

Growth through Experimentation

Alain Gabler*

Markus Poschke[†]

PRELIMINARY AND INCOMPLETE

Abstract

Recent empirical research has documented the importance of shocks to firm-specific productivity, but has provided only limited evidence on their sources. This paper proposes and analyzes purposeful experimentation by firms as a source of such shocks and models industry dynamics in such a setting. We thereby make two contributions. The first is conceptual and consists in providing a micro-foundation to the stochastic process for firm-level productivity typically specified in the macroeconomic literature with firm heterogeneity. The second consists in quantifying the importance of experimentation for aggregate productivity growth to which experimentation, as a generalized form of R&D, contributes. We show that in a setting that allows for growth through experimentation, through market selection among firms and because of other sources, 45% of aggregate productivity growth can be attributed to experimentation. As a consequence, allocative distortions may not just reduce the level of productivity, but also have a substantial effect on growth rates.

JEL codes: E24, L16, O40.

*Université Laval

[†]McGill University

Keywords: endogenous growth, firm dynamics, selection, experimentation, entry and exit.

1 Introduction

Recent empirical research has documented the importance of shocks to firm-specific productivity, but has provided only limited evidence on their sources. The two findings that productivity varies a lot across firms even in narrowly defined industries, in large part due to firm-specific shocks to productivity, and that entry and exit rates are large, positively correlated across industries, and make a substantial contribution to aggregate productivity growth have had an important impact on the macroeconomic literature.¹ In particular, it has been recognized that distortions of allocative efficiency among heterogeneous firms have potentially large aggregate consequences and thereby can contribute to explaining differences in per capita income across countries.²

What are the sources of these firm-specific productivity shocks? This paper proposes and analyzes purposeful experimentation by firms as a source of such shocks and links them to aggregate growth. It is known from firm-level analyses that firms can influence the risk they take (see e.g. Coles, Daniel, and Naveen 2006) and that experimenting with new products and processes is a defining feature of innovation at the firm level. For instance, every year, about 25% of consumer goods for sale are either new or will be discontinued the next year, at least 40% of new goods are sold only for a single year, and plants adopt only between half and a third of the technologies they try (McGuckin, Streitwieser, and Doms 1996; Broda and Weinstein 2010; see also Lentz

¹For some important contributions to this empirical literature, see e.g. Baldwin (1995); Geroski (1995); Sutton (1997); Caves (1998); Foster, Haltiwanger, and Krizan (2001, 2006); Hsieh and Klenow (2009); Gabaix (2010); Syverson (forthcoming).

²See e.g. Hopenhayn and Rogerson (1993); Alvarez and Veracierto (2001); Restuccia and Rogerson (2008); Barseghyan (2008); Hsieh and Klenow (2009); Poschke (2009, 2010); Moscoso Boedo and Mukoyama (2010); Midrigan and Xu (2010).

and Mortensen 2008 and Bernard, Redding, and Schott 2010). There is a broad management literature that interprets this process of churning at different levels as “innovation through experimentation” (see e.g. Thomke 2003). The finding that R&D outcomes are very uncertain (Doraszelski and Jaumandreu 2009) also supports such an interpretation. Finally, it has been documented that the variance of idiosyncratic shocks differs across countries and industries and has been increasing over the last two decades.³ Interestingly, Castro, Clementi, and Lee (2009) show that the variance of idiosyncratic shocks is larger in industries in which there is more product turnover and in industries with higher R&D intensity. All of this suggests that to some degree, firms deliberate expose themselves to “productivity risk” in order to improve their productivity, but can control the extent of this risk by choosing their experiments.

The first contribution of this paper consists in modelling experimentation at the firm level to provide very simple micro-foundations for a stochastic process for firm-level productivity. In existing heterogeneous-firm work in macroeconomics, this is taken as exogenous (see e.g. Hopenhayn 1992; Alvarez and Veracierto 2001; Samaniego 2006; Luttmer 2007; Restuccia and Rogerson 2008). If it is instead endogenous, analyses of the impact of e.g. micro-level distortions on aggregate productivity that presuppose an exogenous process miss the effect of distortions on risk-taking and therefore miss part of the effect on aggregate growth. Therefore, modeling experimentation is important.

We model experimentation in a very simple way: in the setting of a heterogeneous-firm model in the tradition of Hopenhayn (1992), we allow firms to experiment with their production process every period. The experiment is modelled as drawing a random innovation to the firm’s productivity.⁴ (We also allow for additional shocks the firm

³See Haltiwanger, Jarmin, and Schank (2003); Castro, Clementi, and Lee (2009); Bartram, Brown, and Stulz (2009); Bartelsman, Gautier, and de Wind (2010) and Faggio, Salvanes, and Van Reenen (2007); see also Haskel and Martin (2002) and Dunne, Foster, Haltiwanger, and Troske (2004).

⁴When productivity is measured as revenue productivity, as is the case in almost all data sets used in productivity measurement, fluctuations in product quality or consumer tastes are indistinguishable from productivity fluctuations. For this reason, our setting in terms of productivity risk and

cannot influence.) Firms can choose how risky or crazy they want their experiment to be; riskier experiments are draws from a distribution with a higher variance. Firms are not forced to stick with the outcomes of failed experiments; they can undo experiments that reduce their productivity, possibly at a cost or after a delay. Because of this option (think of not implementing R&D findings or pulling an unsuccessful new product off the market), the expected value of experimenting is positive and increases in the riskiness of the experiment. This is balanced by a higher cost of conducting and undoing failed ambitious experiments compared to more incremental/marginal ones so that in equilibrium, firms choose experiments with limited risk. We first derive some results in a simple analytical model and then integrate the experimentation process into a full quantitative model.

As a consequence of experimentation, a firm's productivity is stochastic. It may sometimes decline, either because it takes time to undo failed experiments or because it is costly to do so. This is in line with observations from the empirical literature that the productivity of new projects or the profitability of new products are hard to forecast, resulting in high entry and exit rates of young firms and products. Even once in place, it takes time to quantify the change; see e.g. Jovanovic (1982) or Abbring and Campbell (2005). The particular nature of the stochastic process generated by our model, with full persistence of innovations to the upside and reversion to the mean after innovations to the downside, is new; the empirical literature has focussed on symmetric processes for productivity. The AR(1) processes typically estimated there have the disadvantage of implying that productivity improvements are always temporary;⁵ the process generated by the model in contrast allows for persistence of improvements but mean reversion after

experimentation with processes can also be interpreted in terms of experimentation with products.

