

Trade Liberalization with Endogenous Borrowing Constraints

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ABSTRACT

A recent empirical literature has documented that credit availability is a significant barrier for firm-level exports. We develop a dynamic general equilibrium trade model with heterogeneous monopolistic competitive firms and imperfect credit markets due to limited contract enforceability. We show that this model is consistent with the findings of the empirical literature. We ask if credit constraints reduce gains from a tariff reduction. In a calibrated example, we find that the percentage change in steady state consumption is in an economy with limited enforcement is approximately equal to the change in an equivalent one with perfect credit markets. We conclude that the presence of financial constraints at the firm level does not reduce the aggregate gains from a tariff reduction. This is because the credit constraints respond to profit opportunities. When tariffs are reduced, exporters are more profitable, which allows them to borrow more. In an equivalent economy where credit constraints are exogenous, there is a 10% smaller increase in consumption from tariff reduction.

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1. Introduction

A recent, growing empirical literature has documented that firm-level financial constraints are a significant barrier to firm-level export decisions (see Manova (2010) for an excellent survey of this literature). Across a number of different countries and different definitions of financial constraints, the availability of financing is a significant determinant of trade on both the extensive and intensive margins. This paper analyzes the consequences of this empirical regularity for a tariff reduction. Our central question is whether or not financial constraints affect the aggregate welfare gain following a tariff reduction. That is, do financial constraints, which limit the ability of firms to export, also reduce the increase in aggregate exports that follow a tariff reduction? In a calibrated example, we show that the presence of financial constraints does not reduce the gains from a tariff reduction. This is due to the fact that the financial constraints respond to the profit opportunities of firms. When tariffs are reduced, credit constraints endogenously relax allowing firms to increase their exports.

We develop a dynamic general equilibrium model that has two main features: firm-level export decisions as in Melitz (2003), and limited contract enforcement as in Albuquerque and Hopenhayn (2004) and Kehoe and Levine (1993). Firms must incur a fixed cost to operate and to enter export markets. To finance these costs and to remunerate factors of production before revenues are realized, firms enter into long-term contracts with competitive financial intermediaries. These contracts must be self-enforcing and specify plans for production, repayment and export status. The firm's scale of operation starts low, and as the firm repays the intermediary their scale expands and they may enter the export market. In finite time, they are able to produce at an unconstrained level.

We show that our model is consistent with the main findings of the empirical literature

on trade and finance: (a) the probability that a firm is an exporter is decreasing in the credit constraints that it faces, and (b) the scale at which firms export is decreasing in the credit constraints it faces. We then use the model to assess the effects of credit market imperfections on the response of the economy to a tariff reduction.

We first show analytically that, in a partial equilibrium environment, credit constrained exporters increase their scale by a greater proportion than unconstrained exporters following a tariff reduction. Both constrained and unconstrained exporters increase their scale following a tariff reduction because the marginal revenue from exporting has increased. However, constrained exporters have the additional effect of relaxed borrowing constraints arising from the increase in the value of the exporting firm. When the firm's net present value has increased, default becomes less attractive and borrowing constraints are relaxed. This second effect is not present if a firm's financial constraint is not binding. Hence the constrained firm's scale increases by a greater proportion than an otherwise identical unconstrained firm. It should be emphasized that the level of production for the constrained firm is always lower than for an unconstrained firm. The previous argument shows that the percentage gap between the constrained and unconstrained firm is reduced by the trade liberalization, not that the scale of the constrained firm actually surpasses that of the unconstrained firm.

In general equilibrium, the trade reform can potentially have a perverse effect on non-exporters. The increase in real wages that follows a bilateral trade liberalization will make the debt limit tighter for firms that do not export by exactly the opposite argument of that presented above. If the real wage goes up the value of the non-exporting firm declines, which makes default more attractive and borrowing constraints tighten. This generates winners and losers in general equilibrium. Borrowing constraints may loosen for exporters and tighten for

non-exporters.

To determine the total impact of these general equilibrium effects we conduct a quantitative exercise. We calibrate the model using Colombian data and compare the gains from a symmetric, bilateral tariff reduction in both full and limited enforcement economies. In the limited enforcement economy the percentage steady state change in consumption is higher than in an analogue economy with full enforcement - although the difference is very small. We conclude that the presence of financial constraints at the firm level do not reduce aggregate gains from a tariff reduction. Moreover, the model delivers some new cross-sectional implications: a tariff reduction leads to a change in the distribution of firm-level credit. Exporting firms have easier access to credit, while firms that produce domestically may have restricted access. This amplifies the increase in firm-level size variance following a trade liberalization implied by the usual Melitz model.

Throughout this paper we are thinking of credit market frictions as technological. It is clear that welfare (i.e., steady state consumption) is lower in economies with worse financial market frictions. The point we want to make is not about how consumption *levels* vary with changes in financial market quality, but how *changes* in consumption vary with financial market quality when those changes come from a tariff reduction. Our main result is that financial market quality does not affect changes in steady state consumption¹.

We demonstrate that the endogeneity of financial constraints is key for our results by comparing the limited enforcement economy to an equivalent economy with exogenous debt limits. This economy is exactly the same as the baseline limited enforcement economy except that debt limits do not change when the economy undergoes a trade liberalization. That is,

¹See Figure 8.

financial constraints do not relax in response to the changes in firm-level profits. We show that the economy with exogenous constraints exhibits a smaller gain in welfare than does the full enforcement economy. Moreover, this difference in welfare changes increases as financial market quality declines². We interpret this as evidence that the endogenous relaxation of borrowing constraints is the driving force behind our main result.

Next, we will discuss the related literature. Then in Section 2, we survey the empirical literature and establish some stylized facts. In Section 3, we describe our model. In Section 4, we characterize the stationary equilibrium of the model and show that it is consistent with the stylized facts from Section 2. In Section 5, we consider the effect of a trade liberalization. Section 6 concludes.

Related Literature

Our work is related to Melitz (2003) which, building on the seminal work of Hopenhayn (1992), analyzes (in a static environment) the effect on productivity of a reduction in trade barriers in a two country world with monopolistically competitive firms, and to dynamic extensions, such as those analyzed in Alessandria and Choi (2007). The purpose of this paper is to see how open economy models of those types, which explicitly model firm-level heterogeneity, are affected by financial frictions. The paper that is most similar to this is Chaney (2005), which introduces liquidity constraints in a Melitz (2003) model. This paper differs from ours in two respects. First, the goal of his paper is to provide a mechanism that can account for the observed low response of trade balance to exchange rate fluctuations and the high elasticity for the demand of foreign goods required by trade models to account for

²See Figure 9.

change in trade flows after trade liberalization episodes³. He does not analyze the impact of a tariff reduction, the central theme of our paper. Second, Chaney (2005) model is static, while the life-cycle dynamic of a firm is central in our model.

This paper is related to the literature that studies how the micro-level details of an economy affect the aggregate response to a tariff reduction. We analyze the impact of financial frictions while Atkeson and Burstein (2010) consider the role of firm-level innovation, entry and exit decisions. As in our paper, they find that adding these have little change on the welfare implications of the model despite the fact that they generate different cross-sectional implications. Similarly Arkolakis, Costinot and Rodriguez-Clare (2011) show for a class of trade models (including Krugman (1980) and Melitz (2003)) that the aggregate welfare gains from a tariff reduction only depend on changes in aggregate trade shares and it is independent of cross-sectional reallocation. Our paper reaches the same conclusion, but we perform a different experiment: we show that for a given tariff reduction the change in aggregate trade share is the same in the two economies we consider.

Kambourov (2009) considers how labor market frictions, such as firing costs, affect the increase in welfare following a tariff reduction. He finds that the failure to liberalize labor markets at the same time as trade can lead to a significant loss in welfare on the transition path, and in the steady state the change is the same with and without labor market reform. Our mechanism and exercise are quite different from this. The exercise in Kambourov (2009) compares the exogenous reduction in one friction (tariffs) to the exogenous, simultaneous reduction in two frictions (tariffs and firing costs). In our exercise, we exogenously reduce tariffs and show that a second friction (enforcement constraints) endogenously relaxes. As

³This is what Ruhl (2008) termed the "international elasticity puzzle".

an extension we compare that to the case when the enforcement frictions are not allowed to relax.

Our work builds on Albuquerque and Hopenhayn (2004), who characterize the optimal long term contract between a firm and a financial intermediary in an environment characterized by limited contract enforceability and limited liability on the part of the firm. They show how the implied firm's dynamic is consistent with facts about firm growth and survival. Our paper extends their work, embedding the contracting problem in a general equilibrium trade model with a discrete choice of operating on a second market. Other authors, for instance Cooley, Marimon and Quadrini (2004), have investigated the aggregate implication of limited contract enforceability in general equilibrium for a closed economy. The model we develop is very similar to that in Wang (2011), which was developed independently. She finds strong empirical support for the prediction of the model. However, she does not address the implications for trade liberalization, which is our main goal.

Our paper is closely related to Jermann and Quadrini (2007). They analyze the impact upon aggregates of a news shock in a closed economy with a unique final good, decreasing return to scale technology, and limited enforcement of intertemporal contracts. They find that the prospect of future productivity growth can increase current productivity by relaxing the firms' borrowing constraint, allowing younger firms to borrow more and operate at a scale closer to the efficient level. Our model shares the same mechanism: a reduction in trade costs increases the profitability for exporting firms, making the option of defaulting on the outstanding debt less attractive, thus increasing the amount of credit they can raise and their scale. Our model differs from theirs in two important ways. First, our model has distributional aspects that are not present in their model. In their model, all firms benefit

from the positive news shock. In ours, a trade liberalization instead creates winners and losers: the enforcement constraint is relaxed for exporters and tightened for non-exporters due to general equilibrium effects. Second, Jermann and Quadrini (2007) consider only an increase in the prospect of future productivity that it is never realized. In our model, a tariff reduction generates an actual increase in productivity, generating general equilibrium effects that can potentially overturn their result.

This paper is importantly different from the literature that analyzes trade between countries that differ in the quality of financial markets, such as Antras and Caballero (2009). They analyze how countries with differing quality of financial markets are differentially impacted by opening to trade with one another. In this literature, heterogeneity in the quality of credit markets is key. In our model, we illustrate our mechanism by analyzing countries that are identical in financial quality and see how does credit availability endogenously responds to a tariff reduction.

2. Stylized Facts

We will now survey some facts from a growing literature on the relationship between the availability of financing and firm-level export decisions. The main finding of this literature is that credit constrained firms are less likely to export and export at lower levels than firms that have more access to credit, even controlling for differences in productivity. The goal of this paper is to determine what the implications of these facts are for a tariff reduction. In later sections we will develop a model that is consistent with these facts and use that model to determine how taking account of these facts affects the computed change in welfare from a trade liberalization.

A number of recent papers have explored the relationship between firm-level access to credit markets and export decisions of firms, using firm-level data from a variety of countries and using many different measures to proxy for the ability of a firm to obtain credit. We will be specific about the measure of financial constraints used in each so that we can compare them to the definition of financial constraints present in the model developed in later sections. For a dataset of Belgian firms, Muûls (2008) shows that the probability of exporting and export levels are positively related to credit scores (assumed to reflect the firm's availability of credit). Also, it is shown that more productive firms have higher credit scores. Similar results are shown by Berman and Hericourt (2010) and Greenaway et al (2007) using a dataset of 9 emerging economies, and a sample of UK firms respectively. Both of them use leverage ratios⁴ as a measure of the firm's financial position, and they find that it is negatively correlated with firm's exports. Lastly, Minetti and Zhu (2010), using Italian data, and Gorodnichenko and Schnitzer (2010), using firm-level data across 27 countries, have survey responses as a measure of financial availability. All of these show that export decisions are hampered by lack of credit availability.

Manova (2008a) examines the role of financial constraints across a large sample of sectors and countries. She shows that credit constraints hamper a country's exports on both the extensive and intensive margins. She shows that this is consistent with a model in which high productivity firms have enough access to credit to operate at their efficient scale, while lower productivity firms have restricted access to credit. This will be a feature of the model developed in later sections.

⁴In particular, Berman and Hericourt (2010) use total debt over total assets, while Greenaway et al. (2007) use short term debt over current assets.

We interpret these results as being consistent with two broad facts across many countries, and many definitions of financial constraints:

Fact A: *The probability that a firm is an exporter is decreasing in the credit constraints that it faces.*

Fact B: *The scale at which firms export is decreasing in the credit constraints it faces.*

The following sections will build up a model that is consistent with these facts. We will then use the model to analyze the aggregate and cross-sectional effects of a trade liberalization on firms that face credit constraints and make export decisions.

3. Model

Time is discrete and denoted by $t = 0, 1, \dots$. There are two symmetric countries, home and foreign, and variables for the foreign country are denoted with a f . Each country is populated by a unit measure of identical households, competitive final good producers, and monopolistic competitive firms producing an intermediate differentiated product. Each period a mass of potential monopolistic competitive firms is born, and existing firms face an exogenous exit probability. Firms have to borrow from competitive intermediaries to pay an initial fixed cost, a fixed cost to export, and to finance working capital. Financial contracts are not perfectly enforceable in the sense that in any period the manager of the firm may choose to default on their debt and abscond with the firm's working capital. Therefore, long term contracts must be written so that it is never optimal for the manager to default. There is no aggregate uncertainty.

