

War Signals: A Theory of Trade, Trust and Conflict*

Dominic Rohner[†], Mathias Thoenig[‡], Fabrizio Zilibotti[§]

February 9, 2011

Abstract

We construct a dynamic theory of civil war and social conflict hinging on cross-community beliefs (*trust*) and business links (*trade*). The model economy is populated by two groups that can engage in mutually beneficial trade. Trade necessitates specific investments featuring strategic complementarities and thick-market externalities. One group does not know the average propensity to trade of the other group. Since conflict disrupts trade, the onset of a conflict signals that the aggressor has a low propensity to trade. Agents observe the history of warfare and update their beliefs over time, and transmit them to the next generation. Low trust reduces investments on both sides, thereby increasing the probability of future wars. Along the equilibrium path, war is a stochastic process whose frequency depends on the state of endogenous beliefs.

The theory bears some testable predictions. First, the probability of future civil wars increases after each conflict episode. Second, a sequence of "accidental" conflicts can lead to the permanent breakdown of trust, plunging a society into a state of recurrent conflicts (a *war trap*). This situation is irreversible and is characterized by weak cross-community trade links even in peace times. War traps are robust to additional sources of social learning, such as people learning from the direct observation of the history of cross-community trade.

The incidence of conflict can be reduced by policies abating cultural barriers, fostering human capital and targeting beliefs. Coercitive peace policies such as peacekeeping forces or externally imposed regime changes have instead no persistent effects.

JEL classification: D74, D83, O15, Q34.

Keywords: beliefs, civil war, conflict, cultural transmission, ethnic fractionalization, human capital investments, learning, matching, peacekeeping, stochastic war, strategic complementarity, trade.

1 Introduction

Over 16 million people are estimated to have died due to civil conflicts in the second half of the 20th century (cf. Fearon and Laitin, 2003). Such conflicts are geographically highly concentrated.

*We thank Simon Alder for valuable research assistance. We are grateful for comments to Daron Acemoglu, Philippe Aghion, Alberto Alesina, Roland Benabou, Julien Hugonnier, Gerard Pedro-i-Miguel, Maria Saez-Marti, Pierre Yared, as well as to seminar and conference participants at the CEPR conference on Political Economy of Development, NBER Summer Institute, University of Oslo, University of Lausanne and University of Zurich. Dominic Rohner acknowledges financial support from the Swiss National Science Foundation (grant no. 100014-122636). Fabrizio Zilibotti acknowledges financial support from the ERC Advanced Grant IPCDP-229883.

[†]Department of Economics, University of Zurich. Email: drohner@iew.uzh.ch.

[‡]Department of Economics, University of Lausanne. Email: mathias.thoenig@unil.ch.

[§]Department of Economics, University of Zurich. Email: zilibott@iew.uzh.ch.

For instance, as many as 68 percent of all outbreaks in this same time period took place in countries where multiple conflicts were recorded.¹ This persistence has sprung a large body of research aiming to identify which institutional failures make some countries prone to conflict. Yet, weak institutions are unlikely to be the whole story. For instance, various studies show that democracy has no systematic effect on the risk of civil war after controlling for ethnicity and GDP per capita.² Moreover, several developing countries with relatively solid institutions plunge into recurrent conflicts, whereas other countries with weak institutions and high ethnic fractionalization never experience civil conflicts. In this paper, we propose a theory based on asymmetric information and social learning, arguing that inter-community distrust and pessimism about the viability of peaceful trade can make societies fall into vicious spirals of violence and civil conflicts. This can occur in spite of otherwise good economic fundamentals.³

Our theory has two building blocks. The first is a relationship between trade and civil conflict. Since conflict disrupts cross-community business relationships (hereafter, *trade*), the expected gains from trade are the opportunity cost of staging war. Thus, good trade opportunities deter war. Conversely, when such opportunities are scant, conflict is a likely outcome. The second is a relationship between trust and specific human capital investments that enable trade. Many bilateral business relationships involving members of different communities (e.g., seller-buyer, employer-employee, supplier-producer, lender-borrower) require specific investments on both sides. How much each community is prepared to invest depends then on the belief about the propensity of the other communities to invest (hereafter, *trust*). Therefore, trade relies on trust.

The two building blocks together imply a negative correlation between trust and war. Causality runs both ways: On the one hand distrust between communities or ethnic groups reduces trade, thereby increasing the probability of civil conflict. On the other hand, war erodes cross-community trust. Consistent with this prediction, Figure 1 shows that an average country-level measure of trust is negatively correlated with the frequency of civil wars during the period 1981-2008.⁴ This correlation is robust to control for democracy and other covariates.⁵

The link between trust, specific investments and business relationships is related to a large body

¹This number is based on the sample covered in Collier and Hoeffler (2004). See, among others, Collier and Hoeffler (2004), Collier, Hoeffler and Rohner (2009), Quinn, Mason and Gurses (2007), and Walter (2004), who have found that past wars are strong predictors of future wars.

²See, e.g., Fearon and Laitin (2003), Montalvo and Reynal-Querol (2005), and Collier and Rohner (2008)

³Columbia, India, Turkey, Sri Lanka and the Philippines fare relatively well in terms of democracy and other institutional indicators, conditional on their stage of development. Yet, they are prone to civil conflicts. Interestingly, the average level of trust as measured by the World Value Survey is significantly lower in these countries than in the average non-OECD country (0.16 vs. 0.22). On the opposite front, Bhutan, Cameroon, Gabon, Kazakhstan, Togo, China and Vietnam have low scores on democracy and high ethnic fractionalization, but no recent history of civil war. Data on trust are only available for China and Vietnam among these countries. The average trust is 0.51, even larger than in the average OECD country.

⁴Trust is a dummy variable and takes a value of 1 if "yes" is replied to question A165 ("can most people be trusted?") of the World Values Survey (2010). The civil war data is from PRIO (2010).

⁵In Rohner, Thoenig and Zilibotti (2010) we find that trust is especially affected by experiencing war during childhood. To the extent to which earlier age is especially "formative" in terms of beliefs and values, this is broadly consistent with the view that war erodes trust.

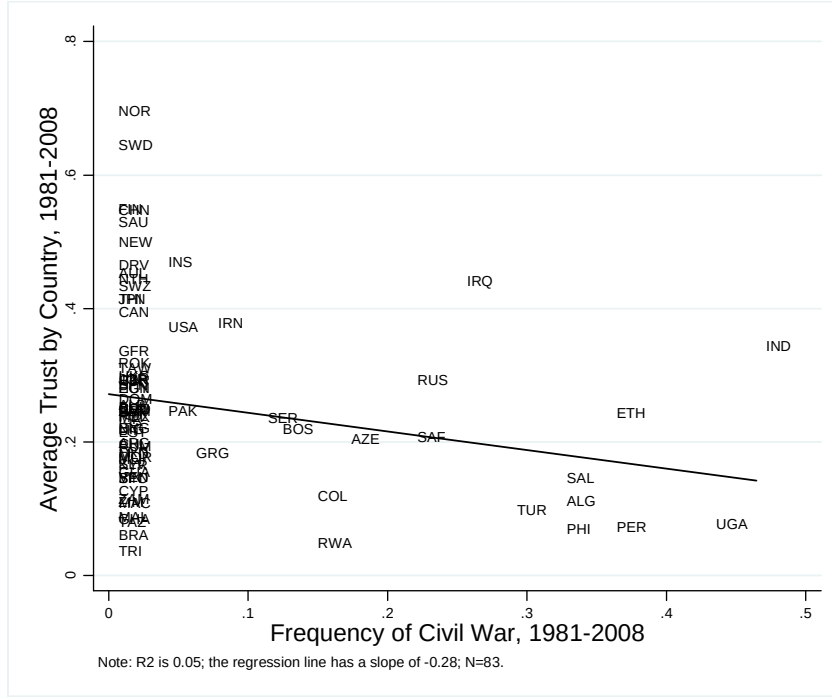


Figure 1: Country-Level Trust and Frequency of Civil War

of literature on contractual incompleteness where successful economic relationships hinge on various forms of bilateral investments. The salience of this issue in the context of cross-community trade is emphasized by Dixit (2003). In Hauk and Saez Marti (2002) and Tabellini (2009) the investment leads to the adoption of pro-social norms preventing opportunistic behavior; in Greif (1994), and Rauch (1999) it leads to the development of a social network where reputation and retaliation can be enforced; finally, in Dewatripont and Tirole (2005) it leads to the acquisition of communication tools, such as the other group's customs and language.⁶ Felbermayr and Toubal (2010) and Guiso, Sapienza and Zingales (2009) provide evidence that at the country-pair level international trade is correlated positively with bilateral trust, suggesting a causal link from trust to trade. Systematic within-country direct evidence of the same nature is less easily available since it is difficult to measure within-country trade.

The war-deterring effect of trade is documented empirically by Martin, Mayer and Thoenig (2008). While most of their evidence is about international trade, a number of case studies document that inter-ethnic trade has a similar effect within countries. For instance, Jha (2008) studies Hindu-Muslim

⁶The high importance and market value of such tools is witnessed by the large-scale advertising campaign launched by HSBC in 2002 branded "Never underestimate the importance of local knowledge". In this campaign HSBC highlights its unique ability to operate as a "truly local organization in each of the markets that it serves". The series of ads emphasize the key role for inter-cultural business relationships of a good knowledge of the system of customs, norms and social conventions.

interactions using town-level data for India. He finds that during Medieval times in India's trade ports Hindus and Muslims could provide each other with complementary services, and argues that this led to religious tolerance and a lower level of political violence in Medieval trade ports than in other Indian towns. Interestingly, such situation persists today. The trust-trade-war-trust circle appear to have been important in the 1994 conflict of Rwanda. Throughout the 1980s inter-ethnic trust was high and sustained symbiotic business relationships, cooperation in agricultural production associations and mixed rotating savings groups involving both Hutus and Tutsis (Ingelaere, 2007; Pinchotti and Verwimp, 2007). Survey data indicate that trust plunged as of October 1990, after localized fighting erupted in northern Rwanda between the Rwandan Patriotic Front (RPF), a rebel group formed from Tutsi refugees in Uganda, and the Hutu-dominated government of Habyarimana (Ingelaere, 2007). The collapse of trust was followed by waning trade and business links between the communities, until inter-ethnic cooperation ceased altogether at the outset of the 1994 genocide.⁷ Even many years after the conflict the average inter-ethnic trust levels are significantly lower than in the 1980's (Ingelaere, 2007) and also inter-ethnic trade is persistently lower (Colletta and Cullen, 2000). Similar feedback effects between trust and conflict have for example been observed for Cambodia, Guatemala, and Somalia (Colletta and Cullen, 2000).

We formalize our ideas with the aid of a dynamic model in which the economy is populated by two groups that can engage in mutually beneficial trade relations. Agents wishing to trade with the other group must undertake a human-capital investment. There are strategic complementarities and thick-market externalities across groups: the proportion of investors in a group increases the expected return to investments in the other group by increasing the probability for an investor to find a trading partner. Investment costs are heterogenous both within and across groups. The key parameter is a group-specific fixed effect, which pins down the average investment cost (*propensity to trade*), and about which information is asymmetric: one group ignores the average propensity to trade of the other group. The belief about such propensity is our measure of trust. In this environment, a group with a high propensity to trade has a high opportunity cost in staging war. Thus, the onset of a conflict signals that the aggressor has a low propensity to trade, or is little trustworthy. Agents update their beliefs over time, and trust is transmitted between generations. Low trust reduces investments on both sides thereby decreasing trade and the opportunity cost of future wars.

Our theory bears a number of testable predictions. First, war is a stochastic process whose realization reduces trust and inter-ethnic trade. This is consistent with the empirical correlations discussed above. Second, after each civil war episode, the probability that a country falls again into a civil war in future goes up. This is consistent with the empirical evidence that peace duration reduces significantly the risk of future civil war, even after controlling for country fixed effects (Martin, Mayer and Thoenig 2008b). Third, "war accidents", e.g., an aggression initiated by a belligerent minority of

⁷Colletta and Cullen, (2000:45) find that while vertical (within-group) social capital remained intact, "conflict deeply penetrated such forms of horizontal social capital as exchange, mutual assistance, collective action, trust and the protection of the vulnerable. [...] The use of credit in exchanges was common in preconflict Rwanda. This practice has diminished over time, in part due to decreased levels of trust as a consequence of warfare".

a group against the will of the majority of the group itself, may lead to the permanent breakdown of peaceful relationships across groups. More precisely, we show that repeated such episodes can make a society plunge into a state of recurrent conflicts (a *war trap*) where inter-ethnic trade relationships are weak even in peace times. Interestingly, such war traps arise from the information asymmetry and cultural transmission of beliefs, and may occur even in societies where peace and trade would characterize equilibrium in a full-information environment.

War traps are not the sole possible long-run outcomes. A luckier sequence of realizations of peace episodes can drive the economy into a steady state where low conflict and thriving trade are common fare. In such steady state, even occasional conflicts do not destroy trust. The theory allows us to characterize the probability distribution of different long-run scenarios as function of parameters and initial conditions.

The benchmark model relies on a strong restriction on the information set of agents: agents only learn the propensity of the other group through the observation of the peace-and-war history. This simplification aids tractability. However, in an extension we relax the informational assumptions and allow traders to acquire direct information about the other group's type. This information is transmitted to future generations within each family, but is subject to decay over time. We show that learning traps are robust to such environment, although the possibility for families to acquire knowledge through trade reduces the region of the parameter space such that traps arise.

Finally, we discuss policy implications of our analysis and their relationship with the empirical literature on conflict. Our theory has three main implications. First, increasing the returns from inter-ethnic trade reduces the scope for recurrent wars. Thus, policies abating barriers, e.g., educational policies promoting the knowledge of several national languages, as well as subsidies to human-capital accumulation (especially if focused on aspects of human capital that facilitate inter-ethnic trade) can reduce conflicts. Second, similar to other papers (see, e.g., Torvik, 2002, Acemoglu *et al.*, 2010) our theory indicates that the availability of windfall gains from war (e.g., natural resources that are easy to expropriate and exploit without relying on inter-ethnic cooperation) fuel war recurrence. International measures such as the boycott of regimes taking control of resources through ethnic violence can reduce the return to war. Third, and perhaps most interesting, our results emphasize the importance of policies targeting beliefs. For instance, credible campaigns documenting and publicizing success stories of inter-ethnic business relationships, joint ventures, etc. can shift beliefs in a desirable direction and reduce the probability of future conflicts. To the opposite, policies trying to *impose* peace through coercion – e.g., peacekeeping forces or externally-imposed regime changes – have ultimately no persistent effects. This is consistent with empirical studies in the conflict literature, including Luttwak (1999) and Sambanis (2008) that we discuss in more detail below.

Our paper relates to a number of different streams of economic literature. Our learning traps are related to the literature on herding, social learning, and informational cascades. This includes Banerjee (1992); Bikhchandani, Hirshleifer and Welch (1992); Ely and Valimaki (2003), Fernandez (2007) and Piketty (1995). The theory is also related to the theoretical literature on supermodular games

with strategic complementarities (Baliga and Sjostrom, 2004; Chamley, 1999; Chassang and Padro i Miquel, 2008 and Cooper and John, 1988). While most of these papers emphasize the possibility of static multiplicity, in our paper we constrain parameters to yield a unique equilibrium under perfect information.⁸ The dynamic nature of the model of conflict is related to Yared (2010). The importance of luck and persistent effects of negative shock link our contribution with Acemoglu and Zilibotti (1997). Also related to our research are the recent papers Aghion et al. (2010) and Aghion, Algan and Cahuc (2010) focusing on the relation between public policy, on the one hand, and beliefs and norms of cooperation in the labor market, on the other hand.⁹

The paper is also related more generally to the economic literature studying civil war and conflict. Some existing theories focus on institutional failures, such as weak state capacity and weak institutions (Besley and Persson, 2009, 2010; Fearon, 2005). In Besley and Persson (2009) the lack of checks and balances implies that rent-sharing strongly depends on who is in power, thereby strengthening incentives to fight. According to Collier and Hoeffler (2004) poverty plays a key role in conflict, as low income reduces the opportunity cost of fighting, while Esteban and Ray (2008) argue that ethnic polarization can favor the collective action needed for appropriation by generating the right mix of capital and labor for the groups, and Caselli and Coleman (2010) view ethnicity as a mechanism to enforce coalition membership.¹⁰ While explaining why some countries are more prone to conflicts than others, most such theories do not explain why a civil war today makes future conflict more likely. An exception is Acemoglu, Ticchi and Vindigni (2010) who argue that in weakly-institutionalized states civilian governments have incentives to select small and weak armies to prevent coups. This has the undesired consequence of making it harder for the state to end insurgency and rebellion. Collier and Hoeffler (2004) argue that current conflict makes conflict recurrence more likely due to the existence of conflict-specific capital, like cheap military equipment.

The plan of the paper is the following. Section 2 presents the benchmark model of inter-ethnic trade and conflict. Section 3 extends it to a dynamic environment where beliefs are transmitted across generation, and derives the main results. Section 4 presents a major extension where agents can learn from the observation of trade history together with warfare history. Section 5 discusses some policy implications. Section 6 concludes and discusses avenues for future research. The proofs of Lemmas, Propositions and Corollary are in the appendix, unless specified otherwise.

⁸ Among these papers, Chamley (1999) is the closest to us as he also studies coordination in a dynamic setting with learning and strategic complementarities. However, in his model the dynamics are driven by exogenous changes in the unobservable fundamentals and the possibility of persistence and absorbing states with learning traps is absent.

⁹ Aghion, Algan and Cahuc (2010) document a negative empirical correlation between the quality of labor relations and state regulation of the minimum wage. They explain this evidence with the aid of a model in which agents learn about the quality of labor relations, and where state regulation prevents workers from learning through experimentation. Their model features multiple equilibria: one characterized by good labor relations, and another characterized by low trust and strong minimum wage regulation.

¹⁰ Another stream of literature views civil wars as failure of bargaining processes due to private information (Fearon, 1995), commitment problems (Powell, 2006), issue indivisibilities or political bias of leaders (Jackson and Morelli, 2007).

2 The Static Model

2.1 Setup

The model economy is populated by a continuum of risk-neutral individuals who belong to two "ethnic groups" of unit mass, A and B . The interaction between the two groups are described by a two-stage game. First, group A decides whether to stage war against group B . Next, inter-ethnic trade may occur. No economically interesting decisions are made under the shadow of war. In case of peace, each member of the two groups can engage in bilateral inter-ethnic trade. Pre-requisite of trade is a human capital investment enabling agents to deal with the other ethnic group.¹¹ More precisely, after investments are sunk, each agent in group A is randomly matched with an agent in group B . Trade occurs only if both agents in a match have acquired human capital. In this case, each trading partner receives a return z , where we assume that $0 < z < 1$.

Investment decisions are based on a comparison between costs and benefits. Part of the return to human capital investments is the ability to trade with the other group. We define ι to be the difference between the investment cost and the part of the return that is unrelated to trade. Such net cost is heterogenous across agents, reflecting individual shocks to ability and investment opportunities. We assume ι to be *i.i.d.* across agents, and to be drawn from a probability density function (p.d.f.), $f^J : \mathbb{R} \rightarrow \mathbb{R}^+$, where $J \in \{A, B\}$. Note that the support of the p.d.f.'s may include negative values, implying that some agents invest in human capital even in the absence of inter-ethnic trade. We denote by $F^J : \mathbb{R} \rightarrow [0, 1]$ the corresponding cumulative distribution function (c.d.f.). Group A can be of two types: $f^A \in \{f^+, f^-\}$, and accordingly $F^A \in \{F^+, F^-\}$. We introduce two assumptions that are maintained throughout the rest of the paper.

Assumption 1 *There exists $\varepsilon > 0$ such that the p.d.f.'s $f^B(\iota)$, $f^+(\iota)$ and $f^-(\iota)$ are non-decreasing in the subrange $\iota \in [0, z + \varepsilon]$.*

Assumption 2 *The c.d.f. F^- first-order stochastically dominates the c.d.f. F^+ .*

Assumption 1 is introduced for technical reasons that will be explained later. Intuitively, it requires that, at least in the interval $[0, z + \varepsilon]$, there are fewer people with a low (or negative) than with a high net investment cost. Assumption 2 captures a fundamental feature of the model. Since investment costs are a barrier to trade, we say that group A has a high propensity to trade (A is of the *high* type) when $F^A = F^+$, and has a low propensity to trade (A is of the *low* type) when $F^A = F^-$. Instead, we assume that F^B has a unique realization. Such asymmetry is introduced for simplicity, in order to avoid to deal with a multidimensional learning process.

