

Precautionary Saving over the Business Cycle*

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Abstract

In this paper, we present a tractable model of time-varying precautionary saving behaviour due to changes in uninsured unemployment risk. In our model, agents facing incomplete markets and borrowing constraints respond to changes in labour market conditions by altering their buffer stock of precautionary wealth, with a direct impact on current consumption. The model is calibrated to match the evidence on the share of “permanent income consumers” and the distribution of wealth in the U.S. economy. We find a large, but relatively short-lived, impact of the precautionary motive on aggregate consumption in response to a typical labour market shock. The implications of the precautionary motive for consumption Euler equation tests are also discussed.

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1 Introduction

How important are changes in precautionary asset accumulation for the propagation of business cycle shocks? In this paper, we attempt to answer this question by constructing a tractable model of time-varying precautionary saving behaviour driven by endogenously countercyclical changes in unemployment risk. Because households are assumed to be imperfectly insured against this risk, they rationally respond to such changes by altering their buffer stock of precautionary wealth. This in turn amplifies the consumption response to shocks and, via general-equilibrium effects, propagates to all other macroeconomic aggregates.

Our motivation for investigating the role of precautionary saving over the business cycle is primarily based on earlier empirical evidence, which points out to a significant role for the precautionary motive in explaining the accumulation of wealth by individuals and their variations over time. Empirical studies that focus on the cross-sectional dispersion of wealth suggest that households facing higher income risk accumulate more wealth, all else equal (Carroll, 1994; Carroll and Samwick, 1997, 1998; Carroll et al., 2003). This argument has been extended to the time-series dimension by Carroll (1992), Gourinchas and Parker (2001) and Parker and Preston (2005), who argue that changes in precautionary wealth accumulation may substantially amplify consumption fluctuations. We take stock of their results and construct a general equilibrium model in which the strength of the precautionary motive is explicitly related to the extent of unemployment risk (the main source of income fluctuations for most households), and the latter back to the optimal job opening policy of firms.

Although students of the business cycle now routinely incorporate involuntary unemployment into business cycle analysis (via the introduction of labour search and matching frictions a la Diamond-Mortensen-Pissarides), the precautionary motive for accumulating assets has been remarkably absent from these analyses. Indeed, most papers follow the original construct of Mertz (1995) and Andolfatto (1996) by assuming complete insurance against unemployment risk. This assumption implies that a representative agent allocates economywide resources between consumption and savings while facing only aggregate risk, so that households' idiosyncratic transitions across employment statuses do not independently affect consumption-saving plans. Clearly, the ubiquity of the complete unemployment insurance assumption is based on the analytical tractability that it allows, rather than by a strong economic prior about the effectiveness of insurance markets. As is well known, the lack of perfect cross-household insurance against individual risk often implies a considerable

amount of household heterogeneity, because the decision (state) of every household typically turns out to depend on the whole history of shocks that this household has faced (see, e.g., Huggett, 1993; Aiyagari, 1994; Krusell and Smith, 1998). In this respect, the contribution of this paper is partly methodological in that it provides a tractable search-and-matching model of equilibrium unemployment with incomplete consumption insurance.

From a substantive point of view, our paper is an attempt at identifying and clarifying the channels by which incomplete insurance may amplify the impact of aggregate (productivity) shocks via their impact on individual uncertainty and precautionary saving. We focus the impact recessionary shocks, which have the effect of slowing down firms' job creation and hence raising the duration of unemployment spells. Imperfect income insurance implies that households fear such spells, and all the more so that they are long-lasting. Consequently, a negative shock strengthens the precautionary motive for holding wealth, causing aggregate consumption to fall substantially more (and hence the saving rate to fall substantially less) than under complete markets. This phenomenon is consistent with the idea that the precautionary motive substantially propagates recessions by amplifying the consumption response to the initial shock. We also present evidence of this propagation mechanism. More specifically, we find that an expected deterioration of labour market conditions has a significant impact on realised consumption growth, once the usual determinants of consumption growth have been controlled for (i.e., the interest rate and current income growth).

It is useful to compare the properties of our model with those proposed by Krusell et al. (2010) and Nakajima (2010), who construct computational, heterogeneous agents models featuring both labour search and matching frictions and incomplete unemployment insurance.¹ Both papers find the behaviour of their incomplete-market economies to be quite similar to that of the complete-market one. An important difference between our paper and theirs is that they assume that wages are determined via Nash bargaining. Under the calibration of the search and matching model proposed by Shimer (2005), Nash bargaining implies high wage flexibility, little employment volatility, and hence a small effect of productivity shocks on idiosyncratic unemployment risk. Under the "small surplus" calibration strategy proposed by Hagedorn and Manovskii (2008), employment is highly volatile but unemployment is almost not costly, due to the high value of home production (or leisure.) In

¹Gomes, Greenwood and Rebelo (2001) also combine incomplete markets with labour search, but use the Lucas-Prescott modelling of the labour markets, rather than the Mortensen-Pissarides framework that we share with Krusell et al. and Nakajima.

either case, aggregate shocks have a very limited impact either on the size or on the cost of idiosyncratic labour income fluctuations, implying that the precautionary motive for holding wealth is bound to be weak in the first place. The present paper analyses the alternative (and yet unexplored) possibility that unemployment fluctuations have realistic magnitudes whilst at the same time being relatively costly to the workers –a configuration that we obtain by imposing a substantial degree of wage rigidity, as in Hall (2005, 2009). In consequence, households in our model have good reasons to fear unemployment and hence to form a substantial buffer stock of precautionary wealth against *ex ante*. Relatedly, their targeted stock changes substantially over the business cycle, with significant effects on aggregates.

From a methodological point of view, we exhibit a particular class of incomplete-market models with aggregate shocks whose behaviour can be summarised by a small-dimensional dynamic system and is thus amenable to standard solution techniques (e.g., log-linearisation). This is possible because the equilibrium on which our analysis is based has two key features. First, it endogenously generates a small amount of cross-sectional household heterogeneity, rather than the large-scale heterogeneity implied by most heterogeneous-agent models.² Second, agents in the model do not achieve full self-insurance (despite precautionary wealth accumulation) and hence experience a discontinuous drop in individual income and consumption when falling into unemployment. Because the impact of this fall on the (ex ante) precautionary motive is of first-order magnitude, it is properly accounted for by a linear(ised) asset accumulation rule.

The remainder of the paper is organised as follows. The following section introduces the model. It starts by describing firms’ decisions about investment and hiring, then goes on to describe households’ decisions about consumption and saving, and finally characterises the equilibrium that results from their interactions. Section 3 further specifies the model by endogenously limiting the cross-sectional heterogeneity in household wealth, thereby making the model tractable. In section 4, the model is calibrated and impulse-response functions are drawn and discussed, with particular attention being paid to the response of aggregate consumption to aggregate shocks. Section 5 shows a piece of evidence supporting the countercyclicality of precautionary saving over the business cycle. Section 6 concludes.

²The present paper builds on Algan et al. (2009) and Challe and Ragot (2010), who also use equilibria featuring limited cross-sectional heterogeneity. However, these papers focus on aggregate (policy) shocks that do not affect the variance of idiosyncratic volatility.

2 The model

The economy is populated by a continuum of (heterogenous) households, indexed by i and uniformly distributed along the unit interval, as well as a representative firm. Households differ in two respects: their degree of patience and their access to credit markets, both of which affect their desired asset holdings. All households rent out labour and capital to the representative firm, which latter produces the unique (all-purpose) good in the economy. The good and asset markets are perfectly competitive. However, asset markets are incomplete and the labour market is imperfectly competitive, as we describe below.

2.1 Production and employment

Timing and technologies The representative firm produces output, Y_t , out of capital K_t and employment n_t according to the production function $Y_t = z_t G(K_t, n_t)$, where $G(\cdot, \cdot)$ exhibits positive, decreasing marginal products and constant returns to scale (CRS), and where $\{z_t\}_{t=0}^{\infty}$, $z_t > 0 \forall t$, is a stationary aggregate productivity process with date t history $z^t = \{z_i\}_{i=0}^t$. Defining $k_t \equiv K_t/n_t$ and $g(k_t) \equiv G(k_t, 1)$ gives $Y_t = z_t n_t g(k_t)$. Capital depreciates at a rate $\mu \in [0, 1]$. The capital stock is owned by the households and rented out to the firm in every period; as usual, the supply of capital results from date $t-1$ households' asset holding decisions, and the demand for it from date t firm's decision (conditional on z_t), with the price of capital freely adjusting to clear the market.

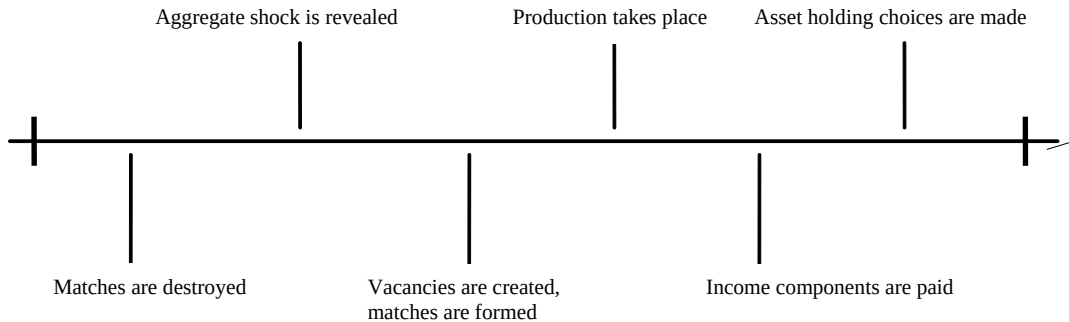


Figure 1: Sequence of events at date t .