⁵For instance, Hall (1987); Evans (1987); Sutton (1997); Blundell and Bond (2000); Ábrahám and White (2006) and Lee and Mukoyama (2008) estimate an autoregressive process for firm-level TFP and find an autoregressive coefficient below 1. (Distinguishing this from a unit root is not always easy, see e.g. Hall and Mairesse 2005.)

other shocks or failed experiments and could thus reconcile persistence of productivity improvements with apparent mean reversion. In addition, while symmetric processes are appealing for shocks hitting the firm from outside, e.g. due to changes in consumer tastes, the asymmetric process proposed here reflects that firms have some control over their production process or the appeal of their product portfolio. The asymmetry also helps to identify the component of productivity movements due to experimentation and to separate it from shocks coming from other sources.

The possibility to reject failed experiments implies that in expectation, an experimenting firm's productivity grows. In promoting productivity, experimentation is akin to R&D and can in fact be interpreted as a generalized form of R&D, which after all essentially is directed experimentation.⁶ Our approach is more general than the typical modeling of R&D for two reasons. Firstly, it allows capturing the activity of the large portion of firms which do not report patenting or R&D spending but still innovate. (These are non-negligible; see also Francois and Lloyd-Ellis, 2003; Klette and Kortum, 2004; Syverson, forthcoming, .) Secondly, in our setting, policies that do not affect the cost of R&D or its rate of return but create adjustment costs still affect growth by penalizing risky experiments.

Aggregating, experimentation at the firm level translates into growth in aggregate productivity, with the idiosyncratic risk present at the firm level smoothed out. Unsuccessful firms slowly fall behind as others' productivity progresses, until they find that

⁶More formally, compared to the endogenous growth literature with R&D, our modelling strategy implies that a firm's costly innovation activities always yield a result (not only with some probability), but that this result may well be unsatisfactory (e.g. not a productivity improvement). Moreover, in our setting, investing more in innovation activities raises the expected productivity increase because new, better outcomes become attainable, not because the probability of making an innovation of fixed size increases.

Kortum (1997) allows for stochastic R&D outcomes when modeling research at the frontier. Yet, predictions for the frontier do not help explain firm dynamics for the bulk of firms. Doraszelski and Jaumandreu (2009) propose a setting where firms engage in R&D with stochastic outcomes resulting in a stochastic process for productivity, which they estimate using the Spanish ESEE firm-level data set. Their paper is empirical and focusses on the firm level; it does not consider aggregate implications.

profits do not cover fixed costs anymore and they exit. Coupling this with entry and closing the model as in Luttmer (2007) and Gabler and Licandro (2007) results in a balanced growth path. In these papers, the growth rate depends on the (exogenous) variance of idiosyncratic shocks and on the (endogenous) rates of entry and exit. A larger variance of firm-level productivity shocks (crazier experiments) raises the aggregate productivity growth rate. Intuitively, this occurs because firms always have the option to exit after negative shocks. Then an increase in variance is beneficial for productivity growth.⁷ This is true even if the average shock does not improve a firm’s productivity. In the present setting, this effect is even stronger, as failed experiments can be undone while continuing operations. Thus, by allowing for a more realistic response to idiosyncratic shocks and by endogenizing their variance, our setting allows to quantify experimentation and its contribution to growth. It also helps to understand the determinants of variation in experimentation across industries and over time. Doing so provides a more complete picture of the contribution of firm-level purposeful improvement activity to aggregate growth than R&D or patent figures could give.

The second contribution of the paper exploits this and consists in quantifying the contribution of experimentation to aggregate productivity growth by calibrating the model to U.S. data on firm dynamics such as firm and job turnover rates, survival hazards, and the contribution of entry and exit to aggregate productivity growth.

Preliminary results obtained in a version of the model where undoing failed experiments is restricted to be costless suggest that 45% of aggregate productivity growth can be attributed to experimentation.

⁷In this spirit, the process of entry and exit, which involves a lot of “churn” and exhibits positive correlation of entry and exit rates across industries, can also be interpreted in terms of experimentation, as proposed by Haltiwanger, Jarmin, and Schank (2003) and by Bartelsman, Perotti, and Scarpetta (2008): entry can result in spectacular success (sometimes) or in failure (often), is associated with large uncertainty about success, and therefore is essentially an experiment. This view fits in naturally with the importance of entry and exit for aggregate productivity growth documented by Foster, Haltiwanger, and Krizan (2001, 2006).

We then analyze the impact of policy on experimentation, and its explanatory power for cross-country differences. [In progress.] Because ambitious experiments may require a firm to adjust its factors a lot, policies which penalize factor adjustments not only impede the efficient allocation of resources among firms, as abundantly analyzed in the literature, but also discourage experimentation in the first place. This fits qualitatively with evidence comparing Germany with the U.S. (Haltiwanger, Jarmin, and Schank, 2003). Using measures of such policies, we verify how much of the variation in idiosyncratic risk across countries and industries reported in the empirical literature can be explained by the model. This in turn allows assessing the contribution of differences in regulation to differences in growth rates across countries. (This is in contrast to e.g. Moscoso Boedo and Mukoyama (2010), who assess the impact of entry and labor market regulation on productivity levels.)

Because the way we model experimentation is designed to fit well in a macro model, it is quite distinct from the theoretical literature on experimentation (see e.g. Bolton and Harris, 1999; Keller, Rady, and Cripps, 2005; Acemoglu, Bimpikis, and Ozdaglar, 2011). These papers consider two- or multi-armed bandits. These correspond to the choice between discrete projects with returns following unknown stochastic processes which can be learned by experimentation. Our setting in contrast can be interpreted either as experimenting with changing an existing project or with adding a new project to a portfolio, where the stochastic process for returns is known and can be influenced. The two types of models thus correspond to very different settings, involving different types of uncertainty.

The paper is organized as follows. In Section 2, we present a simple model of experimentation that admits analytical solutions. Section 3 contains the full quantitative model. The quantitative analysis is described in Section 4, and Section 5 concludes.