The collective decision of group A about staging war is taken by unanimity rule before individual members know the realization of their individual cost ι . The gains from trade that are foregone by

¹¹We interpret this investment as the familiarization with the customs of the other community, such as learning a foreign language, becoming aware of informal rules and traditions, getting in touch with external networks, etc.

staging war are denoted by $\hat{S}^k \in [S^{\min}, S^{\max}]$, where $k \in \{+, -\}$. This is the opportunity cost of war. The benefit of war is assumed to be a stochastic variable denoted by $\tilde{V} \in \{V_L, V, V_H\}$ whose realization is observed by group A before it takes its decision. $\tilde{V}$ is interpreted as the value of grabbing a resource over which group B has property rights, net of the military and psychological costs associated with war. We will maintain throughout the paper the following assumption.

Assumption 3 $V_L < S^{\min} < V < S^{\max} < V_H$.

The intermediate realization, V , is the most frequent one, and is referred to as *business as usual* (BAU). Under BAU, staging war is profitable if $V > \hat{S}^k$, and unprofitable otherwise. The high-benefit-of-war realization V_H corresponds to a situation in which the military cost of making war is exceptionally low, implying that the benefit of war exceeds its opportunity cost. The low-benefit-of-war realization V_L corresponds to a polar-opposite scenario in which such cost is exceptionally high, e.g., due to a failure to solve the collective action problem.¹² As $\hat{S}^k \geq S^{\min}$, peace necessarily occurs when $\tilde{V} = V_L$; likewise, as $\hat{S}^k \leq S^{\max}$, war necessarily occurs when $\tilde{V} = V_H$. We refer to the infrequent realizations V_H and V_L as a *war shock* and a *peace shock*, with probabilities $\lambda_W < 1/3$ and $\lambda_P < 1/3$, respectively. Hence, the probability of BAU is $1 - \lambda_W - \lambda_P > 1/3$. This stylized model is in accordance with the recent literature that views the onset of war as "stochastic" (Gartzke, 1999), due to stochastic shocks to coordination costs of rebellion (Collier and Hoeffler, 1998), or to rebel capability (Gates, 2002; Buhaug, Gates and Lujala, 2009).

2.2 Perfect information

To establish a benchmark, we first consider the case in which group A's type is public knowledge. In this case, war spoils trade but conveys no information. Consider the investment problem during peace. Due to random matching, the expected gain from trade for an investor in group A is $z \cdot n_B$, whereas the expected gain from trade for an investor in group B is $z \cdot n_A$. Thus, all agents with $\iota \leq zn_B$ (resp. $\iota \leq zn_A$) in group A (resp. group B) invest. The Nash equilibrium conditional on group A's type ($k \in \{A, B\}$) is given then by the fixed point

$$\{n_A^k, n_B^k\} = \{F^k(zn_B^k), F^B(zn_A^k)\} \quad (1)$$

¹²The stochastic process can alternatively be driven by shocks to the political process or psychological costs of conflict. When $\tilde{V} = V_H$, the perceived cost of staging war is low, due to an explosion of hatred (Gurr, 1970) or due to the capture of the political process by a biased political elite (Jackson and Morelli, 2007). To the opposite, a temporary political moderation or a high reluctance to start a conflict would lead to $\tilde{V} = V_L$.

Yet another interpretation is that there are shock to the beliefs of group A about the net benefits of war which are driven by the acquisition of private information. Let us assume that $\tilde{V}$ is drawn from a cumulative density function $H(\cdot)$ known by groups A and B such that $V \equiv E[\tilde{V}] = \int \tilde{V} dH(\tilde{V})$. With probability ζ the group A receives some binomial private signal $s \in \{s_W, s_P\}$, with a binomial parameter γ , and consequently updates its private beliefs. The signals s_W and s_P are, respectively, a good and a bad signal on $\tilde{V}$, since the posterior c.d.f. verifies the following first-order dominance criterion $V_H \equiv \int \tilde{V} dH(\tilde{V} | s = s_W) > V > V_L \equiv \int \tilde{V} dH(\tilde{V} | s = s_P)$. To sum up, the expected benefit of war is V_H with probability $\lambda_W = \zeta \times \gamma$, V_L with probability $\lambda_P = \zeta \times (1 - \gamma)$, and V with probability $(1 - \zeta)$. This alternative model emphasizes the role of private information in the process of war, arguably a key mechanism among the rational theories of war (Fearon, 1995).

The strategic complementarity in investments may lead to multiple Nash equilibria. Since static equilibrium multiplicity in games of strategic complementarities is well understood and is not the main focus of this paper, we restrict attention to p.d.f.'s that yield a unique Nash equilibrium for each k . Assumption 1 is sufficient (though not necessary) to ensure that the Nash equilibrium is unique under perfect information. This restriction allows us to focus more sharply on the dynamic interaction between belief formation and warfare.

The trade surplus accruing to group A is given by the product between the measure of successful trade relationships ($n_A^k \cdot n_B^k$) and the return to trade (n) minus the aggregate investment cost. Since the optimality of the investment decisions of group A implies that $n_A^k = F^k(zn_B^k)$ and that the threshold cost is $\iota = zn_B^k$, the trade surplus can be expressed as a function of the proportion of investors in group B:

$$\hat{S}^k(n_B^k) = z \cdot F^k(zn_B^k) \cdot n_B^k - \int^{zn_B^k} \iota dF^k(\iota) \quad (2)$$

Note that the previous equation implies that necessarily $\hat{S}^k \in [S^{\min}, S^{\max}]$ with $S^{\min} = -\int^z \iota dF^+(\iota)$ and $S^{\max} = z$.

Proposition 1 *Under Assumptions 1-2 and perfect information, the Nash Equilibrium of the investment/trade continuation game conditional on $k \in \{+, -\}$ exists and is unique.*

The equilibrium investments are given by $\{n_A^-, n_A^+, n_B^-, n_B^+\}$ consistent with equation (1), where $n_A^- \leq n_A^+$ and $n_B^- \leq n_B^+$.

The equilibrium trade surplus accruing to group A is given by $\hat{S}^k(n_B^k)$ – as described by (2) – evaluated at the equilibrium value of n_B^k . Moreover, $\hat{S}^- \leq \hat{S}^+$.

Moving backwards to the war decision, three cases are possible. If either $V < \hat{S}^- < \hat{S}^+$ or $\hat{S}^- < \hat{S}^+ < V$, group A's type has no effect on the probability of war. The most interesting case is when $\hat{S}^- < V < \hat{S}^+$. In this case, the low type stages war while the high type retains peace under BAU. Under this condition, war is more frequent when $k = -$ (probability is $1 - \lambda_P$) than when $k = +$ (probability is λ_W).

2.3 Asymmetric Information

In the rest of the paper we assume that group B can observe neither group A's type nor the realization of $\tilde{V}$.¹³ Under these assumptions staging war signals a low propensity to trade, although the signal is not perfectly revealing. For instance, if $S^- < V < S^+$ and war is staged, group B cannot be sure that A is of the low type, since war may have erupted due to a war shock.

We denote by π_{-1} the common prior belief held by agents in group B that $k = +$. Beliefs are common knowledge. After observing war or peace, group B updates its beliefs using Bayes' rule. We

¹³The assumption that there is no asymmetric information about group B's type is for tractability, as two-sided learning would complicate the analysis. Note that $\tilde{V}$ is neither observable ex-ante nor verifiable ex-post to group B. Otherwise, the process of belief updating would be more complicated.

denote by (π_W, π_P) the posterior probability that group A is of the high type conditional on war and peace, respectively.

The timing of the game is the following.

1. The *war stage*: all agents in group B receive the prior belief π_{-1} , all agents in group A observe the state $\tilde{V}$, and group A decides whether to stage war or keep peace.
2. The *investment/trade stage*: agents in group B update their beliefs. If there is war, there are no further choices and all agents receive their payoffs. If there is peace, all agents in both groups draw ι from the distribution of net costs, and each of them decides in a decentralized way whether to invest. Finally, the two groups are randomly matched to trade, gains from trade are realized, and consumption occurs.

The equilibrium concept is Perfect Bayesian Equilibria (PBE).

Definition 1 *A strategy for an agent in population A specifies for each of her possible types, $k \in \{+, -\}$ and for each state $\tilde{V} \in \{V_L, V, V_H\}$, a war action ("stage war" or "keep peace"), and, for each possible realization of the investment cost, ι , an investment action ("invest" or "not invest"). A strategy for an agent in population B specifies an "investment action" ("invest" or "not invest") for each of the possible realizations of the investment cost, ι . A PBE is a strategy profile, a belief system and a triplet $(n_A^-, n_A^+, n_B) \in [0, 1]^3$ such that: (i) in the investment/trade continuation game all agents choose their investment so as to maximize the expected pay-off given the posterior beliefs after peace (π_P) and the realization of the net investment cost (ι) ; (n_A^-, n_A^+, n_B) yields the associated measure of agents who optimally invest in group A for each type, $k \in \{+, -\}$, and for group B, respectively. (ii) all agents in group A choose unanimously the probability of staging war on group B so as to maximize their expected utility, given group A's type (k), the state ($\tilde{V}$) and beliefs (π_{-1}) , (iii) beliefs are updated using Bayes' rule.*

2.3.1 Investment/Trade Continuation Game

We solve the PBE backwards, starting from the Nash equilibrium of the investment/trade continuation game under peace. Since the investments of agents in group A are subject to no uncertainty, group A's reaction function continues to be given by $F^k(zn_B)$, with $k \in \{+, -\}$. However, since n_A depends on the unknown type, group B faces some uncertainty, and its reaction function becomes $F^B(zE_B(n_A | \pi_P)) = F^B(z[\pi_P n_A^+ + (1 - \pi_P) n_A^-])$. The equilibrium proportions of investors in the two groups satisfy the following fixed point:

$$\{n_A^-, n_A^+, n_B\} = \{F^-(zn_B), F^+(zn_B), F^B(z[\pi_P F^+(zn_B) + (1 - \pi_P) F^-(zn_B)])\} \quad (3)$$

Proposition 2 characterizes the set of Nash equilibria of the investment/trade game.

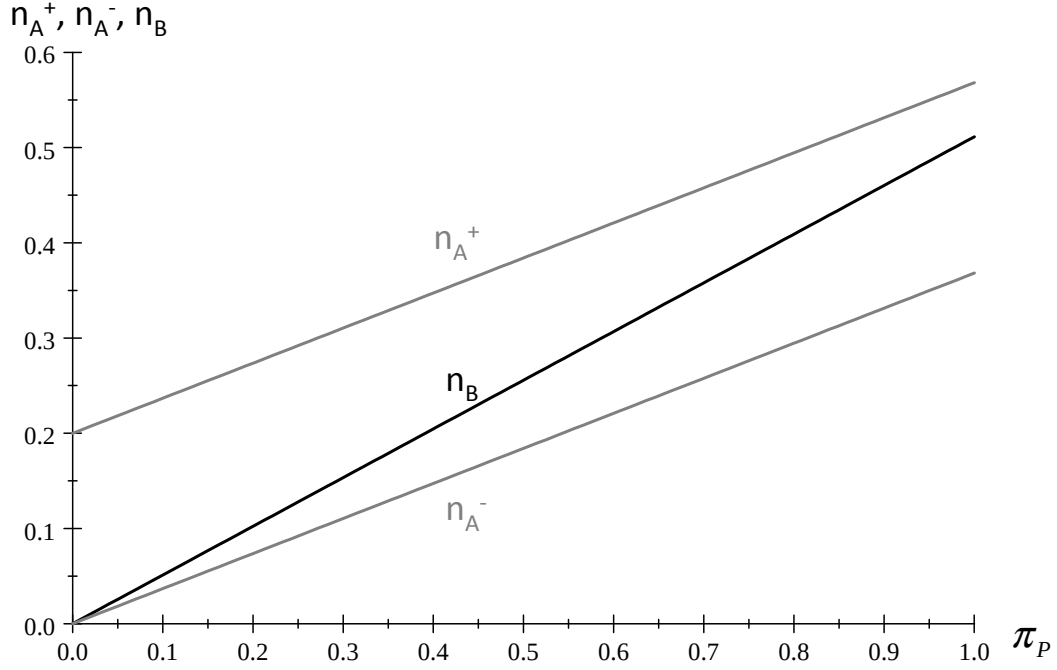


Figure 2: Investment as function of beliefs

Proposition 2 *Given a posterior belief $\pi_P \in (0, 1)$, the Nash Equilibrium of the investment/trade continuation game conditional on $k \in \{+, -\}$ exists and is unique.*

The equilibrium investments are given by $\{n_A^-(\pi_P), n_A^+(\pi_P), n_B(\pi_P)\}$ implicitly defined by equation (3). $n_A^-(\pi_P)$, $n_A^+(\pi_P)$ and $n_B(\pi_P)$ are continuous and weakly increasing. Moreover, $n_A^-(\pi_P) \leq n_A^+(\pi_P)$.

The equilibrium trade surplus accruing to group A, $\hat{S}^k(n_B(\pi_P))$, is given by (2) and is weakly increasing in π_P . Moreover, $\hat{S}^-(n_B(\pi_P)) \leq \hat{S}^+(n_B(\pi_P))$.

Note that trust affects the investments of both groups, due to the strategic complementarity. Pessimistic beliefs (i.e., low π_P), induce agents in group B to expect that only few agents in group A will invest, determining a low n_B . In turn, a low n_B reduces the proportion of investors in group A, whatever its true type $k \in \{-, +\}$. As a result both $\hat{S}^+$ and $\hat{S}^-$ are increasing in trust. Figure 2 plots the equilibrium relationships $n_B(\pi_P)$ and $(n_A^-(\pi_P), n_A^+(\pi_P))$ in the case of a uniform distribution of investment costs.¹⁴

¹⁴In particular, we set $z = 0.9$ and assume a uniform distributions of investment costs on the following supports: $F^B \sim [0, 1]$, $F^+ \sim [-0.25, 1]$, $F^- \sim [0, 1.25]$.

2.3.2 War Decision and PBE

In this section, we analyze the decision of group A of whether to stage war. As discussed above, such decision is based on a comparison between the opportunity cost of war, given by (2), and the stochastic realization of its benefit, $\tilde{V}$. Since $\hat{S}^+$ and $\hat{S}^-$ depend on posterior beliefs, we must first characterize the belief updating process. To this aim it is useful to rescale beliefs in term of likelihood ratio and to introduce some new notation.

Notation 1 (i) $r_W(r_{-1})$ and $r_P(r_{-1})$ denote the mapping from prior to posterior likelihood ratios conditional on war and peace, respectively, where $r_{-1} \equiv \pi_{-1}/(1 - \pi_{-1})$ and $r_s \equiv \pi_s/(1 - \pi_s)$ for $s \in \{W, P\}$.

(ii) $\sigma^+(r_{-1})$ and $\sigma^-(r_{-1})$ denote the probability that peace is maintained under BAU by the high and low type respectively.

(iii) $S^k(\pi_P) = \hat{S}^k(n_B(\pi_P))$.

Proposition 2 and all ensuing results in the previous section can be expressed in terms of this new notation by replacing π_P by $r_P/(1 + r_P)$ in each expression. Bayes' rule implies that¹⁵

$$\ln r_P(r_{-1}) = \ln r_{-1} + \ln \frac{\lambda_P + (1 - \lambda_W - \lambda_P) \sigma^+(r_{-1})}{\lambda_P + (1 - \lambda_W - \lambda_P) \sigma^-(r_{-1})}, \quad (4)$$

$$\ln r_W(r_{-1}) = \ln r_{-1} - \ln \frac{1 - \lambda_P - (1 - \lambda_W - \lambda_P) \sigma^-(r_{-1})}{1 - \lambda_P - (1 - \lambda_W - \lambda_P) \sigma^+(r_{-1})}, \quad (5)$$

where $\sigma^k(r_{-1})$ is the key choice variable:¹⁶

$$\sigma^k(r_{-1}) = \begin{cases} 0 & \text{if } S^k\left(\frac{r_P(r_{-1})}{1+r_P(r_{-1})}\right) < V \\ \in [0, 1] & \text{if } S^k\left(\frac{r_P(r_{-1})}{1+r_P(r_{-1})}\right) = V \\ 1 & \text{if } S^k\left(\frac{r_P(r_{-1})}{1+r_P(r_{-1})}\right) > V \end{cases}. \quad (6)$$

Intuitively, peace (war) is chosen with probability one under BAU whenever $S^k > V$ ($S^k < V$). If $S^k = V$ agents are indifferent, and the Nash equilibrium may involve mixed strategies. The existence of the PBE follows immediately from Proposition 2 (proof in the text).

Proposition 3 *A PBE exists and is fully characterized by the set of equations (2), (3), (4), (5), (6), given a prior belief π_{-1} and the definitions in Notation 1.*

¹⁵ After peace, the posterior is given by $\ln r_P = \ln r_{-1} + \ln \Lambda_P^+ / \Lambda_P^-$ where Λ_P^k represents the probability of observing peace if the true type is $k \in \{+, -\}$. Peace signal is observed with certainty under a peace shock (an event of probability λ_P) or with probability $\sigma^k(r_{-1})$ under BAU (an event of probability $1 - \lambda_P - \lambda_W$). Hence, $\Lambda_P^k \equiv \lambda_P + (1 - \lambda_W - \lambda_P) \sigma^k(r_{-1})$. Similarly, the after-war posterior is $\ln r_W = \ln r_{-1} + \ln \Lambda_W^+ / \Lambda_W^-$ where $\Lambda_W^k \equiv \lambda_W + (1 - \lambda_W - \lambda_P) (1 - \sigma^k(r_{-1}))$.

¹⁶ For instance, if under BAU the high type finds it optimal to keep peace ($\sigma^+(r_{-1}) = 1$) while the low type finds it optimal to stage war ($\sigma^-(r_{-1}) = 0$), then $r_P = ((1 - \lambda_W) / \lambda_P) \cdot r_{-1}$, where the updating factor after peace is given by the probability of no war shock divided by the probability of a peace shock. Conversely, $r_W = (\lambda_W / (1 - \lambda_P)) \cdot r_{-1}$, where the updating factor after war is given by the probability of a war shock divided by the probability of no peace shock.

To see how beliefs are updated along the equilibrium path, note that when either $V < S^-(\pi_P)$ or $V > S^+(\pi_P)$ the probability of war is independent of group A's type, as in the former case both types retain peace under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 1$) while in the latter case both types stage war under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 0$). Therefore, when either of the inequalities above holds, the occurrence of war or peace does not affect beliefs. On the contrary, war/peace is informative whenever $S^-(\pi_P) \leq V \leq S^+(\pi_P)$ – where one inequality is necessarily strict. In this case, the low type would stage war whereas the high type would preserve peace under BAU ($\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$). Thus, peace increases the trust of group B towards group A, while war decreases it. More formally, $S^-(\pi_P) \leq V \leq S^+(\pi_P) \Leftrightarrow \pi_P > \pi_{-1} > \pi_W$. We refer to this situation as an informative PBE.

Definition 2 *Given π_{-1} (and, hence, r_{-1}), a PBE is "informative" iff $\sigma^+(r_{-1}) > \sigma^-(r_P)$, or identically, $r_P(r_{-1}) > r_{-1} > r_W(r_{-1})$. A PBE is "uninformative" (or a "learning trap") iff $\sigma^+(r_{-1}) = \sigma^-(r_P)$, or identically $r_P(r_{-1}) = r_{-1} = r_W(r_{-1})$.*

Figure 3 plots the functions S^+ and S^- as functions of the after-peace beliefs for the particular example of Figure 2, given a particular value of the parameter V . In the case represented in the figure, war/peace is informative if and only if $\pi_P \geq \underline{\pi}_P$. In contrast, when $\pi_P < \underline{\pi}_P$, war/peace is uninformative. Note that S^+ and S^- are functions of the posterior π_P , which is endogenous. We now discuss how the equilibrium mapping from prior to posterior.

