

Optimal taxation and constrained inefficiency in an infinite-horizon economy with incomplete markets*

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Abstract

How should capital and labor be taxed when individuals' labor income is subject to uninsurable idiosyncratic risks? To address this question, we develop a tractable infinite horizon model with incomplete markets and consider a dynamic optimal taxation problem with linear taxes on the wage and interest income. We derive two general principles for public policy in such an environment: (i) providing an insurance for the idiosyncratic income risks; and (ii) allocating tax burdens efficiently over time. The first principle calls for taxing the labor income. The second principle clarifies when accumulating government debt is welfare improving, and also when the tax rate on physical capital needs to be strictly positive in the long run. We also calibrate our model to the U.S. economy and find that the presence of idiosyncratic income risks significantly affects the optimal tax rates and the optimal amount of the government debt.

Keywords: incomplete markets; constrained inefficiency; optimal taxation; Ramsey equilibrium.

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1 Introduction

In this paper we study how capital and labor income should be taxed or subsidized when individuals face uninsurable idiosyncratic shocks to their labor income. The purpose of this paper is to explore the basic principles for optimal linear taxation in a dynamic incomplete markets model.

A related question has been extensively studied in the complete-markets/ representative-agent frameworks.¹ With incomplete asset markets, it is often argued that capital should be taxed, because capital tends to be ‘over-accumulated’ in such an economy provided that individuals are prudent.² In his seminal contribution, Aiyagari (1995) argues that capital should be taxed, and a number of numerical analyses following up his work also conclude that capital should be taxed, which appear to reinforce this argument.

This line of reasoning is not correct, however. Notice that over-accumulation of capital here simply means that capital is accumulated more in a competitive equilibrium of the incomplete-markets economy than in the first-best allocation (assuming prudence).³ The argument asserts the property of a second-best equilibrium by comparing a sub-optimal equilibrium and the first-best allocation. On the other hand, whether or not capital should be taxed concerns the property of a second-best equilibrium, where one must seek for a best scenario *within* the set of equilibria with taxes, which typically does not contain the first best allocation. Therefore, one cannot determine whether or not capital should be taxed by simply comparing the level of equilibrium savings under incomplete markets with the first-best level.⁴

In fact, Aiyagari’s (1995) result is mute to the common argument above. Above all, he does not solve the second best problem. Instead, what is done in Aiyagari (1995) is to provide some conditions under which capital will be accumulated unboundedly unless the tax rate on capital income is strictly positive in the long run. In particular, two conditions are crucial for this result. First, consumption goods and leisure are perfect substitutes in the utility function of individuals.⁵ Second, the government provides public goods in such a way that the before-tax interest rate would be equal to the time discount rate at a steady state of the optimal-tax equilibrium. But we should note that under these assumptions it is not clear why capital stock should be bounded, and thus the existence of a steady state itself is a non-trivial question. In this sense Aiyagari’s (1995) result does not establish that the optimal capital tax is strictly positive in the long run.

¹Examples for the complete-markets/representative-agent setups include, among others, Stokey and Lucas (1983), Judd (1985), Chamley (1986), Zhu (1992), Chari, Christiano, and Kehoe (1994), Jones, Manuelli, and Rossi (1997). Examples for the incomplete-markets setups are Aiyagari (1995), İmrohoroglu (1998), Domeij and Heathcote (2004), Conesa, Kitao and Krueger (2009), etc.

²Ljungqvist and Sargent (2004, pp. 535-536) is an example of this type of argument: “(T)he optimal capital tax in a heterogeneous-agent model with incomplete insurance markets is actually positive, even in the long run. A positive capital tax is used to counter the tendency of such an economy to overaccumulate capital because of too much precautionary saving.”

³With endogenous labor supply decisions, capital may be accumulated less under incomplete markets even if individuals are prudent. For this, see, for instance, Marcet, Obiols-Homs, and Weil (2007).

⁴Using a two-period model, Gottardi, Kajii and Nakajima (2009) make it clear that whether or not capital should be taxed in an incomplete-markets economy has nothing to do with whether or not its equilibrium savings are larger than in the first-best allocation.

⁵As demonstrated by Marcet, Obiols-Homs, and Weil (2007), if consumption and leisure are not perfect substitutes, long-run savings remain finite even when the interest rate equals the time discount rate.

In order to understand the nature of optimal-tax equilibrium, the second best problem needs to be worked out explicitly. However, even numerically, it is in general difficult to solve the second best problem under incomplete markets because of the curse of dimensionality problem inherent in such an economy. In this respect, important progress has been made by such studies as Aiyagari and McGrattan (1998), İmrohoroglu (1998), Domeij and Heathcote (2004), Conesa, Kitao and Krueger (2009), and Fukushima (2010), among others. Nevertheless, none of them solve the fully dynamic second best problem. For instance, a typical assumption made in this literature is that the social planner attempts to maximize only the average welfare at the steady state. The solution to such a problem ignores welfare gains/losses during transition to the steady state, and can in principle be very different from the solution to the second best problem which takes them into account. Among the papers listed above, Domeij and Heathcote (2004) consider transition, but they restrict fiscal policies so that the tax rates are constant over time.

In this paper, in order to explicitly study the optimal taxation problem, we focus on a highly stylized but tractable model of incomplete markets. As a result, we show in a very transparent fashion when and why capital and labor should be taxed/subsidized. It reveals two general principles about the nature of optimal taxation under incomplete markets, which are different from the aforementioned common line of argument. Furthermore, because of the tractability of the model, we obtain a numerical solution to the optimal (linear) taxation problem without making ad hoc assumptions as in the literature.

Now we shall outline our findings. Our model is an incomplete-markets version of the endogenous growth model studied by Jones and Manuelli (1990), and is closely related to Krebs (2003). Individuals have access to three types of assets: bonds, physical capital, and human capital. The first two assets are riskfree, but the accumulation of human capital is subject to idiosyncratic shocks, which are identically and independently distributed both across individuals and across periods. As in Aiyagari (1995), we restrict attention to linear taxes on labor and capital income.⁶ Given that all individuals have identical Epstein-Zin-Weil preferences, we show that the competitive equilibrium can be characterized in a simple way. Our model differs from the standard one in that there is no labor/leisure choice and the labor productivity of an individual is determined by his/her investment in human capital. But the individuals exhibit prudence and in the laissez-faire equilibrium without taxes, the capital/labor ratio is greater than the first-best level in our setup, like in the standard model.

After setting up the optimal taxation problem formally, we start with a simple case where the government does not consume goods, and keeps balanced budget in each period. In this case, we show that, under a set of plausible assumptions, *subsidizing* the interest income and *taxing* the wage income makes everyone better off, starting from the laissez-faire equilibrium. That is, the government should increase the capital/labor ratio. Notice that this result cannot be understood with the common argument. But there is a simple economic reason: the welfare will be improved if risky income is insured, so the first principle of optimal taxation is that the government should tax risky source of income to subsidize less risky source of income. In our model, it is the labor

⁶Here we consider only linear taxes for the sake of tractability and clarity. It is, of course, a very important line of future research to examine how robust our findings are when non-linear taxes are allowed as in Kocherlakota (2005).

productivity which is risky, and taxing the wage income reduces the after-tax price of labor.⁷ Under the balanced budget of the government, the revenue from taxing the wage income must be distributed back to the private sector by subsidizing the interest income.

Next we allow for the government to borrow and lend, and consider a dynamic optimal taxation problem. The insurance principle above still indicates that the wage income should be taxed other things being equal, but the government is no longer restricted to use the capital taxation for subsidizing since it can allocate the tax burdens intertemporally.

We provide two theoretical results. The first result clarifies to what extent government debt is beneficial: it shows in particular that when the government's consumption is small enough, the government should borrow.⁸ To understand this, recall that the optimal capital tax is negative in the case of balanced budget with no government purchase, and so the after-tax rate of return on physical capital is greater than its before-tax rate. In addition, because of risk aversion, the after-tax rate of return on human capital is greater than that on physical capital. As a result, the average rate of return in the private sector is greater than the rate of return faced by the government, i.e., the private sector is more efficient. Thus there will be a gain if the government borrows to reduce taxes at the margin to encourage the private sector to accumulate more wealth.⁹ By continuity, this continues to be true with a sufficiently small level of government purchases.

The result above indicate that the balanced budget is almost never optimal, but it does not tell us how much the government should borrow or lend. The second result then describes the condition that the tax rates must satisfy at the steady state of the optimal tax equilibrium. Roughly speaking, the condition says that the government's return must be equated and the private sector returns, after adjusted for the effect of changing saving rate. Intuitively, such a parity must hold when the tax burdens are efficiently allocated intertemporally since at the margin, a transfer of wealth from the government to the private sector results in a direct effect of increasing private sector wealth as well as an indirect effect through possible change in saving rate.

The condition implies in particular, when the intertemporal elasticity of substitution is unity and hence the saving rate is invariant of the rate of returns, there should be no difference in the rates of returns between the government and the private sector at the steady state of the optimal-tax equilibrium. Since the after-tax rate of return on human capital must be greater than that on physical capital, the parity of returns holds only when the tax rate on physical capital is strictly positive. Consequently, the optimal tax rate on physical capital is strictly positive in the long run. This observation reveals the second principle for the optimal taxation: for efficient intertemporal allocation of tax burdens, the government should tax riskless asset in the private sector to keep the returns of the government bond in parity with the average private sector returns.

Finally we calibrate our model to the U.S. economy. Thanks to the tractability of our model,

⁷Taxing the wage income also has an offsetting pecuniary externality effect: It tends to reduce the aggregate stock of human capital, which increases the (before-tax) wage rate. In our model, the direct effect dominates the indirect, pecuniary externality effect so that a tax on the wage income reduces the after-tax wage rate.

⁸We assume that the amount of taxes paid by each individual in a given period depends only on his/her current labor and capital incomes. This makes our question here well defined. See Bassetto and Kocherlakota (2004) more on this issue.

⁹In our model, one also needs to take into account how the timing of taxes affects the saving rate, in order to determine the optimal allocation of tax burdens. However, as we shall see, evaluated locally around the equilibrium obtained under the balanced budget restriction, this effect vanishes.

we are able to obtain a numerical solution to the optimal taxation problem relatively easily. The parameter for the idiosyncratic income risk is chosen based on the evidence provided by Meghir and Pistaferri (2004), and the rest of the parameters are set as in Chari, Christiano and Kehoe (1994). We find that the presence of idiosyncratic labor-income risks significantly affects the optimal tax rates and the optimal amount of the government debt. They are also very sensitive to the degree of relative risk aversion. For instance, the steady-state debt-output ratio is about -100 percent when the coefficient of risk aversion is one, and it is about 200 percent when the coefficient of risk aversion is 9. Under our baseline calibration with the coefficient of risk aversion equal to three, the welfare gain of adopting the Ramsey policy amounts to a permanent increase in consumption of all individuals by 0.85 percent (taking into account of the transition).

The rest of the paper is organized as follows. In Section 2, the model economy is described, and the benchmark equilibrium is derived. Section 3 discusses the constrained inefficiency of the benchmark equilibrium. Section 4 analyzes the optimal taxation problem when the government is required to have balanced budget in each period. Section 5 considers the dynamic optimal taxation problem and discusses the main result of this paper. Section 6 describes the numerical results. Section 7 concludes.

2 Model economy

In this section we describe the model economy. Time is discrete and indexed by $t = 0, 1, 2, \dots$. The economy consists of a government, a continuum of individuals, and perfectly competitive firms that produce a single, homogeneous product. There is no aggregate risk in the model and so all the aggregate variables are non-random. Consequently, the market clearing prices are non-random throughout. The government needs to purchase an exogenously given amount of output in each period, which is financed by issuing debt and collecting taxes. Regarding taxes, we restrict attention to linear taxes on wages and interests. Negative taxes (i.e., subsidies) are also allowed. In this section we describe a competitive equilibrium associated with a given fiscal policy. The optimal fiscal policy is discussed later.

2.1 Firms

In each period, a single commodity is produced by perfectly competitive firms, using physical and human capital as inputs. All firms have identical production technology, described by a Cobb-Douglas production function:

$$y = F(k, h) = Ak^\alpha h^{1-\alpha}, \quad (1)$$

where y is the level of output, A is a constant, k is the input of physical capital, and h is the input of human capital. In particular, there is no productivity shock in the technology.

Let K_{t-1} and H_{t-1} denote, respectively, the aggregate stock of physical and human capital at the beginning of period t . Market clearing requires that the quantities of the factors demanded by the firms equal to these values. Hence, the aggregate amount of output produced in period t is therefore given by

$$Y_t = F(K_{t-1}, H_{t-1}) = AK_{t-1}^\alpha H_{t-1}^{1-\alpha}.$$

The profit maximization condition with market clearing implies that the before-tax rental rate of physical capital in period t equals the marginal product of physical capital in that period, $F_{k,t}$, where

$$F_{k,t} \equiv \frac{\partial F(K_{t-1}, H_{t-1})}{\partial K_{t-1}}$$

Similarly, the before-tax wage rate per efficiency unit of labor in period t is the marginal product of human capital:

$$F_{h,t} \equiv \frac{\partial F(K_{t-1}, H_{t-1})}{\partial H_{t-1}}$$

2.2 Individuals

There is a continuum of individuals. In every period, individuals consume the consumption good, supply one unit of raw labor inelastically, and invest in three kinds of assets: risk-free bond, physical capital, and human capital. The level of human capital of each individual determines the “efficiency units” of his/her labor.

Each individual $i \in [0, 1]$ has Epstein-Zin-Weil preferences over random sequences of consumption, which are defined recursively by

$$u_{i,t} = \left\{ (1 - \beta)(c_{i,t})^{1 - \frac{1}{\psi}} + \beta [E_t(u_{i,t+1})^{1 - \gamma}]^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right\}^{\frac{\psi}{\psi - 1}}, \quad t = 0, 1, \dots \quad (2)$$

where $u_{i,t}$ is the level of utility of individual i in period t , E_t is the conditional expectation operator at time t , $c_{i,t}$ is his/her consumption in period t , $\beta \in (0, 1)$ is the discount factor, ψ is the elasticity of intertemporal substitution, and γ is the coefficient of relative risk aversion.