A. Household Problem

The representative household in each country is endowed with L units of time in every period. He chooses final good consumption c_t , physical capital k_{t+1} , and bond holdings b_{t+1} to maximize his lifetime utility

$$(1) \quad \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\beta \in (0, 1)$ is the discount factor, subject to the sequence of budget constraints

$$p_t (c_t + k_{t+1}) + q_t b_{t+1} \leq w_t L + r_t k_t + p_t (1 - \delta_k) k_t + b_t + \Pi_t + T_t$$

where p_t is the price of the final good, w_t is the wage, r_t is the rental rate on capital, q_t is the intertemporal price, δ_k is the depreciation rate of capital, Π_t are profits from the monopolistic competitive firms, and T_t are lump-sum transfers from the government (revenue from tariffs). The problem for the stand-in household in the foreign country is symmetric.

B. Final Good Producers

The final good in the home country is produced using the following CES aggregator:

$$(2) \quad y_t = \left[\omega \int_I y_{dt}(i)^{\frac{\sigma-1}{\sigma}} di + (1 - \omega) \int_{I_{xt}^f} y_{xt}^f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

where I_t is the set of active domestic firms at time t , I_{xt}^f is the set of foreign firms that export at t , $y_{dt}(i)$ is the output of firm i in I_t , $y_{xt}^f(i)$ is the output of firm i in I_{xt}^f . The final good in the foreign country is produced analogously. The parameter ω can be either thought of as

controlling for the home bias in consumption or as standing for iceberg transportation cost.

The elasticity of substitution is $\sigma \geq 1$.

Final goods producers are competitive. A representative firm solves the following static problem:

$$\max p_t y_t - \int_{I_t} p(i) y_{dt}(i) di - \int_{I_{xt}^f} (1 + \tau_t) p(i) y_{xt}^f(i) di$$

subject to (2). One can then derive the inverse demand functions faced by domestic and foreign producers for the intermediated good i :

$$(3) \quad p_{dt}(y(i)) = p_t \omega y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}}$$

$$(4) \quad p_{xt}(y(i)) = \frac{p_t}{(1 + \tau_t)} (1 - \omega) y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}}$$

C. Monopolistic Competitive Firms and Financial Contracts

In each country, in every period, a mass $1 - \delta$, $\delta \in (0, 1)$, of potential new firms are born. Each firm produces a differentiated intermediate good. At birth the firm draw its productivity z from a distribution Γ . For simplicity, we assume that z remains constant through time. The firm can potentially produce the differentiated good using the following technology:

$$(5) \quad y = zF(k, l) = zk^\alpha l^{1-\alpha}, \quad \alpha \in (0, 1)$$

where l and k are the labor and capital employed by the firm, and y is total output produced. The firm faces the inverse demand functions (3) and (4). A firm faces an exogenous probability of exit probability $1 - \delta$ in each period.

To start operating, the firm must pay a fixed entry cost f_e . Moreover, conditional on entry, the firm can access the foreign market by paying a fixed cost of f_x . Fixed costs are paid in terms of the final good.

Financial Contract

The firm is owned by the representative household⁵. At birth a firm has zero assets; to operate it has to borrow from an intermediary to pay (i) the fixed cost $p_t f_e$, (ii) eventually $p_t f_x$, and (iii) to pay for the working capital, that is remunerate factors of production, $w_t l_t + r_t k_t$, before it sells its output. As in Albuquerque and Hopenhayn (2004), the firm enters into a long-term contract with a competitive intermediary. Productivity is public information, at any t , if profitable, the intermediary offers a long term contract $\{d_{t+s}, k_{t+s}, l_{t+s}, y_{dt+s}, y_{xt+s}, x_{t+s}\}_{s=0}^{\infty}$ that specifies for all $t+s$ dividend payments to the firm's equity holder, d_{t+s} , labor and capital employed by the firms, l_{t+s} and k_{t+s} , production for the domestic and export market y_{dt+s} and y_{xt+s} , and $x_{t+s} \in \{0, 1\}$, the decision to enter the export market, where we use the convention that $x_{t+s} = 1$ means that a firm is exporting. The first time that $x_{t+s} = 1$ the intermediary has to pay the sunk cost $p_t f_x$.

Credit contracts are *not enforceable*: in each period the firm can default on its outstanding debt and working capital loan. Define $v_t \equiv \sum_{s=0}^{\infty} Q_{t,t+s} \delta^s d_{t+s}$ to be the equity value of the firm at t , where $Q_{t,t+s} \equiv \prod_{h=0}^s q_{t+h}$. A contract must satisfy the following *enforcement*

⁵Nothing changes (in a stationary equilibrium) if we instead assume that firms are owned by a risk neutral entrepreneur with the same discount factor as the representative household.

constraint $\forall s$:

$$v_{t+s} \geq \theta (w_{t+s}l_{t+s} + r_{t+s}k_{t+s})$$

that is, the equity in the firm must be greater than a multiple of the firm's working capital. Intuitively, the firm can default on its outstanding debt and abscond with a fraction of the working capital loan, $w_t l_t + r_t k_t$. The parameter $\theta \geq 0$ can be derived from primitives as a solution to a renegotiation problem between the firm and the intermediary - see Jermann and Quadrini (2009) - or it can be thought of as a reduced form parameter as in Kiyotaki and Moore (1997). θ indexes the quality of the credit market: if $\theta = 0$ long term contract are perfectly enforceable; the higher θ , the more stringent is the enforcement problem. We further impose a *limited liability constraint*:

$$\forall s, d_{t+s} \geq 0$$

requiring that dividends payments are always non-negative: the intermediary cannot ask the equity holder to make a positive contribution to the firm. This is consistent with the entrepreneur having no external wealth.

The objective of the intermediary is to maximize the discounted expected payments from the firms:

$$\sum_{s=0}^{\infty} Q_{t,t+s} \delta^s \left\{ \begin{array}{l} p_{dt+s}(y_{dt+s})y_{dt+s} + x_{t+s}p_{xt+s}(y_{xt+s})y_{xt+s} - d_{t+s} \\ -w_{t+s}l_{t+s} - r_{t+s}k_{t+s} - \mathbf{1}\{s = \inf_s s \text{ s.t. } x_{t+s} = 1\} p_{t+s}f_x \end{array} \right\} - p_t f_e$$

As in Albuquerque and Hopenhayn (2004), we can write the intermediary's problem recursively with the productivity, the equity value for the firm, v , and the export status as state variables. An intermediary solves $W_{0t}(z) = \max\{\tilde{W}_{0t}(z), 0\}$ with

$$(6) \quad \tilde{W}_{0t}(z) = \max_v W_t^{nx}(v, z) - p_t f_e \quad \text{s.t.} \quad v \geq v_{0t}(z)$$

where $v_{0t}(z)$ is the equilibrium equity value for a new-born firm with productivity z at time t , and $W_t^{nx}(v, z)$ is the value function for a domestic firm (a firm that has not yet paid the sunk cost for starting to export) with promised utility v and productivity level z :

$$(7) \quad W_t^{nx}(v, z) = \max_{d, v', k, l, y_d, y_x, x \in \{0, 1\}} p_{dt}(y_d)y_d + xp_{xt}(y_x)y_x - w_t l_t - r_t k_t - d - xp_t f_x \\ + q_t \delta [xW_{t+1}^x(v', z) + (1-x)W_{t+1}^{nx}(v', z)]$$

subject to the feasibility constraint (5), the promise keeping constraint

$$(8) \quad d + q_t \delta v' \geq v$$

the enforcement constraint

$$(9) \quad d + q_t \delta v' \geq \theta(w_t l + r_t k)$$

and the limited liability constraint

$$(10) \quad d \geq 0$$

where $x = 1$ if the firm decides to export in that period, and is zero otherwise. For an exporter (who paid f_x in a previous period) the value function $W_t^x(v, z)$ is:

$$(11) \quad W_t^x(v, z) = \max_{d, v', k, l, y_d, y_x} p_{dt}(y_d)y_d + p_{xt}(y_x) - w_t l - r_t k + q_t \delta W_{t+1}^x(v', z)$$

subject to (5), (8), (9) and (10). We will denote the policy functions associated with the above problems as $\{d_t^{nx}, y_{dt}^{nx}, y_{xt}^{nx}, l_t^{nx}, k_t^{nx}, x_t, v_t^{nx}\}_{t=0}^\infty$ for non-exporters and $\{d_t^x, y_{dt}^x, y_{xt}^x, l_t^x, k_t^x, v_t^x\}_{t=0}^\infty$ for exporters. The discrete decision to start exporting may induce non-concavity in the value function for a domestic firm W^{nx} , hence it could be optimal to introduce lotteries in the equity value. In our numerical simulation we abstract from lotteries and we check that they are indeed not necessary by checking the concavity of W^{nx} .

D. Equilibrium Conditions

We now report the feasibility conditions for the home country. Analogous relations must hold for the foreign country. To do so, we need to keep track of the evolution of the measure of operating firms over (v, z, s) , where $s \in \{x, nx\}$ is the firm's export status (x if an exporter and nx if a non-exporter). Denote by λ_t^{nx} and λ_t^x the measure of non-exporting and exporting firms over (v, z) respectively, and let $\lambda_t = (\lambda_t^{nx}, \lambda_t^x)$. The evolution over time for λ_t^{nx} and λ_t^x is given by: $\forall (Z, V) \in B(\mathbb{R}_+) \times B(\mathbb{R}_+)$

$$(12) \quad \begin{aligned} \lambda_{t+1}^x(Z, V) = & \delta \int 1 \{v_t^{x'}(v; z) \in V, z \in Z\} d\lambda_t^x + \\ & + \delta \int 1 \{x_t(v; z) = 1, v_t^{d'}(v; z) \in V, z \in Z\} d\lambda_t^{nx} + \\ & + (1 - \delta) \int_Z 1 \{x_t(v_{0t}(z); z) = 1, v_t^{nx'}(v_{0t}(z); z) \in V\} d\Gamma \end{aligned}$$

$$\begin{aligned}
(13) \quad \lambda_{t+1}^{nx}(Z, V) &= \delta \int 1 \{x_t(v; z) = 0, v_t^{nx'}(v; z) \in V, z \in Z\} d\lambda_t^{nx} \\
&\quad + (1 - \delta) \int_Z 1 \{x_t(v_{0t}(z); z) = 0, v_t^{nx'}(v_{0t}(z); z) \in V\} d\Gamma
\end{aligned}$$

where we set $v_{0t}(z)$ to zero if not profitable to pay the set-up cost.

Market clearing in the final good market requires that

$$(14) \quad y_t = c_t + k_{t+1} - (1 - \delta)k_t + y_{ft}$$

where y_{ft} is the total amount of fixed cost paid in period t :

$$(15) \quad y_{ft} = \int x_t(v; z) f_x d\lambda_t^{nx} + (1 - \delta) \int_Z [f_e + x_t(v_{0t}(z), z) f_x] d\Gamma$$

The labor market feasibility is given by

$$(16) \quad L = \int l_t^x(v; z) d\lambda_t^x + \int l_t^{nx}(v; z) d\lambda_t^{nx} + (1 - \delta) \int_Z l_t^{nx}(v_{0t}(z), z) d\Gamma$$

for the rental capital market by

$$(17) \quad k_t = \int k_t^x(v; z) d\lambda_t^x + \int k_t^{nx}(v; z) d\lambda_t^{nx} + (1 - \delta) \int_Z k_t^{nx}(v_{0t}(z), z) d\Gamma$$

For the bond market to clear, it must be that

$$(18) \quad b_t + b_t^f = B_t + B_t^f$$

where B_t is the amount of bonds held by the intermediary to finance its operations. Without loss of generality, since the two countries are identical, we assume that there is full home bias in lending. That is, the home intermediary is lending to domestic firms only. B_t is given by

$$(19) \quad B_t = \int W_t d\lambda_t$$

Finally, aggregate dividend distributions are given by

$$(20) \quad \Pi_t = \sum_{i \in \{nx, x\}} \int d_t^i(v; z) d\lambda_t^i + (1 - \delta) \int_Z d_t^{nx}(v_{0t}(z), z) d\Gamma$$

and lump-sum transfers by

$$(21) \quad T_t = (1 - \tau_t) \left[\int p_{xt} \left[y_{xt}^{xf}(v; z) \right] y_{xt}^{xf}(v; z) d\lambda^{xf} + \int p_{xt} \left[y_{xt}^{nxf}(v; z) \right] y_{xt}^{nxf}(v; z) d\lambda^{nxf} \right. \\ \left. + (1 - \delta) \int p_{xt} \left[y_{xt}^{nxf}(v_{0t}^f(z); z) \right] y_{xt}^{nxf}(v_{0t}^f(z); z) d\Gamma \right]$$

We are now able to define a symmetric equilibrium for the economy in which the two countries start with the same initial distribution of firms, the same bond holding, the same capital stock, and with equal tariffs. In such an economy, prices will be equal in both countries.