Notation 2 *Let*

$$\underline{r}^*(V) \equiv \begin{cases} \frac{(S^+)^{-1}(V)}{1-(S^+)^{-1}(V)} & \text{if } V \geq S^+(0) \\ 0 & \text{if } V < S^+(0) \end{cases}, \quad \underline{r}(V) \equiv \frac{\lambda_P}{1-\lambda_W} \underline{r}^*(V) \quad (7)$$

$$\bar{r}(V) \equiv \begin{cases} \frac{(S^-)^{-1}(V)}{1-(S^-)^{-1}(V)} & \text{if } V \leq S^-(1) \\ \infty & \text{if } V > S^-(1) \end{cases}, \quad \bar{r}^*(V) \equiv \frac{\lambda_P}{1-\lambda_W} \bar{r}(V), \quad (8)$$

where $\underline{r}^*(V) < \underline{r}(V) < \bar{r}(V) < \bar{r}^*(V)$.

Intuitively, $\underline{r}^*(V)$ is the threshold posterior belief such that both types stage war under BAU if $r_P \leq \underline{r}^*(V)$. As long as $r_{-1} \geq \frac{\lambda_P}{1-\lambda_W} \underline{r}^*(V)$, the posterior can be larger or equal to $\underline{r}^*(V)$. Likewise, $\bar{r}(V)$ is the threshold posterior belief such that both types retain peace under BAU if $r_P \geq \bar{r}(V)$. As long as $r_{-1} \geq \frac{\lambda_P}{1-\lambda_W} \bar{r}(V)$, the posterior can be larger or equal to $\bar{r}(V)$. Given these definitions, the following Lemma can be established.

Lemma 1 *An uninformative PBE exists if and only if either $r_{-1} \leq \underline{r}^*(V)$ or $r_{-1} \geq \bar{r}(V)$. Informative PBE exist if and only if $r_{-1} \in [\underline{r}(V), \bar{r}(V)]$. If $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$, then there are multiple PBE. Otherwise, the PBE is unique.*

Uninformative PBE are associated to either very pessimistic or very optimistic priors. Intuitively, when trust is very low (high), trade opportunities are scant (abundant) and both the high and the low

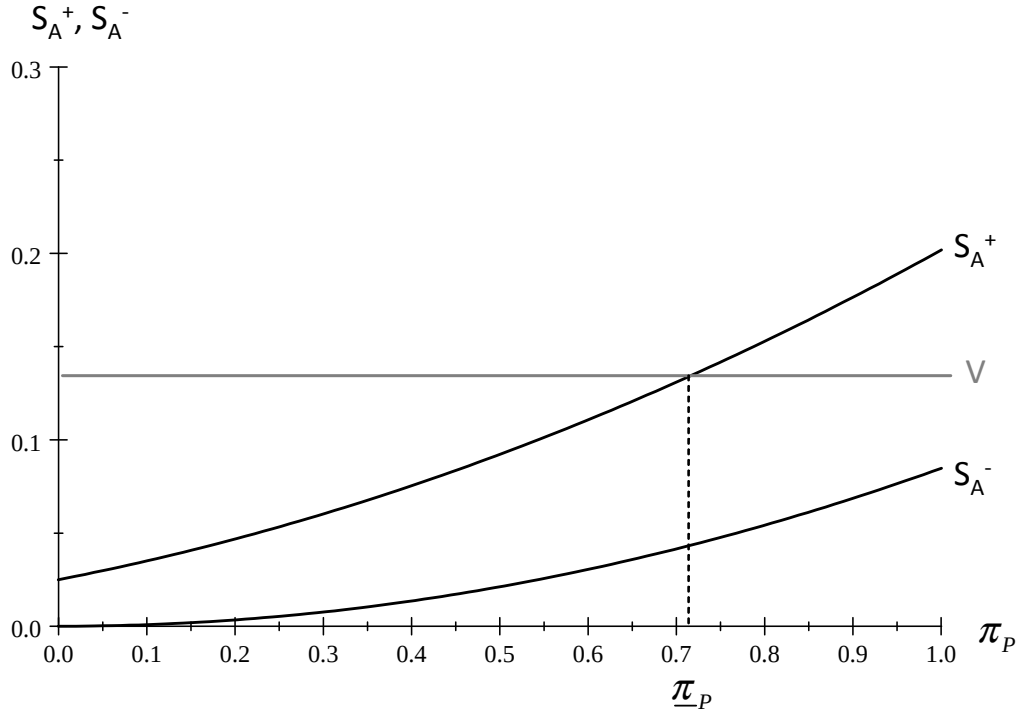


Figure 3: Surplus from trade and gains from war as function of the posterior belief

type stage war (keep peace) under BAU. Figure 4 provides an illustration of Lemma 1. Informative PBE arise in an intermediate range of beliefs (although, note, the range may be open to the right as in the case in Figure 3). Two ranges of priors have special properties: $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$ and $r_{-1} \in [\bar{r}(V), \bar{r}^*(V)]$. When $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$, the mapping from priors to posteriors yields multiple PBE, of which one is uninformative and two are informative. When $r_{-1} \in [\bar{r}(V), \bar{r}^*(V)]$, the mapping from priors to posteriors yields a unique PBE, but this involves randomization of the low type ($\sigma^-(r_{-1}) \in (0, 1)$).¹⁷

3 The Dynamic Model

In this section, we extend the analysis to a dynamic economy populated by overlapping generations of two-period lived agents. In the first period of their lives (*childhood*) agents make no economic choice, and receive the beliefs (which are common knowledge) from their parents' generation. In the second period (*adulthood*) agents make all economic decisions. Those in group A decide whether to stage war. Then, adult agents update their beliefs, make investment decisions and (if there is no war) trade, and transmit their updated beliefs to their children. The dynamics of beliefs are the driving force of the

¹⁷See the proof for formal details. The Appendix also provides an intuitive discussion of the set of PBE in these two ranges (see Figure ??).

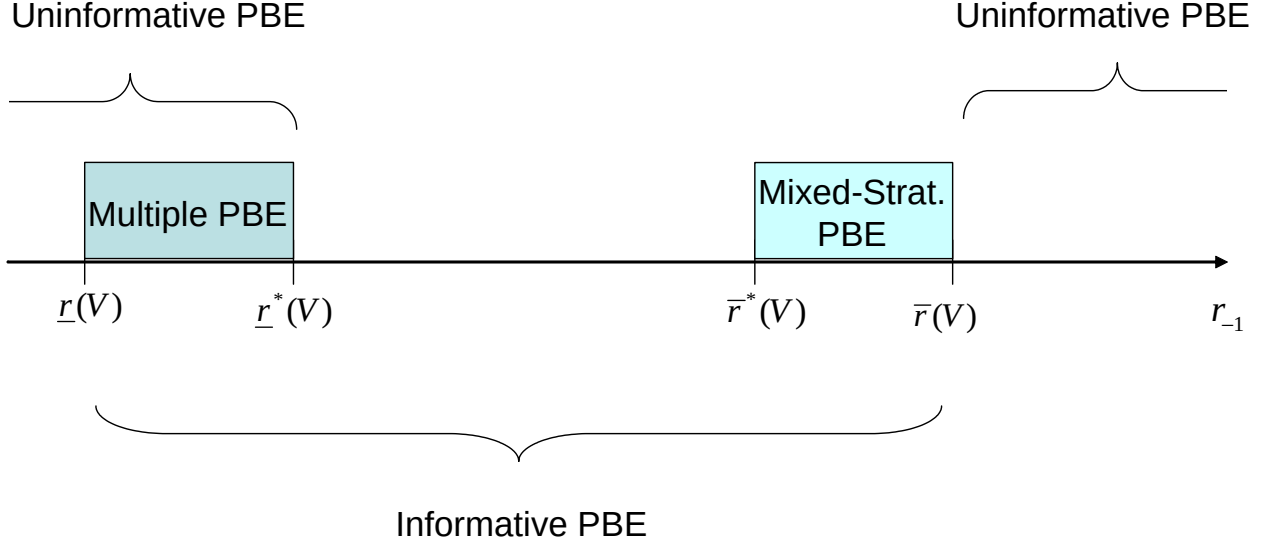


Figure 4: Uninformative and informative PBE

stochastic process of war/peace and trade.

Definition 3 *A Dynamic Stochastic Equilibrium (DSE) is a sequence of PBE with an associated sequence of beliefs such that, given an initial likelihood ratio r_0 the posterior likelihood ratio at t is the prior likelihood ratio at $t + 1$, for all $t \geq 0$.*

For the sake of the dynamic analysis, the multiplicity of PBE described in Lemma 1 is a source of uninteresting technical complications. While none of our results depends on a specific selection criterion, we make the following convenient assumption.

Assumption 4 *In the range of prior beliefs such that multiple PBE exists, the most informative equilibrium is selected.*

Since the rest of our analysis emphasizes the possibility for economies to fall into uninformative equilibria, this is a conservative selection criterion.

It is useful to distinguish between two cases. In the first case, the value of war is high ($V > S^-(1)$), and the DSE can converge to an uninformative PBE with pessimistic beliefs, but not to an uninformative PBE with optimistic beliefs. In the second case, the value of war is lower ($V \in [S^+(0), S^-(1)]$), and the DSE can converge with positive probability to both an uninformative PBE with pessimistic beliefs, and an uninformative PBE with optimistic beliefs.

3.1 High Value of War

The following proposition characterizes the dynamic equilibrium when the value of war is high (the proof follows from Lemma 1 and its proof).

Proposition 4 Assume $V > S^-(1)$ and the selection criterion of Assumption 4. Let $\underline{r}(V)$ be defined as in (7). The DSE is characterized as follows:

The PBE at time t is unique and given by Proposition 3, after setting $r_{-1} = r_{t-1}$. In particular, if $r_{t-1} < \underline{r}(V)$, then both types choose war under BAU ($\sigma^+(r_{t-1}) = \sigma^-(r_{t-1}) = 0$), and the PBE is uninformative. If $r_{t-1} \geq \underline{r}(V)$, then the low type chooses war while the high type chooses peace under BAU ($\sigma^+(r_{t-1}) = 1$ and $\sigma^-(r_{t-1}) = 0$), and the PBE is informative.

The equilibrium law of motion of beliefs is given by the following stochastic process:

$$\ln r_t = \begin{cases} \ln r_{t-1} & \text{if } r_{t-1} \in [0, \underline{r}(V)] \\ \ln r_{t-1} + (1 - I_{WAR,t}) \ln \left(\frac{1-\lambda_W}{\lambda_P} \right) - I_{WAR,t} \ln \left(\frac{1-\lambda_P}{\lambda_W} \right) & \text{if } r_{t-1} > \underline{r}(V) \end{cases} \quad (9)$$

where $I_{WAR} \in \{0, 1\}$ is an indicator function of war, with the following conditional probability

$$\Pr(I_{WAR,t} = 1 | r_{t-1}) = \begin{cases} 1 - \lambda_P & \text{if } r_{t-1} \in [0, \underline{r}(V)] \\ I^- \cdot (1 - \lambda_P) + (1 - I^-) \cdot \lambda_W & \text{if } r_{t-1} > \underline{r}(V) \end{cases}. \quad (10)$$

where $I^- \in \{0, 1\}$ is an indicator functions of $\{k = -\}$.

The stochastic process (9) is represented in Figure 5. Note that, conditional on r_{t-1} , the realizations of r_t are independent of k . However, the probability of peace and war do depend on k , as in equation (10).

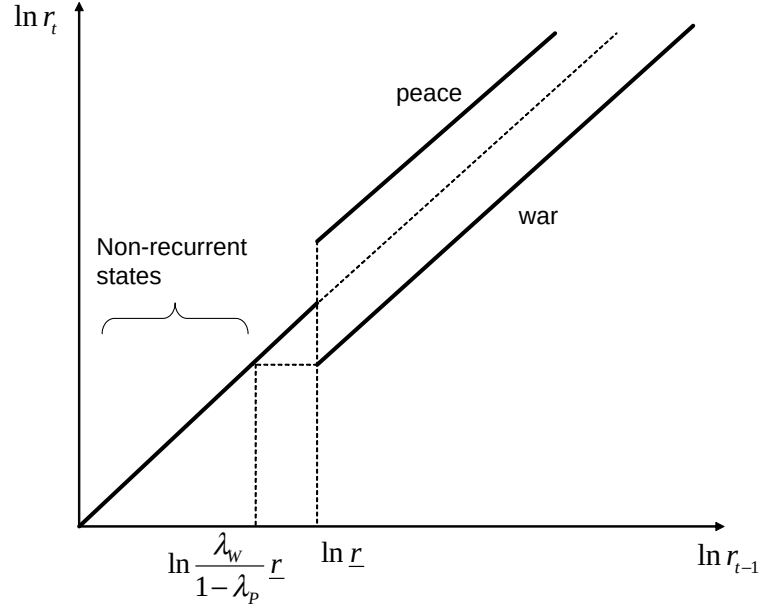


Figure 5: Dynamics of beliefs

Suppose, first, that the true state of nature is $k = -$. In this case, the probability of war is high for all levels of r_{t-1} – see equation (10). Interestingly, group B never learns for sure that A has a low propensity to trade, as learning comes to a halt as soon as r falls below the barrier $\underline{r}(V)$. To the opposite, a low-probability long sequence of peace episodes could make group B converge almost surely to the false belief that $k = +$. However, we will see that when $k = -$ the probability that such incorrect learning occurs is zero.

Consider, next, the case in which $k = +$. In this case, if the economy starts with $r_0 > \underline{r}(V)$, the probability of war is low. Yet, an unlucky sequence of war shocks can spoil trust inducing a fall in r . As the barrier $\underline{r}(V)$ is crossed, the probability of war jumps from λ_W to $1 - \lambda_P$. Moreover, agents rationally stops updating its belief which gets stuck to a low level. In particular, even a long sequence of peace episodes is viewed by group B as uninformative, since they must arise from peace shocks.

We introduce now a formal definition of a learning trap.¹⁸

Definition 4 *A war-dominated learning trap (WDLT) is a set of states, $\Omega_{WDLT} \subset R^+$, such that if $r_t \in \Omega_{WDLT}$ then $\forall s \geq t, r_s = r_t$, and the incidence of war is high, $\Pr(I_{WAR,s} = 1) = 1 - \lambda_P$, for all continuation paths $[r_s]_{s=t}^\infty$.*

It follows immediately from Proposition 4 that $\Omega_{WDLT} = [0, \underline{r}(V)]$.

Since, given any $r \notin \Omega_{WDLT}$ there exists a finite number of war episodes leading into Ω_{WDLT} , the economy falls into the WDLT with a positive probability. Does it mean that the DSE necessarily converge in probability to the WDLT? Even in the case in which group A is of the high type? Or can group B eventually learn that $k = +$ when this is the true state? The answer is not straightforward, as Figure 5 suggests. On the one hand, when $r > \underline{r}(V)$, peace is common fare, so there is a high probability that trust increases over time. Moreover, this process never comes to a halt, since there is no upper barrier and r_t can grow without bound. On the other hand, any war is informative as long as $r > \underline{r}(V)$. Thus, whatever level of trust has been achieved, a sufficiently long sequence of war shocks can destroy it and drive the economy into the WDLT. Thankfully, this need not be the case. We show below that while the equilibrium stochastic process can lead r_t to cross the barrier $\underline{r}(V)$, it can also alternatively bring r_t infinitely far from it ending up with an almost correct learning, $r_t \rightarrow +\infty$. A positive probability is associated with each of the two long-run scenarios.

From a technical standpoint, the stochastic process for $\ln r_t$ is an asymmetric random walk with a drift. Given an initial condition $\ln r_0 \geq \ln \underline{r}(V)$, and an unobserved true state of nature $k \in \{+, -\}$, a key step towards the characterization of the long-run distribution is the determination of the *first passage time* of the random walk below the barrier $\ln \underline{r}(V)$: $T = \min\{t; \ln r_t \leq \ln \underline{r}(V)\}$. If this stopping time T is finite, it means that learning stops in T and priors are trapped in the WDLT. If the stopping time T is infinite, then learning takes place at each period in time and one must study under which condition the process converges toward perfect learning. Applying existing results in stopping-time theory (see, e.g., Williams (1991), ch.10), we establish the following proposition.

¹⁸Some of the states in the WDLT are non-recurrent, namely, they cannot be reached unless they are chosen as initial conditions. Figure 5 shows the lower bound to the set of recurrent states, $\frac{\lambda_W}{1-\lambda_P} \underline{r}(V)$.

Proposition 5 Assume that $V > S^-(1)$ and let $r_0 \notin \Omega_{WDLT}$. Then:

- (i) If $k = -$, the DSE enters the WDLT in finite time almost surely: $\Pr\{\exists T < \infty \mid r_T \in \Omega_{WDLT}\} = 1$.
- (ii) If $k = +$, the DSE enters the WDLT in finite time with probability $\Pr\{\exists T < \infty \mid r_T \in \Omega_{WDLT}\} = P_{WDLT}(r) \in (0, 1)$. With probability $1 - P_{WDLT}(r)$, the DSE converges to perfect learning, i.e., $r_t \rightarrow \infty$, and to a low war incidence, $\Pr(I_{WAR,t} = 1) \rightarrow \lambda_W$. The probability $P_{WDLT}(r)$ has the following bounds:

$$0 < \frac{\lambda_W}{1 - \lambda_P} \frac{\underline{r}(V)}{r_0} < P_{WDLT}(r) \leq \frac{\underline{r}(V)}{r_0} < 1.$$

The intuition behind the Proposition 5 is the following: When the true state of nature is $k = -$ the stochastic process of beliefs cannot stay forever in the region of the informative equilibrium. If this were the case, the agents would observe an infinite number of realizations of the war/peace process. Then, by the strong law of large numbers, the empirical frequency of war/peace would converge to the underlying probabilities $(1 - \lambda_P, \lambda_P)$. Hence, agents would learn that the state of nature is $-$, namely $\ln r_t \rightarrow -\infty$. However, this would imply that at some finite T , r_T falls below $\underline{r}(V)$ and the economy enters the WDLT. When the true state of nature is $k = +$, a positive-probability set of finite sequences of wars drives r_t below $\underline{r}(V)$. In this case, group B stops learning and the economy is trapped in a WDLT. However, the probability of falling into a WDLT is less than unity. With the complement probability no such sequence is realized, and r never exits the region $[\underline{r}(V), \infty)$. In this case, group B observes an infinite number of realizations of the war/peace process, and the strong law of large numbers ensures then that the empirical frequency of war/peace converges to the underlying probabilities $(\lambda_W, 1 - \lambda_W)$. Thus, group B ultimately learns that the true state of nature is almost surely $k = +$.

For general parameter values, we can only provide bounds to the probability of falling into the WDLT. Remarkably, the expression of the bounds is very simple. Both the lower and upper bound decrease with the distance between the prior and the barrier: the larger the state of trust, the less likely it is that the barrier will ever be hit. Interestingly, the probability of ever falling into a WDLT decreases after a sequence of peace episodes. Thus, peace fosters trust and decreases the probability of falling into the war trap. Conversely, a few war incidents increase the risk of an irreversible crisis. The lower bound of $P_{WDLT}(r)$ also increases with $\lambda_W / (1 - \lambda_P)$. This is intuitive, as this ratio is inversely related to the informational value of the war/peace signal. If this ratio were unity, the two states of nature would be observationally equivalent and there would be no learning. More generally, the larger the ratio, the lower the learning speed. In terms of the deep parameters, a higher probability of war and peace shocks generates signal jamming, thereby increasing the lower bound probability for the economy to fall into a trap. Moreover, as it should be expected, since $\underline{r}(V)$ is non-decreasing in V , the probability for the economy to fall into a WDLT is non-decreasing in the value of war, V .

A direct consequence of Proposition 5 is that after a war the probability for an economy to enter a trap increases. $I_{WAR,t} = 1$ implies $r_{t+1}/r_t = \frac{\lambda_W}{1-\lambda_P} < 1$. Hence we get

$$1 < \frac{Pr \{ \exists T < \infty, r_T \in \Omega_{WDLT} \mid I_{WAR,t} = 1, r_t \}}{Pr \{ \exists T < \infty, r_T \in \Omega_{WDLT} \mid r_t \}} < \left(\frac{1 - \lambda_P}{\lambda_W} \right)^2$$

In the particular case in which $\lambda_W = \lambda_P = \lambda$, we can obtain an exact characterization of $P_{WDLT}(r)$:

Corollary 1 *Assume that $V > S^-(1)$, $\lambda_W = \lambda_P = \lambda < 1/3$ and let $r_0 \notin \Omega_{WDLT}$. Then, $P_{WDLT}(r) = \left(\frac{\lambda}{1-\lambda} \right)^{\Delta_0}$ where $P_{WDLT}(r)$ is defined as in Proposition 5 and $\Delta_0 \equiv \lceil \ln(r_0/\underline{r}(V)) / \ln((1-\lambda)/\lambda) \rceil$. Moreover, if T denotes the expected first passage time T into the trap, then $E(T \mid T < \infty) = \Delta_0 / (1 - 2\lambda)$.*

The term Δ_0 yields the count of the net number of wars (i.e., number of wars minus number of peace episodes) which are needed to drive the initial prior r_0 below $\underline{r}(V)$. The corollary is consistent with the general discussion of Proposition 5. In particular, $P_{WDLT}(r)$ increases with the noise term λ . Moreover, after a war the probability of entering into the trap increases by a factor $(1-\lambda)/\lambda > 1$.