Let $b_{i,t-1}$, $k_{i,t-1}$ and $h_{i,t-1}$ denote, respectively, the quantities of risk-free bond, physical capital, and human capital that individual i holds at the end of period $t - 1$. To capture the idea that labor income is subject to uninsurable idiosyncratic shocks, we assume that the human capital of individual i is affected by a random shock parameter, $\theta_{i,t}$, at the beginning of each period t . We assume that $\theta_{i,t}$, $i \in [0, 1]$, $t = 1, \dots$, are identically and independently distributed across individuals and across periods, with unit mean. Thus the actual amount of human capital of individual i available at the beginning of each period t is equal to $\theta_{i,t}h_{i,t-1}$. We further assume that the law of large number applies, so that the aggregate stock of human capital at the beginning of period t is not random: that is, the following relation holds with probability one:

$$\int_0^1 \theta_{i,t} h_{i,t-1} di = \int_0^1 h_{i,t-1} di = H_{t-1}.$$

We suppose that both physical and human capital are accumulated by investing output after the private shock has been observed; that is, the amount of capital is determined first by the time t shock and then by depreciation, and new investment takes place. Let $\iota_{k,i,t}$ and $\iota_{h,i,t}$ denote, respectively, the investment in physical and human capital of individual i in period t . Then the two types of capital evolve as, for $t = 1, 2, \dots$,

$$k_{i,t} = \iota_{k,i,t} + (1 - \delta_k)k_{i,t-1} \quad (3)$$

$$h_{i,t} = \iota_{h,i,t} + (1 - \delta_h)\theta_{i,t}h_{i,t-1} \quad (4)$$

where δ_k and δ_h are the depreciation rates of physical and human capital, respectively.

We assume that idiosyncratic shocks $\theta_{i,t}$ are the only sources of uncertainty, hence there is no aggregate uncertainty in the economy. Consequently, the market returns of production factors are not random. Both labor and capital income are subject to linear taxes. It follows that the risk-free bond and physical capital are perfect substitutes and therefore the after-tax rate of return of the physical capital must be equal to the risk free rate in equilibrium. Let $r_{k,t}$ denote the after-tax rental rate of physical capital (and hence the risk-free rate), and w_t the after-tax wage rate. The after-tax gross rates of return on the two types of capital are given in equilibrium by:

$$\begin{aligned} R_{k,t} &= 1 - \delta_k + r_{k,t}, \\ R_{h,t} &= 1 - \delta_h + w_t. \end{aligned}$$

Then the flow budget constraint of individual i is written as, for $t = 1, 2, \dots$,

$$c_{i,t} + \iota_{k,i,t} + (1 - \delta_k) k_{i,t-1} + \iota_{h,i,t} + (1 - \delta_h) \theta_{i,t} h_{i,t-1} + b_{i,t} = R_{k,t} k_{i,t-1} + R_{h,t} \theta_{i,t} h_{i,t-1} + R_{k,t} b_{i,t-1} \quad (5)$$

Individuals may borrow so that $b_{i,t}$ can be negative, but the holdings of capital are non-negative: $k_{i,t} \geq 0$ and $h_{i,t} \geq 0$ are required for all periods and under all contingencies.

Let $x_{i,t}$ be the total wealth of individual i at the beginning of period t after the time t shock $\theta_{i,t}$ has been realized: that is,

$$x_{i,t} \equiv R_{k,t}(k_{i,t-1} + b_{i,t-1}) + R_{h,t}\theta_{i,t}h_{i,t-1}$$

The amount of borrowing is restricted by the natural debt limit:

$$x_{i,t+1} \geq 0, \quad (6)$$

for all periods and at all contingencies.

To sum up, given the initial wealth $x_{i,0} > 0$ and a sequence of prices $\{r_{k,t}, w_t\}_{t=0}^{\infty}$, each individual i maximizes the lifetime utility $u_{i,0}$, defined by (2) subject to the flow budget constraints (7) and the debt limit (6).

This optimization problem can be complex in principle, since individual's choice variables depend on the history of shocks and individuals have different histories of shocks. But thanks to the particular form of the utility function (2) as well as the fact that the shocks are permanent and hence the current wealth is the discounted present value of future individual income stream, there is a tractable characterization of the utility maximizing choices, which we shall summarize below.

First, equations (3)-(5) can be combined together to obtain:

$$x_{i,t+1} = (1 - \eta_{c,i,t}) \{R_{k,t+1}(1 - \eta_{h,i,t}) + R_{h,t+1}\theta_{i,t+1}\eta_{h,i,t}\} x_{i,t} \quad (7)$$

where

$$\begin{aligned} \eta_{c,i,t} &\equiv \frac{c_{i,t}}{x_{i,t}} \\ \eta_{h,i,t} &\equiv \frac{h_{i,t}}{b_{i,t} + k_{i,t} + h_{i,t}} \end{aligned}$$

with initial condition $x_{i,0} > 0$. That is, the optimization problem can be equivalently written as a problem of choosing a sequence of the rate of consumption out of his/her wealth, $\eta_{c,i,t}$, and the portfolio between the human capital and the riskless assets (physical capital and risk-free bond),

$(\eta_{h,i,t}, 1 - \eta_{h,i,t})$ per unit of investment for every $t = 0, 1, \dots$, given $x_{i,0}$. By construction, the original choice values are given iteratively starting with $x_{i,0}$ from the following equations, $t = 0, 1, \dots$:

$$\begin{aligned} c_{i,t} &= \eta_{c,t} x_{i,t} \\ k_{i,t} + b_{i,t} &= (1 - \eta_{c,t})(1 - \eta_{h,t}) x_{i,t} \\ h_{i,t} &= (1 - \eta_{c,t}) \eta_{h,t} x_{i,t} \end{aligned} \quad (8)$$

As is well known,¹⁰ the optimal choice of the portfolio in this type of utility maximization problem is reduced to a static problem which is independent of all the other choice variables. Specifically, define the certainty-equivalent rate of return ρ associated with the after-tax rental rate r_k , after-tax wage rate w , as follows:

$$\rho(r_k, w, \eta_h) \equiv \left\{ E((1 - \delta_k + r_k)(1 - \eta_h) + \theta(1 - \delta_h + w)\eta_h)^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}. \quad (9)$$

It can be shown that at any time period, for any level of initial wealth hence for any individual, an optimal portfolio is given by a solution to the following maximization problem given the prevailing rates r_k and w :

$$\max_{\eta'_h \geq 0} \rho(r_k, w, \eta'_h). \quad (10)$$

Since $\rho(r, w, \eta_h)$ is strictly concave in η_h , the solution to this maximization problem is unique if it exists. Given this, we have the following simple characterization of utility maximization.

Lemma 1. *Given a sequence of prices, $\{r_{k,t}, w_t\}_{t=0}^\infty$, for any individual i , a utility maximizing sequence of portfolio and rate of consumption are characterized by the following rule: for the portfolio, at any time $t = 1, \dots$,*

$$\eta_{h,t} = \arg \max_{\eta'_h \geq 0} \rho(r_{k,t+1}, w_{t+1}, \eta'_h), \quad (11)$$

and for the rate of consumption,

$$\eta_{c,t} = \left\{ 1 + \sum_{s=0}^{\infty} \prod_{j=0}^s \left(\beta^\psi \rho_{t+1+j}^{\psi-1} \right) \right\}^{-1}, \quad (12)$$

where ρ_{t+1} , $t = 0, \dots$, denotes the optimized certainty-equivalent rate of return between periods t and $t + 1$ which is

$$\rho_{t+1} \equiv \max_{\eta'_h \geq 0} \rho(r_{k,t+1}, w_{t+1}, \eta'_h), \quad (13)$$

Moreover, the time t utility level is given by

$$u_{i,t} = v_t x_{i,t},$$

where

$$v_t = (1 - \beta)^{\frac{\psi}{\psi-1}} \left\{ 1 + \sum_{s=0}^{\infty} \prod_{j=0}^s \left(\beta^\psi \rho_{t+1+j}^{\psi-1} \right) \right\}^{\frac{1}{\psi-1}}. \quad (14)$$

¹⁰See, for instance, Epstein and Zin (1991), and Angeletos (2007).

Notice in particular that since the right hand sides of (11) and (12) are independent of index i , this result implies that all the individuals in the economy choose the same rate of consumption, $\eta_{c,t}$, and the same portfolio, $\eta_{h,t}$, in each period in equilibrium. The differences across individuals appear in the level of utility, but notice that the level of utility is the level of wealth of the individual multiplied by a common constant v_t . Thus in particular, the average level of utility is simply the average wealth multiplied by v_t , hence to determine the level of welfare we only need to find the parameter v_t and the average wealth.

Note that from (12) and (14) time t utility per wealth v_t and time t consumption share $\eta_{c,t}$ are related as

$$\eta_{c,t} = (1 - \beta)^\psi v_t^{1-\psi}. \quad (15)$$

Finally, we will need to know how the aggregate supplies of the two capitals change as the environment changes to study the effects of government policies. Lemma 1 says that the ratio of the aggregate supplies is determined by a solution to the maximization problem (10). Since ρ is a concave function of η_h , an interior solution for (10) is characterized by the first order condition. So define $\Phi : \mathbb{R}_+^3 \rightarrow \mathbb{R}$ by the rule:

$$\begin{aligned} \Phi(r_k, w, \eta_h) \equiv E \Big[& \left\{ (1 - \delta_k + r_k)(1 - \eta_h) + \theta(1 - \delta_h + w)\eta_h \right\}^{-\gamma} \\ & \times \left\{ \theta(1 - \delta_h + w) - (1 - \delta_k + r_k) \right\} \Big]. \end{aligned} \quad (16)$$

Then $\Phi(r_k, w, \eta_h) = 0$ corresponds to the first order condition. Especially for comparative static exercises, we will be concerned with the signs of the derivatives of Φ , which in general depends on the property of the shock variable. We shall assume the following throughout the analyses.

Assumption 1. *For the function $\Phi : \mathbb{R}_+^3 \rightarrow \mathbb{R}$ defined in equation (16), The derivatives of Φ at $\Phi = 0$ have the following signs:*

$$\frac{\partial \Phi}{\partial r_k} < 0, \quad \frac{\partial \Phi}{\partial w} > 0, \quad \text{and} \quad \frac{\partial \Phi}{\partial \eta_h} < 0.$$

In fact $\frac{\partial \Phi}{\partial \eta_h} < 0$ readily follows from the concavity of the certainty equivalent function, so the main part of this assumption is $\frac{\partial \Phi}{\partial r_k} < 0$ and $\frac{\partial \Phi}{\partial w} > 0$. It can be shown that these are satisfied when $\gamma \leq 1$. When $\gamma > 1$, these appear to depend on the distribution of θ_i .

2.3 Government

The government purchases an exogenously given amount of output, G_t , in each period t . It is financed by collecting taxes and issuing debt. Following the common assumption in the literature, we assume that government purchases do not yield utility to individuals.

Let B_{t-1} be the government debt outstanding at the beginning of period t . Denote by $\tau_{k,t}$ and $\tau_{h,t}$ the effective tax rates on the returns of physical and human capital at time t , respectively. For a given sequence of aggregate stocks, $\{K_t, H_t\}_{t=0}^\infty$, the flow budget constraint of the government in period t is given by

$$B_t + \tau_{k,t} F_{k,t} K_{t-1} + \tau_{h,t} F_{h,t} H_{t-1} = G_t + R_{k,t} B_{t-1}, \quad (17)$$

where the initial stock of debt, B_{-1} , is given with $B_{-1} = \int_0^1 b_{i,-1} di$.

A fiscal policy $\{\tau_{k,t}, \tau_{h,t}, B_t\}_{t=0}^\infty$ is said to be feasible (under $\{K_t, H_t\}_{t=0}^\infty$) if the flow budget constraint (17) is satisfied for every $t = 0, 1, \dots$, and

$$\lim_{t \rightarrow \infty} \left(\prod_{j=1}^t R_{k,j}^{-1} \right) B_t = 0. \quad (18)$$

2.4 Competitive equilibrium

The initial conditions of the economy are $\{b_{i,-1}, k_{i,-1}, h_{i,-1}, \theta_{i,0} : i \in [0, 1]\}$, $K_{-1} = \int_0^1 k_{i,-1} di$, $B_{-1} = \int_0^1 b_{i,-1} di$, and $H_{-1} = \int_0^1 h_{i,-1} di$. An allocation is a collection of stochastic processes $\{c_{i,t}, x_{i,t}, b_{i,t}, k_{i,t}, h_{i,t} : i \in [0, 1]\}_{t=0}^\infty$, where for each i , $\{c_{i,t}, x_{i,t}, k_{i,t}, h_{i,t}\}_{t=0}^\infty$ are stochastic processes adapted to the filtration generated by the process of idiosyncratic shocks $\{\theta_{i,t}\}_{t=0}^\infty$.

