Limited participation and International Risk-Sharing

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Abstract

The "consumption real exchange rate anomaly" is one of the most subborn puzzle in open economy macroeconomics. We make substantial progress by introducing limited participation in an otherwise standard model. In particular, one of our methodological contribution is to show how we can solve for the characteristics and the dynamics of optimal portfolios in such a setup.

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1 Introduction

In most of our open economy models relative consumptions and real exchange rate verify the following relation:

$$\left(\frac{C_{H,t}}{C_{F,t}}\right)^{-\sigma} = \frac{P_{H,t}}{P_{F,t}}$$

This relationship is strongly rejected by the data: this is the well known consumption real exchange rate anomaly (otherwise called the 'Kollmann-Backus-Smith' puzzle). For most developed countries, the correlation between relative consumption and the real exchange rate is close to zero or of the wrong sign (see Corsetti, Dedola and Leduc (2007) for recent evidence in the case of the US). This anomaly is stubborn: existing models with endogenous portfolio choice cannot break the link between consumption and real exchange rate, even with (locally) incomplete markets (due to a restricted number of assets or a large number of shocks; see Coeurdacier; Kollmann, Martin (2007), Benigno and Kucuk-Tuger (2008)).

In this paper, we propose that adding limited participation together with investment (or 'news') shocks can potentially help to solve the consumption real exchange rate anomaly, without the extreme assumption of either financial autarky or a unique international riskless bond[1] (see Corsetti, Dedola and Leduc (2007), Heathcote and Perri (2002), Kollmann (1996) and Baxter and Crucini (1995)). Limited participation seems to be a realistic feature of the data and allows us to introduce some market incompleteness, while investment (or 'news') shocks can generate potentially large wealth transfers and endogenous demand shocks. Kollmann (2009) introduces limited participation and hand to mouth consumers in a simple two-country static model. He shows that this brings the theory somewhat closer to the data. Our paper adds a new level of realism (and complexity) by introducing a dynamic setting and allowing non participants in the equity markets to have bank accounts (invest in bonds)

First we spell out intuitively how limited participation can help to account for the data when there are two types of shocks. Conditional on TFP shocks, it is very unlikely that limited participation will be enough to account for the consumption real exchange rate anomaly. A positive TFP shock at Home increases the relative wealth of non-participants (with respect to participants of both countries and to Foreign non-participants) and leads to an increase in Home consumption

[1] Note that with one riskless bond, accounting for the consumption real exchange rate anomaly is still difficult; one needs to assume very persistent TFP shocks.

(even more if the shock is very persistent). The increase in the supply of Home goods deteriorates the terms-of-trade and the Home real exchange rate depreciates (although less than in the complete markets case due to raise in Home demand induced by the increase in wealth of Home non-participants). Relative consumption and real exchange rate still moves in opposite direction (unless the wealth effect dominates such that despite an increase in the supply, the Home terms-of trade appreciates as in Corsetti, Dedola and Leduc (2007) which is unlikely.

However, conditionally on investment (or 'news') shocks, it is possible that the relative consumption of Home versus Foreign non-participants and the Home real exchange rate move in the same direction (note that the relative consumption of Home versus Foreign participants will always be linked by the Backus-Smith condition as participants face a complete menu of assets). Thus, the correlation between aggregate consumption and the real exchange rate could be of either sign, depending on the relative importance of both groups in aggregate consumption. Here is the mechanism: when investment becomes more productive at Home, the demand for Home goods raises and the terms-of-trade appreciates. While Home participants reduce consumption and make financial transfers to Foreign participants in these states, the consumption of Home non-participants increases: indeed, the appreciation of the terms-of-trade increases wages today at Home, while the increase in the Home capital stock increases Home wages tomorrow (and thus, because investment is financed by stockholders): as a consequence, Home non-participants makes large capital gains on their non-financial wealth and this increase in wealth translates in higher consumption today (note that they might also make capital gains on their local bonds depending the dynamic of the terms-of-trade). The size of this wealth effect will of course be larger 1) if shocks are highly persistent and 2) if the elasticity of substitution between Home and Foreign goods is not too low (indeed for an elasticity smaller than unity, the increase in production tomorrow translates into a fall of future wages). Finally, because the wealth increase of non-participants and the demand for Home goods for investment affects the terms-of-trade in the same direction, this will tend to make exchange rates much more volatile. We now turn to the details of the model.

2 The model

There are two countries, Home (H) and Foreign (F) (countries are symmetric *ex-ante*). Country $i = H, F$ produces one good using labor and capital and household consume both goods. There is trade in goods. Markets are perfectly competitive. Each country is populated by two types of

households: participants (denoted by p) and non-participants (denoted by np). Participants can freely trade any type of financial assets (Home and Foreign stocks and bonds) while non-participants are restricted to trade the local risk-free bond. The model economy is composed of two countries: the home country (H), and the foreign country (F). Each country produces one good using labor and capital. Markets are assumed to be perfectly competitive.

2.1 Households

The household sector is populated by two types of agents. Each type is composed of a large number of identical, infinitely lived consumers as described in Güvenen [36]. Stockholders, who are superscripted by p (for participants), are a constant share $\mu \in (0,1)$ of the totality of the households. The rest of the household do not have access to the stockmarket and hold only riskless bonds, hence the notation np (for non-participants).

Participant households The Bellman equation for the home country households that have access to the stockmarket is given by the following expression.

$$\begin{aligned} V^{H,p}(\mathbf{s}_t^{H,p}) \quad = \quad & \max_{\{\mathbf{c}_t^{H,p}\}} \left\{ \frac{(C_{H,t}^p)^{1-\sigma}}{1-\sigma} - \theta \frac{l_{H,t}^{1+\omega}}{1+\omega} + \beta E_t\big[V^{H,p}\big(\mathbf{s}_{t+1}^{H,p}\big)\big] \right\} \\ s.t. \quad & w_{H,t} l_{H,t} + (p_{H,t}^S + d_{H,t}) S_{H,t}^H + (p_{F,t}^S + d_{F,t}) S_{F,t}^H \\ & + (p_{H,t}^b + p_{H,t}) b_{H,t}^{H,p} + (p_{F,t}^b + p_{F,t}) b_{F,t}^{H,p} \\ & - P_{H,t} C_{H,t}^p - p_{H,t}^S S_{H,t+1}^H - p_{F,t+1}^S S_{F,t+1}^H \\ & - p_{F,t}^b b_{F,t+1}^{H,p} - p_{H,t}^b b_{H,t+1}^{H,p} \geq 0 \end{aligned} \tag{1}$$

where C_H^p, l_H, w_H denote respectively the participant household's consumption, leisure and wage, and

$$\begin{aligned} \mathbf{s}_t^{H,p} \quad &= \quad \{S_{H,t}^H, b_{H,t}^{H,np}, b_{H,t}^{H,p}, k_t^H, k_t^F, z_t^H, z_t^F, \chi_t^H, \chi_t^F\} \\ \mathbf{c}_t^{H,p} \quad &= \quad \{C_{H,t}^p, l_{H,t}, S_{H,t+1}^H, b_{H,t+1}^{H,p}\} \end{aligned}$$

are the state and control variables of the participant household's dynamic optimization problem. In this notation, we already take into account the symmetry condition, which will be discussed later. We denote by $S_{j,t+1}^i$ the number of shares of stock j held by the country i participants at the end of period, while $b_{j,t+1}^{i,\mathcal{K}}$ represents claims held by i (at the end of period t) to future unconditional payments of good j held by household of type $\mathcal{K} = \{p, np\}$. An additional assumption is that

non-participants cannot trade foreign bonds, hence $b^{i,np}_{j,t} = 0$ for $j \neq i$. The structural parameters σ, $\omega \geq 0$, θ, and β are respectively the risk aversion, the inverse of the Frisch elasticity of labor supply, the preference weight of leisure in utility[2] and the discount factor.