The firm's hiring decisions are impeded by search and matching frictions a la Diamond-Mortensen-Pissarides. The timing of job destruction and creation, which we summarise in Figure 1, is as follows. At the very beginning of date t , a fraction ρ of existing matches are

destroyed, leaving the firm with $(1 - \rho) n_{t-1}$ employees and producing a job seekers' pool of size $1 - (1 - \rho) n_{t-1}$ (i.e., the number of unemployed at the end of date $t - 1$, $1 - n_{t-1}$, *plus* those who lose their job at the beginning of date t , ρn_{t-1}). Given its knowledge of $1 - (1 - \rho) n_{t-1}$ and z_t , the firm posts v_t vacancies, at cost $c > 0$ each, and a fraction λ_t of which are filled in the current period. Hence, total employment at date t is:

$$n_t = (1 - \rho) n_{t-1} + \lambda_t v_t. \quad (1)$$

The vacancy filling rate λ_t is related to the vacancy opening policy of the firm via the matching technology. The number of matches M_t formed at date t is assumed to depend on both the size of the job seekers' pool and the number of posted vacancies, v_t , according to the function $M_t = M(1 - (1 - \rho) n_{t-1}, v_t)$, which is increasing and strictly concave in both arguments and has CRS. Thus, the vacancy-filling rate satisfies $\lambda_t = M_t/v_t = m(\theta_t)$, where $\theta_t \equiv v_t / (1 - (1 - \rho) n_{t-1})$ is the market tightness ratio, and where the function $m(\theta_t) \equiv M(\theta_t^{-1}, 1)$ is strictly decreasing in θ_t .

Employment contracts. The issue of wage formation is a much debated one in the context of labour search and matching models. In particular, Hall (2005, 2009) and Shimer (2010) have made it clear that a broad set of wage setting schemes (of which period-by-period generalised Nash bargaining is only one example) generate gains from trade. We assume here that the households and the firm split the match surplus according to bilaterally efficient dynamic contracts that are negotiated at the time of the match and implemented as planned for the duration of the match. As is well known, the existence of a match surplus arising from search costs implies that there are many feasible such contracts because the bargaining set of households and the firm is nonempty. Following Hall (2009) and Stevens (2004), we restrict our attention to a simple class of dynamic contracts whereby the firm pays the worker its full marginal product, except at the time of the match when the worker is paid below marginal product. The profit flow extracted by the firm on new matches motivates –and finances– the payment of vacancy opening costs, but existing matches generate no pure profits thereafter. Formally, this arrangement is equivalent to a “fee contract” in which any matched worker i enjoys the wage $w_t = z_t G_2(K_t, n_t)$ at any point in time but pays a fixed fee $\psi_t > 0$ to the firm at the time of hiring; that is, the worker is actually paid $\tilde{w}_t = w_t - \psi_t > 0$ during the probation period, which is assumed to last for one period, and the full wage thereafter. We let ψ_t respond to the aggregate states, i.e., $\psi_t = \psi(z_t)$, $\psi'(\cdot) > 0$. As discussed by Hall (2009), the response of the initial wage discount to productivity reflects the degree of

inertia in employees' compensation and hence the strength of the job-creation response to underlying driving forces. More specifically, a high elasticity of ψ_t w.r.t. z_t is associated with a more muted response of newly employed's compensation $\tilde{w}_t$ to z_t , which tends magnify the firm's job creation response –and hence the size of the employment response– to the shock.

The instantaneous profit flow of the representative firm at date t is then given by:

$$\Pi_t = z_t G(K_t, n_t) - n_t w_t - (R_t - 1 + \mu) K_t + v_t (\lambda_t \psi_t - c), \quad (2)$$

subject to (1), and taking n_{t-1} , λ_t , R_t , as well as the contract $(\tilde{w}_t, w_t)$, as given. The optimal choice of capital per employee gives:

$$k_t = g'^{-1} \left(\frac{R_t^K - 1 + \mu}{z_t} \right). \quad (3)$$

On the other hand, from (2) the firm expands vancancy openings until $z_t G_2(K_t, n_t) \lambda_t - \lambda_t w_t + \lambda_t \psi_t - c = 0$. Since $G_2(K_t, n_t) = w_t$ in the class of contracts under consideration, the economywide vacancy-filling rate that results from these openings is:

$$\lambda_t = \frac{c}{\psi(z_t)} \equiv \lambda(z_t), \quad (4)$$

From (2)–(4) and the CRS assumption, the firm makes no pure profits in equilibrium (i.e., old matches generate no profit, while the quasi-rent extracted from new matches is exhausted in the payment of vacancies costs.). From the matching function specified above, the tightness ratio that results from the optimal vacancy policy of the firm is $\theta_t = m^{-1}(\lambda_t)$. Hence, the job-finding rate in this economy is:

$$f_t = \lambda_t \cdot m^{-1}(\lambda_t) \equiv f(z_t). \quad (5)$$

where f is continuously increasing in z_t . With our assumed timing and job destruction and creation processes, households lose their job at the beginning of the period with probability ρ , but job losers get rematched with the firm within the same quarter with probability f_t . It follows that the quarter-to-quarter separation rate is:

$$s_t = \rho(1 - f_t). \quad (6)$$

Finally, the change in employment from date $t-1$ to date t is $\Delta n_t = f_t(1 - n_{t-1}) - s_t n_{t-1}$. Using (5)–(6) and rearranging, we find the law of motion for total employment to be:

$$n_t = (1 - \rho)(1 - f(z_t))n_{t-1} + f(z_t). \quad (7)$$

There are several advantages to the dynamic contracts assumed here, and the equilibrium unemployment dynamics that results, relative to the more familiar approach to surplus sharing whereby bargaining takes place in every period and the firm extracts quasi-rents from all surviving matches. One of them is that the problem of the firm turns out to be static, and hence unambiguously well defined whoever holds the liabilities of the firm. Indeed, our model will feature asset holders having heterogeneous pricing kernels and who may thus disagree about the appropriate investment and job-opening policies of the firm, because they value future profits (relative to current ones) differently. By having the firm unambiguously maximise current profits only, our specification allows us to sidestep such corporate governance issues whilst making full use of the search-and-matching framework to model the unemployment risk faced by individuals. Another advantage of this specification is that the value of a match to the firm, and hence the number of vacancy openings and implied equilibrium job-finding rate, $f(z_t)$, only depend on the current aggregate state z_t . This property will allow us in the quantitative exercise below to treat $\{f_t\}$ as the forcing sequence summarising the unemployment risk faced by individuals –the only object of interest in this paper–, and then to flexibly “back up” this sequence by an appropriate choice of $\{z_t\}$ and $\psi(z_t)$. Finally, it implies that there is only one outside asset in the economy –claims to the capital stock–, which substantially simplify the exposition.

2.2 Households’ behaviour

Every household i is endowed with one unit of labour, which is supplied inelastically to the representative firm if the household is employed. Employed households earn w_t (minus the down payment $\psi(z_t)$ for newly matched employees), while unemployed households earn a fixed home production income $\delta > 0$. The population is divided into two subgroups, *impatient* (i.e., high-discounting) households with subjective discount factor $\beta \in (0, 1)$ and *patient* ones with subjective discount factor $\beta^p \in (\beta, 1)$. The former occupy the subinterval $[0, \Omega]$, $\Omega \in [0, 1)$, while the latter cover the complement interval $(\Omega, 1]$. Impatient households are thus in proportion Ω in the economy.

Impatient households. Impatient households maximise their expected life-time utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i)$, $i \in [0, \Omega]$, where c_t^i is the consumption of a such typical household i at date t and $u(\cdot)$ the period utility function of impatient households, which satisfies $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$. We restrict the set of assets that impatient households have access to in

two ways. First, we assume that these households cannot issue assets contingent on their employment status –that is, there is not unemployment insurance scheme, either public or private.³ Second, we assume they cannot borrow against future income.⁴ Given these restrictions, together with the fact that the representative firm makes zero profits at all times (see Section 2.1), the only asset that can be used to smooth out idiosyncratic labour income fluctuations are claims to the capital stock. We denote by e_t^i household i 's employment status at date t , with $e_t^i = 1$ if the household is employed and zero otherwise. Moreover, we use the indicator variable ν_t^i to indicate whether an employed worker at date t was separated from the firm at the beginning of the period, with $\nu_t^i = 1$ if this is the case and 0 otherwise. The budget and non-negativity constraints faced by an impatient household i are then:

$$a_t^i + c_t^i = e_t^i (w_t^i - \nu_t^i \psi_t) + (1 - e_t^i) \delta + R_t a_{t-1}^i, \quad (8)$$

$$c_t^i, a_t^i \geq 0. \quad (9)$$

where a_t^i is household i 's holdings of claims to the capital stock of the representative firm by the end of date t . The Euler condition for impatient households is:

$$u'(c_t^i) = \beta E_t(u'(c_{t+1}^i) R_{t+1}) + \varphi_t^i, \quad (10)$$

where φ_t^i is the Lagrange coefficient associated with the borrowing constraint $a_t^i \geq 0$, with $\varphi_t^i > 0$ if the constraint is binding and $\varphi_t^i = 0$ otherwise. Condition (10), together with the given initial asset holdings a_{-1}^i and the terminal condition $\lim_{t \rightarrow \infty} \beta^t E_t u'(c_t^i) = 0$, fully characterise the optimal asset holdings of impatient households.