Given the initial condition and a sequence of government purchases, $\{G_t\}_{t=0}^\infty$, a competitive equilibrium is defined by a price system $\{r_{k,t}, w_t\}_{t=0}^\infty$, a fiscal policy $\{\tau_{k,t}, \tau_{h,t}, B_t\}_{t=0}^\infty$, and an allocation $\{c_{i,t}, x_{i,t}, b_{i,t}, k_{i,t}, h_{i,t} : i \in [0, 1]\}_{t=0}^\infty$ such that (a) for each $i \in [0, 1]$, given the price system, $\{c_{i,t}, x_{i,t}, k_{i,t}, h_{i,t}\}_{t=0}^\infty$ solves the utility maximization problem; (b) profit maximization occurs (and the prices reflect taxes); that is,

$$r_{k,t} = (1 - \tau_{k,t})F_k(K_{t-1}, H_{t-1}), \quad \text{and} \quad w_t = (1 - \tau_{h,t})F_h(K_{t-1}, H_{t-1}),$$

for all $t \geq 0$, where

$$\begin{aligned} K_{t-1} &= \int_0^1 k_{i,t-1} di, \\ H_{t-1} &= \int_0^1 \theta_{i,t} h_{i,t-1} di = \int_0^1 h_{i,t-1} di; \end{aligned}$$

and (c) all markets clear:

$$C_t + G_t + K_t + H_t = (1 - \delta_k)K_{t-1} + (1 - \delta_h)H_{t-1} + F(K_{t-1}, H_{t-1}), \quad (19)$$

$$B_t = \int_0^1 b_{i,t} di \quad (20)$$

where $C_t = \int_0^1 c_{i,t} di$; and (d) the government policy is feasible.

Recall that by Lemma 1, there is an associated sequence of $\{\eta_{ct}, \eta_{ht}\}_{t=0}^\infty$ for an equilibrium allocation, which is common across all the individuals. For this reason, the aggregate dynamics of a competitive equilibrium can be succinctly summarized by the average wealth and the sequence $\{\eta_{ct}, \eta_{ht}\}_{t=0}^\infty$ as follows. Let X_t denote the average amount of wealth at the beginning of period t :

$$X_t \equiv \int_0^1 x_{i,t} di$$

Then X_t evolves as

$$X_{t+1} = R_{x,t+1}(1 - \eta_{c,t})X_t, \quad t = 0, 1, 2, \dots$$

where $R_{x,t+1}$ is the equilibrium average rate of return of individual portfolios: for $t = 0, 1, 2, \dots$

$$\begin{aligned} R_{x,t+1} &\equiv R_{k,t+1}(1 - \eta_{h,t}) + R_{h,t+1}\eta_{h,t}, \\ &= [1 - \delta_k + (1 - \tau_{k,t})F_{k,t}](1 - \eta_{h,t}) + [1 - \delta_h + (1 - \tau_{h,t})F_{h,t}]\eta_{h,t}. \end{aligned}$$

The aggregate amounts of consumption, physical capital and human capital are given, respectively, as follows: for $t = 0, 1, 2, \dots$,

$$\begin{aligned} C_t &= \eta_{c,t} X_t, \\ K_t &= (1 - \eta_{c,t})(1 - \eta_{h,t}) X_t - B_t, \\ H_t &= (1 - \eta_{c,t}) \eta_{h,t} X_t. \end{aligned}$$

Finally, the average utility level is given for $t = 0, 1, 2, \dots$ by

$$U_t = \int_0^1 u_{i,t} di = v_t X_t.$$

2.5 Benchmark equilibrium with no taxes

As a benchmark, let us consider the case in which the government does not purchases goods, and it does not issue debt nor impose any taxes:

$$G_t = B_t = \tau_{k,t} = \tau_{h,t} = 0, \quad \text{for all } t \geq 0, \quad \text{and} \quad b_{i,-1} = 0, \quad \text{for all } i \in [0, 1]. \quad (21)$$

In this case, the competitive equilibrium has a very simple structure. The aggregate economy is always on a balanced growth path, although each individual's consumption fluctuates stochastically over time.

To see these, first notice that in the benchmark economy with (21), from Lemma 1, $\eta_{h,t}$ must maximize the certainty equivalent function ρ where the rates must be consistent with profit maximization and market clearing. This means that if we set $r_{k,t} = F_k(1 - \eta_{h,t}, \eta_{h,t})$ and $w_t = F_h(1 - \eta_{h,t}, \eta_{h,t})$, the first-order condition for maximization of ρ must be met: that is, for every $t = 0, 1, 2, \dots$,

$$\Phi[F_k(1 - \eta_{h,t}, \eta_{h,t}), F_h(1 - \eta_{h,t}, \eta_{h,t}), \eta_{h,t}] = 0, \quad (22)$$

where Φ is given in (16). Note that $F_k(1, 0) = F_h(0, 1) = 0$ and $F_k(0, 1) = F_h(1, 0) = +\infty$, and that $\lim_{\eta_h \rightarrow 0} F_h(1 - \eta_h, \eta_h)h = \lim_{\eta_h \rightarrow 1} F_k(1 - \eta_h, \eta_h)(1 - \eta_h) = 0$. Furthermore, under Assumption 1, $\frac{\partial}{\partial \eta_h} \Phi[F_k(1 - \eta_h, \eta_h), F_h(1 - \eta_h, \eta_h), \eta_h] < 0$ whenever $\Phi = 0$, so it follows there exists a unique $\hat{\eta}_h \in (0, 1)$ that satisfies (22). The uniqueness implies that $\eta_{h,t} = \hat{\eta}_h$ must hold for every t .

Set $\hat{F}_k = F_k(1 - \hat{\eta}_h, \hat{\eta}_h)$, $\hat{F}_h = F_h(1 - \hat{\eta}_h, \hat{\eta}_h)$, and let $\hat{\rho}$ to be the associated certainty-equivalent rate of return:

$$\hat{\rho} = \rho\left((1 - \delta_k + \hat{F}_k)(1 - \hat{\eta}_h), (1 - \delta_h + \hat{F}_h)\hat{\eta}_h, \hat{\eta}_h\right). \quad (23)$$

The argument above combined with Lemma 1 has shown the following characterization result.

Proposition 2. *Suppose that Assumption 1 holds, and consider the benchmark economy with (21). Let $\hat{\eta}_h \in (0, 1)$ be the solution to (22), and $\hat{\rho}$ be the associated certainty-equivalent rate of return defined in (23). Suppose that*

$$\beta^\psi \hat{\rho}^{\psi-1} < 1.$$

Then a unique competitive equilibrium of the benchmark economy is generated by $\hat{\eta}_h$ and $\hat{\eta}_c \equiv 1 - \beta^\psi \hat{\rho}^{\psi-1}$, which are common across i , through (7) and (8). Thus the aggregate variables C_t , K_t , H_t , and X_t all grow at the same rate g_x , which is given by

$$\hat{g}_x \equiv (1 - \hat{\eta}_c) \hat{R}_x$$

where

$$\hat{R}_x \equiv (1 - \delta_k + \hat{F}_k)(1 - \hat{\eta}_h) + (1 - \delta_h + \hat{F}_h)\hat{\eta}_h$$

The level of utility of each individual evolves as

$$u_{i,t} = \hat{v}x_{i,t}$$

where

$$\hat{v} \equiv \left[\frac{(1 - \beta)^\psi}{1 - \beta^\psi \hat{\rho}^{\psi-1}} \right]^{\frac{1}{\psi-1}}.$$

Note that the uniqueness stated in Proposition 2 that any (interior) equilibrium must be of the form above. In what follows, this laissez-faire equilibrium without government purchases is referred to as the benchmark equilibrium, and the value of a variable in the benchmark equilibrium is denoted by a hat (^) over the variable.

3 Constrained inefficiency

In this section we consider the benchmark equilibrium with no government purchases and no taxes obtained in Section 2.5, and demonstrate that if a social planner can force individuals to invest less in physical capital and more in human capital, then all individuals are made better off. In this sense, the competitive equilibrium of our benchmark economy is constrained inefficient, and it involves over-accumulation of physical capital.¹¹

Assume that there are no government purchases and no taxes as in (21). Suppose that a social planner can directly control a deterministic sequence $\{\eta_{h,t}\}_{t=0}^\infty$ of portfolio choices. Consider a hypothetical economy where each consumer is constrained to choose $\{\eta_{h,t}\}_{t=0}^\infty$, that is, $k_{i,t}$ and $h_{i,t}$ must be chosen to satisfy

$$\eta_{h,t} = \frac{h_{i,t}}{k_{i,t} + h_{i,t}}, \quad t = 0, 1, 2, \dots, \quad (24)$$

but all the other variables are determined as in the benchmark economy, namely, by the utility maximization, profit maximization, and market clearing conditions. Formally, define an $\{\eta_{h,t}\}_{t=0}^\infty$ -constrained equilibrium as a collection of an allocation $\{c_{i,t}, k_{i,t}, h_{i,t}, x_{i,t} : i \in [0, 1]\}_{t=0}^\infty$ and a price system $\{r_{k,t}, w_t\}_{t=0}^\infty$ such that (a) for each i , given $\{r_{k,t}, w_t\}_{t=1}^\infty$ and $\{\eta_{h,t}\}_{t=0}^\infty$, $\{c_{i,t}, k_{i,t}, h_{i,t}, x_{i,t}\}$ solves the utility maximization problem of individual i with the additional constraint given by (24); (b) the prices satisfy $r_{k,t} = F_k(K_{t-1}, H_{t-1})$ and $w_t = F_h(K_{t-1}, H_{t-1})$, where K_{t-1} and H_{t-1} are the aggregate stocks of physical and human capital at the beginning of period t ; and (c) all markets clear. The benchmark equilibrium is by definition an $\{\hat{\eta}_{h,t}\}_{t=0}^\infty$ -constrained equilibrium. We say that an $\{\eta_{h,t}\}_{t=0}^\infty$ -constrained equilibrium is constrained efficient if there exist no sequences $\{\eta'_{h,t}\}_{t=0}^\infty$ such that all individuals are better off in the $\{\eta'_{h,t}\}_{t=0}^\infty$ -constrained equilibrium than in the $\{\eta_{h,t}\}_{t=0}^\infty$ -constrained equilibrium.

Given $\{\eta_{h,t}\}_{t=0}^\infty$, the associated constrained equilibrium can be constructed as follows, utilizing Lemma 1. First, market clearing and profit maximization imply,

$$r_{k,t+1} = F_k(1 - \eta_{h,t}, \bar{\eta}_{h,t}), \quad \text{and} \quad w_{t+1} = F_h(1 - \eta_{h,t}, \bar{\eta}_{h,t}). \quad (25)$$

¹¹The constrained inefficiency analysis here is closely related to the previous work by Davila, Hong, Krusell, and Ríos-Rull (2005).

Next, define the common individual certainty-equivalent rate of return between t and $t + 1$ as

$$\begin{aligned}\rho_{t+1} &= \rho(r_{k,t+1}, w_{t+1}, \eta_{h,t}), \\ &= \rho(F_k(1 - \eta_{h,t}, \eta_{h,t}), F_h(1 - \eta_{h,t}, \eta_{h,t}), \eta_{h,t}),\end{aligned}\tag{26}$$

where $\rho(r_k, w, \eta_h)$ is defined in (9). In our environment, even when $\{\eta_{h,t}\}_{t=0}^\infty$ is fixed exogenously, once the prices are fixed in this way, then the consumers' problem of choosing $\{\eta_{c,t}\}_{t=0}^\infty$ is the same as in the competitive equilibrium. Therefore, a $\{\eta_{h,t}\}_{t=0}^\infty$ -constrained equilibrium is then characterized as in the next proposition.

Proposition 3. *Consider the benchmark economy with (21). Given $\{\eta_{h,t}\}_{t=0}^\infty$, define $\{\rho_{t+1}\}_{t=0}^\infty$ as in (26). Suppose that the associated infinite sum, $\sum_{t=0}^\infty \prod_{j=0}^t (\beta^\psi \rho_{j+1}^{\psi-1})$, is well defined and takes a finite value. Then an associated constrained equilibrium exists: The prices and the certainty equivalents are determined by (25)-(26), and all the other endogenous variables are determined as in Section 2.4.*

By construction, the benchmark equilibrium given in Proposition 2 is the constrained equilibrium associated with $\eta_{h,t} = \hat{\eta}_h$ for all t . For each i and t , consider infinitesimal changes from $\hat{\eta}_h$:

$$\eta_{h,t} = \hat{\eta}_h + d\eta_{h,t},$$

where $d\eta_{h,t}$ may differ across periods and so $\eta_{h,t}$ may be time dependent although $\hat{\eta}_h$ is not.

Here we ask whether or not such infinitesimal changes in common portfolios, $\{d\eta_{h,t}\}$, can make all individuals better off. To answer this question, first notice in Lemma 1 that each individual's lifetime utility in period 0 is given by

$$u_{i,0} = v_0 x_{i,0}$$

where

$$v_0 = (1 - \beta)^{\frac{\psi}{\psi-1}} \left\{ 1 + \sum_{t=0}^\infty \prod_{j=0}^t (\beta^\psi \rho_{j+1}^{\psi-1}) \right\}^{\frac{1}{\psi-1}}.$$

where ρ_{j+1} is the certainty-equivalent rate of return between periods j and $j + 1$, which is common for all individuals as given in (26).

This relation provides a few observations. First of all, the lifetime utility of each individual monotonically increases with the realized certainty-equivalent rate of return in each period: that is,

$$\frac{\partial v_0}{\partial \rho_{t+1}} > 0, \quad \text{for all } t \geq 0.\tag{27}$$

Therefore, it follows that a sufficient condition for every individual i 's welfare to improve is that the certainty-equivalent rate of return, ρ_{t+1} , increases for all t . Secondly, for each t , ρ_{t+1} depends only on $\eta_{h,t}$ as shown in (26). Thirdly, evaluated at $\eta_{h,t} = \hat{\eta}_h$, $\eta_{h,t}$ does not have any first-order effect on $\rho_{i,t+1}$, since $\hat{\eta}_h$ is maximizing the certainty equivalent in the individual utility maximization (the envelope property):

$$\left. \frac{\partial \rho(r_{k,t+1}, w_{t+1}, \eta_{h,t})}{\partial \eta_{h,t}} \right|_{\eta_{h,i,t} = \hat{\eta}_h} = 0.$$

Thus, $d\eta_{h,t}$ has a first-order effect on ρ_{t+1} only through its effect on the prices, $r_{k,t+1}$ and w_{t+1} . The next proposition shows that a reduction in $\eta_{h,t}$ from $\hat{\eta}_h$ makes all individuals better off.¹²

Proposition 4. *The competitive equilibrium of the benchmark economy is constrained inefficient. Reducing the proportion of investment on physical capital improves the welfare of all individuals: for all $i \in [0, 1]$ and all $t \geq 0$,*

$$\left. \frac{dv_{i,0}}{d\eta_{h,t}} \right|_{\eta_{h,t}=\hat{\eta}_h} > 0.$$

That is, the benchmark economy involves over-accumulation of physical capital in the constrained-efficiency sense.