2 A Simple Model of Endogenous Experimentation

In this section we present a simple model in which productivity growth results from experimentation by firms. We model this process of experimentation by assuming that firms are hit by idiosyncratic productivity shocks whose variance they can choose. We interpret these as experiments with random outcomes through which firms try to improve their productivity. Firms can choose how conservative or “crazy” an experiment is; this is reflected in the variance of the shock they receive. If the result of an experiment is not as desired, a firm can revert to its previous level of productivity. We assume that experimentation is costly in terms of current output. The simple specification chosen here, together with some convenient functional form assumptions, allows for an analytical solution for firm behavior and the growth rate. As usual, simplicity has benefits, but also costs. Therefore, we numerically explore a generalized model with some additional realistic features in the Section 3.

The economy is populated by a representative household and by a continuum of measure 1 of goods-producing firms. In this section, we abstract from entry and exit. There also is a sector of perfectly competitive financial intermediaries. Time is discrete.

2.1 Preferences

Household preferences are given by

$$\begin{aligned}\sum_{t=0}^{\infty} \beta^t U(c_t) &= \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \quad \text{for } \sigma > 0, \sigma \neq 1 \\ &= \sum_{t=0}^{\infty} \beta^t \log(c_t) \quad \text{for } \sigma \rightarrow 1,\end{aligned}$$

where $\beta \in (0,1)$. Households can consume or save by investing in shares of output-producing firms via a sector of perfectly competitive financial intermediaries. In equi-

librium, these intermediaries don't make profits, all hold the market portfolio, and, absent aggregate uncertainty, pay a net return r on consumers' investments. A household's budget constraint then is

$$c_t = w_t l + a_t (1 + r) - a_{t+1},$$

where a_t denotes assets held at the beginning of period t and household labor supply is constant at l .

The Euler equation for the accumulation of assets is

$$c_t^{-\sigma} = \beta E [(1 + r) c_{t+1}^{-\sigma}]. \quad (1)$$

For future use, also define ρ such that $\beta = \frac{1}{1+\rho}$.

2.2 The Problem of the Firm

Firms produce output with the production function

$$y(z, s) = [z \cdot \theta(s)]^\alpha l(z)^{1-\alpha} \quad (2)$$

and sell it in a competitive market. We normalize the price of output to 1. Firms differ in their productivity z and choose their labor input $l(z)$ as a function of it. The term $\theta(s)$ indicates a disruption cost of experimentation. This is analogous to Holmes, Levine, and Schmitz (2008), who assume the presence of similar “switchover disruption costs” in technology adoption in their analysis of the link between competition and productivity. Profits then are

$$\Pi(z, w, s) = [z \cdot \theta(s)]^\alpha l(z, s)^{1-\alpha} - w l(z, s), \quad (3)$$

and optimal $l(z, s)$ satisfies

$$\alpha [z \cdot \theta(s)]^\alpha l(z, s)^{-\alpha} = w. \quad (4)$$

Substituting (4) into (3) yields

$$\Pi(z, w, s) = \alpha z \cdot \theta(s) \left(\frac{1 - \alpha}{w} \right)^{\frac{1 - \alpha}{\alpha}}. \quad (5)$$

Each period, the firm chooses how much to experiment. Experimenting implies drawing an innovation ε from a distribution with *cdf* $\Phi_s(\varepsilon)$, resulting in a new level of productivity $z' = \varepsilon z$. The firm can choose the risk it takes in its experiment. This choice is represented by the parameter s , which is related to the variance of ε . In this section, we assume that ε follows a logistic distribution with mean μ and variance $s^2\pi^2/3$. The firm can conduct one experiment per period, implying that the choice of s can be adjusted every period. Choosing riskier experiments is costly as it is disruptive of current production: we assume that $\theta(0) = 1, \theta'(s) < 0, \theta''(s) < 0$, so the cost of experimenting is convex.

Firms can discard the results of unsuccessful experiments, i.e. those resulting in draws of $\varepsilon < 1$, which would imply a fall in productivity if the result of the experiment was adopted. In this case, the firm still suffers the disruption cost of the experiment, but can directly revert to its previous technology with productivity z . Later on, we will consider that reverting to the old technology may be costly or only possible with a delay, either because it takes time to measure ε or because changes brought about by the experiment are costly to undo. In that case, the acceptance threshold may be below 1.

The value of a firm is

$$\begin{aligned} V(z, w) &= \max_s \left(\Pi(z, w, s) + \frac{1}{1+r} \left[\int_{-\infty}^1 V(z, w) d\Phi_s(\varepsilon) + \int_1^{\infty} V(z', w) d\Phi_s(\varepsilon) \right] \right) \\ &= \max_s \left(\Pi(z, w, s) + \frac{1}{1+r} \left[V(z, w) \Phi_s(1) + \int_1^{\infty} V(z\varepsilon, w) d\Phi_s(\varepsilon) \right] \right). \end{aligned}$$

If we assume that $V(z, w) = \zeta(w)z$, i.e. linear in z just as the profit function, we have

$$\zeta(w)z = \max_s \left(\Pi(1, w, s)z + \frac{1}{1+r} \left[\zeta(w)z\Phi_s(1) + \int_1^{\infty} \zeta(w)\varepsilon z d\Phi(\varepsilon) \right] \right).$$

From (5),

$$\zeta(w) = \max_s \left(\theta(s) \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} + \zeta(w) \frac{1}{1+r} \left[\Phi_s(1) + \int_1^{\infty} \varepsilon d\Phi(\varepsilon) \right] \right). \quad (6)$$

The first-order condition for s is

$$\theta'(s) \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} + \zeta(w) \frac{\gamma'(s)}{1+r} = 0,$$

where $\gamma_z(s) = \Phi_s(1) \cdot 1 + \int_1^{\infty} \varepsilon d\Phi(\varepsilon)$ is the expected gross growth rate of z given a choice of s . Inserting this condition into (6), we obtain

$$-\frac{\theta'(s)}{\theta(s)} = \frac{\gamma'_z(s)}{1+r-\gamma_z(s)}, \quad (7)$$

which implies that the marginal cost of experimentation (left-hand side) should equal its expected marginal capitalized benefit (right-hand side).⁸

With a logistic distribution for ε with mean $\mu = 1$ and variance $s^2\pi^2/3$, $\gamma_z(s) =$

⁸As usual, this requires restricting parameters such that $\gamma_z - 1 < r$ for boundedness. A sufficient condition for the second order condition for a maximum to be satisfied is that γ''_z is non-positive, which holds for logistic innovations ε .