DEFINITION 1. *Given an initial symmetric distribution of firms $\lambda_0 = \lambda_0^f$, capital stock $k_0 = k_0^f$ and bonds holdings $b_0 = b_0^f$, and a sequence $\{\tau_t, \tau_t^f\}_{t=0}^\infty$ such that $\tau_t = \tau_t^f$, a symmetric equilibrium for the deterministic economy consists of (i) prices $\{p_t, w_t, r_t, q_t\}_{t=0}^\infty$, (ii) household's allocations $\{c_t, b_{t+1}, k_{t+1}\}_{t=0}^\infty$, (iii) firms decision rules $\{d_t^{nx}, y_{dt}^{nx}, y_{xt}^{nx}, l_t^{nx}, k_t^{nx}, x_t, v_t^{fd}\}_{t=0}^\infty$ and $\{d_t^x, y_{dt}^x, y_{xt}^x, l_t^x, k_t^x, v_t^{fx}\}_{t=0}^\infty$, (iv) initial conditions for new-entrants $\{v_{0t}\}_{t=0}^\infty$, (v) Inverse*

demand functions $\{p_{xt}, p_{dt}\}_{t=0}^{\infty}$, (vi) measure of firms $\{\lambda_t\}_{t=0}^{\infty}$, (vii) dividend distributions $\{\Pi_t\}_{t=0}^{\infty}$, lump-sum transfers $\{T_t\}_{t=0}^{\infty}$, and $\{y_{ft}\}_{t=0}^{\infty}$ and analogous object for the foreign country, such that:

1. Households' allocation solves the household's problem (1)
2. Firms' decision rules are optimal for (7) and (11)
3. Equity value for new-entrants satisfy the zero-profit condition for the lender

$$(22) \quad W_t(v_{0t}(z); z) = p_t f_d$$

4. Inverse demand functions are given by (3) and (4)
5. Final good, labor, rental capital, bonds markets clear, that is (14), (16), (17), (18) hold, where y_{ft} is given by (15)
6. The measures of firms evolve according to (12) and (13)
7. Aggregate profits and lump-sum transfers are given by (20) and (21) respectively.

In most of our analysis we will focus on a *symmetric stationary equilibrium* for the economy, or on a transition from one stationary equilibrium to another.

4. Symmetric Stationary Equilibrium

We now characterize the symmetric stationary equilibrium for the economy, and we show that the model is able to account for the features of the life-cycle path of a firm summarized in section 2: (a) the probability that a firm is an exporter is decreasing with measures of firm-level financial constraints, (b) firms' sales and exports grow over time and are decreasing in the credit constraints it faces.

A. Firm's Problem Characterization

In a stationary equilibrium, all equilibrium objects are constant over time. Therefore, we will drop the dependence on time in this section. Moreover, we use the final good as the numeraire, $p = 1$. Before proceeding to characterize the solution to the firm-intermediary's problem, we consider a relaxed problem, dropping the enforcement constraint, or equivalently, letting $\theta = 0$. It is easy to see that the production decisions are independent of the firm's equity value, and solve the following static problem:

$$(23) \quad \Pi^*(z) = \max_{l, k, y_d, y_x, x} \omega y^{1/\sigma} y_d^{1-1/\sigma} + x \frac{1-\omega}{1+\tau} y^{1/\sigma} y_x^{1-1/\sigma} - x(1-q\delta)f_x - wl - rk$$

subject to

$$y_d + xy_x \leq zF(k, l)$$

Given prices w, r, q , tariff τ and aggregate final output y , denote the solutions to this problem with $l^*(z), k^*(z), y_d^*(z), y_x^*(z), x^*(z)$. These would be the firms decision rules in a standard Melitz (2003) model. For any productivity level z , define

$$v^*(z) \equiv \theta [wl^*(z) + rk^*(z)]$$

to be the minimal firm's equity value for which the full-enforcement production plan satisfies the enforcement constraint (9).

We now turn to the limited enforcement case, $\theta > 0$. The following proposition fully

characterizes the evolution of a firm over time. The proof is relegated to the appendix⁶.

PROPOSITION 1. (i) $\exists$ cut-off productivity level z_d s.t. if $z \geq z_d$ the firm enters;

(ii) $\forall z$, if $v \leq v^*(z)$ then $d(v, z) = 0$ (with minor qualification), if $v > v^*(z)$ then $d(v, z) \in [0, v^*(z) - \delta q v^*(z)]$

(iii) $\exists$ cut-off productivity level $z_x \geq z_d$ s.t. the firm will eventually export iff $z \geq z_x$;

(iv) $\forall z \geq z_x \exists \tilde{v}(z) \in [v_0(z), v^*(z)]$ s.t firm export iff $v \geq \tilde{v}(z)$;

Proof. Appendix.

Part (i) simply states that a firm must be sufficiently productive to find it profitable to pay the set-up cost. Part (ii) states a standard result for limited enforcement economies: rewards are back-loaded because they to help relax the enforcement constraint. Given the linearity of the objective function for the equity holder, until the equity value reaches the point at which the enforcement constraint does not bind, $v^*(z)$, the firm does not distribute dividends. Thus, the evolution for the equity is given by $v_t = v_0/(q\delta)^t$ for $t \leq T^*(z)$, $T^*(z) = \lfloor -\log(v^*(z)/v_0(z))/\log(q\delta) \rfloor$, where $\lfloor \cdot \rfloor$ is the floor operator. In finite time $T^*(z)$, a firm will reach its unconstrained level. For $t > T^*(z)$, despite indeterminacy in the firm's dividend policy, it must be that $v_t \geq v^*(z)$, so the firm is unconstrained. Part (iii) states that only more productive firms will eventually start to export, as in Melitz (2003). Part (iv) states that the firm's export status depends on both productivity and equity value. Only above a certain level of equity it is profitable to pay the sunk cost to start exporting. If v is too low, the value of entering the export market is small, as their operating scale is

⁶The characterization extends Albuquerque and Hopenhayn (2004) to an environment with a discrete choice of increasing the number of markets in which the firm operates.

constrained by their equity value. Over time, v grows, increasing the value of exporting, until it reaches $\tilde{v}(z)$ and the firm enters the export market. The result is illustrated in Figure 1 which shows how the decision of a domestic firm to start exporting depends on productivity and equity value.

After the initial productivity there is uncertainty (except for exogenous exit) and firms are fully characterized by their productivity and their age. A firm will start exporting at age $\tilde{T}(z) = \lfloor -\log(\tilde{v}(z)/v_0(z))/\log(q\delta) \rfloor$. We can prove the following proposition:

PROPOSITION 2. *If $z' > z \geq z_x$ then $\frac{\tilde{v}(z')}{v_0(z')} \leq \frac{\tilde{v}(z)}{v_0(z)}$, implying $\tilde{T}(z') \leq \tilde{T}(z)$.*

Proof. Appendix.

Then more productive firms start exporting earlier in the life cycle. Using data from the *Colombian Survey of Manufactures*, estimating firm-level productivity by applying the method of Levinsohn and Petrin (2003) industry-by-industry we found that this prediction of the model is consistent with the data (see Figure 2 for a comparison of the model with the data).

Given the evolution of equity and the decision to start export, one can easily derive the firm's production plans as a function of firm's age s :

$$l_s = \begin{cases} \min \left\{ \frac{(1-\alpha)v_s}{\theta w}, l_d^* \right\} & \text{if } s < \tilde{T} \\ \frac{(1-\alpha)v_s}{\theta w} & \text{if } \tilde{T} \leq s < T^* \\ l^* & \text{if } s \geq T^* \end{cases}, \quad k_t = \begin{cases} \min \left\{ \frac{\alpha v_s}{\theta r}, k_d^* \right\} & \text{if } s < \tilde{T} \\ \frac{\alpha v_s}{\theta r} & \text{if } \tilde{T} \leq s < T^* \\ k^* & \text{if } s \geq T^* \end{cases}$$

and $y_{xs} = 0$ if $s < \tilde{T}$ or $y_{xs} = zF(l_s, k_s)/(1 + \phi)$ if $s \geq \tilde{T}$, $y_{ds} = zF(l_s, k_s)$ if $s < \tilde{T}$ or

$y_{ds} = zF(l_s, k_s)\phi/(1 + \phi)$ if $s \geq \tilde{T}$ where $\phi \equiv ((1 + \tau)\omega/(1 - \omega))^\sigma$ is the ratio of domestic over foreign production.

The typical life-cycle path predicted by the model for total sales, export sales, labor and capital employed by a firm is illustrated in Figure 3 for different level of productivity. In the first panel, we show it for a non-exporter, $z < z_x$. In the second panel, we display the path for a firm that does not start to export from the beginning, but it starts after 4 periods. Notice how this firm first reaches the domestic optimal scale in period 2, then it waits 2 periods before it starts to export. Finally, the third panel shows the dynamics for a very productive firm that start to export from the first period.

B. Connection with the Empirical Evidence

We now show that our model can account for the main facts emerging from the empirical literature on credit constraints and firms' export decisions. First, consider Fact A: the probability of being an exporter is found to be higher for firms that are less financially constrained. Some studies, such as Gorodnichenko and Schnitzer (2010) and Minetti and Zhu (2010), use survey responses to generate indicator variables about whether or not firms would like to acquire more credit at market rates. In our model, an entrepreneur with productivity a will respond affirmatively to such a question if and only if $v(z) < v^*(z)$. Then, by part (iii) and (iv) of Proposition 1, we have that the probability of a firm being an exporter is increasing in v if $z \geq z_x$. We can then conclude that if we run the same logit regression using data simulated by our model we would find that, controlling for productivity, a credit constrained firm ($v(a) < v^*(a)$) is less likely to be an exporter. That is, we would find the negative relation between credit constraints and the probability of being an exporter as is

found in the data. We would obtain a similar result if we would instead use a different proxy for credit constraint, such as a measure of firm's leverage (such as the ratio of debt to assets). In the model, a firm's debt to asset ratio (suppose the firm's asset is the capital stock⁷) is decreasing in the firm's equity value. Therefore we would find that, controlling for firm's productivity, the probability of being an exporter is inversely related to the firms' leverage ratio, as found by Berman and Hericourt (2010).

Second, we emphasized how the model is able to deliver firm's sales and exports growing over time, following the growth in firm's equity value (see Figure 3). Hence, if one would regress export sales generated by the model on the two measures of credit availability described in the previous paragraph, controlling for firm's productivity, one would get the negative relationship found in the data between export sales and measures of the tightness of credit constraints. This is what we have labeled Fact B.

Summing up, our stylized model can replicate facts A and B in section 2. We turn now to analyze how the economy reacts to a bilateral tariff reduction.

5. Effects of a Trade Liberalization

In this section we are interested in evaluating if credit market imperfections, which we show limit the ability of firms to export in the data and in the model, can reduce the welfare benefits from a bilateral tariff reduction. This might occur because firms are not able to take advantage of the new profit opportunity due to poor access to credit. On the other hand, when tariffs are reduced, the borrowing constraints of exporting firms endogenously relax. We use a calibrated example to determine the relative sizes of these effects.

⁷Here we are thinking of an alternative decentralization of the same equilibrium allocation with firms owning the capital stock.

First, we prove that, in a partial equilibrium environment, a tariff reduction increases the scale of an exporter by more in the limited enforcement economy than in the full enforcement economy. Second, we conduct a quantitative exercise to consider the general equilibrium effects. In our calibrated example we show that in the limited enforcement economy the change in aggregate consumption, output and exports from the steady state in which both countries have equal, positive tariffs to the steady state in which neither country imposes tariffs is within one or two basis points of what we obtain in an economy with perfect credit market, or equivalently from a static Melitz (2003) model. We also compare our economy with one with exogenous debt limits. We find that the welfare gains are higher in the endogenous case, since the relaxation of the debt limit helps the exporters to take advantage of the trade reform. However, we find that credit conditions are relatively worse for firms that do not export in the endogenous case as compared to the exogenous one. Finally, we show that the above conclusions are unchanged if we consider the transitional dynamics from the high to the low tariff stationary equilibrium.

A. Partial Equilibrium

In this subsection we study how changes in τ , y , and w affect the decision rules from the individual firm problem with limited enforcement and with perfect credit markets. This exercise will allow us to analyze the direct effect of a tariff reduction with different enforcement technologies abstracting from general equilibrium effects.

We first show that, in response to a decrease in tariffs, constrained exporters exhibit a larger response than do unconstrained exporters. Consider the firm's problem defined in (6),(7), (11), and treat τ as a parameter. We want to compare this problem with the one of a

firm which faces no enforcement constraint at all ($\theta = 0$) which reduces the decision problem to the static one defined in (23). We assume that all quantities taken as given by the firm (tariff, output, wage, rental rate on capital, and fixed costs) are the same in both problems.

We can prove the following proposition:

PROPOSITION 3. *Let $\tau > \tau'$ and $z \geq z_x$. If $\frac{v_0(z, \tau)}{v^*(z, \tau)} < 1$ then $\frac{v_0(z, \tau')}{v^*(z, \tau')} > \frac{v_0(z, \tau)}{v^*(z, \tau)}$.*

Proof. Appendix.

This proposition states that if an exporter is initially constrained, after a tariff reduction, its initial equity value, v_0 , increases more than the minimal equity value necessary to produce at full scale, v^* . We know that the input usage of a constrained firm of age s is proportional to its equity value $v_0/(q\delta)^s$ and that v^* is proportional to a firm's optimal input usage (the input usage under perfect credit markets). Hence, a corollary to the above proposition is that, after a tariff reduction, the scale of constrained exporters increases by a greater proportion than does the scale of unconstrained exporters. This is what we will refer to as *amplification*. This is due to the fact that the constrained exporter increases their scale for two reasons: (i) the direct effect of increased marginal revenue from exporting, and (ii) the relaxation of borrowing constraints from the increase in the present value of the firm. Since the value of the firm has increased, the entrepreneur will realize more dividends in the future, which relaxes the enforcement problem. Also, because credit markets are competitive, all of these gains are realized by the entrepreneur. Hence, the value to the entrepreneur increases by a greater proportion than does the value of the firm.

Furthermore, in general equilibrium, a bilateral tariff reduction will induce an increase in the aggregate output level. Aggregate output appears as a demand parameter in the firm's

problem and it has a positive effect on the inverse demand faced by the firm. We can show that an increase in aggregate output has a larger effect on a financially constrained firm than on an identical firm that does not face borrowing constraints. Formally:

PROPOSITION 4. *Let $y < y'$. If $\frac{v_0(z,y)}{v^*(z,y)} < 1$ then $\frac{v_0(z,y')}{v^*(z,y')} > \frac{v_0(z,y)}{v^*(z,y)}$.*

Proof. Appendix.

