Economic and Politico-Economic Equivalence of Fiscal Policies*

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Abstract

Extending established neutrality results, we derive conditions under which an economic state and a sequence of policy choices are economically equivalent to another such pair, in the sense that both pairs support the same competitive equilibrium. We then extend this "economic" neutrality result to the political sphere where policy instruments are chosen sequentially. In particular, we derive conditions under which a state (summarizing economic conditions as well as predetermined policy choices) and a policy regime (summarizing admissible policy instruments and values for these instruments) are politico-economically equivalent to another such pair, in the sense that both pairs give rise to the same equilibrium allocation.

We apply the conditions for politico-economic equivalence in the context of well-known models with the aim to understand when a change of policy regime is non-neutral even if policies in these regimes are equivalent from a purely economic perspective. Focusing specifically on the comparison of pay-as-you-go financed social security regimes versus regimes with explicit government debt, our analysis contributes towards a theory of social security reform.

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1 Introduction

Important results in public economics and macroeconomics establish equivalence classes of exogenous policies. These results state that a given equilibrium allocation may be supported by a class of policies with each member of the class making use of different policy instruments or assigning different values to the policy instruments. For example, in a simple model of household choices, policies relying on different combinations of consumption, capital-income and labor-income taxes form equivalence classes, and in the standard overlapping-generations model, pay-as-you go policies imposing labor-income taxes and old-age benefits are equivalent to policies relying on taxes and explicit government debt.

When policies are endogenous, equivalence classes defined over exogenous policies loose some of their relevance. Of interest in endogenous-policy environments is not only whether different policies support the same allocation but also whether different institutional environments, or policy regimes, give rise to politico-economic equilibria that support the same allocation. Consider for example a social security regime allowing for pay-as-you-go policies and a debt regime allowing for policies relying on taxes and explicit government debt. While an individual pay-as-you-go policy may belong to the same economic equivalence class as a specific debt policy, the two regimes may not necessarily give rise to politico-economic equilibria supporting the same allocation. In fact, the observed disagreement among policy makers about the benefits of "privatizing" social security (and debt financing the transition phase) suggests that a shift from a social security regime to a debt regime may significantly change the equilibrium allocation.

The purpose of this paper is to establish conditions for the politico-economic equivalence of policy regimes and to examine whether these conditions are satisfied in the context of well-known models in the literature. The paper makes four contributions. First, it derives conditions for politico-economic equivalence: given a state and a policy regime with known equilibrium policy functions, these conditions guarantee that a different policy regime with unknown equilibrium policy functions gives rise to the same equilibrium allocation as the first regime. Second, since these conditions often are easier to check than it is to directly characterize equilibrium in settings with high dimensional state and policy spaces, the paper provides a useful tool for researchers interested in more complex environments with endogenous policy choice.

Third, the paper contributes to the small but growing literature on government debt in politico-economic equilibrium (see the literature review below). Based on the theoretical results in the paper we assess whether the politico-economic equilibria in a set of well-known models may be reinterpreted as politico-economic equilibria in models featuring other policy instruments. For the most part, we focus on policy regimes with pay-as-you-go policies on the one hand and tax-and-debt policies on the other. We find that, sometimes such a reinterpretation is valid while often it may not. Finally, the paper therefore takes a step towards a theory of social security reform. By highlighting the factors potentially undermining politico-economic equivalence, it rationalizes why interest groups might favor or oppose the privatization of social security even if at first sight, they do not seem to benefit or loose from such a change of regime.

We derive the results in the context of a dynamic framework with households and firms as well as a government with access to general taxes, transfers and debt. Fundamental to our approach is the notion of economic equivalence. An economic state and a sequence of policy choices are economically equivalent to another such pair if both pairs support the same competitive equilibrium. Extending results in the literature (Barro, 1974; Sargent, 1987; Rangel, 1997; Bassetto and Kocherlakota, 2004; Niepelt, 2005), we prove that an economic state and a sequence of policy choices that support an equilibrium are economically equivalent to another such pair if each household's budget set is unchanged under the two pairs and if the second pair satisfies the government's dynamic budget constraint subject to equilibrium prices and quantities under the first pair.

Building on this economic-equivalence result, we establish conditions for politico-economic equivalence. A policy space characterizes the institutional environment of political decision makers, defining the set of policy instruments they may use in a given period. A policy regime is a policy space subject to numerical restrictions on the policy instruments. A state (consisting of an economic state and a political state, the latter reflecting predetermined policy instruments) and a policy regime are politico-economically equivalent to another such pair if both pairs support politico-economic equilibria and if the economic state and equilibrium policy in one politico-economic equilibrium is economically equivalent to the economic state and equilibrium policy in the other. We show that three conditions need to be satisfied to guarantee politico-economic equivalence. The logic underlying these conditions relies on a simple comparison of choice sets faced by political decision makers.

The first condition guarantees that the state and policy spaces can be compared across regimes (this is not guaranteed, since policy instruments and commitment structures generally differ across policy regimes). For any state in the "initial" policy regime, an associated state is a state in the "new" policy regime such that the state and the sequence of equilibrium policy choices in the initial regime is economically equivalent to the associated state together with some combination of policy instruments in the policy space of the new regime. The first condition requires that for any state in the initial policy regime, there exists effectively one and only one associated state in the new policy regime.[1] If this condition is satisfied, the policy space of the new policy regime allows to implement competitive equilibria that are equilibrium outcomes in the initial policy regime. One can therefore define equivalent (continuation) policy functions that map states in the new regime into values for contemporaneous and future policy instruments in the policy space of the new regime. These equivalent (continuation) policy functions effectively replicate in the new regime the (continuation) policy functions in the initial regime.

The second condition for politico-economic equivalence guarantees that, conditional on two associated states, political decision makers in the new regime can "do as much as" in the initial regime. It requires that, if equivalent continuation policy functions determine subsequent policy choices, then it must be possible in the new regime to implement the competitive equilibrium along the equilibrium path in the initial regime. The third condition for politico-economic equivalence guarantees that political decision makers in the new regime cannot "do more than" in the initial regime. It requires that, if equivalent contin-

[1] If multiple states are associated with the same state, they must all support the same competitive equilibrium.

uation policy functions determine subsequent policy choices, then it must not be possible in the new regime to implement competitive equilibria that cannot be implemented in the initial regime by means of a one-period deviation.

The second and third condition imply that, conditional on equivalent continuation policy functions, the set of competitive equilibria among which political decision makers in the new regime may choose in a given period effectively matches the set in the initial regime. At the same time, the first and third condition imply that, conditional on states in the two regimes being associated in a given period, states in the subsequent period are associated as well if political decisions in the new regime are governed by the equivalent policy function. As a consequence, the three conditions (which can be extended to accommodate non-fundamental state variables of the type supporting trigger strategies) imply politico-economic equivalence.

We apply the conditions for politico-economic equivalence in the context of well known models in the literature in order to shed light on the question whether a change of policy regime alters the equilibrium allocations in those models. The changes of policy regime that we consider are of two types. In the context of Bassetto and Kocherlakota's (2004) model, we analyze the implications of allowing taxes to depend not only on current but also on lagged income. In the context of Cooley and Soares's (1999), Tabellini's (2000), Boldrin and Rustichini's (2000), Forni's (2005) and Gonzalez-Eiras and Niepelt's (2008) models, we assess politico-economic equivalence of the social security regimes analyzed in these papers on the one hand and regimes with taxes and government debt on the other.

We find that some existing theories of social security in politico-economic equilibrium may alternatively be interpreted as politico-economic theories of government debt and in that sense, we contribute to the small but growing literature on government debt in politico-economic equilibrium (Battaglini and Coate, 2008; Díaz-Giménez, Giovanetti, Marimon and Teles, 2008; Song, Storesletten and Zilibotti, 2007; Niepelt, 2009). For other theories of social security in politico-economic equilibrium, such an interpretation is not valid. In those cases, the politico-economic equivalence conditions are violated because economically equivalent policies do not satisfy numerical admissibility restrictions on policy instruments or because they differ with respect to the flexibility they provide for political decision makers ex post. These negative findings suggest that prominent politico-economic theories of intergenerational redistribution are not robust to changing the set of policy instruments considered. At the same time, they contribute to a theory of social security reform by rationalizing why interest groups might favor the privatization of social security even if at first sight, they do not seem to benefit or loose from such a change of regime.

The paper is structured as follows. Section 2 lays out the model. In Sections 3 and 4, we derive the economic and politico-economic equivalence results, respectively. Section 5 contains the applications and Section 6 concludes with a discussion of the implications for social security reform and the applicability of our results to a wider class of policies.

2 The Model

We consider an infinite-horizon deterministic, discrete-time economy with time indexed by $t = 0, 1, \ldots$.[2] The economy is inhabited by a government, firms (potentially including an external sector), and households.

Let $\mathcal{I}$ denote the set of households, and let $\mathcal{I}_t \subseteq \mathcal{I}$ denote the set of households alive in period t. For any household $i \in \mathcal{I}$, let i_1 and i_T denote the first period and last period of household i's lifetime, respectively.

There are L commodities in each period. Let $e_t^i \in \mathbb{R}_+^L$ and $x_t^i \in \mathbb{R}_+^L$ denote endowment and consumption vectors, respectively, of household i in period t and let $X_t^i \subset \mathbb{R}_+^L$ denote the household's consumption set. For example, e_t^i might include household i's time endowment, and x_t^i might include consumption of leisure or goods. Household preferences in period t are described by the function $\Omega_t^i : X_{i_1}^i \times \cdots \times X_{i_T}^i \rightarrow \mathbb{R}$ for all $i \in \mathcal{I}$ with $i_1 \leq t \leq i_T$. Let x_t denote the vector of consumption choices by all households $i \in \mathcal{I}_t$ in period t, and let x^i denote the consumption profile of household i over i's lifetime. Let b_t^i denote household i's holdings of maturing government debt in period t and let a_t^i denote household i's financial wealth net of government debt but including discounted firm profits, if they exist (all in terms of the numeraire). Household i's total financial wealth, f_t^i, is given by $f_t^i \equiv a_t^i + b_t^i z_t$ with z_t denoting the repayment rate on government debt, to be discussed below.

Let $\mathcal{J}$ denote the set of firms. Let $y_t^j \equiv (y_{1,t}^j, y_{2,t}^j) \in \mathbb{R}^{2L}$ denote a production plan of firm $j \in \mathcal{J}$ in period t and let $Y_t^j \subset \mathbb{R}^{2L}$ denote the production set of the firm in period t. A production plan y_t^j lists firm j's net input-outputs in period t (the first L elements, corresponding to $y_{1,t}^j$) as well as the resulting net input-outputs at the beginning of the following period (the second L elements, corresponding to $y_{2,t}^j$).[3] Given a predetermined $y_{2,t-1}^j$, firm j's sequence $(y_t^j, y_{t+1}^j, \ldots)$ constitutes a *production path* if $y_s^j \in Y_s^j$ for all $s \geq t$. This production path generates a sequence of net inputs-outputs of the L commodities equal to $\{y_{2,s-1}^j + y_{1,s}^j\}_{s \geq t}$.

Let q and r denote the pre-tax prices in the economy. In particular, the vector q_t denotes the period-t prices of the L commodities in terms of the numeraire; and $r_{t,s}$ denotes the period-t price of the numeraire in period s in terms of the numeraire in period t, that is, the inverse of the gross interest rate between periods t and s, $s \geq t$. The vector r_t denotes the term structure of interest rates in period t, and ${}_t r$ denotes the term structures of interest rates in periods preceding t.

It is useful to partition x^i as $x^i = ({}_t x^i, x_t^i, x^{it})$ with ${}_t x^i \equiv (x_{i_1}^i, x_{i_1+1}^i, \ldots, x_{t-1}^i)$ and $x^{it} \equiv (x_{t+1}^i, x_{t+2}^i, \ldots, x_{i_T}^i)$. Vectors defined over all households as well as the prices q are similarly partitioned. In particular, $x = ({}_t x, x_t, x^t)$ with ${}_t x \equiv (x_0, x_1, \ldots, x_{t-1})$ and $x^t \equiv (x_{t+1}, x_{t+2}, \ldots)$.

Let $g_t^i({}_t x^i, x_t^i, {}_t q, q_t, {}_t r, f_t^i)$ denote the tax imposed on household i in period t, defined in terms of the numeraire. Taxes may depend on the household's financial wealth as well as

[2] The extension to the stochastic case is immediate. If the number of states in each period is finite, and with some adjustments to notation, t can alternatively be interpreted as indexing histories.

[3] See Mas-Colell, Whinston and Green (1995, ch. 20.C). For example, if the firm uses goods k and labor l to produce new goods k' (and if goods and labor are the only commodities), then $y_t^j = (-k, -l, k', 0)$.

on its consumption choices and prices in the current or previous periods.[4] They may also depend on the household's endowments, but for ease of notation we do not list current or past endowments as arguments of the tax function. Let $g^i \equiv \{g^i_s(\cdot)\}_{s=i_1}^{i_T}$ and denote the profiles of tax functions across households by $g \equiv \{g^i\}_{i\in\mathcal{I}}$.[5] Examples of tax functions $g^i_t(\cdot)$ include labor income, consumption or capital income taxes. A proportional tax on contemporaneous labor income can be represented as $g^i_t(l^i_t, w_t) = \tau^w_t w_t l^i_t$ where τ^w_t, w_t, l^i_t denote the tax rate, wage, and labor supply (that is, the time endowment minus leisure consumption), respectively. A proportional tax on contemporaneous consumption can be represented as $g^i_t(c^i_t, q^c_t) = \tau^c_t q^c_t c^i_t$ with τ^c_t, q^c_t, c^i_t denoting the tax rate, price of the good, and consumption, respectively. A proportional tax on the net return on savings can be represented as $g^i_t({}_tr, f^i_t) = \tau^k_t f^i_t(1 - r_{t-1,t})$ with τ^k_t denoting the tax rate.

A government *policy* consists of a sequence of vectors of government purchases, profiles of tax functions imposed on firms, profiles of tax functions imposed on households, g, and sequences of government debt issuance and redemption (defined in terms of the numeraire). Let b_{t+1} denote the amount of government debt issued in period t and maturing in period $t+1$, and let v_t denote the price of debt at issuance.[6] The policies we consider differ with respect to the profiles of tax functions imposed on households, g, as well as the government debt policy (b, z), but not with respect to taxes imposed on firms or government purchases. To simplify the notation, we therefore assume that the latter two instruments are not employed at all. Accordingly, a policy p can be represented as $p = (g, (b, z))$. Let $p_{\geq t}$ denote the policy instruments applied under policy p in period t or later, $p_{\geq t} \equiv (g_{\geq t}, (b, z)_{\geq t}) \equiv \{\{g^i_s(\cdot)\}_{i\in\mathcal{I}_s}, (b_{s+1}, z_s)\}_{s\geq t}$.