$1 + s \ln 2$. Assuming that the disruption cost function takes the form $\theta(s) = (\bar{s} - s)^q / \bar{s}^q$, $s \in [0, \bar{s}]$, $q \in (0, 1)$, this implies

$$\frac{q}{\bar{s} - s} = \frac{\ln 2}{r - s \ln 2}.$$

The optimal choice of experimentation for an individual firm then is

$$s = \frac{\bar{s} \ln 2 - qr}{(1 - q) \ln 2}$$

and its expected net growth rate of z is

$$\gamma_z - 1 = s \ln 2 = \frac{\bar{s} \ln 2 - qr}{1 - q}.$$

The degree of experimentation increases in $\bar{s}$ and falls in r . This is of course typical for an investment decision. (Given q , raising $\bar{s}$ reduces the cost of the investment.) The derivative of s with respect to q has the sign of $\bar{s} \ln 2 - r$, which is positive for most plausible parameter values. Higher q implies that the marginal cost of experimenting more becomes flatter for low s and thereby pushes the optimal choice of s closer to $\bar{s}$.

With all firms choosing the same s and conducting independent experiments, there is no aggregate uncertainty, and the growth rate of aggregate productivity equals the expected productivity growth rate of each firm, $\alpha(\gamma_z - 1)$.⁹ This implies an aggregate output growth rate g of

$$\alpha(\gamma_z - 1) = \alpha s \ln 2 = \alpha \frac{\bar{s} \ln 2 - q\rho}{1 + (\alpha\sigma - 1)q},$$

⁹For details on aggregation, see the appendix.

an interest rate

$$1 + r = (1 + \alpha s \ln 2)^\sigma (1 + \rho) = 1 + \rho + \alpha \sigma s \ln 2$$

and a firm value

$$\zeta(w)z = \alpha \left(\frac{1 - \alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \theta(s) \frac{1 + r}{r - \gamma_z} = \alpha \left(\frac{1 - \alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \theta(s) \frac{1 + \rho + \alpha \sigma s \ln 2}{\rho + (\alpha \sigma - 1) s \ln 2},$$

where the last equality in each equation uses a log approximation to r . Given the positive relationship of r with ρ and σ stemming from the Euler equation, experimentation intensity and growth fall with ρ . The growth rate increases with α because the direct effect of higher α that translates a given γ_z into a higher output growth rate at the firm level outweighs the general equilibrium effect operating through a higher interest rate.

This simple model allows for a balanced growth path, albeit one where the variance of the firm productivity distribution grows without bound. In the full model, this can be corrected by considering entry and exit. The endogenously generated aggregate growth rate depends on patience and risk attitudes, on the importance of technology relative to inputs in generating output (α), and on factors related to the cost of experimentation ($\bar{s}, q$). The last three factors may differ across industries, and can therefore contribute to explaining cross-industry differences in experimentation.

These growth rate determinants and their comparative statics are similar to those found in other endogenous growth models. What is particular here is that innovation is very “informal” and occurs so to speak on the shop floor, not in the R&D department. R&D can of course be seen as a special case of our perspective on innovation, consisting in directed experimentation using dedicated resources. In fact, our setting essentially is a generalization of common ways of modeling R&D: An experimenting firm’s expected productivity improvement increases in s since higher s implies that a productivity

improvement is larger, conditional on being an improvement. The probability of the experiment being successful is not affected by s in the simplest specification. (This changes slightly once experiments are costly to reverse; see below.) This is in contrast to the standard specification of knowledge production through R&D in the endogenous growth literature, where more innovative activity raises the probability of obtaining a given productivity improvement. If only the expected productivity improvement mattered, the two settings would be very close. However, given the variance of firm-level productivity, its variation across countries and industries, and the uncertainty of R&D success found in the data, the typical R&D specification appears to abstract from many relevant factors that our specification can capture. In particular, if the process of improving productivity entails potentially substantial fluctuations in input use, as it does if experimentation is important and there are costs of undoing experiments, not only the expected value of higher productivity matters for how much innovation activity is undertaken, but adjustment costs encountered in the process also affect the choice of how much to experiment. This factor of course is absent from the typical R&D specification.

2.3 Other Sources of Shocks

Of course, a firm's experimentation with products or technology may not be the only source of innovations to its (measured) productivity. There may also be idiosyncratic shocks affecting it coming from changes in customers' tastes or from exogenous shocks to its technology. This section shows that allowing for such additional shocks does not affect the basic conclusions of this section.

Suppose that every period, after a firm's productivity is modified by the firm's experiment, its productivity is also hit by an exogenous multiplicative innovation u with *cdf* $F(u)$. Because this is uncorrelated with the level of z , it does not break

linearity of firm value in z ; there just is added uncertainty. A firm's problem as given in (6) then becomes

$$\zeta(w) = \max_s \left(\theta(s) \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} + \zeta(w) \frac{1}{1+r} \left[\Phi_s(1) + \int_1^\infty \varepsilon d\Phi(\varepsilon) \right] \int u dF(u) \right). \quad (8)$$

The first order condition for s then is

$$\theta'(s) \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} + \zeta(w) \frac{\gamma'_z(s)}{1+r} \int u dF(u) = 0,$$

implying

$$-\frac{\theta'(s)}{\theta(s)} = \frac{\gamma'_z(s)}{1+r-\gamma_z(s)} \int u dF(u).$$

If the expected value of u is 1, which is a rather natural assumption, this equation reduces to 7 above. Then the solutions for s , r and the growth rate are unaffected.¹⁰

3 The Full Model

The simple model in the preceding section abstracted from several realistic features of the full model we use in the quantitative analysis. The full model is presented in this section. Apart from allowing for more realistic but less tractable functional forms, we enhance the simple model by endogenous entry and exit of firms and by allowing for a cost of undoing experiments.