The proof and the logic of this proposition are nearly identical to the previous one. It is important, though, to note that this result affects all firms, not only exporters. The above results describe a channel through which financially constrained economies can actually realize relatively higher gains from trade than an economy with perfect financial markets.

However, an increase in wages after a tariff reduction will have a larger effect in the constrained economy than in the unconstrained one. This is because it reduces the stream of dividends for the firm, making the option to default more attractive, hence tightening the enforcement constraint. This result is summarized by the following proposition:

PROPOSITION 5. *Let $w < w'$. If $\frac{v_0(z,w)}{v^*(z,w)} < 1$ then $\frac{v_0(z,w')}{v^*(z,w')} < \frac{v_0(z,w)}{v^*(z,w)}$.*

Proof. Appendix.

Again, the proof of this proposition follows a nearly identical argument to that of the previous propositions. Keeping everything else constant, when the wage rate increases (as happens following a trade liberalization), all constrained firms reduce their scale by a greater amount than does an identical firm in an unconstrained economy.

In general equilibrium, following a reduction in tariffs, total output and wages both increase. Hence, the reduction in tariffs and increase in output relax the borrowing constraints

of exporters, while the increase in wages tightens them. For non-exporters, only the increase in output works in their favor, while the increase in wages tightens their constraint. The interplay of these channels will determine the aggregate effect of the tariff reduction. The relative sizes of these effects can only be determined in general equilibrium.

As an example, if all firms are exporters ($f_x = 0$) and the entry cost is instead paid in terms of labor then we can show analytically that the welfare gains from a trade liberalization (percentage increase in steady state consumption) in the limited enforcement economy are the same as in the economy with perfect credit markets. In this case the three effects identified above exactly cancel out in general equilibrium. For the general case we need to rely on numerical simulations.

B. General Equilibrium: Quantitative Exercise

To evaluate the effects of a trade liberalization in general equilibrium, we conduct the following quantitative exercise: we calibrate our model and we analyze the response to a bilateral, unforeseen reduction of tariff from 10% to zero. We then compare the result for our limited enforcement to an identical economy with full enforcement. We also contrast our results with the ones from a model with *exogenous debt limit*. In particular, we will pick debt limits such that the associated equilibrium allocation in the high tariff steady state is identical to the one in our model with endogenous debt limits.

Calibration

To calibrate our model we use the *Colombian Survey of Manufactures*, which is described in detail in Roberts (1996)⁸. We choose the parameters for our model as follows. A first set of parameters is set to either match some steady state targets, or, otherwise, follow the literature. We consider the period to be one year, so we set β such that the annual real interest rate is equal to 4%. We set α to .3 to have that the labor share plus one half of total profits is approximately 65% of GDP⁹. We use annual capital depreciation of 5%, as is standard in the literature. There is a lot of disagreement in the literature about the appropriate value for the elasticity of substitution in the CES aggregator (see for instance Ruhl (2008)). In our baseline calibration we set σ to be 3, at the lower end of the spectrum. Tariffs are set to be equal to 10%, approximately the average effectively applied tariff rate for Colombia in 1989 according to UN-TRAINS. The probability of exogenous exit for a firm, $1 - \delta$, is set to match the average age of firms that we find in the Colombian dataset.

We assume that the ex-ante productivity distribution Γ is distributed log-normal(m, s). We are then left with $m, s, \theta, \omega, f_e$ and f_x . We normalize m to zero and set f_e to 0.1. Ideally, one would set θ using firm-level balance sheet data, but such information is not available in the Colombian dataset. We set θ to be seven¹⁰ and we then check if the model is broadly consistent with the life-cycle path of the firm. In particular, we look at the ratio of average labor used by firms of age 15 to that used by firms of age 1¹¹, and the ratio of the average

⁸We are considering Colombian firms because of data availability, not because we are interested in looking at trade between developing countries or countries with low financial market quality.

⁹In an economy with monopolistic competition and Cobb-Douglas technology the labor share is not equal to α , because one has to account for profits.

¹⁰This number is in line with what used by Jermann and Quadrini (2009), which use 5 for the US, a country with better financial institutions and contract enforcement than Colombia.

¹¹We only target half of the firm-level growth in labor usage because there are likely to be other contributing factors besides financial constraints.

productivity of age 1 new exporters to overall average productivity. Finally, we calibrate f_x , ω and s using three moments from the data: the standard deviation of productivity for operating firms¹², the percentage of firms that export, and the export percentage of GDP. Exports as a percentage of GDP are calculated from IMF IFS, and is the average for the years 1981-1991 (the years covered by the firm-level data). We estimate firm-level productivity by applying the method of Levinsohn and Petrin (2003) industry-by-industry on the Colombian data, then use that to calculate the standard deviation of firm-level productivity. The fraction of firms that export also comes from the Colombian data.

Table 1 in the Appendix summarizes the parameters we use and Table 2 reports how the model is able to match the chosen targets in the data. We next use the calibrated model to predict what will happen after an unforeseen bilateral reduction of tariffs from 10% to zero.

Steady State Comparison

We first compare the steady state with no tariffs to one with 10% tariffs in both countries. The first line of Table 3 in the Appendix reports the percentage change in consumption, output, capital stock and exports for the limited enforcement economy. With limited enforcement, the 10% tariff reduction results in a steady state increase of consumption of 0.81%, in line with the results in the literature¹³, while exports increases by approximately 30%. We are interested in confronting our result with what one would obtain from (i) an economy with perfect credit markets and (ii) with an economy with exogenous debt limits.

¹²The distribution of operating firms does not coincide with Γ , the distribution of productivity draws, because low productivity firms do not find profitable to pay the set-up cost f_d .

¹³See Alessandria and Choi (2007).

The second and third row of Table 3 report the change in aggregates for an economy with perfect credit markets. In particular, the second row shows the results we obtain using the same parameters that we use in the benchmark limited enforcement economy, other than θ . The comparison between the first and second row in Table 3 can be thought as a comparative statics exercise, varying θ from its calibrated value to zero (perfect credit market). The percentage change in consumption, output, capital and the export to GDP ratio are very similar, the welfare change in the limited enforcement economy is only 1 basis point higher than with full enforcement. One problem with this comparison is that the two economies are different in terms of observables. As illustrated in the third column of Table 2, the export to GDP ratio is approximately the same, but the two economies differ slightly in the fraction of exporting firms and in the standard deviation of productivity for operating firms.

To address this issue, the third row of Table 3 reports the steady state change for a full enforcement economy calibrated to match the same moments we use to calibrate the limited enforcement economy¹⁴. See Table 1 and 2 in the Appendix for the parameters used, and for the model fit to the data. Also in this case the percentage change in consumption is approximately the same with perfect credit markets as in our benchmark economy (only 2 basis points lower).

Finally, we consider the effects of a tariff reduction in an economy with exogenous debt limits. To allow for a clear comparison with the economy with endogenous borrowing constraints, we choose the debt limit such that the two economies are identical in the high tariff

¹⁴Precisely, we set s, ω and f_x to match the standard deviation of productivity for operating firms, the percentage of firms that export, the export percentage of GDP, while maintaining all other parameters unchanged. See Table 1 and 2, column (b).

steady state. More precisely, in the exogenous debt limit economy, firms face productivity and age dependent limits to the amount of intra-period debt they can raise to finance their working capital. A formal definition for the firm's problem and how we construct the debt limits can be found in Section D of the Appendix. The last row of Table 3 shows that the steady state change in consumption following the considered tariff reduction is approximately 12% lower with exogenous debt limits than with endogenous ones. The same is true for the other aggregates we considered: output, capital, and exports.

Figure 4 shows the steady state change in the ratio v_0/v^* in our model after the considered tariff reduction. This ratio measures the tightness of the borrowing constraints. If v_0/v^* is greater or equal to 1, then the firm is not constrained from the beginning. A lower ratio implies that it will take more time for the firm to be able to operate at full scale. After a trade liberalization, in the new steady state, the enforcement constraint becomes tighter for non-exporters, while it relaxes for old exporters. This is consistent with what we proved in a partial equilibrium context in the previous subsection. The exact opposite happens in the economy with exogenous debt limit. For new exporters, the constraint is tighter in both cases, because there is a discrete jump in their optimal scale: with high tariffs they were producing for the domestic market only, but now they have to produce for two markets. This distributional implication of the model can potentially be used to test the theory using data from a trade liberalization episode.

We next perform a sensitivity analysis to check the robustness of our findings. We consider different values for the enforcement parameter $\theta \in \{3, 7, 10\}$, the elasticity of substitution $\sigma \in \{2, 3, 4\}$, and the set-up cost $f_d \in \{0.05, 0.1, 0.2\}$. In Figure 7 we report the comparison of the percentage steady state change in consumption under the different para-

meterizations that we use.

In the first panel we compare the model with endogenous borrowing constraints to the model with perfect credit markets. Different parameterizations lead to different implications for the size of the welfare gain that follows a 10% tariff reduction, but the graph shows that the difference in gains for the full and limited enforcement economy is always very small. The mean percentage difference in the percentage change in consumption is -0.5%. The largest difference between the two is for parameters $\sigma = 2$, $\theta = 10$, $f_d = .1$ when the limited enforcement economy has a 6.6% smaller gain from trade than does the full enforcement (a 1.99% change instead of a 2.13% change). Different parameters imply different levels of gains from a tariff reduction, but those gains do not depend on financial market quality.

In the second panel we compare the gains predicted by the model with full enforcement to those from the model with exogenous debt limits. We see that the exogenous debt limits model exhibits systematically lower gains from a tariff reduction. The mean difference in gains from the trade liberalization between full enforcement and exogenous debt limits is -11.2%. The key parameter driving the difference is θ . When $\theta = 10$ the mean difference is -22%, meaning that nearly a quarter of the gains from the liberalization are lost due to the inability of financial constraints to adjust. Most of the points that are close to the 45° line have $\theta = 3$. At that parameterization the borrowing constraint is binding for very few firms so that the economy with full enforcement and the economy with exogenous debt limits are nearly identical. We conclude from this that financial market quality can only be an important barrier to gains from a trade liberalization if financial markets do not respond to profit opportunities.

Transitional Dynamics

We now show that the above conclusions are unchanged if we consider the transitional dynamics from the high to the low tariff stationary equilibrium. To do so, we need to specify a functional form for the households' period utility function. Figure 5 shows the transition path (normalized by the high tariff steady state level) for the calibrated limited enforcement economy and linear preferences. We do not report the transition for the full enforcement economy because it is essentially the same in this case. With infinite intertemporal elasticity of substitution, despite the non-trivial dynamic for the distribution of firms λ , the aggregates converges in 2 periods to their low tariff steady state levels.

Figure 6 reports the transition path with log preferences for the economy with limited enforcement and the economy with perfect credit markets (same parameterization). The two transition paths are very similar. The change in consumption and exports are always greater in the limited enforcement economy than in the full enforcement one. Both economies overshoot their new steady state value for consumption during the transition.

In summary, we find that after a tariff reduction the percentage change in steady state consumption in an economy with limited enforcement differs from that in an equivalent one with perfect credit markets by only a few basis points. We interpret this difference as being inconsequential. We reach the same conclusion if we look at the transition path from the high tariff steady state to the no-tariff one. We can then conclude that the presence of finance constraints at the firm level does *not* reduce the aggregate gains from a tariff reduction in our calibrated model. The endogeneity of the credit constraint is crucial for our result: if credit constraints do not respond to profit opportunities then the gains from a tariff reduction are lower than in a benchmark economy with perfect credit markets.

6. Concluding Remarks

In this paper we have built a dynamic general equilibrium trade model with limited enforceability of credit contract based on Melitz (2003) and Albuquerque and Hopenhayn (2004). The model is consistent with the main findings of the empirical literature on trade and financial constraints. We then use the model to assess the effects of credit market imperfections on the response of the economy to a tariff reduction. In a partial equilibrium environment, credit constrained exporters increase their scale by a greater proportion than unconstrained exporters following a tariff reduction because it helps to relax the borrowing constraint that exporters face. In general equilibrium, the trade reform can potentially have a perverse effect for non-exporters: the increase in real wage that follows a trade liberalization episode will make tighter the debt limit for firms that do not export. In a calibrated example, we find that the percentage change in steady state consumption is approximately the same in an economy with limited enforcement than in an equivalent one with perfect credit markets. We conclude that the presence of credit constraints at the firm level does *not* reduce the aggregate gains from a tariff reduction. The endogeneity of the credit constraint is key for our result: if credit constraints do not respond to profit opportunities then the gains from a tariff reduction are lower than in a benchmark economy with perfect credit markets.

Our theoretical result is not inconsistent with the finding that in countries with less developed financial markets exports are less sensitive to real exchange rate fluctuations¹⁵. In fact, we are considering a permanent reduction in trade barriers, while real exchange rate shocks are only transitory and uncertain. Extending our model to include transitory shocks could be an interesting direction for future research.

¹⁵See Manova (2010) and references therein.

Our model has abstracted from two potentially important considerations. First, we do not consider how trade cost affects the incentives for firms to invest in innovation. Studying the interaction between investment in innovation and trade cost in a model with limited enforcement can be fruitful since investments in innovation are not easy to collateralize and, hence, are difficult to finance.