Similarly foreign country households that participate to the stockmarket have the following dynamic optimization problem

$$\begin{aligned} V^{F,p}(\mathbf{s}^{F,p}_t) \quad = \quad & \max_{\{\mathbf{c}^{F,p}_t\}} \left\{ \frac{(C^p_{F,t})^{1-\sigma}}{1-\sigma} - \theta \frac{l^{1+\omega}_{F,t}}{1+\omega} + \beta E_t \left[V^{F,p}(\mathbf{s}^{F,p}_{t+1}) \right] \right\} \\ s.t. \quad & w_{F,t} l_{F,t} + (p^S_{F,t} + d_{F,t}) S^F_{F,t} + (p^S_{H,t} + d_{H,t}) S^F_{H,t} \\ & + (p^b_{F,t} + p_{F,t}) b^{F,p}_{F,t} + (p^b_{H,t} + p_{H,t}) b^{F,p}_{H,t} \\ & - P_{F,t} C^p_{F,t} - p^S_{F,t} S^F_{F,t+1} - p^S_{H,t} S^F_{H,t+1} \\ & - p^b_{H,t} b^{F,p}_{H,t+1} - p^b_{F,t} b^{F,p}_{F,t+1} \geq 0 \end{aligned} \tag{2}$$

where $\mathbf{s}^{F,p}_t = \mathbf{s}^{H,p}_t$ and $\mathbf{c}^{F,p}_t = \{C^p_{F,t}, l_{F,t}, S^F_{F,t}, b^{F,p}_{F,t+1}\}$.

Non-participant households Households that do not have access to the stockmarket have the same preferences as the participant households. However due to their inability to buy and sell stocks, they will have a slightly modified budget constraint. Non-participant households from the home country will have the following optimization program

$$\begin{aligned} V^{H,np}(\mathbf{s}^{H,np}_t) \quad = \quad & \max_{\{\mathbf{c}^{H,np}_t\}} \left\{ \frac{(C^{np}_{H,t})^{1-\sigma}}{1-\sigma} - \theta \frac{l^{1+\omega}_{H,t}}{1+\omega} + \beta E_t \left[V^{H,np}(\mathbf{s}^{H,np}_{t+1}) \right] \right\} \\ s.t. \quad & w_{H,t} l_{H,t} + (p^b_{H,t} + p_{H,t}) b^{H,np}_{H,t} - P_{H,t} C^{np}_{H,t} - p^b_{H,t} b^{H,np}_{H,t+1} \geq 0 \end{aligned} \tag{3}$$

where $\mathbf{s}^{H,np}_t = \mathbf{s}^{H,p}_t$ and $\mathbf{c}^{H,np}_t = \{C^{np}_{H,t}, l_{H,t}, b^{H,np}_{H,t+1}\}$.

Foreign and non-participant households have the following dynamic optimization program

$$\begin{aligned} V^{F,np}(\mathbf{s}^{F,np}_t) \quad = \quad & \max_{\{\mathbf{c}^{F,np}_t\}} \left\{ \frac{(C^{np}_{F,t})^{1-\sigma}}{1-\sigma} - \theta \frac{l^{1+\omega}_{F,t}}{1+\omega} + \beta E_t \left[V^{F,np}(\mathbf{s}^{F,np}_{t+1}) \right] \right\} \\ s.t. \quad & w_{F,t} l_{F,t} + (p^b_{F,t} + p_{F,t}) b^{F,np}_{F,t} - P_{F,t} C^{np}_{F,t} - p^b_{F,t} b^{F,np}_{F,t+1} \geq 0 \end{aligned} \tag{4}$$

where $\mathbf{s}^{F,np}_t = \mathbf{s}^{H,p}_t$ and $\mathbf{c}^{F,np}_t = \{C^{np}_{F,t}, l_{F,t}, b^{F,np}_{F,t+1}\}$.

2.2 Firms

Each country is characterized by a representative firm that uses the same type of Cobb-Douglas technology. In the current version there are no investment adjustment costs, hence the problem

[2]This parameter will be used to calibrate the steady state value for l

that the representative firm faces is static. If we include investment adjustment costs the problem would be dynamic. The firm chooses inputs of labour, $l_{H,t}$, capital $k_{H,t}$ and investment, $I_{H,t}$ to solve the following expression, which gives the the general form of the dynamic optimization program of the firm

$$W^H(\mathbf{s}_t^H) = \max_{\{\mathbf{c}_t^H\}} \left\{ y_{H,t} - w_{H,t} l_{H,t} - I_{H,t} + E_t \left[\varphi_{t,t+1}^{H,p} W^H(\mathbf{s}_{t+1}^H) \right] \right\} \tag{5}$$

where

$$\begin{aligned} \mathbf{s}_t^H &= \mathbf{s}_t^{H,p} \\ \mathbf{c}_t^H &= \{I_{H,t}, w_{H,t}\} \end{aligned}$$

collects respectively the state and control variables of the firms dynamic program. This problem is solved subject to the production technology used by the firm and given by

$$y_{H,t} = z_{H,t} k_{H,t}^{\alpha} l_{H,t}^{1-\alpha}$$

and the law of motion for the capital stock, which is given by

$$\chi_{H,t} I_{H,t} = k_{H,t+1} - (1-\delta) k_{H,t}$$

where $\chi_{H,t}$ is an investment-specific shock in country H.

The pricing kernel of the participant households used at date t to value date $t+1$ payoffs of firms, is given by

$$\varphi_{t,t+1}^{H,p} \equiv \beta E_t \left[\left(\frac{C_{H,t+1}^p}{C_{H,t}^p} \right)^{-\sigma} \left(\frac{P_{H,t}}{P_{H,t+1}} \right) \right]$$

We assume that $\varphi_{t,t+1}^{H,p}$ equals the intertemporal marginal rate of substitution of the home country participants since they are holding the stocks.

Similarly the dynamic problem for the foreign country's firm is given by

$$W^F(\mathbf{s}_t^H) = \max_{\{\mathbf{c}_t^H\}} \left\{ y_{F,t} - w_{F,t} l_{F,t} - I_{F,t} + E_t \left[\varphi_{t,t+1}^{F,p} W^F(\mathbf{s}_{t+1}^F) \right] \right\} \tag{6}$$

with

$$\begin{aligned} y_{F,t} &= z_{F,t} k_{F,t}^{\alpha} l_{F,t}^{1-\alpha} \\ \chi_{F,t} I_{F,t} &= k_{F,t+1} - (1-\delta) k_{F,t} \\ \varphi_{t,t+1}^{F,p} &\equiv \beta E_t \left[\left(\frac{C_{F,t+1}^{p}}{C_{F,t}^{p}} \right)^{-\sigma} \left(\frac{P_{F,t}}{P_{F,t+1}} \right) \right] \end{aligned}$$

Table (1) summarizes all the variables in the model by specifying the control and state variables.