Let $q_t \cdot (x^i_t - e^i_t)$ represent household i's net expenditure in period t before taxes and in terms of the numeraire. The *choice set* of household i as of period t, $\mathcal{B}^i_t(f^i_t, {}_tx^i, q, r, g^i)$, is given by[7]

$$\mathcal{B}^i_t(f^i_t, {}_tx^i, q, r, g^i) = \\ \left\{ (x^i_t, x^{it})| \begin{array}{l} (x^i_t, x^{it}) \in X^i_t \times \cdots \times X^i_{i_T}; \\ \sum_{s=t}^{i_T} r_{t,s} \left(q_s \cdot (x^i_s - e^i_s) + g^i_s({}_sx^i, x^i_s, {}_sq, q_s, {}_sr, f^i_s) \right) \leq f^i_t; \\ f^i_{s+1} = \frac{r_{t,s}}{r_{t,s+1}} \left(f^i_s - q_s \cdot (x^i_s - e^i_s) - g^i_s({}_sx^i, x^i_s, {}_sq, q_s, {}_sr, f^i_s) \right), s \geq t \end{array} \right\}, \quad (1)$$

that is households may freely save or borrow. By convention, $(a^i_t, b^i_t) = (0, 0)$ for $i \in \mathcal{I}$ with $i_1 \geq t$.

The *economic state* in period t comprises several objects. First, the state variables resulting from production in the preceding period, $\{y^j_{2,t-1}\}_{j\in\mathcal{J}}$. Second, households' asset holdings. Third, past consumption choices and prices if these enter as arguments of tax functions. In the most general case, the economic state κ_t therefore is given by $\kappa_t \equiv (\{y^j_{2,t-1}\}_{j\in\mathcal{J}}, \{a^i_t, b^i_t\}_{i\in\mathcal{I}_t}, {}_tx, {}_tq, {}_tr)$.

[4]With sequential decision making of the form considered later, taxes cannot depend on future household choices.

[5]Tax functions imposed on households typically satisfy cross-section restrictions, e.g., $g^i_t(\cdot) = g^j_t(\cdot)$ for all $i, j \in \mathcal{I}$.

[6]For simplicity, we only consider short-term debt.

[7]For ease of notation, we do not list the household's endowments as arguments of the choice set.

Definition 1. A *competitive equilibrium* as of period t conditional on κ_t as well as $p_{\geq t}$, denoted by $\mathrm{CE}(\kappa_t, p_{\geq t})$ for short, consists of prices (q_t, q^t, r_t), household choices (x_t^i, x^{it}) for all $i \in \mathcal{I}$ with $i_T \geq t$, and production paths $\{\{y_s^j\}_{j\in\mathcal{J}}\}_{s\geq t}$ such that

i. household choices are optimal:

$$(x_t^i, x^{it}) \in \arg \max_{(\hat{x}_t^i, \hat{x}^{it}) \in \mathcal{B}_t^i(a_t^i + b_t^i z_t, {}_t x^i, q, r, g^i)} \Omega_t^i({}_t x^i, \hat{x}_t^i, \hat{x}^{it}) \text{ for all } i \in \mathcal{I} \text{ with } i_T \geq t;$$

ii. production paths are optimal:

$$y_s^j \in \arg \max_{\hat{y}_s^j \in Y_s^j} q_s \cdot \hat{y}_{1,s}^j + r_{s,s+1}(q_{s+1} \cdot \hat{y}_{2,s}^j) \text{ for all } j \in \mathcal{J}, s \geq t;$$

iii. the resource constraints are satisfied:

$$\sum_{i\in\mathcal{I}_s} (x_s^i - e_s^i) \leq \sum_{j\in\mathcal{J}} (y_{2,s-1}^j + y_{1,s}^j) \text{ for all } s \geq t;$$

iv. government budget constraints are satisfied:

$$\sum_{i\in\mathcal{I}_s} g_s^i({}_s x^i, x_s^i, {}_s q, q_s, {}_s r, f_s^i) = b_s z_s - z_{s+1} \frac{r_{t,s+1}}{r_{t,s}} b_{s+1} \text{ for all } s \geq t, \tag{2}$$

where $b_t \equiv \sum_{i\in\mathcal{I}_t} b_t^i$, as well as the no-Ponzi-game condition.

Three remarks are in order. First, the government budget constraint (2) uses the fact that in equilibrium, government debt prices equal

$$v_s = z_{s+1} \frac{r_{t,s+1}}{r_{t,s}} \text{ for all } s \geq t.$$

To simplify the notation, we do not include government debt prices in the definition of competitive equilibrium.

Second, the competitive equilibrium conditions together with the budget set (1) determine equilibrium net asset positions of households over time, $\{\{f_s^i\}_{i\in\mathcal{I}_s}\}_{s\geq t+1}$, but not the composition of households' portfolios, $\{\{a_s^i, b_s^i\}_{i\in\mathcal{I}_s}\}_{s\geq t+1}$, since the after-tax returns on all assets are identical in equilibrium.

Finally, note from the definition of $\mathcal{B}_t^i(\cdot)$ as well as from equation (2) that there exists a continuum of debt quantities and repayment rates corresponding to a given equilibrium allocation. In particular, consider the competitive equilibrium $\mathrm{CE}(\kappa_t, p_{\geq t})$ and let $\{\alpha_s\}_{s\geq t}$ be a sequence of strictly positive scalars. Then, the competitive equilibria $\mathrm{CE}(\kappa_t, p_{\geq t})$ and $\mathrm{CE}(\kappa_t', p_{\geq t}')$ coincide if the initial debt holdings $\{b_t^i\}_{i\in\mathcal{I}_t}$ in κ_t are replaced by $\{\alpha_t b_t^i\}_{i\in\mathcal{I}_t}$ in κ_t' and the sequences $\{b_{s+1}, z_s\}_{s\geq t}$ in $p_{\geq t}$ are replaced by the sequences $\{\alpha_{s+1} b_{s+1}, z_s/\alpha_s\}_{s\geq t}$ in $p_{\geq t}'$. We return to this issue later.

3 Economic Equivalence

This Section defines conditions under which pairs of policy instruments and state variables support the same competitive equilibrium. Let $\kappa_t^i \equiv (a_t^i, b_t^i, {}_t x^i, {}_t q, {}_t r)$ denote the *economic state of individual i*.

Lemma 1. Suppose that two tax-profiles-cum-repayment-rate, $(g_{\geq t}^i, z_t)$ and $(g_{\geq t}^{i'}, z_t')$ respectively, give rise to the same choice set for household i as of period t, conditional on two economic states of individual i, κ_t^i and $\kappa_t^{i'}$ respectively:

$$\mathcal{B}_t^i(a_t^i + b_t^i z_t, {}_t x^i, q, r, g_{\geq t}^i) = \mathcal{B}_t^i(a_t^{i'} + b_t^{i'} z_t', {}_t x^{i'}, ({}_t q', q_t, q^t), ({}_t r', r_t), g_{\geq t}^{i'}). \tag{3}$$

Then, $(g_{\geq t}^i, z_t)$ together with κ_t^i supports the same household choices (x_t^i, x^{it}) as does $(g_{\geq t}^{i'}, z_t')$ together with $\kappa_t^{i'}$.

Proof. Immediate. □

Lemma 1 states a "partial equilibrium indifference" condition at the level of an individual household. For policy changes to be "irrelevant" in general equilibrium (as defined in Definition 2 below), this partial equilibrium condition must hold for all households. Moreover, it must be true that aggregate equilibrium conditions are equivalently satisfied under both policies. Assumption 1 and Proposition 1 below specify conditions under which this is the case.

Definition 2. An economic state and set of policy instruments, $(\kappa_t, p_{\geq t})$, is *economically equivalent* to another state and set of policy instruments, $(\kappa_t', p_{\geq t}')$, if

i. $(\kappa_t, p_{\geq t})$ supports a competitive equilibrium $\text{CE}(\kappa_t, p_{\geq t})$;

ii. $(\kappa_t', p_{\geq t}')$ supports a competitive equilibrium $\text{CE}(\kappa_t', p_{\geq t}')$;

iii. the two competitive equilibria are identical.

Assumption 1. Policy does not affect production sets.

Proposition 1. Consider an economic state and set of policy instruments, $(\kappa_t, p_{\geq t})$, that support a competitive equilibrium $\text{CE}(\kappa_t, p_{\geq t})$, and consider a new economic state and set of policy instruments, $(\kappa_t', p_{\geq t}')$. Suppose that κ_t' and $p_{\geq t}'$ satisfy the following conditions:

i. predetermined net input-outputs coincide across economic states:

$$y_{2,t-1}^j = y_{2,t-1}^{j'} \text{ for all } j \in \mathcal{J};$$

ii. at equilibrium prices, $(\kappa_t, p_{\geq t})$ and $(\kappa_t', p_{\geq t}')$ imply identical choice sets for all households as of period t:

$$\text{(3) holds in period } t \text{ for all } i \in \mathcal{I} \text{ with } i_T \geq t; \tag{4}$$

iii. at equilibrium prices and quantities, κ_t' and $p_{\geq t}'$ satisfy the government budget constraints:

$$\sum_{i\in\mathcal{I}_s} g_s^{i'}\left(({}_t x^{i'}, x_t^i, \ldots, x_{s-1}^i), x_s^i, ({}_t q', q_t, \ldots, q_{s-1}), q_s, ({}_t r', r_t, \ldots, r_{s-1}), f_s^{i'}\right)$$
$$= b_s' z_s' - z_{s+1}' \frac{r_{t,s+1}}{r_{t,s}} b_{s+1}' \text{ for all } s \geq t, \tag{5}$$

where $b_s' \equiv \sum_{i\in\mathcal{I}_s} b_s^{i'}$.

Then, under Assumption 1, $(\kappa_t, p_{\geq t})$ is economically equivalent to $(\kappa_t', p_{\geq t}')$.

Proof. Consider the situation with $(\kappa_t', p_{\geq t}')$ and conjecture that prices (q_t, q^t, r_t) remain unchanged relative to $\text{CE}(\kappa_t, p_{\geq t})$.[8] Then, from Lemma 1 and condition (4), household choices remain optimal under $p_{\geq t}'$ and the first requirement of competitive equilibrium is satisfied. From Assumption 1, firm choices continue to constitute production paths. Since prices are unchanged, these choices also remain optimal such that the second requirement of competitive equilibrium is satisfied. Due to unchanged predetermined net input-outputs, the resource constraints continue to be satisfied and the third requirement of competitive equilibrium is met. From Equation (5), the new policy satisfies the government budget constraints and the fourth requirement of competitive equilibrium is satisfied.

We conclude that $(\kappa_t', p_{\geq t}')$ supports a competitive equilibrium that coincides with $\text{CE}(\kappa_t, p_{\geq t})$. □

In the equilibrium supported by the new policy, government debt continues to pay the required return since $v_s' = z_{s+1}' \frac{r_{t,s+1}}{r_{t,s}}$ for all $s \geq t$. As a consequence, households continue to be indifferent between holding government debt or privately issued debt. To finance their original consumption plans households adjust their savings in a period by the amount corresponding to the differential tax payments under the two policies. Summed over all households, demand for government debt therefore matches supply.

Proposition 1 summarizes the Ricardian equivalence result and related neutrality theorems discussed in the literature. Consider for example a model of a representative agent that values consumption and leisure, saves and borrows at the market interest rate, sells labor and capital services to firms, purchases consumption goods from firms, and lends funds to the government. This model satisfies Assumption 1, implying that tax collections and government deficits may be shifted over the entire horizon of the economy without affecting the equilibrium allocation as long as the choice set $\mathcal{B}_t(\cdot)$ of the representative household remains unchanged.[9] In particular, with non-distorting taxes, economic equivalence of two fiscal policies (subject to initial states) only requires the present discounted value of taxes to be the same across policies; this is the standard Ricardian equivalence result (see Barro, 1974).

With overlapping generations of representative households rather than a single infinitely lived household, and under Assumption 1, Proposition 1 implies that a change in

[8] Recall that debt prices are not included in the set of equilibrium objects.

[9] See Ghiglino and Shell (2000), Bassetto and Kocherlakota (2004) and the discussion in Section 5.

the timing of tax collections and deficits leaves the allocation unaffected as long as the choice set of each cohort remains unchanged. In particular, certain debt-and-tax policies are equivalent to pay-as-you-go financed transfer policies. Consider for example an environment with two-period lived overlapping generations. Let $p_{\geq t} = (g_{\geq t}, (\mathbf{0}, z)_{\geq t}) = ((\tau, \sigma)_{\geq t}, (\mathbf{0}, z)_{\geq t})$ be a policy without explicit government debt that supports the competitive equilibrium $\text{CE}(\kappa_t, p_{\geq t})$. Here, τ denotes social security contributions paid by workers and $-\sigma$ denotes benefits paid to retirees. Let $p'_{\geq t} = ((\tau', \sigma')_{\geq t}, (b', z')_{\geq t})$ be another policy with explicit government debt. Proposition 1 implies that $(\kappa'_t, p'_{\geq t})$ is economically equivalent to $(\kappa_t, p_{\geq t})$ if κ'_t and κ_t have the same predetermined net input-outputs and if (4) and (5) are satisfied. Letting $\mathcal{I}_t^r$ and $\mathcal{I}_t^w$ denote the sets of retirees and workers in period t, respectively, and dropping the arguments of tax functions, condition (4) requires

$$\sigma_t^i(\cdot) = \sigma_t^{i\prime}(\cdot) \text{ for all } x_t^i \in X_t^i, i \in \mathcal{I}_t^r$$

and

$$\tau_s^i(\cdot) - \frac{r_{t,s+1}}{r_{t,s}}\sigma_{s+1}^i(\cdot) = \tau_s^{i\prime}(\cdot) - \frac{r_{t,s+1}}{r_{t,s}}{\sigma_{s+1}^i}'(\cdot) \text{ for all } x^i \in X_s^i \times X_{s+1}^i, i \in \mathcal{I}_s^w, s \geq t.$$

Moreover, condition (5) requires

$$\sum_{i \in \mathcal{I}_s} \tau_s^{i\prime}(\cdot) + \sigma_s^{i\prime}(\cdot) = b'_s z'_s - z'_{s+1}\frac{r_{t,s+1}}{r_{t,s}}b'_{s+1}$$

for all $s \geq t$, where taxes are evaluated at equilibrium quantities and prices. Conditional on appropriately defined states κ_t and κ'_t, any pay-as-you-go financed social security policy without explicit debt, $((\tau, \sigma)_{\geq t}, (\mathbf{0}, z)_{\geq t})$, is therefore economically equivalent to policies relying on contributions, benefits, and explicit debt, $((\tau', \sigma')_{\geq t}, (b', z')_{\geq t})$ (see, e.g., Sargent (1987, ch. 8), Rangel (1997) and Niepelt (2005)). In particular, there exists such an equivalent policy that involves no benefits after period t.[10]

4 Politico-Economic Equivalence

Our aim in this Section is to establish conditions under which the economic equivalence result of Proposition 1 extends to situations where policy is sequentially chosen to maximize some objective function. More specifically, we are interested in the conditions under which economic equivalence of two policies (subject to appropriately defined initial states) implies that the second policy constitutes a politico-economic equilibrium if the first policy does so.

