

The Efficiency of the Global Markets for Final Goods and Productive Capabilities

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This Draft: December 14, 2010

Abstract

Despite integration of financial and goods markets, borders still impose considerable friction on the flow of goods. This paper shows that a decline in output shock variance might increase the time of mean reversion despite declining border frictions. It also proposes a new method of estimating border cost from time-series data only, without relying on within-country variation. The estimate of border cost builds on nonlinearities in the real exchange rate process. The paper applies this method to the real exchange rate of final goods and a novel measure of the real exchange rate for productive capabilities. Relocation costs differ substantially between final goods and productive capabilities. During the years 1974–2008, a relocation reduces productive capability by 18% for the median country pair, whereas final goods by only 13%.

Acknowledgments: For comments and suggestions I thank Rick Bond, Frank Diebold, Bernard Dumas, Urban Jermann, Karen Lewis, and Frank Schorfheide, and as well as participants of the 2010 conference on Microeconomic Sources of Real Exchange Rate Behavior at Vanderbilt University, and of seminars at Boston College, Erasmus Universiteit Rotterdam, Georgetown University, the University of North Carolina at Chapel Hill, Purdue University, Università Bocconi, University of Pennsylvania, CEMFI, the Oxford-Man Institute of Quantitative Finance, the Bank of Canada, the European Central Bank, the Board of Governors of the Federal Reserve System, and at the Federal Reserve Bank of New York.

Keywords: Border effect, Capital goods, Indirect inference, Real exchange rate, PPP, Technology Spillover, Threshold model

JEL classification: F21, F31, C51

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1 Introduction

During the past three decades not only final goods, but also capital goods, have become increasingly mobile. Despite globalization of financial and goods markets borders constitute a considerable friction for flows of goods. The extent of these frictions varies considerably: For final goods, on the one hand, frictions may be limited to plain transportation costs. For capital goods, on the other hand, frictions may comprise a very complex relocation if the good needs to be adapted to the local environment or if its use requires training. Differences in standards, culture, skill, and local technology levels bring about relocation costs that far exceed the cost of transportation.

Previous studies of the market frictions embodied in borders focus on *final goods*. These goods are similarly useful in all countries and do not require an excessive amount of localization, which is why I also refer to them as unbundled goods. Existing estimates of market frictions rely on trade flows (e.g. McCallum 1995, Anderson & van Wincoop 2003) and on the real exchange rate of consumer goods (e.g. Broda & Weinstein 2008, Engel & Rogers 1996, Gorodnichenko & Tesar 2009). The latter price-based studies share the same empirical approach, comparing the within-country variation of prices with their cross-country variation. This approach requires a detailed dataset of prices at many different locations (e.g. cities) in each country. Unfortunately, such detailed data is available only for few countries and/or narrowly defined sets of goods, which makes this approach infeasible for goods with only one price per country, for example for goods traded on centralized national markets.

For this reason no estimates exist of the relocation costs of complex goods, such as capital goods or *productive capabilities*. Productive capabilities comprise, for example, machinery, technology, and human capital embedded in the productive sector. The price of these capabilities is typically determined on national stock markets, thus is unique to each country. In this paper I propose a method to estimate relocation costs from only one price series per country. My method exploits the time series variation of prices and does not require within-country variation. Whereas this method is ideally suited to real exchange rates derived from financial market prices, it is by no means limited to them. Another area of application are disaggregated price indices, e.g. by industry, which are only available at the national level.

In this paper I apply my method to final goods and productive capabilities. The latter markets deserve particular attention, as international flows of final goods are largely liberalized. The cost of adjusting productive capability by means of investing, i.e. the cost of converting final goods into productive capabilities and vice versa, is considerable, and unbundling and rebundling of productive capabilities for the sole purpose of relocating them

is often economically unfeasible. Thus the cost of *directly* relocating productive capabilities (Burstein & Monge-Naranjo 2009, Keller & Yeaple 2009) becomes decisive for the efficiency of their global allocation. This allocation is a key determinant of relative output and ultimately of relative welfare (Lucas 1990, Hsieh & Klenow 2007).

Several key questions emerge: How costly is the relocation of productive capabilities? Does this cost differ by country pair, and what share of this cost can be attributed to national borders? How do these relocation costs relate to the cost of transferring final goods? The answer to each of these questions has direct implications for relative market efficiency and for determining which market would most benefit from additional effort to reduce these frictions.

A natural approach to these questions might start with data on goods' allocations among countries, which is available unfortunately only as a rough estimate, especially for productive capabilities. I therefore choose an indirect approach. Dumas (1992), as well as Sercu, Uppal & van Hulle (1995), formally derive the link between relocation cost, allocations of goods and the real exchange rate.¹ As already observed by Heckscher (1916), under non-zero relocation cost large deviations from purchasing power parity (PPP) can persist. Their fundamental insight allows me to analyze the real exchange rate in lieu of the allocation of goods across countries.

An important innovation of my approach is the estimation of relocation cost directly from the variation of the real exchange rate over time. Unlike previous studies, I do not rely exclusively on differences of price index levels between countries. Instead, I build on continuous time models of international finance, which predict more than just bounds on sustainable price differences between two countries. My identification of relocation cost is based on the models prediction of the evolution of drift and diffusion of the real exchange rate over time. My model, similar to Dumas (1992), predicts that both the conditional drift and diffusion follow a nonlinear process. A parsimonious approximation, which captures this nonlinearity, is the exponential smooth transition autoregressive (ESTAR) model. I utilize ESTAR as a natural auxiliary model in an indirect inference framework, which allows me to translate nonlinearity into relocation costs.

Because the conventional definition of a real exchange rate is based on the price index for unbundled goods, e.g. the consumer (CPI) or wholesale price index (WPI), its behavior over time reflects the frictions in the market for unbundled goods. Such a real exchange rate contains no information on productive capabilities. For this reason, I construct a novel

¹Costs of international trade have also been included in international business cycle models, e.g. Backus, Kehoe & Kydland (1992), Obstfeld & Rogoff (2001), and Ravn & Mazzenga (2004).

measure of the real exchange rate of productive capabilities based on financial market data, which allows me to compare productive capabilities with final goods.

I build on several strands of earlier literature. First, nonlinearity in real exchange rates is a well-documented phenomenon.² Prakash & Taylor (1997) and Obstfeld & Taylor (1997), for example, estimate a model with a hard cut-off between regimes.³ Closer to my approach are studies that use a smooth transition between regimes, as in the ESTAR specification.⁴ Michael, Nobay & Peel (1997), for example, use a sample of monthly WPI data of four countries in the 1920s and annual data for two countries over 200 years. Taylor, Peel & Sarno (2001) work with monthly CPI data for five countries over the period 1973–1996. Modeling real exchange rates by an ESTAR model, Kilian & Taylor (2003) find predictability of the nominal exchange rate at horizons of two years or more. All these studies report strong support for a nonlinear specification, with a random walk dominating over short horizons. However, these studies do not make further use of this nonlinearity to quantify the relocation cost, nor are their results applicable to productive capabilities.

Second, my work is related to studies of mean reversion of international stock markets. Studies focusing on relative nominal international stock prices, such as Richards (1995) and Balvers, Wu & Gilliland (2000), find evidence of transitory country-specific effects in long-run relative stock returns. No one has yet explored the mean reversion of a real capital market-based measure, such as the real exchange rate for productive capabilities.

Third, Caselli & Feyrer (2007) show the importance of price of and availability of complementary factors, such as technology (Eaton & Kortum 1999) or human capital (Lucas 1990), for differences in the marginal product of capital across countries. Because my definition of productive capabilities contains these complementary factors, my estimates of border cost shed light on the causes of sustained country heterogeneity in stocks of these factors.

This paper is organized as follows: In Section 2 I derive the methodology, including the model and the estimation and inference procedure. I introduce productive capabilities in Section 3. Section 4 presents and compares the empirical results for final goods with the results for productive capabilities. I conclude with a summary in Section 5.

²The importance of a nonlinear specification is highlighted by the weak support for mean-reversion achievable with linear models (Adler & Lehman 1983, Frankel & Rose 1996, Lothian & Taylor 1996, Rogoff, Froot & Kim 2001). Froot & Rogoff (1995) and Sarno (2005) provide useful surveys. These studies imply extremely slow overall speeds of mean reversion (Rogoff 1996, Murray & Papell 2005). Nonlinear models provide a natural explanation for both observations. The real exchange rate mean-reverts whenever it has wandered far away from parity, but follows a random walk when it is close to parity.

³This threshold autoregressive model was introduced by Tong (1990) and Balke & Fomby (1997).

⁴See Taylor (2005) for a survey. At the sectoral level, Imbs, Mumtaz, Ravn & Rey (2003) find nonlinearity for two thirds of the sectors in their sample.

2 Methodology

In this section I first discuss a model of endogenous capital accumulation with depreciation shocks and relocation cost, which captures key features of the real exchange rate data. I then describe a model-inspired estimation method, which achieves computational feasibility despite a tight link to the model.

2.1 Modeling Real Exchange Rates with Frictions

The following model describes a particular segment of a two-country economy subject to arbitrage trade. The model economy consists of two countries, which are separated by a border. There is only one good in the economy,⁵ which I interpret as *either* a final good *or* a productive capability.

I assume complete financial markets, i.e. all necessary securities are available and international financial flows are unconstrained.⁶ The counterpart of financial markets in the real economy, the market for goods, is subject to frictions. Relocating a generic good from one country to another entails a cost, $1 - r \in (0, 1)$, that is, of every unit relocated only r percent arrive.

Because any transfer of goods across the border entails an “iceberg” loss of $1 - r$, the good’s location matters.⁷ Accordingly, I mark parameters and quantities of the foreign country with an asterisk (*). The stock of goods, K , can be either consumed, c , or invested in a constant returns to scale production with productivity α . The stock of goods in each country is subject to zero-mean country-idiosyncratic depreciation shock, $\sigma_{11}dz$ and $\sigma_{22}dz^*$, where dz and dz^* are increments of a standard Brownian motion process.⁸

The marginal rates of substitution differ between countries at almost every instance because of the cost of crossing the border. Because the relocation cost is the only friction hindering the movement of goods, this cost bounds the possible valuation differences between countries. No relative valuation outside of the interval $[r; 1/r]$ can persist, because this would

⁵This rules out trade due to specialization or love-of-variety – only trade for arbitrage reasons is possible. See appendix A for a larger model that also embeds trade due to specialization.

⁶Completeness seems to accurately describe financial markets among developed countries over the past 30 years, considering the almost immeasurable variety of financial instruments. Between developed countries, movements of financial capital were largely free from legal restrictions. See e.g. Allen & Gale (1994).

⁷This loss reflects shipping costs for final goods, and costs for transferring technology as in Keller & Yeaple (2009) for productive capabilities. This model does not intend to explain standard trade flows. See appendix A.

⁸A more general one-good economy is described in appendix C.1.

trigger immediate, risk-free, and profitable transfers of goods, dX , from a low-price to a high-price country, until the relative valuation has returned back into this interval.⁹

Owing to the assumption of complete financial markets, the decentralized two-country problem is equivalent to the planner's problem¹⁰

$$V(K, K^*) = \max_{\substack{c(t), c^*(t), \\ \Xi(r)}} E_0 \int_0^\infty e^{-\rho u} \left(\frac{q}{\gamma} c(u)^\gamma + \frac{2-q}{\gamma} c^*(u)^\gamma \right) du \quad (1)$$

s.t.

$$\begin{aligned} dK(t) &= [\alpha K(t) - c(t)] dt + K(t) \sigma_{11} dz(t) - dX(t) + r dX^*(t) \\ dK^*(t) &= [\alpha^* K^*(t) - c^*(t)] dt + K^*(t) \sigma_{22} dz^*(t) + r dX(t) - dX^*(t) \end{aligned}$$

where $K(0)$ and $K^*(0)$ are given, $c(t), c^*(t), K(t), K^*(t), X(t), X^*(t) \geq 0 \ \forall t$, and where $\rho > 0$ denotes the discount rate, $1 - \gamma > 0$ the risk aversion and q the welfare weight of the home country. The relocation of goods is captured by $X(t)$, which is an adapted, non-negative, right-continuous, nondecreasing stochastic process. Ξ denotes the open region in the (K, K^*) space in which no goods are transferred, i.e. where $dX = 0$ and $dX^* = 0$. Note that $\Xi(r)$ is chosen optimally, and thus dX and dX^* are endogenous. Due to the homogeneity of the value function, Ξ is fully characterized by the minimal and maximal imbalance levels, $\underline{\omega}$ and $\bar{\omega}$.¹¹

The symmetric version of this model has been developed by Dumas (1992).¹² I extend this model by country heterogeneity in the ability to produce new goods (Hsieh & Klenow 2007), which can result in persistent price differences between countries. Making the model asymmetric complicates it considerably, but helps explaining key features of the real exchange rate, for example, a non-zero unconditional mean of its drift.

I define the *imbalance* of goods as $\omega = K/K^*$.¹³ Substituting $V(K, K^*) = K^* I(\omega)$,

⁹Obviously, the economy is always in equilibrium. Thus there is no “mispricing” of stocks or other assets which in some finance models constitutes an arbitrage opportunity that triggers e.g. foreign direct investment (Baker, Foley & Wurgler 2009) or investment (Polk & Sapienza 2009), or where imperfect capital markets cause foreign direct investment to be linked with e.g. currency movements (Froot & Stein 1991).

¹⁰Basak & Croitoru (2007) show that a decentralized economy with country-specific bonds and a claim on the dividend flow of one country can equivalently be solved by (1).

¹¹Appendix C provides detailed calculations and discussion of many of the results in this section.

¹²The models of Sercu et al. (1995) and O'Connell & Wei (2002) predict a no-arbitrage band as well. Versions of this model are used by e.g. Uppal (1993) to analyze the effect of home bias in consumption on portfolio choice, and by Dumas & Uppal (2001) to assess the benefit of international financial integration.

¹³Any global shock to both countries (not modelled) has no effect on ω ; its effect cancels out. Because

and using the homogeneity property of the value function, I obtain a second-order ordinary differential equation, which governs the imbalance process in periods of no relocations.

$$\begin{aligned}
0 = & \frac{1-\gamma}{\gamma} q^{\frac{1}{1-\gamma}} I'(\omega)^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} (2-q)^{\frac{1}{1-\gamma}} [\gamma I(\omega) - \omega I'(\omega)]^{\frac{\gamma}{\gamma-1}} \\
& + \left[\alpha^* \gamma - \rho + \frac{1}{2} \sigma_{22}^2 \gamma (\gamma - 1) \right] I(\omega) \\
& + [\alpha - \alpha^* - (\gamma - 1) \sigma_{22}^2] \omega I'(\omega) \\
& + \frac{1}{2} (\sigma_{11}^2 + \sigma_{22}^2) \omega^2 I''(\omega)
\end{aligned} \tag{2}$$

By optimal choice of the boundary of Ξ , the unknown function $I(\omega)$ must satisfy value matching and smooth pasting conditions at both boundaries at all times. The value matching condition requires equalization of the value of the marginal good at the moment of relocation, e.g. for the upper boundary, $\bar{\omega}$,¹⁴

$$V_K(K, K^*) = r V_{K^*}(K, K^*).$$

The smooth pasting conditions require

$$V_{KK}(K, K^*) = r V_{KK^*}(K, K^*),$$

and

$$V_{K^*K}(K, K^*) = r V_{K^*K^*}(K, K^*).$$

Substituting for the value function I can express these conditions in terms of the unknown functional $I(\omega)$.

$$\frac{I'(\bar{\omega})}{\gamma I(\bar{\omega})} = \frac{r}{1 + r\bar{\omega}} \tag{3}$$

$$\frac{I''(\bar{\omega})}{\gamma I(\bar{\omega})} = \frac{r^2(\gamma - 1)}{(1 + r\bar{\omega})^2} \tag{4}$$

The differential equation (2) with boundary conditions (3) and (4) and the analogous conditions for the lower boundary can now be solved numerically for the function $I(\omega)$. I determine the optimal boundaries by guessing values for $\underline{\omega}$ and $\bar{\omega}$ and iterating both forward toward some intermediate imbalance level $\omega_0 \in (\underline{\omega}, \bar{\omega})$ using the Bulirsch-Stoer method

the imbalance ω is the only state variable, $\sigma_{11}dz$ and $\sigma_{22}dz^*$ reflect the country-idiosyncratic component of total shocks only.

¹⁴The conditions for the lower boundary are analogous.

(Stoer & Bulirsch 2005) augmented by a Richardson extrapolation.¹⁵ Richardson's deferred approach to the limit corrects for the estimated error stemming from finite accuracy. A solution has been found if the values of $(I'(\omega_0), I''(\omega_0))$ obtained by iterating from the upper boundary are equal to the values obtained by iterating from the lower boundary, i.e. if $\overline{I'(\omega_0)} = \underline{I'(\omega_0)}$ and $\overline{I''(\omega_0)} = \underline{I''(\omega_0)}$. Otherwise I retry with a new guess for the pair of boundaries. In a second step I use the results from the Bulirsch-Stoer method as starting values in a Newton algorithm for two point boundary value problems.¹⁶

[Table 1 about here.]