Market-clearing in goods and asset markets and symmetry conditions The market-clearing conditions are given by;

$$\begin{aligned} c_{H,t}^{H,p} + c_{H,t}^{F,p} + c_{H,t}^{H,np} + c_{H,t}^{F,np} + i_{H,t}^{H} + i_{H,t}^{F} &= y_{H,t} \\ c_{F,t}^{F,p} + c_{F,t}^{H,p} + c_{F,t}^{F,np} + c_{F,t}^{H,np} + i_{F,t}^{F} + i_{F,t}^{H} &= y_{F,t} \\ S_{H,t}^{H} + S_{H,t}^{F} &= 1 \\ S_{F,t}^{F} + S_{F,t}^{H} &= 1 \\ b_{H,t}^{H,p} + b_{H,t}^{H,np} + b_{H,t}^{F,p} &= b_H \\ b_{F,t}^{F,p} + b_{F,t}^{F,np} + b_{F,t}^{H,p} &= b_F \end{aligned}$$

Together with the symmetry conditions, we get

$$\begin{aligned} S_F^H &= S_H^F \\ S_F^F &= S_H^H \\ b_F^{F,np} &= b_H^{H,np} \\ b_F^{H,p} &= b_H^{F,p} \\ b_F^{F,p} &= b_H^{H,p} \end{aligned}$$

which significantly reduces the dimension of the state space of the model.

2.3 Dynamic equations

The (non linear) dynamic equations together with the market clearing conditions are given by the following equations.

The welfare based consumer price index is given by

$$\begin{aligned} P_{H,t} &= \left[a(p_{H,t})^{1-\phi} + (1-a)(p_{F,t})^{1-\phi}\right]^{\frac{1}{1-\phi}} \\ P_{F,t} &= \left[a(p_{F,t})^{1-\phi} + (1-a)(p_{H,t})^{1-\phi}\right]^{\frac{1}{1-\phi}} \end{aligned} \tag{7}$$

The investment price index defined as

$$\begin{aligned} P^I_{H,t} &= \left[a_I(p_{H,t})^{1-\phi_I} + (1-a_I)(p_{F,t})^{1-\phi_I}\right]^{\frac{1}{1-\phi_I}} \\ P^I_{F,t} &= \left[a_I(p_{F,t})^{1-\phi_I} + (1-a_I)(p_{H,t})^{1-\phi_I}\right]^{\frac{1}{1-\phi_I}} \end{aligned} \tag{8}$$

with

$$\begin{aligned} 1 &= P_{H,t} + P_{F,t} \\ q_t &= \frac{P_{H,t}}{P_{F,t}} \end{aligned} \tag{9}$$

In period t, country $i = \{H, F\}$ produces $y_{i,t}$ units of good i according to the production function

$$y_{i,t} = z_{i,t}(k_{i,t})^{\alpha}(l_{i,t})^{1-\alpha} \tag{10}$$

Total factor productivity, $z_{i,t}$, is an exogeneous random variable, given by

$$\log(z_{i,t+1}) = \rho_z \log(z_{i,t}) + \varepsilon^z_{i,t+1} \tag{11}$$

The exogeneous shock to investment efficiency is a positive random variable given by

$$\log(\chi_{i,t+1}) = \rho_\chi \log(\chi_{i,t}) + \varepsilon^\chi_{i,t+1} \tag{12}$$

The law of motion of the capital stock is

$$k_{i,t+1} = (1-\delta)k_{i,t} + I_{i,t}\chi_{i,t} \tag{13}$$

Country $i = H, F$ wage bill is given by

$$w_{i,t}l_{i,t} = (1-\alpha)p_{i,t}y_{i,t} \tag{14}$$

A share α of the country's output, net of physical investment spending is paid as a dividend to shareholders

$$d_{i,t} = \alpha p_{i,t}y_{i,t} - P^I_{i,t}I_{i,t} \tag{15}$$

Households who participate in the stock market in the respective countries face the following budget constraint

$$\begin{aligned}
0 &= \mu w_{H,t} l_{H,t} + (p^S_{H,t} + d_{H,t}) S^H_{H,t} + (p^S_{F,t} + d_{F,t}) S^H_{F,t} \\
&\quad + (p^b_{H,t} + p_{H,t}) b^{H,p}_{H,t} + (p^b_{F,t} + p_{F,t}) b^{H,p}_{F,t} \\
&\quad - p^S_{H,t} S^H_{H,t+1} - p^S_{F,t} S^H_{F,t+1} - p^b_{F,t} b^{H,p}_{F,t+1} - p^b_{H,t} b^{H,p}_{H,t+1} \\
&\quad - P_{H,t} C^p_{H,t} \\
0 &= \mu w_{F,t} l_{F,t} + (p^S_{F,t} + d_{F,t}) S^F_{F,t} + (p^S_{H,t} + d_{H,t}) S^F_{H,t} \\
&\quad + (p^b_{F,t} + p_{F,t}) b^{F,p}_{F,t} + (p^b_{H,t} + p_{H,t}) b^{F,p}_{H,t} \\
&\quad - p^S_{F,t} S^F_{F,t+1} - p^S_{H,t} S^F_{H,t+1} - p^b_{H,t} b^{F,p}_{H,t+1} - p^b_{F,t} b^{F,p}_{F,t+1} \\
&\quad - P_{F,t} C^p_{F,t}
\end{aligned} \tag{16}$$

The pricing kernel used at date t to value $t+1$ payoffs is defined as

$$\begin{aligned}
\varphi^{H,np}_{t,t+1} &\equiv \beta E_t \left[\frac{C^{np}_{H,t+1}}{C^{np}_{H,t}} \right]^{-\sigma} \left[\frac{P_{H,t}}{P_{H,t+1}} \right] \\
\varphi^{F,np}_{t,t+1} &\equiv \beta E_t \left[\frac{C^{np}_{F,t+1}}{C^{np}_{F,t}} \right]^{-\sigma} \left[\frac{P_{F,t}}{P_{F,t+1}} \right] \\
\varphi^{H,p}_{t,t+1} &\equiv \beta E_t \left[\frac{C^{p}_{H,t+1}}{C^{p}_{H,t}} \right]^{-\sigma} \left[\frac{P_{H,t}}{P_{H,t+1}} \right] \\
\varphi^{F,p}_{t,t+1} &\equiv \beta E_t \left[\frac{C^{p}_{F,t}}{C^{p}_{F,t}} \right]^{-\sigma} \left[\frac{P_{F,t}}{P_{F,t+1}} \right]
\end{aligned} \tag{17}$$

Household who do not participate in the stock market in the respective countries face the following budget constraint

$$\begin{aligned}
0 &= (1-\mu) w_{H,t} l_{H,t} + (p^b_{H,t} + p_{H,t}) b^{H,np}_{H,t} - P_{H,t} C^{np}_{H,t} - p^b_{H,t} b^{H,np}_{H,t+1} \\
0 &= (1-\mu) w_{F,t} l_{F,t} + (p^b_{F,t} + p_{F,t}) b^{F,np}_{F,t} - P_{F,t} C^{np}_{F,t} - p^b_{F,t} b^{F,np}_{F,t+1}
\end{aligned} \tag{18}$$