Proof. It is sufficient to show that $\frac{d\rho_{t+1}}{d\eta_{h,t}} > 0$, evaluated at $\eta_{h,t} = \hat{\eta}_h$. By the utility maximization in the constrained equilibrium, notice that we can write $\rho_{t+1} = \left(E[R_{x,i,t+1}^{1-\gamma}]\right)^{\frac{1}{1-\gamma}}$, where $R_{x,i,t+1} = (1 - \delta_k + r_{k,t+1})(1 - \eta_{h,t}) + \theta_{i,t+1}(1 - \delta_h + w_{t+1})\eta_{h,t}$. It follows that

$$\left. \frac{d\rho_{t+1}}{d\eta_{h,t}} \right|_{\eta_{h,t}=\hat{\eta}_{h,t}} = (\hat{\rho}_{t+1})^\gamma \cdot E \left[(R_{x,i,t+1})^{-\gamma} \cdot \left\{ \frac{dr_{k,t+1}}{d\eta_{h,t}}(1 - \hat{\eta}_{h,t}) + \theta_{i,t+1} \frac{dw_{t+1}}{d\eta_{h,t}} \hat{\eta}_{h,t} \right\} \right]$$

Since $r_{k,t+1} = F_k(1 - \eta_{h,t}, \eta_{h,t})$ and $w_{t+1} = F_h(1 - \eta_{h,t}, \eta_{h,t})$ for any $\eta_{h,t}$, we have from the homogeneity of the production function, for any $\eta_{h,t}$,

$$\frac{dr_{k,t+1}}{d\eta_{h,t}}(1 - \eta_{h,t}) + \frac{dw_{t+1}}{d\eta_{h,t}}\eta_{h,t} = 0.$$

Hence we obtain

$$\left. \frac{d\rho_{i,t+1}}{d\eta_{h,t}} \right|_{\eta_{h,t}=\hat{\eta}_h} = (\hat{\rho}_{t+1})^\gamma \cdot E \left[(R_{x,i,t+1})^{-\gamma} \cdot (\theta_{i,t+1} - 1) \right] \cdot \hat{\eta}_h \frac{dw_{t+1}}{d\eta_{h,t}}.$$

Under the assumption that $E(\theta_{i,t+1}) = 1$,

$$E \left[(R_{x,i,t+1})^{-\gamma} \cdot (\theta_{i,t+1} - 1) \right] = \text{Cov} \left[(R_{x,i,t+1})^{-\gamma}, (\theta_{i,t+1} - 1) \right] < 0.$$

Since $w_{t+1} = F_h(1 - \eta_{h,t}, \eta_{h,t})$, so $\frac{dw_{t+1}}{d\eta_{h,t}} < 0$ holds. Therefore,

$$\left. \frac{d\rho_{t+1}}{d\eta_{h,t}} \right|_{\eta_{h,t}=\hat{\eta}_{h,t}} > 0.$$

□

This proposition shows that the ratio of physical capital to human capital is too high in the benchmark incomplete-markets equilibrium: taking the structure of the asset markets as given, reducing investment ratio in physical capital is welfare improving. This should not be confused with the simple observation that the physical-human capital ratio is larger than that in the complete market setting. Indeed, as shown by Hong, Davila, Krusell, and Ríos-Rull (2005), and Gottardi, Kajii, and Nakajima (2009), incomplete-markets economies may involve under-accumulation of physical capital in general.

¹²The fact that the benchmark equilibrium exhibits time independence is not important for this result. It can be readily checked even if it were time dependent, the rest of the argument goes through.

Proposition 4 has a simple intuition. From the planner's point of view, individuals are exposed to too much risk in the benchmark incomplete-markets equilibrium. If the planner changes the portfolio of individuals, $\eta_{h,t}$, marginally from the equilibrium level, it affects their welfare through its effect on prices, $r_{k,t+1}$ and w_{t+1} (because of the envelope property). Now suppose that the planner increases $\eta_{h,t}$ from the equilibrium level. Then it raises the rental rate $r_{k,t+1}$ and reduces the wage rate w_{t+1} . Note that the labor income of each individual is subject to uninsurable idiosyncratic risk, and his/her capital income is not. It follows that such a change in the factor prices tend to reduce the amount of risk that each individual faces. In this sense, reducing the investment in physical capital effectively provides an insurance. This is why increasing $\eta_{h,t}$ from the equilibrium level achieves a Pareto improvement in our model economy.

Notice that a similar result on the constrained inefficiency also obtains in a more common environment where there is no human capital accumulation and the labor productivity of each individual follows an exogenously specified stochastic process. For instance, Gottardi, Kajii, and Nakajima (2009) consider such a model with two time periods, and show that a competitive equilibrium with incomplete markets involves too much physical capital when individuals are ex ante identical.

4 Taxation: Local analysis around a competitive equilibrium

The previous section shows that the laissez-faire equilibrium in our benchmark economy accumulates physical capital too much relative to human capital in the sense of constrained inefficiency, around a competitive equilibrium. Does that mean that around a competitive equilibrium the government should tax physical capital (interest income) and subsidize human capital (wage income) to make this ratio smaller?

The answer to this question depends on what kind of fiscal instruments are available to the policy maker. In this section we consider a relatively simple case where the government has balanced budget at all times:

$$B_t = 0, \quad \text{for all } t, \quad \text{and} \quad b_{i,-1} = 0, \quad \text{for all } i \in [0, 1].$$

In this case we show that the the answer to the above question turns out to be the opposite: The government should subsidize capital and tax labor.

For the sake of comparison with the previous section, let us start with the case without government purchases:

$$G_t = 0, \quad \text{for all } t.$$

The the balanced budget requirement implies that

$$\tau_{k,t} F_{k,t} K_{t-1} + \tau_{h,t} F_{h,t} H_{t-1} = 0, \quad \text{for all } t.$$

where $F_{k,t} = F_k(K_{t-1}, H_{t-1})$ and $F_{h,t} = F_h(K_{t-1}, H_{t-1})$. With the Cobb-Douglas technology (1),

$$\frac{F_{k,t} K_{t-1}}{F_{h,t} H_{t-1}} = \frac{\alpha}{1 - \alpha}$$

holds in equilibrium at any t . Therefore the government budget constraint may be replaced by

$$\tau_{h,t} = -\frac{\alpha}{1-\alpha}\tau_{k,t}, \quad \text{for all } t.$$

A competitive equilibrium with balanced budget of the government is characterized as in Section 2.4 under the additional conditions that $G_t = B_t = 0$ and $\alpha\tau_{k,t} + (1-\alpha)\tau_{h,t} = 0$ for all t .

Now consider the benchmark equilibrium in Section 2.5, which can be viewed as the tax equilibrium with $\tau_{k,t} = \tau_{h,t} = 0$ for all t . And we ask whether or not changing $\tau_{k,t}$ from zero is welfare improving for each t . Note that Assumption 1 guarantees that $\partial\eta_{h,t}/\partial r_{k,t+1} < 0$ and $\partial\eta_{h,t}/\partial w_{t+1} > 0$ in the utility maximization problem. Given a pair of the tax rates, (τ_k, τ_h) , the associated utility maximizing portfolio $\eta_h(\tau_k, \tau_h)$ is defined implicitly as

$$\Phi[(1-\tau_k)F_k(1-\eta_h, \eta_h), (1-\tau_h)F_h(1-\eta_h, \eta_h), \eta_h] = 0, \quad (28)$$

where the function Φ is defined in (16). Using this function $\eta_h(\tau_k, \tau_h)$, define the induced certainty-equivalent rate of return as

$$\rho(\tau_k, \tau_h) \equiv \rho[(1-\delta_k + r_k(\tau_k, \tau_h))(1-\eta_h(\tau_k, \tau_h)) + (1-\delta_h + w(\tau_k, \tau_h))\theta\eta_h(\tau_k, \tau_h)], \quad (29)$$

where

$$\begin{aligned} r_k(\tau_k, \tau_h) &\equiv (1-\tau_k)F_k[1-\eta_h(\tau_k, \tau_h), \eta_h(\tau_k, \tau_h)], \\ w(\tau_k, \tau_h) &\equiv (1-\tau_h)F_h[1-\eta_h(\tau_k, \tau_h), \eta_h(\tau_k, \tau_h)]. \end{aligned}$$

With the balanced budget requirement, $\alpha\tau_k + (1-\alpha)\tau_h = 0$, the effect of τ_k on the after tax return r_k at $\tau \equiv (\tau_k, \tau_h) = 0$ is given by

$$\left. \frac{dr_k}{d\tau_k} \right|_{\tau=0} = -F_k + (-F_{kk} + F_{kh}) \left(\frac{\partial\eta_h}{\partial\tau_k} - \frac{\alpha}{1-\alpha} \frac{\partial\eta_h}{\partial\tau_h} \right).$$

where all derivatives are evaluated at $\tau = 0$. The first-term $(-F_k)$ reflects the the direct effect of the capital-income tax, and the second term reflects the indirect effect due to the pecuniary externality. The following lemma shows that direct effect dominates the indirect effect.

Lemma 5. *Suppose that Assumption 1 holds. Then*

$$\left. \frac{dr_k}{d\tau_k} \right|_{\tau=0} < 0.$$

Proof. Appendix. □

We can define the coefficient for the lifetime utility v_0 in equilibrium as a function of $\{(\tau_{k,t}, \tau_{h,t})\}_{t=0}^{\infty}$:

$$v_0((\tau_{k,t}, \tau_{h,t})_{t=0}^{\infty}) \equiv (1-\beta)^{\frac{\psi}{\psi-1}} \left\{ 1 + \sum_{t=0}^{\infty} \prod_{j=0}^t (\beta^{\psi} \rho(\tau_{k,1+j}, \tau_{h,1+j})^{\psi-1}) \right\}^{\frac{1}{\psi-1}},$$

where $\rho(\tau_k, \tau_h)$ is defined as in (29). Taxing physical capital is welfare improving if this function is increasing in $\tau_{k,t}$ while $\tau_{h,t}$ is determined to meet the government budget constraint. Interestingly enough, the following proposition establishes that subsidizing capital makes everyone better off.

Proposition 6. Suppose that $B_t = G_t = 0$ for all t and $b_{i,-1} = 0$ for all $i \in [0, 1]$. Suppose also that Assumption 1 holds. Then, for all t ,

$$\left(\frac{\partial v_0}{\partial \tau_{k,t}} + \frac{\partial v_0}{\partial \tau_{h,t}} \frac{d\tau_h}{d\tau_k} \right) \Big|_{\tau=0} < 0,$$

where $d\tau_h/d\tau_k = -\alpha/(1-\alpha)$. Therefore, if a sequence of taxes $\{\tau_{k,t}, \tau_{h,t}\}_{t=0}^\infty$ with $\alpha\tau_{k,t} + (1-\alpha)\tau_{h,t} = 0$ for all t subsidizes capital, $\tau_{k,t} < 0$, and hence taxes labor, $\tau_{h,t} = -\alpha\tau_{k,t}/(1-\alpha) > 0$ for every t , it improves the welfare of all individuals over the benchmark equilibrium, if these taxes are small enough.

Proof. In the proof here, unless otherwise stated, all derivatives are evaluated at the benchmark competitive equilibrium with zero taxes, $\tau_k = \tau_h = 0$. Since the equilibrium in the benchmark economy is invariant of t , the argument is the same for every t , hence we shall omit t . Also to simplify the notation, we write $d/d\tau_k$ to mean that $\partial/\partial\tau_k - \alpha/(1-\alpha)\partial/\partial\tau_h$, i.e., $d/d\tau_k$ takes into account the government budget constraint. Since the lifetime utility is increasing in ρ_t for each t , it suffices to show that $d\rho/d\tau_k < 0$. Note first that for the before tax returns,

$$\begin{aligned} \frac{dF_h}{d\tau_k} &= (F_{hh} - F_{hk}) \frac{d\eta_h}{d\tau_k} \\ &= (F_{kk} - F_{kh}) \frac{1 - \eta_h}{\eta_h} \frac{d\eta_h}{d\tau_k} \\ &= -\frac{1 - \eta_h}{\eta_h} \frac{dF_k}{d\tau_k} \end{aligned}$$

where we have used the fact that $-F_{hk} + F_{hh} = (F_{kk} - F_{kh})(1 - \eta_h)/\eta_h$. Then, evaluated at $\tau_k = \tau_h = 0$, for after tax returns,

$$\begin{aligned} \frac{dw}{d\tau_k} &= \frac{F_k(1 - \hat{\eta}_h)}{F_h \hat{\eta}_h} w + \frac{dF_h}{d\tau_k} \\ &= F_k \frac{1 - \hat{\eta}_h}{\hat{\eta}_h} + (F_{kk} - F_{kh}) \frac{1 - \hat{\eta}_h}{\hat{\eta}_h} \frac{d\eta_h}{d\tau_k} \\ &= -\frac{1 - \hat{\eta}_h}{\hat{\eta}_h} \left(\frac{dF_k}{d\tau_k} - F_k \right) \\ &= -\frac{1 - \hat{\eta}_h}{\hat{\eta}_h} \frac{dr_k}{d\tau_k}, \end{aligned} \tag{30}$$

which means in particular $\frac{dw}{d\tau_k} > 0$ by Lemma 5. Using the envelope property, we obtain

$$\frac{d\rho}{d\tau_k} = \hat{\rho}^\gamma E \left[\hat{R}_{x,i,\theta_i}^{-\gamma} \left\{ (1 - \hat{\eta}_h) \frac{dr_k}{d\tau_k} + \theta_i \hat{\eta}_h \frac{dw}{d\tau_k} \right\} \right]$$

where $\hat{R}_{x,i,\theta_i} \equiv (1 - \delta_k + \hat{r}_k)(1 - \hat{\eta}_h) + (1 - \delta_h + \hat{w})\theta_i \hat{\eta}_h$. Using (30), we have

$$\frac{d\rho}{d\tau_k} = \hat{\rho}^\gamma E \left[\hat{R}_{x,\theta}^{-\gamma} (\theta - 1) \right] \hat{\eta}_h \frac{dw}{d\tau_k} < 0,$$

where the inequality follows from the fact that $E[\hat{R}_{x,\theta}^{-\gamma}(\theta - 1)] < 0$ and $dw/d\tau_k > 0$. $\square$

The intuition for this result is again very simple. Suppose that the government is to change the tax rates, τ_k and τ_h , marginally from zero under the balanced budget constraint. Due to the envelope property, such a change in the tax rates affects the utility of each individual only through

its effects on the after-tax factor prices, r_k and w . As we have discussed in the previous section, individuals are exposed to too much risk in their labor income in the benchmark incomplete-markets equilibrium. In the constrained inefficiency result, we have seen that reducing the investment in physical capital lowers such risk by increasing r_k and decreasing w . Here, the planner uses taxes rather than controlling the portfolios of individuals. Then the way to reduce the amount of idiosyncratic risk that individuals face is to tax the labor income and subsidize the capital income, that is, to make $\tau_h > 0$ and $\tau_k < 0$. In this way, the planner can reduce the (after-tax) wage rate. Thus, Proposition 4 and Proposition 6 are perfectly consistent with each other. Both attempt to provide an insurance for uninsurable labor income risk using the policy instruments available to the planner.