The upper panel of table 1 reports the maximum sustainable imbalance as a function of risk and risk aversion for modest relocation cost ($r = 0.9$) in a symmetric world. The imbalances are the larger, the lower the risk aversion and the higher the risk. The lower panel of the same table shows that a higher relocation cost ($r = 0.75$) implies larger sustainable imbalances for any level of risk aversion and risk. The corresponding hittimes, given in table 2, shoot up even more.

[Table 2 about here.]

The tables reveal that, given some degree of risk aversion, the maximum sustainable imbalance approaches an asymptote as risk grows to infinity. For risk aversion larger than unity there exists a maximum risk level, beyond which the differential equation has no solution. If two countries differed in their productivity, then the sustainable abundance of goods in the more productive country would be higher, and correspondingly lower in the less productive country.

I are now able to define the *real exchange rate*, p , as the ratio of the marginal values of the good in two countries, i.e.

$$p(\omega) = \frac{V_K(K, K^*)}{V_{K^*}(K, K^*)} = \frac{I'(\omega)}{\gamma I(\omega) - \omega I'(\omega)}. \quad (5)$$

Note that the real exchange rate depends only on the imbalance, ω , but not on the stocks, K and K^* , themselves. Therefore, the real exchange rate tracks the *relative* scarcity of goods

¹⁵Press, Teukolsky, Vetterling & Flannery (2001, p.718ff) describe this technique in detail.

¹⁶Press et al. (2001, p.376ff) present a similar algorithm. This second step improves precision. Whereas this Newton algorithm is very effective in fine-tuning a raw solution, it cannot be recommended as a stand-alone routine. Without Bulirsch-Stoer as a first step, the Newton algorithm would frequently fail to find the solution.

– a scarcity due to the frictions in the goods market, which emphasizes the strong impact of relative productivity on the real exchange rate.

Using Ito’s lemma, drift and diffusion of the natural logarithm of the real exchange rate process

$$d\ln(p) = \mu_p(p)dt + \sigma_p(p)dz$$

can be written as a function of ω , $I(\omega)$, $I'(\omega)$, $I''(\omega)$. The drift of $\ln(p)$ at the boundaries with $\sigma_{11} = \sigma_{22} = \sigma$ is given by the following proposition.

Proposition 1 (*Drift of Real Exchange Rate at the Boundary*) *The drift of the real exchange rate at the upper boundary is*

$$\mu_p(\omega = \bar{\omega}) = \alpha^* - \alpha + \frac{\bar{\omega} - 1/r}{\bar{\omega} + 1/r}(1 - \gamma)\sigma^2,$$

and at the lower boundary

$$\mu_p(\omega = \underline{\omega}) = \alpha^* - \alpha + \frac{\underline{\omega} - r}{\underline{\omega} + r}(1 - \gamma)\sigma^2.$$

Proof: See appendix C.3.

For realistic parameter values the mean reversion at the boundary gains in strength with shock diffusion, σ^2 , and with risk aversion, $1 - \gamma$.¹⁷ The drift towards the center therefore shrinks as the economy is hit by smaller shocks. It further approaches $\alpha^* - \alpha$ as the economies get perfectly integrated ($r \rightarrow 1$). Therefore, because financial integration coincided with a period of “great moderation” (Kim & Nelson 1999, Stock & Watson 2002), it is not surprising that exchange rates do not appear more mean reverting today than in the past.

[Figure 1 about here.]

A key feature of my model is that drift and diffusion of the the real exchange rate vary systematically with its level. The upper panel of figure 1 shows the drift of two real exchange rate processes with different relocation costs. The process represented by the dashed line results from high relocation cost ($r = 0.78$), whereas the process represented by the solid line reflects low relocation cost ($r = 0.88$). Both processes share the property that a deviation from parity entails a drift of opposite sign. The diffusion, shown in the lower panel, decreases as real exchange rate deviations from parity become large. The real exchange rate process is

¹⁷The necessary condition for this to hold is $\underline{\omega} < r$ and $\bar{\omega} > 1/r$. Table 1 shows that this is satisfied except for very small σ in combination with $\gamma \leq 0$.

therefore mean reverting at the boundaries of Ξ , but indistinguishable from a random walk close to parity. Clearly, increases in relocation cost not only widen the range of sustainable deviations from PPP, but also lower the drift at all real exchange rate levels. Likewise, they increase diffusion at all real exchange rate levels.

[Figure 2 about here.]

I now take a closer look at the high relocation cost scenario. Figure 2 compares the drift of the real exchange rate in a world, in which productivities are equal ($\alpha = \alpha^*$, dashed line), with the drift in a world, in which productivities differ ($\alpha < \alpha^*$, solid line). The solid line shows that small differences in growth rates can shift the reversion target level away from PPP ($\ln(p) = 0$) to a considerably different level ($\ln(p) \approx -0.15$).¹⁸ The tiny offset of the zero-drift point from the origin stems from the unequal country weights ($q \neq 1$).

2.2 Indirect Inference

My model has no closed form solution. Drift and diffusion are time-varying and functions of the unobserved imbalance, ω . In principle the coefficients on $I'(\omega)^{\frac{\gamma}{\gamma-1}}$, $(\gamma I(\omega) - \omega I'(\omega))^{\frac{\gamma}{\gamma-1}}$, $I(\omega)$, $\omega I'(\omega)$, and $\omega^2 I''(\omega)$ of differential equation (2) can be estimated by maximum likelihood (Brown & Hewitt 1975) after solving the nonlinear filtering problem of $\omega(t)$ based on the observations $p(t)$ (Bensoussan 1992). However, because the differential equation for p is not available in closed form, this calculation of ω_0 from $p(\omega_0)$ is computationally very costly and practically infeasible. Further, even after filtering and obtaining the series of ω , the closed form of the density of the conditional likelihood is not available because of the discreteness of the data. In contrast, going in the opposite direction, i.e. calculating $p(\omega_0)$ from ω_0 by (5), is a simple task, because the functional's value at ω_0 , $I(\omega_0)$, is a by-product of calculating ω_0 via (2). More productive is therefore a method that does not require calculating the conditional distribution of $p(t)$ given $p(t-1)$ directly from the original model.

This estimation problem ideally suits the indirect inference procedure, introduced by Smith (1993), Gouriéroux, Monfort & Renault (1993) and Gallant & Tauchen (1996). Indirect inference replaces the hard-to-evaluate likelihood function of the original model with the likelihood function of an auxiliary model which is easier to estimate. Importantly, the

¹⁸If the productivity gap between two countries is extremely wide, then the drift can have the same sign at almost all real exchange rate levels. In this case the real exchange rate process is therefore divergent for half of its support, although it is still bounded by Ξ .

auxiliary model must pick up the key features of interest of the data, in particular nonlinearity in drift and diffusion. One can then generate independent simulated data sets from the structural model for various parameters, estimate the auxiliary model with these simulated data, and repeat this procedure until parameters are found for which the estimates of the auxiliary model based on the simulated data are close by some metric to the estimates based on the actual data. Further, indirect inference does not require the hard calculation of the unobservable ω from the observed p . It allows solving and simulating the model for ω , and then calculating the implied real exchange rate, $p(\omega)$.

2.2.1 Auxiliary Model

The crucial decision in indirect inference is choosing an appropriate auxiliary model. For the problem at hand, a natural auxiliary model is the ESTAR model of Haggan & Ozaki (1981) and Teräsvirta (1994). Whereas the process of the real exchange rate implied by my structural model is complicated and not available in closed form, its key feature, the smooth transition from a divergent to a mean reverting regime, can parsimoniously be modeled by the ESTAR specification. The ESTAR model has the following standard form:

$$\begin{aligned}
p_t - p_{t-1} &= (1 - \Phi(\theta; p_{t-d} - \mu)) \left(\beta_0 + (\beta_1 - 1)p_{t-1} + \sum_{j=1}^m \beta_j p_{t-j} \right) + \\
&+ (\Phi(\theta; p_{t-d} - \mu)) \left(\beta_0^* + (\beta_1^* - 1)p_{t-1} + \sum_{j=1}^m \beta_j^* p_{t-j} \right) + \epsilon_t \\
\epsilon_t &\sim N(0, \sigma_t^2)
\end{aligned} \tag{6}$$

The transition function $\Phi(\theta; p_{t-d} - \mu)$, parametrized by the transition lag d and the transition parameter $\theta \geq 0$, governs the smooth transition between the inner autoregressive process with parameters β_i and the outer autoregressive process with parameters β_i^* , and is specified as

$$\Phi(\theta; p_{t-d} - \mu) = 1 - \exp(-\theta (p_{t-d} - \mu)^2).$$

I generalize the standard ESTAR to allow for conditional variance dynamics which my structural model (1) predicts. My specification uses a second transition function $\tilde{\Phi}(\tilde{\theta}; p_{t-d} - \mu)$, which smoothly moves between an inner regime variance σ_1^2 and an outer regime variance

σ_2^2 .¹⁹ In addition to time-varying conditional variance my specification restricts the outer regime mean dynamics.

$$\begin{aligned}
p_t - p_{t-1} &= (1 - \Phi(\theta; p_{t-d} - \mu)) \left((\beta_1 - 1)p_{t-1} + \sum_{j=1}^m \beta_j p_{t-j} \right) + \\
&\quad - \Phi(\theta; p_{t-d} - \mu) p_{t-1} + \epsilon_t \\
\sigma_t^2 &= \left(1 - \tilde{\Phi}(\tilde{\theta}; p_{t-d} - \mu) \right) \sigma_1^2 + \left(\tilde{\Phi}(\tilde{\theta}; p_{t-d} - \mu) \right) \sigma_2^2 \\
\tilde{\Phi}(\tilde{\theta}; p_{t-d} - \mu) &= 1 - \exp\left(-\tilde{\theta}(p_{t-d} - \mu)^2\right) \\
\epsilon_t &\sim N(0, \sigma_t^2)
\end{aligned}$$

I follow Teräsvirta (1994) in specifying the transition lag, d , and number of autoregressive terms, m , based on the nonlinearity test of Granger & Teräsvirta (1993), where I restrict $d \leq 12$ and $2 \leq m \leq 4$.

Michael et al. (1997) argue that real exchange rates based on CPI can be modeled by an ESTAR process. The same applies to real exchange rates based on productive capabilities. Table 3 shows that this real exchange rate has a nonstationary inner regime for most country pairs and a stationary outer regime for all country pairs.²⁰

[Table 3 about here.]

2.2.2 Discretization, Simulation and Optimization

Although the ESTAR estimates reveal nonlinearity in the data, they do not provide a natural interpretation of the autoregressive coefficients. Furthermore, they lack a natural benchmark for the transition parameter, θ . By estimating my structural model I can now ask whether θ is “big” or “small”. In my indirect inference framework ESTAR assumes then the role of the auxiliary model.

Inspection of the estimation equations (2) together with (3) and (4) reveals that in a country-by-country estimation not all parameters can be identified. Importantly, however,

¹⁹Studies that allow for time-varying conditional variance in an ESTAR setup are scarce. A notable exception are Lundbergh & Teräsvirta (2006), who augment an ESTAR-type model with a GARCH variance process. For the *nominal* exchange rate of the Swedish krona and the Norwegian krone against a currency basket in the 1980s they find only a very weak decline of the conditional variance at the boundaries of the target zone set by the central bank.

²⁰Individual coefficients of the mean equation are often insignificant. Conversely, coefficient estimates of the variance equation are typically significant, often with a higher variance in the outer regime. If an outer variance regime cannot be identified, I impose a fixed variance regime $\sigma_t^2 = \sigma_1^2$ as in the standard ESTAR setup.

the real exchange rate process may be asymmetric, reflecting differences in productivity between countries. For example, the real exchange rate process may be close to one boundary most of the time, and hardly ever reach the other. To allow for this possibility I keep the productivity differential $\Delta\alpha = \alpha - \alpha_{USA}$ as a parameter to be estimated. I calibrate $\alpha_{USA} = 0.180$ to the average growth rate of capital stock of the USA.²¹ I assume equal variance $\sigma_{11} = \sigma_{22}$ and fix the discount rate at $\rho = 0.07$. A first stage estimation gives on average $\gamma \approx -0.5$. Because in my structural model γ applies to both the USA and each foreign country, I subsequently reestimate the model with γ fixed at this average to ensure the same U.S. risk aversion in each country pair.

Estimation requires the efficient simulation of many discrete trajectories of p for a given parameter set. I first simulate the $\tilde{\omega} = \ln(\omega)$ process, which can be done with higher precision because the diffusion of this process is constant. Wagner & Platen (1978) and Platen (1981) introduce an Itô-Taylor scheme which strongly converges at rate 1.5.²² Using the fact that the noise in my model is additive and the diffusion of $\tilde{\omega}$ is constant, this scheme can be written as

$$\begin{aligned}\tilde{\omega}_{t+1} &= \tilde{\omega}_t + \mu_{\tilde{\omega}}(\tilde{\omega}_t)\Delta + \sigma_{\tilde{\omega}}\Delta z_t + \mu'_{\tilde{\omega}}(\tilde{\omega}_t)\sigma_{\tilde{\omega}}\Delta y_t \\ &+ \frac{1}{2} \left(\mu_{\tilde{\omega}}(\tilde{\omega}_t)\mu'_{\tilde{\omega}}(\tilde{\omega}_t) + \frac{1}{2}\sigma_{\tilde{\omega}}^2\mu''_{\tilde{\omega}}(\tilde{\omega}_t) \right) \Delta^2,\end{aligned}$$

where $\Delta z_t = \sqrt{\Delta}u_1$, $\Delta y_t = \frac{1}{2}\Delta^{3/2} \left(u_1 + \frac{1}{\sqrt{3}}u_2 \right)$, and $u_1 \sim N(0, 1)$ and $u_2 \sim N(0, 1)$ are independent.²³

Next, I calculate the process of the real exchange rate from the process of imbalances, using the interim results $(I(\omega), I'(\omega))$ obtained in the calculation of the imbalance process.

In summary, I proceed with inference by the following steps:

1. Estimate the ESTAR specification based on actual data by quasi maximum likelihood. Denote the set of parameter estimates by $A_0 = \{\theta, \tilde{\theta}, \beta_i, \beta_i^*, \mu, \sigma_1, \sigma_2\}$.
2. Draw sequences of random numbers u_1 and u_2 .
3. Pick starting values for the parameters of the structural model, $B = \{\gamma, r, \Delta\alpha, \sigma, q\}$.

²¹I use annual 1974–2007 data from the World Development Indicators provided by the Worldbank to calculate α^* by

$$\alpha^* = \frac{GDP_T - GDP_0}{\sum_{t=0}^{T-1} (GDP_t - C_t)}.$$

²²This convergence excels the rate of 1.0 achieved by the well-known Milstein (1974) scheme.

²³See also Kloeden & Platen (1999, p.351).

4. Solve the differential equation for optimal boundaries, $\bar{\omega}$ and $\underline{\omega}$, using the Bulirsch–Stoer method, augmented by subsequent Newton optimization.
5. Simulate $S = 500$ paths of the imbalance process, ω_t , by the Itô-Taylor scheme for 649 periods based on the structural model. The first 240 periods, or 20 years, are used as burn-in period. They are discarded to ensure that the results are independent from the starting values of the process. This leaves me with $T = 409$ simulated values.
6. Calculate the price process, $p(\omega_t)$, from the imbalance process.
7. Compute the indirect inference estimate of B by minimizing the distance between the data-based and the simulation-based estimate, measured by the score criterion

$$\begin{aligned} \hat{B} &= \underset{B}{\operatorname{argmin}} \left[\sum_{s=1}^S \sum_{t=1}^T \frac{\partial \ln f^{ESTAR}}{\partial A} (p_t^s(B) | p_{t-1}^s, A_0) \right]' \\ &\times \Omega \times \left[\sum_{s=1}^S \sum_{t=1}^T \frac{\partial \ln f^{ESTAR}}{\partial A} (p_t^s(B) | p_{t-1}^s, A_0) \right], \end{aligned}$$

where Ω is a nonnegative, symmetric weighting matrix.

It is important to stress the key advantages of this model and estimation approach. Not only does one time series per country suffice to estimate the relocation cost, but also does this approach exploit the information in the time series at each point in time. In contrast to an extremum estimator or a hard cut-off threshold model, with this approach drift and diffusion reveal information about the location of the threshold – and thus border cost – at *each* instant. That is, the structural features of the economy, such as relocation cost, which bind real allocations only in the long run, are mirrored at a much higher frequency in the drift and diffusion of exchange rates.

3 Productive Capabilities

As shown in appendix B, my one-good model of arbitrage-induced trade (1) is a reduced form of a two-good model featuring both final goods and productive capabilities. The reduced one-good model allows therefore for two interpretations of the generic good: a final good interpretation and a productive capability interpretation. Under the final good interpretation K is a consumption good, such as “grain”, which reproduces itself at rate α and can be

consumed. Under the productive capability interpretation the good is “knowledge”, which also reproduces itself, and is, via an (not modelled) production process, converted into a consumable, e.g. “movies”. The two-good model reveals that the dynamics of the exchange rate for each good separately can be described by the one-good model.