The firms' first order condition is given by

$$\begin{aligned}
\frac{P^I_{H,t}}{\chi_{H,t}} &= E_t \varphi^{H,p}_{t,t+1} \left[p_{H,t+1} \alpha \frac{y_{H,t+1}}{k_{H,t+1}} + (1-\delta) \frac{P^I_{H,t+1}}{\chi_{H,t+1}} \right] \\
\frac{P^I_{F,t}}{\chi_{F,t}} &= E_t \varphi^{F,p}_{t,t+1} \left[p_{F,t+1} \alpha \frac{y_{F,t+1}}{k_{F,t+1}} + (1-\delta) \frac{P^I_{F,t+1}}{\chi_{F,t+1}} \right]
\end{aligned} \tag{19}$$

The Euler equations (for the participating agents) with respect to the two stocks, and the Home and Foreign good bonds is given by

$$\begin{aligned} 1 &= E_t\left[\varphi^{F,p}_{t,t+1} R^S_{H,t+1}\right] \\ 1 &= E_t\left[\varphi^{F,p}_{t,t+1} R^b_{H,t+1}\right] \\ 1 &= E_t\left[\varphi^{H,p}_{t,t+1} R^S_{F,t+1}\right] \\ 1 &= E_t\left[\varphi^{H,p}_{t,t+1} R^b_{F,t+1}\right] \end{aligned} \tag{20}$$

$R^S_{i,t+1}$ and $R^b_{i,t+1}$, for $i = \{H, F\}$ are the gross returns of stock i, and of the ggod-i bond, respectively

$$\begin{aligned} R^S_{H,t+1} &= \frac{p^S_{H,t+1} + d_{H,t+1}}{p^S_{H,t}} \\ R^S_{F,t+1} &= \frac{p^S_{F,t+1} + d_{F,t+1}}{p^S_{F,t}} \\ R^b_{H,t+1} &= \frac{p^b_{H,t+1} + p_{H,t+1}}{p^b_{H,t}} \\ R^b_{F,t+1} &= \frac{p^b_{F,t+1} + p_{F,t+1}}{p^b_{F,t}} \end{aligned} \tag{21}$$

The Euler equations (for the non-participating agents) with respect to the Home and Foreign good bonds is given by

$$\begin{aligned} 1 &= E_t\left[\varphi^{H,np}_{t,t+1} R^b_{H,t+1}\right] \\ 1 &= E_t\left[\varphi^{F,np}_{t,t+1} R^b_{F,t+1}\right] \end{aligned} \tag{22}$$

From the cost minimization problem, we get

$$\begin{aligned} i^H_{H,t} &= a_I \left[\frac{p_{H,t}}{P^I_{H,t}}\right]^{-\phi_I} I_{H,t} \\ i^F_{F,t} &= a_I \left[\frac{p_{F,t}}{P^I_{F,t}}\right]^{-\phi_I} I_{F,t} \\ i^H_{F,t} &= (1-a_I) \left[\frac{p_{F,t}}{P^I_{H,t}}\right]^{-\phi_L} I_{H,t} \\ i^F_{H,t} &= (1-a_I) \left[\frac{p_{H,t}}{P^I_{F,t}}\right]^{-\phi_I} I_{F,t} \end{aligned} \tag{23}$$

From the first order conditions of the households' decision problems, we get

$$\begin{aligned}
c_{H,t}^{H,p} &= a\left[\frac{p_{H,t}}{P_{H,t}}\right]^{-\phi} C_{H,t}^{p} \\
c_{H,t}^{H,np} &= a\left[\frac{p_{H,t}}{P_{H,t}}\right]^{-\phi} C_{H,t}^{np} \\
c_{F,t}^{F,p} &= a\left[\frac{p_{F,t}}{P_{F,t}}\right]^{-\phi} C_{F,t}^{p} \\
c_{F,t}^{F,np} &= a\left[\frac{p_{F,t}}{P_{F,t}}\right]^{-\phi} C_{F,t}^{np} \\
c_{F,t}^{H,p} &= (1-a)\left[\frac{p_{F,t}}{P_{H,t}}\right]^{-\phi} C_{H,t}^{p} \\
c_{F,t}^{H,np} &= (1-a)\left[\frac{p_{F,t}}{P_{H,t}}\right]^{-\phi} C_{H,t}^{np} \\
c_{H,t}^{F,p} &= (1-a)\left[\frac{p_{H,t}}{P_{F,t}}\right]^{-\phi} C_{F,t}^{p} \\
c_{H,t}^{F,np} &= (1-a)\left[\frac{p_{H,t}}{P_{F,t}}\right]^{-\phi} C_{F,t}^{np}
\end{aligned} \tag{24}$$

Market clearing conditions are given by

$$\begin{aligned}
c_{H,t}^{H,p} + c_{H,t}^{F,p} + c_{H,t}^{H,np} + c_{H,t}^{F,np} + i_{H,t}^{H} + i_{H,t}^{F} &= y_{H,t} \\
c_{F,t}^{F,p} + c_{F,t}^{H,p} + c_{F,t}^{F,np} + c_{F,t}^{H,np} + i_{F,t}^{F} + i_{F,t}^{H} &= y_{F,t} \\
S_{H,t}^{H} + S_{H,t}^{F} &= 1 \\
S_{F,t}^{F} + S_{F,t}^{H} &= 1 \\
b_{H,t}^{H,p} + b_{H,t}^{H,np} + b_{H,t}^{F,p} &= b_H \\
b_{F,t}^{F,p} + b_{F,t}^{F,np} + b_{F,t}^{H,p} &= b_F
\end{aligned} \tag{25}$$

[3]

[4]

2.4 Parametrization

2.4.1 Standard structural parameters.

The share, a, of consumption spending devoted to the local good is set to 0.85. The share of capital in the production function, α, is set to 0.4. The depreciation rate of capital, δ, is 0.1. The discount factor, β, is set to 0.96. The inverse of the Frisch elasticity parameter is 0.5. The local bias in investment spending is given by $a_I = 0.85$. In the current version, labor supply is exogeneous and equal to one, with $l_1 = l_2 = 1$.

2.4.2 Stock market participation rate.

Finally the share of the households that participate in the stock market is given by μ. The parametrization of μ is fixed following the work by Guiso et al. [35] who present the stock holdings in different countries (see Table 3).

2.4.3 Sensitivity analysis

The following parameters will take different values over the sensitivity analysis: the coefficient of relative risk aversion, σ, the elasticity of substitution between the 2 goods, ϕ, the elasticity of substitution between domestic and imported investment inputs, ϕ_I, the standard error of the exogeneous $i.i.d.$-shock in the autoregressive investment shock, σ_χ, the standard error of the exogeneous $i.i.d.$-shock in the autoregressive productivity process, σ_z, the persistence parameter in the autoregressive investment shock, ρ_χ, the persistence parameter in the autoregressive productivity process, ρ_z, the correlation in the productivity shocks between the two countries, $\sigma_{z_1 z_2}$ and the correlation in the investment specific shocks between the two countries, $\sigma_{\chi_1 \chi_2}$.

2.4.4 (Constant) Portfolio holdings

We present one version of the model in which portfolio holdings (S_H^H, $b_H^{H,np}$ and $b_H^{H,p}$) are assumed to be constant over time. As a consequence they will be treated as parameters of the model. However the value of these parameters will be determined endogeneously unlike the rest of the parameters that are set exogeneously.

In the following sections we will introduce the simplifying assumptions for solving this model. These include the assumption on symmetry, risk aversion and elasticity of consumption. The solutions to the model (for different parametrizations) are given in Table (**??**). As it will be shown later, these simplifying assumptions will significantly reduce the dimension of the state space from 12 to 6.