Without loss of generality, we focus on the case where political decisions are taken in every period.[11] We assume that political decision makers act in the beginning of a period,

[10] According to some authors, pay-as-you-go financed social security policies are *not* equivalent to policies relying on contributions, benefits, and debt (see, e.g., Birkeland and Prescott, 2007). These authors reach this different conclusion because they restrict the set of available policy instruments such that condition (4) cannot be met.

[11] If political decisions are taken once and for all, economic equivalence trivially extends to the political sphere. If political decisions are taken more than once, but not in every period, then the analysis in the text applies if periods are redefined appropriately.

before the private sector. Let p_t denote the *policy choice* by political decision makers in period t that is, the choice of values for the policy instruments in $p_{\geq t}$ that are under the control of political decision makers in period t; let p^t denote the *continuation policy choice* by subsequent political decision makers; and let ${}_tp$ denote the choices by policy makers in preceding periods (potentially including some initial, predetermined values for policy instruments). A policy p can then be partitioned as $p = ({}_tp, p_t, p^t)$.

Let $\mathcal{P}_t$ denote the set of *admissible* policy choices in period t. The restrictions embedded in $\mathcal{P}_t$ specify the policy instruments under the control of political decision makers in period t (and thus, the degree of commitment) as well as restrictions on the numerical values of those instruments. A *policy regime* is defined by $\mathcal{P} \equiv \prod_t \mathcal{P}_t$ which can be partitioned as $\mathcal{P} = ({}_t\mathcal{P}, \mathcal{P}_t, \mathcal{P}^t)$. The *policy space* in period t, $\mathcal{Q}_t$, is defined as the superset of $\mathcal{P}_t$ that results if restrictions on the numerical values of the policy instruments in $\mathcal{P}_t$ are dropped. The policy space $\mathcal{Q} \equiv \prod_t \mathcal{Q}_t$ can be partitioned as $\mathcal{Q} = ({}_t\mathcal{Q}, \mathcal{Q}_t, \mathcal{Q}^t)$.

Recall from the discussion after Definition 1 that, if government debt constitutes a policy instrument, there exists a continuum of debt quantities and repayment rates corresponding with a given set of prices, taxes, household choices and production paths. Without loss of generality, we eliminate this indeterminacy by fixing government debt per debt holder (if debt is a policy instrument) at some strictly positive exogenous sequence, $\{\bar{b}_t\}_{t\geq 0} > 0$. Adopting this normalization does not constrain the effective choice set of policy makers—with $\bar{b}_t > 0$, the amount of resources transferred to bond holders can be controlled by the choice of repayment rate—nor does it constrain the ownership structure of government debt and thus, the relative exposure of different groups of households to public debt.

If political decision makers can commit to certain future policy instruments then some of the policy instruments employed from period t onwards, $p_{\geq t}$, are predetermined. We denote such predetermined policy instruments by ${}_tp_{\geq t}$, implying $p_{\geq t} = ({}_tp_{\geq t}, p_t, p^t)$. The predetermined policy instruments ${}_tp_{\geq t}$ include *committed tax functions*, $\{g_t^{ic}(\cdot)\}_{i\in\mathcal{I}_t}$, that are imposed on households in period t but are chosen by political decision makers in an earlier period, and they include the repayment rate on contemporaneously maturing debt, z_t, if political decision makers can issue debt and commit to repay.

Let μ_t denote the set of state variables in period t that determine (together with $\mathcal{P}$) which competitive equilibria are implementable. The elements contained in μ_t are fully described by κ_t and ${}_tp_{\geq t}$ but they differ depending on the policy regime in place. First, with explicit government debt but without commitment to debt repayment, $\mu_t = (\kappa_t, {}_tp_{\geq t}) = (\kappa_t, \{\{g_s^{ic}(\cdot)\}_{i\in\mathcal{I}_s}\}_{s\geq t})$. In this case, the asset ownership structures $\{a_t^i, b_t^i\}_{i\in\mathcal{I}_t}$ (contained in κ_t) are separately included in μ_t because they determine the extent to which political decision makers may affect the relative wealth positions of households by choosing the repayment rate z_t. Second, with explicit government debt and with commitment to debt repayment, political decision makers cannot affect the financial wealth of households ex post implying $\mu_t = (\kappa_t \setminus \{a_t^i, b_t^i\}_{i\in\mathcal{I}_t}, \{f_t^i\}_{i\in\mathcal{I}_t}, \{\{g_s^{ic}(\cdot)\}_{i\in\mathcal{I}_s}\}_{s\geq t})$. Finally, in a setup without government debt, $\mu_t = (\kappa_t \setminus \{b_t^i\}_{i\in\mathcal{I}_t}, \{\{g_s^{ic}(\cdot)\}_{i\in\mathcal{I}_s}\}_{s\geq t})$ and b_t^i (as well as $\bar{b}_t$) equals zero for all $i \in \mathcal{I}_t$ and all t.

By construction, the state variables contained in μ_t and the policy choices $p^{t-1} = (p_t, p^t)$ completely characterize a competitive equilibrium. With some slight abuse of notation, we can therefore denote a competitive equilibrium by $\mathrm{CE}(\mu_t, p^{t-1})$ rather than $\mathrm{CE}(\kappa_t, p_{\geq t})$. Accordingly, we modify Definition 2 of economic equivalence. For convenience, we also extend the Definition of economic equivalence to sets:

Definition 3. A state and sequence of policy choices, (μ_t, p^{t-1}), is *economically equivalent* to another state and sequence of policy choices, (μ'_t, p'^{t-1}), if

i. (μ_t, p^{t-1}) supports a competitive equilibrium $\mathrm{CE}(\mu_t, p^{t-1})$;

ii. (μ'_t, p'^{t-1}) supports a competitive equilibrium $\mathrm{CE}(\mu'_t, p'^{t-1})$;

iii. the two competitive equilibria are identical.

A state and a set of sequences of policy choices, $(\mu_t, \{p^{t-1}\})$, is *economically equivalent* to another state and set of sequences of policy choices, $(\mu'_t, \{p'^{t-1}\})$, if

i. for every $p^{t-1} \in \{p^{t-1}\}$ such that (μ_t, p^{t-1}) supports a competitive equilibrium, there exists a $p'^{t-1} \in \{p'^{t-1}\}$ such that (μ_t, p^{t-1}) is economically equivalent to (μ'_t, p'^{t-1});

ii. for every $p'^{t-1} \in \{p'^{t-1}\}$ such that (μ'_t, p'^{t-1}) supports a competitive equilibrium, there exists a $p^{t-1} \in \{p^{t-1}\}$ such that (μ_t, p^{t-1}) is economically equivalent to (μ'_t, p'^{t-1}).

An admissible continuation policy choice $p^t \in \mathcal{P}^t$ is *feasible* conditional on μ_{t+1} if p^t supports a competitive equilibrium $\mathrm{CE}(\mu_{t+1}, p^t)$. Let $\mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$ denote the set of admissible and feasible continuation policy choices conditional on μ_{t+1}. An admissible policy choice $p_t \in \mathcal{P}_t$ is feasible conditional on μ_t if there exists an admissible continuation policy $p^t \in \mathcal{P}^t$ such that $p^{t-1} = (p_t, p^t)$ supports a competitive equilibrium $\mathrm{CE}(\mu_t, p^{t-1})$. Let $\mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$ denote the set of admissible and feasible policy choices conditional on μ_t. Every admissible and feasible continuation policy choice at time 0, $p^{-1} = (p_0, p^0) \in \mathcal{P}^{-1}(\mu_0)$, and the allocation it supports correspond with a sequence of the state, $\{\mu_t\}_{t\geq 0}$. This sequence need not be unique.[12] Let $\mathcal{M}_t$ denote the set of values that the state may take in period t across all such admissible and feasible continuation policy choices.

Sequential decision making implies that policy choices in period t are functions of the economy's history. We assume that this history is only relevant insofar as it constrains the set of competitive equilibria as of period t conditional on μ_t that can be supported by admissible continuation policies. (Below, we will relax this assumption and consider an enlarged state space allowing for trigger strategies.) Accordingly, the state in the program of political decision makers in period t is given by μ_t, and the *policy function* $p_t(\cdot)$ is a mapping from $\mathcal{M}_t$ into $\bigcup_{\mu_t \in \mathcal{M}_t} \mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$. Similarly, a *continuation policy function* $p^t(\cdot)$ is a mapping from $\mathcal{M}_{t+1}$ into $\bigcup_{\mu_{t+1} \in \mathcal{M}_{t+1}} \mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$. To streamline notation, we define continuation policy functions not only for $t \geq 0$ but also for $t = -1$.

In line with the maintained assumption that policy does not enter household preferences, we assume that the objective function maximized by political decision makers is a function of consumption x only.[13]

[12]The sequence is not unique if the cross section of asset holdings is indeterminate as might be the case in a policy regime with debt and without commitment to debt repayment.

[13]This assumption can be relaxed. Crucially, the objective function must not depend on policy choices.

Assumption 2. The political objective function in period t is given by $\Omega_t(x)$.

We are now ready to state the definition of politico-economic equilibrium.

Definition 4. A *politico-economic equilibrium* as of period t conditional on $\mu_t \in \mathcal{M}_t$ as well as policy regime $\mathcal{P}$, denoted as PEE$(\mu_t, \mathcal{P})$ for short, consists of a sequence of policy functions $\{p_s(\cdot)\}_{s\geq t}$, a sequence of continuation policy functions $\{p^s(\cdot)\}_{s\geq t-1}$, policy choices $p^{\star t-1}$, prices $(q_t^\star, q^{\star t}, r_t^\star)$, household choices $(x_t^\star, x^{\star t})$, and production paths $\{\{y_s^{j\star}\}_{j\in\mathcal{J}}\}_{s\geq t}$ such that

i. policy functions are optimal subject to continuation policy functions:

$$p_s(\mu_s) \in \arg\max_{p_s\in\mathcal{P}_s(\mu_s)} \Omega_s({}_s x, x_s, x^s) \text{ s.t. } p^s = p^s(\mu_{s+1}) \text{ for all } \mu_s \in \mathcal{M}_s, s \geq t,$$

where (x_s, x^s) and μ_{s+1} correspond with CE$(\mu_s, (p_s, p^s))$;

ii. continuation policy functions are consistent with actual policy choices:

$$p^{s-1}(\mu_s) = (p_s(\mu_s), p^s(\mu_{s+1})) \text{ for all } \mu_s \in \mathcal{M}_s, s \geq t,$$

where μ_{s+1} corresponds with CE$(\mu_s, p^{s-1}(\mu_s))$;

iii. equilibrium policy choices $p^{\star t-1}$ are generated by the continuation policy function:

$$p^{\star t-1} = p^{t-1}(\mu_t);$$

iv. $(q_t^\star, q^{\star t}, r_t^\star)$, $(x_t^\star, x^{\star t})$ and $\{\{y_s^{j\star}\}_{j\in\mathcal{J}}\}_{s\geq t}$ constitute CE$(\mu_t, p^{\star t-1})$.

Returning to the motivating question, consider an "initial" policy regime $\mathcal{P}$ with associated politico-economic equilibrium PEE$(\mu_0, \mathcal{P})$, and a "new" policy regime $\mathcal{P}'$. We are interested in conditions that need to be satisfied to guarantee politico-economic equivalence as specified in the following Definition:

Definition 5. A state and policy regime, $(\mu_t, \mathcal{P})$, is *politico-economically equivalent* to another state and policy regime, $(\mu'_t, \mathcal{P}')$, if

i. $(\mu_t, \mathcal{P})$ supports a politico-economic equilibrium PEE$(\mu_t, \mathcal{P})$ with policy choices $p^{\star t-1}$;

ii. $(\mu'_t, \mathcal{P}')$ supports a politico-economic equilibrium PEE$(\mu'_t, \mathcal{P}')$ with policy choices $p^{'\star t-1}$;

iii. $(\mu_t, p^{\star t-1})$ is economically equivalent to $(\mu'_t, p^{'\star t-1})$.

Note that politico-economic equivalence is defined with respect to pairs of a state and policy regime whereas economic equivalence was defined with respect to pairs of a state and policy. This difference arises because policy is exogenous in competitive equilibrium but endogenous (and shaped by the policy regime) in politico-economic equilibrium.

A sufficient condition for politico-economic equivalence is that the choice set of political decision makers in the new regime satisfies two requirements. On the one hand, this choice set must be "sufficiently large" in the sense that political decision makers in the new regime can support those competitive equilibria that political decision makers in the initial regime choose to implement, on or off the equilibrium path. On the other hand, the choice set in the new regime must not be "too large." In particular, political decision makers in the new regime must not be able to support competitive equilibria that cannot be supported in the initial regime. If both requirements are satisfied, then political decision makers in the new regime will implement policies that support the same competitive equilibrium as in the initial regime. Conditions 2 and 3 below formalize these two requirements.

To compare the set of competitive equilibria that political decision makers may implement in the two regimes it is necessary to render states comparable across the regimes. Condition 1 below assures that this is possible.

Definition 6. For a state $\mu_t \in \mathcal{M}_t$ in the policy regime $\mathcal{P}$, an *associated state* μ'_t in the policy regime $\mathcal{P}'$ satisfies

i. ${}_t p'_{\geq t}$ is part of a ${}_t p' \in {}_t\mathcal{Q}'$;

ii. there exists a $p^{'t-1} \in \mathcal{Q}^{'t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to $(\mu'_t, p^{'t-1})$.

The set of states in the policy regime $\mathcal{P}'$ that are associated with $\mu_t \in \mathcal{M}_t$ is denoted $\tilde{\mathcal{M}}'_t(\mu_t)$.

The first part of Definition 6 requires that the predetermined policy instruments lie in the policy space of the new regime. Similarly, the continuation policy choice $p^{'t-1}$ in the second requirement in Definition 6 is constrained by the policy space $\mathcal{Q}'$ and not by the policy regime $\mathcal{P}'$. That is, while the policy choices must contain policy instruments available in the new regime, the numerical values of these instruments do not need to satisfy the admissibility restrictions present in the new regime. If $\tilde{\mathcal{M}}'_t(\mu_t)$ is empty for some μ_t, then the policy instruments in the new policy regime are not flexible enough to support the equilibrium allocation given μ_t in the initial policy regime, even disregarding numerical restrictions on the instruments.

Condition 1. The following holds true for all t:

i. $\mathcal{M}'_t \subseteq \cup_{\mu_t \in \mathcal{M}_t} \tilde{\mathcal{M}}'_t(\mu_t)$;

ii. $\tilde{\mathcal{M}}'_t(\mu_t) \neq \emptyset$ for all $\mu_t \in \mathcal{M}_t$;

iii. if $\mu_t, \hat{\mu}_t \in \mathcal{M}_t$ and $\mathrm{CE}(\mu_t, p^{t-1}(\mu_t)) \neq \mathrm{CE}(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$ then $\tilde{\mathcal{M}}'_t(\mu_t) \cap \tilde{\mathcal{M}}'_t(\hat{\mu}_t) = \emptyset$.