3.1 Costly Reversal of Experiment Outcomes

Arguably, experiments may not just disrupt current production, but may also be costly to undo. Therefore, suppose that undoing an experiment carries a positive disruption

¹⁰In this simple analytical model, there is no growth through selection stemming from u because there is no entry and exit.

cost $\kappa(z, s)$ which depends positively on the degree of experimentation s and the firm's current productivity z . We assume that $\kappa(z, s)$ is positive and monotonically increasing in both arguments. A continuing firm's problem then becomes

$$V(z, w) = \max_s \left\{ \Pi(z, w, s) + \frac{1}{1+r} E \max[V(z, w) - \kappa(z, s), V(\varepsilon z, w)] \right\}.$$

Because $V(z, w)$ is monotonically increasing in z , a threshold policy is optimal and firms will undo experiments with outcomes ε below some threshold $\underline{\varepsilon}$ defined by

$$V(z, w) - \kappa(z, s) = V(\underline{\varepsilon} z, w).$$

That is, for $\varepsilon = \underline{\varepsilon}$, the value of undoing the experiment (left-hand side) is equal to the value of using the newly found technology. The threshold $\underline{\varepsilon}$ is a function of z , s and w . Since $\kappa(z, s) \geq 0$, it follows that $\underline{\varepsilon} \leq 1$: only experiments which lead to lower productivity will be rejected.

We assume the following functional form for the cost of undoing the experiment:

$$\kappa(z, s) = \kappa z s^\nu,$$

with $\nu > 1$. Assuming that the firm's value function is linear, that is, $V = z\zeta(w)$, the threshold value will be

$$\underline{\varepsilon}(s, w) = 1 - \frac{\kappa s^\nu}{\zeta(w)},$$

which is decreasing in κ (a higher cost of undoing makes rejection less likely), and increasing in ζ (a higher value of being productive makes it more costly to maintain a failed experiment). The probability of having to reject the outcome of an experiment is $\Phi_s(\underline{\varepsilon}(s, w))$. For symmetric distributions, this implies that the rejection probability

is less than $1/2$ for $\kappa > 0$. The problem then becomes

$$V(z, w) = \max_s \left(\Pi(z, w, s) + \frac{1}{1+r} \int \left\{ \begin{aligned} &\int_{-\infty}^{\underline{\varepsilon}(s, w)} [V(zu, w) - \kappa z s^\nu] d\Phi_s(\varepsilon) \\ &+ \int_{\underline{\varepsilon}(s, w)}^{\infty} V(\varepsilon zu, w) d\Phi_s(\varepsilon) \end{aligned} \right\} dF(u) \right),$$

which implies (using the specification of the profit function in equation 5):

$$\zeta(w) = \max_s \left(\theta(s) \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} + \frac{1}{1+r} \{ \zeta(w) \gamma(s) - \Phi_s(\underline{\varepsilon}(s)) \kappa s^\nu \} \int u dF(u) \right),$$

where $\gamma(s) = \Phi_s(\underline{\varepsilon}) + \int_{\underline{\varepsilon}}^{\infty} \varepsilon d\Phi(\varepsilon)$ again is the expected gross growth rate of z for a given degree of experimentation s . The first-order condition satisfies

$$\frac{\zeta(w) \gamma'(s)}{1+r} \int u dF(u) = -\theta'(s) \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} + \frac{1}{1+r} \frac{\partial [\Phi_s(\underline{\varepsilon}(s)) \kappa s^\nu]}{\partial s} \int u dF(u).$$

At the optimum, the expected marginal benefit of experimenting, which consists in the increase in the expected growth rate (left-hand side), is equal to its marginal cost, which includes the increase in the disruption cost $\theta(s)$ and the increase in the expected cost of reverting to the old technology (right-hand side). s affects the expected cost of reversion in two ways: by making reversion more likely, and by making it more costly. Since the choice of s affects both the shape of the distribution of ε and its integration threshold, there is no closed-form solution for s .

3.2 Productivity Dynamics

The model has implications for the evolution of firm-specific productivity. In particular, it predicts that z evolves as follows:¹¹

$$z' = \begin{cases} z\varepsilon & \text{if } \varepsilon \geq \underline{\varepsilon} = 1 - \kappa s^\nu / \zeta, \\ z & \text{otherwise.} \end{cases}$$

Since $\underline{\varepsilon} < 1$, experimentation can result in persistent reductions in productivity; however, while losses are bounded on the downside by the possibility of reversion, they are unbounded on the upside.

More realistically, reverting to the old technology may take time, either because of physical constraints, or because the firm does not directly observe its productivity and needs to produce for some time before assessing whether the experiment was successful (as e.g. in Jovanovic, 1982). A tractable case here is the one where the firm needs to use the new technology for one period before it can revert to the old one. Here, the problem is

$$V(z, w) = \max_s \left\{ \Pi(\varepsilon z, w) + \frac{1}{1+r} E \max [V(z, w) - k(z, s), V(\varepsilon z, w)] \right\}.$$

While the reversion delay may affect the choice of s , it does not affect the expected value of the firm in the next period, and hence has no impact on the reversion threshold $\underline{\varepsilon}$. The law of motion for z will therefore not change (except through a change in the

¹¹The remaining few paragraphs abstract from the additional shock u for notational simplicity.

choice of s). However, observed productivity is equal to

$$z_t \varepsilon_t = \begin{cases} z_{t-1} \varepsilon_{t-1} \varepsilon_t & \text{if } \varepsilon_{t-1} \geq \underline{\varepsilon} = 1 - \kappa s^\nu / \zeta, \\ z_{t-1} \varepsilon_t & \text{otherwise.} \end{cases}$$

If the productivity which is observed at time $t - 1$, $z_{t-1} \varepsilon_{t-1}$, is very low ($\varepsilon_{t-1} < \underline{\varepsilon}$), the firm will reject the experiment and revert to its previous technology. Because of this, there is now an additional asymmetry: productivity can fall, but if it falls a lot (below $z \underline{\varepsilon}$), it will rebound quickly back to its original level. If it falls slightly, it may stay low. The corresponding growth rate is

$$g_t = \frac{z_t \varepsilon_t}{z_{t-1} \varepsilon_{t-1}} = \begin{cases} \varepsilon_t & \text{if } \varepsilon_{t-1} \geq \underline{\varepsilon} = 1 - \kappa s^\nu / \zeta, \\ \varepsilon_t / \varepsilon_{t-1} & \text{otherwise.} \end{cases}$$

When ε_{t-1} (and therefore g_{t-1}) is low, g_t is high.

As a consequence, productivity improvements are fully persistent, as are small declines, while large declines will be reversed. When this asymmetric process is observed in the data, the level of productivity may therefor appear to exhibit mean reversion.