3.2 Low Value of War

In this section we analyze the case in which the value of war is low, $V \in [S^+(0), S^-(1)]$.¹⁹ The main new implication of this case is that there are two learning traps, one with frequent and one with rare wars. A particular example is represented in Figure 6. In the range $\pi_P \leq \bar{\pi}_P$ the implications are qualitatively identical to those in Figure 3. However, in the range $\pi_P \geq \bar{\pi}_P$, the trade surplus are for both types larger than the value of war, $S^+(\pi_P) > S^-(\pi_P) > V$, such that even the low type chooses peace under BAU. In such range, the equilibrium is uninformative and peace prevails even if group A has a low propensity to trade.

As before, the process of revision of beliefs is characterized by equations (4)-(5) whereas the mapping of prior beliefs into equilibrium strategies is characterized by (6). There is however a range of priors in the neighborhood of the threshold $\bar{\pi}_P$ where the PBE has some noteworthy features. Different from the lower threshold $\underline{\pi}_P$, this region features no multiple PBE. However, for a range of priors $r \in [\bar{r}^*(V), \bar{r}(V)]$ the unique PBE has the low type indifferent between war and peace. In this case, group A chooses a mixed strategy in the war game under BAU, $\sigma^-(r_t) \in (0, 1)$. In such a range, the informativeness of the observation of war or peace decreases as we increase r until we reach $\bar{r}(V)$. As $r_{t-1} \geq \bar{r}(V)$, $\sigma^- \rightarrow 1$. The intuition for why the PBE involves randomization is as follows. First, recall that in this region $\sigma^+ = 1$. Then, if $\sigma^- = 0$, peace would be highly informative. Fast updating would increase the trade surplus, making group A regret staging war. Conversely, if $\sigma^- = 1$, peace would be uninformative. The absence of belief updating would keep the trade surplus low, making group A regret retaining peace.

¹⁹We do not study the case in which $V \in [0, S^+(0)]$. This case is the mirror image of the high- V case, and features learning traps with frequent peace but no WDLT. The region of parameters that sustain this type of equilibrium is thin for reasons that will become clearer in later sections.

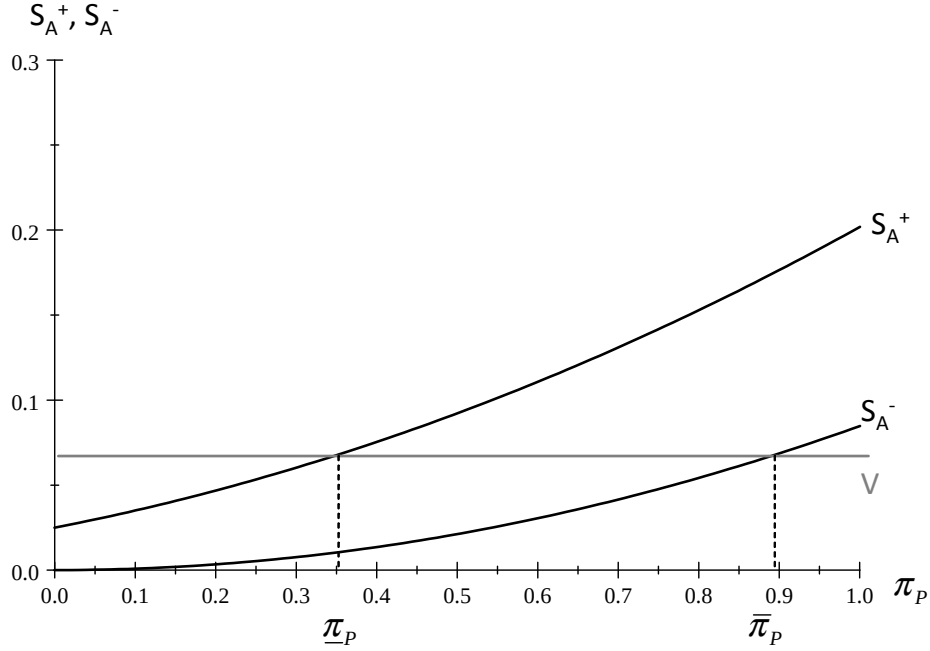


Figure 6: Surplus from trade and gains from war as function of beliefs in the presence of two traps

We can now state the analogue of Proposition 4 for the low- V case (the proof follows from lemma 1 and its proof).

Proposition 6 Assume $V \in [S^+(0), S^-(1)]$ and the selection criterion of Assumption 4. Let $\bar{r}(V) \equiv \frac{(S^-)^{-1}(V)}{1-(S^-)^{-1}(V)}$ and $\bar{r}^*(V) \equiv \frac{\lambda_P}{1-\lambda_W} \bar{r}(V)$. The DSE is characterized as follows: The PBE at time t is unique and characterized by Proposition 3 after setting $r_{-1} = r_{t-1}$. In particular, if $r_{t-1} < \bar{r}^*(V)$ the DSE is characterized as in Proposition 4. If $r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)]$, the high type chooses peace while the low type randomizes the war/peace choice under BAU ($\sigma^+(r_{t-1}) = 1$ and $\sigma^-(r_{t-1}) = \hat{\sigma}^-(r_{t-1}) = \frac{(1-\lambda_W)^{\frac{r_{t-1}}{\bar{r}(V)} - \lambda_P}}{1-\lambda_W - \lambda_P} \in [0, 1]$), and the PBE is informative. Finally, if $r_{t-1} > \bar{r}(V)$, then both types choose peace under BAU ($\sigma^+(r_{t-1}) = \sigma^-(r_{t-1}) = 1$), and the PBE is uninformative. Given an initial condition r_0 , the equilibrium law of motion of beliefs is given by the following stochastic process:

$$\ln r_t = \begin{cases} \ln r_{t-1} & \text{if } r_{t-1} \in [0, \underline{r}(V)) \cup [\bar{r}(V), \infty) \\ \ln r_{t-1} + (1 - I_{WAR,t}) \ln \left(\frac{1-\lambda_W}{\lambda_P} \right) - I_{WAR,t} \ln \left(\frac{1-\lambda_P}{\lambda_W} \right) & \text{if } r_{t-1} \in [\underline{r}(V), \bar{r}^*(V)] \\ (1 - I_{WAR,t}) \ln \bar{r}(V) + I_{WAR,t} \ln \frac{\lambda_W \bar{r}(V) r_{t-1}}{\bar{r}(V) - r_{t-1} (1-\lambda_W)} & \text{if } r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)] \end{cases} \quad (11)$$

where $I_{WAR,t} \in \{0, 1\}$ is an indicator function of War at date t with the following conditional proba-

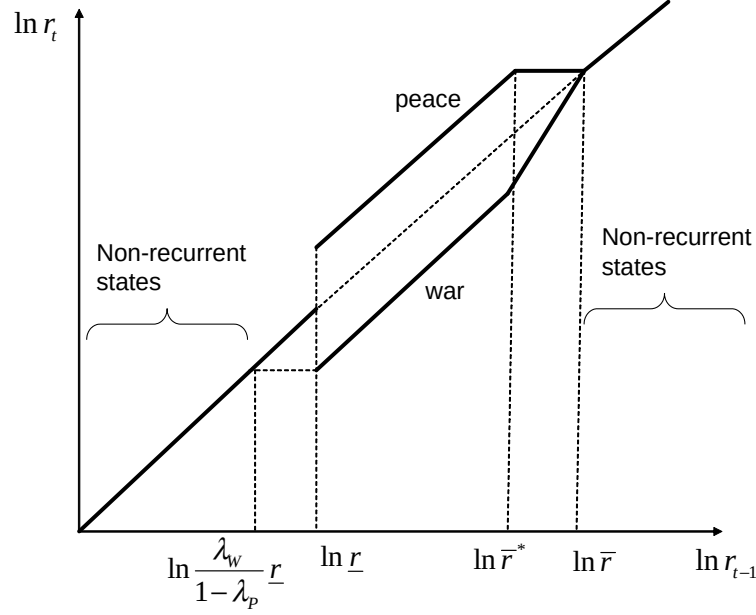


Figure 7: Dynamics of beliefs with two traps

bility

$$\Pr(I_{WAR,t} = 1 | r_{t-1}) = \begin{cases} 1 - \lambda_P & \text{if } r_{t-1} \in [0, \underline{r}(V) [\\ I^- \cdot (1 - \lambda_P) + (1 - I^-) \cdot \lambda_W & \text{if } r_{t-1} \in [\underline{r}(V), \bar{r}^*(V)] \\ I^- \cdot (1 - \lambda_P - (1 - \lambda_P - \lambda_W) \hat{\sigma}^-(r_{t-1})) + (1 - I^-) \cdot \lambda_W & \text{if } r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)] \\ \lambda_W & \text{if } r_{t-1} \in]\bar{r}(V), \infty] \end{cases} \quad (12)$$

where $I^- \in \{0, 1\}$ is an indicator function of $\{k = -\}$.

Figure 7 illustrates the equilibrium dynamics of beliefs, as given by equation (11). The main difference with respect to Figure 5 is that in the high-prior region there is no learning, since peace is preferred by group A even when it is of a low type. Note that if the economy first enters the range $r_{t-1} \in [\bar{r}^*(V), \bar{r}(V)]$, and then peace prevails for another period, beliefs get stuck to $r_{t+s} = \bar{r}(V)$ for all $s \geq 0$. Namely, $\bar{r}(V)$ is an absorbing state. Larger r are non-recurrent states, which can only be reached if the economy starts there.

Definition 5 A *peace-dominated learning trap (PDLT)* is a set of states $\Omega_{PDLT} \subset R^+$ such that, if $r_t \in \Omega_{PDLT}$ then $\forall s \geq t, r_s = r_t$ and the incidence of war is low, $\Pr(I_{WAR,s} = 1) = \lambda_W$, for all continuation paths $[r_s]_{s=t}^\infty$.

It follows from Proposition 6 and the definition of a PDLT that $\Omega_{PDLT} = [\bar{r}(V), \infty)$.

Given an initial prior in the informative region ($r_0 \in [\underline{r}(V), \bar{r}(V)]$), the economy starts in an informative equilibrium and there are learning dynamics. Eventually, the economy gets stuck into either of the two traps. As before, we can characterize the long-run probability distribution.

Proposition 7 *Assume that $V \in [S^+(0), S^-(1)]$ and let $r_0 \in [\underline{r}(V), \bar{r}(V)]$. Then, in both states of nature, $k = +$ and $k = -$, the DSE exits the informative equilibrium regime almost surely, and learning comes to a halt in finite time. The final belief is such that with probability $P_W(r) > 0$ the economy is in a WDLT and with probability $1 - P_W(r) > 0$ it is in a PDLT.*

The intuition behind this proposition is the same as in the discussion of the high- V case. In both states of nature, the process of priors cannot stay forever in the informative equilibrium regime. Otherwise agents could observe an infinite number of realizations of the war/peace process. Thus, by virtue of the strong law of large numbers, the empirical frequency of war/peace would converge to the true underlying probabilities, which is either $(1 - \lambda_P, \lambda_P)$, if $k = -$, or $(\lambda_W, 1 - \lambda_W)$ if $k = +$. This would enable agents to learn the true state of nature.²⁰

4 Learning from Trade

In the analysis so far, the information set of group B was limited to the history of warfare. However, the inference of agents in group B about the propensity to trade of group A could be improved upon if they observed directly part of the trade history. For instance, if public records existed of the outcome of past inter-ethnic trade, group B could infer $k \in \{-, +\}$ exactly. While our analysis imposes strong restrictions on the information set, the perfect-information scenario is not realistic either, since in reality cross-community trade and business links are typically decentralized and hardly distinguishable from intra-community links.

In this section, we expand the information set available to group B. In particular, we allow agents in group B to retain *some* memory of the information acquired through their individual family trade history. To retain tractability, we make the simplifying assumption that as soon as an agent invests and attempts to trade, she has the opportunity to observe the true k .²¹ This knowledge is not useful to the trader herself (it arrives too late to guide her investment decision), but can be transmitted to the offspring. In this environment, without further assumptions, all families would end up learning perfectly k . To prevent the informational friction from vanishing in the long run, we also assume that the inter-generational transmission of private information is subject to stochastic breakdowns: with

²⁰Contrary to the previous case we cannot provide a closed-form characterization of the probability P_W . The reason is that the stochastic process (11) is not a random walk, due to the behavior of the stochastic process in the region $r_{t-1} \in [\underline{r}(V), \bar{r}^*(V)]$.

²¹The assumption that a trader learns the exact value of k may appear too drastic. Note, though, that this is an adversary assumption, since our goal is to show that learning traps are a robust outcome even if one increases the extent of information available in societies. Thus, assuming that private learning through trade is very effective plays against our result. Moreover a model in which private learning from trade history is less drastic would be complicated to analyze, as the distribution of private signals would become a state variable whose evolution would be hard to keep track.

an exogenous Poisson probability θ , the child of an informed parent fails to be transmitted the hard information about k . Thus, θ is an inverse measure of the efficiency of learning from trade history: $1/\theta$ is the average number of generations within the same family which do not experience a memory loss.

Given our set of assumptions, in every period there is both a hard information inflow (uninformed families that engage in trade learn k) and an exogenous outflow. In war times, nobody trades and the net inflow is negative. In peace times, the net inflow can be positive. This model captures in a stylized fashion the notion that information depreciates: If trade was intense in the far past, but it waned in more recent times, the information gathered through past trade fades away. This representation is tractable, since the heterogeneity of information sets within group B is reduced to a two-point distribution, consisting of perfectly informed agents on the one hand and agents who only observe the warfare history on the other hand.

As in the benchmark model, we solve the game backwards, starting from the investment/trade continuation game after peace. The distribution of beliefs in group B is now more complicated. Besides uninformed agents who still hold a public posterior belief conditional on the observation of peace/war, $\{\pi_P, \pi_W\}$, there is now a share of perfectly informed agents. We define by β^- and β^+ , respectively, this share of informed agents conditional on group A type being $k = -$ and $k = +$. Recall that all agents in group A know the type. However, agents in group B ignore it, and thus the uninformed in this group cannot tell whether the share of informed agents is $\beta = \beta^-$ or $\beta = \beta^+$. Different from the benchmark model, the aggregate investment of group B is now type-contingent too, as some agents in group B know k . More formally, agents in group A have perfect information and so observe n_B^k and their reaction function continues to be given by $n_A^k = F_A^k(zn_B^k)$. In group B a share β^k of the agents take their investment decisions under perfect information while a share $1 - \beta^k$ takes a decision based on their common public belief π_P . For $k \in \{-, +\}$ the reaction functions of group B are now given by $n_B^k = \beta^k \cdot zn_A^k + (1 - \beta^k) \cdot zE[n_A | \pi_P]$ where $E[n_A | \pi_P] = \pi_P \cdot n_A^+ + (1 - \pi_P) \cdot n_A^-$.

Proposition 8 *Under assumption 1, for a given $(\pi_P, \beta^-, \beta^+) \in [0, 1]^3$, the Nash Equilibrium of the investment/trade continuation game exists and is the unique 4-tuple $\{n_A^-, n_A^+, n_B^-, n_B^+\} \in [0, 1]^4$ such that $n_A^k(\pi_P, \beta^-, \beta^+) = F_A^k(zn_B^k(\pi_P, \beta^-, \beta^+))$ and $n_B^k(\pi_P, \beta^-, \beta^+)$ is the implicit solution of the following fixed-point equation*

$$n_B^k = \beta^k F^B \left(z F^k \left(zn_B^k \right) \right) + (1 - \beta^k) F^B \left[z \pi_P F^+(zn_B^+) + z(1 - \pi_P) F^-(zn_B^-) \right]. \quad (13)$$

The investment decision of agents in group A, $n_A^k(\pi_P, \beta^-, \beta^+)$, depends on both β^- and β^+ , despite the fact that group A knows its type. Indeed, both β^- and β^+ affect the investment of the uninformed agents in group B who ignore the true type. Due to the strategic complementarity, then, β^- and β^+ also affects the investment of group A and of the informed in group B.

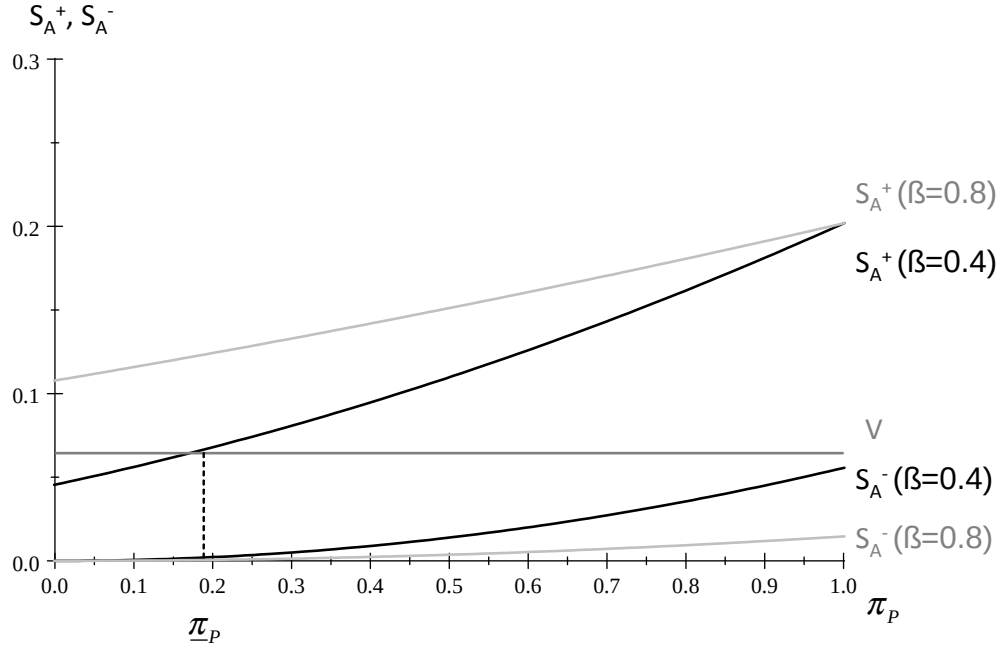


Figure 8: Surplus from trade with exogenous β

4.1 Exogenous β

In order to build some useful intuition, we consider first an economy in which the proportion of informed agents is exogenous. Clearly, in this case $\beta^+ = \beta^- = \beta$ in equations (13). For a given after-peace belief π_P , the static equilibrium and the associated trade surplus now depend both on the belief and on the share of informed agents: $S^-(\pi_P; \beta)$ and $S^+(\pi_P; \beta)$.

Lemma 2 *Under assumption 1, for a given $(\pi_P, \beta) \in [0, 1]^2$, the equilibrium exists and is a unique 4-uplet $(n_A^-, n_A^+, n_B^-, n_B^+) \in [0, 1]^4$ which is continuous and non-decreasing in π_P . Moreover the trade surplus $S^-(\pi_P, \beta), S^+(\pi_P, \beta)$ are continuous and non decreasing in π_P ; S^- is non increasing in β ; S^+ is non decreasing in β . And we have $S^-(\pi_P, \beta) \leq S^+(\pi_P, \beta)$.*

$\begin{matrix} (+) & (-) & & (+) & (+) \end{matrix}$

Lemma 2 has the intuitive result that $\partial S^- / \partial \beta \leq 0$, while $\partial S^+ / \partial \beta \geq 0$. Consequently, the wedge between the two surpluses increases in β , $\partial(S^+ - S^-) / \partial \beta \geq 0$. Intuitively, as the share of informed agents increases, the equilibrium outcomes in the two states of nature become more separated, approaching the perfect information equilibrium as $\beta \rightarrow 1$. Such a divergence between the two trade surplus functions makes war and more informative for any given π_P . This in turn makes learning traps harder to sustain. Figure 8 is drawn for the same distribution of investment costs and parameter values as in figure 6. Hence, for the benchmark case of $\beta = 0$ the surplus S^+ and S^- would be identical as in figure 6, where both a WDLT and a PDLT exist. Initially, increasing β simply reduces the range of posteriors consistent with the existence of two traps. A further increase in

β rules out the PDLT (as shown by the black lines in figure 8 capturing $\beta = 0.4$), and an even further increase eventually also rules out the WDLT (as shown by the light grey lines in figure 8 capturing $\beta = 0.8$). The result that the range of sustainability of learning traps falls with β is general.