Finally, by considering two symmetric countries, we abstracted from the interaction between trade reform and international capital flows. Manova (2008b) finds that the effects of equity market liberalizations on exports are higher in economies with lower financial market quality, suggesting that there is a complementarity between liberalizing trade and the capital account with credit market frictions. We view this interaction as an interesting topic for future research.

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8. Appendix

A. Proof of Proposition 1

For convenience rewrite $W^{nx}(v, z)$ and $W^x(v, z)$ in sequential form as

(24)

$$W^{nx}(v_0, z) = \sup_{T \in \mathbb{N}, \{v_{t+1}, d_t, y_{dt}, l_t, k_t\}} \sum_{t=0}^T (\delta q)^t \{p_d(y_{dt})y_{dt} - wl_t - rk_t - d_t\} + (\delta q)^T \{W^x(v_T, z) - f_x\}$$

subject to $\forall t = 0, 1, \dots, T$

$$y_{dt} \leq z k_t^\alpha l_t^{1-\alpha}$$

$$v_t \leq d_t + \delta q v_{t+1} \quad \lambda_t$$

$$\theta(wl_t + rk_t) \leq d_t + \delta q v_{t+1} \quad \gamma_t$$

$$d_t \geq 0 \quad \eta_t$$

$$(25) \quad W^x(v, z) = \max_{d, v', k, l, y_d, y_x,} \sum_{t=0}^{\infty} (q\delta)^t \{p_d(y_{dt})y_{dt} + p_x(y_{xt})y_{xt} - wl_t - rk_t - d_t\}$$

subject to $\forall t$

$$y_{dt} + y_{xt} \leq z k_t^\alpha l_t^{1-\alpha}$$

$$d_t + q_t \delta v_{t+1} \geq v_t \quad \lambda$$

$$d_t + q_t \delta v_t \geq \theta(w_t l + r_t k) \quad \gamma$$

$$d_t \geq 0 \quad \eta$$

For notational convenience, define also

$$\Pi^x(v, z) = \max_{y_d, y_x, l, k} p_d(y_d) y_d + p_x(y_x) y_x - w l - r k$$

subject to

$$y_d + y_x \leq z F(k, l)$$

$$v \geq \theta(w l + r k)$$

and

$$\Pi^{nx}(v, z) = \max_{y_d, l, k} p_d(y_d) y_d - w l - r k$$

subject to

$$y_d \leq z F(k, l)$$

$$v \geq \theta(w l + r k)$$

(i) There exists of a cut-off productivity level z_d s.t. if $z \geq z_d$ the firm enters. This is obvious as $W^{nx}(v, z)$ is strictly increasing in z .

(ii) To show that $\forall z$, if $v \leq v^*(z)$ then $d(v, z) = 0$ (with minor qualification), if $v > v^*(z)$ then $d(v, z) \in [0, v^*(z) - \delta q v^*(z)]$, consider first the following lemma.

LEMMA 1. $\forall z, \exists \underline{v}, v^*$ such that

- (i) for $v > v^*$ W^x is linear and decreasing, in particular, $W^x(v) = \frac{\Pi^x(v^*)}{1-q\delta} - v$,
- (ii) $\forall v \in [0, \underline{v}]$ $W^x(v)$ is constant
- (iii) for $v \in (\underline{v}, v^*]$ W^x is str. decreasing and str. concave.

Proof. (i) Let $v^* \equiv \theta(wl^* + rk^*)$. The enforcement constraint is not binding for all $v \geq v^*$, then define the following solution to the problem: $d(v) \in (0, v - \delta q v^*)$ and $v'(v) = \frac{v - d(v)}{\delta q} \geq v^*$, $l(v) = l^*, k(v) = k^*$: it is clearly feasible and it attains the optimum for the unconstrained problem - dropping the enforcement constraint and non-negativity for dividends - therefore it is optimal for the constrained problem, and $W^x(v) = \frac{\Pi^x(v^*)}{1-q\delta} - v$.

(ii) Notice that $W^x(v)$ is non-increasing in v because of monotonicity of the constraint set in v . We want to show that there is a flat region of the Pareto frontier. Consider auxiliary problem:

$$\tilde{W}(v) = \max_{v', d', y_d, y_x, l, k} p_d(y_d)y_d + p_x(y_x)y_x - wl - rk - d + \delta q \tilde{W}^x(v')$$

subject to

$$v = d + \delta q v'$$

and (5), (9) and $d \geq 0$. $\tilde{W}$ is concave, since the period return function is concave and the constraint set is convex. $\tilde{W}(0) = 0$ and it is increasing at 0. We showed in (i) that for $v \geq v^*$, $\tilde{W}(v) = W^x(v) = \frac{\Pi^x(v^*)}{1-q\delta} - v$ and therefore decreasing. Then, by concavity of $\tilde{W}$ it must exist a unique $\underline{v} \in (0, v^*)$ s.t. $\tilde{W}(\underline{v}) \geq \tilde{W}(v) \forall v$, where the uniqueness come from the strict concavity of $\tilde{W}$ over $(0, v^*)$.

Using the above result about the auxiliary problem, and the monotonicity of the constraint set in the original problem, we have that $\forall v \leq \underline{v} \ W(v) = \tilde{W}(\underline{v})$ as wanted.

Finally (iii) is implied by the fact that for $v \in [\underline{v}, v^*]$ $W(v) = \tilde{W}(v)$ which is str. concave and strictly decreasing. Q.E.D.

For an exporter (dropping the dependence from z for simplicity), by the previous lemma (25) is a convex problem, the FOCs for the above problem requires that

$$v' : \quad W_1^x(v') + \lambda + \gamma = 0$$

$$d : \quad -1 + \lambda + \gamma + \eta = 0$$

where λ , γ and η are the Lagrange multipliers on the promise keeping, enforcement and limited liability constraints respectively. The envelope condition is

$$W^{x'}(v) = -\lambda$$

First, let $v < \delta q v^*$, which implies that $\gamma > 0$. We want to show that $\eta > 0$. By the previous lemma part (iii), we have that $W'(v') \in (0, -1)$ because for any feasible choice of d , $v' < v^*$.

Then combining the FOCs we have

$$1 > -W^{x'}(v') = \lambda + \gamma = 1 - \eta$$

which implies $\eta > 0$ as desired.

Now suppose $v \geq \delta q v^*$. By previous lemma part (ii) we have that $W^{x'}(v') = -1$ if $d \in [0, v - \delta q v^*]$, $W^{x'}(v') > -1$ if $d > v - \delta q v^*$. Suppose for contradiction that $d > v - \delta q v^* > 0$, which implies that $\eta = 0$. Then from the FOCs

$$1 > -W^{x'}(v') = \lambda + \gamma = 1 - \eta = 1$$

which is a contradiction. Then we have proven part (ii) for an exporter.

Consider now the problem for a domestic firm (24). If it will never export, then the same logic developed for the exporter case goes through. Now assume that a firm will eventually export conditional on surviving. We show that either (a) it will not distribute dividends until $T + 1$ and $v_{T+1} < v^*$ or (a) it will never distribute dividends until $v_t \geq \delta q v_d^*$ and $v_{T+1} \geq v^*$, where $T + 1 = \min_{\tau \in N} \tau$ s.t. $\frac{v_0}{(\delta q)^\tau} \geq v^*$. Fix T . The problem is convex.

Take FOCs to get:

$$\begin{aligned} d_t & : & -(\delta q)^t + \lambda_t + \gamma_t + \eta_t &= 0 \\ v_{t+1} & : & \delta q \lambda_t - \lambda_{t+1} + \delta q \gamma_t &= 0 \\ v_{T+1} & : & (\delta q)^{T+1} W^{x'}(v_{T+1}) + \delta q \lambda_T + \delta q \gamma_T &= 0 \end{aligned}$$

If $v_{T+1} < v^*$, then we know $-W^{x'}(v_{T+1}) < 1$ then from last FOCs

$$\lambda_T + \gamma_T < (\delta q)^T$$

and the first tells us that

$$\lambda_T + \gamma_T = (\delta q)^T - \eta^T$$

then $\eta^T > 0$. Combine the first two FOCs to get

$$\eta_t = \frac{\eta_{t+1} + \gamma_{t+1}}{\delta q}$$

since all of the $\gamma_{t+1} \geq 0$ then one can show inductively that for all t , $\eta_t > 0$, which proves (a).

Finally, suppose that $v_{T+1} \in (v^*, \frac{v^*}{\delta q})$. If $v_t < \delta q v_d^*$ then $\gamma_{t+1} > 0$ (by a similar argument as for the exporter) and hence by $\eta_t = \frac{\eta_{t+1} + \gamma_{t+1}}{\delta q}$ it follows that $\eta_t > 0$ and so $d_t = 0$, establishing (b).

We now prove part (iii) and (iv). By part (ii) we can set $d_t = 0$ without loss for all t and rewrite the intermediary's problem as follows:

$$\begin{aligned} W^{nx}(v, z) &= \max \left\{ \Pi^{nx}(v, z) + q\delta W^d \left(\frac{v}{q\delta}, z \right); \Pi^x(v, z) - f_x + q\delta W^x \left(\frac{v}{q\delta}, z \right) \right\} \\ W^x(v, z) &= \Pi^x(v, z) + q\delta W^x \left(\frac{v}{q\delta}, z \right) \end{aligned}$$

A firm will eventually reach v^* , because $\{v_t\} = \left\{ \frac{v_0}{(\delta q)^t} \right\}$. Then, for $v' \geq v^*$ a domestic firm

with inside equity value v' will start exporting iff

$$\frac{\Pi^{x*}(z)}{1 - \delta q} - \frac{\Pi^{nx*}(z)}{1 - \delta q} \geq f_x$$

as in a standard Melitz model. Since the LHS is strictly increasing in z , there exists a cut-off a_x s.t. the above condition holds for all $a \geq a_x$. Consider

$$\begin{aligned} W^x(v, z) - W^{nx}(v, z) &= \Pi^x(v, z) + q\delta W^x\left(\frac{v}{q\delta}, z\right) \\ &\quad - \max\left\{\Pi^{nx}(v, z) + q\delta W^{nx}\left(\frac{v}{q\delta}, z\right); W^x(v, z) - f_x\right\} \\ &= \min\left\{\Pi^x(v, z) - \Pi^{nx}(v, z) + q\delta\left(W^x\left(\frac{v}{q\delta}, z\right) - W^{nx}\left(\frac{v}{q\delta}, z\right)\right); f_x\right\} \\ &= \min\left\{\Delta\Pi(v, z) + q\delta\left(W^x\left(\frac{v}{q\delta}, z\right) - W^{nx}\left(\frac{v}{q\delta}, z\right)\right); f_x\right\} \end{aligned}$$

The following lemma shows that the value of becoming an exporter increases (weakly) with v .

LEMMA 2. (a) $\forall z$ $W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in v , and (b) $\forall v$ $W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in z .

Proof. Define $T : C(\mathbb{R}_+ \times \mathbb{R}_+) \rightarrow C(\mathbb{R}_+ \times \mathbb{R}_+)$ as

$$Tf(v, z) = \min\left\{\Delta\Pi(v, z) + q\delta f\left(\frac{v}{q\delta}, z\right); f_x\right\}$$

where $C(\mathbb{R}_+ \times \mathbb{R}_+)$ is the space of continuous and bounded functions. T satisfies the Blackwell's sufficient conditions for a contraction mapping. Then T is a contraction, and $W^x - W^{nx}$ is its unique fixed point.

To prove (a), let $C''(\mathbb{R}_+ \times \mathbb{R}_+)$ be the set of continuous, bounded and weakly increasing function in their first argument. $C''(\mathbb{R}_+ \times \mathbb{R}_+)$ is a closed set, hence by Corollary 3.1 in Stokey, Lucas and Prescott (1989) it suffices to show that $\forall f \in C''(\mathbb{R}_+ \times \mathbb{R}_+) \quad Tf \in C''(\mathbb{R}_+ \times \mathbb{R}_+)$ to prove that $W^x - W^{nx}$ is increasing in its first argument. Fix z , let $f \in C''(\mathbb{R}_+ \times \mathbb{R}_+)$ and $v' > v$:

$$\begin{aligned} Tf(v', z) &= \min \left\{ \Delta\Pi(v', z) + q\delta f\left(\frac{v'}{q\delta}, z\right); f_x \right\} \\ &\geq \min \left\{ \Delta\Pi(v, z) + q\delta f\left(\frac{v}{q\delta}, z\right); f_x \right\} = Tf(v, z) \end{aligned}$$

as wanted, because $\Delta\Pi(v, z)$ is increasing in v , and f is weakly increasing by assumption. Then we established (a). The exact same argument can be used to prove (b) noticing that $\Delta\Pi(v, z)$ is increasing in z also. Q.E.D.

Now, notice that $0 \leq W^x(v, z) - W^{nx}(v, z) \leq f_x$ since it is always feasible for a domestic firm to start exporting, and $\forall z, \forall v \geq v^*(z)$, $W^x(v, z) - W^d(v, z)$ is constant. Thus, if $z \leq z_x$ a firm will never export since for all v $W^x(v, z) - W^{nx}(v, z) \leq W^x(v^*, z) - W^{nx}(v^*, z) < f_x$. Vice versa, if $z \geq z_x$, then the firm will eventually export, proving (iii).

To prove (iv), just notice that if $z \geq z_x$ the firm will eventually export, the fact that $W^x(v, z) - W^{nx}(v, z)$ is increasing in v implies that it exist a unique threshold $\tilde{v}(z)$ such that a firm will export iff $v \geq \tilde{v}(z)$.