Note that Proposition 6 only concerns with the local property, and it does not guarantee that the optimal tax rate on physical capital is indeed negative under the balanced budget of the government. In what follows, when needed, we simply assume that it is indeed the case. A sufficient condition for this is that $\rho(\tau_k, -\alpha/(1-\alpha)\tau_k)$ defined in (29) has a unique local maximum. (This property is satisfied in all the numerical examples we have considered.)

Assumption 2. *The function $\rho(\tau_k, -\alpha/(1-\alpha)\tau_k)$ has a unique local maximum so that the optimal tax rate on physical capital is negative when $G_t = B_t = 0$ for all t .*

5 Optimal taxation with government debt

In this section we allow the government to borrow or lend and consider a dynamic optimal taxation problem. With complete markets, Judd (1985) and Chamley (1986) have shown that the optimal tax rate on physical capital is zero in the steady state. In addition, Jones, Manuelli and Rossi (1997) demonstrate that when there is human capital accumulation in such a model, the optimal tax rates on physical and human capital are both equal to zero in the steady state. In this section we illustrate how the structure of optimal taxation changes under incomplete asset markets. With incomplete markets, it is in general beneficial for the government to tax labor and physical capital even if there are no government purchases. Specifically, we provide two theoretical results in this section. First, accumulating government debt improves welfare as long as government purchases are small enough. Second, the steady state optimal taxation on physical capital is strictly positive under standard preference restrictions, even without government purchases.

5.1 Ramsey problem

Recall that for given an exogenous sequence of purchases, $\{G_t\}_{t=0}^\infty$, a competitive equilibrium determines a sequence of aggregate capital stocks, $\{K_t, H_t\}_{t=0}^\infty$, and a fiscal policy of the government, as in Section 2.4. The latter is a sequence of taxes and debt, $\{\tau_{k,t}, \tau_{h,t}, B_t\}_{t=0}^\infty$ that satisfies (17) and (18).

It is convenient to normalize aggregate variables in terms of the total wealth X_t , so define for each t

$$k_t \equiv \frac{K_t}{X_t}, \quad h_t \equiv \frac{H_t}{X_t}, \quad b_t \equiv \frac{B_t}{X_t}, \quad g_t \equiv \frac{G_t}{X_{t-1}}.$$

With these normalized variables, the government's flow budget constraint can be rewritten as

$$g_t + (1 - \delta_k + r_{k,t})b_{t-1} = (1 - \eta_{c,t-1})R_{x,t}b_t + F(k_{t-1}, h_{t-1}) - r_{k,t}k_{t-1} - w_t h_{t-1}$$

We assume that the government takes the sequence of normalized purchases, $\{g_t\}_{t=0}^\infty$, as exogenously given throughout the analysis.

As is standard in the literature, in order to rule out a trivial solution, we assume that the tax rates in the initial period are set exogenously as:

$$\tau_{k,0} = \bar{\tau}_{k,0}, \quad \text{and} \quad \tau_{h,0} = \bar{\tau}_{h,0},$$

hence the initial debt level is given exogenously as well. In other words, in the initial period $t = 0$, k_{-1} , h_{-1} , b_{-1} , $\tau_{k,0}$, and $\tau_{h,0}$ are predetermined. It follows that $r_{k,0}$, w_0 , and b_0 are predetermined as well.

The dynamic optimal taxation problem is to choose $\{b_{t+1}, r_{k,t+1}, w_{t+1}\}_{t=0}^\infty$ as well as all the other endogenous variables so as to maximize equilibrium utility level

$$\max v_0 = (1 - \beta)^{\frac{\psi}{\psi-1}} \left\{ 1 + \sum_{t=0}^{\infty} \prod_{j=0}^t \left(\beta^\psi \rho_{1+j}^{\psi-1} \right) \right\}^{\frac{1}{\psi-1}} \quad (31)$$

subject to the equilibrium constraints which guarantee that the endogenous variables constitute a competitive equilibrium (i.e., the intended fiscal policy is indeed part of the competitive equilibrium):

$$\begin{aligned} \eta_{h,t} &= \arg \max_{\eta'_h} \rho(r_{k,t+1}, w_{t+1}, \eta'_h) \\ \rho_{t+1} &= \max_{\eta'_h} \rho(r_{k,t+1}, w_{t+1}, \eta'_h) \\ R_{x,t+1} &= (1 - \delta_k + r_{k,t+1})(1 - \eta_{h,t}) + (1 - \delta_h + w_{t+1})\eta_{h,t} \\ \eta_{c,t} &= \left\{ 1 + \sum_{s=0}^{\infty} \prod_{j=0}^s \left(\beta^\psi \rho_{t+1+j}^{\psi-1} \right) \right\}^{-1} \\ k_t &= (1 - \eta_{c,t})(1 - \eta_{h,t}) - b_t \\ h_t &= (1 - \eta_{c,t})\eta_{h,t} \\ g_{t+1} + (1 - \delta_k + r_{k,t+1})b_t &= (1 - \eta_{c,t})R_{x,t+1}b_{t+1} + F(k_t, h_t) - r_{k,t+1}k_t - w_{t+1}h_t \\ \lim_{t \rightarrow \infty} \left\{ \prod_{j=1}^t (1 - \delta_k + r_{k,j})^{-1} (1 - \eta_{c,j-1})R_{x,j} \right\} b_t &= 0 \end{aligned}$$

given b_0 . We shall call this maximization problem as the Ramsey problem, and the resulting equilibrium as the Ramsey equilibrium.

It is convenient to divide the Ramsey problem in two steps. The first step is to find optimal after-tax prices for a given sequence of debt and a given sequence of rates of consumption, assuming that the given sequences are consistent with a competitive equilibrium with taxes. Then such conditionally optimal prices will be functions of a sequence of debt and a sequence of rates of consumption. The second step is then to find an optimal sequence of debt and the rate of consumption taking into account these functional relations, as well as the rest of equilibrium conditions.

Since there is a one-to-one relation between the rate of consumption and the utility parameter v_t when utility is maximized (see (14) in Lemma 1), the first step is equivalent to choose after-tax factor prices $\{r_{k,t+1}, w_{t+1}\}_{t=0}^{\infty}$ to maximize the certainty equivalent function given a pair of sequences $\{b_t, v_t\}_{t=0}^{\infty}$. That is, this step can be expressed as, for each fixed t , solving the following problem:

$$\tilde{\rho}(b_t, b_{t+1}, v_t) \equiv \max_{\{r_{k,t+1}, w_{t+1}, \eta_{c,t}, \eta_{h,t}\}} \rho(r_{k,t+1}, w_{t+1}, \eta_{h,t}) \quad (32)$$

subject to

$$\begin{aligned} g_{t+1} + (1 - \delta_k + r_{k,t+1})b_t &= (1 - \eta_{c,t})R_{x,t+1}b_{t+1} + F[(1 - \eta_{c,t})(1 - \eta_{h,t}) - b_t, (1 - \eta_{c,t})\eta_{h,t}] \\ &\quad - r_{k,t+1}[(1 - \eta_{c,t})(1 - \eta_{h,t}) - b_t] - w_{t+1}(1 - \eta_{c,t})\eta_{h,t} \\ \eta_{c,t} &= (1 - \beta)^{\psi} v_t^{1-\psi} \\ \eta_{h,t} &= \arg \max_{\eta_h} \rho(r_{k,t+1}, w_{t+1}, \eta_h) \\ R_{x,t+1} &= (1 - \delta_k + r_{k,t+1})(1 - \eta_{h,t}) + (1 - \delta_h + w_{t+1})\eta_{h,t} \end{aligned} \quad (33)$$

The second step is then to choose $\{b_{t+1}, v_{t+1}\}_{t=0}^{\infty}$ so as to maximize v_0 given b_0 :

$$\max_{\{v_{t+1}, b_{t+1}\}_{t=0}^{\infty}} v_0 \quad (34)$$

subject to the remaining equilibrium constraints

$$v_t^{\psi-1} = (1 - \beta)^{\psi} + \beta^{\psi} \rho_{t+1}^{\psi-1} v_{t+1}^{\psi-1} \quad (35)$$

$$\rho_{t+1} = \tilde{\rho}(b_t, b_{t+1}, v_t) \quad (36)$$

Regarding the function $\tilde{\rho}(b, b', v)$ defined in (32), the following simple observation is useful.

Lemma 7. *Assume that $g_{t+1} = 0$. Consider the function $\tilde{\rho}(b, b', v)$ defined in (32). If $b = b' = 0$, the first order effect of v is zero everywhere: that is,*

$$\frac{\partial \tilde{\rho}}{\partial v} = 0, \quad \text{if } b = b' = 0.$$

Proof. When $g_{t+1} = 0$ and $b_t = b_{t+1} = 0$, notice that the first equation in the constraints for the maximization problem (32) becomes, taking advantage of the homogeneity of F , $0 = F[(1 - \eta_{h,t}), \eta_{h,t}] - r_{k,t+1}(1 - \eta_{h,t}) - w_{t+1}\eta_{h,t}$. Thus variables v and $\eta_{c,t}$ appear only in the equation $\eta_{c,t} = (1 - \beta)^{\psi} v_t^{1-\psi}$, hence any change in v is completely absorbed by a change $\eta_{c,t}$. So the value of the objective function remains unchanged when v changes, so the result follows. $\square$

5.2 Desirability of the government debt

Before discussing the solution to the Ramsey problem, we examine the sense in which the government debt is welfare improving with incomplete markets. Here we show that starting from zero government debt, increasing the amount of government debt is welfare improving as long as the amount of government purchases are small enough.

To see the welfare effect of the government bond, consider the optimal taxation problem with the restriction that

$$b_t = 0, \quad \text{for all } t \neq T + 1. \quad (37)$$

To simplify the argument, assume also that the (normalized) amount of government purchases is constant over time:

$$g_t = g, \quad \text{for all } t. \quad (38)$$

Let us denote the variables in the optimal tax equilibrium with $b_t = 0$ for all t by the superscript o , that is, v^o , etc. Thus,

$$v^o \equiv \left[\frac{(1 - \beta)^\psi}{1 - \beta^\psi (\rho^o)^{\psi-1}} \right]^{\frac{1}{\psi-1}}$$

where $\rho^o = \tilde{\rho}(0, 0, v^o)$.

Denote the variables in the optimal tax equilibrium under the restriction given by (37) as $v_t(b_{T+1})$, etc. Thus,

$$v_t(b_{T+1}) = \left\{ (1 - \beta)^\psi + \beta^\psi \rho_{t+1}(b_{T+1})^{\psi-1} v_{t+1}(b_{T+1})^{\psi-1} \right\}^{\frac{1}{\psi-1}}$$

where

$$\rho_{t+1}(b_{T+1}) = \tilde{\rho}(b_t, b_{t+1}, v_t(b_{T+1}))$$

with $b_t = 0$ for all $t \neq T + 1$. Notice that

$$\begin{aligned} v_t(b_{T+1}) &= v^o, \quad \forall t \geq T + 2, \\ \rho_t(b_{T+1}) &= \rho^o, \quad \forall t \neq T + 1, T + 2 \end{aligned}$$

where the second equality follows from Lemma 7.

The next proposition states that having a positive amount of debt, $b_{T+1} > 0$, is welfare improving as long as government purchases are sufficiently small.

Proposition 8. *Suppose that $g_t = g$ for all t , and that Assumptions 1 and 2 hold. Consider the optimal tax equilibrium under the balanced budget requirement: $b_t = 0$ for all t . Then increasing b_{T+1} from zero for a given period $T + 1$ improves the lifetime utility of all individuals if g is sufficiently small.*

Proof. Appendix. □

To obtain an intuition for this result, consider the case where $g = 0$. Let R_x^o and F_k^o be the average rate of return and the marginal product of capital in that equilibrium, respectively. Then, R_x^o is related to the benefit enjoyed in period $T + 1$ of increasing b_{T+1} , and $1 - \delta_k + F_k^o$ is to its cost incurred in period $T + 2$. To see this, consider the optimal tax equilibrium under the balanced-budget restriction, $b_t = 0$ for all t . Given equation (31), whether or not increasing b_{T+1} from zero is welfare improving depends on how it affects $\{\rho_{t+1}\}$. In an optimal tax equilibrium, $\rho_{t+1} = \tilde{\rho}(b_t, b_{t+1}, v_t)$, but Lemma 7 says that $\tilde{\rho}$ is locally independent of v_t when $b_{t+1} = 0$. This greatly simplifies the argument. We only have to look at the partial derivatives of $\tilde{\rho}(b_T, b_{T+1}, v_T)$ and $\tilde{\rho}(b_{T+1}, b_{T+2}, v_{T+1})$ with respect to b_{T+1} evaluated at $b_T = b_{T+2} = 0$. Let us denote those derivatives by ρ_2^o and ρ_1^o , respectively. The derivative ρ_2^o is positive and measures the benefit in

$T + 1$ of increasing the government debt, which is primarily due to an associated tax cut. The derivative ρ_1^o is negative and can be interpreted as the cost of the increase in taxes in period $T + 2$ that is required to redeem b_{T+1} .