Symmetry The assumption about symmetry allows us to reduce the dimension of the space on which the portfolio choice is defined. This means that for a given S_H^H, we determine the following

stockholdings

$$\begin{aligned} S_H^F &= 1 - S_H^H \\ S_F^H &= 1 - S_H^H \\ S_F^F &= S_H^H \end{aligned} \tag{26}$$

For a given bond holding $b_H^{F,p}$, we can determine

$$b_F^{H,p} = b_H^{F,p} \tag{27}$$

Similarly for a given $b_H^{H,np}$, we derive

$$\begin{aligned} b_H^{H,p} &= -(b_H^{F,p} + b_H^{H,np}) \\ b_F^{F,p} &= b_H^{H,p} \\ b_F^{F,np} &= b_H^{H,np} \end{aligned} \tag{28}$$

This has allowed us to reduce the dimension in the *parameter*-space (since the portfolio is treated as parameters) from 10 to 3.

Closed-form solution for the portfolio – Case 1: $\sigma = 1$ and $\phi = 1$ In the previous subsection we have seen how the symmetry condition reduced the dimension of the space on which the portfolio choice was defined. In the special case of log-utility ($\sigma = 1$) and with an elasticity for consumption parameter set equal to 1 ($\phi = 1$), an analytical solution can be found for the stock holding of the households. This result is shown in Coeurdacier, Kollmann and Martin [16] for the case in which all households participate in the stock market. The adapted expression for the case with limited participation is given by

$$S_H^H = \frac{1}{2}\left[\frac{(2a-1)\mu(1-\alpha)}{1-(2a-1)\left[(1-\mu)(1-\alpha)+\alpha\right]} + 1\right] \tag{29}$$

Once we have defined S_H^H, we can find the rest of the portfolio components using the following expressions

$$\begin{aligned} S_H^F &= 1 - S_H^H \\ S_F^H &= S_H^F \\ S_F^F &= S_H^H \end{aligned} \tag{30}$$

For a given $b_H^{H,np}$, we can find

$$\begin{aligned} b_H^{H,p} &= -\frac{b_H^{H,np}}{2}\left[\frac{1-(2a-1)}{1-(2a-1)[(1-\mu)(1-\alpha)+\alpha]}\right] \\ b_H^{F,p} &= -\left(b_H^{H,p}+b_H^{H,np}\right) \\ b_F^{F,np} &= b_H^{H,np} \\ b_F^{H,p} &= b_H^{F,p} \\ b_F^{F,p} &= b_H^{H,p} \end{aligned} \tag{31}$$

and

$$b_H^{H,np} = (b_H^{F,p}-b_H^{H,p})\cdot\left(\frac{(\mu(1-\alpha)+(2S-1)\alpha)(2a-1)}{1-(1-\mu)(1-\alpha)(2a-1)}\right)^{-1} \tag{32}$$

Numerical solution for the portfolio – Case 2: $\sigma \neq 1$ and $\phi \neq 1$ As soon as we go beyond the log-utility case, we will not be able to derive a closed-form solution for any portfolio component. In order to solve the model with the optimal portfolio choice in this case, we need to rely on a numerical and iterative algorithm which is based on the Euler equation error.

In this case expressions (29) and (31) cannot be used. We will instead discretize the space on which the portfolios are defined and use choose the optimal (constant) portfolio choice such that the Euler equation residuals are minimized[5].

3 Solving the model

This section presents the solution method that we have adopted to solve the model which has been previously exposed. We start by presenting the general approach and illustrate the method using a simple small-dimensional model.

3.1 General solution method

Let us assume that we solve the previous model using standard projection methods. This would imply that the decision rules are defined as a linear combination of a finite set of basis functions. The coefficients associated with each basis function are determined following the projection method that we have chosen (i.e., collocation method, Galerkin method).

[5] The general idea here is to discretize the space on which portfolios are defined and to solve the model for each combination of the portfolio. In order to determine the optimal choice among the set of feasible portfolio choices, we choose the configuration that minimizes the Euler equation error.

Using the notation for the decision rules as in Heathcote and Perri [40] we end up looking for the following functions

$$\begin{aligned} \lambda_{H,t+1} &= g_1\left(\lambda_{H,t}, \lambda_{F,t}, X_t\right) \\ \lambda_{F,t+1} &= g_2\left(\lambda_{H,t}, \lambda_{F,t}, X_t\right) \\ X_{t+1} &= g_3\left(\lambda_{H,t}, \lambda_{F,t}, X_t, \varepsilon_{t+1}\right) \\ Y_t &= g_4\left(\lambda_{H,t}, \lambda_{F,t}, X_t\right) \end{aligned} \tag{33}$$

where $\lambda_H,\ \lambda_F \in \mathbb{R}^{n_1}$ are the variables of local portfolio components held by home and foreign residents, respectively. $X \in \mathbb{R}^n$ is the vector of non portfolio state variables (i.e. productivities, investment-specific shocks, and capital stock). $Y \in \mathbb{R}^m$ is the vector of non portfolio control variables (i.e. consumption, investment, labor)

Let us now assume that we have a simpler model where the components of $(\lambda_H, \lambda_F, X, Y)$ are all scalars and not vectors as defined earlier. Then the approximation of expression (33) will be given by

$$\lambda_{H,t+1} = \sum_{j=0}^{N_1} \sum_{k=0}^{N_1} \sum_{m=0}^{N_1} a_{jkm} P_j(\lambda_{H,t}) P_k(\lambda_{F,t}) P_m(X_t) \tag{34}$$

and similar functions will be obtained for $\lambda_{F,t+1}$, X_{t+1}, and Y_t.

Choice of the basis functions. The functions $P(\cdot)$ are ordinary polynomials, and more specifically we use here *complete basis* functions such that

$$P_N(x) \equiv \left\{ x_1^{i_1}, \ldots, x_m^{i_m} \,|\, \sum_{l=1}^{m} i_l \leq N, i_l \geq 0, l = 1, \ldots, m \right\}$$

In other words, we will always have $j + k + m \leq N$ in expression (34), where N is the order of the approximation.

Choice of the projection method. The choice of the coefficients a_{jkm} in expression (34) are determined depending on the projection method that is used. We use in the current version the *least squares* method, which is known to be the less efficient method. But it turns out to be the easiest to implement here. In this approach we are choosing the coefficients a_{jkm} such that the residuals of the Euler equation will be minimized.

3.2 Hybrid projection-perturbation method with constant portfolios

In the current setup we assume that labor is exogeneous and constant, and portfolios are also constant, but they are fixed at a level that satisfies the optimality conditions.

The method is based on an iterative approach. The criteria used for determining the equilibrium portfolio is the *Euler equation error* given by the first order conditions for the household's portfolios. These errors are obtained by plugging back the approximated decision rules in the original problem.

This criteria is comparable to the one used in Lettau and Ludvigson [60], where the authors show that the standard asset pricing model with consumption generates large unconditional Euler equation errors when evaluated on cross-sections of stock returns. Instead of comparing various models, we will instead use this criteria to choose the optimal portfolio choice for a given parametrization of the model. In the current setup the *free parameters* are the constant portfolios and these are chosen such that they will minimize the Euler equation errors.