In summary, this subsection has shown that learning traps are robust to the assumption that an exogenous share of the population is informed about the type of group A, as long as the share of informed agents is not too large.

4.2 Endogenous β

In this section, we consider economies with an endogenous proportion of informed agents who acquire information through trade and transmit it to their offspring. This extension increases complexity considerably as there are now three state variables to keep track of, $(\pi_t, \beta_t^+, \beta_t^-) \in [0, 1]^3$. The PBE Definition 1 is modified in three respects. First, a strategy for an agent in group B specifies an "investment action" for each of her possible types, *informed* or *uninformed*, and for each of the possible realizations of the investment cost. Second, the PBE is defined up to a triplet, $(\pi_{t-1}, \beta_{t-1}^+, \beta_{t-1}^-) \in [0, 1]^3$. Third, the triplet $(n_{At}^-, n_{At}^+, n_{Bt})$ is replaced by the 4-tuple $(n_{At}^-, n_{At}^+, n_{Bt}^+, n_{Bt}^-)$, where (n_{Bt}^+, n_{Bt}^-) yields the measure of agents who optimally invest in group B for each type $k \in \{+, -\}$.

The share of informed agents evolves according to the following law of motion:

$$\beta_t^k = (1 - \theta) \left[n_{Bt-1}^k + \left(1 - n_{Bt-1}^k \right) \beta_{t-1}^k \right]. \quad (14)$$

The set of informed agents at t consists of children either of traders or of informed non-traders, conditional on no memory loss. The DSE is then modified as follows. As in the analysis of the benchmark model, we define the equilibrium in terms of the state variable $r \equiv \pi / (1 - \pi)$.

Definition 6 *A DSE is a sequence of PBE with an associated sequence of beliefs and measure of informed agents such that, given an initial condition $(\pi_0, \beta_0^+, \beta_0^-)$ the posterior belief at t is the prior belief at $t+1$ and the law of motion of β_t^+ and β_t^- is given by (14).*

As before, it is convenient to characterize the DSE in terms of likelihood ratios, r , rather than in term of π . With this in mind, we extend the definition of learning trap to the new environment.

Definition 7 *A WDLT (PDLT) is a set of states, $\Omega_{WDLT} \subset \mathbb{R}^+ \times [0, 1]^3$ ($\Omega_{PDLT} \subset \mathbb{R}^+ \times [0, 1]^3$), such that if $(r_t, \beta_t^+, \beta_t^-) \in \Omega_{WDLT}$ (if $(r_t, \beta_t^+, \beta_t^-) \in \Omega_{PDLT}$) then $\forall s \geq t, r_s = r_t$, and the incidence of war is high (low), $\Pr(I_{WAR,s} = 1) = 1 - \lambda_P$ ($\Pr(I_{WAR,s} = 1) = \lambda_W$), for all continuation paths $[r_s, \beta_s^+, \beta_s^-]_{s=t}^\infty$.*

When the economy is in a learning trap, the belief sequence is stationary irrespective of any realization of the war process. Note that we do not require the stationarity of β_t^+ and β_t^- for an economy to be in a learning trap. Our aim here is to characterize the parameter range of θ 's which is compatible with the existence of the learning traps, given the remaining parameters. This is not

straightforward, since the equilibrium path is governed by a three-dimensional stochastic process $(r_t, \beta_t^-, \beta_t^+)$ which admits no closed-form solution. For tractability we make the following simplifying assumption:

Assumption 5 ι^B is uniformly distributed on $[0, 1]$ and ι^A is uniformly distributed on $[-x_A, 1 - x_A]$ with $x_A \in \{-x, +x\}$ and $x < 1/2$.

Note that this assumption is nested in assumption 1 when $z < 1 - x$. However, the results of the next two propositions are valid for any $z \in (0, 1)$.

We can establish a sharp characterization result, summarized in the following Proposition.

Proposition 9 (i) Assume V such that $S^+(0) < V < \min\{S^+(1), 1/2\}$. A WDLT exists if and only if $\theta \geq \theta_W \equiv zx/(1 + zx)$; (ii) A PDLT exists if and only if $V < S^-(1)$ and $\theta \geq \theta_P \equiv 1 - (z^2 - x - \sqrt{2V}) / (z^3 - z^3\sqrt{2V})$; (iii) we have $\theta_P > \theta_W$.

To see the intuition, note first that if families never forget, i.e., $\theta = 0$, then the economy necessarily converges to perfect learning. The intuition is straightforward. Since $\lambda_P > 0$, the probability of peace is bounded away from zero. Conditional on peace, there is some trade, and this induces learning from new families. It is then easy to show that the process converges to the full-information equilibrium. Imposing a lower bound on θ has similar effects to imposing a lower bound on β when this is exogenous (see previous section). In particular, when $\theta > 0$ there exists an upper bound to β^+ and β^- corresponding to the limit of a sequence of repeated peace realizations. This limit yields an upper bound to the share of informed agents conditional to group A type, denoted by β_∞^+ and β_∞^- . Consider, now, a case in which $k = +$ and the state at $t - 1$ is $(r_{t-1}, \beta_\infty^+, \beta_{t-1}^-)$. Suppose that in this state, both the high and the low type would stage war under BAU, implying that $r_t = r_{t-1}$ under both peace and war. Then, $(r_{t-1}, \beta_\infty^+, \beta_{t-1}^-) \in \Omega_{WDLT}$. Intuitively, the share of informed agents cannot increase, since it is already at its upper bound. If such share falls, investments will fall, strengthening further the incentive for group A to stage war. Thus, uninformed agents never learn, and the economy is in a WDLT.

Interestingly, WDLT are more robust to private learning from trade history than PDLT. More formally, $\theta_W < \theta_P$. The key differences between the two traps is that in a PDLT (i.e. $V < S^- < S^+$), the belief is optimistic and so many agents invest and trade; this increases the diffusion of private information; therefore the state-contingent equilibrium trade surplus (S^-, S^+) tend to be more and more separated and this potentially restores the informativeness of the war/peace process (i.e. $S^- < V < S^+$) making the PDLT not sustainable anymore. In the case of a WDLT, the same mechanism is at work but it is dampened because, beliefs being more pessimistic, the level of trade is smaller and so is its informational externality. To sum up, this result implies that the two sources of learning - trade history and warfare history - are complements.

The differential robustness of WDLT and PDLT to learning from trade can be substantial. Let us consider a situation where on average the propensity to trade in the good state of nature is 10 percent

larger than in the bad state of nature (i.e. $x = 0.05$). Let us also assume that in the benchmark situation without private learning from trade ($\theta = 1$) there is a PDLT and a WDLT of equal size (i.e., $z = 1, V = (1 - x)^2/2$). With such parameter configuration the thresholds are equal to $\theta_W = 0.047$ and $\theta_P = 0.5$. In words, family memory should last on average no more than two generations for the PDLT to vanish (i.e., $1/\theta_P = 2$), while the WDLT is sustained as long as memory persists on average for up to twenty one generations (i.e., $1/\theta_W = 21$).

Proposition 9 established an existence result for learning traps. The next Proposition establishes that economies starting in an informative equilibrium, $(r_0, \beta_0^+, \beta_0^-) \notin \Omega_{WDLT}$, can actually fall in WDLT with a positive probability as long as the WDLT is non empty (i.e. $\theta > zx/(1 + zx)$). To this purpose, we identify a finite time-passage T , corresponding to a non-zero measure subset of continuation paths over the period $0, \dots, T$, such that $(r_T, \beta_T^-, \beta_T^+) \in \Omega_{WDLT}$. Basically these paths include a sequence of war shocks which manages to drive r_T into a range of sufficiently pessimistic beliefs. Moreover, by disrupting trade, such sequence depletes the share of informed agents β_0^+ such that $\beta_0^+/\beta_T^+ = 1/(1 - \theta)^T < 1$. When the pace of decrease of the informational externality of trade, $1/(1 - \theta)$, is larger than the informativeness of war, $\frac{1-\lambda_P}{\lambda_W}$, this sequence of war shocks is able to drive the economy into the WDLT.²²

Proposition 10 *Assume $\theta > \max\left(1 - \frac{\lambda_W}{1-\lambda_P}, \frac{zx}{1+zx}\right)$. Suppose $(r_0, \beta_0^+, \beta_0^-) \notin \Omega_{WDLT}$. Then, the economy falls into a WDLT in finite time with a strictly positive probability, $\Pr\{\exists T < \infty, (r_T, \beta_T^+, \beta_T^-) \in \Omega_{WDLT}\} > 0$.*

To sum up, learning traps are robust to the presence of a positive share of informed agents. However, as the share of informed people increases (i.e., as we lower θ), learning traps with incorrect beliefs become harder to sustain. Eventually, for θ sufficiently small, such learning traps are ruled out. WDLT are more robust than PDLT to private learning through trade. Economies starting in informative equilibria can fall into learning traps even though agents learn through trade.

5 Policy Implications

In this section we outline some comparative statics and policy implications of our theory. Our model implies that *larger individual returns from trade (i.e., larger z)* make human capital investments more attractive, thereby increasing the expected trade surplus (equation (2) shows that $\partial S^+(\pi_P)/\partial z \geq 0$ and $\partial S^-(\pi_P)/\partial z \geq 0$). Thus, policies subsidizing inter-group trade push up the opportunity cost of war, narrowing on the one hand the range of beliefs for which WDLT occur, and enlarging on the other hand the range of beliefs for which PDLT arise (more formally, this corresponds to an upward rotation of $S^+(\pi_P)$ and $S^-(\pi_P)$ in the Figures 3 and 6). This prediction is in line with the empirical results of

²²Proving convergence to a PDLT is harder. We conjecture that convergence may occur under more restrictive conditions. On the one hand, peace must occur to make beliefs more optimistic over time. On the other hand, this would reveal to an increasing share of group B that group A is of the low type.

Horowitz (2000) on affirmative action and ethnic conflict. He finds that preferential programs aiming at better integrating less advanced ethnic groups in the national economy reduced the potential for conflict in various countries such as India, Indonesia, Malaysia and Nigeria. Since trade typically thrives in fast-growing economies, our theory is also broadly consistent with the empirical finding that high economic growth reduces the risk of war recurrence (Sambanis, 2008; Walter, 2004). Our setting theory also provides a rationale to *subsidize human capital investment* which reduce inter-ethnic barriers. Public education initiatives promoting for example the knowledge of several national languages can lower the obstacles to inter-group trade. This is in line with the empirical findings that higher education expenditures and enrollment rates decrease the risk of civil wars (Thyne, 2006).

Unsurprisingly, *larger windfall gains from war* (i.e., larger $\tilde{V}$) expand the range of beliefs such that the economy can plunge in a WDLT. This is in line with the empirical findings that more abundant natural resources hinder lasting recovery and fuel war recurrence (see, e.g., Doyle and Sambanis, 2000; Fortna, 2004; Sambanis, 2008). International measures such as embargoes on arms exports to or natural resource imports from regimes arising from ethnic aggression could limit trust depletion and war recurrence.

Our theory has more subtle implications about the effectiveness of *international peacekeeping*. The model predicts that international peacekeeping efforts that limit themselves to "stopping the shooting" will only have a short-run effect on political stability. To reach a long-run impact, e.g. to get a country out of a WDLT, peacekeeping must be complemented first and replaced later by trade- and trust-enhancing measures. In fact, the prolonged insistence on external peacekeeping may be detrimental, as it may undermine the externality of peace on learning and trust. In other words, local groups may attribute peace to the presence of foreign troops, and fail to update their beliefs about the propensity to trade of other communities. These predictions are in line with the conclusion of a study on survival of peace duration by Sambanis (2008: 30): "*UN missions have a robust positive effect on peacebuilding outcomes, particularly participatory peace, but the effects occur mainly in the short run and are stronger when peacekeepers remain.*" Indeed, he finds that the effect becomes non-significant once UN troops have left and that long-run enduring peace depends crucially on economic development and the rebuilding of institutions, not on past UN peacekeeping.

Similar conclusions are reached by Luttwak (1999: 37) who argues that simple peacekeeping – without trade-promoting or trust-resoring measures – does not lead to lasting peace, but just interrupts hostilities that will recur once the UN troops leave: "*(Peacekeeping), perversely, can systematically prevent the transformation of war into peace. The Dayton accords are typical of the genre: they have condemned Bosnia to remain divided into three rival armed camps, with combat suspended momentarily but a state of hostility prolonged indefinitely... Because no path to peace is even visible, the dominant priority is to prepare for future war rather than to reconstruct devastated economies and ravaged societies.*"

Our theory also suggests that policies *targeting beliefs directly* may be important, especially when there is no fundamental reason for persistent distrust and war. If the state of the world was $k = +$,

there may sometimes be ways to credibly communicate this to the population (e.g., by documenting and publicizing successful episodes of inter-ethnic business cooperation). There is empirical evidence that inter-group prejudices can be reduced by targeted media exposure (cf. Paluck, 2009; Paluck and Green, 2009). According to Paluck's (2009) findings the listeners exposed to the "social reconciliation" radio soap opera in Rwanda were significantly more likely to find it "not naive to trust" and to feel empathy for other Rwandans than the control group exposed to a "health" radio soap opera. This is related to the economic literature on cultural transmission arguing the importance of campaigns shifting beliefs. For instance, Hauk and Saez-Marti (2002) argue that even temporary educational campaigns, such as the anti-corruption campaigns run in Hong Kong since the 1970s, can be effective. Like in their paper, our theory suggests that the success of such campaigns need not rely on psychological elements.

6 Conclusion

The economic theory of civil conflicts is rooted in the rational choice paradigm. In contrast, a number of political scientists emphasize the notion of *grievance* (e.g. Gurr, 1970; Sambanis, 2001). This view is supported by empirical studies showing that wars tend to reoccur more frequently if they are associated to grievances and ethnic identities (Doyle and Sambanis, 2000; Licklider, 1995). In a recent survey article, Blattman and Miguel (2010) argue that incorporating such factors in economic models is one of the big challenges of theory. In this paper, we take a first step in this direction. In this paper, we have proposed an economic theory where asymmetric information and cultural transmission of beliefs explain why societies can plunge into recurrent civil conflicts. In our theory conflicts are not a mere explosion of irrational grievances, but are associated with the collapse of trust, a notion that is closely connected to that of grievance.²³ The persistent effects of conflict on trust, and the possible emergence of irreversible vicious circles, is explained by a rational belief updating process under imperfect information.

We emphasize the link between trade and war, which has been highlighted in the recent literature as an important factor explaining international conflicts. We believe the link trust-trade-war to be even more salient in the analysis of inter-community conflicts within societies, where business relationships (e.g., seller-buyer, employer-employee, supplier-producer, lender-borrower) are very decentralized and do not need the mediation of institutions that can aggregate and diffuse information.

While in the theory presented in this paper agents are perfectly rational, we expect that integrating more explicit psychological aspects into the theory may cast additional light on the issues at hand. In

²³For instance, Downes (2006) writes: "*The key issues concern the adversary's intentions... The process of fighting a war gives both belligerents plentiful evidence of the adversary's malign intentions. Beyond the normal costs of conflict, civil wars are often characterized by depredations against civilians including ethnic cleansing, massacre, rape, bombing, starvation, and forced relocation. These factors produce deep feelings of hostility and hatred, and make it hard for former belligerents to trust each other. Belligerents have little reason to believe their opponent's intentions suddenly have become benign... Moreover, even if the adversary's intentions seem benign now, what guarantee is there that they will not change in future? These issues are of critical importance.*"

some work in progress (Rohner, Thoenig and Zilibotti, 2010) we find that children who are exposed to war in tender age suffer from a permanent deficit of trust, and that the effect is significantly larger than for adults exposed to war. To the extent to which the earlier age is especially "formative" in terms of beliefs and values, this is broadly consistent with the view that war erodes trust. We plan to extend the theory to emphasize the formative nature of the earlier childhood as in Doepke and Zilibotti (2008). We also plan to study how war affects trust within countries differentially in regions where inter-community relations have different intensities.

Understanding both institutions and trust is important to get at the roots of the phenomenon of conflict within societies. Like Aghion et al. (2010), we believe that the two factors are not independent, and that institutions can matter through their effect on the trust-building process. Studying this connection is left to future research, too.

References

- [1] Acemoglu, Daron, Michael Golosov, Aleh Tsyvinski and Pierre Yared, 2010, "A Dynamic Theory of Resource Wars", Mimeo, MIT.
- [2] Acemoglu, Daron, Davide Ticchi, and Andrea Vindigni, 2010, "Persistence of Civil Wars", *Journal of the European Economic Association* 8: 664–676.
- [3] Acemoglu, Daron, and Fabrizio Zilibotti, 1997, "Was Prometheus Unbound by Chance? Risk, Diversification, and Growth", *Journal of Political Economy* 105: 709-751.
- [4] Aghion, Philippe, Yann Algan, and Pierre Cahuc, 2010, "Civil society and the state: The interplay between cooperation and minimum wage regulation". Forthcoming in the *Journal of the European Economic Association*.
- [5] Aghion, Philippe, Yann Algan, Pierre Cahuc, and Andrei Shleifer, 2010, "Regulation and Distrust", *Quarterly Journal of Economics* 125: 1015-1049.
- [6] Baliga, Sandeep, and Tomas Sjöström, 2004, "Arms Races and Negotiations", *Review of Economic Studies* 71: 351-369.
- [7] Banerjee, Abhijit, 1992, "A simple model of herd behavior", *Quarterly Journal of Economics* 107: 797-817
- [8] Besley, Timothy, and Torsten Persson, 2009, "The Logic of Political Violence", forthcoming in *Quarterly Journal of Economics*.
- [9] Besley, Timothy, and Torsten Persson, 2010, "State Capacity, Conflict and Development", *Econometrica* 78: 1-34.
- [10] Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch, 1992, "A theory of fads, fashion, custom, and cultural change as information cascades", *Journal of Political Economy* 100: 992-1026.
- [11] Blattman, Chris, and Edward Miguel, 2010, "Civil War", *Journal of Economic Literature* 48: 3-57.
- [12] Buhaug, Halvard, Scott Gates, and Päivi Lujala, 2009, "Geography, Rebel Capability, and the Duration of Civil Conflict", *Journal of Conflict Resolution* 53: 544-69.