Using the homogeneity of $F(k, h)$, let us rewrite the government's flow budget constraint as

$$g + (1 - \delta_k + F_k)b_t = (1 - \eta_{c,t})R_{x,t+1}b_{t+1} + (F_{k,t+1} - r_{k,t+1})(1 - \eta_{c,t})(1 - \eta_{h,t}) + (F_{h,t+1} - w_{t+1})(1 - \eta_{c,t})\eta_{h,t}$$

where $F_{k,t+1} = F_k((1 - \eta_{c,t})(1 - \eta_{h,t}) - b_t, (1 - \eta_{c,t})\eta_{h,t})$, and $F_{h,t+1} = F_h((1 - \eta_{c,t})(1 - \eta_{h,t}) - b_t, (1 - \eta_{c,t})\eta_{h,t})$. It follows that the benefit of the increase in b_{T+1} , ρ_2^o , is proportional to the after-tax average rate of return, R_x^o . This is natural because this is the average rate that individuals earn using the proceeds from the tax cut in $T + 1$. The cost of the increase in b_{T+1} , ρ_1^o , is proportional to the (before-tax) rate of return on the government debt, $1 - \delta_k + F_k^o$. Whether or not increasing b_{T+1} is beneficial depends on the sign of their discounted sum. In fact, we can show that the benefit of increasing b_{T+1} dominates if and only if

$$R_x^o > 1 - \delta_k + F_k^o. \quad (39)$$

The reason why (39) holds when $g = 0$ is simple. As is discussed in Section 4, when $g = 0$, the optimal tax rates under the balanced-budget of the government satisfy $\tau_k^o < 0$ and $\tau_h^o > 0$. Note also that since the investment in the human capital is risky, the after-tax rate of return on human capital must be greater than the after-tax rate of return on physical capital. It then follows from risk aversion that

$$\begin{aligned} R_x^o &= [1 - \delta_k + (1 - \tau_k^o)F_k^o](1 - \eta_h^o) + [1 - \delta_h + (1 - \tau_h^o)F_h^o]\eta_h^o \\ &> 1 - \delta_k + (1 - \tau_k^o)F_k^o \\ &> 1 - \delta_k + F_k^o \end{aligned}$$

Thus, when $g = 0$, (39) holds and having a positive amount of government debt is welfare improving. Extending the argument to the case where g is sufficiently small is straightforward because of the continuity.

5.3 Ramsey steady state

Now consider the Ramsey problem given by (31). As we have seen, the Ramsey problem in our economy can be decomposed into two steps as given by (32) and (34). Here we focus on what needs to be true at a steady state, i.e., along a balanced growth path.¹³

Consider the choice of b_{t+1} , that is, the (normalized) amount of government debt issued in period $t + 1$, keeping b_s fixed for all $s \neq t + 1$. From the structure of the second-step problem (34) it is straightforward to see that b_{t+1} affects the initial utility coefficient v_0 only through its effect on v_t , the utility coefficient in period t . In turn, the evolution of the utility coefficients (35)

¹³A presumption here is that a solution to the Ramsey problem converges to a steady state. We do not have a formal proof for this, but in all the numerical results reported later, such a convergence takes place in just one period. Note also that we use the terms a “steady state” and a “balanced growth path” interchangeably given the fact that a balanced growth path in the original economy is obtained as a steady state with the adequately normalized variables.

shows that b_{t+1} affects v_t by changing the product term $\rho_{t+1}v_{t+1}$. Then, given the determination of the certainty-equivalent rate of return ρ_{t+1} as in (36), the effects of an increase in b_{t+1} on v_t are summarized as follows: (i) First, it increases ρ_{t+1} because an increase in the government debt issued in period $t + 1$ implies a reduction in the tax rates in that period. This is a benefit of an increase in b_{t+1} ; (ii) Second, it reduces ρ_{t+2} because the increase in b_{t+1} require an increase in the tax rates in the next period, $t + 2$. This is a cost of an increase in b_{t+1} . (iii) Finally, the change in b_{t+1} may also affect the saving rates in periods $t + 1$ and $t + 2$. A change in the saving rate also affects the certainty-equivalent rates of return.¹⁴ The optimality requires, of course, the marginal effect of b_{t+1} on v_t must be zero. The next proposition shows that, at the steady state, this condition is characterized by a relatively simple equation.

Proposition 9. *In the steady state of the Ramsey equilibrium, the following condition holds:*

$$R_x = (1 - \delta_k + F_k) (1 - \beta^\psi \rho^{\psi-2} \rho_v v)^{-1} \quad (40)$$

with $\rho_v \equiv \frac{\partial \tilde{\rho}}{\partial v}$, where $\tilde{\rho}$ is the function defined in (32).

Proof. Appendix. □

An intuition for this result is given as follows. As shown in Appendix, at the steady state, the first-order condition for b_{t+1} becomes

$$\rho_2 + \frac{\beta^\psi \rho^{\psi-1}}{1 - \beta^\psi \rho^{\psi-2} \rho_v \rho} \rho_1 = 0, \quad (41)$$

where $\rho_2 \equiv \frac{\partial}{\partial b'} \tilde{\rho}(b, b', v)$ and $\rho_1 \equiv \frac{\partial}{\partial b} \tilde{\rho}(b, b', v)$. This equation shows the three effects of b_{t+1} on v_t described in the above paragraph: The first term ρ_2 represents the benefit of an increase in b_{t+1} on v_t (effect i); ρ_1 in the second term is its cost (effect ii); the factor $\beta^\psi \rho^{\psi-1}$ on the second term represents effective discounting; and finally the factor $(1 - \beta^\psi \rho^{\psi-2} \rho_v \rho)^{-1}$ is the effect due to the change in the saving rate (effect iii). It follows from the definition of $\tilde{\rho}(b, b', v)$ in the first-step problem (32) that at the steady state

$$\frac{\rho_2}{\rho_1} = \frac{(1 - \eta_c) R_x}{1 - \delta_k + F_k}. \quad (42)$$

Finally note that the saving rate at the steady state satisfies $1 - \eta_c = \beta^\psi \rho^{\psi-1}$. Then, combining (41) and (42) yields the condition (40).

As a result, (40) can be understood as follows: the term R_x represents the marginal benefit of increasing the government debt; the term $1 - \delta_k + F_k$ expresses its discounted marginal cost; and the term $(1 - \beta^\psi \rho^{\psi-2} \rho_v v)^{-1}$ shows the effect due to the change in the saving rate. Intuitively, R_x is the average rate of return earned in the private sector, and $1 - \delta_k + F_k$ is the rate of return for the government (that is, the before-tax rate on the risk free asset). Thus, the steady-state condition (40) says that, after the adjustment due to the change in the saving rate, the rates of return earned by the private sector and by the government should be equal at the steady state.

Notice that for a special case of $\psi = 1$, v_0 in (31) becomes

$$\ln v_0 = \sum_{t=0}^{\infty} \beta^{t+1} \ln \rho_{t+1} \quad (43)$$

¹⁴Remember that this effect through the saving rate is reflected in that the function $\tilde{\rho}$ defined in (32) has an argument v_t .

and $\eta_{c,t}$ becomes a constant irrespective of the sequence of market rates:

$$\eta_{c,t} = 1 - \beta$$

In this case, therefore, the function $\tilde{\rho}$ defined in (32) becomes independent of v . So in this case there is no effect of changing rates, so the two returns should be equal with no adjustment. Indeed, this can be confirmed by setting $\rho_v = 0$ in (40). More importantly, the resulting equation implies that the steady-state tax rate on physical capital is positive in our incomplete-markets economy.

Corollary 10. *Consider the case of $\psi = 1$. Then, in the steady state of the Ramsey equilibrium, the following condition holds:*

$$R_x = 1 - \delta_k + F_k \quad (44)$$

It implies that the optimal tax rate on physical capital at the steady state is positive:

$$\tau_k > 0$$

Proof. The condition (44) immediately follows from (40). To see $\tau_k > 0$, notice that because of *risk aversion*, the rate of return on human capital must be greater than the rate of return of the risk-free assets. That is,

$$1 - \delta_k + r_k < R_x < 1 - \delta_h + w$$

Then condition (44) implies that

$$\tau_k > 0.$$

□

Corollary 10 can be directly related to the previous results obtained by Judd (1985), Chamley (1986), Jones, Manuelli, and Rossi (1997), among others, which show that the optimal tax rate on physical capital at the steady state is zero. In our setup, if shocks to human capital of individuals were insurable so that human capital is effectively a riskless asset, then the three assets (risk-free bonds, physical capital, and human capital) must all yield the same rate of return, that is,

$$1 - \delta_k + r_k = 1 - \delta_h + w = R_x$$

It then follows from (44) that

$$\tau_k = 0.$$

6 Numerical result

In this section we calibrate our model based on some empirical evidence on the U.S. economy, and examine how market incompleteness affects the structure of the Ramsey taxation.

6.1 Baseline calibration

Suppose that $\theta_{i,t} \in \{1 + \bar{\theta}, 1 - \bar{\theta}\}$, each occurring with equal probability. Also, suppose that the normalized amount of government purchases is constant over time, $g_t = g$ for all t . Then the set of parameters of our model economy is given by $\{\beta, \psi, \gamma, A, \alpha, \delta_k, \delta_h, g, \bar{\theta}\}$. The baseline values for these parameters are set as follows. First, we set $\psi = 1$ and $\gamma = 3$, that is, the intertemporal

elasticity of substitution is unity, and the coefficient of relative risk aversion is three. Second, the capital share of income is set to 0.36, and the depreciation rates of physical and human capital are both 0.06: $\alpha = 0.36$ and $\delta_k = \delta_h = 0.06$. To set the values for the remaining parameters, consider a balanced growth path with $\tau_{k,t} = \tau_{h,t} = \tau$ and $b_t = b$ for all t . They are set so that the associated balanced growth path replicates the following features of the U.S. economy: (i) government purchases are 18 percent of GDP; (ii) the government debt is 51 percent of GDP; (iii) the capital-output ratio is 2.7; (iv) the growth rate of GDP is 1.6 percent; (v) the variance of the permanent shock to individual labor earnings is 0.0313. Here, the first four facts are based on Chari, Christiano and Kehoe (1994), and the last fact is taken from Meghir and Pistaferri (2004).¹⁵ The baseline parameter values are summarized in Table 1.

6.2 Results

Table 2 shows the tax rates and the government debt at the Ramsey steady state. For reference, the third column shows the corresponding values under the baseline policy: the debt-to-output ratio of 51 percent and the growth rate of 1.6 percent are the values calibrated to the U.S. economy, which imply the (uniform) tax rate of 19.95 percent. The fourth column describes the Ramsey steady state under the baseline calibration. The fifth column demonstrates the Ramsey steady state when there are no government purchases.

As shown in Proposition 10, the capital tax rate at the Ramsey steady state is strictly positive. Quantitatively, it is not a trivial amount even when there are no government purchases. The wage tax rate at the Ramsey steady state is also positive. This is due to the fact that taxing the wage income tends to reduce the idiosyncratic risk that individuals face. As suggested by Proposition 8, the amount of the government debt at the Ramsey steady state depends on the level of government purchases. Under the baseline parameter values, the debt-to-output ratio at the Ramsey steady state is almost zero (0.19 percent). However, when there are no government purchases, this value increases to 202.6 percent.

The transitional dynamics of the Ramsey equilibrium in our model is very simple. Under the Ramsey policy, the economy converges to the steady state as quickly as possible. It is illustrated in Figure 1. It plots the debt-to-output ratio, b_t/y_t , and the two tax rates, $\tau_{k,t}$ and $\tau_{h,t}$, in the Ramsey equilibrium (31) where the initial condition b_0 is equal to the value in the baseline steady state.¹⁶ As shown there, the convergence to the steady state in this case is immediate: The government imposes a sufficiently high tax rates in period 1 so that the level of debt in period 1, b_1 , attains the steady-state level for the Ramsey equilibrium. Then the economy is on the balanced growth path afterwards.

How much benefits do individuals in our economy obtain by moving from the baseline polity to the Ramsey policy. It is measured by the rate of permanent increase in consumption of each individual that makes him/her indifferent between the two policies. Note that, as can be seen in Lemma 1, everyone agrees about this ratio, and it is indeed given by the ratio of v_0 under the policies in comparison. Table 3 shows the result. When we only compare the steady state associated with

¹⁵It is also consistent with the evidence reported by Storesletten, Telmer and Yaron (2004).

¹⁶Thus, implicitly, the date 0 tax rates are restricted to be the values under the baseline policy: $\tau_{k,0} = \tau_{h,0} = 0.1995$.

the baseline policy and the Ramsey policy, the welfare gain of adopting the Ramsey policy amounts to about 8.7 percent increase in each individual's consumption. But this number ignores the cost of transition. When the transition is taken into account, the gain gets substantially smaller, 0.85 percent, which is nevertheless a significant amount.