The first part of Condition 1 requires that every state in the new policy regime can be associated with a state in the initial regime, and the second part requires that for every state in the initial regime, there is a state in the new regime that can be associated with it. The third part of the Condition requires that a state in the new policy regime can be associated with more than one state in the initial regime only if the latter induce identical competitive equilibria.

We can then define an *equivalent continuation policy function* $\tilde{p}'^{t-1}(\cdot)$ that maps the state μ'_t which is associated with μ_t into a continuation policy choice $\tilde{p}'^{t-1}(\mu'_t) \in \mathcal{Q}'^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to $(\mu'_t, \tilde{p}'^{t-1}(\mu'_t))$. Similarly, we can define an *equivalent policy function* $\tilde{p}'_t(\cdot)$ that maps the state μ'_t into a policy choice $\tilde{p}'_t(\mu'_t) \in \mathcal{Q}'_t$ that corresponds to the time-t component of $\tilde{p}'^{t-1}(\mu'_t)$. Both functions have domain $\cup_{\mu_t \in \mathcal{M}_t} \tilde{\mathcal{M}}'_t(\mu_t)$. If policy instruments in the new policy regime are redundant then the equivalent continuation policy function and the equivalent policy function generally are correspondences rather than functions. For simplicity, we disregard this possibility when stating the following conditions.

Condition 2 formalizes the requirement that the choice set of political decision makers in the new regime be sufficiently large:

Condition 2. The following holds true for all $\mu'_t \in \mathcal{M}'_t$ and all t:

i. $\tilde{p}'_t(\mu'_t) \in \mathcal{P}'_t$.

Condition 3 formalizes the requirement that the choice set not be too large. It stipulates that every competitive equilibrium supported by μ'_t, an admissible period-t policy choice in the new policy regime and the economically equivalent continuation policy function, can also be supported in the initial regime:

Condition 3. The following holds true for all $\mu'_t \in \mathcal{M}'_t$ and all t, where $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t), \mu_t \in \mathcal{M}_t$:

i. If there exists a $p'_t \in \mathcal{P}'_t$ such that $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1})))$ supports the competitive equilibrium $\text{CE}(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1})))$ corresponding with μ'_{t+1}, then there exists a $p_t \in \mathcal{P}_t$ such that $(\mu_t, (p_t, p^t(\mu_{t+1})))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1})))$ where μ_{t+1} corresponds with the competitive equilibrium $\text{CE}(\mu_t, (p_t, p^t(\mu_{t+1})))$.

Note that under Condition 1, the μ_{t+1} and μ'_{t+1} in Condition 3 satisfy $\mu'_{t+1} \in \tilde{\mathcal{M}}'_{t+1}(\mu_{t+1})$ because economic equivalence of $(\mu_t, (p_t, p^t(\mu_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1})))$ with $p'_t \in \mathcal{P}'_t$ implies ${}_{t+1}p' \in {}_{t+1}\mathcal{Q}'$ as well as economic equivalence of $(\mu_{t+1}, p^t(\mu_{t+1}))$ and $(\mu'_{t+1}, \tilde{p}'^t(\mu'_{t+1}))$.

We can now state the politico-economic equivalence result:

Proposition 2. Consider a state and policy regime, $(\mu_0, \mathcal{P})$ with $\mu_0 \in \mathcal{M}_0$, that support a politico-economic equilibrium $\text{PEE}(\mu_0, \mathcal{P})$, and consider a new state and policy regime, $(\mu'_0, \mathcal{P}')$ with $\mu'_0 \in \tilde{\mathcal{M}}'_0(\mu_0)$. Suppose that Conditions 1–3 are satisfied. Then, under Assumptions 1–2, $(\mu_0, \mathcal{P})$ is politico-economically equivalent to $(\mu'_0, \mathcal{P}')$.

Proof. We show that there exists a politico-economic equilibrium in the new regime that consists of the policy and continuation policy functions $\{\tilde{p}'_t(\cdot), \tilde{p}'^{t-1}(\cdot)\}_{t\geq 0}$, the policy choices $p'^{\star -1} \equiv \tilde{p}'^{-1}(\mu'_0)$, and the same prices, household choices and production paths as in $\text{PEE}(\mu_0, \mathcal{P})$.

Conjecture that in the new regime in period t, political decision makers as well as the private sector expect future policy choices to be determined according to the continuation policy function $\tilde{p}'^t(\cdot)$. (From Condition 1, this function is well defined over the domain $\mathcal{M}'_{t+1}$). We claim that, under this conjecture, the policy function in the new regime is given by $\tilde{p}'_t(\cdot)$. To verify the claim by contradiction, suppose instead that the policy function is given by another function, $\pi'_t(\cdot)$ say, such that for some $\mu'_t \in \mathcal{M}'_t$ with $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$, $\mu_t \in \mathcal{M}_t$, the allocation in $\text{CE}(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1})))$ is strictly preferred over the allocation in $\text{CE}(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1})))$ (where μ'_{t+1} corresponds with the respective equilibrium) and $\pi'_t(\mu'_t) \in \mathcal{P}'_t$. From Condition 3, there exists an admissible policy choice $\pi_t \in \mathcal{P}_t$ in the initial regime such that $(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1})))$ is economically equivalent to $(\mu_t, (\pi_t, p^t(\mu_{t+1})))$ (where μ_{t+1} corresponds with the latter equilibrium). By definition of the policy function, $\text{CE}(\mu_t, p^{t-1}(\mu_t))$ is preferred (at least weakly) over $\text{CE}(\mu_t, (\pi_t, p^t(\mu_{t+1})))$. From Assumption 2, political decision makers in the new regime share this preference. From Condition 2, political decision makers in the new regime can support the former equilibrium by choosing $\tilde{p}'_t(\mu'_t)$ rather than $\pi'_t(\mu'_t)$. This establishes the desired contradiction and thus, verifies the claim.

We conclude that for all $\mu'_t \in \mathcal{M}'_t$ and all t, political decision makers in the new regime implement policy choices according to the policy function $\tilde{p}'_t(\cdot)$ if agents expect the continuation policy function $\tilde{p}'^t(\cdot)$. We show next that such expectations are consistent with equilibrium. As noted earlier, $\mu'_t \in \tilde{\mathcal{M}}'_t(\mu_t)$ as well as economic equivalence of $(\mu_t, p^{t-1}(\mu_t))$ and $(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1})))$ with $p_t(\mu_t) \in \mathcal{P}_t$ and $\tilde{p}'_t(\mu'_t) \in \mathcal{P}'_t$ implies that $\mu'_{t+1} \in \tilde{\mathcal{M}}'_{t+1}(\mu_{t+1})$. By induction, the above argument for period t therefore extends to subsequent periods and the conjectured expected continuation policy functions are consistent with the policy functions governing actual policy choices. Accordingly, the functions $\tilde{p}'_t(\cdot)$ and $\tilde{p}'^t(\cdot)$ satisfy the conditions of politico-economic equilibrium.

Economic equivalence of $(\mu'_0, (\tilde{p}'_0(\mu'_0), \tilde{p}'^0(\mu'_1)))$ and $(\mu_0, (p_0(\mu_0), p^0(\mu_1)))$ (where μ'_1 and μ_1 correspond with the respective equilibria) implies that the equilibrium policy choices in the new policy regime support the same competitive equilibrium as in the old policy regime. The result then follows. $\square$

Proposition 2 can be extended to accommodate trigger strategies sustained by non-fundamental state variables. Let ξ_t represent such a non-fundamental state variable and let $\mathcal{S}_t \subseteq \mathcal{P}^{t-1}$ denote a "proposed policy" or "suggested policy" that political decision makers in period t are confronted with. A proposed policy contains a single p^{t-1} if it prescribes a unique admissible policy choice in each period. Otherwise, the proposed policy contains multiple p^{t-1}. The non-fundamental state variable takes the value one if political decision makers in earlier periods implemented policies consistent with the proposed policies they were confronted with, and zero otherwise.

In addition to the state variables contained in μ_t, political decision makers in period t inherit a proposed policy $\mathcal{S}_t$ as well as the non-fundamental state variable ξ_t. Given ξ_t and $\mathcal{S}_t$, the policy choice p_t determines ξ_{t+1} according to the law of motion

$$\xi_0 = 1 \text{ and } \xi_{t+1} = \begin{cases} 1 & \text{if } \xi_t = 1 \text{ and } \exists p^t \in \mathcal{P}^t : (p_t, p^t) \in \mathcal{S}_t \\ 0 & \text{otherwise} \end{cases} . \tag{6}$$

Depending on the institutional setup, political decision makers in period t may choose to modify the policy suggested to successive political decision makers. Let $\mathbf{S}_{t+1}(\mathcal{S}_t)$ denote the set of proposed policies that political decision makers can choose from if they themselves are confronted with the proposed policy $\mathcal{S}_t$. Moreover, let $\langle\mathcal{S}_t\rangle \subseteq \mathcal{P}^t$ denote the policy choices contained in the set $\mathcal{S}_t$ that apply in period $t+1$ or later, $\langle\mathcal{S}_t\rangle = \{p^t | \; \exists p_t \in \mathcal{P}_t : (p_t, p^t) \in \mathcal{S}_t\}$. If political decision makers in period t may not modify future suggestions embedded in the proposed policy, then the choice set $\mathbf{S}_{t+1}(\mathcal{S}_t)$ contains a single element, $\langle\mathcal{S}_t\rangle$, and subsequent political decision makers are confronted with essentially the same suggestion as contemporaneous ones, $\mathcal{S}_{t+1} = \langle\mathcal{S}_t\rangle$. If, in contrast, the institutional setup allows political decision makers in period t to choose among a set of proposed policies, then $\mathbf{S}_{t+1}(\mathcal{S}_t)$ is a subset of the power set $2^{\langle\mathcal{S}_t\rangle}$, that is, it contains several elements each of which is a subset of $\langle\mathcal{S}_t\rangle$. For example, if political decision makers in period t may suggest a particular value for the policy choice in the subsequent period then $\mathbf{S}_{t+1}(\mathcal{S}_t) = \{\{p^t \in \langle\mathcal{S}_t\rangle | \; p_{t+1} = p^{sug}_{t+1}\}\}_{p^{sug}_{t+1} \in \mathcal{P}_{t+1}}$.

A trigger strategy is defined by $\mathbf{S} \equiv (\mathcal{S}_0, \mathbf{S}_1(\cdot), \mathbf{S}_2(\cdot), \mathbf{S}_3(\cdot), \ldots)$. Let $\mathbf{S}_{t+1}$ denote the set of proposed policies in period $t+1$ that can be generated under the trigger strategy. This set is defined recursively as $\mathbf{S}_{t+1} \equiv \bigcup_{\mathcal{S}_t \in \mathbf{S}_t} \mathbf{S}_{t+1}(\mathcal{S}_t)$ with $\mathbf{S}_0 = \mathcal{S}_0$.

A policy regime with trigger strategies is defined by $(\mathcal{P}, \mathbf{S})$. We denote the state in such a policy regime by $\phi_t \equiv (\mu_t, \xi_t, \mathcal{S}_t)$. (Both ξ_t and $\mathcal{S}_t$ become irrelevant state variables if $\xi_t = 0$.) The policy functions $p_t(\cdot)$ and $\mathcal{S}_{t+1}(\cdot)$ map $\Phi_t \equiv \mathcal{M}_t \times \{0,1\} \times \mathbf{S}_t$ into $\cup_{\mu_t \in \mathcal{M}_t} \mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$ and $\mathbf{S}_{t+1}$, respectively, and the continuation policy function $p^t(\cdot)$ maps Φ_{t+1} into $\cup_{\mu_{t+1} \in \mathcal{M}_{t+1}} \mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$. (There is no need to specify the continuation policy function for the choice of proposed policy.)

Definition 7. A *politico-economic equilibrium with trigger strategies* as of period t conditional on $\phi_t \in \Phi_t$ as well as policy regime $(\mathcal{P}, \mathbf{S})$, denoted as PEET$(\phi_t, \mathcal{P}, \mathbf{S})$ for short, consists of a sequence of policy functions $\{p_s(\cdot)\}_{s \geq t}$, a sequence of continuation policy functions $\{p^s(\cdot)\}_{s \geq t-1}$, a sequence of proposed policy functions $\{\mathcal{S}_s(\cdot)\}_{s \geq t+1}$, policy choices $p^{\star t-1}$, prices $(q^\star_t, q^{\star t}, r^\star_t)$, household choices $(x^\star_t, x^{\star t})$, and production paths $\{\{y^{j\star}_s\}_{j \in \mathcal{J}}\}_{s \geq t}$ such that

i. policy and proposed policy functions are optimal subject to continuation policy functions:

$$(p_s, \mathcal{S}_{s+1})(\phi_s) \in \arg \max_{p_s \in \mathcal{P}_s(\mu_s), \mathcal{S}_{s+1} \in \mathbf{S}_{s+1}(\mathcal{S}_s)} \Omega_s({}_s x, x_s, x^s) \text{ s.t. } p^s = p^s(\phi_{s+1}) \text{ for all } \phi_s \in \Phi_s, s \geq t,$$

where (x_s, x^s) and μ_{s+1} correspond with CE$(\mu_s, (p_s, p^s))$ and ξ_{s+1} follows from (6);

ii. continuation policy functions are consistent with actual policy choices:

$$p^{s-1}(\phi_s) = (p_s(\phi_s), p^s(\mu_{s+1}, \xi_{s+1}, \mathcal{S}_{s+1}(\phi_s))) \text{ for all } \phi_s \in \Phi_s, s \geq t,$$

where μ_{s+1} corresponds with CE$(\mu_s, p^{s-1}(\phi_s))$ and ξ_{s+1} follows from (6);

iii. equilibrium policy choices $p^{\star t-1}$ are generated by the continuation policy function:

$$p^{\star t-1} = p^{t-1}(\phi_t);$$

iv. $(q^\star_t, q^{\star t}, r^\star_t)$, $(x^\star_t, x^{\star t})$ and $\{\{y^{j\star}_s\}_{j \in \mathcal{J}}\}_{s \geq t}$ constitute CE$(\mu_t, p^{\star t-1})$.