If, on the one hand, the model is in terms of final goods, then the relocation cost, r , reflects first and foremost the cost of shipping. If, on the other hand, the model is in terms of productive capabilities, then r contains a second component: the cost of relocating organizational structures and knowledge necessary for operating the physical good. The overall relocation cost for productive capabilities can therefore be higher than for final goods, even if the shipping cost for productive capabilities is small. In order to empirically verify this hypothesis, I derive in this section a measure of the real exchange rate for productive capabilities and compare its properties with a standard real exchange rate for final goods.

3.1 Measuring Real Exchange Rates

The model described in section 2 allows estimating the relocation cost directly from real exchange rate time-series data. Typically the real exchange rate is calculated for final goods, and based on the CPI, WPI, or deflators of the gross domestic product (GDP). Each of these excludes a large share of productive capabilities, in particular immaterial goods. To compare the border effect of final goods with that of productive capabilities, I need one real exchange rate series for each.

For final goods I use the standard real exchange rate based on the WPI, which captures the bulkiness and business-to-business nature of these goods in international transactions.

Unfortunately, for productive capabilities no appropriate real exchange rate is readily available. Productive capabilities are factors in operation in the productive sector of the economy. They are typically owned by a firm, need to be combined with other capital goods in order to be fully useful,²⁴ and are often intangible (e.g. in the form of patents and know-

²⁴A major share of productive capabilities, except machinery, can be considered complementary factors. In the case of information technology investment, for example, only about one third of expenditures are invested in hardware and prepackaged software, whereas the rest is spent for complementary capital such as training, support, and custom software, which is a necessary requirement for the hardware to be useful within the productive sector (Basu, Fernald, Oulton & Srinivasan 2004, Kiley 1999). Caselli & Feyrer (2007) highlight the importance of these complementary factors for understanding global capital goods allocations. They find that under a narrow definition of capital stocks (i.e. machinery only) the marginal product of capital across countries does not differ by much, and thus border frictions for these goods are small. It is the other, complementary goods needed in the production process, such as human capital and technology, that explain the difference in the marginal product of capital. They point out that the scarcity of complementary capital goods, which are included in productive capabilities, and their higher cost explain investment into

how, as in Burstein & Monge-Naranjo (2009)). Clearly, none of the aforementioned real exchange rates focuses on productive capabilities, and for the most part they do not contain any productive capabilities at all. I therefore construct a novel real exchange rate series tailored to capture the valuation differences of productive capabilities between countries.

To demonstrate what data I need, I rewrite (5) by raising the fraction to higher terms using the world market price of an uninvested capital good, V_G .

$$p(\omega) = \frac{V_K(K, K^*)K/(V_G K)}{V_{K^*}(K, K^*)K^*/(V_G K^*)} \quad (7)$$

Notice that the market value of all productive capabilities, or invested capital goods, in a given country can be written as

$$M(t) = V_K(K(t), K^*(t)) K(t) e(t),$$

where $e(t)$ is the nominal exchange rate to a numeraire currency. Likewise, book values of all capital goods in a given country are

$$B(t) = V_G K(t) e(t) \varphi,$$

where φ denotes a time-invariant, country-specific accounting constant. The real exchange rate (7) can therefore be written in terms of (inflation-adjusted) market-to-book ratios, M/B , i.e.

$$p(\omega) = \frac{M/B}{M^*/B^*} \frac{\varphi}{\varphi^*}.$$

Stock indices measure the total market *value* of productive capabilities of firms included in this stock index. The underlying *quantity* is captured by the aggregate book values, after adjusting it for the effect of inflation. Normalizing the stock indices by adjusted book values removes the effect of nominal exchange rates and of quantity changes via e.g. retained earnings or international relocation of productive capabilities, because the book values share the currency unit of the market value M , and obtain a measure of the value of one unit of productive capability. For countries with identical accounting standards the real exchange rate of productive capabilities is thus simply the ratio of average market-to-book values.

these countries. My model predicts a similar link between high cost of productive capabilities and low new investment, but with causality running in the opposite direction. Without relocations, the less productive country is short of productive capabilities. Because relocation costs hinder a complete equalization of this imbalance, the price of productive capabilities in the less productive country is higher.

When countries differ in accounting standards or leverage levels, $p(\omega)$ can be corrected for the constant factor $\frac{\varphi}{\varphi^*}$ by setting midrange of $\log(p(\omega))$ to zero.

My use of market-to-book ratios differs from its interpretation in standard q -theory (Hayashi 1982). As the value of productive capabilities changes over time, the value of one unit of productive capability, measured by the market-to-book ratio, changes with it. Despite these fluctuations in market-to-book ratios, Tobin's q in my model is always unity, because the model does not explicitly consider the adjustment cost of investment within a given country.²⁵ In my setup the market-to-book ratio of a single country's stock index has therefore no economic meaning in isolation, but is informative *relative* to other countries' market-to-book ratio. The ratio of two countries' market-to-book ratios measures the relative price of productive capability in these two countries. In my model high relative market-to-book ratios between two countries lower future relative returns. This property of market-to-book ratios in my model is in line with the stylized fact that market-to-book ratios are inversely related to future equity returns (Fama & French 1992, Fama & French 1998, Pontiff & Schall 1998).

My concept of the real exchange rate based on productive capabilities has multiple advantages. Firstly, it allows me to study properties of the market for productive capabilities in isolation from markets for other goods. In combination with my estimation approach, it enables me to estimate relocation costs of productive capabilities from macroeconomic data. With microeconomic data this has not been possible so far, because data on the cost of relocating productive capabilities across borders on a per-project basis is usually not publicly available.²⁶

Secondly, because one component of market-to-book ratios is determined in financial markets, it responds quickly to new information. Accounting regulations (Nexia International 1993) restrict the revaluation of book values of productive capabilities. The rare revaluation of book values paired with frequent quantity adjustments, which makes book values inappropriate as a measure of value, is a virtue for my purposes. It makes, properly adjusted, book values a measure of the quantity of productive capabilities operating in an economy. My approach mitigates problems with CPI and WPI data, which are subject to aggregation bias (Imbs, Mumtaz, Ravn & Rey 2005) and non-synchronous sampling (Taylor 2001).

²⁵Market-to-book values measure the ratio of the market value of equity relative to the book value of equity, i.e. the denominator includes the book value of all productive capabilities (including goodwill) minus the book value of debt. My model does not distinguish between equity and debt. As long as within any given country the debt level is a fixed proportion of equity, dividing by book values provides the desired quantity correction.

²⁶Available data, such as data on FDI, which might be used to identify frictions between countries, measures only financial flows, but not the underlying flow of productive capabilities.

In particular, the valuation component of my real exchange rate, the market values, are synchronously sampled worldwide in centralized markets in a standardized and automated manner. It is collected in real time, and not subject to revisions. Further, aggregation to a country stock index is transparent and largely internationally comparable.

My approach differs in important ways from the study of Engel & Rogers (1996) on the “width of the border”, addressing many concerns pointed out by Gorodnichenko & Tesar (2009). For example, I do not rely on the unconditional variance of the real exchange rate as dependent variable. My estimates are identified by time variation of country aggregates, not by within-country cross-section variation, and are therefore unaffected by differences in within-country price dispersion. Further, the real exchange rate for productive capabilities is – by virtue of the speed of financial markets – less persistent than a real exchange rate based on the CPI, and by its definition immune to pure nominal exchange rate changes.

3.2 Data

I collect data from various editions of Capital International Perspective (1975-2008). This dataset from Morgan Stanley Capital International contains monthly nominal exchange rates, stock price indices and consistently calculated market-to-book ratios for these indices of 18 developed countries for the period December 1974 to December 2008.

[Figure 3 about here.]

The market-to-book ratios vary substantially over time. The solid line in figure 3 shows that the equal-weighted average market-to-book ratio trended upward over the last 30 years, with large transitory upward bursts, but returned close to unity during the 2008 financial crisis. The variation across countries does not show any trend during the same period. In periods of a high market-to-book ratio average, however, the variation increases temporarily. This indicates that except in the recent 2005–2007 boom not all countries participated in these transitory upward bursts. After a few years variation returns back to the long-term base level. This foreshadows mean reversion of relative market-to-book values between countries, and thus of the real exchange rate for productive capabilities.

I correct book values for the effect of inflation, using WPI and CPI data provided by the International Monetary Fund’s (IMF) International Financial Statistics database.²⁷ This

²⁷Inflation adjustment is based on the firm investment cycle. I assume a degressive depreciation schedule at a depreciation rate δ of 10%, i.e. $B_t = \sum_{i=0}^{\infty} \delta^i I_{t-i}$, which lets me calculate the approximate path of investment over time, $I_t = B_t - \delta B_{t-1}$. Adjusted bookvalues are therefore $\tilde{B}_t = \sum_{i=0}^{\infty} \delta^i I_{t-i} \Pi_{t-i}^t$, where

correction adjusts, for example, the original bookvalues for Germany upwards during the high inflation periods of the 1970s.

I calculate the real exchange rate for final goods based on the monthly WPI from the IMF International Financial Statistics database for the same countries and time period. Subsequently, I demean the data and remove any seasonality effects by regression on a set of monthly dummies to account for cyclical activity or reporting.

[Figure 4 about here.]

Figure 4 compares my two measures of the real exchange rate.²⁸ The real exchange rate based on productive capabilities, represented by the solid line, moves less steadily than the real exchange rate for final goods, represented by the dashed line. Quite striking is the difference in evolution of the real exchange rate of productive capabilities during and after the communication technology boom. Canadian productive capabilities, shown in the top panel, reached their 30-year low in value relative to the USA shortly before the peak of the communication technology boom, reflecting the delayed growth of this sector in Canada. In contrast, German productive capabilities in the middle panel reached their all-time low in the after-technology-boom recession, which suggests that the 2002 recession had freed up more productive capabilities in Germany than in the USA.

My model predicts a nonlinear relationship between returns and current levels of the real exchange rate. This kind of relationship is known to exist for final goods (Michael et al. 1997),²⁹ and I find a similar relationship for productive capabilities. I test the real exchange rate series of each of the 153 possible country pairs for ESTAR-type nonlinearity, using a Granger & Teräsvirta (1993)-type test. This test is based on a second-order Taylor approximation of the ESTAR function around $\theta = 0$.³⁰ As expected, real exchange rates based on productive capabilities of most country pairs follow a nonlinear process as well. 52% of country pairs reject linearity at the 5% level, and 76% at the 10% level, in favor of ESTAR-type nonlinearity. As illustration, figure 5 plots two-year changes in the log real

Π_{t-i}^t denotes the WPI price deflator between period t and $t - i$. Appendix D provides graphs of the effect of this book value correction.

²⁸Appendix E presents graphs for additional country pairs.

²⁹Broda & Weinstein (2008) find nonlinearity at the product level. At such a disaggregated level, however, nonlinearity may primarily be driven by strategic pricing: Temporary sale events by manufacturers or correlated sale events of distributors in limited marketing regions are very common. It is the very essence of a sale event that sale prices revert quickly back to their long-run levels. This type of mean reversion at the product level, however, does then not reflect international arbitrage or reversion back to an international PPP.

³⁰Appendix F describes this test in more detail.

exchange rate of productive capabilities against the initial log levels. Both country pairs shown feature random walk behavior close to the parity level and strong mean reversion away from parity. Visual inspection thus already indicates nonlinearity with two regimes.³¹

[Figure 5 about here.]

4 Empirical Results

I apply my estimation procedure to the real exchange rates of both final goods and productive capabilities with the USA as base country. The estimated relocation cost for final goods given in the first column of table 4 ranges from 2% for Canada–USA up to 26% for France–USA, with a median of about 13%. The picture changes dramatically when I shift my focus to productive capabilities. The second column of this table reports for all countries except Austria, France, and UK much larger relocation costs of productive capabilities than of final goods, with a median of about 18%. The variation of these costs across countries is considerably higher as well, ranging from 11% to 54%. But there are also similarities: On the one hand, the direct neighbor of the USA, Canada, has the lowest relocation cost to and from the USA for both classes of goods, 2% for final goods and 11% for productive capabilities. On the other hand, the distant large economy Japan has a high relocation cost for both classes of goods, 22% for final goods and 28% for productive capabilities.

My parameter estimates allow me to graphically compare the drift and diffusion processes. As an example, I discuss here the real exchange rates of the pair Germany–USA. Figure 6 emphasizes the very different border costs for final goods and productive capabilities. The real exchange rate process for final goods, shown by the dashed line, follows a much narrower band with lower diffusion than the process for productive capabilities. The mean reversion properties of final goods resemble those of productive capabilities, but cut off at much tighter boundaries, which limits the maximum mean reversion of the real exchange rate of final goods.

[Figure 6 about here.]

Figures 7 and 8 compare the real exchange rate process for productive capabilities of Germany–USA, shown as dashed line, with other country pairs.

³¹Further, the variance appears higher in the center than in the outer regime, in line with the predictions of my model. This may, however, be an effect of the relatively small number of observations in the outer regime.

[Figure 7 about here.]

The process for Japan–USA in figure 7 moves with a smaller variance within a much wider band. Its mean reversion is extremely small.

[Figure 8 about here.]

In contrast, the solid line in figure 8 displays a country pair with mean reversion as strong as, but much more rapid, than Germany–USA. The shown country pair with a narrow real exchange rate band is UK–USA, which is among the country pairs with the lowest relocation costs for productive capabilities. Both its mean reversion and maximum diffusion are higher than in Germany–USA. The upper panel of this figure also shows that the drift of the real exchange rates at parity is negative, which is an effect of the higher productivity of the UK relative to the USA and the, as a result, relative abundance of goods in the UK. Because the USA itself reproduces goods at a lower rate, only a large abundance of goods in the UK, i.e. a small p , triggers a relocation of goods from the UK to the USA.

Figure 9 compares the relocation cost of final goods with the corresponding cost of productive capabilities directly. Relocation cost for productive capabilities are about 50% higher than for final goods. The close economic ties between the UK and the USA are reflected in a low relocation cost of productive capabilities, even lower than the cost of moving final goods. Norway, Sweden, and Canada display a very pronounced cost pattern. Whereas its trade costs are relatively low, its relocation costs for productive capabilities are much larger, pointing to differences in openness between markets.

Overall, the relocation costs are only weakly correlated between the two types of goods. Besides Canada, the countries UK, Denmark, Germany, Italy, and the Netherlands have low costs for both types of goods. Hongkong, Norway, Spain, and Sweden appear to be somewhat disconnected from the market for productive capabilities.

[Figure 9 about here.]

[Table 4 about here.]

Returning to the left half of table 4, which ranks countries by their cost of transferring final goods to and from the USA, reveals that by far the lowest border cost obtains between the USA and Canada. Only 2% of the quantity of final goods crossing this border is lost in the form of iceberg costs. On the other extreme, a transfer of final goods from Japan to the USA incurs a cost of more than 20%. Besides the distant economy Japan, relatively small or

protected markets such as Switzerland, Belgium, Austria, or France are among the countries with the highest trade cost to and from the USA.

I compare my estimates with the ratio of cost-insurance-freight (*cif*) and free-on-board (*fob*) prices provided by the IMF's International Financial Statistics database for a few countries. For countries in my sample both *cif* and *fob* values of imports are available for Australia, France, Germany, UK, and USA. The average *cif/fob* ratio for these countries fell from approximately 6% in 1975 to approximately 4.5% today. All transfer cost estimates except for Canada and Sweden exceed this level. This emphasizes that transfer cost consists of more than just insurance and freight. Other expenses, such as e.g. administration, customs, distribution, and market access costs devour up to another 20% of the transferred quantity in the case of France and Japan.³²

The right half of table 4 ranks the same countries again, but now by their cost of relocating productive capabilities to and from the USA. This heavily reshuffles the ordering of countries. Hongkong, Norway, and Sweden are economies which are quite open for trade in final goods, but largely closed for transfers of productive capabilities. Borders to Austria and the UK, on the contrary, seem to impose less frictions on productive capabilities than on final goods. The logic behind the ordering seems to very different between the two types of goods. For productive capabilities, European open economies impose the lowest frictions, whereas Asian economies and economies with more government activity tend to show higher frictions. For final goods, however, these factors do not matter. In contrast, small economies and economies with specific market structures appear to impose the higher frictions on final goods markets.

In order to put my estimates for productive capabilities into perspective, I compare them with estimates of adjustment costs. The adjustment cost of capital goods within a country, i.e. the difference between the productive capability's value as part of a firm and the proceeds from selling the dismantled capital good, provide an upper bound on how large relocation costs of productive capabilities within a country can be. In a setup similar to Abel & Eberly (1994) and Abel & Eberly (1996), where the only form of adjustment cost is a gap between buying and selling prices of capital goods, Cooper & Haltiwanger (2006) find for the USA an adjustment loss of about 20%. Many of these intra-US adjustment losses are larger than my estimates for relocation costs. For most country pairs in my sample a direct relocation of productive capabilities is therefore cheaper than a disinvestment and subsequent transfer

³²These border cost estimates are somewhat smaller than the average estimate in the literature. The transfer cost equivalent of the estimate in the survey of Anderson & van Wincoop (2004) is more than 20% for a representative rich country. Gopinath, Gourinchas, Hsieh & Li (2009) find for a grocery chain price differences of 20% for even the US-Canadian border.

as final good, whereas for Hongkong, Norway, and Sweden the border imposes considerable additional friction for productive capabilities.