Patient households. We assume that patient households maximise the lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^p u^p(c_t^i)$, $i \in (\Omega, 1]$, where $\beta^p \in (\beta, 1)$ is their discount factor and $u^p(\cdot)$ their instant utility function, which is assumed to be increasing and strictly concave over $[0, \infty)$. In contrast to impatient households, patient households have complete access to asset markets –including the full set of Arrow-Debreu securities and loan contracts. As noticed by Mertz

³Alternatively, one could interpret the home production parameter δ as the outcome of an unemployment insurance scheme. Our result would be unaltered provided that the scheme is funded by lump sum taxing the employed.

⁴The assumption of no borrowing follows Bewley (1977), Scheinkman and Weiss (1986), Zeldes (1989), Carrol (1991) and more recently Krusell and Smith (1998) and Heathcote (2005), among many others. As discussed by Heathcote, this condition may overstate actual borrowing limits, but also understate it if some of the agent's assets (e.g., durables) may not readily be liquidated to smooth out income shocks.

(1995) and Hall (2009), full insurance implies that these households collectively behave like a large “representative family” in which the family head ensures equal ex post marginal utility of consumption for all its members –despite the fact that the members experience heterogenous employment statuses due to the random process of job creation and destruction. Since consumption is the only argument in the period utility, equal marginal utility implies equal consumption, so the budget constraint of this hypothetical family is:

$$C_t^p + A_t^p + (1 - \Omega) (1 - (1 - \rho) n_{t-1}) \psi_t f_t = R_t A_{t-1}^p + (1 - \Omega) (w_t n_t + (1 - n_t) \delta), \quad (11)$$

where C_t^p and A_t^p denote the consumption and end-of-period asset holdings of the family (both of which must be divided by $1 - \Omega$ to find the per-member analogues). Because patient and impatient households are perfectly symmetric from the point of view of the firm, the family enjoys a share $1 - \Omega$ of aggregate labour income, which also includes home production. In every period, the job seekers pool contains $(1 - \Omega) (1 - (1 - \rho) n_{t-1})$ members of the representative family, a share f_t of which gets to be matched with the firm in the same period; by assumption, every match is associated with a transfer ψ_t from the family to the firm.

The Euler equation summarising the optimal asset allocation of the family is given by:

$$\beta^p E_t [(u^{p'}(C_{t+1}^p) / u^{p'}(C_t^p)) R_{t+1}] = 1. \quad (12)$$

As is now well understood since Becker (1980), Becker and Foias (1981), and more recently Kiyotaki and Moore (1997) and Iacoviello (2005), under heterogenous discount factors and borrowing limits patient households tend to accumulate large quantities of assets at the expense more impatient ones, asymptotically leading the former to hold the entire asset stock. This will not occur in our economy because impatient households have a specific (precautionary) motive for holding wealth –despite their relatively high degree of impatience.

2.3 Market clearing

Since households are uniformly distributed over $[0, 1]$, with a share Ω of impatient households in the economy, clearing of the market for claims to the capital stock is given by:

$$A_{t-1}^p + \int_0^\Omega a_{t-1}^i di = n_t k_t, \quad (13)$$

where the left hand side is total asset holdings by the households at the end of date $t - 1$ and the right hand side the demand for capital by the representative firm at date t . On the

other hand, clearing of the goods market requires:

$$C_t^p + \int_0^\Omega c_t^i di + I_t + cv_t = z_t n_t g(k_t) + (1 - n_t) \delta, \quad (14)$$

where $\int_0^\Omega c_t^i di$ is total consumption by impatient households, $I_t = n_{t+1}k_{t+1} - (1 - \mu) n_{t+1}k_{t+1}$ aggregate investment, cv_t total vacancy costs and $(1 - n_t) \delta$ total home production income.

We define an *equilibrium* of this economy as a sequence of households' decision variables $\{C_t^p, c_t^i, A_t^p, a_t^i\}_{t=0}^\infty$, $i \in [0, \Omega]$, firm's decision variables $\{K_t, v_t\}_{t=0}^\infty$, and aggregate variables $\{\lambda_t, f_t, s_t, n_t, w_t, R_t\}_{t=0}^\infty$ that satisfy the households' and the firm's optimality conditions (3), (4) and (12), together with the market-clearing conditions (13)-(14), given the forcing sequence $\{z_t\}_{t=0}^\infty$ and the initial wealth distribution $(A_{-1}^p, a_{-1}^i)_{i \in [0, \Omega]}$.

3 A minimal cross-sectional heterogeneity equilibrium

As is well known, the joint assumption of incomplete markets and borrowing constraints generally preclude the reduction of the model's dynamics to a small-scale dynamic system. This is because the asset holding decisions of a particular household depend its accumulated wealth, while the later is determined by the entire history of idiosyncratic income shocks. In consequence, the asymptotic cross-sectional distribution of wealth usually has infinitely many states, and hence infinitely many Euler equations would be necessary to exactly describe the behaviour of the economy (Aiyagari, 1994; Krusell and Smith, 1997). In this paper, we circumvent this issue by making specific assumptions about impatient household's period utility and the tightness of the borrowing constraint, which ensure that the cross-sectional distribution of wealth and implied number of household types are both finite.

3.1 Assumptions and conjectured equilibrium

Let us first assume that the instant utility function for impatient households $u(c)$ is i) continuous, increasing and differentiable over $[0, +\infty)$, ii) strictly concave with local relative risk aversion coefficient $\xi(c) = -cu''(c)/u'(c) > 0$ over $[0, c^*]$, where c^* is an exogenous, positive threshold, and iii) linear with slope $\eta > 0$ over $(c^*, +\infty)$ (see Figure 2). Essentially, this utility function (an extreme form of decreasing relative risk aversion) implies that high-consumption (i.e., relatively wealthy) households do not mind moderate consumption fluctuations –i.e., as long as the implied optimal consumption level stays inside $(c^*, +\infty)$ –

but dislike substantial consumption drops –those that would cause consumption to fall inside $[0, c^*]$. In the equilibrium that we are focusing on, “moderate consumption fluctuations” refer to consumption changes triggered by variations in asset and wage incomes *conditional on the agent remaining employed*; in contrast, “substantial consumption drops” refer to those triggered by the large falls in current income that are associated with a *change in employment status from employed to unemployed*. In other words, we are constructing an equilibrium in which:

$$\forall i \in [0, \Omega], e_t^i = 1 \Rightarrow c_t^i > c^*, e_t^i = 0 \Rightarrow c_t^i \leq c^*. \quad (15)$$

As we shall see shortly, one implication of this utility function and consumption rankings is that employed households fear unemployment and consequently engage in precautionary saving behaviour *ex ante* in order to limit (but without being able to fully eliminate) the associated shooting up in marginal utility. As a result, their asset holdings will be well defined despite the fact that these agents are locally risk-neutral.

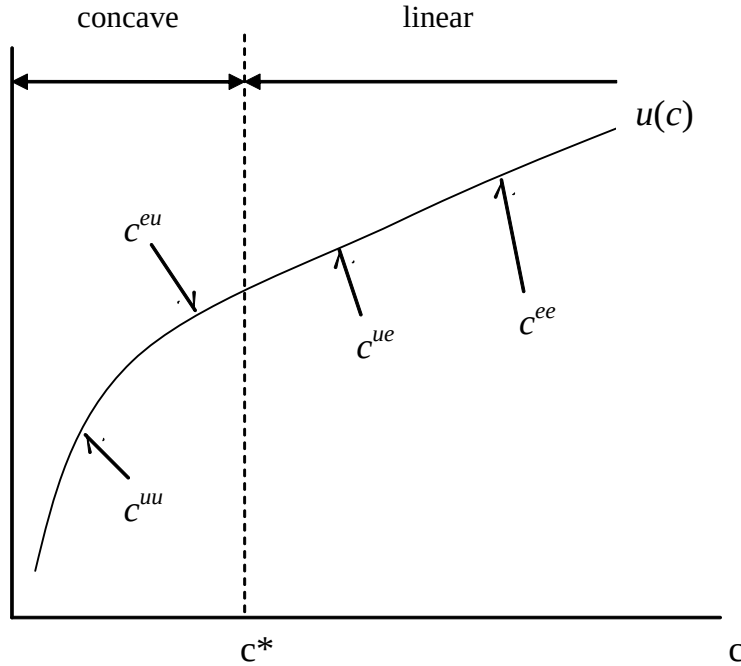


Figure 2: Instant utility function of impatient households

The second feature of the equilibrium that we are constructing is that the borrowing constraint in (9) is binding for all unemployed households (that is, the Lagrange multiplier in (10) is positive), so that their end-of-period asset holdings are zero (rather than negative).