In the presence of trigger strategies, the definitions of politico-economic equivalence and associated states need slight adjustment (cf. Definitions 5 and 6):

Definition 8. A state and policy regime with trigger strategies, $(\phi_t, \mathcal{P}, \mathbf{S})$, is *politico-economically equivalent* to another state and policy regime with trigger strategies, $(\phi'_t, \mathcal{P}', \mathbf{S}')$, if

i. $(\phi_t, \mathcal{P}, \mathbf{S})$ supports a politico-economic equilibrium with trigger strategies $\mathrm{PEET}(\phi_t, \mathcal{P}, \mathbf{S})$ with policy choices $p^{\star t-1}$;

ii. $(\phi'_t, \mathcal{P}', \mathbf{S}')$ supports a politico-economic equilibrium with trigger strategies $\mathrm{PEET}(\phi'_t, \mathcal{P}', \mathbf{S}')$ with policy choices $p'^{\star t-1}$;

iii. $(\mu_t, p^{\star t-1})$ is economically equivalent to $(\mu'_t, p'^{\star t-1})$.

Definition 9. For a state $\phi_t \in \Phi_t$ in the policy regime with trigger strategies $(\mathcal{P}, \mathbf{S})$, an *associated state* ϕ'_t in the policy regime with trigger strategies $(\mathcal{P}', \mathbf{S}')$ satisfies

i. ${}_tp'_{\geq t}$ is part of a ${}_tp' \in {}_t\mathcal{Q}'$;

ii. there exists a $p'^{t-1} \in \mathcal{Q}'^{t-1}$ such that $(\mu_t, p^{t-1}(\phi_t))$ is economically equivalent to (μ'_t, p'^{t-1});

iii. $\xi'_t = \xi_t$;

iv. $\mathcal{S}'_t \subseteq \mathcal{Q}'^{t-1}$;

v. $(\mu_t, \mathcal{S}_t)$ is economically equivalent to $(\mu'_t, \mathcal{S}'_t)$.

The set of states in the policy regime with trigger strategies $(\mathcal{P}', \mathbf{S}')$ that are associated with $\phi_t \in \Phi_t$ is denoted $\tilde{\Phi}'_t(\phi_t)$.

The first two requirements in Definition 9 parallel the requirements in Definition 6. The third requirement postulates that the state variables summarizing adherence to proposed policies in the past be identical across regimes. The final two requirements guarantee that the proposed policies allow for the same set of competitive equilibria across regimes. If $\tilde{\Phi}'_t(\phi_t)$ is empty for some ϕ_t, then the policy instruments or the trigger strategy in the new policy regime are not flexible enough to support the equilibrium allocation given μ_t in the initial policy regime, even disregarding numerical restrictions on the instruments.

Conditions 1–3 may now be adjusted accordingly.

Condition 4. The following holds true for all t:

i. $\Phi'_t \subseteq \cup_{\phi_t \in \Phi_t} \tilde{\Phi}'_t(\phi_t)$;

ii. $\tilde{\Phi}'_t(\phi_t) \neq \emptyset$ for all $\phi_t \in \Phi_t$;

iii. if $\phi_t \equiv (\mu_t, \xi_t, \mathcal{S}_t), \hat{\phi}_t \equiv (\hat{\mu}_t, \hat{\xi}_t, \hat{\mathcal{S}}_t) \in \Phi_t$ and $\mathrm{CE}(\mu_t, p^{t-1}(\phi_t)) \neq \mathrm{CE}(\hat{\mu}_t, p^{t-1}(\hat{\phi}_t))$ then $\tilde{\Phi}'_t(\phi_t) \cap \tilde{\Phi}'_t(\hat{\phi}_t) = \emptyset$.

If Condition 4 is satisfied, we can again define an equivalent continuation policy function $\tilde{p}'^{t-1}(\cdot)$ and equivalent policy function $\tilde{p}'_t(\cdot)$ that map the state ϕ'_t into a continuation policy choice $\tilde{p}'^{t-1}(\phi'_t) \in \mathcal{Q}'^{t-1}$ and a policy choice $\tilde{p}'_t(\phi'_t) \in \mathcal{Q}'_t$, respectively. Both functions have domain $\cup_{\phi_t \in \Phi_t} \tilde{\Phi}'_t(\phi_t)$. Moreover, we can define an *equivalent proposed policy function* $\tilde{\mathcal{S}}'_{t+1}(\cdot)$ that maps the state $\phi'_t \in \cup_{\phi_t \in \Phi_t} \tilde{\Phi}'_t(\phi_t)$ into a proposed policy $\tilde{\mathcal{S}}'_{t+1}(\phi'_t) \subseteq \mathcal{Q}'^t$ by letting $\tilde{\mathcal{S}}'_{t+1}(\phi'_t)$ be the set of continuation policies p'^t in the new policy space such that $(\mu_t, \mathcal{S}_{t+1}(\phi_t))$ is economically equivalent to $(\mu'_t, \tilde{\mathcal{S}}'_{t+1}(\phi'_t))$ with $\phi'_t \in \tilde{\Phi}'_t(\phi_t)$. As before, we disregard the possibility that the equivalent continuation policy function or the equivalent policy function are correspondences.

Condition 5 formalizes the requirement that the choice set of political decision makers in the new regime be sufficiently large. Relative to Condition 2, Condition 5 adds the requirement that the equivalent proposed policy be admissible in the new policy regime:

Condition 5. The following holds true for all $\phi'_t \in \Phi'_t$ and all t:

i. $\tilde{p}'_t(\phi'_t) \in \mathcal{P}'_t$ and $\tilde{\mathcal{S}}'_{t+1}(\phi'_t) \in \mathbf{S}'_{t+1}(\mathcal{S}'_t)$.

Condition 6 formalizes the requirement that the choice set not be too large:

Condition 6. The following holds true for all $\phi'_t \in \Phi'_t$ and all t, where $\phi'_t \in \tilde{\Phi}'_t(\phi_t)$, $\phi_t \in \Phi_t$:

i. If there exists a $p'_t \in \mathcal{P}'_t$ and $\mathcal{S}'_{t+1} \in \mathbf{S}'_{t+1}(\mathcal{S}'_t)$ such that $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ supports the competitive equilibrium $\mathrm{CE}(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ corresponding with $\phi'_{t+1} = (\mu'_{t+1}, \xi'_{t+1}, \mathcal{S}'_{t+1})$, then there exists a $p_t \in \mathcal{P}_t$ and $\mathcal{S}_{t+1} \in \mathbf{S}_{t+1}(\mathcal{S}_t)$ such that

   (i) $(\mu_t, (p_t, p^t(\phi_{t+1})))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ and

   (ii) $(\mu_{t+1}, \mathcal{S}_{t+1})$ is economically equivalent to $(\mu'_{t+1}, \mathcal{S}'_{t+1})$

   where $\phi_{t+1} = (\mu_{t+1}, \xi_{t+1}, \mathcal{S}_{t+1})$ corresponds with the competitive equilibrium $\mathrm{CE}(\mu_t, (p_t, p^t(\phi_{t+1})))$.

Note that under Condition 4, the ϕ_{t+1} and ϕ'_{t+1} in Condition 6 satisfy $\phi'_{t+1} \in \tilde{\Phi}'_{t+1}(\phi_{t+1})$, because of three facts. First, economic equivalence of $(\mu_t, (p_t, p^t(\phi_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ with $p'_t \in \mathcal{P}'_t$ implies ${}_{t+1}p' \in {}_{t+1}\mathcal{Q}'$ as well as economic equivalence of $(\mu_{t+1}, p^t(\phi_{t+1}))$ and $(\mu'_{t+1}, \tilde{p}'^t(\phi'_{t+1}))$, in parallel to the case without trigger strategies. Second, $\xi_{t+1} = \xi'_{t+1}$ because of $\phi'_t \in \tilde{\Phi}'_t(\phi_t)$ and of economic equivalence of $(\mu_t, (p_t, p^t(\phi_{t+1})))$ and $(\mu'_t, (p'_t, \tilde{p}'^t(\phi'_{t+1})))$ on the one hand and $(\mu_t, \mathcal{S}_t)$ and $(\mu'_t, \mathcal{S}'_t)$ on the other. Third, $(\mu_{t+1}, \mathcal{S}_{t+1})$ is economically equivalent to $(\mu'_{t+1}, \mathcal{S}'_{t+1})$ by Condition 6 (ii).

We can now state the extended equivalence result:

Proposition 3. Consider a state and policy regime with trigger strategies, $(\phi_0, \mathcal{P}, \mathbf{S})$ with $\phi_0 \in \Phi_0$, that support a politico-economic equilibrium with trigger strategies $\mathrm{PEET}(\phi_0, \mathcal{P}, \mathbf{S})$, and consider a new state and policy regime with trigger strategies, $(\phi'_0, \mathcal{P}', \mathbf{S}')$ with $\phi'_0 \in \tilde{\Phi}'_0(\phi_0)$. Suppose that Conditions 4–6 are satisfied. Then, under Assumptions 1–2, $(\phi_0, \mathcal{P}, \mathbf{S})$ is politico-economically equivalent to $(\phi'_0, \mathcal{P}', \mathbf{S}')$.

Proof. We show that there exists a politico-economic equilibrium in the new regime that consists of the policy, continuation policy and proposed policy functions $\{\tilde{p}'_t(\cdot), \tilde{p}'^{t-1}(\cdot), \tilde{\mathcal{S}}'_{t+1}(\cdot)\}_{t\geq 0}$, the policy choices $p'^{\star -1} \equiv \tilde{p}'^{-1}(\phi'_0)$, and the same prices, household choices and production paths as in PEET$(\mu_0, \mathcal{P}, \mathbf{S})$.

The logic of the proof follows the one of Proposition 2. Assumption 2 and Conditions 4–6 imply that, for all $\phi'_t \in \Phi'_t$ and all t, political decision makers in the new regime implement policy and proposed policy choices according to the policy and proposed policy functions $\tilde{p}'_t(\cdot)$ and $\tilde{\mathcal{S}}'_{t+1}(\cdot)$ respectively, if agents expect the continuation policy function $\tilde{p}'^t(\cdot)$. Such expectations are consistent with equilibrium. For, as noted above, $\phi'_t \in \Phi'_t(\phi_t)$ and implementation of equivalent (proposed) policies implies $\phi'_{t+1} \in \Phi'_{t+1}(\phi_{t+1})$. By induction, the continuation policy functions are consistent with the policy functions governing actual policy choices. Accordingly, the functions $\tilde{p}'_t(\cdot)$, $\tilde{p}'^t(\cdot)$ and $\tilde{\mathcal{S}}'_{t+1}(\cdot)$ satisfy the conditions of politico-economic equilibrium.

Economic equivalence of $(\mu'_0, (\tilde{p}'_0(\phi'_0), \tilde{p}'^0(\phi'_1)))$ and $(\mu_0, (p_0(\phi_0), p^0(\phi_1)))$ (where ϕ'_1 and ϕ_1 correspond with the respective equilibria) implies that the equilibrium policy choices in the new policy regime support the same competitive equilibrium as in the old policy regime. The result then follows. □

5 Applications

We now turn to several applications of the theoretical framework, based on specific models in the literature. We discuss these applications in order of increasing complexity. In addition to the notation introduced in the previous sections, and unless otherwise noted, we let w_t, l_t and l^i_t denote the wage, labor supply of the representative worker, and labor supply of type i (or household i) in period t, respectively; k_t and c_t the capital stock per worker and consumption, respectively; $\mathcal{I}^w_t$ and $\mathcal{I}^r_t$ the set of workers and retirees, respectively; $g^w_t(\cdot)$ and $g^r_t(\cdot)$ tax functions imposed on workers and retirees, respectively; and ν_t the gross population growth rate in period t. All applications feature a single produced good.

5.1 Bassetto and Kocherlakota (2004)

Bassetto and Kocherlakota (2004) show that changes in the timing of tax collections (and thus, the path of government debt) may leave the equilibrium allocation unaffected even if taxes are distorting. In their two-period model, the government can commit to debt repayment and may levy distorting labor income taxes where second-period taxes can be based on labor income generated in the first or second period. Bassetto and Kocherlakota (2004) show that a policy change does not alter the equilibrium allocation as long as it leaves the mapping from labor supply paths into the present value of lifetime tax payments unchanged. We illustrate their result in the context of a simplified model with proportional labor income taxes. Rather than positing a government spending requirement in the second period, as the authors do, we assume that some government debt is outstanding in the initial period. Moreover, rather than assuming full commitment to all policy instruments as Bassetto and Kocherlakota (2004) do and as would directly imply politico-economic equivalence of regimes, we assume instead that governments can only commit to a subset of the policy instruments.

Table 1 summarizes the two policy regimes that we compare. In line with the assumption made by Bassetto and Kocherlakota (2004), we allow the repayment rate on government debt in period 1 to be negative that is, we allow the government to save. The "initial" regime with taxes on contemporaneous labor income only is characterized in the left column of the table, the "new" regime with taxes on contemporaneous and lagged income in the right column. The repayment rate on initially outstanding debt is pre-committed in both regimes and must be positive while tax rates may be positive or negative; political decision makers choose the first-period tax and second-period repayment rate in both regimes; they choose one second-period tax rate in the initial regime but two tax rates (one of which applies to income generated in the first period) in the new regime; and no trigger strategies are employed. Since there is commitment to debt repayment but not to taxes, the state in the initial regime is given by $\mu_t = (\kappa_t \setminus \{a_t^i, b_t^i\}_{i\in\mathcal{I}_t}, \{f_t^i\}_{i\in\mathcal{I}_t})$; since there is no capital in the economy and taxes do not depend on lagged prices or quantities, $\kappa_t = \emptyset$; and since households are identical and do not hold assets other than government debt, the state variables $\{f_t^i\}_{i\in\mathcal{I}_t}$ can be summarized by $\bar{b}_t z_t$. In the new regime, the situation is identical except that lagged labor income enters κ_1' because taxes may depend on it.

The middle and lower parts of Table 1 summarize the economic-equivalence cross-regime restrictions (see Proposition 1). We refer to these restrictions as "EE restrictions." From the perspective of the initial period, economic equivalence requires identical present values of payments between households and the government across regimes, conditional on wages, the interest rate and labor supply in the initial equilibrium; identical marginal tax rates in both periods; as well as identical government cash flows in both periods.[14] These requirements reduce to $\tau_0 = \tau_0' + r_{0,1}\tau_{1,0}'$; $\tau_1 = \tau_{1,1}'$; $\bar{b}_0 z_0 = \bar{b}_0' z_0'$; and $(\tau_0' - \tau_0)w_0 l_0 = r_{0,1}(\bar{b}_1 z_1 - \bar{b}_1' z_1')$ or the expressions reported in the Table. Since the new regime allows for policy choices satisfying these requirements, Bassetto and Kocherlakota's (2004) finding is verified. From the perspective of period 1, economic equivalence requires $\bar{b}_1 z_1 - \tau_1 w_1 l_1 = \bar{b}_1' z_1' - \tau_{1,0}' w_0' l_0' - \tau_{1,1}' w_1 l_1$ and $\tau_1 = \tau_{1,1}'$ which simplifies to the expressions reported at the bottom of the Table.