The relocation cost parameter, r , the key parameter of interest, is estimated with small standard error. r captures most of the nonlinearity of the process, which is the key feature of the auxiliary model, and the motivation for my estimation strategy. The relocation discounts differ significantly from zero and unity for all country pairs. That is, for both real exchange rates I reject both of the extremes random walk ($r = 0$) and constant ($r = 1$).³³

Until now I have derived a new way of measuring the cost of relocating productive capabilities from one country to another, and found this cost to be larger than the corresponding cost for final goods. But what are the underlying causes for these costs? To answer this question, I regress the natural logarithm of my border cost measures on four explanatory factors commonly used in international trade. The independent variables are distance, the absolute 2003 GDP difference, and a dummy for common language.

[Table 5 about here.]

[Table 6 about here.]

Table 5 reveals that geographic distance affects the relocation cost for final goods, but not for productive capabilities. For every 1000 kilometers of geographic distance, the relocation cost for final goods increases by more than one percent of the transferred quantity. But as McCallum (1995) and Engel & Rogers (1996) have pointed out, there is more to a border than just geographic distance, and this is what matters for productive capabilities. This is captured by the relocation cost for productive capabilities: The left column of table 6 reveals that the cost reflected in prices of productive capabilities have additional explanatory power of the merger and acquisition (M&A) activity with the U.S., even after controlling for distance, language, and GDP. The relocation cost of final goods, however, does not explain

³³All other parameters are estimated with much larger standard errors, and help primarily in fitting the model. The averages of these estimates are, however, of some interest on their own: The median estimated productivity differential relative to the USA, $\Delta\alpha$, is +0.02 for final goods, but less than zero for productive capabilities. This might indicate that relative to the other countries in the sample the USA is better at producing productive capabilities, e.g. ideas, than at producing final goods. Likewise, the median welfare weight, q , on the foreign country for final goods is above one, whereas it is only 0.9 for productive capabilities. Interpreting these weights as a reflection of initial endowments, this indicates that the USA started with higher endowments of productive capabilities than its peers. Finally, the mean volatility estimate, σ , for final goods of 0.46 is considerably smaller than the estimate for productive capabilities of 0.55. Clearly, the creation process of productive capabilities (Schumpeter (1942)'s "creative destruction") involves more risk. Appendix G provides detailed results for all 17 country pairs.

M&A activity, simply because M&A does not involve the relocation of physical goods, but often a transfer of knowledge.

Having established mean reversion as a property of the real exchange rate, I want to know how long full mean reversion to the parity level takes. I therefore calculate the time that it takes the real exchange rate process on average to hit a boundary when started at a balanced level, and the time it takes to revert to this level if started at either boundary. Despite mean reversion, these so-called hittimes are for many countries two years and more. An extreme case is Japan-USA in table 8; these hittimes are, in the words of Rogoff (1996), “glacial”. However, there are notable exceptions. Table 7 lists the hittimes for the example Germany-USA, which are obtained by integrating over the appropriately scaled speed density (Karlin & Taylor 1981). The real exchange rate of final goods takes about 1.5 years from the upper boundary to a balanced level and vice versa. Despite the higher relocation cost this time interval for productive capabilities is only about 2/3 that long. This stems from the higher variance for productive capabilities, which illustrates that a narrow range of the real exchange rate is *not* equivalent to fast mean reversion. In fact, a narrow range for p does not necessarily imply a narrow range for ω . The deterministic drift weakens as the real exchange rate approaches the balanced level, and it requires a random shock to push the exchange rate through its zero-drift level. Likewise, only large random shocks push the exchange rate to the boundary despite a mean reverting drift.

[Table 7 about here.]

[Table 8 about here.]

Furthermore, if country pairs are asymmetric, then the mean reversion time depends on which boundary the process starts. In my example Germany-USA, returning to a balanced level from a situation where the USA enjoys maximum abundance takes about as long as returning from the opposite boundary. For Japan-USA, in contrast, the mean reversion times differ considerably depending on the boundary. The reason is the productivity differential, which makes e.g. keeping an excess amount of final goods in Japan attractive, and which makes the stock of final goods in the Japan grow faster than in the USA.

The hittime distribution is very skewed. Figure 10 shows that a large number of shocks to PPP which decay quickly within, say, a year, is offset by a smaller number of extremely slowly decaying deviations, with hittimes of five years and more. Hittimes also vary considerably between countries. Taken together this may explain the inconclusiveness of the data about halftimes of mean reversion documented by Kilian & Zha (2002).

[Figure 10 about here.]

In summary, relocation of productive capabilities across a border is more costly than relocation of final goods. Borders affect productive capabilities more than final goods. In particular, geographic distance increases the relocation cost for productive capabilities, but relative economic size seems to matter as well. In effect, returning from a maximum imbalance to a balanced position takes several years and this long time span obtains despite the real exchange rate process clearly not following a random walk.

5 Conclusion

Nonlinearities in the real exchange rate process carry useful foreign exchange market information. Based on a novel approach, which allows to estimate border cost directly from a single real exchange rate time series, I find borders to hinder the relocation of productive capabilities considerably more than of final goods. This indicates that markets for productive capabilities and final goods are at different stages of integration.

My approach confirms that the real exchange rate follows an overall stationary, nonlinear process, which is indistinguishable from a random walk close to parity, and provides two important new insights: First, as further integration of the world economy continues to shrink the barriers to trade, the range of values assumed by the real exchange rate will shrink as well. But if volatility shrinks at the same time, mean reversion might in fact *not* accelerate, despite shrinking barriers. Thus no matter how easy it eventually will become to relocate goods, the random walk behavior in the inner regime will ensure that reversion to PPP will remain slow. Second, even if relative prices never reach the arbitrage points, these unobserved points can nevertheless be estimated from their effect on the dynamics of the price process far away from the arbitrage points. Thus border cost can be estimated from time series only, without observing any arbitrage activity.

Three important implications for the forecasting of real exchange rates can be taken from this study. First, there is long-run predictability in line with Kilian & Taylor (2003) despite short-run random walk behavior. Second, my work emphasizes that for a meaningful forecast the *definition* of the real exchange rate must be tailored to the market of interest. Third, and importantly, border cost, endowment shocks, and its determining factors affect the drift of real exchange rate and might therefore help in forecasting exchange rates.

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Table 1: Maximum Sustainable Imbalance ($\bar{\omega}$)

(a) for Small Relocation Cost

Risk av. ($1 - \gamma$)	Risk ($\sigma_{11} = \sigma_{22}$)					
	0^+	0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	1.23	1.64	2.63	3.25	3.29	3.30
1	1.11	1.41	2.11	2.56	2.59	2.59
1.5	1.07	1.34	1.94	2.29	n.a.	n.a.
2	1.05	1.31	1.86	2.13	n.a.	n.a.
3	1.04	1.28	1.75	n.a.	n.a.	n.a.

Parameter values $r = 0.9$, $\rho = 0.07$, $\alpha = \alpha^* = 0.1$, $q = 1$

(b) for High Relocation Cost

Risk av. ($1 - \gamma$)	Risk ($\sigma_{11} = \sigma_{22}$)					
	0^+	0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	1.78	2.32	3.95	5.88	6.06	6.13
1	1.33	1.69	2.76	4.12	4.26	4.31
1.5	1.21	1.54	2.46	3.53	n.a.	n.a.
2	1.15	1.47	2.31	3.21	n.a.	n.a.
3	1.10	1.41	2.15	n.a.	n.a.	n.a.

Parameter values $r = 0.75$, $\rho = 0.07$, $\alpha = \alpha^* = 0.1$, $q = 1$

Table 2: Expected Time until First Transfer

(a) for Small Relocation Cost

risk av. $1 - \gamma$	0^+	risk σ				
		0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	∞	123.99	31.03	2.62	0.68	0
1	∞	51.47	17.62	1.70	0.45	0
1.5	∞	37.13	14.33	1.35	n.a.	n.a.
2	∞	31.22	12.70	1.15	n.a.	n.a.
3	∞	26.01	10.94	n.a.	n.a.	n.a.

Parameter values $r = 0.9$, $\rho = 0.07$, $\alpha = \alpha^* = 0.1$, $q = 1$,
starting at balanced level $\omega_0 = 0$

(b) for High Relocation Cost

risk av. $1 - \gamma$	0^+	risk σ				
		0.02	0.1	0.5	1	∞
0	∞	∞	∞	∞	∞	∞
0.5	∞	529.07	57.88	5.61	1.50	0
1	∞	147.31	28.55	3.73	1.03	0
1.5	∞	84.09	22.18	3.06	n.a.	n.a.
2	∞	61.04	19.39	2.70	n.a.	n.a.
3	∞	43.30	16.78	n.a.	n.a.	n.a.

Parameter values $r = 0.75$, $\rho = 0.07$, $\alpha = \alpha^* = 0.1$, $q = 1$,
starting at balanced level $\omega_0 = 0$

Table 3: Stationarity of ESTAR Regimes

	inner regime (AR)		outer regime (AR $\times \Phi$)	
	avg. root	non- stationary	avg. root	non- stationary
final goods	1.009	29%	1.068	0
productive capabilities	0.999	53%	1.080	0

Table 4: Relocation Cost vs. USA

Final Goods: r_{WPI}		Productive Capabilities: r_{CAP}	
Canada	0.98 (0.00)	Canada	0.89 (0.02)
Sweden	0.98 (0.00)	UK	0.89 (0.00)
Norway	0.93 (0.01)	Germany	0.85 (0.02)
Hongkong	0.91 (0.01)	Austria	0.84 (0.05)
Germany	0.89 (0.02)	Switzerland	0.84 (0.01)
Netherlands	0.89 (0.01)	Italy	0.84 (0.03)
Italy	0.89 (0.07)	Denmark	0.83 (0.07)
Denmark	0.88 (0.02)	Netherlands	0.82 (0.00)
UK	0.87 (0.01)	Belgium	0.82 (0.09)
Singapore	0.86 (0.01)	Australia	0.79 (0.02)
Spain	0.86 (0.02)	France	0.75 (0.01)
Australia	0.84 (0.02)	Singapore	0.73 (0.04)
Switzerland	0.84 (0.02)	Japan	0.72 (0.01)
Belgium	0.82 (0.02)	Sweden	0.64 (0.06)
Austria	0.79 (0.01)	Spain	0.49 (0.73)
Japan	0.78 (0.02)	Norway	0.47 (0.02)
France	0.74 (0.06)	Hongkong	0.46 (0.11)

Inflation-adjusted bookvalues, demeaned and deseasonalized, 1974:12–2008:12.
 ESTAR as auxiliary model with $2 \leq p \leq 4$ and $d \leq 12$ chosen by nonlinearity test.
 Standard errors in parentheses.

Table 5: Trade Components of Relocation Cost

	productive capabilities $\log(r_{CAP})$	final goods $\log(r_{WPI})$
Distance	-0.17 (0.21)	-0.11* (0.06)
Common language	0.06 (0.14)	0.06 (0.04)
Δ GDP	-0.05 (0.06)	0.03 (0.02)
Constant	0.32 (0.63)	-0.35 (0.18)
Adj. R^2	-0.11	0.20

Dependent variable: $\ln(r)$, independent variables: distance (in 10000 km), common language (1 if common language, 0 otherwise), absolute GDP difference (trillion USD at PPP, 2003). Sample size $N = 17$.
Standard errors in parentheses. (* significant at the 10% level)

Table 6: Relocation Cost as Determinant of Crossborder Mergers

	Mergers $\log(r_{CAP})$	Mergers $\log(r_{WPI})$
Relocation cost	753** (325)	-1137 (1341)
Distance	-2066*** (267)	-2318*** (317)
Common language	1622*** (161)	1732*** (202)
Total GDP	358*** (74)	364*** (93)
Constant	-2003*** (940)	-2287 (1116)
Adj. R^2	0.91	0.88

Dependent variable: total mergers with a US firm on one side, independent variables: $\ln(r)$, distance (in 10000 km), common language (1 if common language, 0 otherwise), total GDP (trillion USD at PPP, 2003). Sample size $N = 17$. Standard errors in parentheses. (* significant at the 10% level, ** at 5% level, *** at 1% level)

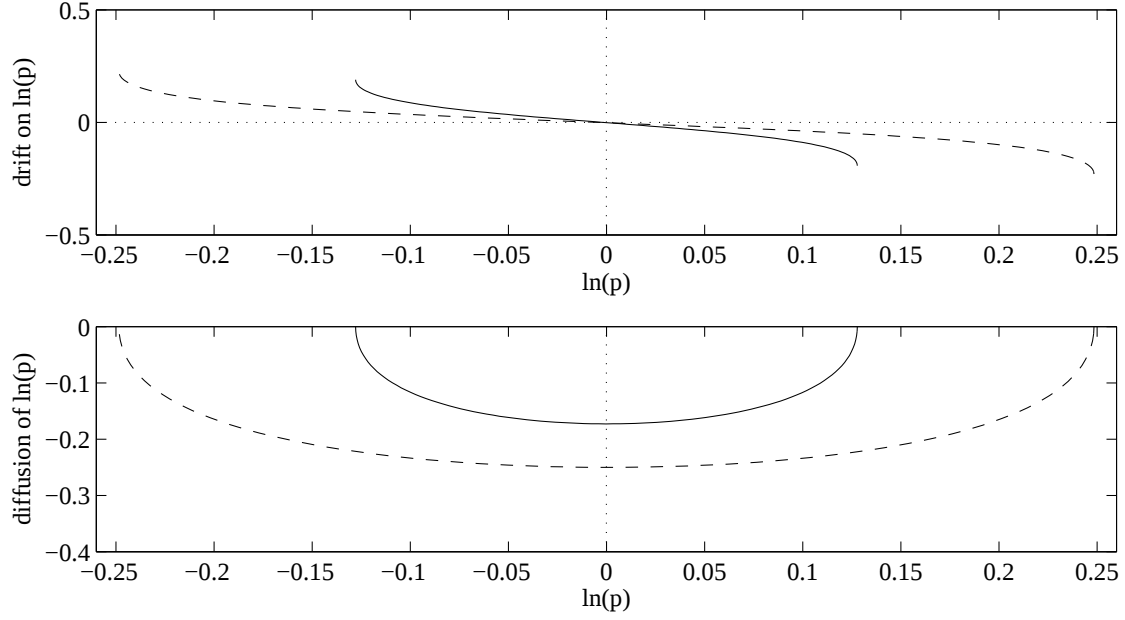
Table 7: Hittimes in years, Example Germany–USA

first hit of	starting at	final goods	productive capabilities
any boundary	ω_0	1.3	1.0
upper boundary $\bar{\omega}$	ω_0	4.2	3.0
lower boundary $\underline{\omega}$	ω_0	4.2	3.0
balanced level ω_0	$\bar{\omega}$	1.4	0.9
balanced level ω_0	$\underline{\omega}$	1.4	1.1
balanced level $\omega_0 = 0$			

Table 8: Hittimes in years, Example Japan–USA

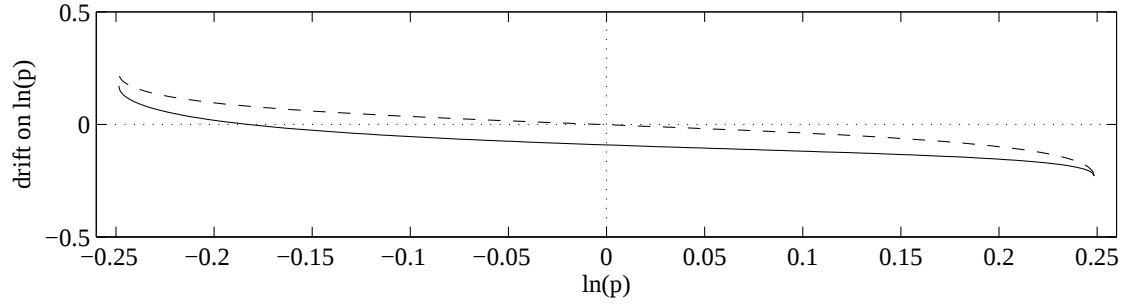
first hit of	starting at	final goods	productive capabilities
any boundary	ω_0	3.8	10.6
upper boundary $\bar{\omega}$	ω_0	11.7	126.8
lower boundary $\underline{\omega}$	ω_0	14.0	131.0
balanced level ω_0	$\bar{\omega}$	6.1	9.3
balanced level ω_0	$\underline{\omega}$	3.1	56.0
balanced level $\omega_0 = 0$			

Figure 1: Drift and Diffusion of Price Process, Effect of Relocation Cost



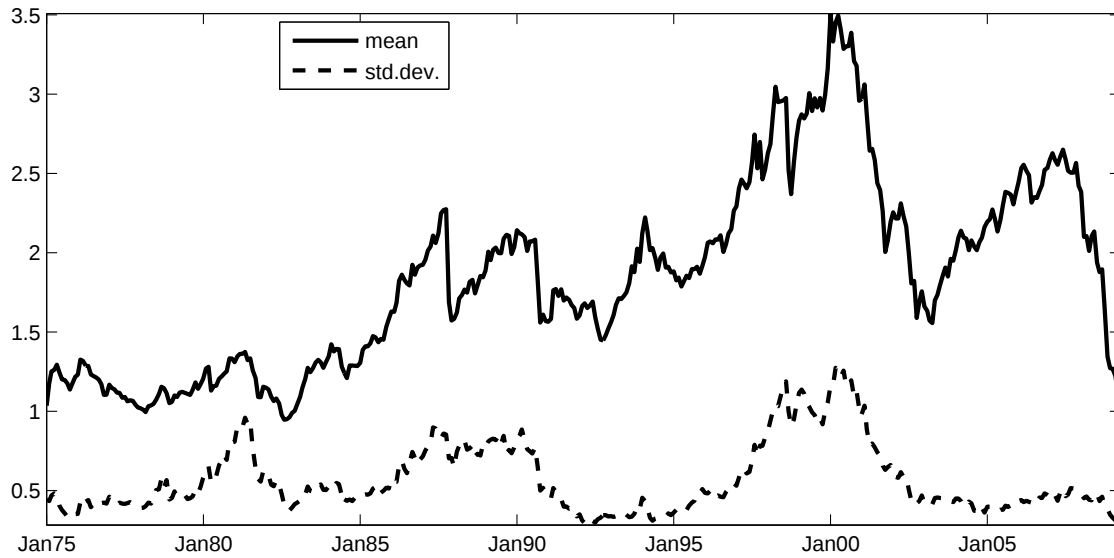
The upper panel shows the drift of the natural logarithm of the real exchange rate process as a function of the natural logarithm of the current real exchange rate, $\ln(p)$. The lower panel shows the corresponding (signed) diffusion. It compares the process for a low relocation cost ($r = 0.88$, solid line) with the process for a high relocation cost ($r = 0.78$, dashed line). The other parameter values used for this graph are $\gamma = -0.5$, $\rho = 0.07$, $q = 0.9$, $\alpha = \alpha^* = 0.18$, and $\sigma_{11} = \sigma_{22} = 0.59$.