In short, the equilibrium that we are constructing has the following property:

$$\forall i \in [0, \Omega], \quad e_t^i = 0 \Rightarrow E_t \left(\beta u' \left(c_{t+1}^i \right) R_{t+1} / u' \left(c_t^i \right) \right) < 1 \text{ and } a_t^i = 0. \quad (16)$$

Equations (15)–(16) have direct implications for the optimal asset holdings of employed households. By construction, a household who is employed at date t has asset wealth $a_t^i R_{t+1}$ at the beginning of date $t + 1$. If the household falls into unemployment at date $t + 1$, it liquidates assets after having collected asset incomes, so the household enjoys consumption

$$c_{t+1}^i = \delta + a_t^i R_{t+1} \quad (17)$$

and marginal utility $u' \left(\delta + a_{t+1}^i R_{t+1} \right)$. On the other hand, if this household stays employed, it enjoys marginal utility η , as it did in period t (by equation (15)). Therefore, if employed agents' consumption is higher than c^* while unemployed agents consumption is lower than c^* , then the optimal asset holding a_t^i of a typical employed household i must satisfy the following Euler equation:

$$\eta = \beta E_t \left[\left((1 - s_{t+1}) \eta + s_{t+1} u' \left(\delta + a_t^i R_{t+1} \right) \right) R_{t+1} \right] \quad (18)$$

where marginal utility at date $t + 1$ (inside the expectations operator) has been broken into the two possible employment statuses that this household may experience at that date, weighted by their probabilities of occurrence. Note that equation (18) uniquely pins down a_t^i as a function of aggregate variables *only*. This in turn implies that asset holdings are symmetric across employed agents –and hence independent of their employment history up to date $t - 1$. We may thus write:

$$\forall i \in [0, \Omega], \quad e_t^i = 1 \Rightarrow a_t^i = a_t > 0. \quad (19)$$

Equations (16) and (19) show that in this equilibrium the cross-sectional distribution of wealth has two states, so that the economy effectively has exactly *four* types of impatient households –since from (8) the type of an agent depends on both beginning- and end-of-period wealth. We call these types “ ij ”, $i, j = e, u$, where i (j) refers to the household's employment status in the previous (current) date. For example, a “ ue ” household is currently employed but was unemployed in the previous period, and its consumption at date t is c_t^{ue} , and so on. The ranking of consumption level for these households is depicted in Figure 1.

Since employed households share the same Euler equation, we rewrite (18) as

$$E_t \left(M_{t+1} R_{t+1} \right) = 1, \quad (20)$$

where M_{t+1} is the common pricing kernel of employed, impatient households:

$$M_{t+1} = \beta \left[1 - s_{t+1} + s_{t+1} \left(\frac{u'(\delta + a_t R_{t+1})}{\eta} \right) \right]. \quad (21)$$

Equations (20)–(21) clarify the importance of the risk of falling into unemployment for the determination of precautionary asset accumulation. Holding asset returns fixed, an increase in the employment exit probability s_{t+1} tends to raise M_{t+1} (since $u'(c_t^{eu})/\eta > 1$ by (15)), so $u'(c_{t+1}^{eu})$ must go down (i.e., $c_{t+1}^{eu} = \delta + a_t R_{t+1}$ must go up) for the Euler equation to hold. This is achieved by raising date t asset holdings, a_t , in (21). Finally, from (16) and (19), total asset holdings by impatient households is:

$$\int_0^\Omega a_t^i di = \Omega n_t a_t, \quad (22)$$

which can be substituted into the market-clearing condition (13).

3.2 Steady state and existence conditions

We shall work out the conditions for our equilibrium with minimal cross-sectional heterogeneity to exist under the maintained assumption that aggregate shocks have small magnitude. Hence, these conditions will be satisfied in the stochastic equilibrium provided that they are so in the steady state.

Steady state. In the steady state, the real interest rate is determined by the discount rate of the most impatient agents, i.e., $R^* = 1/\beta^p$ (see Becker, 1983). From (3) and (7), the steady state levels of employment and capital per employee are:

$$n^* = \frac{1}{1 - \rho + \rho/f(1)}, \quad k^* = g'^{-1} \left(\frac{1}{\beta^p} - 1 + \mu \right). \quad (23)$$

The central variable in our model is the level of asset holdings that employed, impatient households choose to hold as a buffer against unemployment risk. Substituting (6) into (20)–(21) and rearranging, we find these steady state asset holdings to be

$$a^* = \beta^p u'^{-1} \left(\frac{\eta(\beta^p/\beta + \rho(1 - f(1)) - 1)}{\rho(1 - f(1))} \right) - \beta^p \delta \quad (24)$$

Finally, from (13) and (22), steady state (total) asset holdings by patient households are $A^{p*} = n^*(k^* - \Omega a^*)$. The other relevant steady state values directly follow.

Existence conditions. The equilibrium described so far requires two sets of conditions to be satisfied. First, the ranking of consumption levels for impatient households in (15) must be satisfied in equilibrium. For this to be the case, the consumption level of eu households, c_t^{eu} , must be lower than that of ue households, c_t^{ue} . From the budget constraint (8) and the asset holdings conditions (16) and (19), we have $c_t^{eu} = \delta + a_{t-1}R_t$ and $c_t^{ue} = w_t - a_t - \psi_t$, so the equilibrium requires

$$\delta + a_{t-1}R_t < w_t - a_t - \psi_t$$

at all dates. Second, the borrowing limit must be effectively binding for all unemployed, impatient households –so that (16) is satisfied. Such a household leaves unemployment in the next period with probability $f(z_{t+1})$, and in this case enjoy marginal utility η (by condition (15)); it remains unemployed with complementary probability, in which case it enjoys marginal utility $u'(\delta)$ (by (15) and the budget constraint (8)). Unemployed households can be of two types, uu and eu , and requires both to be borrowing-constrained; however, since $c_t^{uu} = \delta < c_t^{eu} = \delta + a_{t-1}R_t$ (and hence $u'(c_t^{uu}) > u'(c_t^{eu})$), a necessary and sufficient condition for both types to be constrained is:

$$u'(\delta + a_{t-1}R_t) > \beta E_t((f(z_{t+1})\eta + (1 - f(z_{t+1}))u'(\delta))R_{t+1})$$

for all t . Evaluating the latter to conditions at the steady state and noting that under the employment contract assumed above w^* is the marginal product of capital, i.e., $w^* = g(k^*) - k^*g'(k^*)$, the steady state counterpart of the latter two inequalities are:

$$\delta + a^*R^* < g(k^*) - k^*g'(k^*) - a^* - \psi(1), \quad (25)$$

$$u'(\delta + a^*R^*) > \beta(f(1)\eta + (1 - f(1))u'(\delta))R^*, \quad (26)$$

Substituting (23) into (25)–(26) and using the fact that $R^* = 1/\beta^p$, we obtain the following existence proposition.

Proposition 1. A sufficient condition for the minimal heterogeneity equilibrium described above to exist is:

$$u'^{-1} \left[\frac{\eta(\beta^p/\beta + \rho(1 - f(1)) - 1)}{\rho(1 - f(1))} \right] < \min \left[\frac{g(k^*) - k^*g'(k^*) - \psi(1) - \beta^p\delta}{1 + \beta^p}, u'^{-1} \left(\frac{\beta(f(1)\eta + (1 - f(1))u'(\delta))}{\beta^p} \right) \right],$$

where k^* is given by (23).

The inequality in Proposition 1 can straightforwardly be checked once specific values are assigned to the deep parameters of the model. As we argue next, it is satisfied for plausible such values.

4 Dynamics

4.1 Baseline model and benchmarks

The dynamics of the model described above is summarised by five equations: the Euler equations for patient and impatient households (equation (12) and (20), respectively), the budget constraint of patient households (equation (11)), the law of motion for employment (equation (7)) and the market-clearing condition for claims to the capital stock (equation (13), with total assets held by impatient households given by (22) and capital per worker given by (3)). The corresponding five-dimensional vector of unknown sequences is:

$$X_t = \begin{bmatrix} A_t^p & C_t^p & a_t & n_t & R_t \end{bmatrix}',$$

and the forcing variable is $\{z_t\}_{t=0}^\infty$. Given X_t , we may use the market-clearing condition for goods (equation (14)) to infer total consumption by impatient households, $\int_0^\Omega c_t^i di$, as well as total consumption,

$$C_t = C_t^p + \int_0^\Omega c_t^i di.$$

Since we want to understand the specific role of uninsured unemployment risk and precautionary saving for the determination of aggregate consumption, it may be useful to compare this model with two alternative models where the precautionary motive is shut down. There are two natural benchmarks against which our model can be compared. The first one is the representative agent model, where all agents are homogenous and have full access to asset markets –despite the presence of idiosyncratic labour income risk. This economy is similar to that analysed by Mertz (1995) and Andolfatto (1996), except for our specific assumptions regarding the type of labour contracts between the households and the firm. We obtain this economy by setting $\Omega = 0$ in (11), observing that clearing of the capital market then only requires $A_{t-1}^p = n_t k_t$, and finally removing $\{a_t\}_{t=0}^\infty$ from the set of unknowns to be determined.