3.3 Firm Entry and Exit

Firm turnover is substantial in real economies. Since it is robustly related to productivity dynamics, it is important to consider it in our analysis. Suppose that firms pay a fixed operating cost of κ_f labor units each period, and can costlessly exit at the beginning of the period, before the new value of z is known.¹² The value of a firm then is

$$W(z, w) = \max[0, EV(z, w)],$$

¹²This is isomorphic to allowing for a scrap value of the firm.

where $V(z, w)$ is the value of staying in the market given by

$$V(z, w) = \max_s E \left\{ \Pi(\varepsilon zu, w) + \frac{1}{1+r} \max[W(zu, w) - k(z, s), W(\varepsilon zu, w)] \right\}. \quad (9)$$

The expectation is taken over ε and u . The optimal strategy is to choose a threshold level z_x below which the firm will exit. The optimal threshold then implies

$$EV(z_x, w) = 0.$$

There is free entry, and entering costs κ_e units of labor. Denote the productivity distribution of entrants by η and that of incumbents by μ . Entering firms imperfectly imitate incumbents; to model this, assume that $\eta(z) = \mu(ez), e \geq 1$, so that the expected productivity of entrants is weakly below that of incumbents.

The law of motion of the productivity distribution is

$$\mu' = Q\mu + \eta,$$

where Q is the productivity transition operator. It is defined by the firms' optimal choices of exit and of s .

In the following, we restrict our analysis to the balanced growth path of this economy. This is defined as a situation where all aggregate variables grow at constant rates.

4 Quantitative Analysis

In this section, the model is calibrated to U.S. data on firm dynamics such as firm and job turnover rates, survival hazards, and the contribution of entry and exit to aggregate

productivity growth. While we focus on the full model for quantitative results, it is useful to also consider the analytical model where the connections between some parameters are more explicit.

4.1 Analytical Model

Parameters to be calibrated are $\rho, \alpha, \sigma, \bar{s}, q$.¹³ Standard parameters can be set in the usual way: to obtain a labor income share of two thirds, set $\alpha = 1/3$. Suppose that the coefficient of relative risk aversion σ is 2. Combined with an average annual growth rate in per capita consumption of 1.8% and a long-run average return on capital of 6.5%, the Euler equation on the balanced growth path then implies that ρ is 0.027.

$\bar{s}$ can then be pinned down via the bound on γ_z . For boundedness, the net productivity growth rate needs to be smaller than the rate of return on capital r . This implies that s needs to be smaller than $0.065 / \ln 2 = 0.0938 = \bar{s}$. It then follows from the optimality condition for s that $q = (\bar{s} \ln 2 - s) / (r - s \ln 2) = (r - s) / (r - (\gamma_z - 1)) = 0.83$. This implies that $\theta(s)$ is concave down as required, but only weakly so.

This strategy presupposes that growth is entirely due to experimentation, which probably is an overstatement. An alternative calibration strategy avoids using growth rate information from the data. This allows inferring how much growth is due to experimentation. Here, the crucial part is finding a number for q or, equivalently, for s . Information on the variance of firm-specific productivity is helpful but not sufficient, as that has to be split between the part due to experimentation and, σ_u , the part attributable to the exogenous shock.¹⁴ With entry and exit, information on job

¹³Without entry and exit, σ_u does not affect behavior and therefore does not need to be calibrated. In principle, the mean of the distribution from which innovations are drawn could also be calibrated. As shown below, slightly less than 50% of new products remain on the market for only one year. This is consistent with a mean close to 1, as assumed in Section 2 above.

¹⁴For example, according to Campbell, Lettau, Malkiel, and Xu (2001), the annual standard deviation of individual stock returns is about 0.3 for firms listed on NYSE or NASDAQ and argue that most of this is due to idiosyncratic shocks. In our model, value is proportional to productivity, so the

Table 1: Parameterization

Parameter	Value	Parameter	Value
σ	2	σ_e	3.72
β	.997	σ_u	.29
α	.64	φ	1.32
$\bar{s}$	.0264	κ_f	.0016
q	.05	κ_e	140

turnover, firm turnover and product/project survival can do the job because it extracts information from the asymmetry induced by experimentation (see below).

4.2 Full model

We now turn to the quantitative evaluation of a version of the model with entry, exit, and disruption costs to experimentation. We assume that the idiosyncratic productivity shocks, as well as the productivity of new firms, are log-normally distributed with mean zero. The length of a time period is set to one month.

The parameters which are set based on a-priori information are a coefficient of risk-aversion σ of 2; a discount factor β of .997, and a labor share α of 64%. The remaining parameters which need to be set are $\bar{s}$ and q , which determine the upper bound for experimentation and the shape of the disruption cost from experimentation, respectively; the variance of the exogenous productivity shocks for entrants, σ_e , and for incumbents, σ_u ; the parameter determining the relative productivity of entrants, φ ; and the fixed costs of production, κ_f , and entry, κ_e .

standard deviation of productivity growth is the same as that of stock returns. A standard deviation of 0.3 implies an aggregate growth rate of more than 11% if shocks are entirely due to experimentation. The observed long-run productivity growth rate of about 1.8% instead is consistent with an s of 0.026 or a standard deviation of firm-level productivity growth due to experimentation of 0.05. Thus, abstracting from the asymmetry in shocks induced by experimentations, these numbers can be interpreted as suggesting that about one sixth of shocks to firm-level productivity could be due to experimentation.

Table 2: Targeted Moments

Moment	Data	Model
Yearly exit rate, incumbents	11%	11%
4-year survival rate, new firms	63%	63%
Relative productivity, new firms	99%	99%
Contribution of entry/exit to growth	60%	60%
Average firm size	26.4	26.4
Contribution of growth through experimentation	—	45%

These latter parameters are set such as to replicate the following moments which are observed in U.S. data : a yearly exit rate of incumbents of 11% (Foster, Haltiwanger, and Krizan, 2001); a 4-year survival rate rate of new firms of 63% (Bartelsman, Haltiwanger, and Scarpetta, 2004); a relative productivity of firms less than five years old of 99% (Foster, Haltiwanger, and Krizan, 2001); a contribution of entry and exit to aggregate productivity growth of 60% (Luttmer, 2007); and an average firm size of 26.4 employees.

As in the analytical model, $\bar{s}$ is set such that the condition for boundedness is fulfilled, that is, the net productivity growth rate needs to be smaller than the rate of return on capital r . The latter is obtained from the Euler equation for consumption (1).