- [13] Caselli, Francesco, and Wilbur John Coleman II, 2010, "On the Theory of Ethnic Conflict", Mimeo, London School of Economics.
- [14] Chamley, Christophe, 1999, "Coordinating Regime Switches", *Quarterly Journal of Economics* 114: 869-905.
- [15] Chassang, Sylvain and Gerard Padro i Miquel, 2008, "Conflict and Deterrence under Strategic Risk", mimeo, Princeton University and LSE.
- [16] Collier, Paul and Anke Hoeffler, 1998, "On economic causes of civil war", *Oxford Economic Papers* 50: 563-73.
- [17] Collier, Paul and Anke Hoeffler, 2004, "Greed and grievance in civil war", *Oxford Economic Papers* 56: 563-95.
- [18] Collier, Paul, Anke Hoeffler, and Dominic Rohner, 2009, "Beyond Greed and Grievance: Feasibility and Civil War", *Oxford Economic Papers* 61: 1-27.
- [19] Collier, Paul and Dominic Rohner, 2008, "Democracy, Development, and Conflict", *Journal of the European Economic Association* 6: 531-40.
- [20] Colletta, Nat and Michelle Cullen, 2000, *Violent Conflict and the Transformation of Social Capital: Lessons from Cambodia, Rwanda, Guatemala, and Somalia*, Washington: World Bank.
- [21] Cooper, Russell, and Andrew John, 1988, "Coordinating Coordination Failures in Keynesian Models", *Quarterly Journal of Economics* 103: 441-463.
- [22] Dewatripont, Mathias, and Jean Tirole, 2005, "Modes of Communication", *Journal of Political Economy* 115: 1217-1238.
- [23] Dixit, Avinash Kamalakar, 2003, "Trade Expansion and Contract Enforcement", *Journal of Political Economy* 111: 1293-1317.
- [24] Doepke, Matthias, and Fabrizio Zilibotti, 2008, "Occupational Choice and the Spirit of Capitalism", *Quarterly Journal of Economics* 123: 747-793.
- [25] Downes, Alexander, 2006, "More Borders, Less Conflicts? Partition as a Solution to Ethnic Civil Wars", *SAIS Review* XXVI: 49-61.
- [26] Doyle, Michael, and Nicholas Sambanis, 2000, "International Peacebuilding: A Theoretical and Quantitative Analysis", *American Political Science Review* 94: 779-801.
- [27] Ely, Jeffrey, and Juuso Valimaki, 2003, "Bad Reputation", *Quarterly Journal of Economics* 118: 785-813.
- [28] Esteban, Joan and Debraj Ray, 2008, "On the Salience of Ethnic Conflict", *American Economic Review* 98: 2185-202.
- [29] Fearon, James, 1995, "Rationalist Explanations for War", *International Organization* 49: 379-414.
- [30] Fearon, James, 2005, "Primary Commodity Exports and Civil War", *Journal of Conflict Resolution* 49: 483-507.
- [31] Fearon, James, and David Laitin, 2003, "Ethnicity, Insurgency, and Civil War", *American Political Science Review* 97: 75-90.
- [32] Felbermayr, Gabriel, and Farid Toubal, 2010, "Cultural Proximity and Trade", *European Economic Review* 54: 279-293 .

- [33] Fernandez, Raquel, 2007, Culture as Learning: The Evolution of Female Labor Force Participation over a Century. Mimeo, New York University.
- [34] Fortna, Virginia, 2004, "Does Peacekeeping Keep Peace? International Intervention and the Duration of Peace After Civil War", *International Studies Quarterly* 48: 269-292.
- [35] Gartzke, Erik, 1999, "War Is in the Error Term", *International Organization* 53: 567-87.
- [36] Gates, Scott, 2002, "Recruitment and Allegiance: The Microfoundations of Rebellion", *Journal of Conflict Resolution* 46: 111-30.
- [37] Greif, Avner, 1994, "Cultural beliefs and the organization of society: A historical and theoretical reflection on collectivist and individualist societies", *Journal of Political Economy* 102: 912-950.
- [38] Guiso, Luigi, Paola Sapienza, and Luigi Zingales, 2009, "Cultural biases in Economic Exchanges?", *Quarterly Journal of Economics* 124: 1095-1131.
- [39] Gurr, Ted, 1970, *Why Men Rebel*, Princeton: Princeton University Press.
- [40] Hauk, Esther and Maria Saez-Marti, 2002, "On the Cultural Transmission of Corruption", *Journal of Economic Theory* 107: 311-15.
- [41] Horowitz, Donald, 2000, *Ethnic Groups in Conflict*, Berkeley: University of California Press (2nd edition).
- [42] Ingelaere, Bert, 2007, "Living the Transition: A Bottom-up Perspective on Rwanda's Political Transition", Discussion Paper, University of Antwerp.
- [43] Jackson, Matthew and Massimo Morelli, 2007, "Political Bias and War", *American Economic Review* 97: 1353-1373.
- [44] Jha, Saumitra, 2008, "Trade, institutions and religious tolerance: evidence from India", mimeo, Stanford University.
- [45] Licklider, Roy, 1995, "The consequences of negotiated settlements in civil wars, 1945-1993", *American Political Science Review* 89: 681-690.
- [46] Luttwak, Edward, 1999, "Give War a Chance", *Foreign Affairs* July/August: 36-44.
- [47] Martin, Philippe, Thierry Mayer, and Mathias Thoenig, 2008, "Make Trade not War?", *Review of Economic Studies*, 75: 865-900.
- [48] Martin, Philippe, Thierry Mayer, and Mathias Thoenig, 2008b, "Civil Wars and International Trade", *Journal of European Economic Association* 6: 541-550.
- [49] Montalvo, José, and Marta Reynal-Querol, 2005, "Ethnic Polarization, Potential Conflict, and Civil Wars", *American Economic Review* 95: 796-816.
- [50] Paluck, Elizabeth Levy, 2009, "Reducing Intergroup Prejudice and Conflict Using the Media: A Field Experiment in Rwanda", *Journal of Personality and Social Psychology* 96: 574-87.
- [51] Paluck, Elizabeth Levy, and Donald Green, 2009, "Prejudice Reduction: What Works? A Review and Assessment of Research and Practice", *Annual Review of Psychology* 60: 339-67.
- [52] Piketty, Thomas, 1995, "Social Mobility and Redistributive Politics", *Quarterly Journal of Economics* 110: 551-584.

- [53] Pinchotti, Shanley and Philip Verwimp, 2007, "Social Capital and the Rwandan Genocide: A Micro-Level Analysis", HiCN working paper 30.
- [54] Powell, Robert, 2006, "War as a Commitment Problem", *International Organization* 60: 169-203.
- [55] Prio, 2010, "UCDP/PRIO Armed Conflict Dataset", <http://www.prio.no/CSCW/Datasets/Armed-Conflict/>
- [56] Quinn, J. Michael, David Mason and Mehmet Gurses, 2007, "Sustaining the Peace: Determinants of Civil War Recurrence", *International Interactions* 33: 167-193.
- [57] Rauch, James, 1999, "Networks Versus Markets in International Trade", *Journal of International Economics* 48: 7-35
- [58] Rohner, Dominic, Mathias Thoenig, and Fabrizio Zilibotti, 2010, "War experience and Trust", mimeo, University of Zurich and University of Lausanne.
- [59] Sambanis, Nicholas, 2001, "Do Ethnic and Nonethnic Civil Wars Have the Same Causes?", *Journal of Conflict Resolution* 45: 259-282.
- [60] Sambanis, Nicholas, 2008, "Short- and Long-Term Effects of United Nations Peace Operations", *World Bank Economic Review* 22: 9-32.
- [61] Shreve, Steven, 2004, *Stochastic Calculus for Finance I*, Berlin: Springer.
- [62] Tabellini, Guido, 2009, "Culture and institutions: Economic development in the regions of Europe", *Journal of the European Economic Association* 8: 677-716.
- [63] Thyne, Clayton, 2006, "ABC's, 123's, and the Golden Rule: The Pacifying Effect of Education on Civil War, 1980-1999", *International Studies Quarterly* 50: 733-54.
- [64] Torvik, Ragnar, 2002, "Natural resources, rent seeking and welfare", *Journal of Development Economics* 67: 455-70.
- [65] Walter, Barbara, 2004, "Does Conflict Beget Conflict? Explaining Recurring Civil War", *Journal of Peace Research* 41: 371-88.
- [66] World Values Survey, 2010, <http://www.worldvaluessurvey.org/>
- [67] Yared, Pierre, 2010, "A Dynamic Theory of War and Peace", *Journal of Economic Theory* 145: 1921-1950

Appendix

A Multiple PBE and Mixed-Strategy PBE in Section 2.3.2

Consider Figure 9. The left-hand panel illustrates a case in which $r_{-1} \in (\underline{r}(V), \underline{r}^*(V))$ and the mapping from prior to posterior induces multiple PBE. The figure displays the relationship between two endogenous variables: the war choice for the high type (σ^+) and the posterior conditional on peace (r_P). The black solid step function shows the optimal war choice for a high type according to equation (6) – recall that in this range $\sigma^- = 0$. Note that $S^+(r_P/(1+r_P)) = V$ at $r_P = \underline{r}^*(V)$, implying that the high type is indifferent between war and peace, hence, any randomization between war and peace is optimal. The grey schedule yields the Bayesian updating, corresponding to equation (4). The crossing points pin down three PBE, corresponding to different self-fulfilling posteriors. The intuition for the multiplicity of equilibria is the following. Suppose agents believe peace to be informative (uninformative). Then, $r_P > \underline{r}^*(V)$ ($r_P = r_{-1} < \underline{r}^*(V)$), the trade surplus is larger (smaller) than the expected benefit of war, and peace (war) is strictly the optimal choice. This fulfils the expectation that peace is informative (uninformative). A third equilibrium in mixed strategies exists, corresponding to the point where the grey schedule intersects the horizontal segment of the black schedule. The mixed equilibrium is not stable to small perturbations of beliefs. The multiplicity disappears when $r_{-1} < \underline{r}(V)$, as the grey curve is shifted down and crosses the step function only once, at $\sigma^+ = 0$. Likewise, there is no multiplicity when $r_{-1} > \underline{r}^*(V)$, as the grey curve only crosses the step function at $\sigma^+ = 1$. Therefore, multiple PBE only arise for a small set of the prior belief space.

The right-hand panel illustrates a case in which $r_{-1} \in (\bar{r}^*(V), \bar{r}(V))$. In this case, the mapping from prior to posterior induces a unique PBE involving randomization of the low type between war and peace (the high type chooses peace with unit probability). The black solid step function shows in this case the optimal war and peace choice for a low type according to equation (6) – recall that in this range $\sigma^+ = 1$. In this case, $S^-(r_P/(1+r_P)) = V$ at $r_P = \bar{r}(V)$, implying that any randomization between war and peace is optimal to the low type. In this case, however, only the interior crossing point is a PBE. To see why the corners are not equilibria, suppose agents believe peace to be informative (uninformative). Then, $r_P > \bar{r}(V)$ ($r_P = r_{-1} < \bar{r}(V)$), the trade surplus is larger (smaller) than the expected benefit of war, and peace (war) is strictly the optimal choice. However, this does not fulfil the expectation that peace is informative (uninformative), since $\sigma^+ = \sigma^- = 1$ ($\sigma^+ = 1$ and $\sigma^- = 0$). Therefore, the mixed-strategy equilibrium is the only PBE. Moreover, this equilibrium is stable to small perturbations of beliefs. Increasing (decreasing) r_{-1} increases (decreases) the probability that the low type retains peace. When $r_{-1} \geq \bar{r}(V)$ ($r_{-1} \leq \bar{r}^*(V)$) the equilibrium features pure strategies, is uninformative (informative) and entails $\sigma^+ = \sigma^- = 1$ ($\sigma^+ = 1$ and $\sigma^- = 0$).

B Proof of Lemmas and Propositions

B.1 Proof of Proposition 1

We start by proving existence for a given $k \in \{-, +\}$. Equation (1) implies that

$$n_B^k = \tilde{F}^k(n_B^k) \equiv F^B(zF^k(zn_B^k)) \quad (15)$$

where $\tilde{F}^k$ is a continuous function with the following properties: (i) $\tilde{F}^k(0) \geq 0$ and $\tilde{F}^k(1) < 1$; (ii) $\tilde{F}^k(n_B)$ is increasing and convex in n_B . Property (i) follows from Assumption 1. Property (ii)

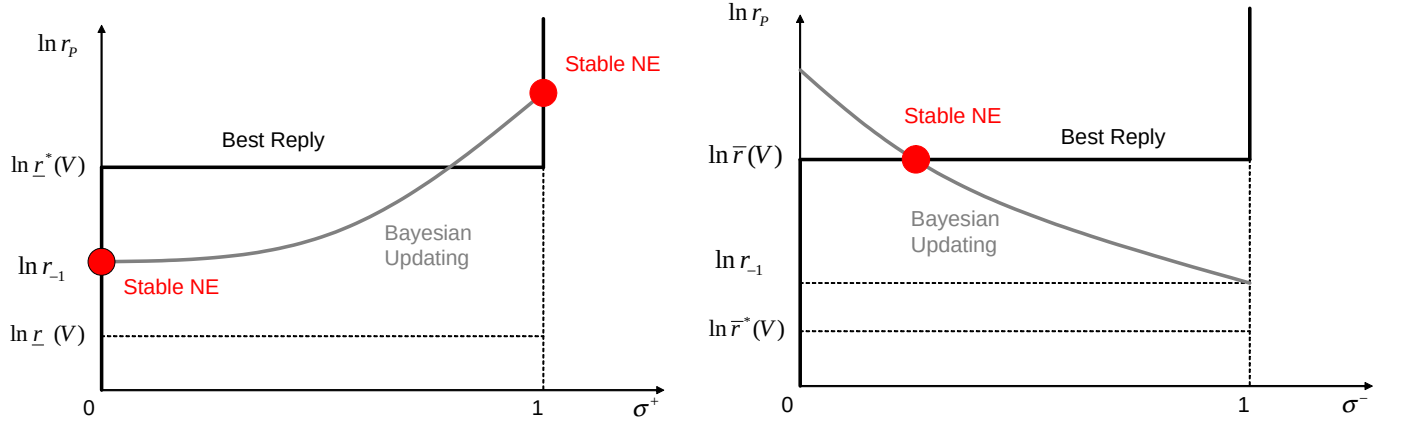


Figure 9: Multiple PBE and Mixed-Strategy PBE

follows from the fact that (due to the standard properties of p.d.f.) $\tilde{F}^k$ is a continuous, non-decreasing transformation of a convex p.d.f. that is continuous, nondecreasing and convex, where convexity follows from Assumption 1. Given property (i) and the continuity of $\tilde{F}^k$, the intermediate value theorem guarantees that there exists $n_B^k \in (0, 1)$ such that $n_B^k = \tilde{F}^k(n_B^k)$.

Properties (i) and (ii) guarantee jointly that the fixed point n_B^k implicitly defined by (15) is unique. To prove uniqueness we proceed by contradiction. Let assume that there exists a second fixed point $\hat{n}_B^k = \tilde{F}^k(\hat{n}_B^k)$. Without loss of generality we assume $n_B^k < \hat{n}_B^k$. The fixed point $\hat{n}_B^k \in [n_B^k, 1]$ can be written as the following convex combination of the interval bounds: $\hat{n}_B^k = \frac{1-\hat{n}_B^k}{1-n_B^k} \times n_B^k + \frac{\hat{n}_B^k-n_B^k}{1-n_B^k} \times 1$. Applying to $\hat{n}_B^k$ the convexity criterion of $\tilde{F}^k$ yields

$$\tilde{F}^k(\hat{n}_B^k) \leq \frac{1-\hat{n}_B^k}{1-n_B^k} \tilde{F}^k(n_B^k) + \frac{\hat{n}_B^k-n_B^k}{1-n_B^k} \tilde{F}^k(1) \quad (1)$$

From definition of the fixed points $(n_B^k, \hat{n}_B^k)$ this inequality yields $\hat{n}_B^k \leq \frac{1-\hat{n}_B^k}{1-n_B^k} n_B^k + \frac{\hat{n}_B^k-n_B^k}{1-n_B^k} \tilde{F}^k(1)$. This leads to $\tilde{F}^k(1) \geq 1$, which contradicts property (i).

Given the existence of a unique fixed point n_B^k for a given $k \in \{-, +\}$ the existence and uniqueness of $n_A^k = F^k(zn_B^k)$. Thus, equation (1) has a unique fixed point. Finally, Assumption 2 implies that $(n_A^-, n_B^-) \leq (n_A^+, n_B^+)$. Let us now turn to the equilibrium value of the trade surplus $\hat{S}^k$ for $k \in \{-, +\}$. Integrating by parts (2) yields

$$\hat{S}^k(n_B^k) = zF^k(zn_B^k)n_B^k - \int^{zn_B^k} \iota dF^k(\iota) = \int^{zn_B^k} F^k(\iota) d\iota \quad (16)$$

As F^- first-order stochastically dominates F^+ , then $\int^{zn_B^k} F^+(\iota) d\iota \geq \int^{zn_B^k} F^-(\iota) d\iota$. We conclude that $\hat{S}^- \leq \hat{S}^+$.

B.2 Proof of Proposition 2

We start by proving existence. Equation (3) implies that

$$n_B = \tilde{F}^B(n_B, \pi_P) \equiv F^B(z[\pi_P F^+(zn_B) + (1-\pi_P) F^-(zn_B)]), \quad (17)$$

where $\tilde{F}^B$ is a continuous function with the following properties: (i) For all π_P , $\tilde{F}^B(0; \pi_P) \geq 0$ and $\tilde{F}^B(1; \pi_P) < 1$; (ii) $\tilde{F}^B(n_B; \pi_P)$ is increasing and convex in n_B , (iii) $\tilde{F}^B(n_B; \pi_P)$ is increasing in π_P . Property (i) follows from Assumption 1. Property (ii) follows from the fact that (due to the standard properties of p.d.f.) $\tilde{F}^B$ is a continuous, non-decreasing transformation of convex combination of p.d.f. that are themselves continuous, nondecreasing and convex in n_B , where convexity follows from Assumption 1. Property (iii) follows from Assumption 2. Given property (i) and the continuity of $\tilde{F}^B$, the intermediate value theorem guarantees that, for any $\pi_P \in [0, 1]$ there exists $n_B \in (0, 1)$ such that $n_B = \tilde{F}^B(n_B; \pi_P)$.

Properties (i), (ii) and (iii) guarantee jointly that the mapping $n_B(\pi_P)$ implicitly defined by (17) is unique and is monotonically increasing. To prove uniqueness we proceed by contradiction. Let assume that there exists a second fixed point $\hat{n}_B = \tilde{F}^B(\hat{n}_B; \pi_P)$. Without loss of generality we assume $n_B < \hat{n}_B$. The fixed point $\hat{n}_B \in [n_B, 1]$ can be written as the following convex combination of the interval bounds: $\hat{n}_B = \frac{1-\hat{n}_B}{1-n_B} \times n_B + \frac{\hat{n}_B-n_B}{1-n_B} \times 1$. Applying to $\hat{n}_B$ the convexity criterion of $\tilde{F}^B$ yields

$$\tilde{F}^B(\hat{n}_B; \pi_P) \leq \frac{1-\hat{n}_B}{1-n_B} \tilde{F}^B(n_B; \pi_P) + \frac{\hat{n}_B-n_B}{1-n_B} \tilde{F}^B(1; \pi_P)$$

From definition of the fixed points $(n_B, \hat{n}_B)$ this inequality yields $\hat{n}_B \leq \frac{1-\hat{n}_B}{1-n_B} n_B + \frac{\hat{n}_B-n_B}{1-n_B} \tilde{F}^B(1; \pi_P)$. This leads to $\tilde{F}^B(1; \pi_P) \geq 1$, which contradicts property (i).

Given the existence of a unique function $n_B(\pi_P)$, the existence and uniqueness of n_A^- and n_A^+ such that $n_A^- = F^-(zn_B(\pi_P)) = n_A^-(\pi_P)$ and $n_A^+ = F^+(zn_B(\pi_P)) = n_A^+(\pi_P)$ follows immediately. Thus, equation (3) has a unique fixed point and defines a unique triplet of equilibrium functions. Finally, Assumption 2 implies that $(n_A^-(\pi_P), n_B^-(\pi_P)) \leq (n_A^+(\pi_P), n_B^+(\pi_P))$.

Let us now turn to the equilibrium value of the trade surplus S^k for $k \in \{-, +\}$. Integrating by parts (2) yields

$$\begin{aligned} S^k(\pi_P) &\equiv zn_A^k(\pi_P) n_B(\pi_P) - \int^{zn_B(\pi_P)} \iota dF^k \\ &= zn_A^k(\pi_P) n_B(\pi_P) - \left[\iota F^k \right]^{zn_B(\pi_P)} + \int^{zn_B(\pi_P)} F^k(\iota) d\iota \\ &= zn_A^k(\pi_P) n_B(\pi_P) - zn_B(\pi_P) F^k(zn_B(\pi_P)) + \int^{zn_B(\pi_P)} F^k(\iota) d\iota \end{aligned}$$

From (3) we get that at equilibrium $n_A^k = F^k(zn_B)$. Combined with the previous equation this gives

$$\hat{S}^k(n_B(\pi_P)) = zF^k(zn_B(\pi_P)) n_B - \int^{zn_B(\pi_P)} \iota dF^k(\iota) = \int^{zn_B(\pi_P)} F^k(\iota) d\iota \quad (18)$$

Given that F^k is non negative and $n_B(\pi_P)$ is non decreasing in π_P we conclude that $\hat{S}^k(\pi_P)$ is non decreasing in π_P . Moreover F^- first-order stochastically dominates F^+ ; $\forall \iota, F^+(\iota) \geq F^-(\iota)$. Hence $\int^{zn_B(\pi_P)} F^+(\iota) d\iota \geq \int^{zn_B(\pi_P)} F^-(\iota) d\iota$. We conclude that $\forall \pi_P \in [0, 1], \hat{S}^-(\pi_P) \leq \hat{S}^+(\pi_P)$.