Baseline model	Borrowing constraint only	Representative agent
Patient households		
$\beta^p E_t[(u^{p'}(C_{t+1}^p)/u^{p'}(C_t^p))R_{t+1}] = 1$		
$C_t^p + A_t^p + (1 - \Omega)(1 - (1 - \rho)n_{t-1})\psi(z_t)f(z_t)$		Same with $\Omega = 0$
$= R_t A_{t-1}^p + (1 - \Omega)(w_t n_t + (1 - n_t)\delta)$		
Impatient households		
$\beta E_t[1 + \rho(1 - f(z_{t+1})) \times$ $(\frac{u'(\delta + a_t R_{t+1}) - \eta}{\eta})R_{t+1}] = 1$	$a_t = 0$	a_t not defined
Labour & asset markets		
$n_t = (1 - \rho)(1 - f(z_t))n_{t-1} + f(z_t)$		
$R_t =$ $z_t g'(\frac{\Omega n_{t-1} a_{t-1} + A_{t-1}^p}{n_t}) + 1 - \mu$	$R_t = z_t g'(\frac{A_{t-1}^p}{n_t}) + 1 - \mu$	$R_t = z_t g'(\frac{A_{t-1}^p}{n_t}) + 1 - \mu$

Table 1. Alternative model specifications. Notes: The endogenous variables are, for all three models, A_t^p , C_t^p , n_t and R_t , and also a_t for the baseline model.

The second natural benchmark against which our economy can be compared is the intermediate case in which households enjoy full access to asset markets, but are heterogeneous in terms of asset holding behaviour (due to discount factor differences) and cannot borrow against future income. This economy closely resembles that analysed by Kiyotaki and Moore (1997), the main differences between the two frameworks being i) our explicit modelling of unemployment and ii) our assumption that neither future labour income nor holdings of outside assets can back up repayment promises. Formally, the economy with borrowing constraints only is one with two representative families (one patient, one impatient) and in which full consumption insurance prevails within each family. Since the impatient family would like to borrow from the patient one, but is prevented from doing so by a binding borrowing constraint, its optimal consumption plan is corner and leads current income to be entirely consumed in every period. In other words, full consumption insurance amongst the impatient eradicates the need to form precautionary saving ex ante and we hence $a_t = 0 \forall t$ (instead of being given by equation (10)), while the market-clearing condition for capital is similar to that of the representative agent economy. For ease of reference, Table 1 summarises the three specifications that we have just discussed.

4.2 Calibration

Preferences and household shares The first set of parameters to calibrate are preferences parameters. We set the discount factor of impatient households β^p , which governs the steady state real interest rate, to the standard value of 0.99. Iacoviello (2005) discusses the evidence on the cross-sectional distribution of discount factors and accordingly sets that of impatient households, β , to 0.95; we follow him here and refer the reader to his paper for the motivating evidence. The period utility of patient households is of the CRRA form, i.e., $u^p(c) = c^{1-\sigma}/(1-\sigma)$, $\sigma > 0$. We choose a utility function for impatient households, $u(c)$, that is as close as possible as that of patient household –despite the fact that it must include a linear portion to produce our equilibrium with limited cross-sectional heterogeneity. Accordingly, we first assume that $u(c) = c^{1-\sigma}/(1-\sigma)$ for $c \in [0, c^*]$. Regarding the linear part, we proceed as follows. First, we note that the marginal utility of impatient asset holders (i.e., η in (24)) governs desired steady state asset holdings a^* . Now, the great majority of these households are of the *ee* type, i.e., they were employed in the previous period and are still so in the current period. We thus set η so that the marginal utility of *ee* households is the same as if their behaviour was governed by the utility function of patient households. That is, we set:

$$\eta = u^{p'}(c^{ee}) = (w + a^*(1 - 1/\beta^p))^{-\sigma}.$$

The latter equation indicates that the appropriate value of η depends on a^* . Since a^* also depends on η (by (24)), we jointly solve for the fixed point (a^*, η) using a (rapidly converging) iterative procedure. In our baseline calibration we set $\sigma = 1$. Given the other parameters of the model, we obtain $\eta = 0.49$. The last parameter to calibrate in $u(\cdot)$ is c^* , which we set slightly below c^{ue} , again to minimise the distance between the two period utility functions. Note that while the implied period utility function for impatient households is continuous and concave, it is not differentiable all over $[0, \infty)$; however, it can be made so by “smooth pasting” the two portions of the function in an arbitrarily small neighborhood of c^* .

We set the share of impatient households, Ω , to 1/2. Campbell and Mankiw (1989) estimate that about one half of U.S. households do not behave as permanent-income consumers. While in their model those who do not adopt a simple “rule-of-thumb” consumption rule, this clearly should be interpreted as a shortcut for rational consumption behaviour under (unmodelled) borrowing constraints and buffer stock saving behaviour (Mankiw, 2000).⁵ More

⁵For example, Mankiw (2000, p. 121) argues that “the consumption literature on ‘buffer stock saving’ can be seen as providing a richer description of this rule-of-thumb behavior. Buffer stock savers are individuals

recently, Gali *et al.* (2007) find that a similar portion of such households is necessary to account for the output response to government spending shocks in a New Keynesian model with rigid prices and wages.

Production. The production function is $Y_t = z_t K_t^\alpha n_t^{1-\alpha}$, $0 < \alpha < 1$, so that production per employee is $Y_t/n_t = k_t^\alpha$, the full equilibrium wage $w_t = (1 - \alpha) k_t^\alpha$, and the demand for capital per employee

$$k_t = \left(\frac{\alpha z_t}{R_t - 1 + \mu} \right)^{\frac{1}{1-\alpha}}.$$

For the calibration of market production, we use the usual values of $\alpha = 1/3$ and $\mu = 0.025$.

For expositional simplicity we have motivated the income earned while unemployed, δ , as “home production”. However, the dynamics of the model is virtually identical if we assume that δ results from the collection unemployment benefits that are paid lump sum by employed workers (who are a lot more numerous than the unemployed.) In the U.S. income replacement rates vary greatly across households. For a typical earner of the mean wage, the OECD indicators report net replacement ratios ranging from 0.06 to 0.75, depending on the length of unemployment, the type of household (numbers of children and wage earners) and income prior to unemployment; we set δ so that the replacement ratio produced by the model (δ/w^*) is 0.40, approximately in the middle of this range and similar to the value used by Shimer (2005).⁶

Matching. The matching function is assumed to be of the Cobb-Douglas form, i.e., $M_t = \kappa(1 - (1 - \rho)n_{t-1})^\gamma v_t^{1-\gamma}$, $\kappa > 0$, $0 < \gamma < 1$. The probation discount function is assumed to be of the form $\psi(z_t) = \psi \cdot z_t^\xi$, where ψ is the steady state value of the discount and ξ its elasticity w.r.t. total factor productivity. With our assumed matching function, the implied

who have high discount rates and often face binding borrowing constraints. Their savings might not be exactly zero: they might hold a small buffer stock as a precaution against very bad income shocks”. This is precisely what happens in our model.

⁶In an alternative calibration approach, Hagedorn and Manovskii (2008) propose to incorporate the implicit value of leisure into home production income, and accordingly set the corresponding replacement ratio to 95.5%. However controversial this approach may be (see, e.g., Mortensen and Nagypál, 2007; Costain and Reiter, 2008), it suffices to note here that in this case households would suffer unemployment at very small cost, which would give them little reason to save for precautionary purpose in the first place.

job-finding rate function takes the following simple form:

$$f(z_t) = f^* \cdot z_t^{\tilde{\xi}},$$

where $f^* \equiv (\kappa(\psi/c)^{1-\gamma})^{1/\gamma}$ is the steady state job-finding rate and $\tilde{\xi} \equiv \xi(1-\gamma)/\gamma$ its elasticity w.r.t. to total factor productivity. We set $\gamma = 0.5$, which is well within the range of available estimates. We set the steady state discount ψ to be 10% of the full wage, w^* , which gives $\psi = 0.17$. We then calibrate the vacancy cost c so that the quarterly steady state vacancy-filling rate, $\lambda^* = c/\psi$, is equal to 0.71, the value calibrated by Denn Hann et al. (2000).⁷ Shimer (2005) reports a monthly average job-finding rate of 0.45, implying that its quarterly counterpart is $f = 1 - (1 - 0.45)^3 = 0.83$. Since $c/\psi = \lambda^* = 0.71$ in our calibration, we must set $\kappa = \sqrt{0.71 \times 0.83} = 0.77$ in the matching function. The last parameter is ρ , the beginning-of-period probability of match destruction, which we set to 0.3; given the mean job-finding rate, this generates a steady state unemployment rate of 5.8%.

