23
$(\delta_{i,t})$^(0.5)	-1.85	0.601	-3.08	-3.03	-0.68
$\mathcal{Y}_t$	2.32	0.359	6.45	1.61	3.02
p_t	11.20	0.236	47.49	10.74	11.66
e_t	0.92	0.059	15.58	0.81	1.04
ζ	0.17	0.024	-	0.13	0.22
σ	0.07	0.024	-	0.03	0.12

Sector: Apparel

Log-likelihood: -479116.74 **Number of obs:** 317890

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.62	0.046	13.53	0.53	0.72
(s-c)/σ	-0.72	0.046	-15.50	-0.81	-0.63
$(\delta_{i,t})$^(0.5)	0.10	2.216	0.05	-4.24	4.44
$\mathcal{Y}_t$	1.16	0.221	5.24	0.73	1.59
p_t	6.22	0.144	43.18	5.93	6.50
e_t	-0.17	0.036	-4.68	-0.24	-0.10
ζ	0.16	0.026	-	0.11	0.21
σ	0.14	0.030	-	0.08	0.19

Table 11: Detailed results of probit models, simple specification

Sector: All sectors					
Log-likelihood:	-3866482.38			**Number of obs:**	2851320
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	0.63	0.005	116.29	0.62	0.64
(s-c)/σ	-0.93	0.005	-173.03	-0.94	-0.91
$(\delta_{i,t})$^(0.5)	-0.07	0.005	-13.51	-0.08	-0.06
$\mathcal{Y}_t$	0.92	0.037	24.49	0.84	0.99
p_t	9.15	0.036	256.30	9.08	9.22
e_t	-0.09	0.007	-11.72	-0.10	-0.07
ζ	0.09	0.003	-	0.08	0.10
σ	0.10	0.005	-	0.09	0.11
Sector: Food					
Log-likelihood:	-1641924.10			**Number of obs:**	1229052
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	0.48	0.012	39.60	0.46	0.51
(s-c)/σ	-0.70	0.012	-57.76	-0.73	-0.68
$(\delta_{i,t})$^(0.5)	-0.06	0.012	-5.12	-0.09	-0.04
$\mathcal{Y}_t$	0.20	0.058	3.43	0.09	0.31
p_t	8.79	0.072	122.53	8.65	8.93
e_t	0.07	0.013	5.64	0.05	0.10
ζ	0.02	0.006	-	0.01	0.03
σ	0.11	0.009	-	0.09	0.13
Sector: Other Goods and Services					
Log-likelihood:	-176187.87			**Number of obs:**	131933
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	0.70	0.021	33.22	0.66	0.74
(s-c)/σ	-1.06	0.021	-50.56	-1.10	-1.02
$(\delta_{i,t})$^(0.5)	-0.08	0.021	-3.78	-0.12	-0.04
$\mathcal{Y}_t$	1.43	0.173	8.24	1.09	1.77
p_t	10.45	0.183	56.98	10.09	10.81
e_t	-0.36	0.034	-10.52	-0.42	-0.29
ζ	0.12	0.013	-	0.09	0.15
σ	0.08	0.019	-	0.05	0.12
Sector: Education and Recreation					
Log-likelihood:	-212798.73			**Number of obs:**	196576
	Coef.	**Std. Err.**	**t-stat**	**95% Conf. Interval**	
(S-c)/σ	1.45	0.011	132.86	1.43	1.47
(s-c)/σ	-2.12	0.011	-199.96	-2.14	-2.10
$(\delta_{i,t})$^(0.5)	-0.12	0.010	-11.99	-0.14	-0.10
$\mathcal{Y}_t$	6.09	0.127	48.12	5.84	6.34
p_t	11.80	0.076	155.84	11.66	11.95
e_t	0.32	0.020	15.87	0.28	0.36
ζ	0.34	0.005	-	0.33	0.35
σ	0.06	0.007	-	0.04	0.07

Table 11: Detailed results of probit models, simple specification (cont.)