B. Proof of Proposition 2

We will show that if $z' > z$ then $\tilde{v}(z')/v_0(z') \leq \tilde{v}(z)/v_0(z)$, implying $\tilde{T}(z') \leq \tilde{T}(z)$. Let $z' > z \geq z_x$. First, that the fact that $W^{nx}(v, z)$ is strictly increasing in z for all v implies

that $v_0(z') > v_0(z)$, since v_0 is such that $W^{nx}(v_0(z), z) = f_e$. To prove the proposition it is sufficient to show that $\tilde{v}(z') < \tilde{v}(z)$. By the previous lemma $\forall v \ W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in z . Thus, if $W^x(\tilde{v}(z), z) - W^{nx}(\tilde{v}(z), z) = -f_x$ then $W^x(\tilde{v}(z), z') - W^{nx}(\tilde{v}(z), z') \geq -f_x$ since $z' > z$, therefore $\tilde{v}(z') \leq \tilde{v}(z)$ as wanted.

C. Proof of Propositions 3,4 and 5

We will first prove Proposition 3, that is v_0/v^* is decreasing in τ (for exporters). One can show that the period profit of a constrained exporting firm is

$$\Pi^x(v_t; \tau) = D(\tau)y^{1/\sigma} \left(z \left(\frac{\alpha}{r} \right)^\alpha \left(\frac{1-\alpha}{w} \right)^{1-\alpha} \frac{v_t}{\theta} \right)^\gamma - \frac{v_t}{\theta}$$

where $\gamma \equiv 1 - 1/\sigma$, and $D(\tau) \equiv \omega (1 + (\omega(1 + \tau))^{-\sigma} / (1 - \omega))^{1-\gamma}$. In what follows, changes in τ will be analyzed as changes in D , which is decreasing in τ .

Let $\pi(v, D) \equiv \Pi^x(v, \tau) + v/\theta$. Then π is homogeneous of degree 1 in D and is homogeneous of degree γ in v . Recall that v^* is the v that maximizes $\pi(v, D)$:

$$v^* = \theta(D\gamma)^\sigma y \left(z \left(\frac{\alpha}{r} \right)^\alpha \left(\frac{1-\alpha}{w} \right)^{1-\alpha} \right)^{\frac{\gamma}{1-\gamma}}$$

which is homogeneous of degree σ in D .

Suppose that it is profitable to start exporting from the first period. In what follows we will ignore floor operators for tractability. The zero profit condition of the intermediary

pins down v_0 and can be written, using $v_t = \min\{v^*, \frac{v_0}{(q\delta)^t}\}$, as

$$\begin{aligned} f_e + v_0 &= \sum_t (q\delta)^t \Pi_x(v_t, D) = \sum_t (q\delta)^t \dot{\pi}(v_t, x) - \sum_t (q\delta)^t \frac{v_t}{\theta} \\ &= -T^* \frac{v_0}{\theta} - \frac{v_0}{1-q\delta} \frac{1}{\theta} + v_0 \frac{Dv^{*\gamma-1}}{1-q\delta} \dot{\pi}(1, 1) + \frac{1 - (\frac{v_0}{v^*})^{1-\gamma}}{1 - (q\delta)^{1-\gamma}} v_0^\gamma D\dot{\pi}(1, 1) \end{aligned}$$

where $T^* = \frac{\log(v_0) - \log(v^*)}{\log(q\delta)}$. Then dividing through the zero profit condition by v_0 yields:

$$(26) \quad \frac{f_e}{v_0} + 1 = -\frac{T^*}{\theta} - \frac{1}{1-q\delta} \frac{1}{\theta} + \frac{Dv^{*\gamma-1}}{1-q\delta} \dot{\pi}(1, 1) + \frac{1 - (\frac{v_0}{v^*})^{1-\gamma}}{1 - (q\delta)^{1-\gamma}} v_0^{\gamma-1} D\dot{\pi}(1, 1)$$

Now suppose that $D' = \Delta_D D$, and $v'_0 = \Delta_0 v_0$. Hence, $v'^* = \Delta_D^\sigma v^*$. Define $\Delta_t = \frac{\log(\frac{\Delta_0}{\Delta_D^\sigma})}{\log(q\delta)}$. By the previous argument, we can write:

$$(27) \quad \frac{f_e}{\Delta_0 v_0} + 1 = -\frac{T^* + \Delta_t}{\theta} - \frac{1}{1-q\delta} \frac{1}{\theta} + \frac{Dv^{*\gamma-1}}{1-q\delta} \dot{\pi}(1, 1) + \frac{1 - (\frac{\Delta_0 v_0}{\Delta_D^\sigma v^*})^{1-\gamma}}{1 - (q\delta)^{1-\gamma}} \Delta_D \Delta_0^{\gamma-1} v_0^{\gamma-1} D\dot{\pi}(1, 1)$$

Subtracting (26) from (27) yields:

$$\left(\frac{1}{\Delta_0} - 1 \right) \frac{f_e}{v_0} = -\frac{\Delta_t}{\theta} + D\dot{\pi}(1, 1) v_0^{\gamma-1} \frac{(\frac{\Delta_0}{\Delta_D^\sigma})^{\gamma-1} - 1}{1 - (q\delta)^{1-\gamma}}$$

Call the left-hand side (L) and the right-hand side (R).

Next, we will show that (R) is increasing in Δ_D .

$$\begin{aligned} \frac{\partial(R)}{\partial \Delta_D} &= \frac{-\sigma}{-\log(q\delta) \Delta_D} \frac{1}{\theta} + \Delta_0^{\gamma-1} D\dot{\pi}(1, 1) \frac{v_0^{\gamma-1}}{1 - (q\delta)^{1-\gamma}} > 0 \\ \text{iff } \dot{\pi}(\Delta_0 v_0, \Delta_D D) - v_0 \frac{\Delta_0}{\theta} \left(\frac{\sigma(1 - (q\delta)^{1-\gamma})}{-\log(q\delta)} \right) &> 0 \end{aligned}$$

Note that the positivity of period profits implies:

$$\dot{\pi}(\Delta_0 v_0, \Delta_D D) - v_0 \frac{\Delta_0}{\theta} > 0$$

Hence, we need only show that $\left(\frac{\sigma(1-(q\delta)^{1-\gamma})}{-\log(q\delta)} \right) \leq 1$. Notice that this expression is equivalent to $1 - (q\delta)^{1-\gamma} < -\log((q\delta)^{1-\gamma})$. Since, $(q\delta)^{1-\gamma} \in (0, 1)$, this is always satisfied. Hence, (R) is increasing in Δ_D .

Now, suppose that $\Delta_D > 1$. Then, it is obvious that $\Delta_0 > 1$. It is sufficient to show that (L)=(R) implies $\Delta_0 > \Delta_D^\sigma$. Notice that when $\Delta_0 = \Delta_D^\sigma$, (R) = 0 and (L) < 0. Suppose for contradiction that (L)=(R) and $\Delta_0 < \Delta_D^\sigma$. Then suppose Δ_D is reduced to $\Delta_0^{1/\sigma}$. (L) does not depend on Δ_D , and (R) is increasing in Δ_D , so reducing Δ_D implies that (L) > (R). But then $\Delta_D = \Delta_0^{1/\sigma}$, or $\Delta_0 = \Delta_D^\sigma$, so (R)=0 > (L), which is a contradiction. Hence

$$\frac{v'_0}{v^{*'}} = \frac{\Delta_0}{\Delta_D^\sigma} \frac{v_0}{v^*} > \frac{v_0}{v^*}$$

This proves the proposition if the firm starts to export from the first period. The proof is essentially the same if $\tilde{T} > 1$.

The proofs for y and w are almost exactly the same, except that (R) is decreasing in Δ_w .

D. Exogenous Debt Limit Economy

Let $\{\bar{B}_s(z)\}_{s=0}^\infty$ be the sequence of intra-period exogenous debt limits that a firm with productivity z faces, where s is the age of the firm. The problem for a domestic firm can be

written recursively using debt, b , age, and productivity as individual states:

$$(28) \quad V_t^{nx}(b, s, z) = \max_{d, v', k, l, y_d, y_x, x \in \{0,1\}} d + q_t \delta [x V_{t+1}^x(b', s+1, z) + (1-x) V_{t+1}^{nx}(b', s+1, z)]$$

subject to the feasibility constraint (5), the budget constraint

$$d \leq p_{dt}(y_d)y_d + xp_{xt}(y_x)y_x - w_t l - r_t k - xp_t f_x - b + q\delta b'$$

and the intra-period debt limit:

$$w_t l - r_t k \leq \bar{B}_s(a)$$

For an exporter:

$$(29) \quad V_t^x(b, s, z) = \max_{d, v', k, l, y_d, y_x} d + q_t \delta V_{t+1}^x(b', s+1, z)$$

subject to (5), and

$$d \leq p_{dt}(y_d)y_d + p_{xt}(y_x) - w_t l - r_t k - b + q\delta b'$$

$$w_t l - r_t k \leq \bar{B}_s(a)$$

The initial debt is going to be equal to f_e and the firm will decide to operate only if $V_t^{nx}(f_e, 0, z) \geq 0$. The definition for a symmetric equilibrium is analogous to the one for the economy with endogenous debt limits.

The debt limits we used are constructed as follows: $\forall z, \forall s$

$$\bar{B}_s(z) = \theta \frac{v_0(z)}{(q\delta)^s}$$

where $v_0(z)$ is the equilibrium initial equity value in the steady state economy with endogenous debt limits and a positive tariff. Then the positive tariff steady state with endogenous debt limits is an equilibrium for the exogenous one.

E. Tables

Table 1. Parameters Value:

(a) Calibration for the limited enforcement economy

(b) Calibration for the full enforcement economy

Parameter	Symbol	Value (a)	Value (b)
Discount Factor	β	0.9615	0.9615
Cobb-Douglass Parameter	α	0.3	0.3
Capital Depreciation	δ_k	0.05	0.05
Elasticity of Substitution	σ	3	3
Tariff	τ	0.1	0.1
Surviving Probability	δ	0.93	0.93
Set-up fix cost	f_e	0.1	0.1
Enforcement Parameter	θ	7	0
Home Bias	ω	0.610	0.614
Std of Productivity	s	3.05	2.75
Export fix cost	f_x	1.36	0.85

Table 2. Target Statistics: Data and Model for the Baseline Calibration

Target	Model			
	Data	LE (a)	FE (a)	FE (b)
Std. Dev. of Prod. Dist'n	0.87	0.85	0.86	0.85
Exports / GDP	15.1%	15.7%	15.7%	15.1%
% Firms Exporters	12.5%	13.0%	10.2%	12.8%
Other Moments				
Firm Growth, 1 to 15 years (half)	52%	50%	-	-
Avg. Prod. Age 1 Exporters/Avg. Prod.	3.89	4.86	-	-

LE : Limited Enforcement. FE : Full Enforcement

(a) Calibration for the limited enforcement economy

(b) Calibration for the full enforcement economy

Table 3. Steady State Comparison: $\tau = .1$ and $\tau = 0$

	% Change in			
	c	y	k	Exports
Limited Enforcement (a)	0.81	1.02	2.64	30.22
Full Enforcement (a)	0.80	1.01	2.63	30.07
Full Enforcement (b)	0.79	0.99	2.54	30.44
Exogenous Debt Limit (a)	0.72	0.91	2.34	30.04

(a) Calibration for the limited enforcement economy

(b) Calibration for the full enforcement economy

F. Figures

Figure 1. Decision to start to export as a function of equity value and productivity