Shocks and propagation. We must now specify the size and persistence of productivity shocks and the extent to which they propagate to the labour market. We set the persistence of TFP shocks to $\tau = 0.95$ and the standard deviation of innovations to TFP shocks to $\sigma_z = 0.007$, as in Mertz (1995). King and Rebello (1999) have a similar value for σ_z and a slightly higher value for τ , but this does not significantly affects our results.

The labour market response to TFP shocks is indexed by ξ , or equivalently, $\tilde{\xi}$. We set $\xi = 2.5$, so that the model's elasticity of market tightness w.r.t. labour productivity roughly matches the value of 7.56 documented by Pissarides (2009). Indeed, under our matching function, the job finding rate is tied to market tightness according to the function $f_t = \kappa \theta_t^{1-\gamma}$, while our contracts ties the marginal product of labour to the real wage, w_t . With a Cobb-Douglas production function, the average productivity of labour is proportional to the marginal product of labour, so both are equal when expressed as proportional deviations from the steady state. Thus, the relevant elasticity is $\partial \hat{\theta}_t / \partial \hat{w}_t$, with hats denoting proportional deviations from the steady state; setting $\xi = 2.5$ gives $\partial \hat{\theta}_t / \partial \hat{w}_t \simeq 0.7$, only slightly below

⁷Davis, Faberman and Haltiwanger (2010) report aggregate hiring yields (i.e., the flow of realised hires during the month per reported job opening at the end of the previous month) higher than one at monthly frequency. This reflects the fact that vacancies that are immediately filled are not recorded in JOLTS. In our model the vacancy filling rate is bounded above by one, and setting $c = \psi$ to achieve this number does not alter our results.

the estimate of Pissarides (raising ξ above 2.5 would magnify the labour market response to the shock, and hence that of precautionary savings). With this value, the probation wage $w_t = w_t - \psi_t$ remains procyclical, but substantially less so than the marginal product of labour, w_t . This rigidity is responsible for the relatively strong labour market response to shocks, as in Hall (2005 and 2009). Table 2 summarises our functional forms, the calibration of the deep parameters of the model, and the implied steady state.

Functional forms					
Production	$Y_t = z_t K_t^{1/3} n_t^{2/3}, \sigma_z = 0.007, \tau_z = 0.95$				
Matching	$M_t = 0.77 \times (1 - 0.7n_{t-1})^{1/2} v_t^{1/2}$				
Period utility/patient	$u^p(c) = \ln c$				
Period utility/impatient	$u(c) = \begin{cases} \ln c & \text{for } c \leq 1.5 \\ \ln c^* + 0.49(c - c^*) & \text{for } c > 1.5 \end{cases}$				
Wage discount	$\psi_t = 0.21 \times z_t^{2.5}$				
Other parameters	Symbol	Value	Steady state	Symbol	Value
Discount factor/patient	β^p	0.99	Job-finding rate	f^*	0.83
Discount factor/impatient	β	0.95	Unemployment rate (%)	$1 - n^*$	5.79
Share of impatient hous.	Ω	0.50	Vacancy-filling rate	λ^*	0.71
Vacancy cost	c	0.15	Replacement ratio	δ/w^*	0.40
Home production	δ	0.82	Wealth share of impa-		
Depreciation rate	μ	0.025	tient households (%)	$\theta a^*/k^*$	0.52

Table 2. Fonctional forms, parameters and implied steady state values.

One way to check the adequacy of our calibration is to compute the implied share of asset wealth held by impatient households –i.e., the poorest half of the population in our parameterisation–, and to compare it to its empirical counterpart. Our model predicts these households as a whole to hold 0.52% of total wealth. This is a very plausible number. Data from the 2007 Survey of Consumer Finances show that the share of net worth, defined broadly as all assets minus all liabilities, held by the poorest half is 2.5%. However, this

net worth concept is not the adequate wealth measure here because it includes net equity in owner-occupied housing, while the primary residence cannot easily be sold to provide for current consumption.⁸ Removing net equity housing from the computation of the poorest half's wealth share gives 0.52%, our predicted value. Removing some other assets with little fungibility (e.g., vehicles or pension plans) still lowers this share, but the exclusion of some of these items would arguably be more debatable. In any case, the wealth shares implied by our model lies well within the bounds of plausibility. Given the wealth share of the poorest half, the condition stated in Proposition 1 above holds by a large margin. In particular, households who fall into unemployment without enjoying full insurance are predicted to exhaust their buffer stock of precautionary wealth within a quarter, and hence to live entirely out of home production (i.e., unemployment benefits) thereafter.⁹

4.3 Impulse-response analysis

Figure 3 displays the responses to a productivity shock of the job-finding rate (f_t), market tightness (θ_t), employment (n_t), market output (Y_t), investment (I_t), the interest rate (R_t), the consumption of impatient and patient households ($\int_0^\theta c_t^i di$ and C_t^p , respectively), and aggregate consumption (C_t). By construction, patient households behave as ‘permanent-income’ ones, while impatient households are ‘precautionary savers’, with drastically different consumption responses. To summarise, the consumption path of patient households is such that consumption growth tracks the interest rate, as is implied by their Euler equation (see (12)). In contrast, impatient households drastically cut their consumption on impact as they raise their buffer stock of precautionary wealth (i.e., by equation (20)–(21)). Under our assumed AR(1) shock process, the impact of consumption is necessarily short-lived: it is

⁸See the discussion in Wolff (2007), who constructs “nonhome wealth” fractiles on the ground that owner-occupied housing is essentially illiquid. While a number of households have recently returned their houses against the cancelation of their mortgages, this behaviour is arguably a specific feature of the current crisis rather than the rule.

⁹We have also experimented a version of the model in which impatient households hold a somewhat larger fraction of total wealth and where two quarters, rather than one, to fully liquidate fungible wealth when remaining unemployed. The aggregate dynamics implied by this specification turns out to differ only marginally from our baseline case. This is because, for realistic transition rates in the labour market, the fraction of households remaining unemployed for two quarters in a row is very small. Consequently, these households as a whole hold a vanishingly small share of total wealth and thus have a very small impact on aggregate consumption.

strong as precautionary savers adjust their wealth following the large initial fall in the job-finding rate; after this adjustment has been made, current income can again be assigned to consumption expenditures, which rise back (although at a lower level than before the shock). Aggregate consumption reflects the behaviour of both types: it drops sharply on impact, and then rises again and gradually adjust back towards the steady state.

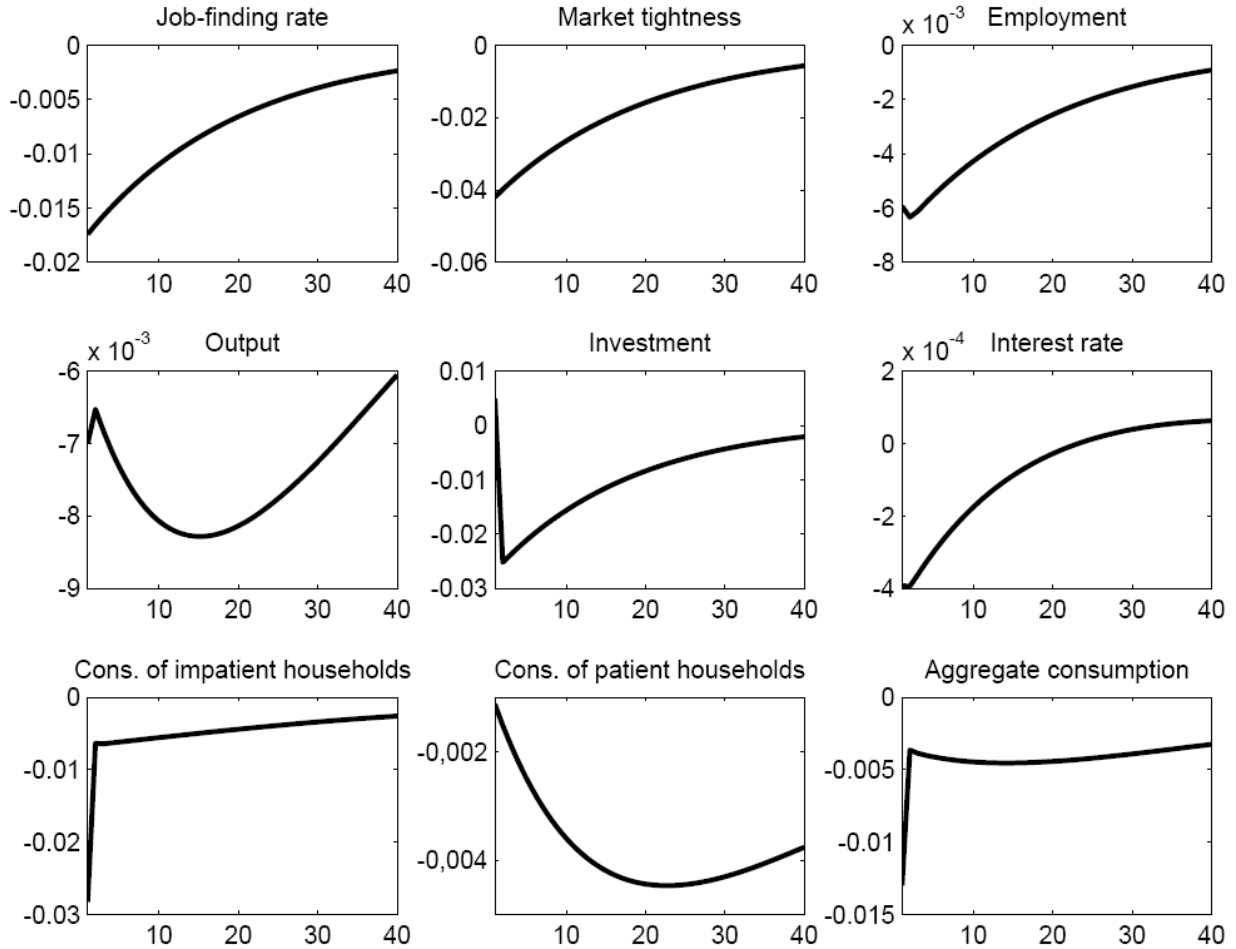


Figure 3: Precautionary saving and borrowing constraints. Notes: All variables are expressed in proportional deviations from the steady state except for rates (i.e., the job-finding rate, the unemployment rate and the interest rate), which are expressed as level deviations from the steady state.