The method can be summarized as follows: We start with a fixed portfolio choice. Given this portfolio, we solve the dynamic optimization problems given in section 1. Based on this solution, we compute the errors of the Euler equation. By doing this over a set of portfolio choices (by discretizing the portion of the state-space relative to the portfolio), we identify the portfolio that minimizes the residuals of the appropriate Euler equations.

The constant portfolios will be treated as standard structural parameters of the model. There is an analogy with the estimation procedure of structural parameters in which we maximize a likelihood function computed using observed data. In our approach the *portfolio parameters* are chosen such that we will minimize a loss function which is computed using the Euler equation errors. We are aware that this approach may have a drawback since the choice of the *portfolio parameters* controls the size of the numerical approximation. For instance there could be values for S and b for which the problem gets highly nonlinear and which would be wrongly rejected by our approach that stops at the second order Taylor expansion.

$$\begin{aligned} \bar{\lambda}_H &= \zeta_0^H \\ \bar{\lambda}_F &= \zeta_0^F \\ X_{t+1} &= g_3\left(X_t, \varepsilon_{t+1}; \underline{\zeta}_0\right) \\ Y_t &= g_4\left(X_t; \underline{\zeta}_0\right) \end{aligned} \tag{35}$$

where the vector of coefficients $\underline{\zeta}_0 = (\zeta_0^H, \zeta_0^F)'$ is determined using the projection method, and the

last two equations are solved using the perturbation method.

3.2.1 Perturbation method.

The system of dynamic equations listed in the previous section, and that describes the model, can be expressed as

$$E_t \left[\mathcal{H}(y_{t+1}, y_t, x_{t+1}, x_t) \right] = 0 \tag{36}$$

where $x_t = [x_{1t}; x_{2t}]$. x_{1t} represents the endogeneous predetermined state variables (k_t^H, k_t^F) and x_{2t} represents the exogeneous state variables $(z_t^H, z_t^F, \chi_t^H, \chi_t^F)$, with

$$x_{2,t+1} = \Lambda x_{2t} + \eta \sigma \epsilon_{t+1} \tag{37}$$

and $y_t = (c_t^H, c_t^F, l_t^H, l_t^F, i_t^H, i_t^F)$.

The solution of this system of equations is given by

$$\begin{aligned} y_t &= g(x_t, \sigma) \\ x_{t+1} &= h(x_t, \sigma) + \eta \sigma \epsilon_{t+1} \end{aligned} \tag{38}$$

For instance the decision rules for next period's capital are given by

$$\begin{aligned} k_{t+1}^H &= h_{k^H}(x_t, \sigma) \\ k_{t+1}^F &= h_{k^F}(x_t, \sigma) \end{aligned}$$

Function $h(\cdot)$ will be a nonlinear function as we use higher order approximations to solve the model for the decision rules. It should be noted that since there is no time-varying volatility involved in the model and since the model is calibrated such that the system will show a limited degree of nonlinearity, we will restrict our approximation to a second-order approximation.

3.2.2 Defining the residuals

The Euler equation errors are given by

$$
\begin{aligned}
e^{f}_{H,t} &\equiv \frac{\chi_{H,t}}{P^{I}_{H,t}} E_t \varphi^{H,p}_{t,t+1} \left[p_{H,t+1} \alpha \frac{y_{H,t+1}}{k_{H,t+1}} + (1-\delta) \frac{P^{I}_{H,t+1}}{\chi_{H,t+1}} \right] - 1 \\
e^{f}_{F,t} &\equiv \frac{\chi_{F,t}}{P^{I}_{F,t}} E_t \varphi^{F,p}_{t,t+1} \left[p_{F,t+1} \alpha \frac{y_{F,t+1}}{k_{F,t+1}} + (1-\delta) \frac{P^{I}_{F,t+1}}{\chi_{F,t+1}} \right] - 1 \\
e^{S,p}_{R,H,t} &\equiv E_t \left[\varphi^{F,p}_{t,t+1} R^{S}_{H,t+1} \right] - 1 \\
e^{b,p}_{R,H,t} &\equiv E_t \left[\varphi^{F,p}_{t,t+1} R^{b}_{H,t+1} \right] - 1 \\
e^{S,p}_{R,F,t} &\equiv E_t \left[\varphi^{H,p}_{t,t+1} R^{S}_{F,t+1} \right] - 1 \\
e^{b,p}_{R,F,t} &\equiv E_t \left[\varphi^{H,p}_{t,t+1} R^{b}_{F,t+1} \right] - 1 \\
e^{b,np}_{R,H,t} &\equiv E_t \left[\varphi^{H,np}_{t,t+1} R^{b}_{H,t+1} \right] - 1 \\
e^{b,np}_{R,F,t} &\equiv E_t \left[\varphi^{F,np}_{t,t+1} R^{b}_{F,t+1} \right] - 1
\end{aligned}
\tag{39}
$$

These Euler equation errors have an economic interpretation and they can be seen as *pricing errors*. The pricing error of a given asset is defined as the difference between its historical mean excess return over the risk-free rate and the risk-premium implied by the model with pricing kernel $\varphi^{i,j}_{t,t+1}$, where $i = \{H, F\}$ and $j = \{p, np\}$ (Lettau and Ludvigson [60]).

Using the approximated non linear decision rules, $h(\cdot)$, we will choose the constant portfolios, which minimize a weighted sum of squared Euler equation errors

$$
\min_{S^H_H, b^{H,np}_H, b^{H,p}_H} \left\{ g_T(S^H_H, b^{H,np}_H, b^{H,p}_H) \equiv \mathbf{w}'_T(S^H_H, b^{H,np}_H, b^{H,p}_H) \mathbf{W} \mathbf{w}_T(S^H_H, b^{H,np}_H, b^{H,p}_H) \right\} \tag{40}
$$

where $\mathbf{W}$ is a positive semi-definite weighting matrix and $\mathbf{w}_T(S^H_H, b^{H,np}_H)$ is the vector of Euler equation errors for each asset with

$$
w^{l,j}_{iT} = \frac{1}{T} \sum_{t=1}^{T} e^{l,j}_{R,i} \tag{41}
$$

Let $\hat{S}^H_H$, $\hat{b}^{H,np}_H$, and $b^{H,p}_H$ denote the $\arg\min g_T(S^H_H, b^{H,np}_H, b^{H,p}_H)$.

There are various ways for computing the expression $w^{l,j}_{iT}$. The notation that we use here suggests that we are deating with time series. In the following section these time series are obtained through simulations from the approximated decision rules.

3.2.3 Accuracy along a simulated path

The simplest way for implementing our method is to compute the approximation errors along a simulated path, starting from the steady state of the model, and generating random draws of the shocks.

For a given constant portfolio choice $(\bar{S}_H^H, \bar{b}_H^{H,np}, \bar{b}_H^{H,p})$ and a given approximated decision rule $(\hat{h}_y, \hat{h}_x)$, we generate a series of points in the state space $x_0, \ldots, x_T$ where $x_t = (k_t^H, k_t^F, z_t^H, z_t^F, \chi_t^H, \chi_t^F)$ and $T = 200$. The initial state is the steady state $x_0 = (\bar{k}_t^H, \bar{k}_t^F, \bar{z}_t^H, \bar{z}_t^F, \bar{\chi}_t^H, \bar{\chi}_t^F)$ and the subsequent state are given by

$$k_{t+1}^H = h_{k^H}(x_t) \tag{42}$$

$$k_{t+1}^F = h_{k^F}(x_t) \tag{43}$$

and the law of motion of z_t^H, z_t^F, χ_t^H, and χ_t^F.