Figure 2: Drift of Price Process, Effect of Productivity Differences



The graph shows the drift of the natural logarithm of the real exchange rate process as a function of the natural logarithm of the current real exchange rate, $\ln(p)$. The dashed line shows the process for the case when both countries are largely identical ($\alpha = \alpha^* = 0.18$). The solid line obtains if country 1 has a higher productivity than country 2 ($\alpha = 0.39 > \alpha^* = 0.18$). The other parameter values used for this graph are $\gamma = -0.5$, $\rho = 0.07$, $q = 0.9$, $r = 0.78$, $\sigma_{11} = \sigma_{22} = 0.59$.

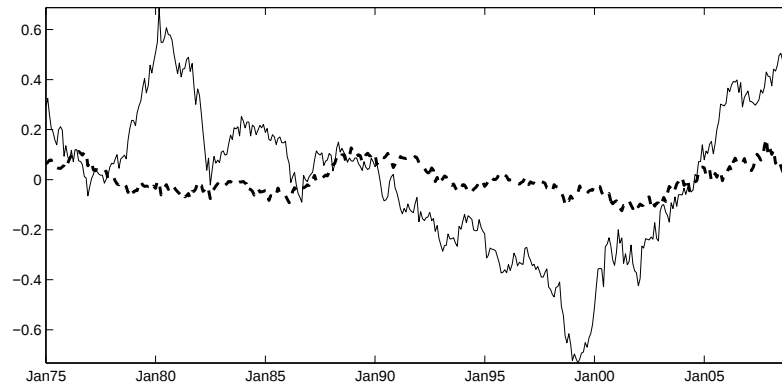
Figure 3: Average and Standard Deviation of Market-to-Book Ratios Across Countries



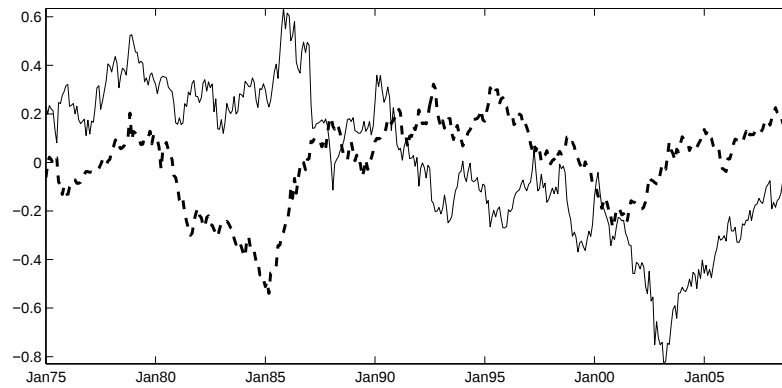
The graph shows the cross-section equal-weight mean and standard deviation of the market-to-book value for all 153 country pairs for the period 1974:12-2008:12.

Figure 4: Real Exchange Rates, 1974–2008

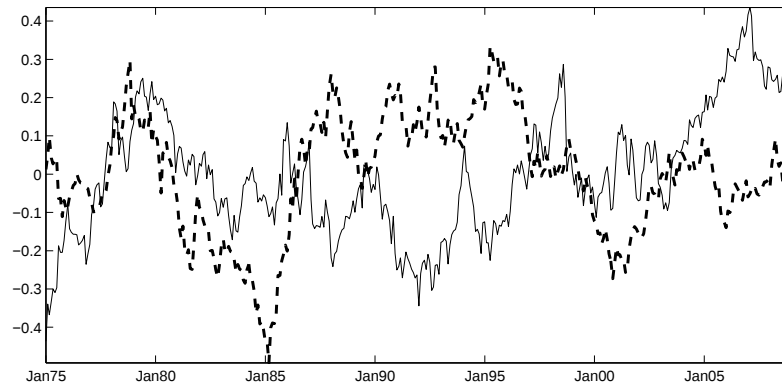
(a) Canada–USA



(b) Germany–USA

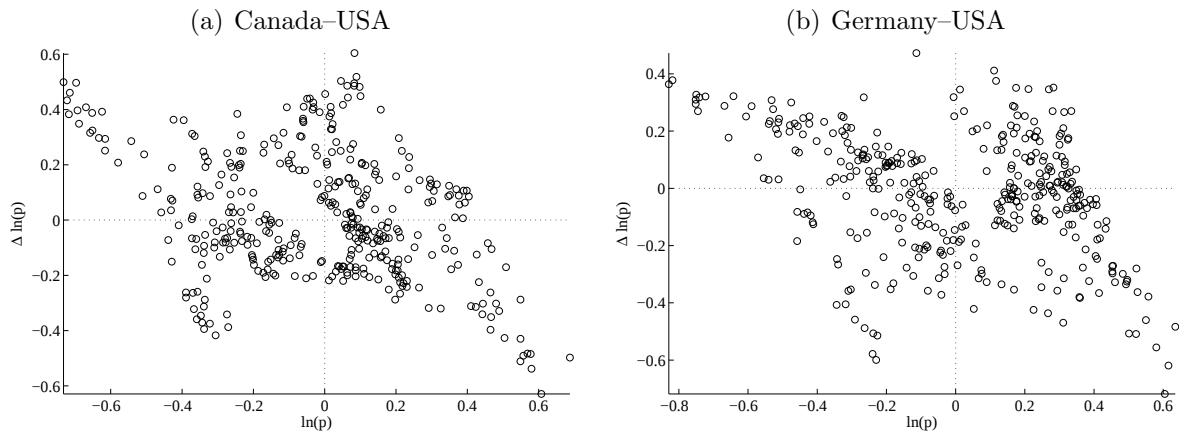


(c) UK–USA



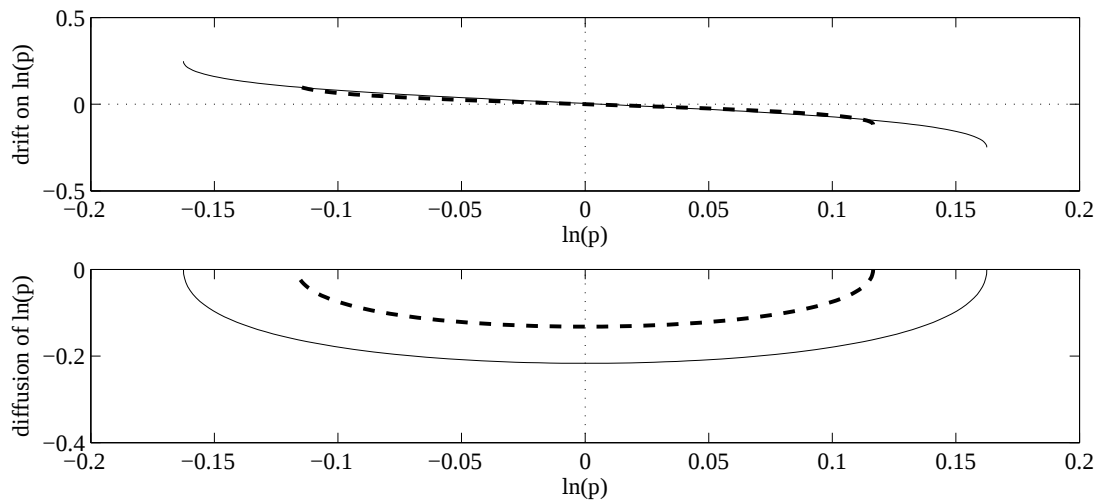
The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the time period 1974:12–2008:12.

Figure 5: 24-month Changes in Real Exchange Rate of Productive Capabilities vs. Initial Levels



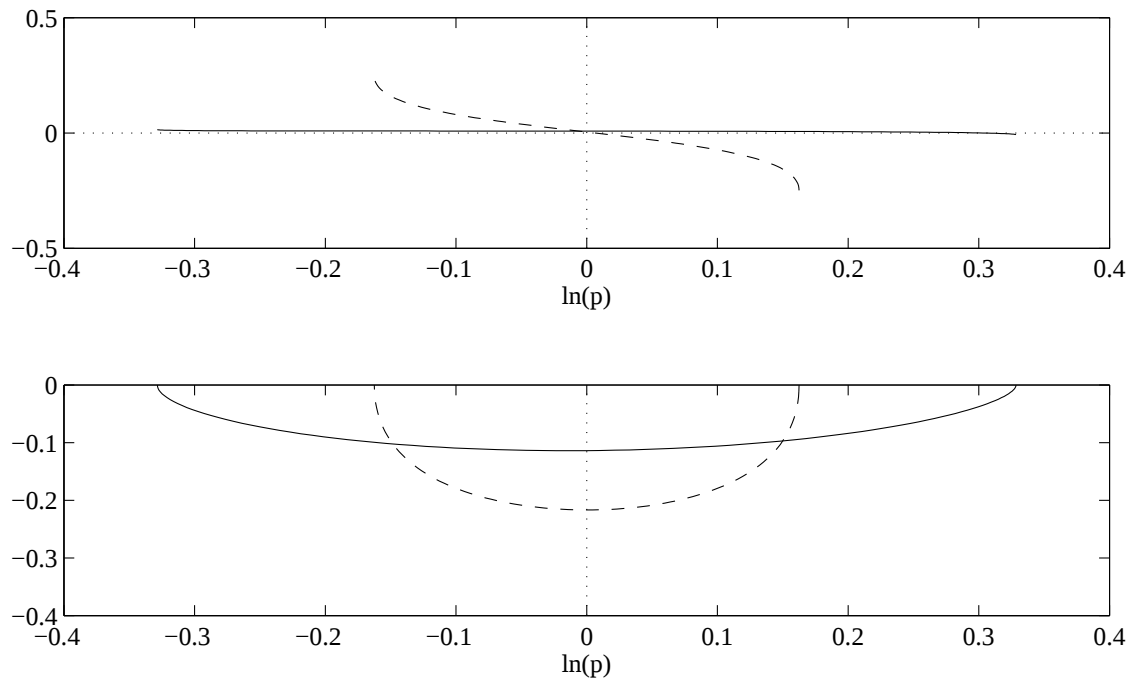
The graphs show the changes in the log real exchange rate of productive capabilities in the two years following a given level for the country pairs Canada-USA and Germany-USA during the period 1974:12-2008:12.

Figure 6: Drift and Diffusion of Real Exchange Rate, Final Goods vs. Productive Capabilities, Germany–USA



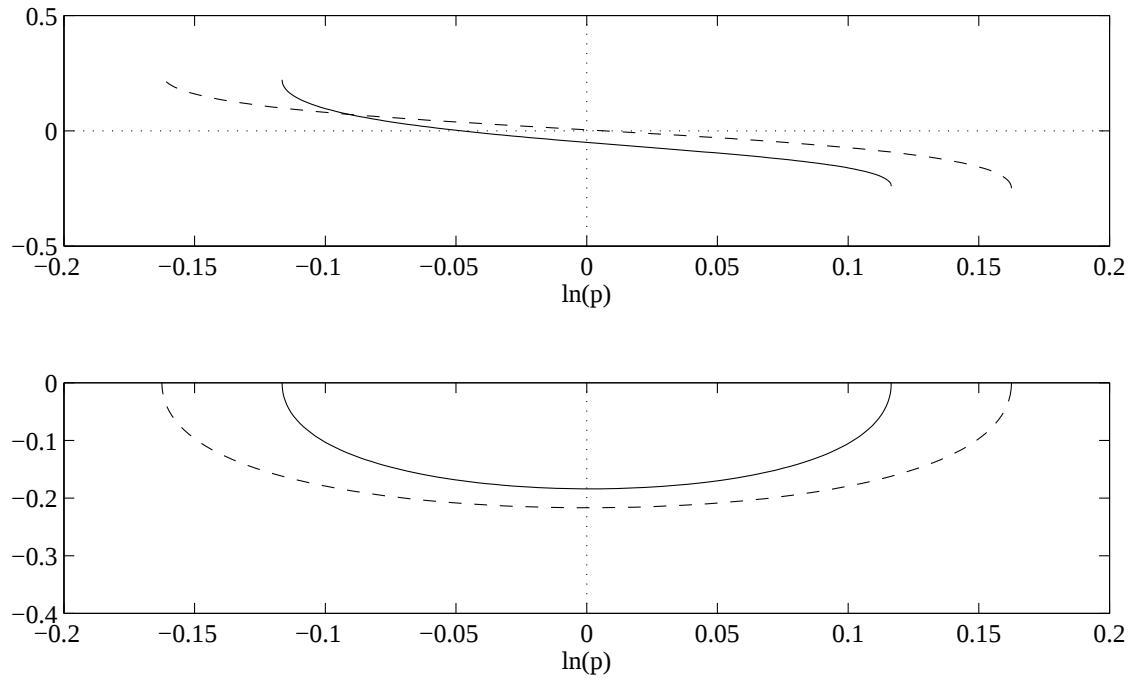
The graph plots the estimated drift and diffusion of the real exchange rate process, based on final goods (dashed line) and based on productive capabilities (solid line), for the pair Germany–USA during the period 1974:12–2008:12. Parameter values are taken from tables 9 and 10. The range of sustainable imbalances for final goods is $\omega \in [0.42; 2.36]$, and for productive capabilities $\omega \in [0.34; 2.51]$.

Figure 7: Drift and Diffusion of Real Exchange Rate based on Productive Capabilities, Japan–USA



The solid line is the estimated drift and diffusion of the real exchange rate of productive capabilities for the country pair Japan–USA during the period 1974:12–2008:12. The dashed line plots the same for Germany–USA. Parameter values are taken from tables 9 and 10. The range of sustainable imbalances is $\omega \in [0.23; 1.84]$.

Figure 8: Drift and Diffusion of Real Exchange Rate based on Productive Capabilities, UK-USA



The solid line is the estimated drift and diffusion of the real exchange rate for productive capabilities for the country pair UK-USA during the period 1974:12-2008:12. The dashed line plots the same for Germany-USA. Parameter values are taken from tables 9 and 10. The range of sustainable imbalances is $\omega \in [0.82; 4.80]$.