Looking at the impulse-response functions implied by the model with borrowing constraints but full insurance amongst impatient households allows us to isolate the specific role of changes in precautionary asset accumulation in affecting aggregate consumption –as opposed to mere binding borrowing constraints. Without the precautionary saving motive aggregate consumption does not suffer the initial negative spike associated with the accu-

mulation of assets by impatient households. The interest path is also mildly altered: it rises more quickly after the shock, relative to the path with precautionary savings. This is because in the latter the higher supply of savings drives the interest rate down, relative to the former. However, because impatient households hold little wealth on average, this effect remains small.

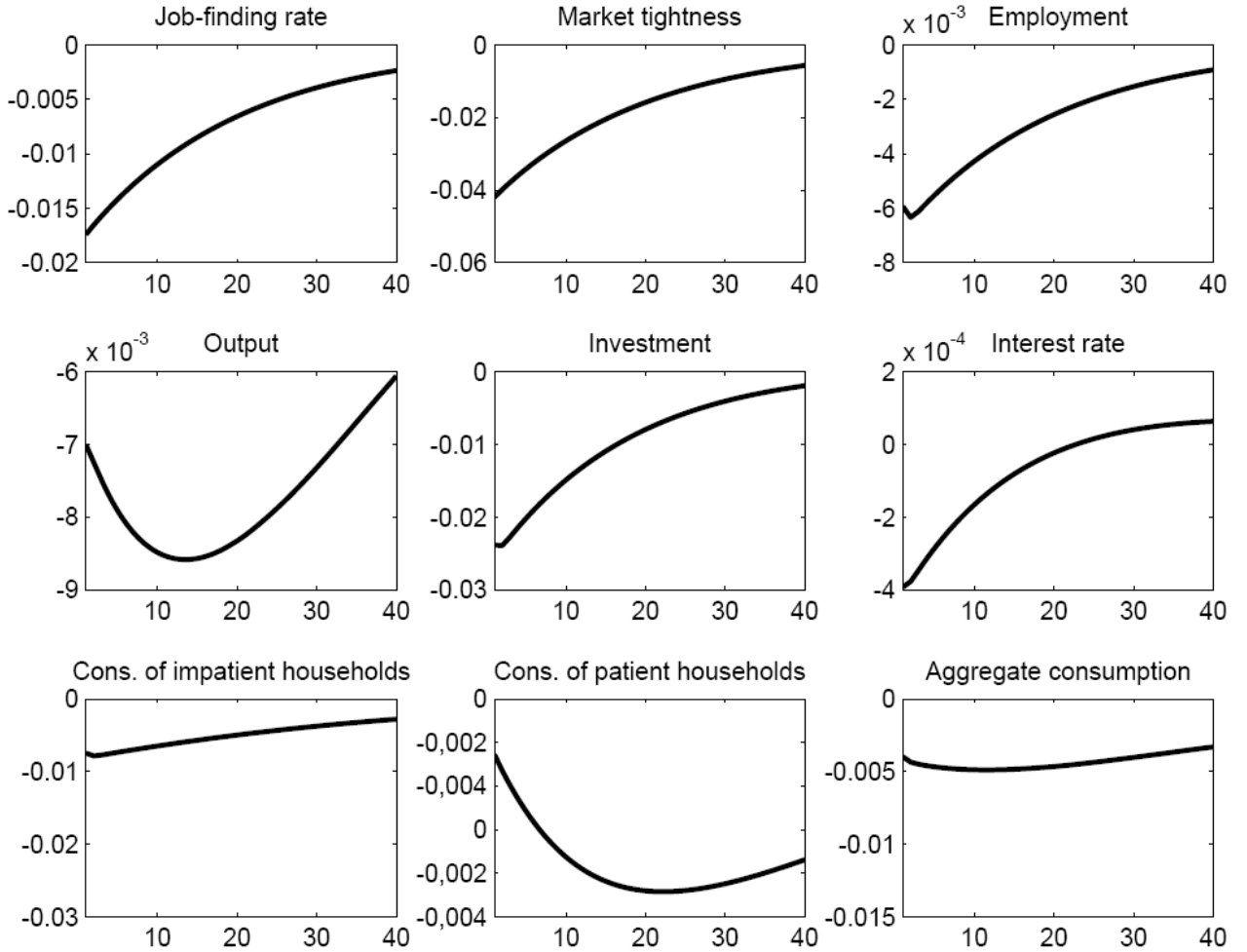


Figure 4: Borrowing constraints only

Finally, we compare our economy with precautionary saving the complete markets analogue, wherein all households are “patient” –essentially a version of the Real Business Cycle model with search and matching frictions a la Mertz (1995). The aggregate consumption responses differ across the two economies mostly by the impact effect. Hence, under our assumed shock process, the change in aggregate consumption specifically due to time-variations in precautionary saving is strong but short-lived.

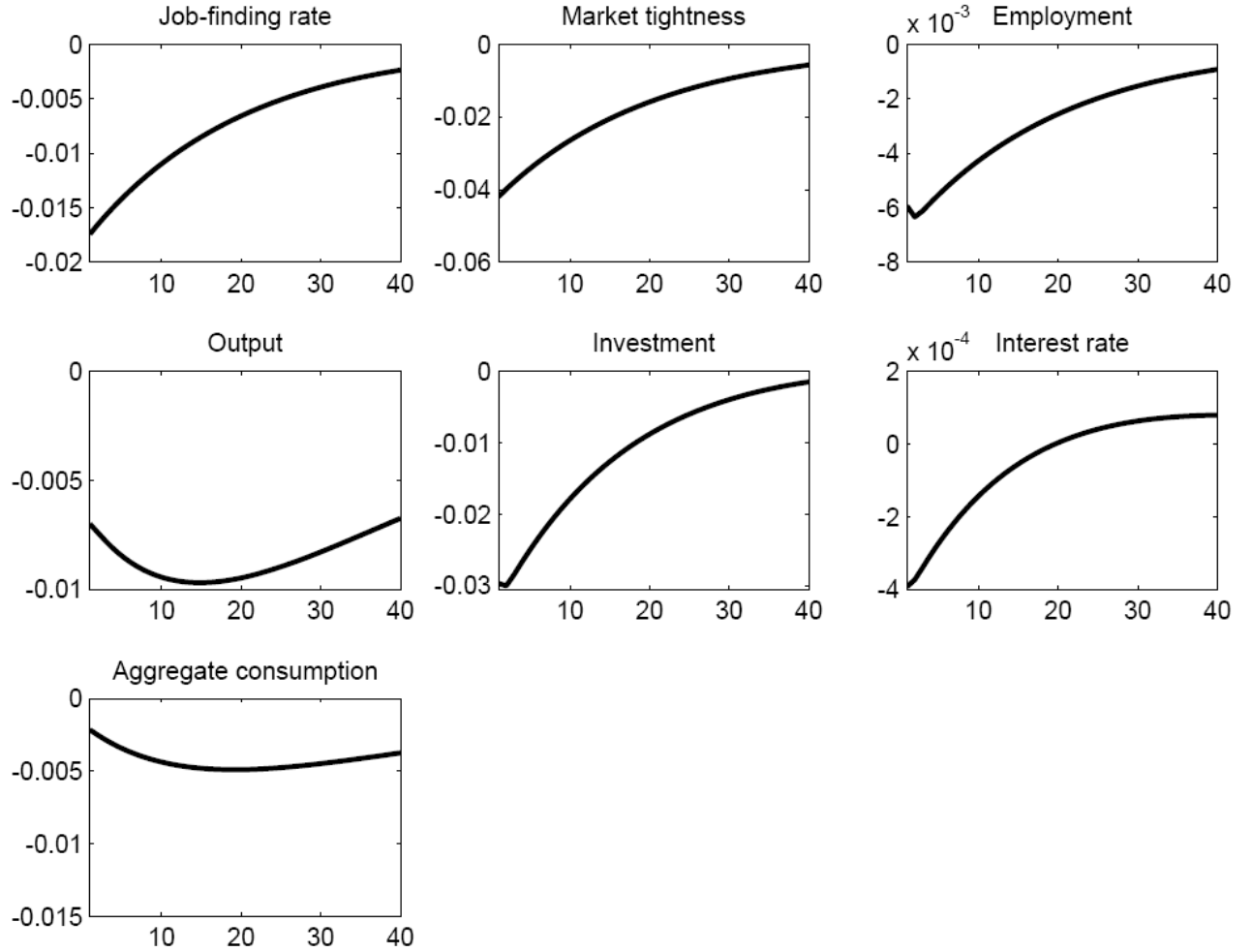
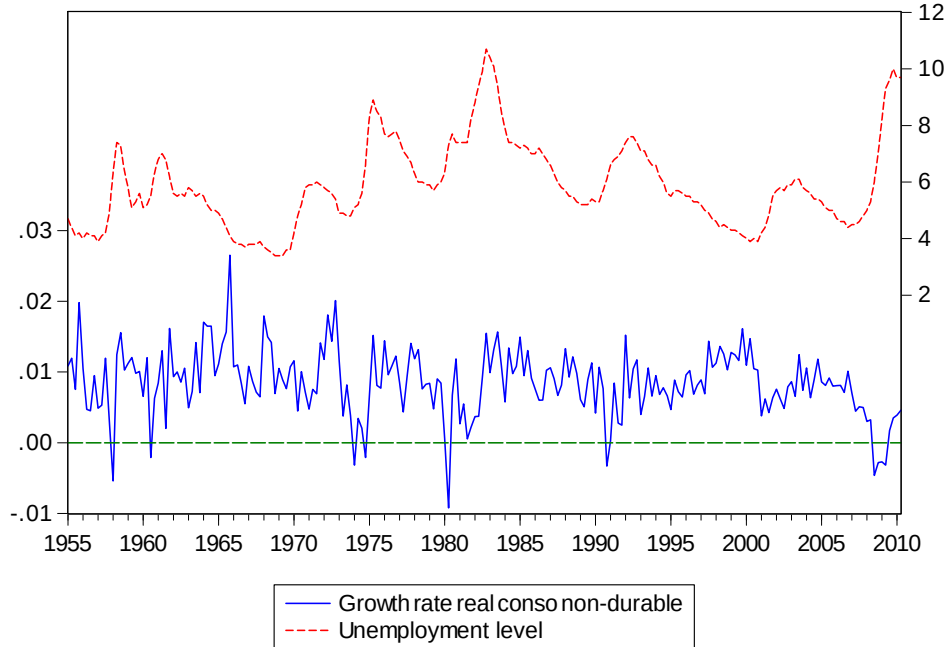