As for politico-economic equivalence, note that for a given μ_1 there exist multiple associated states μ_1' indexed by the values of $w_0' l_0'$ and $\bar{b}_1' z_1'$. Intuitively, for essentially arbitrary values of $w_0' l_0'$ and $\bar{b}_1' z_1'$, households' financial wealth after non-distorting taxes (that is, after imposing the tax rate $\tau_{1,0}'$ which can be chosen appropriately) can be set equal to the value in the initial regime, $\bar{b}_1 z_1$, and the distorting labor income tax rate $\tau_{1,1}'$ then can match the equilibrium tax rate τ_1 implemented in state μ_1. As a consequence of this multiplicity, different states in the initial regime, $\mu_1 = \bar{b}_1 z_1$ and $\hat{\mu}_1 = \bar{b}_1 z_1 + \epsilon$ say with $\epsilon \neq 0$, are associated with the same μ_1' in the new regime although they clearly give rise to different equilibrium allocations in the initial regime. Condition 1 therefore is violated

[14]Formally, economic equivalence requires $\bar{b}_0 z_0 - \tau_0 w_0 l_0 - r_{0,1}\tau_1 w_1 l_1 = \bar{b}_0' z_0' - \tau_0' w_0 l_0 - r_{0,1}\tau_{1,0}' w_0 l_0 - r_{0,1}\tau_{1,1}' w_1 l_1$; $\tau_0 = \tau_0' + r_{0,1}\tau_{1,0}'$ and $\tau_1 = \tau_{1,1}'$; as well as $\bar{b}_0 z_0 - \bar{b}_0' z_0' = (\tau_0 - \tau_0')w_0 l_0 + r_{0,1}(\bar{b}_1 z_1 - \bar{b}_1' z_1')$ and $\bar{b}_1 z_1 - \bar{b}_1' z_1' = (\tau_1 - \tau_{1,1}')w_1 l_1 - \tau_{1,0}' w_0 l_0$.

$g_t(w_t, l_t^i) = \tau_t w_t l_t^i,\ t = 0, 1$	$g'_0(w'_0, l_0^{i'}) = \tau'_0 w'_0 l_0^{i'}$
	$g'_1(w'_0, l_0^{i'}, w'_1, l_1^{i'}) = \tau'_{1,0} w'_0 l_0^{i'} + \tau'_{1,1} w'_1 l_1^{i'}$
$\mathcal{P}_0 = \{(\tau_0, z_1) \in \mathbb{R}^2\},\ z_0$ given	$\mathcal{P}'_0 = \{(\tau'_0, z'_1) \in \mathbb{R}^2\},\ z'_0$ given
$\mathcal{P}_1 = \{\tau_1 \in \mathbb{R}\}$	$\mathcal{P}'_1 = \{(\tau'_{1,0}, \tau'_{1,1}) \in \mathbb{R}^2\}$
$\mathbf{S} = \emptyset$	$\mathbf{S}' = \emptyset$
$\mu_0 = \bar{b}_0 z_0$	$\mu'_0 = \bar{b}'_0 z'_0$
$\mu_1 = \bar{b}_1 z_1$	$\mu'_1 = (w'_0 l'_0, \bar{b}'_1 z'_1)$
EE restrictions, μ'_0	$\bar{b}'_0 z'_0 = \bar{b}_0 z_0$
p'^{-1}	τ'_0 free
	$z'_1 = z_1 \frac{\bar{b}_1}{\bar{b}'_1} + (\tau_0 - \tau'_0) \frac{w_0 l_0}{\bar{b}'_1 r_{0,1}}$
	$\tau'_{1,0} = \frac{\tau_0 - \tau'_0}{r_{0,1}}$
	$\tau'_{1,1} = \tau_1$
EE restrictions, μ'_1	$w'_0 l'_0, \bar{b}'_1 z'_1$ free
p'^0	$\tau'_{1,0} = (\bar{b}'_1 z'_1 - \bar{b}_1 z_1)/w'_0 l'_0$
	$\tau'_{1,1} = \tau_1$

Table 1: Bassetto and Kocherlakota's (2004) model with proportional taxes.

and politico-economic equivalence cannot be guaranteed.

The situation is different in a modified setup where political decision makers in the new regime commit in period 0 to the tax rate $\tau'_{1,0}$ such that all taxes under the control of political decision makers in periods 0 or 1 are distorting, see Table 2. Since the top of the Laffer curve and thus, the maximum amount of feasible debt in period 0 coincides across regimes, $\mathcal{M}'_0 \subseteq \cup_{\mu_0 \in \mathcal{M}_0} \tilde{\mathcal{M}}'_0(\mu_0)$ and Condition 1 is satisfied in period 0. Condition 2 also holds since at least some (among the continuum of) equivalent policies $\tilde{p}'_0(\mu'_0)$ are admissible, for example the one setting $\tau'_{1,0} = 0$. Condition 3 is satisfied as well in period 0. To see this, consider a feasible policy choice in the new regime, $p'_0 = (\tau'_0, z'_1, \tau'_{1,0})$. Conditional on $\mu'_0 = \bar{b}'_0 z'_0$, this policy choice together with the equivalent continuation policy function $\tilde{p}'^0(\cdot)$ supports some allocation with prices $r'_{0,1}, w'_0$ and labor supply l'_0. For Condition 3 to be satisfied, it must be true that $p_0 = (\tau_0, z_1) = (\tau'_0 + r'_{0,1}\tau'_{1,0}, z'_1 \bar{b}'_1/\bar{b}_1 - \tau'_{1,0} w'_0 l'_0/\bar{b}_1)$ is admissible in the initial regime.[15] This is the case.

In period 1, for a given μ_1, there are still multiple associated states μ'_1 indexed by the values of $w'_0 l'_0$ and $\bar{b}'_1 z'_1$. The value of the third state variable in the new regime, $\tau'_{1,0}$, follows from the two others because the associated state must satisfy $\tau'_{1,0} = (\bar{b}'_1 z'_1 - \bar{b}_1 z_1)/w'_0 l'_0$. Since this value of $\tau'_{1,0}$ depends on $\bar{b}_1 z_1$, different states in the initial regime, $\mu_1 = \bar{b}_1 z_1$ and $\hat{\mu}_1 = \bar{b}_1 z_1 + \epsilon$ say with $\epsilon \neq 0$, necessarily are associated with disjoint sets of states in

[15] The two restrictions imply that marginal tax rates on first-period labor income are identical and that government cash flows coincide across regimes.

the new regime, $\tilde{\mathcal{M}}'_1(\mu_1) \cap \tilde{\mathcal{M}}'_1(\hat{\mu}_1) = \emptyset$, implying that Condition 1 is satisfied in period 1. Conditions 2 and 3 are satisfied as well since setting $\tau'_{1,1}$ at value τ_1 necessarily is admissible if τ_1 is admissible, and vice versa. Politico-economic equivalence therefore is guaranteed.

Table 1 applies, except for ...	$\mathcal{P}'_0 = \{(\tau'_0, z'_1, \tau'_{1,0}) \in \mathbb{R}^3\},\ z'_0$ given $\mathcal{P}'_1 = \{\tau'_{1,1} \in \mathbb{R}\}$ $\mu'_1 = (w'_0 l'_0, \bar{b}'_1 z'_1, \tau'_{1,0})$
EE restrictions, μ'_1 p'^0	$w'_0 l'_0, \bar{b}'_1 z'_1$ free, $\tau'_{1,0} = (\bar{b}'_1 z'_1 - \bar{b}_1 z_1)/w'_0 l'_0$ $\tau'_{1,1} = \tau_1$

Table 2: Bassetto and Kocherlakota's (2004) model with proportional taxes and commitment to $\tau'_{1,0}$.

5.2 Forni (2005) and Gonzalez-Eiras and Niepelt (2008)

Gonzalez-Eiras and Niepelt (2008) analyze a two-period lived overlapping generations economy with capital accumulation, elastic labor supply, within-cohort homogeneity and no government debt. The authors consider a pay-as-you-go financed social security system with a proportional labor income tax at rate τ_t funding transfers to retirees, and a second proportional labor income tax at rate θ_t financing a refund to workers. Lump-sum taxes on workers and retirees are excluded implying that tax rates cannot be negative. There is no commitment. Gonzalez-Eiras and Niepelt (2008) assume that policy functions depend on fundamental state variables only. Since admissible taxes do not depend on past quantities or prices, and since newborn households do not hold assets and retirees are homogenous, the state reduces to the capital stock, see the left column of Table 3. Positing a probabilistic-voting objective, Gonzalez-Eiras and Niepelt (2008) show that strictly positive tax rates are sustained in politico-economic equilibrium.

Consider a new policy regime with taxes and debt, see the right column of Table 3. In parallel with the initial regime, political decision makers cannot commit nor employ trigger strategies. Labor income taxes have the same form as in the initial regime while old-age benefits equal zero. Lump-sum taxes are ruled out implying that the tax rate financing the refund to workers must be positive. Government debt policy is specified by the endogenous repayment rate z'_t and the exogenous debt stock per retiree $\bar{b}'_t > 0$ for all t.[16] To streamline notation, we do not distinguish between debt repayment to retirees in periods $t \geq 1$ that previously purchased the debt, and "debt repayment" to retirees in the initial period who did not purchase the debt but simply receive a transfer.

[16] A convenient choice for $\bar{b}'_t$ is $\bar{b}'_t = \tau^\star_t \nu_t w^\star_t l^\star_t$. This value implies that the equilibrium repayment rate equals unity if the economically equivalent debt policy is implemented in equilibrium.

$g_t^w(w_t, l_t^i, l_t) = \tau_t w_t l_t^i + \theta_t w_t (l_t^i - l_t),\ t \geq 0$	$g_t^{w'}(w_t', l_t^{i'}, l_t') = \tau_t' w_t' l_t^{i'} + \theta_t' w_t' (l_t^{i'} - l_t'),\ t \geq 0$
$g_t^r(w_t, l_t) = -\nu_t \tau_t w_t l_t,\ t \geq 0$	$g_0^{r'} = -\bar{b}_0' z_0'$
	$g_t^{r'} = 0,\ t \geq 1$
$\mathcal{P}_t = \{(\tau_t, \theta_t) \in \mathbb{R}_+^2\},\ t \geq 0$	$\mathcal{P}_t' = \{(\tau_t', \theta_t', z_t') \in \mathbb{R} \times \mathbb{R}_+^2\},\ t \geq 0$
$\mathbf{S} = \emptyset$	$\mathbf{S}' = \emptyset$
$\mu_t = k_t,\ t \geq 0$	$\mu_t' = (k_t', \bar{b}_t'),\ t \geq 0$
EE restrictions, μ_t'	$k_t' = k_t$ ($\bar{b}_t'$ exogenous)
p'^{t-1}	$z_s' = \tau_s \nu_s w_s l_s / \bar{b}_s',\ s \geq t$
	$\tau_s' = \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}\nu_{s+1}w_{s+1}l_{s+1}}{w_s l_s},\ s \geq t$
	$\theta_s' = \theta_s + \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}\nu_{s+1}w_{s+1}l_{s+1}}{w_s l_s},\ s \geq t$

Table 3: Gonzalez-Eiras and Niepelt's (2008) model.

Since retirees within a cohort are homogeneous, the cross section of debt holdings among living households is fully characterized by $\bar{b}_t'$. The EE restrictions require identical capital stocks, marginal tax rates, present values of tax payments and government cash flows across regimes.

As for politico-economic equivalence, there exists a unique associated state in the new regime for any state in the initial regime and Condition 1 is satisfied. Condition 2 is satisfied as well since the equivalent policy necessarily is admissible, $\tilde{p}_s'(\mu_s') \in \mathcal{P}_s'$, as τ_s' may be negative. In contrast, Condition 3 is not satisfied, and as a consequence, politico-economic equivalence cannot be guaranteed. To see this, consider a feasible policy choice in the new regime, $p_t' = (\tau_t', \theta_t', z_t')$. Conditional on $\mu_t' = k_t'$ and together with the equivalent continuation policy function $\tilde{p}'^t(\cdot)$, this policy choice supports some allocation with wages and labor supply $w_s', l_s', s \geq t$. Suppose that $\tau_t' + \theta_t'$ is "small" and z_t' "large" (such a choice is admissible and feasible in the new regime). For Condition 3 to be satisfied, it would have to be true that $p_t = (\tau_t, \theta_t)$ with $\theta_t = \tau_t' + \theta_t' - z_t' \bar{b}_t' / (\nu_t w_t' l_t')$ is admissible in the initial regime.[17] But under the maintained assumption, the implied θ_t would be negative, violating the admissibility restriction in the initial regime.

Importantly, this negative result is easily overturned if the assumptions concerning the political or economic environment are slightly modified. We briefly discuss three such modifications. First, consider an institutional setting where political competition leads policy choices to converge to those preferred by the "median" voter (in contrast to Gonzalez-Eiras and Niepelt's (2008) assumption of probabilistic voting). Suppose that this is always a young household, and restrict attention to (the limit of) a finite horizon economy. The equilibrium social security tax rate then satisfies $\tau_t = 0$ in all periods, implying $p^t(\mu_{t+1}) \equiv (\tau^t(\mu_{t+1}), \theta^t(\mu_{t+1})) = (\mathbf{0}, \theta^t(\mu_{t+1}))$ and thus, $\tilde{p}'^t(\mu_{t+1}') \equiv$

[17] The restriction implies that marginal tax rates as well as the disposable incomes of retirees are identical across regimes.

$(\tilde{\tau}'^t(\mu'_{t+1}), \tilde{\theta}'^t(\mu'_{t+1}), \tilde{z}'^t(\mu'_{t+1})) = (\mathbf{0}, \theta^t(\mu_{t+1}), \mathbf{0})$ where μ'_{t+1} is associated with μ_{t+1}. Consider a state μ'_t and an admissible policy choice in the debt regime, $p'_t \equiv (\tau'_t, \theta'_t, z'_t) \in \mathbb{R} \times \mathbb{R}^2_+$. Together with the equivalent continuation policy function $\tilde{p}'^t(\cdot)$, this policy choice is feasible if $(p'_t, \tilde{p}'^t(\mu'_{t+1})) = ((\tau'_t, \mathbf{0}), (\theta'_t, \theta^t(\mu_{t+1})), (z'_t, \mathbf{0}))$ supports a competitive equilibrium. Since $z'_{t+1} = 0$, this requires $z'_t = \tau'_t \nu_t w'_t l'_t / \bar{b}'_t$ to satisfy the government budget constraint. That is, any admissible and feasible policy choice and implied continuation policy in the debt regime is of the form $((z'_t \bar{b}'_t / \nu_t w'_t l'_t, \mathbf{0}), (\theta'_t, \theta^t(\mu_{t+1})), (z'_t, \mathbf{0}))$ with $z'_t \geq 0$. The economically equivalent policy in the social security regime satisfies $\tau_t \geq 0$ and therefore is admissible, implying that Condition 3 is satisfied.

Second, consider Gonzalez-Eiras and Niepelt's (2008) model with probabilistic voting but with inelastic labor supply. The policy instrument θ then is superfluous, see Table 4, and Condition 3 therefore is satisfied.