Figure 9: Relocation Costs of Final Goods vs. Productive Capabilities

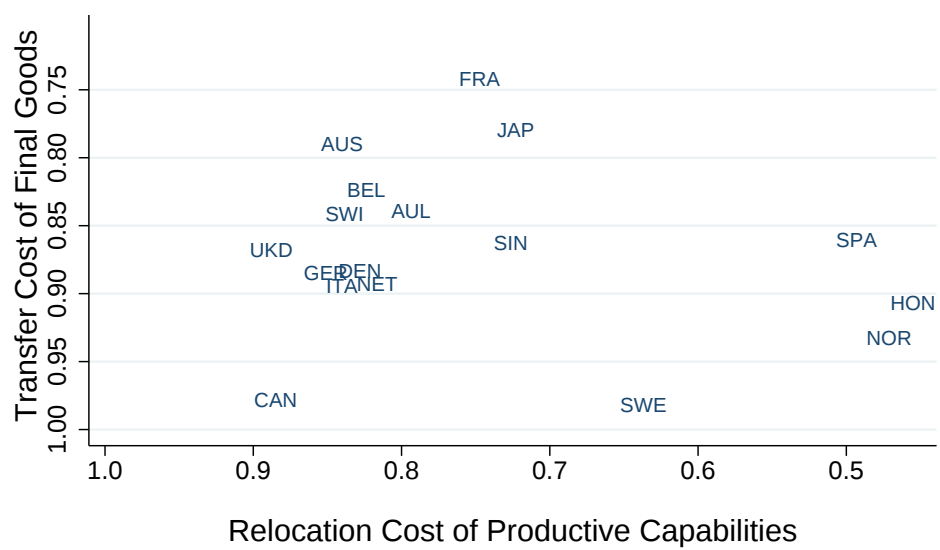
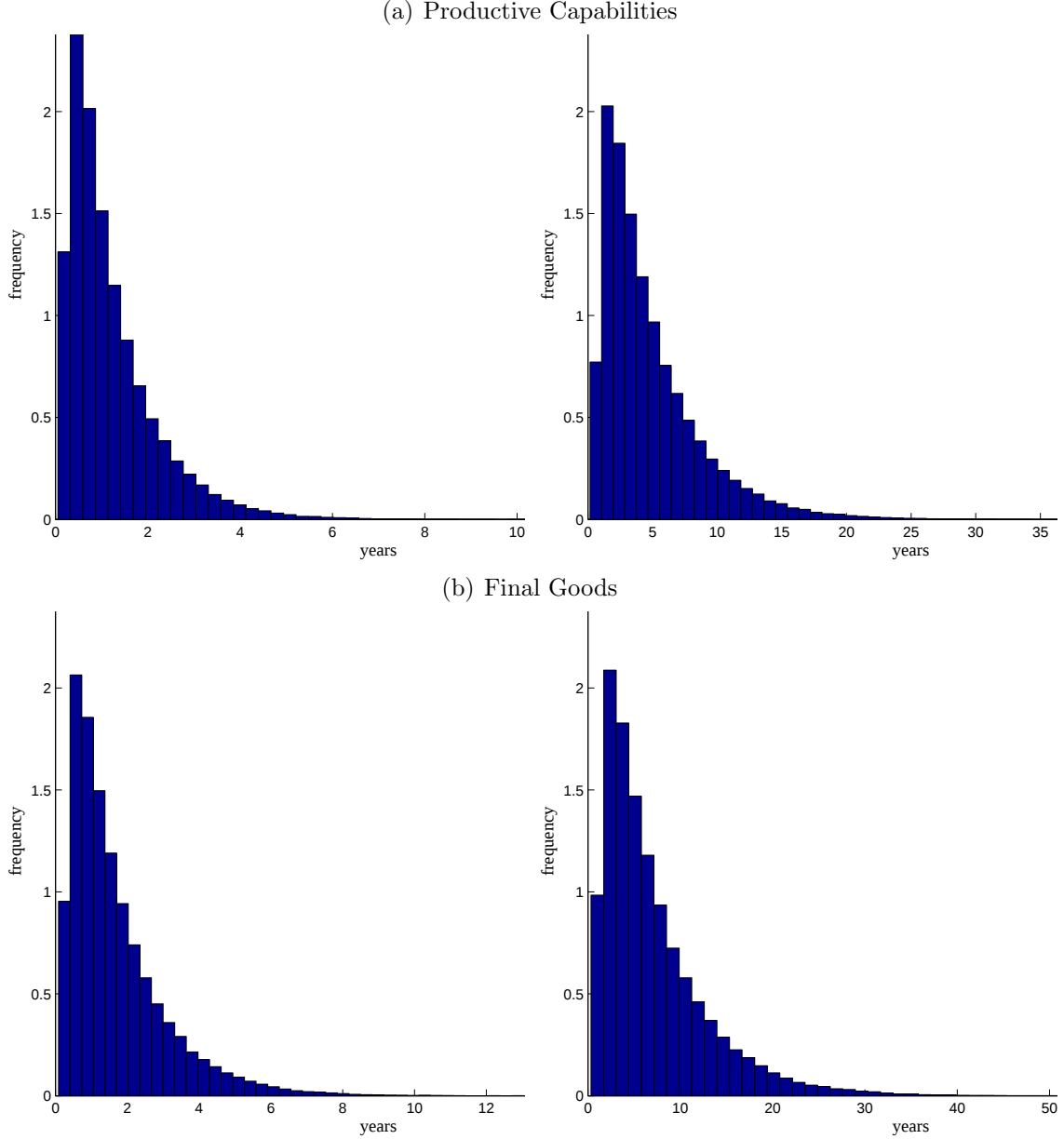


Figure 10: Hittime Distribution for Productive Capabilities, Germany–USA



The histograms show the distribution of the time span until the real exchange rate process reaches a certain level (“hittime distribution”). The upper panels show the hittime distribution of the real exchange rate process for productive capabilities, the lower panels of the real exchange rate process for final goods. The left panels record the first hit of any boundary, when the process starts from a balanced position ($\omega_0 = \sqrt{\bar{\omega}\underline{\omega}}$). The right panels show the first hit of the lower boundary $\underline{\omega}$, when the process starts from the upper boundary $\bar{\omega}$. Each histogram is based on 120000 simulated sample paths with the parameter values taken from tables 9 and 10. Frequencies are measured in 10^4 hits.

A A Model with Trade due to Specialization and Trade due to Arbitrage

This appendix presents a two-country model in which goods are traded for two different reasons. The global undifferentiated good K is produced in both countries, and is traded for arbitrage reasons, whereas the local goods A and B are produced only in country A and B respectively, and are traded due to complete specialization combined with love-of-variety in both countries. Goods located in country B are marked with an asterisk.

$$V(K, K^*, A, A^*, B, B^*) = \max_{\substack{c(t), c^*(t), \\ \Xi(r)}} E_0 \int_0^\infty e^{-\rho u} \left(\frac{q}{\gamma} c_K(u)^\gamma c_A(u)^\gamma c_B(u)^\gamma + \frac{2-q}{\gamma} c_K^*(u)^\gamma c_A^*(u)^\gamma c_B^*(u)^\gamma \right) du$$

s.t.

$$\begin{aligned} dK(t) &= [\alpha_K K(t) - c_K(t)] dt + K(t) \sigma dz(t) - dX_K(t) + rdX_K^*(t) \\ dA(t) &= [\alpha_A A(t) - c_A(t)] dt - dX_A(t) + rdX_A^*(t) \\ dB(t) &= -c_B(t) dt - dX_B(t) + rdX_B^*(t) \end{aligned}$$

$$\begin{aligned} dK^*(t) &= [\alpha_K^* K^*(t) - c_K^*(t)] dt + K^*(t) \sigma dz^*(t) + rdX_K(t) - dX_K^*(t) \\ dA^*(t) &= -c_A^*(t) dt + rdX_A(t) - dX_A^*(t) \\ dB^*(t) &= [\alpha_B B^*(t) - c_B^*(t)] dt + rdX_B(t) - dX_B^*(t) \end{aligned}$$

Storing good B in country A yields no return, it is merely consumed there. Thus absent any fixed cost of shipping and $\alpha_B > 0$ the optimal allocation implies no shipment of good B from country A to B ($dX_B = 0$), hence $rdX_B^* = c_B dt$ and $dB = 0$. Analogously $dX_A^* = 0$, $rdX_A = c_A^* dt$, $dA^* = 0$, and $dA = (\alpha_A A - c_A - \frac{1}{r} c_A^*) dt$. Thus under an optimal consumption plan, goods A and B are shipped at every instant to satisfy the consumption need in the other country. The real exchange rate for these two goods is fixed, for good A $\frac{V_A}{V_{A^*}} = \frac{V_A}{rV_A} = \frac{1}{r}$ and for good B $\frac{V_B}{V_{B^*}} = r$ hold always. Thus for goods which are traded for specialization reasons, there is no time variation in the real exchange rate. As part of an aggregate real exchange rate composed of many goods, these goods affect only the average exchange rate, e.g. the long-run PPP level.

To the contrary, good K is shipped only if due to random shocks this good becomes

very differently valued in the two countries. This trade for arbitrage reasons comes with a systematic time variation in real exchange rates, as shown by equation (17) in appendix C. Because goods A and B contribute nothing to the time-variation of exchange rates, on which my identification approach rests, I drop these goods from my model, and focus my model on goods with arbitrage trade only.

B Two-Good Model

The model described in the paper can be seen as a reduced form of a two-good model. Consider an economy identical to the one described in the paper, but now I explicitly model the difference between a final good, G , and a productive capability, K . Investment, I , the conversion of a final good into a productive capability in the same country carries no cost, but the reverse entails a loss of setup and training costs. These costs are proportional to the amount disinvested, D , so that of one unit of K only $q \in (0, 1]$ units of G remain. Relocation of final goods, X_G , between the two countries entails – as before – a proportional relocation cost of $1 - r_G$, and direct relocation of productive capabilities, X_K , a relocation cost of $1 - r_K$. Only the final good can be consumed in the respective country, and only the productive capability can be used in production.³⁴ The following four equations describe the dynamics of the stock of final goods and productive capabilities in the home and foreign country, respectively:

$$\begin{aligned} dG(t) &= -c(t)dt + G(t)\sigma dz(t) - dX_G(t) + r_G dX_G^*(t) - dI(t) + qdD(t) = 0 \\ dK(t) &= \alpha K(t)dt + K(t)\sigma dz(t) - dX_K(t) + r_K dX_K^*(t) + dI(t) - dD(t) \end{aligned}$$

$$\begin{aligned} dG^*(t) &= -c^*(t)dt + G^*(t)\sigma dz^*(t) - dX_G^*(t) + r_G dX_G^*(t) - dI^*(t) + q^*dD^*(t) = 0 \\ dK^*(t) &= \alpha^* K^*(t)dt + K^*(t)\sigma dz^*(t) - dX_K^*(t) + r_K dX_K^*(t) + dI^*(t) - dD^*(t) \end{aligned}$$

Under the same assumptions as in the model in the paper, in particular $\alpha > 0$, $\alpha^* > 0$, $G(0) = 0$, $G^*(0) = 0$, and non-increasing disinvestment cost coefficients, q and q^* , the stock of final goods in both countries is always zero. This obtains because only the productive capabilities generate output and are no more risky than final goods. Therefore, under continuous time and absent any fixed costs, it is never optimal to disinvest more than what

³⁴Otherwise the notation is the same as in equation (1). In the empirical part, home variables with asterisk represent the USA.

is needed for instantaneous consumption. Any addition to (G, G^*) is either immediately consumed, relocated, or invested. The state vector of this model thus collapses to (K, K^*) .

Despite the zero stock of final goods I can of course calculate the real exchange rate for these goods at all times. The valuation of the final goods is linked to the valuation of productive capabilities via relocation and (dis-)investment by $V_G = \min \left[\frac{V_K}{q}, \frac{V_{K^*}}{q^* r_G} \right]$ and $V_{G^*} = \min \left[\frac{V_{K^*}}{q^*}, \frac{V_K}{q r_G} \right]$.³⁵ These two conditions imply that the real exchange rates for final goods, $p_G(t) \equiv \frac{V_G(t)}{V_{G^*}(t)}$, and for productive capabilities, $p_K(t) \equiv \frac{V_K(t)}{V_{K^*}(t)}$, are linked by

Proposition 2

$$p_G(t) = \min \left[\max \left(p_K(t) \frac{q^*}{q}, r_G \right), \frac{1}{r_G} \right].$$

Proof: From

$$p_G(t) \equiv \frac{V_G(t)}{V_{G^*}(t)} = \frac{\min \left[\frac{V_K}{q}, \frac{V_{K^*}}{q^* r_G} \right]}{\min \left[\frac{V_{K^*}}{q^*}, \frac{V_K}{q r_G} \right]}$$

I get

$$p_G(t) = \begin{cases} r_G & \text{if } \frac{V_K}{q} < \min \left[\frac{r_G V_{K^*}}{q^*}, \frac{V_{K^*}}{r_G q^*} \right] \\ \frac{V_K}{V_{K^*}} \frac{q^*}{q} & \text{if } \frac{r_G V_{K^*}}{q^*} < \frac{V_K}{q} < \frac{V_{K^*}}{r_G q^*} \\ \frac{V_{K^*}}{V_K} \frac{q}{q^*} & \text{if } \frac{r_G V_{K^*}}{q^*} > \frac{V_K}{q} > \frac{V_{K^*}}{r_G q^*} \\ \frac{1}{r_G} & \text{if } \frac{V_K}{q} > \max \left[\frac{r_G V_{K^*}}{q^*}, \frac{V_{K^*}}{r_G q^*} \right] \end{cases},$$

which simplifies to

$$p_G(t) = \begin{cases} r_G & \text{if } \frac{V_K}{V_{K^*}} \frac{q^*}{q} < r_G \\ \frac{V_K}{V_{K^*}} \frac{q^*}{q} & \text{if } r_G \leq \frac{V_K}{V_{K^*}} \frac{q^*}{q} \leq \frac{1}{r_G} \\ \frac{1}{r_G} & \text{if } \frac{V_K}{V_{K^*}} \frac{q^*}{q} > \frac{1}{r_G} \end{cases}.$$

This directly gives the equation stated in the proposition. *q.e.d.*

The proposition shows that the differential behavior of the two real exchange rates stems from a scale factor, $\frac{q^*}{q}$, due to differential disinvestment cost, and from different cutoff points, $(\frac{1}{r_G}, r_G)$ vs. $(\frac{1}{r_K}, r_K)$, due to relocation costs. If $r_G > r_K$ then the link between the two real exchange rates is not only affected by the scale factor, but also by the frequently binding relocation cutoff.

³⁵This holds with equality because $G(t) = G^*(t) = 0 \forall t$.

C One-Good Model

In this appendix I discuss and solve an extension of Dumas (1992). In particular, I introduce nonzero covariance between the country-specific shocks and country-specific productivity.

C.1 General One-Good Model

Suppose z and z^* are two standard Brownian motion processes³⁶ and Ω is a positive definite and symmetric matrix.

$$\begin{pmatrix} d\tilde{z} \\ dz^* \end{pmatrix} = \Omega \begin{pmatrix} dz \\ dz^* \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \begin{pmatrix} dz \\ dz^* \end{pmatrix} = \begin{pmatrix} \sigma_{11}dz + \sigma_{12}dz^* \\ \sigma_{12}dz + \sigma_{22}dz^* \end{pmatrix}$$

Plugging this process of productivity shocks and the first order conditions for c into the Hamilton-Jacobi partial differential equation implies that capital stocks K and K^* follow the differential equation

$$\begin{aligned} 0 = & \frac{1-\gamma}{\gamma} q^{\frac{1}{1-\gamma}} V_K^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} (2-q)^{\frac{1}{1-\gamma}} V_{K^*}^{\frac{\gamma}{\gamma-1}} - \rho V \\ & + V_K \alpha K + V_{K^*} \alpha^* K^* + \frac{1}{2} V_{KK} \sigma_{11}^2 K^2 + V_{KK^*} \sigma_{12}^2 K K^* + \frac{1}{2} V_{K^*K^*} \sigma_{22}^2 K^{*2}. \end{aligned}$$

Using $V(K, K^*) = K^{\gamma} I(\omega)$, the capital imbalance $\omega = K/K^*$ then follows the differential equation

$$\begin{aligned} 0 = & \frac{1-\gamma}{\gamma} q^{\frac{1}{1-\gamma}} I'(\omega)^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} (2-q)^{\frac{1}{1-\gamma}} (\gamma I(\omega) - \omega I'(\omega))^{\frac{\gamma}{\gamma-1}} \\ & + \left[\alpha^* \gamma - \rho + \frac{1}{2} (\sigma_{12}^2 + \sigma_{22}^2) \gamma (\gamma - 1) \right] I(\omega) \\ & + \left[\alpha - \alpha^* + (\gamma - 1) (-\sigma_{22}^2 - \sigma_{12}^2 + \sigma_{12}(\sigma_{11} + \sigma_{22})) \right] \omega I'(\omega) \\ & + \left[\frac{1}{2} (\sigma_{11}^2 + \sigma_{12}^2) + \frac{1}{2} (\sigma_{22}^2 + \sigma_{12}^2) - \sigma_{12}(\sigma_{11} + \sigma_{22}) \right] \omega^2 I''(\omega). \end{aligned} \quad (8)$$

For estimation I use $\sigma_{12} = 0$ and $\sigma_{11} = \sigma_{22} = \sigma$, which leads to equation (2) in the main

³⁶ dz and dz^* may be called white noise with $dz \sim N(0, 1)$, $dz^* \sim N(0, 1)$

text. In the symmetric case $\alpha = \alpha^*$, $\sigma_{11} = \sigma_{22} = \sigma$, $q = 1$, and equation (8) reduces to

$$\begin{aligned}
0 &= \frac{1-\gamma}{\gamma} I'(\omega)^{\frac{\gamma}{\gamma-1}} + \frac{1-\gamma}{\gamma} [\gamma I(\omega) - \omega I'(\omega)]^{\frac{\gamma}{\gamma-1}} \\
&+ (\alpha\gamma - \rho)I(\omega) + \frac{1}{2}\gamma(\gamma-1)(\sigma^2 + \sigma_{12}^2)I(\omega) \\
&- (\gamma-1)(\sigma - \sigma_{12})^2\omega I'(\omega) + (\sigma - \sigma_{12})^2\omega^2 I''(\omega). \tag{9}
\end{aligned}$$

The last three terms capture the effect of a nonzero covariance.