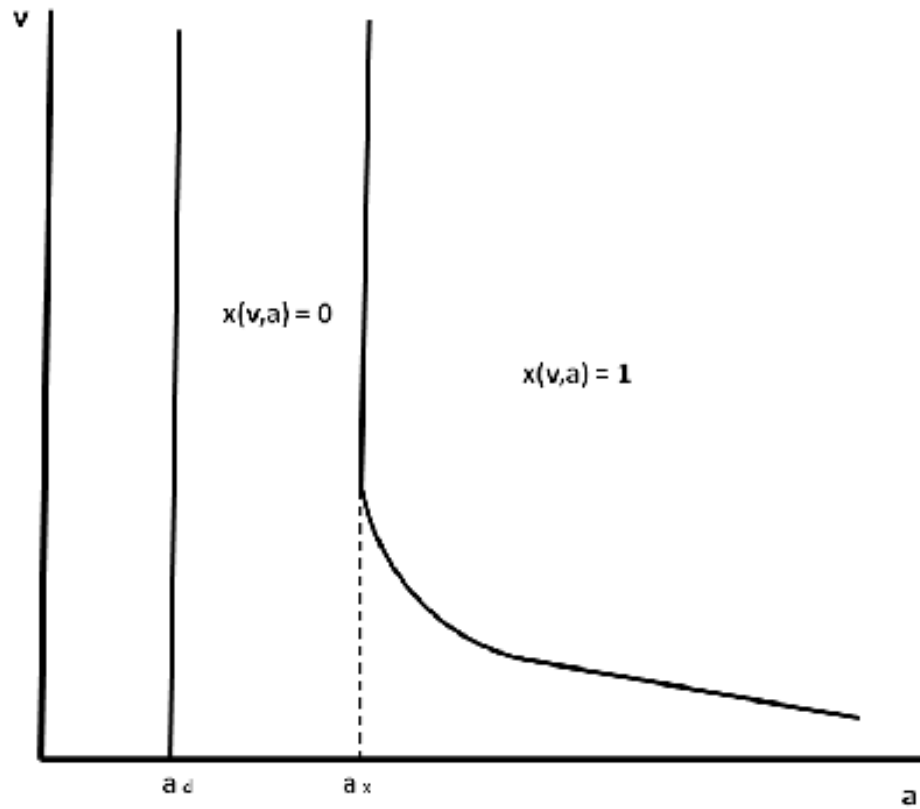


Figure 2. Productivity Distribution for New Exporters by Age, Model and Data

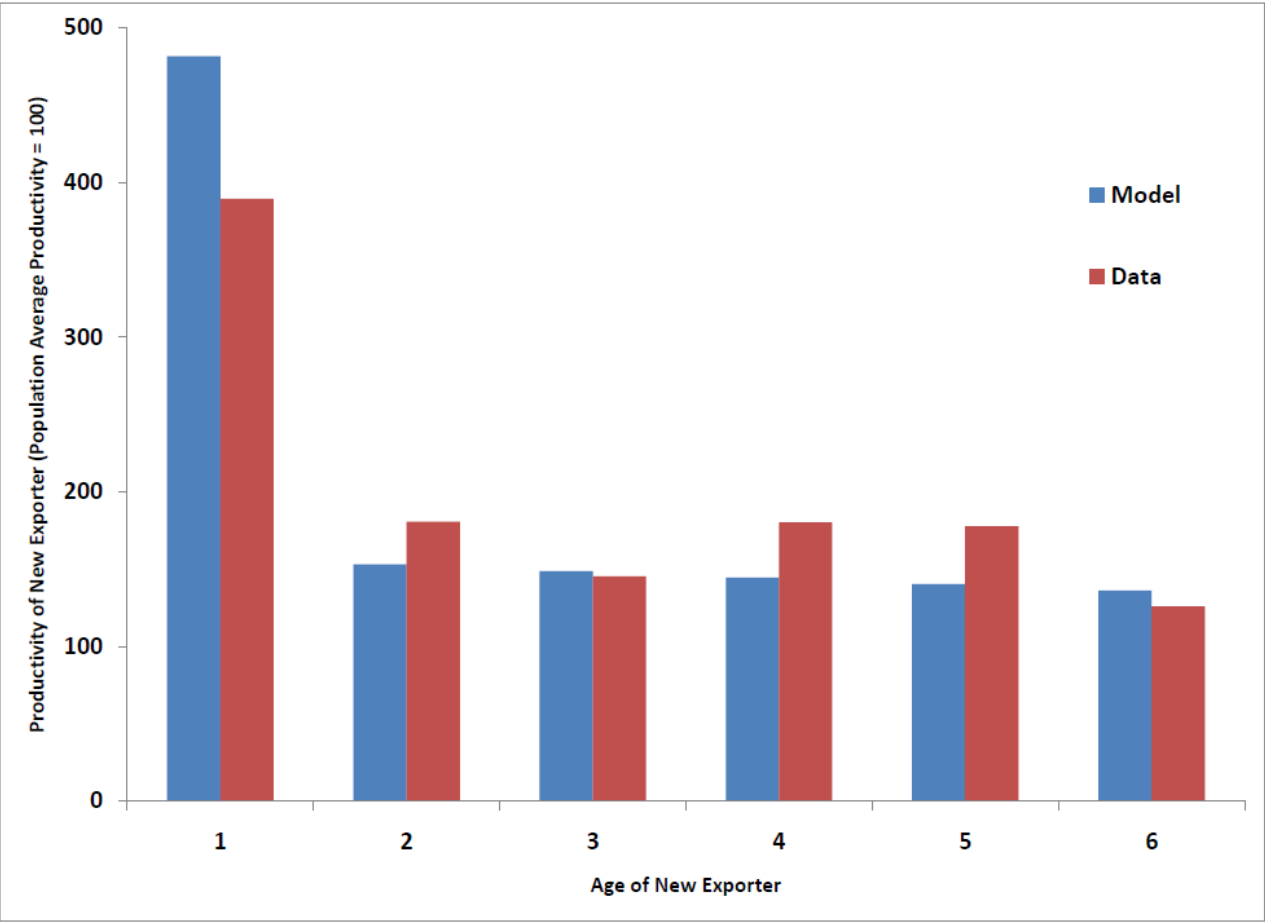


Figure 3. Typical Firm's Life Cycle Production Plans.

(A) Non Exporter, $z < z_x$

(B) Delayed entry in the foreign market

(C) High Productivity

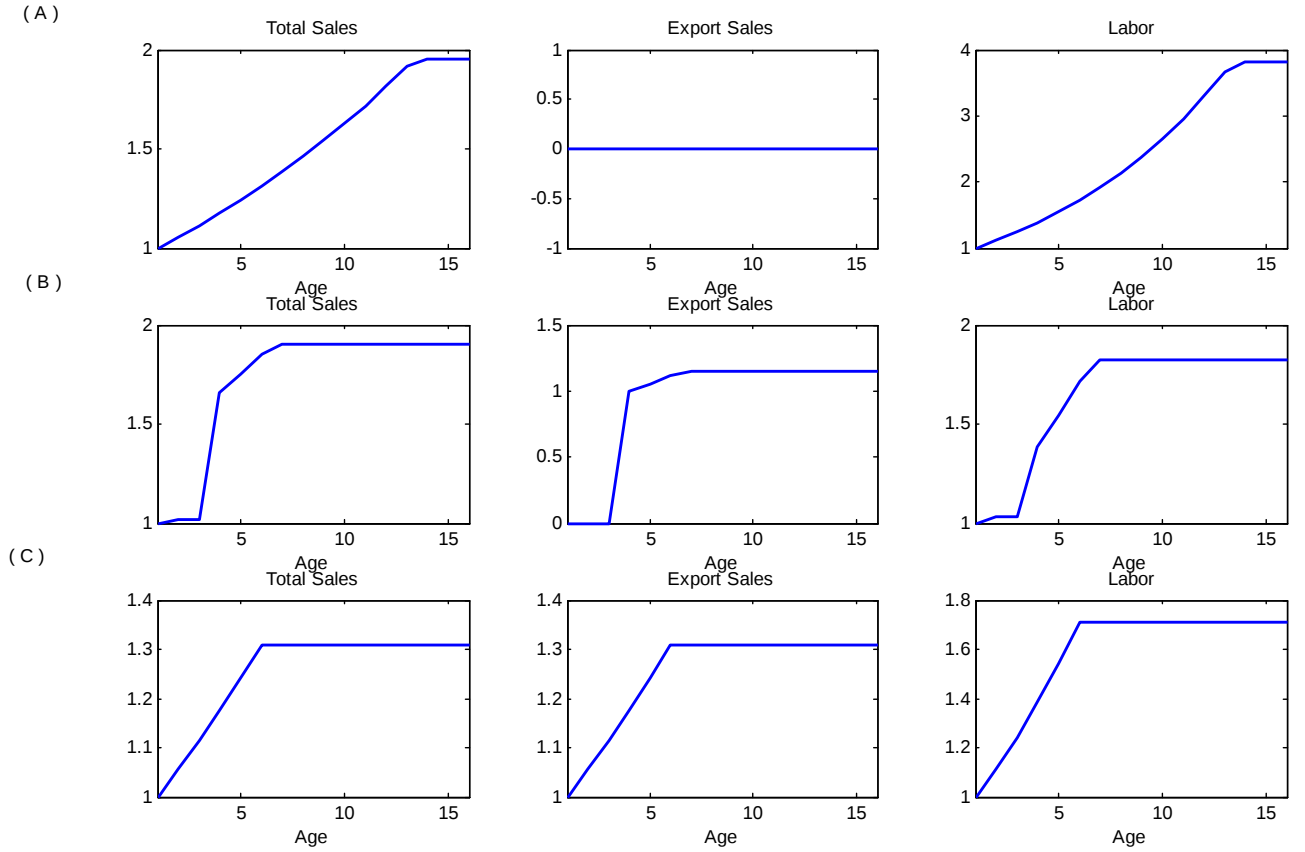


Figure 4. Change in the ratio $\frac{v_0}{v^*}$ from the high tariff steady state to the low tariff steady state.