6.3 Sensitivity analysis

Let us conduct some sensitivity analysis here. Specifically, we examine how the debt-to-output ratio and the tax rates at the Ramsey steady state vary under different values for the risk aversion γ , the intertemporal elasticity of substitution ψ , and the idiosyncratic risk $\bar{\theta}$. For the purpose of normalization, when we change the values of these parameters, we adjust the value of the discount factor β so that the steady-state growth rate under the baseline policy continues to be equal to 1.6 percent.

Figure 2 plots the result for the risk aversion. We can see that the steady-state debt-to-output ratio is very sensitive to the choice of the degree of risk aversion. It is about -100 percent when $\gamma = 1$, and about 200 percent when $\gamma = 9$. Regarding the tax rates, the risk aversion coefficient affects the capital tax rate τ_k much more than the labor tax rate τ_h .

How the Ramsey steady state is affected by the amount of the idiosyncratic risk is shown in Figure 3. Again, the steady-state debt-to-output ratio varies a lot. It is -200 percent when there is no idiosyncratic risk ($\text{std}(\theta) = 0$), and about 60 percent when $\text{std}(\theta) = 0.2$ (it is 0.1585 under our calibration). The two tax rates are very similar when the amount of idiosyncratic risk is moderate ($\text{std}(\theta) < 0.1$), but when it gets large ($\text{std}(\theta) > 0.1$), the labor tax rate τ_h becomes much less sensitive to the change in the amount of idiosyncratic risk.

In contrast to the risk aversion and the amount of idiosyncratic risk, the value of the intertemporal elasticity of substitution does not affect the Ramsey steady state much as illustrated in Figure 4. Any effect that the intertemporal elasticity of substitution has on the Ramsey steady state is offset by our adjustment of β to keep the growth rate under the baseline policy fixed.

7 Conclusion

In this paper we have developed a tractable infinite horizon model with incomplete markets and examined how labor and capital should be taxed when there are uninsurable idiosyncratic shocks to the labor income. Our results can be summarized by the two general principles for public policy in the presence of idiosyncratic income risks. That is, (i) providing (at least partial) insurance against the idiosyncratic risks; and (ii) allocating tax burdens efficiently over time. Here, the first principle calls for taxing the wage income, and the second requires taxing the interest income.

For the sake of tractability and clarity, we have made a number of simplifying assumptions in this paper. As we noted in Introduction, the Ramsey problem is difficult to solve in general, so we contend that the benefit from deriving an explicit solution exceeds the cost of loss of generality. We readily admit however that our results might be sensitive to these assumptions, and it is important to examine the robustness of our findings in more general environments. We therefore conclude with providing two generalizations of particular interest in this respect. The first is to ask if the two principles are valid under a more general, non-linear taxes. Allowing non-linear taxes expand the set of tax equilibria, but typically the first best is not achievable with such taxes. Thus we

speculate that our findings would have some counterpart when non-linear taxes are available for the government.

The second is to study the role of income distribution: taxes change the income distribution across the households, which should have some welfare implication via conflict of interests. We need to generalize our model to address this important question; in our model, each individual's lifetime utility is given by $u_{i,0} = v_0 x_{i,0}$, where $x_{i,0}$ is his/her wealth in period 0 and v_0 is a constant common across all individuals. The objective of the government is therefore to raise this common constant v_0 as much as possible, and hence the income inequality has no welfare consequences.

8 Appendix

Proof of Lemma 5

In this proof, all derivatives are evaluated at the competitive equilibrium with zero taxes, $\tau_k = \tau_h = 0$, unless otherwise stated. First notice that

$$\frac{dr_k}{d\tau_k} = -F_k + (-F_{kk} + F_{kh}) \frac{d\eta_h}{d\tau_k} \quad (45)$$

Next, differentiating the function f in (28) and evaluating it at $\tau_k = 0$, we obtain

$$\left\{ -f_r F_k + f_w \frac{\alpha}{1-\alpha} F_h \right\} d\tau_k + \{ f_r (-F_{kk} + F_{kh}) + f_w (-F_{hk} + F_{hh}) + f_\eta \} d\eta_h = 0$$

Note here that

$$\begin{aligned} \frac{\alpha}{1-\alpha} F_h &= F_k \frac{1-\eta_h}{\eta_h} \\ -F_{hk} + F_{hh} &= \frac{1-\eta_h}{\eta_h} (F_{kk} - F_{kh}) \end{aligned}$$

It then follows that

$$\frac{d\eta_h}{d\tau_k} = \frac{F_k \left(-f_r + f_w \frac{1-\eta_h}{\eta_h} \right)}{(-F_{kk} + F_{kh}) \left(-f_r + f_w \frac{1-\eta_h}{\eta_h} \right) - f_\eta} \quad (46)$$

From (45) and (46), we obtain

$$\begin{aligned} \frac{dr_k}{d\tau_k} &= \frac{F_k f_\eta}{(-F_{kk} + F_{kh}) \left(-f_r + f_w \frac{1-\eta_h}{\eta_h} \right) - f_\eta} \\ &< 0 \end{aligned}$$

The last inequality follows from the fact that $F_k > 0$, $f_\eta < 0$, $-F_{kk} + F_{kh} > 0$, $f_r < 0$ and $f_w > 0$.

Proof of Proposition 8

Suppose that $g = 0$. Consider the optimal tax equilibrium with the restriction that $b_t = 0$ for all $t \neq T+1$. The variables in that equilibrium are denoted as $v_t(b_{T+1})$, etc. Also, denote the variables in the optimal tax equilibrium with $b_t = 0$ for all t by the superscript o , that is, v^o , etc. Remember that

$$v_t(b_{T+1}) = \left\{ (1-\beta)^\psi + \beta^\psi \rho_{t+1}(b_{T+1}) v_{t+1}(b_{T+1})^{\psi-1} \right\}^{\frac{1}{\psi-1}}$$

where

$$\rho_{t+1}(b_{T+1}) = \tilde{\rho}(b_t, b_{t+1}, v_t(b_{T+1}))$$

with $b_t = 0$ for all $t \neq T + 1$. It follows that

$$v_t(b_{T+1}) = v^o, \quad \forall t \geq T + 2, \quad \text{and} \quad \rho_t(b_{T+1}) = \rho^o, \quad \forall t \neq T + 1, T + 2$$

We need to determine the effect of b_{T+1} on $v_0(b_{T+1})$, and it suffices to determine its effect on $v_T(b_{T+1})$:

$$\frac{dv_0}{db_{T+1}} \gtrless 0 \quad \Longleftrightarrow \quad \frac{dv_T}{db_{T+1}} \gtrless 0$$

For this, first let us see how b_{T+1} affects v_{T+1} . Recall that the optimality implies the following relation between v_{T+1} and v_{T+2} :

$$v_{T+1}(b_{T+1}) = \left\{ (1 - \beta)^\psi + \beta^\psi \rho_{T+2}(b_{T+1}) v_{T+2}(b_{T+1})^{\psi-1} \right\}^{\frac{1}{\psi-1}} \quad (47)$$

where $\rho_{T+2}(b_{T+1}) = \tilde{\rho}(b_{T+1}, 0, v_{t+1}(b_{T+1}))$ and $v_{T+2}(b_{T+1}) = v^o$. It follows from Lemma 7 that

$$\frac{\partial \rho_{T+2}}{\partial v_{T+1}} = 0$$

Then differentiating v_{T+1} with respect to b_{T+1} in (47) and evaluating it at $b_{T+1} = 0$ yields

$$\frac{dv_{T+1}}{db_{T+1}} = \beta^\psi (\rho^o)^{\psi-2} \rho_1^o v^o \quad (48)$$

where $\rho_1^o \equiv \partial \tilde{\rho}(b, 0, v^o) / \partial b$ evaluated at $b = 0$.

Next, look at the optimality equation between v_T and v_{T+1} :

$$v_T(b_{T+1}) = \left\{ (1 - \beta)^\psi + \beta^\psi \rho_{T+1}(b_{T+1}) v_{T+1}(b_{T+1})^{\psi-1} \right\}^{\frac{1}{\psi-1}} \quad (49)$$

Again, evaluated at $b_{T+1} = 0$, $\frac{\partial \rho_{T+1}}{\partial v_T} = 0$. Then, given (48), differentiating (49) and evaluating at $b_{T+1} = 0$ yields,

$$\frac{dv_T}{db_{T+1}} = \beta^\psi (\rho^o)^{\psi-2} v^o [\rho_2^o + \beta^\psi (\rho^o)^{\psi-1} \rho_1^o]$$

where $\rho_2^o \equiv \partial \tilde{\rho}(0, b', v^o) / \partial b'$ evaluated at $b' = 0$.

Now remember the definition of the function $\tilde{\rho}(b, b', v)$ in (32). Let $\lambda(b, b', v)$ denote the Lagrange multiplier on the flow budget constraint for the government in the maximization problem there. Then, since $\partial \rho / \partial r_k > 0$ and $\partial \rho / \partial \eta_h = 0$, we have $\lambda > 0$. Thus,

$$\begin{aligned} \frac{\partial \tilde{\rho}}{\partial b} &= -\lambda(b, b', v) [1 - \delta_k + F_k(b, b', v)] < 0 \\ \frac{\partial \tilde{\rho}}{\partial b'} &= \lambda(b, b', v) [1 - \eta_c(b, b', v)] R_x(b, b', v) > 0 \end{aligned}$$

In the optimal tax equilibrium under the balanced-budget requirement, $b_t = 0$ for all t :

$$\eta_c^o = 1 - \beta^\psi (\rho^o)^{\psi-1}$$

It follows that

$$\begin{aligned} \rho_1^o &= -\lambda^o (1 - \delta_k + F_k^o) \\ \rho_2^o &= \lambda^o \beta^\psi (\rho^o)^{\psi-1} R_x^o \end{aligned}$$

Therefore, we obtain

$$\frac{dv_T}{db_{T+1}} = \xi [R_x^o - (1 - \delta_k + F_k^o)] \quad (50)$$

where ξ is a positive constant defined by

$$\xi \equiv \beta^\psi (\rho^o)^{\psi-2} v^o \lambda^o \beta^\psi (\rho^o)^{\psi-1} > 0$$

Under Assumptions 1 and 2, we have

$$\tau_k^o < 0.$$

Because the investment in human capital is risky, its expected return must be greater than the rate of return on physical capital:

$$1 - \delta_k + (1 - \tau_k^o) F_k^o < 1 - \delta_h + (1 - \tau_h^o) F_h^o$$

It follows that

$$\begin{aligned} R_x^o &= [1 - \delta_k + (1 - \tau_k^o) F_k^o] (1 - \eta_h^o) + [1 - \delta_h + (1 - \tau_h^o) F_h^o] \eta_h^o \\ &> 1 - \delta_k + (1 - \tau_k^o) F_k^o \\ &> 1 - \delta_k + F_k^o \end{aligned} \quad (51)$$

From (50) and (51) it follows that $\frac{dv_0}{db_{T+1}} > 0$ at $g = 0$. By continuity, this is also the case with a sufficiently small g .

Proof of Proposition 9

Define the Lagrangian for the problem (34) as

$$v_0 + \sum_{t=0}^{\infty} \lambda_t^v \left\{ (1 - \beta)^\psi + \beta^\psi \tilde{\rho}(b_t, b_{t+1}, v_t)^{\psi-1} v_{t+1}^{\psi-1} - v_t^{\psi-1} \right\}.$$

The first-order condition with respect to b_{t+1} is

$$\lambda_t^v \beta^\psi \tilde{\rho}_{t+1}^{\psi-2} \tilde{\rho}_{2,t+1} v_{t+1}^{\psi-1} + \lambda_{t+1}^v \beta^\psi \tilde{\rho}_{t+2}^{\psi-2} \tilde{\rho}_{1,t+2} v_{t+2}^{\psi-1} = 0, \quad (52)$$

where $\tilde{\rho}_{t+1} \equiv \tilde{\rho}(b_t, b_{t+1}, v_t)$, $\tilde{\rho}_{2,t+1} \equiv \partial \tilde{\rho}(b_t, b_{t+1}, v_t) / \partial b_{t+1}$, and $\tilde{\rho}_{1,t+2} \equiv \partial \tilde{\rho}(b_{t+1}, b_{t+2}, v_{t+1}) / \partial b_{t+1}$.

The first-order condition for v_{t+1} is

$$\lambda_t^v \beta^\psi \tilde{\rho}_{t+1}^{\psi-1} v_{t+1}^{\psi-2} + \lambda_{t+1}^v \beta^\psi \tilde{\rho}_{t+2}^{\psi-2} \tilde{\rho}_{v,t+2} v_{t+2}^{\psi-1} - \lambda_{t+1}^v v_{t+1}^{\psi-2} = 0, \quad (53)$$

where $\tilde{\rho}_{v,t+2} \equiv \partial \tilde{\rho}(b_{t+1}, b_{t+2}, v_{t+1}) / \partial v_{t+1}$.

In a steady-state equilibrium, equation (52) becomes

$$\tilde{\rho}_2 + \frac{\lambda_{t+1}^v}{\lambda_t^v} \tilde{\rho}_1 = 0 \quad (54)$$

whereas equation (53) implies that

$$\frac{\lambda_{t+1}^v}{\lambda_t^v} = \beta^\psi \tilde{\rho}^{\psi-1} \left(1 - \beta^\psi \tilde{\rho}^{\psi-1} \frac{\tilde{\rho}_{vv}}{\tilde{\rho}} \right)^{-1} \quad (55)$$

Now notice that from the definition of $\tilde{\rho}(b, b', v)$ in (32), the derivative $\tilde{\rho}$ with respect to b and b' is given by the derivative of the first constraint times the corresponding multiplier. Then it follows in the steady-state equilibrium,

$$\begin{aligned} -\frac{\tilde{\rho}_1}{\tilde{\rho}_2} &= \frac{1 - \delta_k + F_k}{(1 - \eta_c)R_x} \\ &= \frac{1 - \delta_k + F_k}{\beta^\psi \tilde{\rho}^{\psi-1} R_x}, \end{aligned} \quad (56)$$

where for the second equality, we used $\eta_c = (1 - \beta)^\psi v^{1-\psi}$ and $v^{\psi-1} = (1 - \beta)^\psi + \beta^\psi \rho^{\psi-1} v^{\psi-1}$ from the constraints of (32) and (34), respectively.