The series of shocks $\varepsilon_{z^H,1:T}$, $\varepsilon_{z^F,1:T}$, $\varepsilon_{\chi^H,1:T}$, and $\varepsilon_{\chi^F,1:T}$ are generated with a pseudo-random number generator. The first 50 points in the generated series are discarded, so that the 150 remaining points can be considered as representative of the ergodic distribution[6] of the model variables. On these points, we compute the approximation errors $e_.^i$, with $i = \{H, F\}$, and report the mean and maximum errors (for each equation separately, and for the aggregate across equations).

The expectations are computed using a Monte Carlo approach, as a consequence this there is an analogy between this method and the stochastic PEA.

Because of the presence of a numerical optimizer, the quality of the initial values is relatively important. It turns out that we cannot solve the model for extreme values for the structural parameters if the initial values for the triplet $(S_H^H, b_H^{H,np}, b_H^{H,p})$ are not chosen properly. We rely on the homotopy property to solve this issue. This consists in solving a version of the model for which we have the analytical expression for the portfolio and then we change the structural parameters via small steps.

4 Results

We compute optimal portfolio for the benchmark parametrization given in Table 2. Following this we study the sensitvity of the portfolios to changes in the structural parameters (*see* Table 4-10).

[6] The number of simulation and time period has to be increased, in the future!

Algorithm 1 Errors along the simulated path

- Initialize $\left(S_H^H\right)_0$, $\left(b_H^{H,np}\right)_0$, and $\left(b_H^{H,p}\right)_0$
- Solve the system of non linear dynamic equations using the perturbation method
- Simulate the model and extract time series for $e_{R,H,t}^{S,p}$, $e_{R,H,t}^{b,p}$, $e_{R,F,t}^{S,p}$, $e_{R,F,t}^{b,p}$, $e_{R,H,t}^{b,np}$, and $e_{R,F,t}^{b,np}$ for $t = 1, \ldots, T$
- Compute for $i = \{H, F\}$, $j = \{p, np\}$, and $l = \{H, F\}$

$$w_{iT}^{l,j} = \frac{1}{T} \sum_{t=1}^{T} e_{R,i,t}^{l,j}$$

- Using a numerical optimizer, choose the optimal portfolio such that we will minimize the following expression

$$\min_{S_H^H, b_H^{H,np}, b_H^{H,p}} \left\{ g_T(S_H^H, b_H^{H,np}, b_H^{H,p}) \equiv \mathbf{w}_T'(S_H^H, b_H^{H,np}, b_H^{H,p}) \mathbf{W} \mathbf{w}_T(S_H^H, b_H^{H,np}, b_H^{H,p}) \right\}$$

where $\mathbf{W}$ is a positive semi-definite weighting matrix and $\mathbf{w}_T(S_H^H, b_H^{H,np}, b_H^{H,p})$ is the vector of Euler equation errors for each asset

Table 4 and Table 5 shows the portfolio choices for different values of the risk aversion coefficient and the elasticity of substitution between domestic and foreign good. For a unitary elasticity of substitution ($\phi = 1$), Table 4 shows how portfolios change as we increase the risk aversion parameter. The utility function being of the CRRA-type, the case in which $\sigma = 1$ reduces the model to the log utility case. The exercise that we do here is simply to increase this parameter up to a value of 4 and compute the portfolio choices for a set of intermediary values. It can be seen that higher risk aversion lowers the domestic risky assets held by the home country, while increasing the domestic bond holdings for the non-stockholders from the home country. We do the same exercise for the elasticity of substitution between the two traded goods in Table 5 under the log utility assumption.

In Table 6 and Table 7, we investigate the portfolio choices in the case where the stocks are very small. Portfolios are more sensitive to a variation in the size of the shocks in productivity than they are to the size of investment-specific shocks.

Table 8 and Table 9 summarize the sensitivity of the persistence that the exogeneous shocks have on the portfolio choices. For instance the persistence of the investment-specific shock has almost no impact on the portfolio, while it is sensitive to the persistence of the productivity process.

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Table 1: Description of the variables in the model

Variable types	Home		Foreign		
Control variables					
Quantities					
	C_H^p	C_H^{np}	C_F^p	C_F^{np}	aggregate consumption
	$c_H^{H,p}$	$c_H^{H,np}$	$c_F^{H,p}$	$c_F^{H,np}$	c_j^i: country i's consumption of the good produced by country j
	$c_H^{F,p}$	$c_H^{F,np}$	$c_F^{F,p}$	$c_F^{F,np}$	
	y_H		y_F		y_i: production of country i
	l_H		l_F		labor effort
	I_H		I_F		gross investment in country i
	i_H^H	i_F^H	i_F^F	i_H^F	i_j^i: amount of good j used for investment in country i
	d_H		d_F		dividend paid to shareholders
	$b_F^{H,p}$	$b_F^{F,p}$	$b_F^{F,np}$	b_F	
	$b_H^{H,p}$	$b_H^{F,p}$	$b_H^{H,np}$	b_H	
Prices					
	P_H		P_F		
	p_H		p_F		p_i: price of good i
	P_H^I		P_F^I		
	w_H		w_F		w_i: country i wage rate
	p_H^S		p_F^S		p_i^S: price of stock i
	p_H^b		p_F^b		p_i^b: price of the good-i bond
	R_H^S		R_F^S		R_j^S: gross returns of stock j
	R_H^b		R_F^b		R_j^b: gross return of the good-j bond
State variables					
	k_H		k_F		k_i: country i's stock of capital
	$\boxed{S_H^H}$	S_H^F	S_F^H	S_F^F	S_j^i: number of shares of stock j held by the country i household
	$\boxed{b_H^{H,p}}$	$b_H^{F,p}$	$b_F^{H,p}$	$b_F^{F,p}$	b_j^i: claims held by i
	$\boxed{b_H^{H,np}}$	$b_H^{F,np}$	$b_F^{H,np}$	$b_F^{F,np}$	to future unconditional payments of good j
Exogeneous state variables					
	z_H		z_F		total factor productivity
	χ_H		χ_F		investment efficiency

Table 2: Parametrization of the benchmark model (BENCH)

Parameter	Value	
ω	0.50	inverse of Frisch elasticity parameter
σ	2.00	relative risk aversion
ϕ	2.00	elasticity of substitution between the two goods
ϕ_I	2.00	
a	0.85	share of consumption spending devoted to the local good
δ	0.10	depreciation rate of capital
a_I	0.85	local bias in investment spending
β	0.96	discount factor
α	0.40	capital share
ρ_z	0.75	persistence of productivity process
σ_z	0.0120	standard deviation of productivity process
ρ_χ	0.79	persistence of investment-specific shock
σ_χ	0.0173	standard deviation of investment-specific shock
μ	0.50	participation rate
$\sigma_{z_H z_F}$	0.10	correlation of productivity processes in both countries
$\sigma_{\chi_H \chi_F}$	0.19	correlation of investment-specific shock in both countries
$\bar{l}_H$	1	
$\bar{l}_F$	1	

Table 3: Proportion of households investing in risky assets (Source: Guiso et al. [35])