C.2 Model Solution

Optimal choice of the two boundaries of $\Xi(r)$ requires that the good's valuation, as well as the marginal valuation before and after a relocation must be equal. This imposes three boundary conditions on each side of the differential equation. For the upper boundary these conditions are

$$V_K(K, K^*) = rV_{K^*}(K, K^*), \tag{10}$$

$$V_{KK}(K, K^*) = rV_{KK^*}(K, K^*), \tag{11}$$

$$V_{K^*K}(K, K^*) = rV_{K^*K^*}(K, K^*). \tag{12}$$

By homogeneity of the value function, the latter two conditions are identical. These conditions can be rewritten in terms of $I(\omega)$ as

$$\begin{aligned}
I(\bar{\omega}) &= \frac{1}{\gamma} (1 + r\bar{\omega})^\gamma \left[(1-\gamma) q^{\frac{1}{1-\gamma}} r^{\frac{\gamma}{\gamma-1}} + (1-\gamma) (2-q)^{\frac{1}{1-\gamma}} \right]^{1-\gamma} \\
&\times \left\{ \rho - \alpha^* \gamma - \frac{1}{2}(\sigma_{12}^2 + \sigma_{22}^2)(\gamma-1)\gamma \right. \\
&- \frac{r\bar{\omega}\gamma}{1+r\bar{\omega}} [\alpha - \alpha^* + (\gamma-1)(-\sigma_{22}^2 - \sigma_{12}^2 - \sigma_{12}(\sigma_{11} + \sigma_{22}))] \\
&- \left. \frac{r^2\bar{\omega}^2\gamma(\gamma-1)}{(1+r\bar{\omega})^2} \left[\frac{1}{2}(\sigma_{11}^2 - \sigma_{12}^2) + \frac{1}{2}(\sigma_{22}^2 - \sigma_{12}^2) - \sigma_{12}(\sigma_{11} + \sigma_{22}) \right] \right\}^{\gamma-1}, \tag{13}
\end{aligned}$$

$$\frac{I'(\bar{\omega})}{\gamma I(\bar{\omega})} = \frac{r}{1+r\bar{\omega}},$$

and

$$\frac{I''(\bar{\omega})}{\gamma I(\bar{\omega})} = \frac{r^2(\gamma-1)}{(1+r\bar{\omega})^2}.$$

For the lower boundary I have, analogously,

$$\begin{aligned}
I(\underline{\omega}) &= \frac{1}{\gamma} (r + \underline{\omega})^\gamma \left[(1 - \gamma) q^{\frac{1}{1-\gamma}} + (1 - \gamma) (2 - q)^{\frac{1}{1-\gamma}} r^{\frac{\gamma}{\gamma-1}} \right]^{1-\gamma} \\
&\times \left\{ \rho - \alpha^* \gamma - \frac{1}{2} (\sigma_{12}^2 + \sigma_{22}^2) (\gamma - 1) \gamma \right. \\
&- \frac{\underline{\omega} \gamma}{r + \underline{\omega}} \left[\alpha - \alpha^* + (\gamma - 1) (-\sigma_{22}^2 - \sigma_{12}^2 - \sigma_{12} (\sigma_{11} + \sigma_{22})) \right] \\
&\left. - \frac{\underline{\omega}^2 \gamma (\gamma - 1)}{(r + \underline{\omega})^2} \left[\frac{1}{2} (\sigma_{11}^2 - \sigma_{12}^2) + \frac{1}{2} (\sigma_{22}^2 - \sigma_{12}^2) - \sigma_{12} (\sigma_{11} + \sigma_{22}) \right] \right\}^{\gamma-1},
\end{aligned}$$

$$\frac{I'(\underline{\omega})}{\gamma I(\underline{\omega})} = \frac{1}{1 + r \underline{\omega}},$$

and

$$\frac{I''(\underline{\omega})}{\gamma I(\underline{\omega})} = \frac{\gamma - 1}{(r + \underline{\omega})^2}.$$

In the symmetric case $\alpha = \alpha^*$, $\sigma_{11} = \sigma_{22} = \sigma$, and $q = 1$, with $\sigma_{12} = 0$, I have $\bar{\omega} = 1/\underline{\omega} = \lambda$, $I(\bar{\omega}) = \bar{\omega}^\gamma I(\underline{\omega})$, and (13) reduces to

$$I(\lambda) = \frac{(1 + r\lambda)^\gamma}{\gamma} \left(1 + r^{\frac{\gamma}{\gamma-1}} \right)^{1-\gamma} \left(\frac{\rho - \alpha\gamma}{1 - \gamma} + \frac{\sigma^2 \gamma}{2} \cdot \frac{1 + r^2 \lambda^2}{(1 + r\lambda)^2} \right)^{\gamma-1}.$$

Applying Ito's formula for multiple standard processes I obtain the process of the capital imbalance ω inside of Ξ

$$\begin{aligned}
d\omega &= \left[-\frac{c(t)}{K(t)} + \frac{c^*(t)}{K^*(t)} - \sigma_{12}(\sigma_{11} + \sigma_{22}) + \sigma_{12}^2 + \sigma_{22}^2 + \alpha - \alpha^* \right] \omega dt \\
&+ (\sigma_{11} - \sigma_{12}) \omega dz - (\sigma_{22} - \sigma_{12}) \omega dz^*,
\end{aligned} \tag{14}$$

which in terms of $I(\omega)$ can be written as

$$\begin{aligned}
d\omega &= \left(-\frac{1}{\omega} \left(\frac{I'(\omega)}{q} \right)^{\frac{1}{\gamma-1}} + \left(\frac{\gamma I(\omega) - \omega I'(\omega)}{2 - q} \right)^{\frac{1}{\gamma-1}} - \sigma_{12}(\sigma_{11} + \sigma_{22}) + \sigma_{12}^2 + \sigma_{22}^2 + \alpha - \alpha^* \right) \omega dt \\
&+ \sqrt{(\sigma_{11} - \sigma_{12})^2 + (\sigma_{22} - \sigma_{12})^2} \omega dz',
\end{aligned} \tag{15}$$

where z' is again standard Brownian motion.

From (14), using $p(\omega) = \frac{V_K}{V_{K^*}} = \frac{I'(\omega)}{\gamma I(\omega) - \omega I'(\omega)}$ the process dp can be written as a function

of ω , $I(\omega)$, $I'(\omega)$, $I''(\omega)$, and $I'''(\omega)$. The process for $\ln(p)$ is

$$\begin{aligned}
& d\ln(p(\omega)) \\
&= dt \left\{ \left[-\frac{c}{K} + \frac{c^*}{K^*} + \alpha - \alpha^* + \sigma_{12}^2 + \sigma_{22}^2 - \sigma_{12}(\sigma_{11} + \sigma_{22}) \right] \left[\frac{I''}{I'} - \frac{(\gamma - 1)I' - \omega I''}{\gamma I - \omega I'} \right] \omega \right. \\
&+ \frac{(\sigma_{11} - \sigma_{12})^2 + (\sigma_{12} - \sigma_{22})^2}{2} \\
&\times \left[\frac{I'''I' - I''^2}{I'^2} - \frac{((\gamma - 2)I'' - \omega I''')(\gamma I - \omega I') - ((\gamma - 1)I' - \omega I'')^2}{(\gamma I - \omega I')^2} \right] \omega^2 \Bigg\} \\
&+ dz' \sqrt{(\sigma_{11} - \sigma_{12})^2 + (\sigma_{12} - \sigma_{22})^2} \left[\frac{I''}{I'} - \frac{(\gamma - 1)I' - \omega I''}{\gamma I - \omega I'} \right] \omega. \tag{16}
\end{aligned}$$

The optimality conditions (10), (11), (12), and their counterparts for the lower boundary directly imply that the price level at both boundaries is identical, even if $\bar{\omega} \neq 1/\underline{\omega}$. At the upper boundary $\bar{\omega}$, for example, I calculate

$$p(\bar{\omega}) = \frac{I'(\bar{\omega})}{\gamma I(\bar{\omega}) - \bar{\omega} I'(\bar{\omega})} = \frac{\frac{r\gamma}{1+r\bar{\omega}}}{\gamma - \bar{\omega} \frac{r\gamma}{1+r\bar{\omega}}} = r.$$

In the symmetric case ($\alpha = \alpha^*$, $\sigma_{11} = \sigma_{22} = \sigma$, and $q = 1$) equation (15) reduces to

$$\begin{aligned}
d\omega &= \left(-\frac{1}{\omega} I'(\omega)^{\frac{1}{\gamma-1}} + (\gamma I(\omega) - \omega I'(\omega))^{\frac{1}{\gamma-1}} + (\sigma - \sigma_{12})^2 \right) \omega dt \\
&+ \sqrt{2}(\sigma - \sigma_{12}) \omega dz'.
\end{aligned}$$

Utilizing the useful property of the symmetric case that $I'(1) = \frac{\gamma}{2}I(1)$, I find that at $\omega = 1$ the differential equation (9) implies

$$\begin{aligned}
0 &= (1 - \gamma)I'(1)^{\frac{1}{\gamma-1}} \\
&+ \alpha\gamma - \rho + \gamma(\gamma - 1)\sigma\sigma_{12} + (\sigma - \sigma_{12})^2 \frac{I''(1)}{I(1)}.
\end{aligned}$$

Simplifying further by setting $\sigma_{12} = 0$, the price process (16) reduces to

$$\begin{aligned}
d\ln(p(\omega)) &= dt \left\{ \left[-\frac{c}{K} + \frac{c^*}{K^*} + \sigma^2 \right] \left[\frac{I''}{I'} - \frac{(\gamma-1)I' - \omega I''}{\gamma I - \omega I'} \right] \omega \right. \\
&\quad \left. + \sigma^2 \left[\frac{I''' I' - I''^2}{I'^2} - \frac{((\gamma-2)I'' - \omega I''')(\gamma I - \omega I') - ((\gamma-1)I' - \omega I'')^2}{(\gamma I - \omega I')^2} \right] \omega^2 \right\} \\
&\quad + dz' \sigma \sqrt{2} \left[\frac{I''}{I'} - \frac{(\gamma-1)I' - \omega I''}{\gamma I - \omega I'} \right] \omega. \tag{17}
\end{aligned}$$

This special case allows calculating the exact value of the price level at $\omega = 1$ by

$$p(1) = \frac{\frac{\gamma}{2}}{\gamma - \frac{\gamma}{2}} = 1.$$

C.3 Proof of Proposition 1

Proposition (Drift of Real Exchange Rate at the Boundary) The drift of the real exchange rate at the upper boundary is

$$\mu_p(\omega = \bar{\omega}) = \alpha^* - \alpha + \frac{\bar{\omega} - 1/r}{\bar{\omega} + 1/r} (1 - \gamma) \sigma^2,$$

and at the lower boundary

$$\mu_p(\omega = \underline{\omega}) = \alpha^* - \alpha + \frac{\underline{\omega} - r}{\underline{\omega} + r} (1 - \gamma) \sigma^2.$$

Proof: I calculate the drift of the real exchange rate, $\ln(p)$, at the boundary. Let $\bar{\omega} > 1$ denote the upper imbalance level. One can show that the drift of $d\ln(p) = \mu_p(p)dt + \sigma_p(p)dz$ at $\omega = \bar{\omega}$ with $\sigma_{11} = \sigma_{22} = \sigma$ and $\sigma_{12} = 0$ is

$$\mu_p(r) = \sigma^2 \bar{\omega}^2 \left[\frac{I'''(\bar{\omega})}{I'(\bar{\omega})} (1 + r\bar{\omega}) - (\gamma - 1)(\gamma - 2) \frac{r^2}{1 + r\bar{\omega}} \right].$$

Further,

$$\frac{I'''(\bar{\omega})}{I'(\bar{\omega})} = \frac{\gamma - 1}{\bar{\omega}^2 (1 + r\bar{\omega})^2} \left[r^2 \bar{\omega}^2 (\gamma - 2) - r\bar{\omega} + 1 - \frac{1 + r\bar{\omega}}{\gamma - 1} \frac{\alpha - \alpha^*}{\sigma^2} \right].$$

Therefore the drift of $\ln(p)$ at $\omega = \bar{\omega}$ simplifies to

$$\mu_p(r) = \sigma^2(\gamma - 1) \frac{1 - r\bar{\omega}}{1 + r\bar{\omega}} - \alpha + \alpha^*.$$

q.e.d.

Note that the drift at the boundary depends on ρ only indirectly via $\bar{\omega}$, but directly on the productivity differential $\Delta\alpha = \alpha - \alpha^*$. At $\omega = \bar{\omega} > 1$ I have $V_K < V_{K^*}$ and therefore $p(\lambda) = \frac{V_K}{V_{K^*}} < 1$. For mean reversion to hold I need therefore a positive drift of p , which requires at $\alpha = \alpha^*$ that $1 - r\bar{\omega} < 0$ and therefore $\bar{\omega} > \frac{1}{r}$.³⁷

D Inflation Adjustment of Book Values

Book values record the value of capital goods and productive capabilities at the time of acquisition or production by the firm. In order to properly measure the current value of these goods, that is, in order to match the overall inflation reflected in the market values, I correct the book values for inflation. Figure 11 shows the effect of this inflation correction for Germany, Japan and the USA.

My correction procedure adjusts the original bookvalues (dashed line) for Germany upwards only in the high inflation periods of the 1970s. The new series is shown by the solid line. Persistent inflation periods lead to a substantial upward adjustment in bookvalues, as in the case of the USA in the late 1970s in the lower panel of figure 11. A deflation, as in Japan in the 1990s, has the opposite effect. The corrected book values for Japan in the middle panel of figure 11 are smaller than the original ones.

[Figure 11 about here.]

E Real Exchange Rate Data

Figures 12, 13, and 14 show the natural logarithm of the real exchange rate series in my dataset. The solid line represents the real exchange rate for productive capabilities, and the dashed line the real exchange rate for final goods.

[Figure 12 about here.]

³⁷The values for $\bar{\omega}$ in table 1 show that $\forall \gamma < 0 \exists \sigma_0(\gamma)$ s.t. $\forall \sigma < \sigma_0(\gamma)$ the process of $\ln(p)$ is in fact divergent.

[Figure 13 about here.]

[Figure 14 about here.]

F Nonlinearity Test

I test the null hypothesis of linearity against the alternative of ESTAR-type nonlinearity using a heteroskedasticity-consistent Lagrange-multiplier-type test (Granger & Teräsvirta 1993, Teräsvirta 1994, Franses & van Dijk 2000). For a given transition variable p_{t-d} the test is based on a second order Taylor approximation of the ESTAR model (6)

$$p_t = \beta_0 + \beta_1 q_{t-1} + \beta_2 q_{t-1} p_{t-d} + \beta_3 q_{t-1} p_{t-d}^2 + \varepsilon_t, \quad (18)$$

where $q_{t-1} = (p_{t-1}, p_{t-2}, \dots, p_{t-m})$ denotes the vector of independent AR(m) variables and $\beta_0, \beta_1, \beta_2$ and β_3 are functions of the parameters of the ESTAR model. The null hypothesis of linearity is then $H_0 : \beta_2 = \beta_3 = 0$. A heteroskedasticity-robust test statistic is

$$F = \frac{T - 3m - 1}{2mT} \mathbf{1}' D R (R' R)^{-1} R' D' \mathbf{1},$$

where $\mathbf{1} = (1, \dots, 1)'$, D is a $T \times T$ matrix with the residuals from the linear model on the diagonal, and the matrix R contains the residuals from the regression of the $2m$ interaction terms in (18) on $(1 \ q_{t-1})$. Under the null hypothesis this test statistic is approximately $F(2m, T - 3m - 1)$ distributed.

I determine the lag, d , of the transition variable as the lag which leads to the strongest rejection of linearity (Teräsvirta 1994). When the significance of rejection is similar for multiple lags, I choose the lowest, and the one that is robust against inclusion of additional AR lags.

G Indirect Inference Results

In this appendix I provide details on the other coefficients besides relocation cost for selected countries. All coefficients except the relocation cost are estimated with large standard errors and serve primarily for fitting the model (tables 9 and 10). The shock variance is typically higher for productive capabilities than for final goods.

[Table 9 about here.]

[Table 10 about here.]

Table 9: Indirect Inference Estimates for Final Goods vs. USA

	r	$\Delta\alpha$	σ	q
Australia	0.84 (0.02)	0.31 (4.1)	0.62 (1.2)	1.03 (45)
Austria	0.79 (0.01)	0.16 (0.29)	0.48 (0.06)	0.91 (5.6)
Belgium	0.82 (0.02)	0.13 (0.10)	0.46 (0.14)	1.25 (4.2)
Canada	0.98 (0.00)	0.02 (0.00)	0.16 (0.01)	0.87 (0.15)
Denmark	0.88 (0.02)	0.02 (5.8)	0.47 (0.14)	0.97 (110)
France	0.74 (0.06)	0.01 (0.95)	0.29 (0.17)	1.03 (21)
Germany	0.89 (0.02)	0.00 (7.9)	0.47 (0.07)	1.00 (120)
Hongkong	0.91 (0.01)	-0.00 (0.75)	0.27 (0.07)	0.86 (19)
Italy	0.89 (0.07)	0.04 (27)	0.47 (3.5)	1.25 (400)
Japan	0.78 (0.02)	0.02 (0.02)	0.35 (0.05)	1.08 (0.85)
Netherlands	0.89 (0.01)	0.03 (12)	0.51 (0.53)	1.00 (210)
Norway	0.93 (0.01)	-0.15 (6.5)	0.71 (0.96)	1.15 (180)
Singapore	0.86 (0.01)	-0.03 (0.04)	0.26 (0.05)	0.79 (0.09)
Spain	0.86 (0.02)	0.02 (3.3)	0.44 (0.14)	1.00 (61)
Sweden	0.98 (0.00)	0.00 (0.01)	0.17 (0.02)	1.26 (0.11)
Switzerland	0.84 (0.02)	0.06 (3.3)	0.49 (0.62)	1.28 (43)
UK	0.87 (0.01)	0.05 (2.7)	0.45 (0.50)	1.22 (37)

WPI, demeaned and deseasonalized, 1974:12–2008:12, $\gamma = -0.5$, $\rho = 0.07$. ESTAR as auxiliary model with $2 \leq p \leq 4$ and $d \leq 12$ chosen by nonlinearity test. Standard errors in parentheses.