Combining (54)-(56) yields

$$R_x = (1 - \delta_k + F_k) (1 - \beta^\psi \tilde{\rho}^{\psi-2} \tilde{\rho}_v v)^{-1} \quad (57)$$

This completes the proof of Proposition 9.

When $\psi = 1$, $\rho_v = 0$, which implies that equation (57) becomes

$$R_x = 1 - \delta_k + F_k$$

In this case we have $\tau_k > 0$ as shown in the main text. This completes the proof of Corollary 10.

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Table 1: Baseline parameter values

parameter	value	description
ψ	1	intertemporal elasticity of substitution
γ	3	risk aversion coefficient
A	0.315	coefficient in the production function
α	0.36	share of capital
δ_k	0.06	depreciation rate of physical capital
δ_h	0.06	depreciation rate of human capital
β	0.9511	discount factor
g	0.0256	government purchases as a fraction of total wealth
τ	0.1955	tax rate in the baseline policy ($\tau_{k,t} = \tau_{h,t} = \tau$)
$\bar{\theta}$	0.1585	idiosyncratic shock

Table 2: Steady states

	model notation	baseline	Ramsey	Ramsey with $g = 0$
capital tax rate (%)	τ_k	19.95	19.64	11.56
labor tax rate (%)	τ_h	19.95	14.88	4.99
debt-GDP ratio (%)	$\frac{B_{t-1}}{Y_t}$	51	0.19	202.6
growth rate (%)	$\frac{Y_{t+1}}{Y_t} - 1$	1.6	2.26	3.25

Table 3: Welfare gain of adopting the Ramsey policy

ignoring transition	considering transition
8.7320	0.8494

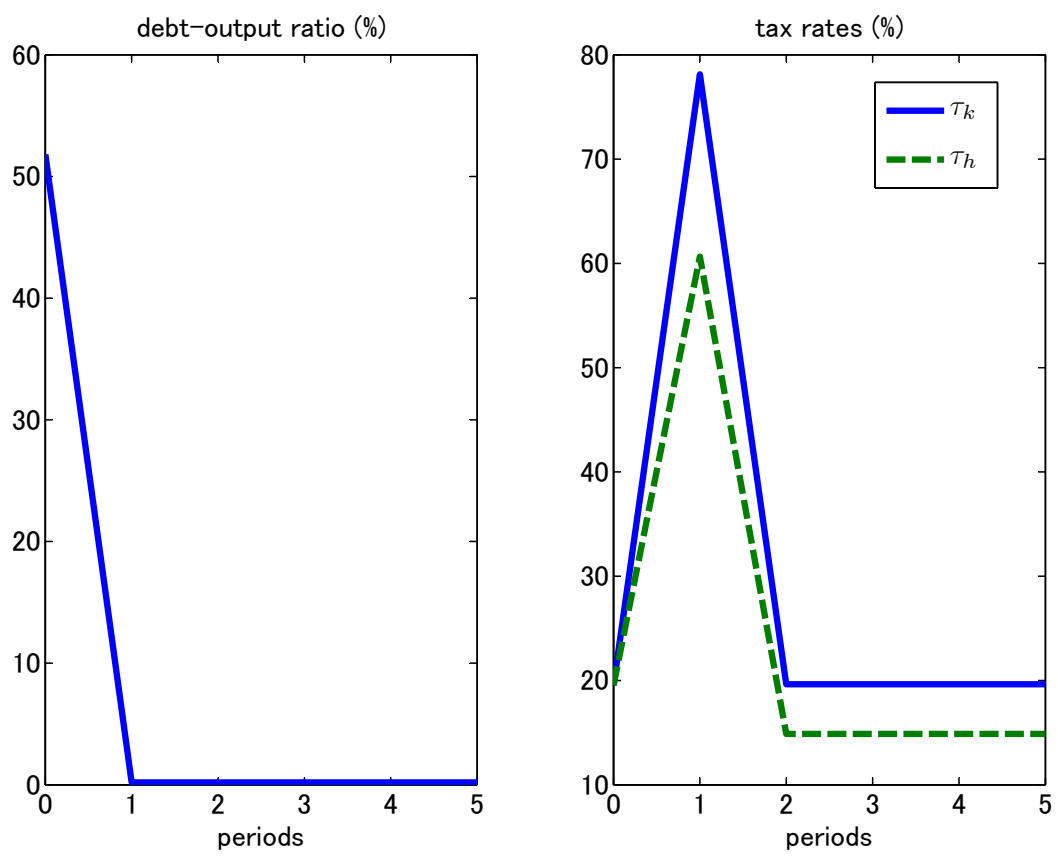


Figure 1: Transitional dynamics.

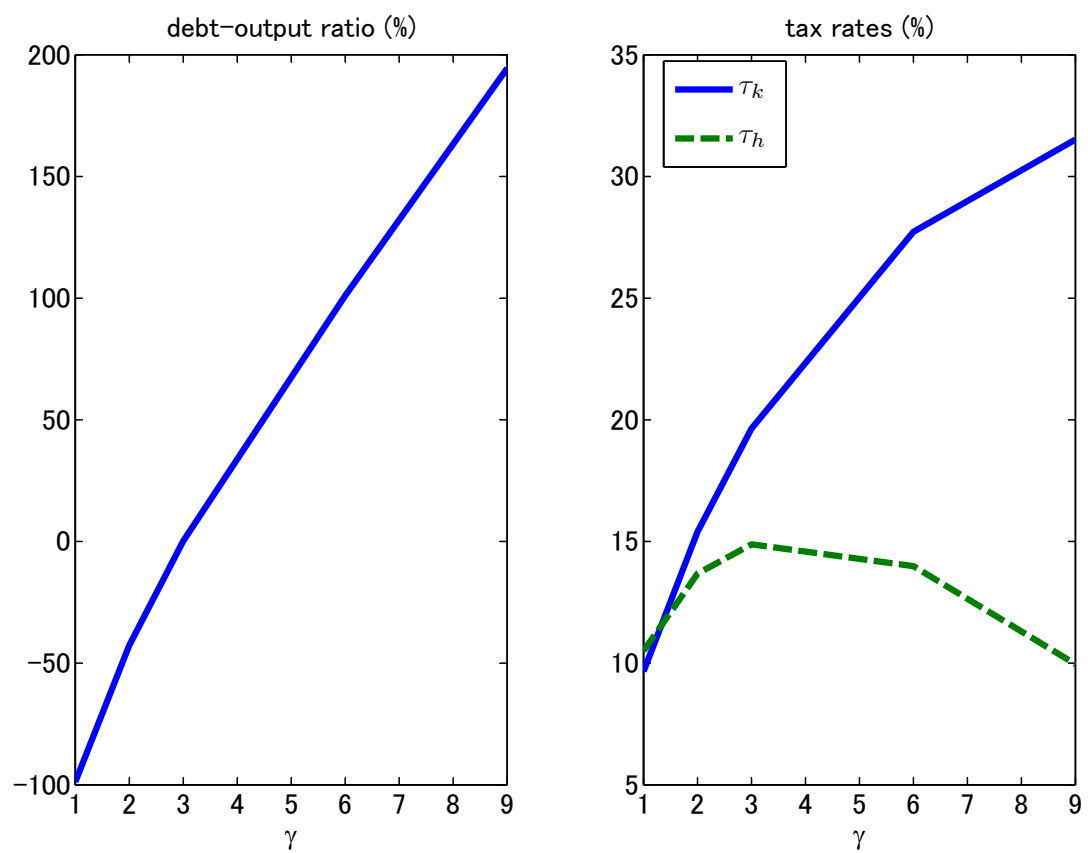


Figure 2: Different values of risk aversion.

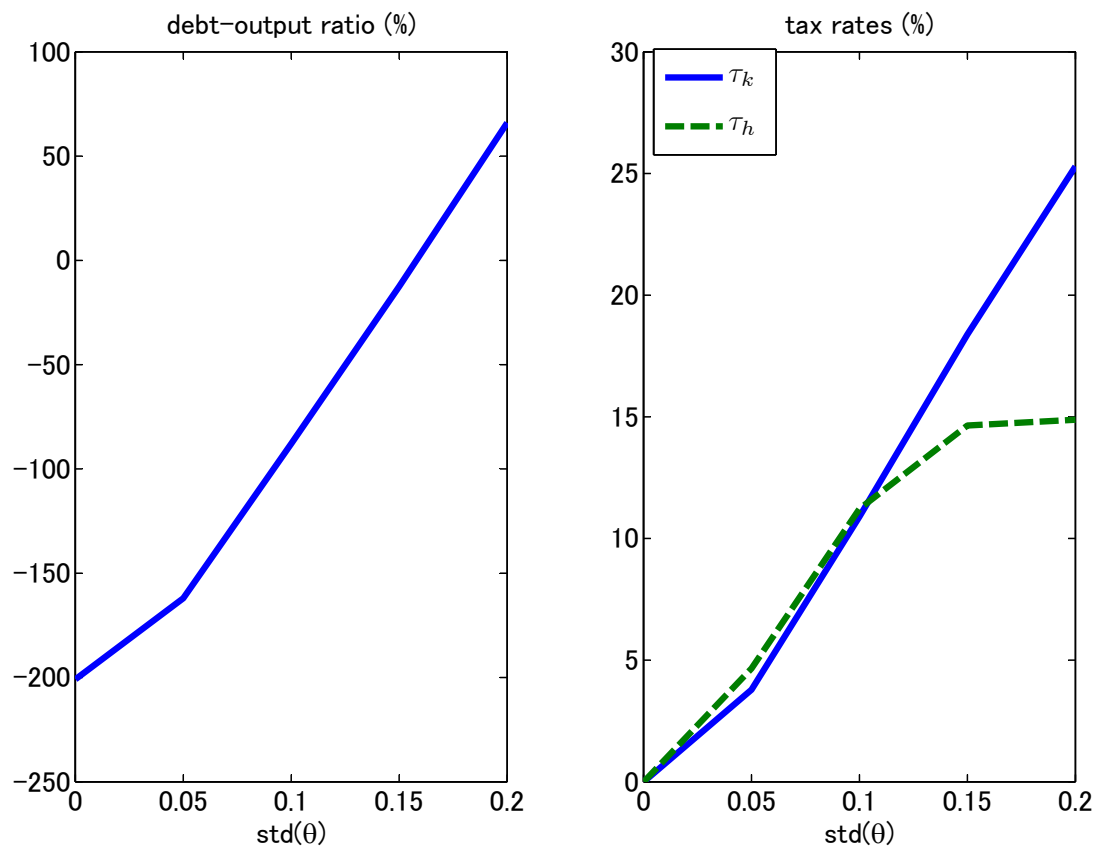


Figure 3: Different values of the idiosyncratic risk.

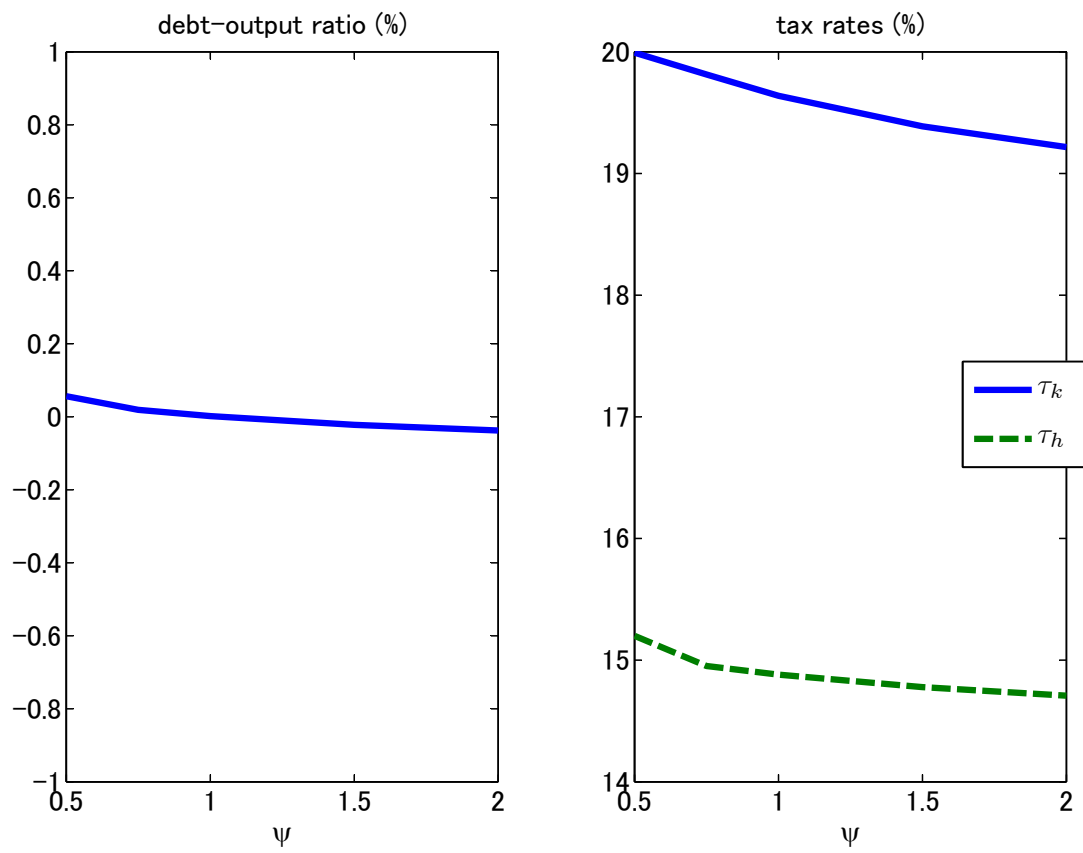


Figure 4: Different values of the intertemporal elasticity of substitution.