	Year	US	UK	NL	DE	IT
Direct stockholding						
	1983	19.1	8.9	-	9.7	-
	1989	16.8	22.6	-	10.3	4.5
	1995	15.2	23.4	11.5	10.5	4.0
	1998	19.2	21.6	15.4	-	7.3
Direct and indirect stockholdings						
	1983	-	-	-	11.2	-
	1989	31.6	-	-	12.4	10.5
	1995	40.4	-	29.4	15.6	14.0
	1998	48.9	31.4	35.1	-	18.7
Risky financial assets						
	1983	-	-	-	13.7	-
	1989	31.9	-	-	17.2	12.0
	1995	40.6	-	21.9	20.2	18.5
	1998	49.2	32.4	27.7	-	22.1
Total risky assets						
	1983	-	-	-	17.8	-
	1989	46.4	-	-	24.1	47.0
	1995	51.6	-	28.4	25.2	46.9
	1998	56.9	-	32.8	-	43.8

Table 4: Portfolio sensitivity [TAB-0]: ($\mu = 0.50$ and $\phi = 1.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$
σ				
1.00	0.8086	0.0223	-0.0236	-11.8299
1.50	0.7720	0.2442	-0.1639	-11.6882
2.00	0.7503	0.4060	-0.2719	-11.5981
2.50	0.7383	0.5074	-0.3408	-11.5264
3.00	0.7305	0.5784	-0.3909	-11.4616
3.50	0.7259	0.6226	-0.4224	-11.3987
4.00	0.7221	0.6608	-0.4507	-11.3361

Table 5: Portfolio sensitivity [TAB-1]: ($\mu = 0.50$ and $\sigma = 1.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$	
ϕ					
1.00	0.8086	0.0223	-0.0236	-11.8299	
1.50	0.8086	0.0223	0.2022	-	
2.00	0.8086	0.0223	0.4276	-11.8288	
2.50	0.8086	0.0223	0.6527	-	
3.00	0.8086	0.0223	0.8772	-11.8257	

Table 6: Portfolio sensitivity [TAB-4]: ($\mu = 0.50$, $\phi = 2.00$ and $\sigma = 2.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$	
σ_χ					
0.0030	0.7267	0.6293	-0.1154	-12.9173	
0.0040	0.7272	0.6223	-0.1086	-12.8335	
0.0050	0.7280	0.6145	-0.1014	-12.7415	
0.0060	0.7289	0.6057	-0.0933	-12.6442	
0.0070	0.7300	0.5958	-0.0842	-12.5439	
0.0080	0.7313	0.5849	-0.0742	-12.4421	
0.0090	0.7326	0.5733	-0.0635	-12.3395	
0.0100	0.7343	0.5580	-0.0494	-12.2410	
0.0173	0.7538	0.3944	0.1088	-11.5914	BENCH

Table 7: Portfolio sensitivity [TAB-5]: ($\mu = 0.50$, $\phi = 2.00$ and $\sigma = 2.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$	
σ_z					
0.001	0.8966	-0.3392	1.1243	-13.3105	
0.002	0.8882	-0.3113	1.0700	-12.9330	
0.003	0.8773	-0.2732	0.9987	-12.6422	
0.004	0.8605	-0.2102	0.8880	-12.3686	
0.005	0.8450	-0.1459	0.7829	-12.1992	
0.006	0.8262	-0.0601	0.6534	-12.0493	
0.007	0.8084	0.0311	0.5269	-11.9363	
0.008	0.7922	0.1242	0.4081	-11.8451	
0.009	0.7784	0.2125	0.3036	-11.7684	
0.010	0.7687	0.2798	0.2286	-11.7102	
0.012	0.7538	0.3944	0.1088	-11.5914	BENCH

Table 8: Portfolio sensitivity [TAB-6]: ($\mu = 0.50$, $\phi = 2.00$ and $\sigma = 2.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$	
ρ_χ					
0.79	0.7538	0.3944	0.1088	-11.5914	BENCH
0.80	0.7539	0.3944	0.1088	-11.6261	
0.81	0.7543	0.3913	0.1122	-11.6616	
0.82	0.7544	0.3913	0.1122	-11.6980	
0.83	0.7549	0.3881	0.1156	-11.7353	
0.84	0.7549	0.3881	0.1156	-11.7735	
0.85	0.7555	0.3845	0.1194	-11.8126	
0.86	0.7558	0.3824	0.1216	-11.8524	
0.87	0.7563	0.3800	0.1243	-11.8930	
0.88	0.7567	0.3771	0.1274	-11.9339	
0.90	0.7573	0.3734	0.1313	-11.9749	

Table 9: Portfolio sensitivity [TAB-7]: ($\mu = 0.50$, $\phi = 2.00$ and $\sigma = 2.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$	
ρ_z					
0.75	0.7538	0.3944	0.1088	-11.5914	BENCH
0.76	0.7594	0.3528	0.1522	-11.6231	
0.77	0.7638	0.3205	0.1865	-11.6473	
0.78	0.7688	0.2862	0.2240	-11.6731	
0.79	0.7741	0.2499	0.2646	-11.7008	
0.80	0.7800	0.2119	0.3082	-11.7308	
0.81	0.7864	0.1727	0.3548	-11.7634	
0.82	0.7931	0.1325	0.4040	-11.7991	
0.83	0.8003	0.0920	0.4553	-11.8383	
0.84	0.8078	0.0517	0.5082	-11.8816	
0.85	0.8156	0.0122	0.5619	-11.9298	
0.86	0.8234	-0.0258	0.6158	-11.9834	
0.87	0.8294	-0.0531	0.6557	-12.0276	
0.88	0.8352	-0.0792	0.6949	-12.0757	
0.89	0.8410	-0.1037	0.7329	-12.1277	
0.90	0.8466	-0.1267	0.7693	-12.1840	

Table 10: Portfolio sensitivity [TAB-8]: ($\mu = 0.50$, $\phi = 2.00$ and $\sigma = 2.00$)

Parameter of interest	S_H^H	$b_H^{H,np}$	$b_H^{H,p}$	$\log(\sum eee)$	
$\sigma_{z_H z_F}$					
0.10	0.7538	0.3944	0.1088	-11.5914	BENCH
0.25	0.7497	0.4251	0.0769	-11.5452	
0.26	0.7495	0.4270	0.0749	-11.5422	
0.27	0.7493	0.4282	0.0736	-11.5392	
0.28	0.7490	0.4302	0.0715	-11.5363	
0.29	0.7485	0.4338	0.0677	-11.5312	
0.30	0.7485	0.4338	0.0677	-11.5282	
0.31	0.7480	0.4375	0.0639	-11.5253	
0.32	0.7480	0.4375	0.0639	-11.5223	
0.33	0.7475	0.4411	0.0601	-11.5194	
0.34	0.7475	0.4410	0.0601	-11.5164	
0.35	0.7470	0.4446	0.0564	-11.5135	
0.36	0.7470	0.4446	0.0564	-11.5106	
0.37	0.7465	0.4481	0.0528	-11.5077	
0.38	0.7465	0.4480	0.0528	-11.5047	
0.39	0.7460	0.4515	0.0492	-11.5019	
0.40	0.7460	0.4515	0.0492	-11.4990	
0.41	0.7455	0.4549	0.0456	-11.4961	
0.42	0.7456	0.4548	0.0456	-11.4932	
0.43	0.7455	0.4548	0.0455	-11.4918	
0.44	0.7451	0.4582	0.0421	-11.4889	
0.45	0.7451	0.4582	0.0421	-11.4875	