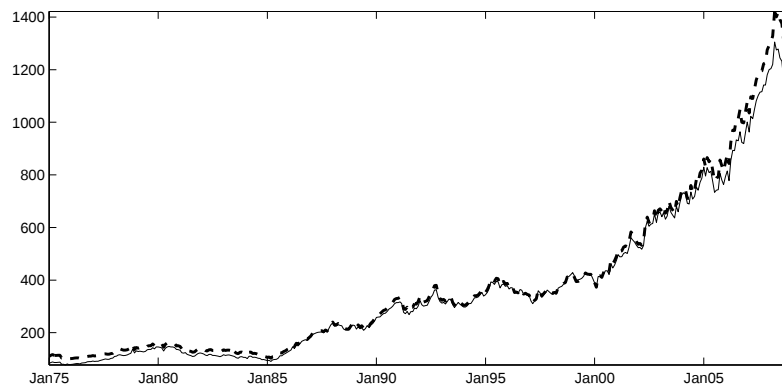
Table 10: Indirect Inference Estimates for Productive Capabilities vs. USA

	r	$\Delta\alpha$	σ	q
Australia	0.79 (0.02)	-0.03 (1.5)	0.52 (0.17)	0.80 (23)
Austria	0.84 (0.05)	0.05 (30)	0.88 (0.29)	1.07 (8800)
Belgium	0.82 (0.09)	-0.10 (13)	0.71 (1.2)	0.52 (490)
Canada	0.89 (0.02)	-0.05 (2.5)	0.54 (0.29)	0.83 (41)
Denmark	0.83 (0.07)	-0.06 (14)	0.68 (0.97)	0.87 (240)
France	0.75 (0.01)	-0.01 (0.16)	0.51 (0.05)	0.88 (1.5)
Germany	0.85 (0.02)	-0.02 (6.9)	0.65 (0.20)	0.90 (170)
Hongkong	0.46 (0.11)	0.04 (2.5)	0.52 (0.59)	1.00 (67)
Italy	0.84 (0.03)	-0.02 (11)	0.84 (0.13)	0.71 (7400)
Japan	0.72 (0.01)	-0.01 (0.03)	0.15 (0.08)	0.77 (0.98)
Netherlands	0.82 (0.00)	0.04 (0.01)	0.25 (0.01)	0.99 (0.04)
Norway	0.47 (0.02)	-0.00 (0.07)	0.42 (0.05)	0.67 (0.66)
Singapore	0.73 (0.04)	0.04 (9.4)	0.63 (0.76)	1.32 (140)
Spain	0.49 (0.73)	0.03 (17)	0.39 (2.9)	1.36 (327)
Sweden	0.64 (0.06)	0.14 (0.39)	0.55 (0.39)	0.58 (6.2)
Switzerland	0.84 (0.01)	-0.08 (2.0)	0.58 (0.31)	1.14 (27)
UK	0.89 (0.00)	0.21 (3.1)	0.68 (0.67)	0.99 (32)

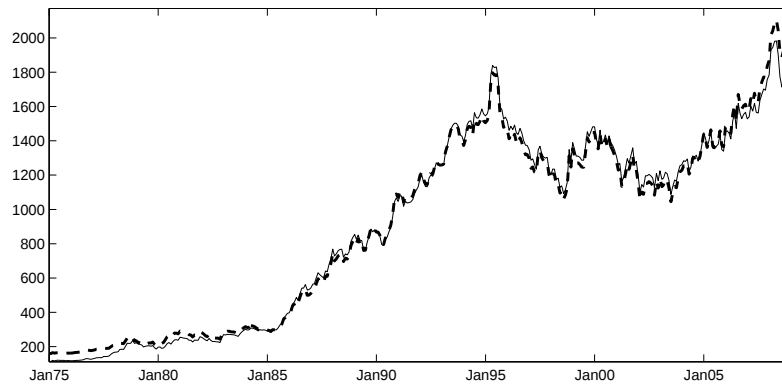
Inflation-adjusted bookvalues, demeaned and deseasonalized, 1974:12–2008:12, $\gamma = -0.5$, $\rho = 0.07$. ESTAR as auxiliary model with $2 \leq p \leq 4$ and $d \leq 12$ chosen by nonlinearity test. Standard errors in parentheses.

Figure 11: Book Value Correction

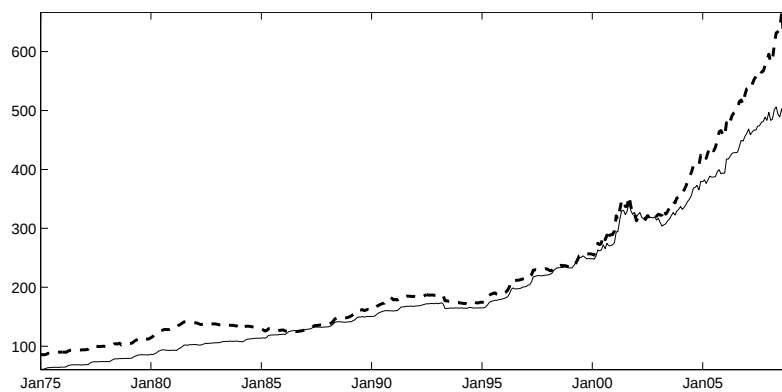
(a) Germany



(b) Japan

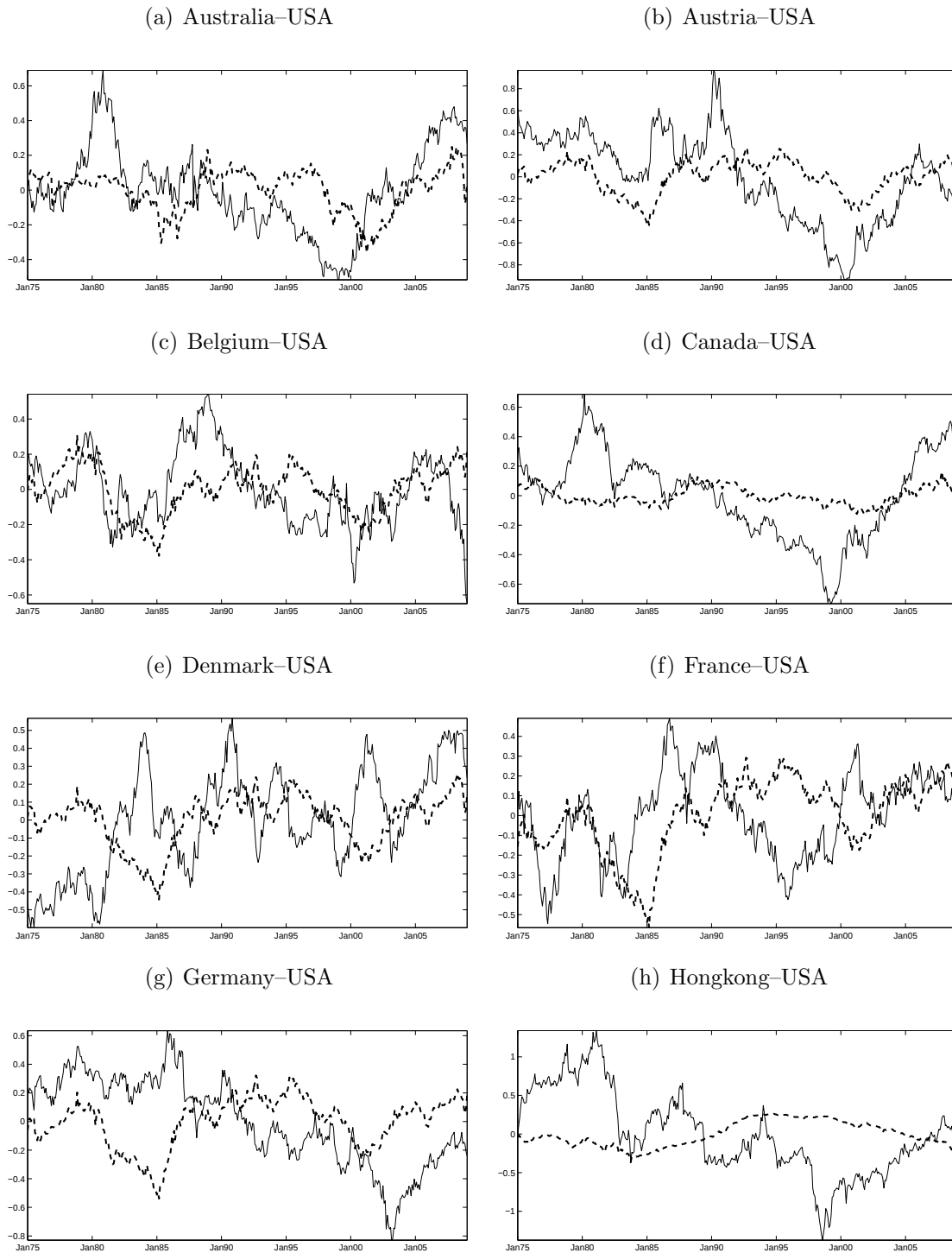


(c) USA



The solid line shows the reported book value for companies included in the respective MSCI country index during 1974:12–2008:12. The dashed line shows the same book value after correcting for inflation.

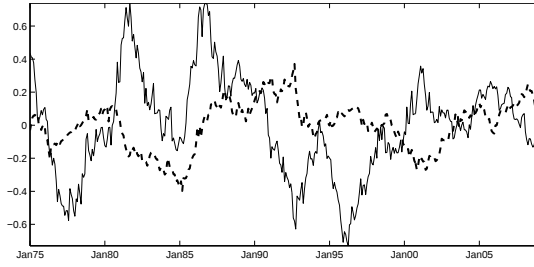
Figure 12: Real Exchange Rates, 1974-2008



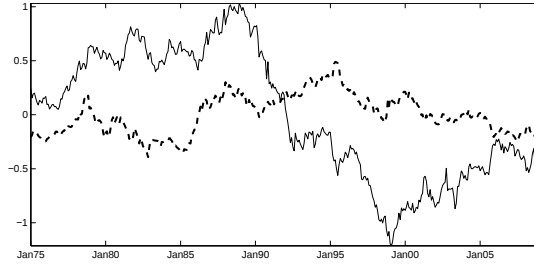
The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the period 1974:12-2008:12.

Figure 13: Real Exchange Rates, 1974-2008 (continued)

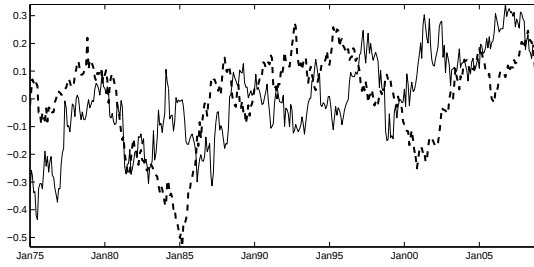
(a) Italy–USA



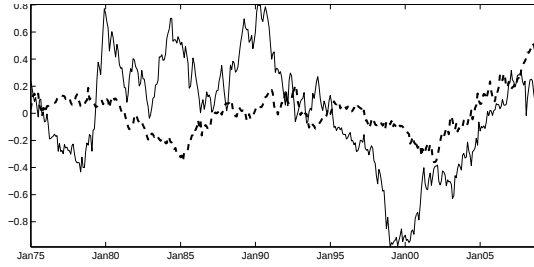
(b) Japan–USA



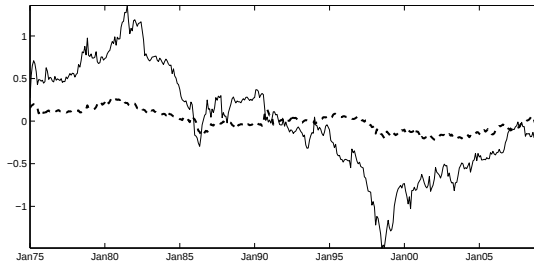
(c) Netherland–USA



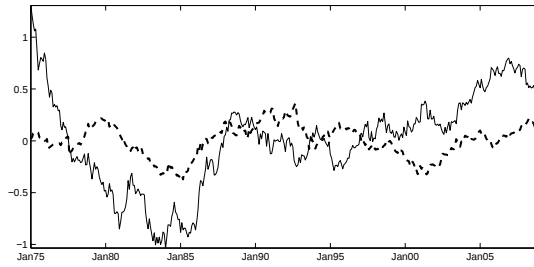
(d) Norway–USA



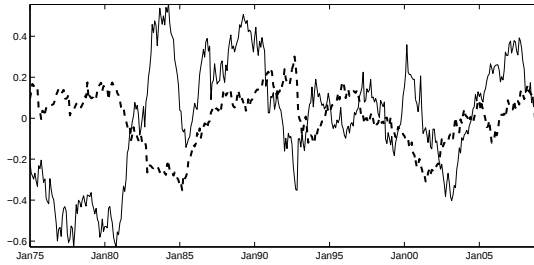
(e) Singapore–USA



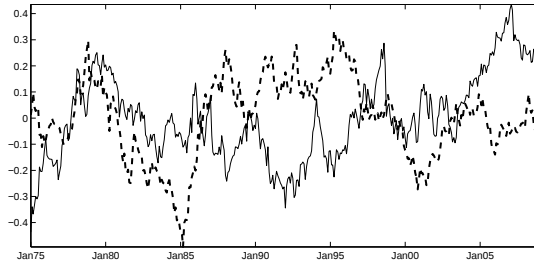
(f) Spain–USA



(g) Sweden–USA



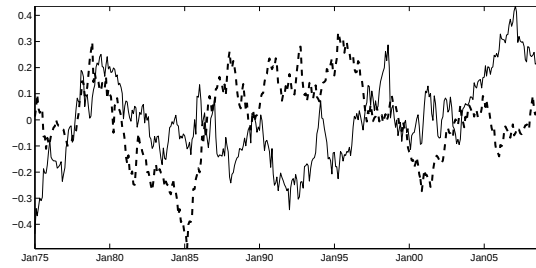
(h) Switzerland–USA



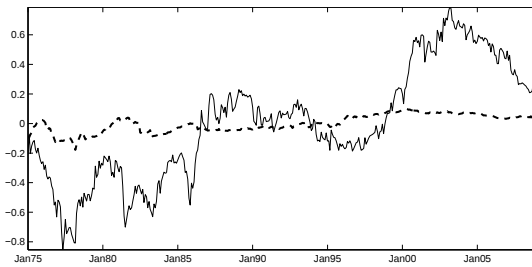
The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the period 1974:12-2008:12.

Figure 14: Real Exchange Rates, 1974-2008 (continued)

(a) United Kingdom–USA



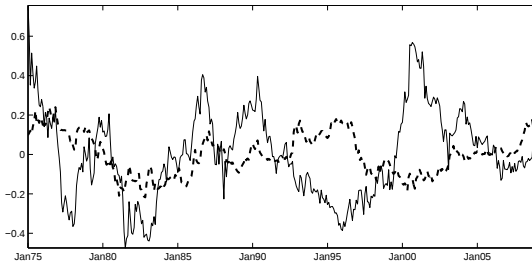
(b) France–Germany



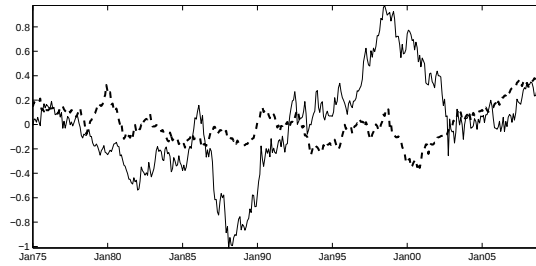
(c) France–Japan



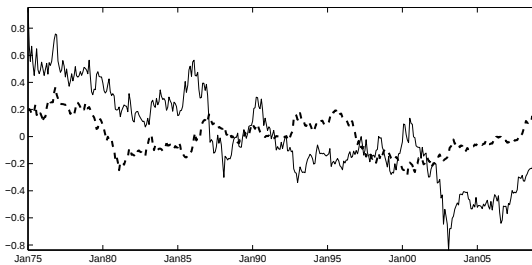
(d) France–United Kingdom



(e) Germany–Japan



(f) Germany–United Kingdom



(g) Japan–United Kingdom



The graphs show the natural logarithm of the real exchange rate for productive capabilities (solid line), and for final goods (dashed line), for the period 1974:12-2008:12.