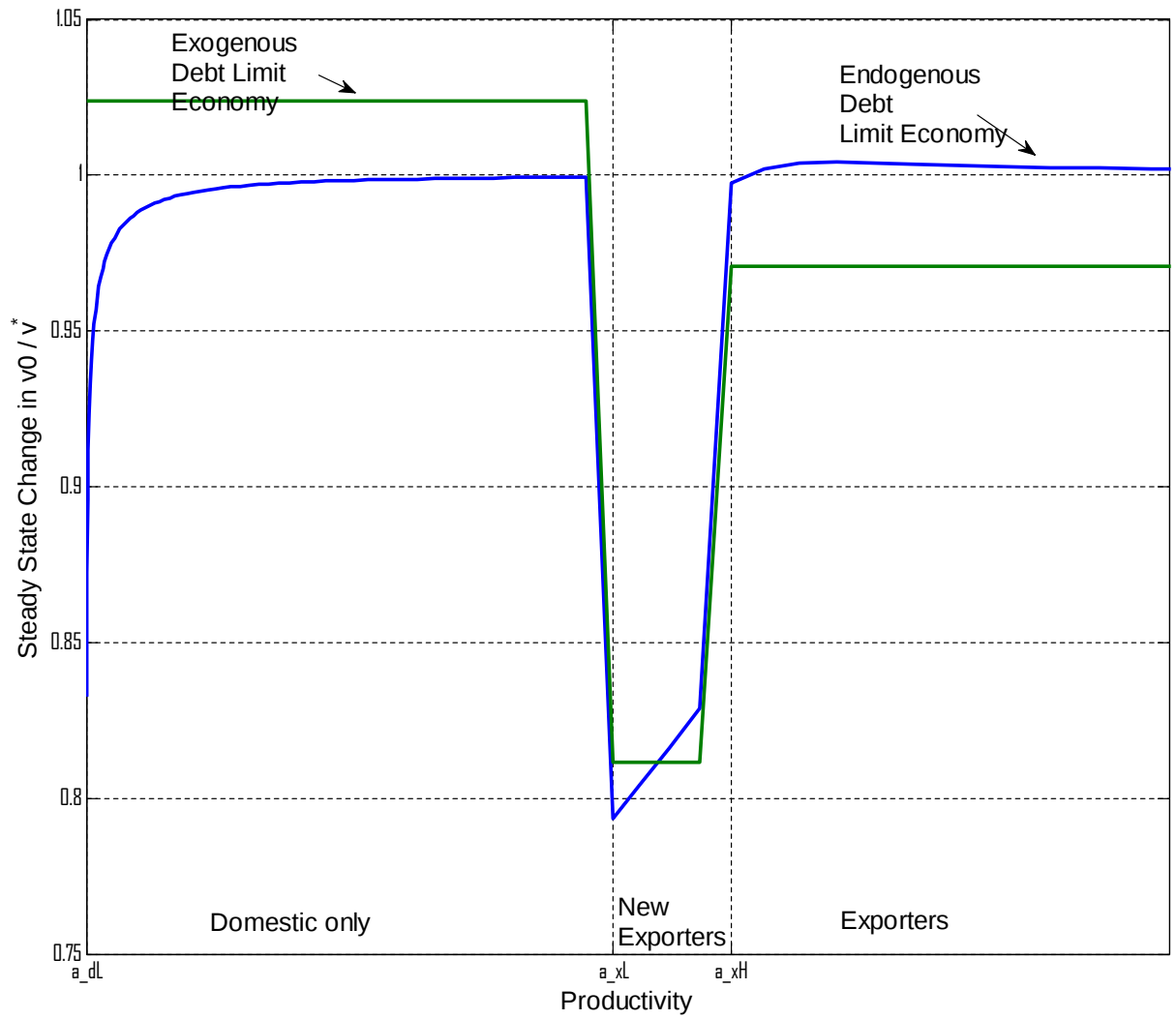


Figure 5. Transition with Linear Preferences

Green line : Steady State

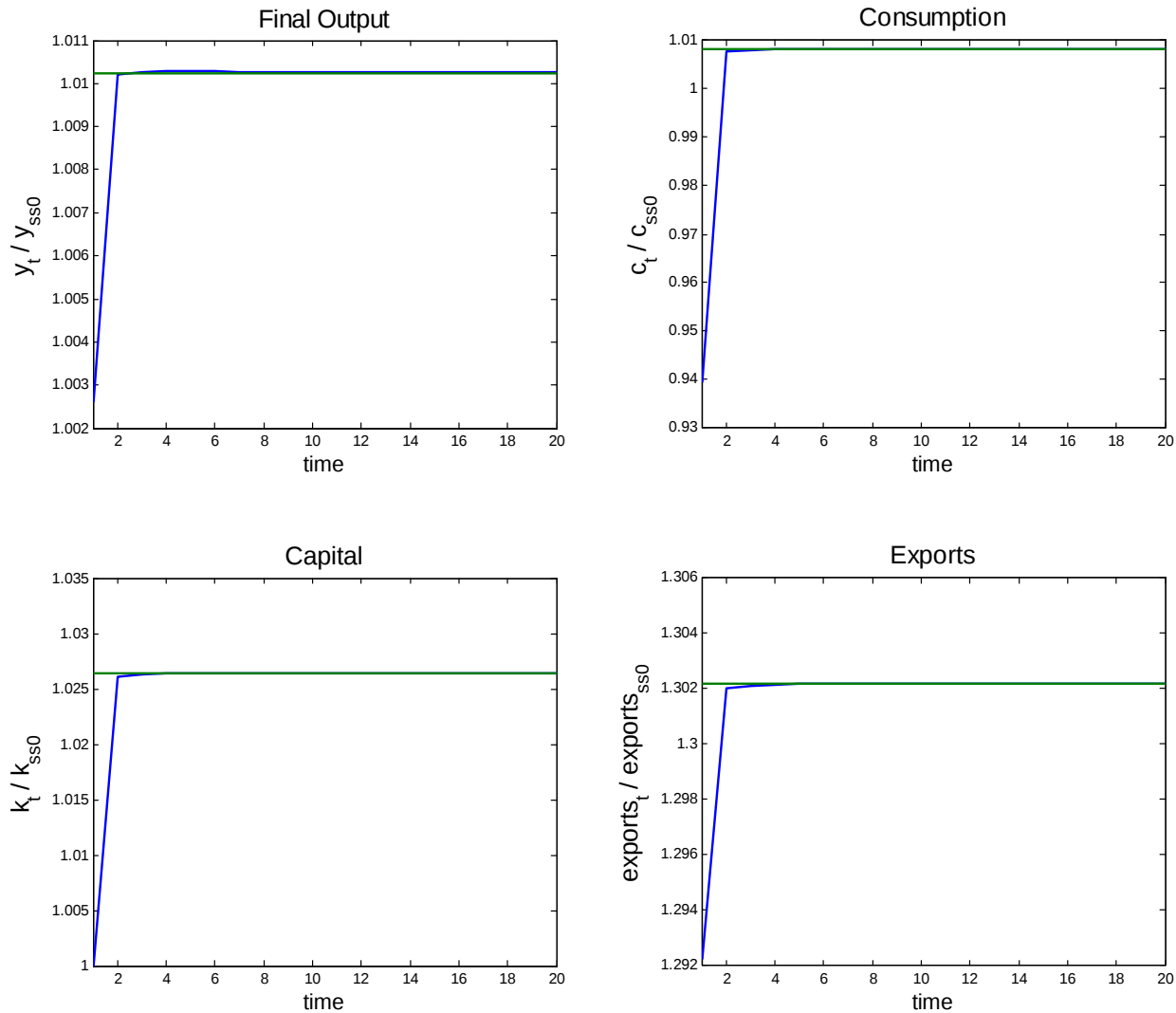


Figure 6. Transition with Log Preferences

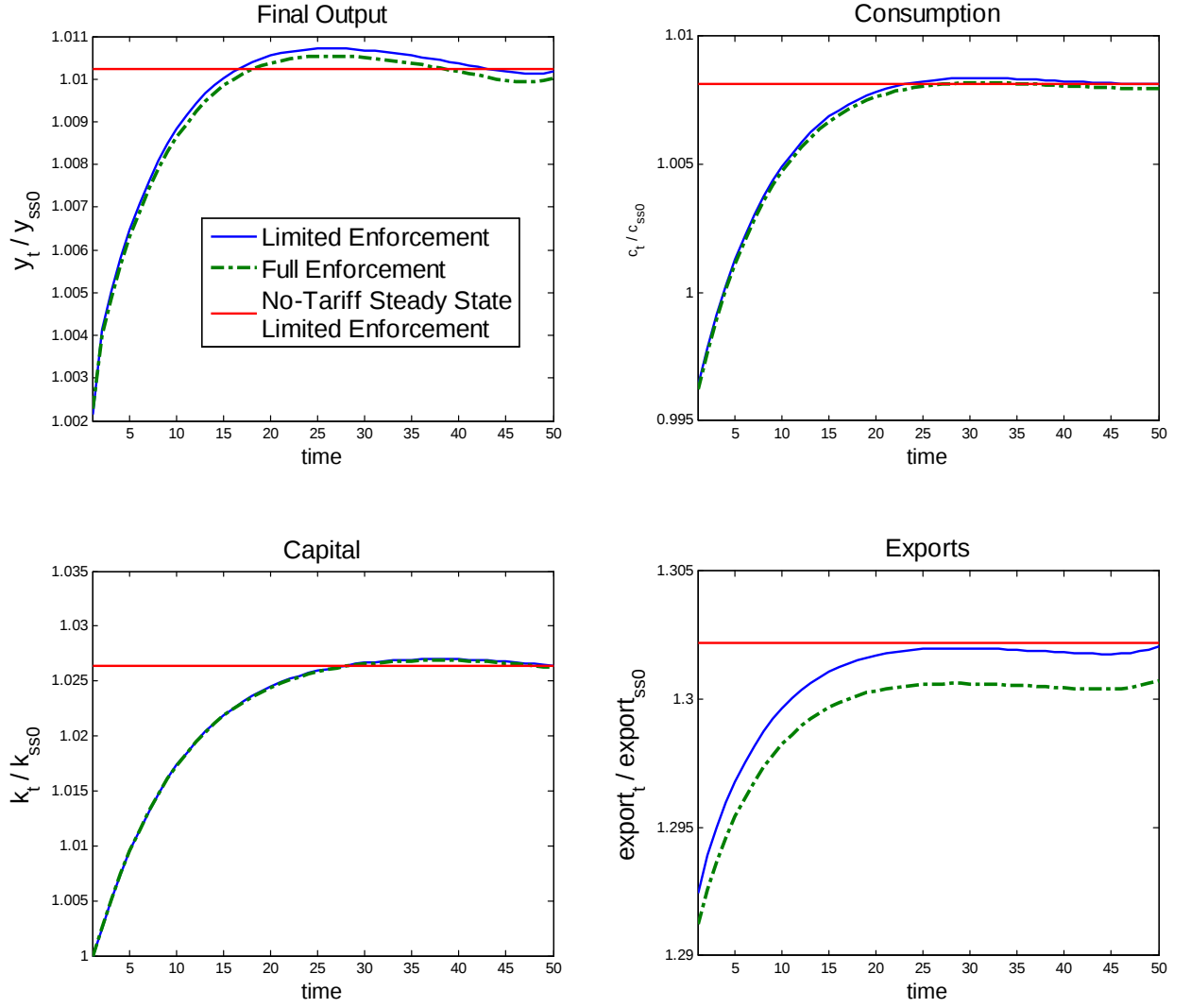


Figure 7. Sensitivity Analysis: Change in Steady State Consumption for a variety of parameters

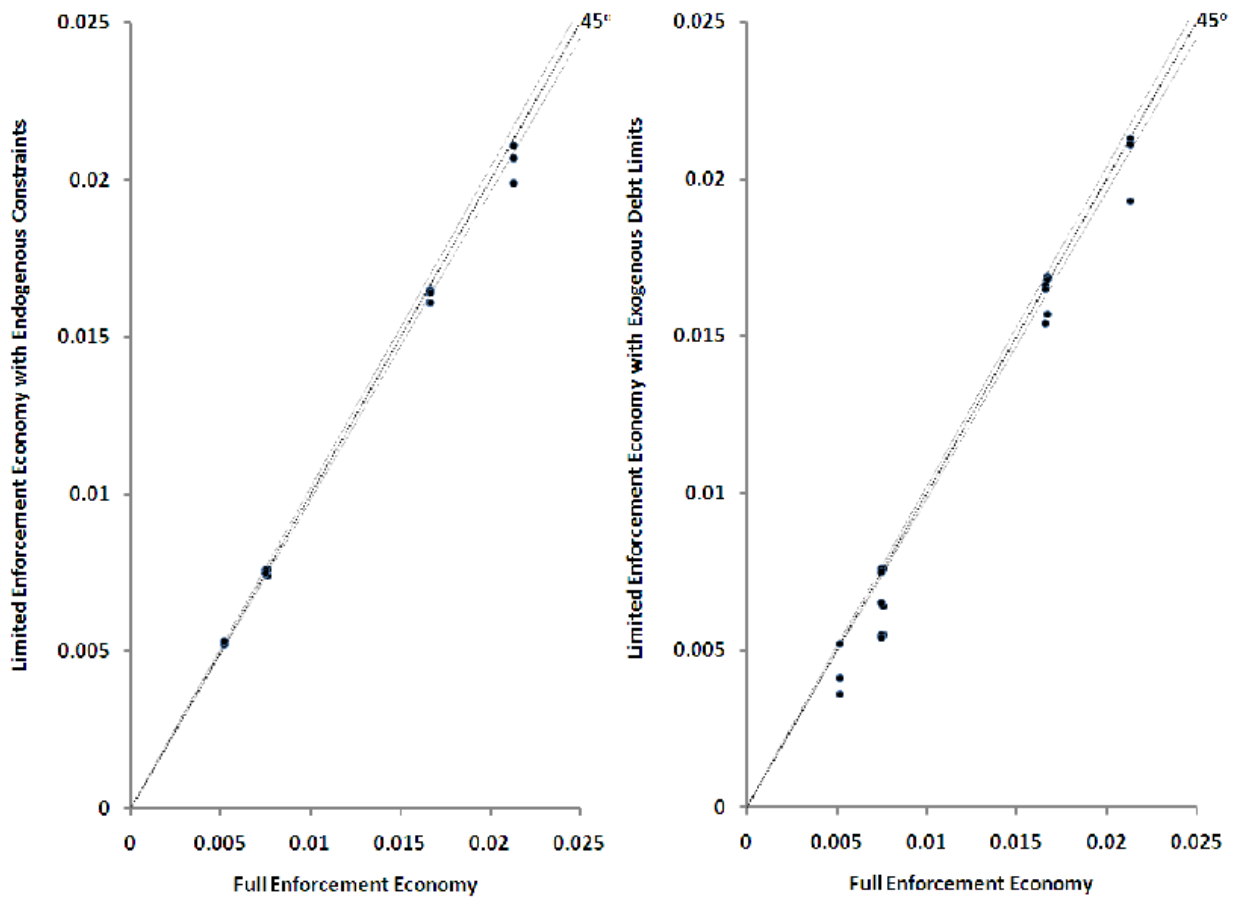


Figure 8. Steady State Consumption Levels with Endogenous Borrowing Constraints

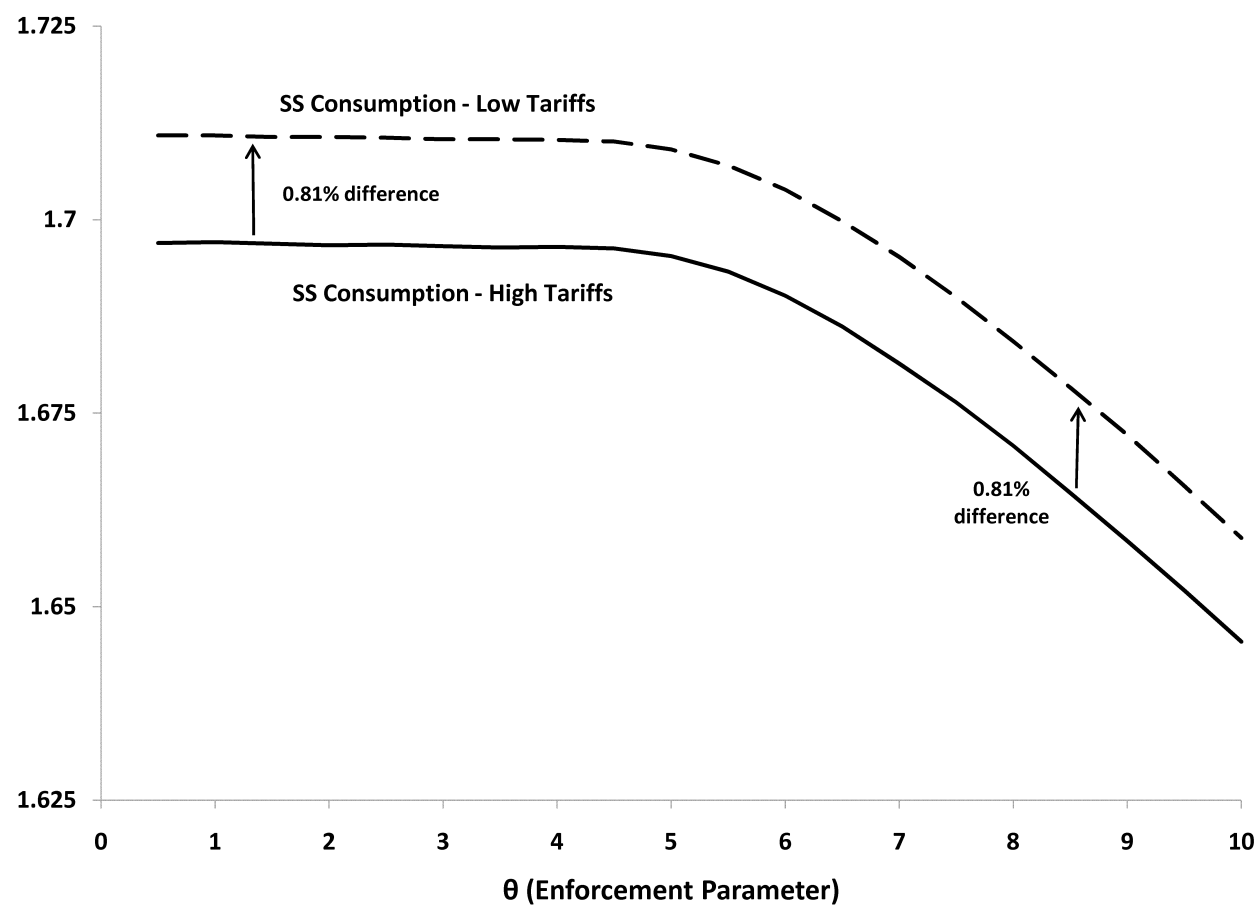


Figure 9. Steady State Consumption Levels with Exogenous Debt Limits