$g_t^w(w_t) = \tau_t w_t,\ t \geq 0$	$g_t^{w'}(w'_t) = \tau'_t w'_t,\ t \geq 0$
$g_t^r(w_t) = -\nu_t \tau_t w_t,\ t \geq 0$	$g_0^{r'} = -\bar{b}'_0 z'_0$
	$g_t^{r'} = 0,\ t \geq 1$
$\mathcal{P}_t = \{\tau_t \in \mathbb{R}_+\},\ t \geq 0$	$\mathcal{P}'_t = \{(\tau'_t, z'_t) \in \mathbb{R} \times \mathbb{R}_+\},\ t \geq 0$
$\mathbf{S} = \emptyset$	$\mathbf{S}' = \emptyset$
$\mu_t = k_t,\ t \geq 0$	$\mu'_t = (k'_t, \bar{b}'_t),\ t \geq 0$
EE restrictions, μ'_t	$k'_t = k_t$ ($\bar{b}'_t$ exogenous)
p'^{t-1}	$z'_s = \tau_s \nu_s w_s / \bar{b}'_s,\ s \geq t$
	$\tau'_s = \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1}}{w_s},\ s \geq t$

Table 4: Gonzalez-Eiras and Niepelt's (2008) model with inelastic labor supply.

Finally, consider the environment with inelastic labor supply (summarized in the left column of Table 4) in the case where a young median voter is politically decisive and the horizon of the economy is infinite. Forni (2005) shows that, in such an environment, an equilibrium may exist for certain parameter values in which the median voter supports strictly positive social security taxes if she anticipates the equilibrium policy function determining future social security benefits to be a decreasing function of the capital stock.[18] As in Gonzalez-Eiras and Niepelt's (2008) model with inelastic labor supply, politico-economic equivalence of the regime considered by Forni (2005) and another regime with taxes and debt (conditional on some initial capital stock) is guaranteed.[19]

[18] See Gonzalez-Eiras (2010) for a characterization of equilibrium beyond the range considered in Forni (2005).

[19] The equilibrium repayment rate on public debt in that alternative regime with debt is a decreasing function of the capital stock.

5.3 Tabellini (2000)

Tabellini (2000) analyzes a two-period lived overlapping generations economy with inelastic labor supply, exogenous labor productivity w_t and heterogeneous time endowments e_t^i among young households, and no capital nor government debt. The cohort average of e_t^i is normalized to unity and $e_t^i \in [\underline{e}, \bar{e}] \subset \mathbb{R}_+$. Tabellini (2000) considers a redistributive social security system with proportional income taxes levied on the young and a lump-sum benefit paid to the old. He assumes that political decision makers cannot commit their successors and do not have access to trigger strategies. Due to the heterogeneity within cohorts, the state includes the cross section of private asset holdings, see the left column of Table 5. Tabellini (2000) shows that, in a median voter framework, a coalition of poor young and old households may sustain a social security system whose size increases with the degree of inequality in e_t^i, but decreases with the constant rate of population growth, ν.[20]

$g_t^w(w_t) = \tau_t w_t e_t^i,\ t \geq 0$	$g_t^{w'}(w_t') = \tau_t' w_t' e_t^i,\ t \geq 0$
$g_t^r(w_t) = -\nu \tau_t w_t,\ t \geq 0$	$g_0^{r'} = -\bar{b}_0' z_0'$
	$g_t^{r'} = 0,\ t \geq 1$
$\mathcal{P}_t = \{\tau_t \in \mathbb{R}_+\},\ t \geq 0$	$\mathcal{P}_t' = \{(\tau_t', z_t') \in \mathbb{R} \times \mathbb{R}_+\},\ t \geq 0$
$\mathbf{S} = \emptyset$	$\mathbf{S}' = \emptyset$
$\mu_t = \{a_t^i\}_{i \in \mathcal{I}_t^r},\ t \geq 0$	$\mu_t' = \{a_t^{i'}, b_t^{i'}\}_{i \in \mathcal{I}_t^r},\ t \geq 0$
EE restrictions	cannot be satisfied for all $i \in \mathcal{I}_s^w,\ s \geq t$

Table 5: Tabellini's (2000) model.

Consider a new policy regime with proportional taxes and debt, no benefits, no commitment and no trigger strategies, see the right column of Table 5. The EE restrictions cannot be satisfied in this regime because it is impossible to replicate in the debt regime the type-dependent budget sets present in the social security regime. Formally, economic equivalence would require, among other conditions, that $\tau_s w_s e_s^i - \frac{r_{t,s+1}}{r_{t,s}} \tau_{s+1} \nu w_{s+1} = \tau_s' w_s e_s^i$ for all $i \in \mathcal{I}_s^w$ and $s \geq t$. For given τ_s, τ_{s+1}, these requirements cannot in general be satisfied. Economic equivalence fails because the tax-and-benefit system in the social security regime amounts to a linear tax function while in the debt regime, only proportional tax functions are admissible.

Consider then the effects of a linear tax function in the debt regime where lump sum taxes are not admissible. The EE restrictions can now be satisfied for all $i \in \mathcal{I}_s^w$ and $s \geq t$, see Table 6. However, Condition 1 iii. is not satisfied. To see this, note that for a given $\mu_t = \{a_t^i\}_{i \in \mathcal{I}_t^r}$ with $\int_i a_t^i \, di = 0$, the set of associated states $\tilde{\mathcal{M}}_t'(\mu_t)$ contains all those states

[20] Tabellini (2000) considers a situation where agents are altruistic towards both their children and parents. He assumes that altruism is sufficiently weak for private transfers not to arise in equilibrium, but sufficiently strong to affect political preferences.

$\mu_t' = \{a_t^{i'}, b_t^{i'}\}_{i\in\mathcal{I}_t^r}$ that satisfy $\int_i a_t^{i'}\ di = 0$, $\int_i (b_t^{i'} - \bar{b}_t')\ di = 0$ and $a_t^{i'} = a_t^i + (1 - b_t^{i'}/\bar{b}_t')\nu\tau_t w_t$ for all $i \in \mathcal{I}_t^r$ where τ_t is the equilibrium tax in the initial regime. Consider two states $\mu_t, \hat{\mu}_t \in \mathcal{M}_t$ with $\{a_t^i\}_i \neq \{\hat{a}_t^i\}_i$. Suppose that the equilibrium policy choices in those states are given by τ_t and $\hat{\tau}_t$, respectively, with $\tau_t \neq \hat{\tau}_t$ such that the equilibrium allocations conditional on μ_t and $\hat{\mu}_t$ differ. Condition 1 iii. then requires that $\tilde{\mathcal{M}}_t'(\mu_t) \cap \tilde{\mathcal{M}}_t'(\hat{\mu}_t) = \emptyset$. But this is not guaranteed. Indeed, a state μ_t' is associated both with μ_t and with $\hat{\mu}_t$ if $\int_i a_t^{i'}\ di = 0$, $\int_i (b_t^{i'} - \bar{b}_t')\ di = 0$, $a_t^{i'} = a_t^i + (1 - b_t^{i'}/\bar{b}_t')\nu\tau_t w_t$ and $a_t^{i'} = \hat{a}_t^i + (1 - b_t^{i'}/\bar{b}_t')\nu\hat{\tau}_t w_t$ for all $i \in \mathcal{I}_t^r$. If $a_t^{i'} = a_t^i \frac{-\hat{\tau}_t}{\tau_t - \hat{\tau}_t} + \hat{a}_t^i \frac{\tau_t}{\tau_t - \hat{\tau}_t}$ and $b_t^{i'} = \bar{b}_t' + (a_t^i - \hat{a}_t^i)\bar{b}_t'/(\nu w_t(\tau_t - \hat{\tau}_t))$ for all $i \in \mathcal{I}_t^r$ then these requirements are simultaneously satisfied and Condition 1 iii. therefore is violated (in contrast to Conditions 1 i. and ii.[21]).

Table 5 applies, except for ...	$g_t^{w'}(w_t') = \tau_t' w_t' e_t^i - T_t',\ t \geq 0$ $\mathcal{P}_t' = \{(\tau_t', T_t', z_t') \in \mathbb{R} \times \mathbb{R}_+^2\},\ t \geq 0$
EE restrictions, μ_t'	$\int_{i\in\mathcal{I}_t^r} (b_t^{i'} - \bar{b}_t')\ di = 0$ $a_t^{i'} = a_t^i + \left(1 - \frac{b_t^{i'}}{\bar{b}_t'}\right)\nu\tau_t w_t,\ i \in \mathcal{I}_t^r$
p'^{t-1}	$z_s' = \tau_s \nu w_s / \bar{b}_s',\ s \geq t$ $\tau_s' = \tau_s,\ s \geq t$ $T_s' = \frac{r_{t,s+1}}{r_{t,s}} \tau_{s+1} \nu w_{s+1},\ s \geq t$

Table 6: Extended version of Tabellini's (2000) model.

One way to overcome this problem is to stipulate that the government debt ownership structure must be symmetric and to restrict the state space $\mathcal{M}_t'$ accordingly. It is then impossible for Condition 1 iii. to be violated (and Conditions 1 i. and ii. continue to be satisfied). Condition 2 is satisfied as well since the equivalent policy is admissible for all admissible sequences $\{\tau_s\}_{s\geq t}$. Politico-economic equivalence then hinges on whether Condition 3 is satisfied. Even with symmetric government debt holdings, this is not the case.[22] For an admissible and feasible policy choice in the debt regime need not satisfy $T_t' = r_{t,t+1}\bar{b}_{t+1}' z_{t+1}'$ as implied by the conditions in Table 6. Intuitively, the ability of political decision makers in the debt regime to independently choose both τ' and T' provides more flexibility than the single independent policy instrument τ in the social

[21] Consider an arbitrary state in the new regime, $\mu_t' = \{a_t^{i'}, b_t^{i'}\}_{i\in\mathcal{I}_t^r}$ satisfying $\int_i a_t^{i'}\ di = 0$ and $\int_i (b_t^{i'} - \bar{b}_t')\ di = 0$ where $i \in \mathcal{I}_t^r$, and fix $\nu\tau_t w_t$ at some positive value that could arise in equilibrium in the initial regime. Consider next the cross-section $\{a_t^i\}_{i\in\mathcal{I}_t^r}$ defined by $a_t^i = a_t^{i'} - (1 - b_t^{i'}/\bar{b}_t')\nu\tau_t w_t$ for all $i \in \mathcal{I}_t^r$. By construction, this cross-section satisfies $\int_i a_t^i\ di = 0$ and therefore is an element of $\mathcal{M}_t$. Moreover, by construction, μ_t' is associated with this element of $\mathcal{M}_t$. It follows that $\mathcal{M}_t' \subseteq \cup_{\mu_t\in\mathcal{M}_t} \tilde{\mathcal{M}}_t'(\mu_t)$. Also, $\tilde{\mathcal{M}}_t'(\mu_t) \neq \emptyset$ for all $\mu_t \in \mathcal{M}_t$.

[22] Asymmetric government debt holdings would also contribute to violating Condition 3, by providing a possibility in the debt regime to transfer type-specific amounts to retirees which is not available in the social security regime.

security regime. This makes clear that the politico-economic equilibrium proposed in Tabellini (2000) hinges on the restriction that social security benefits be independent of contributions and no other means of within-cohort redistribution be available.[23]

In conclusion, to guarantee politico-economic equivalence in the extended version of Tabellini's (2000) model, the admissibility restrictions in the debt regime would have to be tightened, for example by requiring that the government issue the same quantity of government debt to all young households (and these be prevented from trading on secondary markets), and that T_t' coincide with the value of the debt the government issues in period t.

5.4 Boldrin and Rustichini (2000)

The economic environment as well as the admissibility restrictions in Boldrin and Rustichini (2000) parallel those in the version of Gonzalez-Eiras and Niepelt's (2008) model with inelastic labor supply. In contrast to these latter authors, Boldrin and Rustichini (2000) model political choice as a game between successive median voters who play a trigger strategy. Political decision makers are confronted with a suggested policy $\mathcal{S}_t$ consisting of a set of continuation policies τ^{t-1} with τ_t fixed according to the suggestion made by political decision makers in period $t-1$. Their own suggested policy $\mathcal{S}_{t+1} = \{\tau^t \in \langle\mathcal{S}_t\rangle \mid \tau_{t+1} = \tau_{t+1}^{\text{sug}}\}$ is characterized by a particular proposal for the tax rate in the subsequent period, τ_{t+1}^{sug}. The left column of Table 7 summarizes the setup. Boldrin and Rustichini (2000) show that the trigger strategy provides sufficiently strong incentives to support equilibria with strictly positive social security transfers even if the median voter is young.

$g_t^w(w_t) = \tau_t w_t,\ t \ge 0$	$g_t^{w'}(w_t') = \tau_t' w_t',\ t \ge 0$
$g_t^r(w_t) = -\nu_t \tau_t w_t,\ t \ge 0$	$g_0^{r'} = -\bar{b}_0' z_0'$
	$g_t^{r'} = 0,\ t \ge 1$
$\mathcal{P}_t = \{\tau_t \in \mathbb{R}_+\},\ t \ge 0$	$\mathcal{P}_t' = \{(\tau_t', z_t') \in \mathbb{R} \times \mathbb{R}_+\},\ t \ge 0$
$\mathcal{S}_0 = \{\tau \in \mathbb{R}_+^\infty\},\ \mathbf{S}_{t+1}(\mathcal{S}_t) =$	$\mathcal{S}_0' = \{(\tau', z') \in \mathbb{R}^\infty \times \mathbb{R}_+^\infty\},\ \mathbf{S}_{t+1}'(\mathcal{S}_t') =$
$\{\{\tau^t \in \langle\mathcal{S}_t\rangle \mid \tau_{t+1} = \tau_{t+1}^{\text{sug}}\}\}_{\tau_{t+1}^{\text{sug}} \in \mathbb{R}_+},\ t \ge 0$	$\{\{(\tau'^t, z'^t) \in \langle\mathcal{S}_t'\rangle \mid z_{t+1}' = z_{t+1}'^{\text{sug}}\}\}_{z_{t+1}'^{\text{sug}} \in \mathbb{R}_+},\ t \ge 0$
$\xi_t = 0 \ \Rightarrow \tau_s = 0,\ s \ge t$	$\xi_t' = 0 \ \Rightarrow z_s' = 0,\ s \ge t$
$\mu_t = k_t,\ t \ge 0$	$\mu_t' = (k_t', \bar{b}_t'),\ t \ge 0$
EE restrictions, μ_t'	$k_t' = k_t$ ($\bar{b}_t'$ exogenous)
p'^{t-1}	$z_s' = \tau_s \nu_s w_s / \bar{b}_s',\ s \ge t$
	$\tau_s' = \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}\nu_{s+1}w_{s+1}}{w_s},\ s \ge t$

Table 7: Boldrin and Rustichini's (2000) model.

[23]Tabellini (2000) recognizes this in his conclusions. He points out that the proposed equilibrium would break down if an additional policy instrument allowing for within-cohort redistribution were introduced.