Since there is no closed-form solution, the model is solved numerically. Table 1 lists the chosen parameters, while Table 2 lists the targeted moments for the data and the model.

In our model, the implications of firm-specific productivity shocks in terms of aggregate growth varies greatly depending on the source of the shocks. In particular, since unsuccessful experiments can be discarded, shocks which are due to experimentation lead to much higher growth than exogenous productivity shocks of the same size. Indeed, in the calibrated version of the model, 45% of productivity growth is due to

experimentation, although the variance of experiments is several magnitudes smaller than the variance of the exogenous shocks to productivity.

5 Conclusion

Recent empirical research has documented the importance of shocks to firm-specific productivity, but has provided only limited evidence on their sources. This paper proposes and analyzes purposeful experimentation by firms as a source of such shocks and models industry dynamics in such a setting. We thereby make two contributions. The first is conceptual and consists in providing a micro-foundation to the stochastic process for firm-level productivity typically specified in the macroeconomic literature with firm heterogeneity. The implied process for firm-level productivity is theoretically appealing: positive shocks, which may be due to the firm's own purposeful experimentation behavior, may be permanent, whereas negative shocks may be reversed. It remains to test this process in actual firm-level data. The second contribution consists in quantifying the importance of experimentation for aggregate productivity growth to which experimentation, as a generalized form of R&D, contributes. We show that in a setting that allows for growth through experimentation, through market selection among firms and because of other sources, 45% of aggregate productivity growth is due to experimentation.

Our setup also has the potential to explain cross-industry and cross-country differences in experimentation, the variance of idiosyncratic shocks, and growth. It also shows very clearly that factors causing volatility in firm-specific productivity create a tradeoff between their growth benefits and the adjustment costs they imply. Policy can attempt to govern this tradeoff. Therefore, a natural next step (in progress) is to analyze the effects of policies such as employment protection legislation in this setting.

References

- ABBRING, J. H., AND J. R. CAMPBELL (2005): “A Firm’s First Year,” *Tinbergen Institute Discussion Paper*, 046/3.
- ÁBRAHÁM, A., AND T. K. WHITE (2006): “The Dynamics of Plant-level Productivity in US Manufacturing,” *U.S. Bureau of the Census Center for Economic Studies (CES) Working Paper*, CES-WP-06-20.
- ACEMOGLU, D., K. BIMPIKIS, AND A. OZDAGLAR (2011): “Experimentation, Patents, and Innovation,” *American Economic Journal: Microeconomics*, 3(1).
- ALVAREZ, F., AND M. VERACIERTO (2001): “Severance Payments in an Economy with Frictions,” *Journal of Monetary Economics*, 47, 477–498.
- BALDWIN, J. R. (1995): *The Dynamics of Industrial Competition*. Cambridge University Press, Cambridge.
- BARSEGHYAN, L. (2008): “Entry Costs and Cross-Country Differences in Productivity and Output,” *Journal of Economic Growth*, 13(2), 145–167.
- BARTELSMAN, E., J. HALTIWANGER, AND S. SCARPETTA (2004): “Microeconomic Evidence of Creative Destruction in Industrial and Developing Countries,” *World Bank Policy Research Working Paper*, 3464.
- BARTELSMAN, E. J., P. A. GAUTIER, AND J. DE WIND (2010): “Employment Protection, Technology Choice, and Worker Allocation,” *IZA Discussion Paper*, 4895.
- BARTELSMAN, E. J., E. PEROTTI, AND S. SCARPETTA (2008): “Barriers to Exit, Experimentation and Comparative Advantage,” *mimeo, Tinbergen Institute*.
- BARTRAM, S. M., G. BROWN, AND R. M. STULZ (2009): “Why do foreign firms have less idiosyncratic risk than US firms?,” *NBER Working Paper*, 14931.

- BERNARD, A. B., S. J. REDDING, AND P. K. SCHOTT (2010): “Multiple-Product Firms and Product Switching,” *American Economic Review*, 100(1), 70–97.
- BLUNDELL, R., AND S. BOND (2000): “GMM estimation with persistent panel data: an application to production functions,” *Econometric Reviews*, 19(3), 321–340.
- BOLTON, P., AND C. HARRIS (1999): “Strategic experimentation,” *Econometrica*, 67(2), 349–374.
- BRODA, C., AND D. E. WEINSTEIN (2010): “Product creation and destruction: Evidence and price implications,” *American Economic Review*, 100(3), 691–723.
- CAMPBELL, J. Y., M. LETTAU, B. G. MALKIEL, AND Y. XU (2001): “Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk,” *Journal of Finance*, 56(1), 1–43.
- CASTRO, R., G. L. CLEMENTI, AND Y. LEE (2009): “Cross-sectoral variation in firm-level idiosyncratic risk,” *Federal Reserve Bank of Cleveland Working Paper*, 08-12.
- CAVES, R. E. (1998): “Industrial Organization and New Findings on the Turnover and Mobility of Firms,” *Journal of Economic Literature*, 36(4), 1947–1982.
- COLES, J. L., N. D. DANIEL, AND L. NAVEEN (2006): “Managerial incentives and risk-taking,” *Journal of Financial Economics*, 79(2), 431–468.
- DORASZELSKI, U., AND J. JAUMANDREU (2009): “R&D and productivity: Estimating endogenous productivity,” *mimeo, Harvard*.
- DUNNE, T., L. FOSTER, J. HALTIWANGER, AND K. TROSKE (2004): “Wage and Productivity Dispersion In US Manufacturing: The Role of Computer Investment,” *Journal of Labor Economics*, 22(2), 397–430.