B.3 Proof of Lemma 1

We first prove that an uninformative PBE exists if and only if the prior is in either the range $r_{-1} \leq \underline{r}^*(V)$ or $r_{-1} \geq \bar{r}(V)$. Guess that a PBE exists. Since $r_P = r_{-1}$, $r_{-1} \leq \underline{r}^*(V) \Rightarrow r_P \leq \underline{r}^*(V)$ and $r_{-1} \geq \bar{r}(V) \Rightarrow r_P \geq \bar{r}(V)$. Then, by the definitions of $\underline{r}^*(V)$ and $\bar{r}(V)$, both types find it optimal to stage war under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 0$) if $r_{-1} \leq \underline{r}^*(V)$. Likewise, both types retain peace

under BAU ($\sigma^+(r_{-1}) = \sigma^-(r_{-1}) = 1$) if $r_P \geq \bar{r}(V)$. The guess is then fulfilled, proving the "if" part. To prove the "only if" part suppose, to draw a contradiction, that an uninformative PBE exists in the range $r_{-1} \in (\underline{r}^*(V), \bar{r}(V))$. Then, $r_P \in (\underline{r}^*(V), \bar{r}(V))$. However, given a posterior in such range, the good type would retain peace ($\sigma^+(r_{-1}) = 1$) whereas the low type would stage war ($\sigma^-(r_{-1}) = 0$) under BAU, contradicting the assumption that peace is uninformative and that $r_P = r_{-1}$.

Next, we prove that informative PBE exist if and only if $r_{-1} \in [\underline{r}(V), \bar{r}(V)]$. We consider first the subrange $r_{-1} \in [\underline{r}(V), \bar{r}^*(V)] \subset [\underline{r}(V), \bar{r}(V)]$, and prove that in this subrange there exists an informative pure-strategy PBE such that $\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$. Guess that such a PBE exists. Since $r_P = \frac{1-\lambda_W}{\lambda_P} r_{-1}$, then $r_{-1} \in [\underline{r}(V), \bar{r}^*(V)] \Rightarrow r_P \in [\underline{r}^*(V), \bar{r}(V)]$. Then, by the definitions of $\underline{r}^*(V)$ and $\bar{r}(V)$, the high type finds it optimal to retain peace ($\sigma^+(r_{-1}) = 1$) while the low type finds it optimal to stage war ($\sigma^-(r_{-1}) = 0$) under BAU. This fulfils the guess, establishing the existence of an informative pure-strategy PBE in the subrange $r_{-1} \in [\underline{r}(V), \bar{r}^*(V)]$. Next, consider the complementary subrange $r_{-1} \in [\bar{r}^*(V), \bar{r}(V)] \subset [\underline{r}(V), \bar{r}(V)]$. In this subrange, an informative pure-strategy PBE such that $\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$ does not exist, since then $r_P = \frac{1-\lambda_W}{\lambda_P} r_{-1} > \bar{r}(V)$ implying that both types would find it optimal to retain peace, contradicting that $\sigma^+(r_{-1}) = 1$ and $\sigma^-(r_{-1}) = 0$. However, there exists a unique mixed-strategy informative PBE, such that the high type chooses peace ($\sigma^+(r_{-1}) = 1$) while the low type is indifferent between war and peace, and chooses war with probability $\hat{\sigma}^-(r_{-1}) = \frac{(1-\lambda_W)\frac{r_{-1}-\lambda_P}{\bar{r}(V)} - \lambda_P}{1-\lambda_W-\lambda_P}$. Bayes' rule implies then that $r_P = \bar{r}(V)$, fulfilling the guess that the low type is indifferent between war and peace (consequently, war erupts with probability $\lambda_W < 1/3$ if $k = +$ and with probability $1 - \lambda_P - (1 - \lambda_P - \lambda_W)\hat{\sigma}^-(r_{-1}) > \lambda_W$ if $k = -$).

The fact that there are multiple PBE if and only if $r_{-1} \in [\underline{r}(V), \underline{r}^*(V)]$ follows immediately from the analysis above [note that in this range there exist three equilibria, since a mixed-strategy informative equilibrium such that $r_P = \underline{r}^*(V)$ also exists. However, if $r_{-1} \in [\underline{r}^*(V), \bar{r}(V)]$ the informative PBE is unique].

B.4 Proof of Proposition 5

The proof strategy consists of first showing that the stochastic process (9) can be reformulated as an asymmetric random walk with a drift on the real line. Then, applying the properties of Martingale processes, we characterize the probability of the stopping time $Pr\{\exists T < \infty | r_T \in \Omega_{WDLT}\}$. The discrete-time nature of the process introduces some technical complications that would not feature in continuous-time processes. In particular, in discrete time when the random walk has a drift there is a compact set of possible stopping-time values, r_T , and which value in this set is reached depends on the realization of the stochastic process (i.e., r_T is not deterministic). This complication (which would not feature in continuous time) does not arise in the particular case of Corollary 1 in which the random walk has no drift.

The stochastic process (9) can be expressed, after rearranging terms, as

$$Z_t = \delta + Z_{t-1} \pm 1 \text{ with probability } (\rho, 1 - \rho), \quad (19)$$

where $Z_t \equiv \ln r_t/s$, $\delta \equiv d/s < 1$,

$$\rho \equiv 1_{k=+} \times (1 - \lambda_W) + 1_{k=-} \times \lambda_P \quad (20)$$

$$s \equiv \frac{1}{2} \left[\ln\left(\frac{1-\lambda_W}{\lambda_P}\right) + \ln\left(\frac{1-\lambda_P}{\lambda_W}\right) \right] > 0 \quad (21)$$

$$d \equiv \frac{1}{2} \left[\ln\left(\frac{1-\lambda_W}{\lambda_P}\right) - \ln\left(\frac{1-\lambda_P}{\lambda_W}\right) \right] \in]-s, s[. \quad (22)$$

Z_t is a random walk with drift which is defined up to an initial condition $Z_0 \equiv \ln r_0/s$. The process Z_t hits a downward barrier as soon as it falls into the range $]Z^* - 1 + \delta, Z^*]$ where $Z^* \equiv \ln \bar{r}_W(V)/s < Z_0$.

Our next goal is to characterize the first passage time $T \equiv \min\{t; Z^* - 1 + \delta < Z_t \leq Z^* < 0\}$. Our approach generalizes the analysis of Shreve (2004, chap.5) to a random walk with drift. To this aim, we define a family of Martingales $M_t(u)$ which corresponds to a deterministic transformation of Z_t :

$$M_t(u) \equiv e^{u(Z_t - Z^*) - tF(u)} \quad (23)$$

where $u \in \mathbb{R}$ and

$$F(u) \equiv u\delta + \ln(\rho e^u + (1 - \rho)e^{-u}) \quad (24)$$

Using the definitions (20), (21) and (22) we can show that equation $F(u) = 0$ has two roots. One of them is $u = 0$. The other is $u = u^*$, where

$$\begin{aligned} u^* &= -s < 0 \text{ when } k = +, \\ u^* &= s > 0 \text{ when } k = -. \end{aligned} \quad (25)$$

Moreover, $F(u) > 0$ when $k = -$ and $F(u) > 0$ when $k = +$.

The process M_t is a martingale, since

$$M_{t+1} = e^{u(Z_{t+1} - Z^*) - (t+1)F(u)} = e^{u(Z_{t+1} - Z_t)} e^{-F(u)} M_t,$$

where $E_t[M_{t+1}] = M_t e^{-F(u)} E_t[e^{u(Z_{t+1} - Z_t)}] = M_t e^{-F(u)} (\rho e^u + (1 - \rho)e^{-u} + u\delta) = M_t$. Next, let $t \wedge T \equiv \min(t, T)$. Since a Martingale stopped at a stopping time is a martingale, $M_{t \wedge T}$ is a Martingale. Thus, for all $t \in \mathbb{N}$, $M_{0 \wedge T} = E_0[M_{t \wedge T}]$. Hence:

$$e^{u(Z_0 - Z^*)} = E \left[e^{u(Z_{t \wedge T} - Z^*)} e^{-(t \wedge T)F(u)} \right] \quad (26)$$

We will now show that there exists a range of u , $u < \min(u^*, 0)$, such that the process in (26) is bounded as t goes to infinity. To see why note first that $\forall u < 0$ and $\forall t \in [0, \infty)$, $0 \leq e^{u(Z_{t \wedge T} - Z^*)} \leq 1$ since $Z_{t \wedge T} \geq Z^*$. Next, recall that $\forall u < \min(0, u^*)$, $F(u) > 0$. Hence, $\forall t \in [0, \infty)$, $0 < e^{-(t \wedge T)F(u)} < 1$. Since the process is bounded, we can apply the theorem of dominated convergence to (26), implying that, $\forall u < \min(u^*, 0)$,

$$\begin{aligned} e^{u(Z_0 - Z^*)} &= \lim_{t \rightarrow \infty} E \left[e^{u(Z_{t \wedge T} - Z^*)} e^{-(t \wedge T)F(u)} \right] = E \left[\lim_{t \rightarrow \infty} e^{u(Z_{t \wedge T} - Z^*)} e^{-(t \wedge T)F(u)} \right] \\ &= \begin{cases} e^{u(Z_T - Z^*)} \lim_{t \rightarrow \infty} e^{-tF(u)} & \text{if } T < \infty \\ \lim_{t \rightarrow \infty} e^{u(Z_t - Z^*)} e^{-tF(u)} \leq \lim_{t \rightarrow \infty} e^{-tF(u)} = 0 & \text{if } T \rightarrow \infty \end{cases} \end{aligned}$$

This yields

$$e^{u(Z_0 - Z^*)} = E \left[e^{-u(Z^* - Z_T)} 1_{T < \infty} e^{-TF(u)} \right]. \quad (27)$$

By the definition of the stopping time T we have $Z_T \in]Z^* - 1 + \delta, Z^*]$. This implies

$$1 \leq e^{-u(Z^* - Z_T)} < e^{-u(1 - \delta)}. \quad (28)$$

We can at this point prove the following crucial Lemma.

Lemma 3 For $k = -$, $\Pr(T < \infty) = 1$. For $k = +$, $0 < e^{-s(1 - \delta)} e^{-s(Z_0 - Z^*)} < \Pr(T < \infty) \leq e^{-s(Z_0 - Z^*)} < 1$.

Proof. Suppose $k = -$. From our discussion of (24) we have $\forall u < 0, F(u) > 0$. Thus the process $e^{-u(Z^*-Z_T)}1_{T<\infty}e^{-TF(u)}$ is bounded between 0 and $e^{-u(1-\delta)}$. Applying the theorem of dominated convergence to (27) yields, then,

$$\lim_{u \rightarrow 0^-} e^{u(Z_0-Z^*)} = \lim_{u \rightarrow 0^-} E \left[e^{-u(Z^*-Z_T)}1_{T<\infty}e^{-TF(u)} \right] = E \left[\lim_{u \rightarrow 0^-} e^{-u(Z^*-Z_T)}1_{T<\infty}e^{-TF(u)} \right]$$

which is equivalent to

$$\begin{aligned} 1 &= E[1_{T<\infty}] \\ &= \Pr(T < \infty) \end{aligned}$$

Suppose, next, that $k = +$. From our discussion of (24) we have $\forall u < u^* = -s < 0, F(u) > 0$. Thus, $\forall u < u^*$, the process $e^{-u(Z^*-Z_T)}1_{T<\infty}e^{-TF(u)}$ is bounded between 0 and $e^{-u(1-\delta)}$. Applying the theorem of dominated convergence to (27) yields:

$$\lim_{u \rightarrow u^{*-}} e^{u(Z_0-Z^*)} = \lim_{u \rightarrow u^{*-}} E \left[e^{-u(Z^*-Z_T)}1_{T<\infty}e^{-TF(u)} \right] = E \left[\lim_{u \rightarrow u^{*-}} e^{-u(Z^*-Z_T)}1_{T<\infty}e^{-TF(u)} \right]$$

which is equivalent to

$$e^{u^*(Z_0-Z^*)} = E \left[e^{-u^*(Z^*-Z_T)}1_{T<\infty}e^{-TF(u^*)} \right] = E \left[e^{-u^*(Z^*-Z_T)}1_{T<\infty} \right] \quad (29)$$

Premultiplying inequality (28) by $1_{T<\infty}$ we have $E[1_{T<\infty}] \leq E[e^{-u^*(Z^*-Z_T)}1_{T<\infty}] < e^{-u^*(1-\delta)}E[1_{T<\infty}]$. Combined with (25) and (29) this leads to

$$0 < e^{-s(1-\delta)}e^{-s(Z_0-Z^*)} < \Pr(T < \infty) \leq e^{-s(Z_0-Z^*)} < 1$$

■

If $k = -$, Lemma (3) implies that $\Pr\{\exists T < \infty \mid r_T \in \Omega_{WDLT}\} = 1$, proving the first part of Proposition (5). If $k = +$, using the definitions (19), (20), (21) and (22) we can rewrite the chain of inequalities given by $0 < e^{-s(1-\delta)}e^{-s(Z_0-Z^*)} < \Pr(T < \infty) \leq e^{-s(Z_0-Z^*)} < 1$ as $0 < \frac{\lambda_W}{1-\lambda_P} \frac{r(V)}{r_0} < P_{WDLT} \leq \frac{r(V)}{r_0} < 1$. Hence, with probability $0 < 1 - P_{WDLT} < 1$, the process does not enter the trap in finite time and stays in the learning regime. Finally, by the Strong Law of Large Numbers, the process r_t must converge to perfect learning, i.e., $r_t \rightarrow \infty$. This proves the second part of Proposition (5).

B.5 Proof of Corollary 1

When $\lambda_W = \lambda_P = \lambda$, the state space of the stochastic process (9) is isomorphic to $\mathbb{Z}$ (e.g., the termination value of the process is the same after the sequence war-war-peace and after the sequence peace-war-war). This implies that the value of the belief at the stopping time T is deterministically determined by the initial condition: $\pi_T = \bar{\pi}(\pi_0)$ where $\ln(\bar{\pi}(\pi_0)/(1-\bar{\pi}(\pi_0))) \leq \ln \bar{r}_W(V) < \ln(\bar{\pi}(\pi_0)/(1-\bar{\pi}(\pi_0))) - \ln \frac{1-\lambda}{\lambda}$. Since the belief $\pi_t \in [0, 1]$ is a (bounded) Martingale, then

$$\begin{aligned} \forall t, \pi_0 &= E[\pi_t] \\ &= \pi_0 \times E[\pi_t \mid k = +] + (1 - \pi_0) \times E[\pi_t \mid k = -], \end{aligned} \quad (30)$$

The Martingale Convergence Theorem implies that π_t converges almost surely to a random variable π^* . When $k = -$, the Strong Law of Large Numbers implies that $\pi^* = \bar{\pi}(\pi_0)$. When $k = +$, the support of

π^* is equal to the two atoms $\{\bar{\pi}(\pi_0), 1\}$ with a probability distribution $(P_{WDLT}, 1 - P_{WDLT})$. Taking the limit of (30) as $t \rightarrow +\infty$ yields:

$$\forall t, \pi_0 = \pi_0 \times [P_{WDLT} \times \bar{\pi} + (1 - P_{WDLT}) \times 1] + (1 - \pi_0) \times \bar{\pi}(\pi_0),$$

where,

$$P_{WDLT} = \frac{\bar{\pi}(\pi_0) / (1 - \bar{\pi}(\pi_0))}{\pi_0 / (1 - \pi_0)},$$

proving the first part of the corollary (1).

To prove that $E(T \mid T < \infty) = \Delta_0 / (1 - 2\lambda)$, we return to the proof of Proposition 5, and note that when $\lambda_W = \lambda_P = \lambda < 1/3$ the stochastic process Z_t in equation (19) is a random walk without drift, i.e., $\delta = 0$. Moreover, $\rho > 1/2$ iff $k = +$ and $\rho < 1/2$ iff $k = -$. Thus, $Z_t = Z_{t-1} \pm 1$ with probability $(\rho, 1 - \rho)$, where $\rho = 1_{k=+} \times (1 - \lambda) + 1_{k=-} \times \lambda$. As proven above, Z_T (where T denotes the stopping time) is entirely determined by initial conditions: $Z_T = Z^*(Z_0)$ where $Z^*(Z_0) \leq \ln \bar{r}_W(V) / s < Z^*(Z_0) + 1$ and $(Z^*(Z_0) - Z_0) \in \mathbb{Z}^-$. Moreover, $u^* = \ln \frac{1-\rho}{\rho}$ (where $F(u)$ and u are defined by (24) in the proof of Proposition 5, and u^* is the non-zero root of F), implying that u^* is negative (positive) if and only if $k = +$ ($k = -$). Equation (27) becomes, then,

$$\forall u < \min(u^*, 0), e^{u(Z_0 - Z^*)} = E \left[1_{T < \infty} e^{-TF(u)} \right]. \quad (31)$$

Equation (31) is the Laplace transform of the random variable T when $1_{T < \infty} = 1$:

$$\forall F > 0, E \left[1_{T < \infty} e^{-T \cdot F} \right] = e^{u(Z_0 - Z^*(Z_0))}. \quad (32)$$

Differentiating (32) with respect to F yields:

$$E \left[-1_{T < \infty} T e^{-(T+1) \cdot F} \right] = -(Z_0 - Z^*(Z_0)) e^{-u(Z_0 - Z^*(Z_0))} \frac{\partial u}{\partial F}$$

Using (24) leads to

$$E \left[-1_{T < \infty} T e^{-(T+1) \cdot F} \right] = -(Z_0 - Z^*(Z_0)) e^{-u(Z_0 - Z^*(Z_0))} \frac{\rho e^u + (1 - \rho) e^{-u}}{\rho e^u - (1 - \rho) e^{-u}}$$

Applying the dominated convergence theorem when $u \uparrow \min(0, u^*)$ (and so $F \downarrow 0$) yields:

$$E \left[1_{T < \infty} T \right] = (Z_0 - Z^*(Z_0)) e^{-\min(u^*, 0)(Z_0 - Z^*(Z_0))} \frac{1}{2\rho e^{\min(u^*, 0)} - 1} \quad (33)$$

By definition,

$$E \left[1_{T < \infty} T \right] = E \left[T \mid 1_{T < \infty} \right] E \left[1_{T < \infty} \right] = E \left[T \mid T < \infty \right] \Pr \left[T < \infty \right]. \quad (34)$$

Setting $Z_T = Z^*(Z_0)$ equation (29) becomes $\Pr \left[T < \infty \right] = e^{-\min(0, u^*)(Z_0 - Z^*(Z_0))}$. Together with (34) this leads to

$$E \left[1_{T < \infty} T \right] = E \left[T \mid T < \infty \right] e^{-\min(0, u^*)(Z_0 - Z^*(Z_0))}$$

Combining (33) and (34) yields

$$E \left[T \mid T < \infty \right] = \frac{Z_0 - Z^*(Z_0)}{2\rho e^{\min(u^*, 0)} - 1} = \frac{Z_0 - Z^*(Z_0)}{|1 - 2\rho|} = \frac{\Delta_0}{1 - 2\lambda},$$

proving the second part of corollary (1).

B.6 Proof of Proposition 8

For a given $(\pi_P, \beta_t^-, \beta_t^+, z) \in [0, 1]^4$ let denote $G^+(n^-, n^+), G^-(n^-, n^+)$ the RHS of (13). Let define the function G such that $\forall (n^-, n^+) \in [0, 1]^2, G(n^-, n^+) \equiv [G^-(n^-, n^+), G^+(n^-, n^+)]$. An equilibrium of the investment game corresponds to a fixed point of G . Following its definition we see that G is a continuous map from $[0, 1]^2$ to $[0, 1]^2$. And the Brouwer fixed point theorem implies that G has at least one fixed point.