Sector: Housing
Log-likelihood: -479490.92 **Number of obs:** 364112

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.61	0.014	42.01	0.58	0.64
(s-c)/σ	-1.05	0.014	-73.70	-1.07	-1.02
$(\delta_{i,t})$^(0.5)	-0.10	0.014	-7.12	-0.13	-0.07
$\mathcal{Y}_t$	1.36	0.106	12.89	1.16	1.57
p_t	11.02	0.111	98.81	10.80	11.23
e_t	0.11	0.022	5.23	0.07	0.16
ζ	0.11	0.008	-	0.09	0.13
σ	0.08	0.011	-	0.06	0.10

Sector: Medical and Personal Care
Log-likelihood: -678418.63 **Number of obs:** 515849

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.81	0.007	114.48	0.79	0.82
(s-c)/σ	-1.30	0.007	-185.76	-1.31	-1.28
$(\delta_{i,t})$^(0.5)	-0.08	0.007	-12.06	-0.09	-0.07
$\mathcal{Y}_t$	1.24	0.082	15.21	1.08	1.40
p_t	10.06	0.064	158.11	9.94	10.19
e_t	-0.70	0.014	-49.50	-0.73	-0.67
ζ	0.11	0.007	-	0.10	0.12
σ	0.09	0.008	-	0.07	0.10

Sector: Transportation
Log-likelihood: -127016.33 **Number of obs:** 95908

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.60	0.019	31.74	0.56	0.63
(s-c)/σ	-1.21	0.018	-66.77	-1.25	-1.18
$(\delta_{i,t})$^(0.5)	-0.13	0.018	-7.09	-0.16	-0.09
$\mathcal{Y}_t$	1.36	0.211	6.44	0.95	1.78
p_t	14.29	0.189	75.77	13.92	14.66
e_t	0.42	0.039	10.76	0.35	0.50
ζ	0.09	0.013	-	0.06	0.11
σ	0.06	0.015	-	0.03	0.09

Sector: Apparel
Log-likelihood: -430446.42 **Number of obs:** 317890

	Coef.	Std. Err.	t-stat	95% Conf. Interval	
(S-c)/σ	0.62	0.030	20.58	0.56	0.68
(s-c)/σ	-0.72	0.030	-24.04	-0.78	-0.66
$(\delta_{i,t})$^(0.5)	-0.04	0.030	-1.22	-0.10	0.02
$\mathcal{Y}_t$	1.09	0.122	8.90	0.85	1.33
p_t	4.91	0.119	41.39	4.68	5.14
e_t	-0.17	0.025	-6.84	-0.22	-0.12
ζ	0.18	0.018	-	0.15	0.22
σ	0.17	0.026	-	0.12	0.22

Figure 3: Estimated frictionless inflation and actual inflation

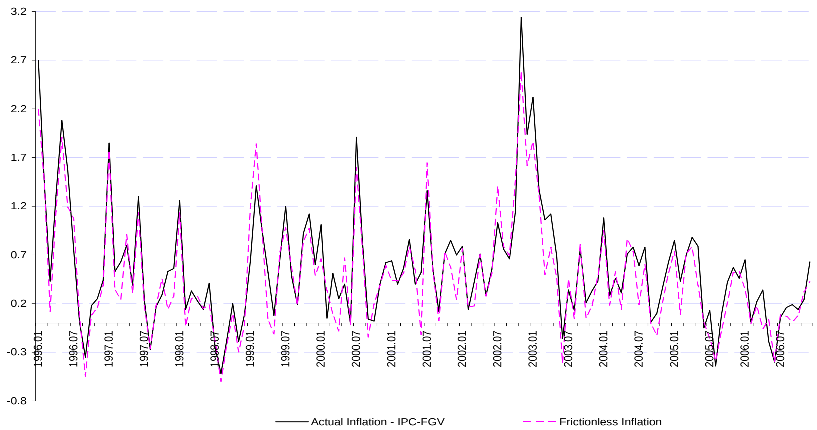

Figure 4: Impulse response of estimated frictionless optimal price, all goods