- EVANS, D. S. (1987): “The Relationship Between Firm Growth, Size, and Age: Estimates for 100 Manufacturing Industries,” *The Journal of Industrial Economics*, 35(4), 567–581.
- FAGGIO, G., K. G. SALVANES, AND J. VAN REENEN (2007): “The evolution of inequality in productivity and wages: Panel data evidence,” *NBER Working Paper*, 13351.
- FOSTER, L., J. HALTIWANGER, AND C. J. KRIZAN (2001): “Aggregate Productivity Growth: Lessons from Microeconomic Evidence,” in *New Developments in Productivity Analysis*, ed. by C. R. Hulten, E. R. Dean, and M. J. Harper, NBER Studies in Income and Wealth. University of Chicago Press, Chicago.
- (2006): “Market Selection, Reallocation, and Restructuring in the U.S. Retail Trade Sector in the 1990s,” *Review of Economics and Statistics*, 88(4).
- FRANCOIS, P., AND H. LLOYD-ELLIS (2003): “Animal Spirits through Creative Destruction,” *American Economic Review*, 93(3), 530–550.
- GABAIX, X. (2010): “The Granular Origins of Aggregate Fluctuations,” *mimeo*, NYU.
- GABLER, A., AND O. LICANDRO (2007): “Endogenous Growth through Selection and Imitation,” *European University Institute Economics Working Paper*, 2007/26.
- GEROSKI, P. A. (1995): “What do we know about entry?,” *International Journal of Industrial Organization*, 13(4), 421–440.
- HALL, B. H. (1987): “The relationship between firm size and firm growth in the US manufacturing sector,” *Journal of Industrial Economics*, 35(4), 583–606.
- HALL, B. H., AND J. MAIRESSE (2005): “Testing for Unit Roots in Panel Data: An Exploration Using Real and Simulated Data,” in *Identification and Inference for*

- Econometric Models: A Festschrift in Honor of Thomas Rothenberg*, ed. by D. W. K. Andrews, and J. H. Stock, pp. 451–479. Cambridge University Press, Amsterdam.
- HALTIWANGER, J., R. JARMIN, AND T. SCHANK (2003): “Productivity, Investment in ICT and Market Experimentation: Micro Evidence from Germany and the U.S.,” *U.S. Bureau of the Census Center for Economic Studies (CES) Research Paper*, 03-06.
- HASKEL, J., AND R. MARTIN (2002): “The UK Manufacturing Productivity Spread,” *Ceriba (Center for Research Into Business Activity at the UK Office of National Statistics) Working Paper*.
- HOLMES, T. J., D. K. LEVINE, AND J. A. SCHMITZ (2008): “Monopoly and the Incentive to innovate when adoption involves switchover disruptions,” *NBER Working Paper*, 13864.
- HOPENHAYN, H. (1992): “Entry, Exit, and Firm Dynamics in Long Run Equilibrium,” *Econometrica*, 60(5), 1127–1150.
- HOPENHAYN, H., AND R. ROGERSON (1993): “Job Turnover and Policy Evaluation: A General Equilibrium Analysis,” *Journal of Political Economy*, 101(5), 915–938.
- HSIEH, C.-T., AND P. J. KLENOW (2009): “Misallocation and Manufacturing TFP in China and India,” *Quarterly Journal of Economics*, 124, 1403–1448.
- JOVANOVIC, B. (1982): “Selection and the Evolution of Industry,” *Econometrica*, 50(3), 649–670.
- KELLER, G., S. RADY, AND M. CRIPPS (2005): “Strategic experimentation with exponential bandits,” *Econometrica*, 73(1), 39–68.

- KLETTE, T. J., AND S. KORTUM (2004): “Innovating Firms and Aggregate Innovation,” *Journal of Political Economy*, 62, 986–1018.
- KORTUM, S. (1997): “Research, patenting, and technological change,” *Econometrica*, 65(6), 1389–1419.
- LEE, Y., AND T. MUKOYAMA (2008): “Entry, Exit and Plant-Level Dynamics over the Business Cycle,” *Federal Reserve Bank of Cleveland Working Paper*, 07-18R.
- LENTZ, R., AND D. T. MORTENSEN (2008): “An empirical model of growth through product innovation,” *Econometrica*, 76(6), 1317–1373.
- LUTTMER, E. (2007): “Selection, Growth and the Size Distribution of Firms,” *Quarterly Journal of Economics*, 122(3), 1103–1144.
- MCGUCKIN, R. H., M. L. STREITWIESER, AND M. E. DOMS (1996): “The Effect of Technology Use on Productivity Growth,” *U.S. Bureau of the Census Center for Economic Studies (CES) Working Paper*, CES 96-2.
- MIDRIGAN, V., AND D. XU (2010): “Finance and Misallocation: Evidence from Plant-level Data,” *mimeo*, NYU.
- MOSCOSO BOEDO, H. J., AND T. MUKOYAMA (2010): “Evaluating the Effects of Entry Regulations and Firing Costs on International Income Differences,” *mimeo*, U Virginia.
- POSCHKE, M. (2009): “Employment protection, firm selection, and growth,” *Journal of Monetary Economics*, 56(8), 1074–1085.
- (2010): “The regulation of entry and aggregate productivity,” *The Economic Journal*, 120(549), 1175–1200.

- RESTUCCIA, D., AND R. ROGERSON (2008): “Policy Distortions and Aggregate Productivity with Heterogeneous Establishments,” *Review of Economic Dynamics*, 11(4), 707–720.
- SAMANIEGO, R. M. (2006): “Employment protection and high-tech aversion,” *Review of Economic Dynamics*, 9(2), 224–241.
- SUTTON, J. (1997): “Gibrat’s Legacy,” *Journal of Economic Literature*, 35(1), 40–50.
- SYVERSON, C. (forthcoming): “What determines Productivity?,” *Journal of Economic Literature*.
- THOMKE, S. H. (2003): *Experimentation Matters: Unlocking the Potential of New Technologies for Innovation*. Harvard Business Press, Cambridge, MA.

Appendix

A Aggregation

The allocation constraint of labor across firms is

$$l = B \int l^*(z) \mu(z) dz,$$

where $l^*(z)$ is the optimal quantity of labor used by firms with productivity z , given their optimal choice of s , and B is the total number of firms. Using the expression for optimal labor implied by (4), we obtain the following expression for the wage rate:

$$w = (1 - \alpha) \left(\frac{\overline{z\theta}}{l} B \right)^\alpha,$$

where $\overline{z\theta}$ is the average productivity in the economy, net of disruption costs. The implied optimal labor force for firms with productivity z is then

$$l^*(z) = \frac{z\theta}{\overline{z\theta}} \frac{l}{B}. \tag{10}$$

Aggregating production (2) across all firms, given the optimal labor usage in (10), we get

$$Y = (\overline{z\theta} B)^\alpha l^{1-\alpha}.$$

The firms' profit function (equation 5) then becomes

$$\Pi(z) = \alpha z \theta \left(\frac{l}{\overline{z\theta} B} \right)^{1-\alpha}.$$