Consider a new policy regime with taxes, debt and a modified trigger strategy where political decision makers choose the repayment rate and taxes, see the right column of Table 7. They are confronted with a suggested policy $\mathcal{S}'_t$ consisting of a set of continuation policies for the repayment rate and the tax rate, (τ'^{t-1}, z'^{t-1}), with z'_t fixed according to the suggestion made by political decision makers in period $t-1$. Their own suggested policy $\mathcal{S}'_{t+1} = \{(\tau'^t, z'^t) \in \langle \mathcal{S}'_t \rangle | z'_{t+1} = z'^{\text{sug}}_{t+1}\}$ is characterized by a particular proposed repayment rate for the subsequent period, z'^{sug}_{t+1}.

For any state ϕ_t in the initial regime, an associated state $\phi'_t \in \Phi'_t$ satisfies $k'_t = k_t$ ($\bar{b}'_t$ is exogenous), $\xi'_t = \xi_t$ and $\mathcal{S}'_t = \{(\tau'^t, z'^t) \in \mathbb{R}^\infty \times \mathbb{R}^\infty_+ | z'_t = \tau^{\text{sug}}_t \nu_t w_t / \bar{b}'_t\}$ where w_t is the equilibrium wage in the initial regime. Because capital stocks are identical across associated states and equilibrium wages are a function of the capital labor ratio only, w_t also constitutes the equilibrium wage in the new regime. The restriction on the repayment rate in $\mathcal{S}'_t$ therefore fully describes the requirement of Definition 9 v. that $(k_t, \mathcal{S}_t)$ be economically equivalent to $(k_t, \mathcal{S}'_t)$.

Condition 4 is satisfied. In particular, Condition 4 i. holds since for any $\phi'_t = (k'_t, \xi'_t, \mathcal{S}'_t) \in \Phi'_t$ there exists a $\phi_t = (k'_t, \xi'_t, \mathcal{S}_t) \in \Phi_t$ with $\mathcal{S}_t$ characterized by $\tau^{\text{sug}}_t = z'^{\text{sug}}_t \bar{b}'_t / (\nu_t w_t)$ such that both states are associated. Condition 4 ii. is satisfied trivially by construction. Finally, Condition 4 iii. is satisfied since $\tilde{\Phi}'_t(\phi_t) \cap \tilde{\Phi}'_t(\hat{\phi}_t) = \emptyset$ as soon as ϕ_t and $\hat{\phi}_t$ differ with respect to at least one of their three elements. Since Condition 4 holds, the equivalent functions $\tilde{p}'_t(\phi'_t)$, $\tilde{p}'^t(\phi'_t)$ and $\tilde{\mathcal{S}}'_{t+1}(\phi'_t)$ are well defined. In particular, $\tilde{\mathcal{S}}'_{t+1}(\phi'_t) = \{(\tau'^t, z'^t) \in \langle \mathcal{S}'_t \rangle | z'^{\text{sug}}_{t+1} = \tau^{\text{sug}}_{t+1} \nu_{t+1} w_{t+1} / \bar{b}'_{t+1}, \tau^{\text{sug}}_{t+1}$ characterizes $\mathcal{S}_{t+1}(\phi_t), \phi'_t \in \tilde{\Phi}'_t(\phi_t)\}$.

Condition 5 is satisfied since the equivalent policy of an admissible policy in the initial regime is admissible as well (see Table 7) and $\tilde{\mathcal{S}}'_{t+1}(\phi'_t) \in \mathbf{S}'_{t+1}(\mathcal{S}'_t)$ by construction. Regarding Condition 6, consider a state $\phi'_t = (k'_t, \xi'_t, \mathcal{S}'_t)$ and an admissible policy choice $p'_t = (\tau'_t, z'_t)$ and suggestion $\mathcal{S}'_{t+1}$ characterized by $z'^{\text{sug}}_{t+1} \geq 0$ such that $\phi'_t, p'_t, \mathcal{S}'_{t+1}$ and $\tilde{p}'^t(\cdot)$ support a competitive equilibrium with wages w'_t, w'_{t+1} and next-period state $\phi'_{t+1} = (k'_{t+1}, \xi'_{t+1}, \mathcal{S}'_{t+1})$. Let the state $\phi_t = (k'_t, \xi'_t, \mathcal{S}_t)$ satisfy $\phi'_t \in \tilde{\Phi}'_t(\phi_t)$ (from Condition 4, such a ϕ_t exists) and consider the policy choice and suggestion in the initial regime, $\tau_t = z'_t \bar{b}'_t / (\nu_t w'_t)$ and $\mathcal{S}_{t+1} = \{\tau^t \in \langle \mathcal{S}_t \rangle | \tau_{t+1} = z'^{\text{sug}}_{t+1} \bar{b}'_{t+1} / (\nu_{t+1} w'_{t+1})\}$ respectively. Both this policy choice and this suggestion are admissible, $\tau_t \in \mathcal{P}_t$ and $\mathcal{S}_{t+1} \in \mathbf{S}_{t+1}(\mathcal{S}_t)$. Condition 6 therefore is satisfied and politico-economic equivalence guaranteed if $(k'_t, (\tau_t, \tau^t(\phi_{t+1})))$ is economically equivalent to $(k'_t, ((\tau'_t, z'_t), (\tilde{\tau}'^t(\phi'_{t+1}), \tilde{z}'^t(\phi'_{t+1}))))$ and $(k'_{t+1}, \mathcal{S}_{t+1})$ is economically equivalent to $(k'_{t+1}, \mathcal{S}'_{t+1})$, where $\phi_{t+1} = (k'_{t+1}, \xi'_{t+1}, \mathcal{S}_{t+1})$. The latter condition is satisfied by construction of $\mathcal{S}_{t+1}$. As to the former, note that it follows from the relationship between τ_t and z'_t on the one hand and the economic equivalence of $(k'_{t+1}, \tau^t(\phi_{t+1}))$ and $(k'_{t+1}, (\tilde{\tau}'^t(\phi'_{t+1}), \tilde{z}'^t(\phi'_{t+1})))$ (which holds by construction) on the other.

5.5 Cooley and Soares (1999)

Cooley and Soares (1999) analyze a four-period lived overlapping generations economy with capital accumulation, within-cohort homogeneity and a pay-as-you-go financed social security system with proportional taxes levied on the labor income of workers and distributed in a lump-sum fashion among retirees. Cooley and Soares (1999) analyze the politico-economic equilibrium under the assumption that the median voter in the initial period chooses a tax rate that serves as time-invariant proposed social security tax rate in all subsequent periods. Successive median voters only choose between implementing the proposed tax rate or dismantling the social security system forever. Numerically solving a calibrated version of their model, Cooley and Soares (1999) find that the median voter is of age two (out of four) and sustains positive intergenerational transfers.

Rather than offering a detailed analysis of the equivalence of the social-security regime in Cooley and Soares's (1999) model and an alternative regime with government debt, we simply make a few observations. First, economic equivalence fails if labor supply is elastic and if only proportional taxes are admissible in the debt regime. This follows from the fact that tax revenue in the debt regime only needs to finance the interest on debt (rather than old age benefits in the social security regime), marginal and average tax rates coincide under a proportional tax, and the allocation depends on marginal tax rates if labor supply is elastic. To overcome this problem, one could decouple marginal and average tax rates in the debt regime by allowing for a linear tax. For simplicity, we assume instead that labor is supplied inelastically.

In this case, second, an economically equivalent state and sequence of policy choices can easily be defined. Consider a $\phi_t = (\{a_t^i\}_{i \in \mathcal{I}_t}, \xi_t, \mathcal{S}_t)$ and $p^{t-1} = \{\tau_s\}_{s \geq t}$ where $\int_i a_t^i \, di$ equals the capital stock in period t and $\mathcal{S}_t$ is characterized by a time invariant tax rate τ^{sug}. Then there exists an economically equivalent $\phi'_t = (\{a_t^{i'}, b_t^{i'}\}_{i \in \mathcal{I}_t}, \xi'_t, \mathcal{S}'_t)$ and $p'^{t-1} = \{\tau'_s, z'_s\}_{s \geq t}$ where $\int_i a_t^{i'} \, di = \int_i a_t^i \, di$ and $\mathcal{S}'_t$ is characterized by a sequence of repayment rates, $\{z_s^{'\text{sug}}\}_{s \geq t}$.

Third, given a state ϕ_t, an associated state must satisfy the restriction $a_t^{i'} + b_t^{i'} z'_t = a_t^i + \Delta_t^i$ for all $i \in \mathcal{I}_t$ where Δ_t^i denotes the present discounted value of benefits minus taxes for household i under the social security regime net of the corresponding present discounted value under the debt regime. This equation imposes a cross restriction on the policy instruments z'_t and $\{\tau'_s\}_{s \geq t}$ (the latter affect Δ_t^i) that parallels a restriction in the context of Tabellini's (2000) model with a linear tax.[24]

This associated-state restriction implies, fourth, that Condition 4 iii. fails, for parallel reasons as in Tabellini's (2000) model with a linear tax. Suppose that two states ϕ_t and $\hat{\phi}_t$ differ with respect to the distribution (but not the mean) of private asset holdings only and that they induce different equilibrium allocations. Following parallel steps to those discussed in the context of Tabellini's (2000) model with a linear tax, one finds that a state ϕ'_t with asset holdings $a_t^{i'} = (a_t^i + \Delta_t^i) \frac{-\hat{z}'_t}{z'_t - \hat{z}'_t} + (\hat{a}_t^i + \hat{\Delta}_t^i) \frac{z'_t}{z'_t - \hat{z}'_t}$ and $b_t^{i'} = (a_t^i + \Delta_t^i - \hat{a}_t^i - \hat{\Delta}_t^i)/(z'_t - \hat{z}'_t)$ for all $i \in \mathcal{I}_t$ is associated both with ϕ_t and $\hat{\phi}_t$, thereby violating Condition 4 iii.

Fifth, even abstracting from this problem (which cannot be circumvented by imposing symmetric debt holdings as in Tabellini's (2000) model with a linear tax), Condition 6 would be violated as well. The allocations that can be implemented in the debt regime depend on the predetermined debt ownership structure. Variations in this structure give rise to variations in the set of redistributive schemes that political decision makers may

[24] In the latter model, all debt holders are retirees who pay no taxes and receive debt repayment that equals the benefits paid under social security such that $\Delta_t^i = \tau_t \nu w_t$ for $i \in \mathcal{I}_t^r$ as well as $z'_t = \tau_t \nu w_t / \bar{b}'_t$.

implement. In the social security regime, the set of such schemes is different since income can only be channeled from workers to retirees.

6 Conclusions

We have derived general conditions for economic and politico-economic equivalence. We have applied these conditions in the context of some well-known models in the literature with the aim to understand why a change of policy regime might matter in politico-economic equilibrium even if policies in these regimes are equivalent from a purely economic perspective.

In Proposition 1, we extended established results about the neutrality of fiscal policy changes and proved that an economic state and a sequence of policy choices are economically equivalent to another such pair, in the sense that both pairs support the same competitive equilibrium, if conditions with respect to the budget sets of all households and the government's dynamic budget constraints are met.

We then turned to the conditions under which a state and a policy regime are politico-economically equivalent to another such pair, in the sense that both pairs give rise to the same equilibrium allocation. Making intensive use of the economic equivalence result, we identified three sufficient conditions with the logic underlying these conditions relying on a simple comparison of choice sets faced by political decision makers.

To operationalize this logic it must be possible to compare across regimes the constraints faced by political decision makers and thus, the state spaces. The first condition for politico-economic equivalence states requirements guaranteeing such comparability. The condition requires, in effect, a one-to-one relationship between states in the initial regime and corresponding or associated states in the new regime. If the condition is satisfied, then it is possible to define equivalent (continuation) policy functions effectively replicating in the new regime the (continuation) policy functions in the initial regime. The second and third conditions for politico-economic equivalence then focus on the admissibility restrictions in the two regimes: it must be possible in the new regime to implement the competitive equilibrium in the initial regime, and it must not be possible in the new regime to implement competitive equilibria that cannot be implemented in the initial regime by means of a one-period deviation. Combined, these conditions imply the desired relation between the choice sets of political decision makers (as shown in Propositions 2 and 3, for environments without and with trigger strategies, respectively).

We have found that the first condition for politico-economic equivalence is violated in the context of several well known models. However, this violation can sometimes be remedied by modifying assumptions regarding the commitment attached to certain instruments (taxes on lagged income in Bassetto and Kocherlakota's (2004) environment) or the debt ownership structure in the new regime (in Tabellini's (2000) environment). Even with the first condition satisfied, these or other environments violate the second and in particular the third condition for politico-economic equivalence.

On the one hand, this is the case if economically equivalent policies do not satisfy numerical admissibility restrictions in the new regime. In the context of Gonzalez-Eiras and Niepelt's (2008) model, for example, political decision makers in the social security regime are unable to replicate the labor supply distortions associated with certain policies in the debt regime. On the other hand, it follows if economically equivalent policies require more flexible policy instruments in one regime than in another, and if this differential flexibility generates differences in the choice sets faced by political decision makers ex post. For example, in Tabellini's (2000) environment with income heterogeneity, proportional taxes and lump sum benefits in the social security regime are economically equivalent to linear taxes in the debt regime. However, this linear tax enlarges the choice set of political decision makers in the debt regime relative to the social security regime and thereby violates the third condition. In environments with income or age heterogeneity as in Tabellini (2000) or Cooley and Soares (1999), the repayment rate on government debt can constitute a very flexible policy instrument too. For in such environments where the government debt ownership structure is non trivial, the repayment rate determines the wealth distribution differently than social security benefits and taxes whose cross sectional structure is fixed.

Our findings suggest that prominent politico-economic theories of intergenerational redistribution are not robust to changing the set of policy instruments considered. At the same time, the findings rationalize why interest groups might favor the privatization of social security even if from an economic-equivalence point of view, a regime change appears irrelevant. For example, in Gonzalez-Eiras and Niepelt's (2008) analysis of the equilibrium in the social security regime, the non-negativity requirement on the purely distorting labor market instrument θ is binding during times with large social security transfers. In a debt regime, in contrast, political decision makers could further reduce this labor supply distortion during these times without having to cut intergenerational transfers. Since the program of political decision makers in the debt regime attains a higher value than in the social security regime, one should expect the political process to push towards a switch from the latter to the former. Arguably, this is what happens in several countries. As our analysis has shown, however, details matter a lot. If assumptions concerning the political or economic environment are modified such that a young median voter is politically decisive or labor supply inelastic then the social security and the debt regime are politico-economically equivalent again.

Within the realm of fiscal policy, our theoretical results can be applied in various other contexts, for example to study the politics of balanced-budget rules or related restrictions on government debt dynamics. More generally, before the background of an appropriately defined equivalence class of policies—be they fiscal, monetary or other—our results can be applied within any model featuring an endogenous choice of such policies.

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