Figure 5: Representative agent

5 Implications for Euler Equation tests

The model developed above gives a central role to unemployment risk in explaining consumption fluctuations. We now wish to check whether this view about the interactions between labour market conditions and consumption-saving plans finds support in the data. The following figure plots the growth rate of non durable consumption of goods and services (deflated by the consumption expenditure price index) from the NIPA accounts against the unemployment rate from the BLS since the post Korean war period. Additional information

about durable goods are provided in the next section.



A striking pattern that emerges is that in most recessions consumption growth fell sharply at the onset of the unemployment wave; in fact, in many cases consumption expenditures fell more than real disposable income. One potential explanation for the drop in the consumption-output ratio is that the fall in current income that arises at the beginning of a recession forecasts even larger income drops, which are in turn discounted back to the present; permanent-income behaviour then elicits a large response of current consumption relative to the mild initial income fall. Another and complementary explanation for this phenomenon, which is consistent with the model developed above, is that part of the initial consumption fall is due to a strengthening of the precautionary motive by households facing rising unemployment risk.

Our purpose is to isolate the specific role of unemployment risk and precautionary saving in the determination of aggregate consumption growth. Permanent income behaviour in frictionless capital markets predicts that expected consumption growth should only depend on expected interest rate, whatever the path of current and future income (Hall, 1978). Flavin (1981) and Campbell and Mankiw (1989) show that consumption growth responds to predictable income growth, an observation that is often interpreted as the outcome of the liquidity constraints faced by a number of individuals. In what follow, we run a consumption growth regression that includes the interest rate, current income and future labour market

conditions as explanatory variables. Interest rate and current income changes are meant to capture the permanent-income and liquidity-constraint components of aggregate consumption growth, as in earlier work. Our model predicts an additional role for changes in future labour market conditions over and above these explanatory variables, which should show up in the corresponding coefficient on aggregate consumption growth.

Consumption is defined as personal consumption expenditures on non durable goods and services deflated by the price index of the personal consumption expenditures. Income is real personal disposable income. The real interest rate is the average three months treasury bill using the price index of the personal consumption expenditures to compute inflation. We use two alternative measures of expected changes in labour market conditions. The first one is the answer to Question 17 in the University of Michigan Survey of Consumers, which records individuals' expectations about future changes in unemployment one year ahead.¹⁰ We use the demeaned relative measure, which increases when individuals think that unemployment will decrease, and denote date t by $UMSC_t$. Our second measure is the expected change in civilian unemployment between the current and the next quarter, $u_{t+1} - u_t$. Both variables are meant to capture the countercyclical nature of idiosyncratic labour income risk, which according to our model should affect the consumption-saving trade-off. The equation being estimated is:

$$\Delta c_t = \mu + \sigma_r r_t + \lambda_y \Delta y_t + \eta_E LMS_t + \varepsilon_t$$

where c_t , y_t and r_t are the logs of consumption, income, and the real interest rate, respectively, and LMS_t a future labour-market condition index, i.e., $LMS_t = UMSC_{t+1}$ or $u_{t+1} - u_t$. That is, we estimate a regression whereby expected changes in future labour market conditions affect *realised* consumption growth. We use lags from 2 to 4 of the explanatory and dependent variables as instruments.¹¹ Our results are summarised in Table 2.

¹⁰ Respondents are asked: "How about people out of work during the coming 12 months – do you think that there will be more unemployment than now, about the same, or less?"

¹¹ We follow Campbell and Mankiw (1989) in excluding first lags of variables as instrument in order to avoid the spurious first-order serial correlation that may results from estimating a discrete-time process for choices that are made in continuous time.

Table 2 - GMM Estimation Consumption Growth rate

	Non-durables and services			Total Consumption		
	(1)	(2)	(3)	(4)	(5)	(6)
μ	0.0046 (0.0000)	0.0070 (0.0000)	0.0062 (0.0000)	0.0027 (0.0000)	0.0064 (0.0000)	0.0067 (0.0000)
r_t	0.0946 (0.2684)	0.1120 (0.0913)	0.0695 (0.1688)	0.1694 (0.1899)	0.1674 (0.0946)	0.1662 (0.0559)
Δy_t	0.4267 (0.001)	0.2197 (0.0198)	0.2365 (0.0236)	0.6031 (0.0004)	0.1558 (0.3161)	0.1205 (0.4541)
$-MHS_t$	-	-0.0001 (0.0000)	-	-	0.0002 (0.0000)	
$u_{t+1} - u_t$	-	-	-0.0061 (0.0003)			-0.0098 (0.0000)
n	194	194	194	194	194	194
$\bar{R}^2$	0.11	0.39	0.38	0.10	0.38	0.38

Notes: In each regression, the nstruments are lags 2 to 4 of the dependent and explanatory variables. The estimation period is 1961Q4-2010Q2.

p -values are given in brackets.

Column (1) is the Mankiw and Campbell (1989)'s regression, and our estimates are in line with theirs. Column (2) includes households expectations about future labour market conditions, using $-UMSC_t$ —that is, a negative point estimates implies that an expected deterioration of the labour market lowers realised consumption growth. This terms turns out to be strongly significant and to significantly raise the fit of the regressin (as measured by the adjusted R^2). Moreover, its inclusion reduces the importance of current income growth in explaining consumption growth by a half, suggesting that a share of the households defined as “rule-of-thumb consumers” by Mankiw and Campbell may be precautionary savers, rather than consumers facing a binding constraint. Column (3), which is the same as (2) except that expected labour market conditions are measured by the (instrumented) change in unemployment, $u_{t+1} - u_t$, yields similar results. Our point estimate in that case suggests that, all else equal, a one-point expected increase in unemployment over the next quarter leads to a fall in current consumption of 0.6%. Column (3) and (4) perform the same exercise for total consumption (i.e., including durable goods in addition to non durables and services) with, as expected, stronger effect on ex post consumption growth. We conclude that Euler equation tests provide support for our model of incomplete markets and precautionary saving.

6 Conclusion

In this paper, we have constructed a small-scale dynamic general equilibrium model with incomplete markets in order to shed light on the precautionary asset holding motive and its likely variations over the business cycle. The key feature of the model is that aggregate (productivity) shocks not only alter aggregate labour income but also the idiosyncratic unemployment risk faced by a subset of individuals. Hence, it is the combination of search frictions in the labour market and incomplete insurance in the asset markets that drives the precautionary motive for holding wealth. The model suggests that consumption growth should be affected by expected changes in future labour market conditions, in addition to the usual determinants (interest rate and realised income growth). We found empirical support for this prediction.

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