To prove uniqueness of the equilibrium we proceed by contradiction. Let us assume that G admits two fixed points $n_0 \equiv (n_0^-, n_0^+)$ and $n_1 \equiv (n_1^-, n_1^+)$. We define $a \equiv (a^-, a^+) \in [0, 1]^2$ and $b \equiv (b^-, b^+) \in [0, 1]^2$ as the intercepts of the line $n_0 n_1$ with the convex hull of $[0, 1]^2$. By definition, the points n_0 and n_1 are included in the segment $[a, b]$. Without loss of generality let us also rank them such that $n_0 \in [a, n_1]$ and $n_1 \in [n_0, b]$. In term of linear combinations we define $(\sigma_0, \sigma_1) \in [0, 1]^2$ such that

$$n_0 = \sigma_0 \times a + (1 - \sigma_0) \times n_1 \quad (35)$$

$$n_1 = \sigma_1 \times n_0 + (1 - \sigma_1) \times b \quad (36)$$

Following assumption 1 we know that $G^-(n^-, n^+)$ and $G^+(n^-, n^+)$ are convex (see the proof of proposition 2). Applying the convexity criterions of G^- and G^+ to (35) and (36) and using the fact that $\{n_0, n_1\}$ are fixed points of G we get

$$n_0^- \leq \sigma_0 \times G^-(a^-) + (1 - \sigma_0) \times n_1^- \quad (37)$$

$$n_1^- \leq \sigma_1 \times n_0^- + (1 - \sigma_1) \times G^-(b^-) \quad (38)$$

$$n_0^+ \leq \sigma_0 \times G^+(a^+) + (1 - \sigma_0) \times n_1^+ \quad (39)$$

$$n_1^+ \leq \sigma_1 \times n_0^+ + (1 - \sigma_1) \times G^+(b^+) \quad (40)$$

The fact that a and b belong to the convex hull of $[0, 1]^2$ implies that the set of equations (35)-(36) and the set of conditions (37)-(40) are not mutually compatible. For example, let us consider the subcase where $a^+ = 1, b^+ = 0$. Following assumption 1 we know that $G^+(1) < 1$ and $G^+(0) \geq 0$. As a consequence equation (39) rewrites as $n_0^+ < \sigma_0 + (1 - \sigma_0) \times n_1^+$ while equation (35) rewrites as $n_0^+ = \sigma_0 + (1 - \sigma_0) \times n_1^+$. A contradiction. The same line of argument applies to the five other generic subcases, namely $(a^+, b^-) = (1, 0), (a^+, b^-) = (1, 1), (a^-, b^-) = (0, 1), (a^+, b^-) = (0, 1), (a^-, b^+) = (0, 0)$. We conclude from this discussion that the equilibrium must be unique.

B.7 Proof of Lemma 2

From continuity of the system (13) we get that the equilibrium value of $(n_A^-, n_A^+, n_B^-, n_B^+)$ is continuous in π_P and β .

Let us first prove that for a given β the equilibrium value $n_B^-(\pi_P, \beta)$ is non decreasing in π_P . We proceed by contradiction. Let us assume that there exists some compact subset of $[0, 1]$ such that $n_B^-(\pi_P, \beta)$ is decreasing in π_P . A look at (13) shows that $n_B^+(0, \beta) < n_B^+(1, \beta)$ and $n_B^-(0, \beta) < n_B^-(1, \beta)$. By continuity of the path $[n_B^-(\pi_P, \beta), n_B^+(\pi_P, \beta)]_{\pi_P \in [0, 1]}$ in the space $[0, 1]^2$ we conclude that there must exist a duplet (π_0, π_1) with $\pi_0 < \pi_1$ such that

$$n_B^-(\pi_0, \beta) = n_B^-(\pi_1, \beta) \quad (41)$$

>From (13) we see that any equilibrium is such that

$$n_B^- - \beta F^B(z F^-(z n_B^-)) = n_B^+ - \beta F^B(z F^+(z n_B^+)) \quad (42)$$

Combining (41) and (42) yields

$$n_B^+(\pi_0, \beta) - \beta F^B(z F^+(z n_B^+(\pi_0, \beta))) = n_B^+(\pi_1, \beta) - \beta F^B(z F^+(z n_B^+(\pi_1, \beta))) \quad (43)$$

Following assumption 1 we know that F^B, F^+, F^- are non decreasing and convex. So equality (43) yields

$$n_B^+(\pi_0, \beta) = n_B^+(\pi_1, \beta) \quad (44)$$

The two conditions (41) and (44) imply that $\pi_0 = \pi_1$. A contradiction.

We deduce from the previous discussion that $n_B^-(\pi_P, \beta)$ is non decreasing in π_P . A similar argument can be applied to show that $n_B^-(\pi_P, \beta)$ is also non increasing in β and $n_B^+(\pi_P, \beta)$ is non decreasing in π_P and β .

Finally it is clear that $\forall (\pi_P, \beta) \in [0, 1]^2, n_B^-(\pi_P, \beta) \leq n_B^+(\pi_P, \beta)$. First for a given β , it is true for $\pi_P = 0$ and $\pi_P = 1$. As a consequence, if it was not true, there would be a π such that $n_B^-(\pi, \beta) = n_B^+(\pi, \beta)$. Using (42) this would imply $F^B(zF^-(zn_B^-)) = F^B(zF^+(zn_B^-))$ which is not compatible with the fact that $F^- \text{ FOSD } F^+$.

The trade surplus $S^k(\pi_P, \beta)$ with $k \in \{-, +\}$ is given by equation (16)

$$S^k(\pi_P, \beta) = \int_{zn_B^k(\pi_P, \beta)}^{zn_B^k(\pi_P, \beta)} F^k(\iota) d\iota$$

Given that $F^- \text{ FOSD } F^+$ and given that $n_B^-(\pi_P, \beta) \leq n_B^+(\pi_P, \beta)$ we get that the trade surplus $S^-(\pi_P, \beta), S^+(\pi_P, \beta)$ are continuous and $S^-(\pi_P, \beta) \leq S^+(\pi_P, \beta)$.

B.8 Proof of Proposition 9

B.8.1 Investment/trade continuation game

We assume that the initial share of informed agents, β_0 , is CK. This implies that the initial condition is such that $\beta_0^+ = \beta_0^- = \beta_0$. For a given triplet $(\pi_P, \beta^+, \beta^-)$ the stage game equilibrium is characterized by (13). Given $\beta_0^+ = \beta_0^- = \beta_0$ it is straightforward to show by forward iteration of (14) that for all continuation paths we have $n_B^- \leq n_B^+$ and $\beta^- \leq \beta^+$. As a consequence, for each $(\pi_P, \beta^+, \beta^-)$ the game equilibrium is given by

In the range (regime A)

$$\begin{cases} \beta^- < 1 - \frac{x}{\pi_P z^2} \\ \beta^+ > \frac{-x + 2xz^2(1 - \pi_P) + (1 - z^2)}{(1 - \pi_P)(1 - z^2 + x)z^2} + \frac{x - (1 - z^2)}{x + (1 - z^2)} \frac{\pi_P}{1 - \pi_P} \beta^- \end{cases}$$

the Nash equilibrium is

$$\begin{cases} n_B^+ = z \frac{-x(1 - \beta^+) + (1 - z^2)\beta^+ + \pi_P(1 + x)(1 - \beta^+) + \pi_P z^2(\beta^+ - \beta^-)}{1 - z^2(1 - \pi_P(1 - \beta^-))} \\ n_B^- = \frac{z\pi_P(1 + x)(1 - \beta^-) - zx}{1 - z^2(1 - \pi_P(1 - \beta^-))} \\ n_A^+ = 1 \text{ and } n_A^- = zn_B^- - x \end{cases}$$

In the range (Regime B)

$$\beta^+ < \min \left\{ \frac{1 - \pi_P z^2 - x}{z^2(1 - \pi_P)}, \frac{1 - 2\pi_P z^2}{z^2(1 - \pi_P)} + \frac{\pi_P}{(1 - \pi_P)} \beta^- \right\}$$

the Nash equilibrium is

$$\begin{cases} n_B^+ = \frac{zx(\beta^+ + (1 - \beta^+)\pi_P)}{1 - z^2(\beta^+ + (1 - \beta^+)\pi_P)} \\ n_B^- = \frac{zx\pi_P(1 - \beta^-)}{1 - z^2(\beta^+ + (1 - \beta^+)\pi_P)} \\ n_A^+ = zn_B^+ + x \text{ and } n_A^- = 0 \end{cases}$$

In the range (regime C)

$$\begin{cases} \beta^- > 1 - \frac{x}{\pi_P z^2} \\ \beta^+ > \frac{1 - \pi_P z^2 - x}{z^2(1 - \pi_P)} \end{cases}$$

the Nash equilibrium is

$$\begin{cases} n_B^+ = z(1 - \beta^+)\pi_P + z\beta^+ \\ n_B^- = z(1 - \beta^-)\pi_P \\ n_A^+ = 1 \text{ and } n_A^- = 0 \end{cases}$$

In the range (regime D)

$$\begin{cases} \beta^+ > \frac{1 - 2\pi_P z^2}{z^2(1 - \pi_P)} + \frac{\pi_P}{(1 - \pi_P)}\beta^- \\ \beta^+ < \frac{-x + 2xz^2(1 - \pi_P) + (1 - z^2)}{(1 - \pi_P)(1 - z^2 + x)z^2} + \frac{x - (1 - z^2)}{x + (1 - z^2)} \frac{\pi_P}{1 - \pi_P}\beta^- \end{cases}$$

the Nash equilibrium is

$$\begin{cases} n_B^+ = xz \frac{-1 + (2 - z^2)\beta^+ + \pi_P(2(1 - \beta^+) + z^2(\beta^+ - \beta^-))}{(1 - z^2)(1 - z^2(\beta^+ - \pi_P(\beta^+ - \beta^-)))} \\ n_B^- = xz \frac{-1 + z^2\beta^+ - \pi_P(z^2(\beta^+ - \beta^-) - 2(1 - \beta^-))}{(1 - z^2)(1 - z^2(\beta^+ - \pi_P(\beta^+ - \beta^-)))} \\ n_A^+ = zn_B^+ + x \text{ and } n_A^- = zn_B^- - x \end{cases}$$

As a consequence, when $\pi_P < x/z^2$, the economy is in regime B iff $\beta^+ < \frac{1 - \pi_P z^2 - x}{z^2(1 - \pi_P)}$. Otherwise it is in regime C. When $x/z^2 < \pi_P < 1/2z^2$, the economy is in regime A iff $\beta^- < 1 - \frac{x}{\pi_P z^2}$ and $\beta^+ > \frac{-x + 2xz^2(1 - \pi_P) + (1 - z^2)}{(1 - \pi_P)(1 - z^2 + x)z^2} + \frac{x - (1 - z^2)}{x + (1 - z^2)} \frac{\pi_P}{1 - \pi_P}\beta^-$. It is in regime B iff $\beta^+ < \min \left\{ \frac{1 - \pi_P z^2 - x}{z^2(1 - \pi_P)}, \frac{1 - 2\pi_P z^2}{z^2(1 - \pi_P)} + \frac{\pi_P}{(1 - \pi_P)}\beta^- \right\}$. It is in regime C iff $\beta^- > 1 - \frac{x}{\pi_P z^2}$ and $\beta^+ > \frac{1 - \pi_P z^2 - x}{z^2(1 - \pi_P)}$. Otherwise it is in regime D. When $\pi_P > 1/2z^2$, the economy is in regime C iff $\beta^- > 1 - \frac{x}{\pi_P z^2}$. It is in regime D when $\beta^+ < \frac{-x + 2xz^2(1 - \pi_P) + (1 - z^2)}{(1 - \pi_P)(1 - z^2 + x)z^2} + \frac{x - (1 - z^2)}{x + (1 - z^2)} \frac{\pi_P}{1 - \pi_P}\beta^-$ and in regime A otherwise. A sufficient condition for regime D to disappear is $\pi_P > \frac{x + (1 - z^2)}{2x}$.

B.8.2 Existence of Ω_{WDLT}

Hereafter we rescale the problem in term of odds ratio $r \equiv \pi_P/(1 - \pi_P)$. We first characterize a subset of the WDLT in the following lemma.

Lemma 4 *Assume $V \in (S^+(0), \min(S^+(1), 1/2))$. For all $\theta \geq zx/(1 + zx)$, there exists $(r_0(\theta), \beta_\infty^+(\theta)) \in \mathbb{R}^+ \times (0, 1]$ such that $\{(r, \beta_0^+, \beta_0^-) \in \mathbb{R}^+ \times [0, 1]^2 \mid 0 \leq r \leq r_0(\theta), 0 \leq \beta_0^+ = \beta_0^- \leq \beta_\infty^+(\theta)\}$ is a non empty subset of Ω_{WDLT} .*

Proof. We first provide a proof of this lemma in the limit case $z = 1$. Then we consider the general case $z < 1$.

Let us consider $\theta \in [0, 1]$ and $(r, \beta) \in \mathbb{R}^+ \times [0, 1]$. We want to provide a sufficient condition for $(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta) \in \Omega_{WDLT}$. From definition (7) $(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta) \in \Omega_{WDLT}$ iff for all continuation paths $[r_t, \beta_t^+, \beta_t^-]_0^\infty$, $r_t = r$ and

$$S^+(r, \beta_t^+, \beta_t^-) < V \quad (45)$$

In all generality the stochastic dynamics of S^+ is difficult to characterize except if $(r, \beta_t^+, \beta_t^-)$ are low enough. Hence we also impose the following additional sufficient condition which guaranties that all continuation paths $[r, \beta_t^+, \beta_t^-]_0^\infty$ evolve within regime B (see Section B.8.1)

$$\begin{cases} \beta_t^+ \leq 1 - (1+r)x \text{ for } r \in \left[0, \frac{x}{1-x}\right] \\ \beta_t^+ \leq 1 - r \text{ for } r \in \left[\frac{x}{1-x}, 1\right] \end{cases} \quad (46)$$

Within regime B the trade surplus is an increasing function of β_t^+

$$S^+(r, \beta_t^+, \beta_t^-) = (n_{Bt}^+ + x)^2 / 2 = \frac{(1+r)^2 x^2 / 2}{(1 - \beta_t^+)^2} \quad (47)$$

and the lom of β_t^+ is derived from (14)

$$\beta_t^+ / (1 - \theta) = (1 - I_{WAR,t-1}) [rx + (1+x)\beta_{t-1}^+] + I_{WAR,t-1}\beta_{t-1}^+ \quad (48)$$

We notice that the threshold β_∞^+ defined by

$$\beta_\infty^+(r, \theta) = xr / \left(\frac{\theta}{1 - \theta} - x \right) \quad (49)$$

corresponds to the fixed-point of (48) when $I_{WAR,t-1} = 0$ for all t . Moreover it is clear from (48) that $\beta_{t-1}^+ \leq \beta_\infty^+$ implies $\beta_t^+ \leq \beta_\infty^+$. We impose an additional sufficient condition on the initial condition, namely that

$$\beta \leq \beta_\infty^+(r, \theta) \quad (50)$$

This implies $\beta_t^+ \leq \beta_\infty^+(r, \theta)$ for all continuation paths $[r, \beta_t^+, \beta_t^-]_0^\infty$. As a consequence for all continuation paths $S^+(r, \beta_t^+, \beta_t^-) \leq S^+(r, \beta_\infty^+, \beta_t^-)$. Combining (47) and (49), we get that the sufficient condition (46) becomes

$$\theta \geq \Sigma(r) \equiv \begin{cases} \frac{(1-x)x}{1/(1+r)-x^2} \text{ for } r \in \left[0, \frac{x}{1-x}\right] \\ \frac{1-x/\sqrt{2V}}{1-(1+x)/\sqrt{2V}+1/(1+r)x} \text{ for } r \in \left[\frac{x}{1-x}, 1\right] \end{cases} \quad (51)$$

and the necessary and sufficient condition (45) becomes

$$\theta \geq \Gamma(r) \equiv \frac{1 - x/\sqrt{2V}}{1 + 1/x(1+r) - (1+x)/\sqrt{2V}} \quad (52)$$

The two functions $\Sigma(r)$ and $\Gamma(r)$ are upward-sloping with $\Sigma(0) = \Gamma(0) = x/(1+x)$. Moreover $V \in [S^+(0), S^-(1)]$ implies $V \leq 1/2$ which in turn implies $\Sigma'(0) < \Gamma'(0)$. By L'Hospital rule this implies that there exists an open neighborhood of $r = 0$ such that $\Sigma(r) < \Gamma(r)$. Moreover, for the set $\{(r, \theta) \in [0, 1] \times [0, 1] \mid \theta \geq \Sigma(r) \text{ and } \theta \geq \Gamma(r)\}$ the two conditions SC and NSC_B are verified. We define $\Gamma^{-1}(\theta)$ for $\theta \in [0, r^*]$ and $\underline{r}_0(\theta) \equiv \Gamma^{-1}(r^*)$ for $\theta \in [r^*, 1]$ and $\beta_\infty^+(\theta) \equiv \beta_\infty^+(\underline{r}_0(\theta), \theta)$ where β_∞^+ is given by (50). Consequently for any $\theta \geq x/(1+x)$, the set $\{0 \leq r \leq \underline{r}_0(\theta), 0 \leq \beta_0^+ = \beta_0^- \leq \beta_\infty^+(\theta)\}$ is non empty. Moreover for all continuation paths $[r, \beta_t^+, \beta_t^-]_0^\infty$ the condition (45) is verified; so $(r, \beta_0^+, \beta_0^-) \in \Omega_{WDLT}$.

Let us consider now the general case $z < 1$). We want to show that the conditions $\Sigma(r)$ and $\Gamma(r)$ still satisfy $\Sigma(0) = \Gamma(0) = x/(r+x)$ and $\Sigma'(0) < \Gamma'(0)$. If correct, by L'Hospital rule, we deduce that

there exists an open neighborhood of $r = 0$ such that $\Sigma(r) < \Gamma(r)$. This allows us to conclude the proof in a similar way than for $z = 1$.

For $z < 1$ the sufficient condition (46) becomes

$$\beta_t^+ \leq \frac{1-x}{z^2} (1+r) - r \quad (53)$$

The condition (47) becomes

$$S^+(r, \beta_t^+, \beta_t^-) \equiv \frac{x^2}{2(1-z^2(\beta_t^+ + (1-\beta_t^+)r/(1+r)))^2} < V \quad (54)$$

The lom of (62) becomes

$$\frac{\beta_t^+}{1-\theta} = (1 - I_{WAR,t-1}) \left[\beta_{t-1}^+ + \frac{xz(1-\beta_{t-1}^+)((1-\beta_{t-1}^+)r/(1+r) + \beta_{t-1}^+)}{1-z^2((1-\beta_{t-1}^+)r/(1+r) + \beta_{t-1}^+)} \right] + I_{WAR,t-1}\beta_{t-1}^+ \quad (55)$$

As a consequence the threshold $\beta_\infty^+(r, \theta)$ is now defined as the root of the second order polynomial $A_1(\beta_\infty^+)^2 + A_2\beta_\infty^+ + A_3 = 0$ with $A_1(r, \theta) \equiv -z^2\left(\frac{\theta}{1-\theta} - \frac{x}{z}\right)$ and $A_2(r, \theta) \equiv \frac{\theta}{1-\theta}(1+r-rz^2) - xz(1-r)$ and $A_3(r, \theta) \equiv -x zr$. For r close to 0, a first order Taylor expansion leads to $\beta_\infty^+(r, \theta) = \left[-A_2(r, \theta) \pm \sqrt{A_2(r, \theta)^2 - 4A_1(r, \theta)A_3(r, \theta)}\right] / 2A_1(r, \theta) \underset{r \sim 0}{\simeq} -A_3(r, \theta)/A_2(r, \theta)$. For $\frac{\theta}{1-\theta} \geq xz$ this yields

$$\beta_\infty^+(r, \theta) \simeq \frac{x zr}{\frac{\theta}{1-\theta} - xz} \quad (56)$$

The conditions $\Sigma(r)$ and $\Gamma(r)$ are obtained by plunging $\beta_\infty^+(r, \theta)$ into (53) and (54). This leads to

$$\theta \geq \Sigma(r) \equiv \frac{(1-x)x}{(1-x)(x+1/z) - rz/(1+r)} \quad (57)$$

$$\theta \geq \Gamma(r) \equiv \frac{1-x/\sqrt{2V}}{\left(1-x/\sqrt{2V}\right)(1+1/xz) - rz/(1+r)x} \quad (58)$$

where condition (58) can be verified if and only if $V > x^2/2 = S^+(0)$.

We want now to analyze the behavior of $\Sigma(r)$ and $\Gamma(r)$ in the neighborhood of $r = 0$. First we notice that $\Sigma(r) = \Gamma(0) = x/(x+z)$. Secondly we get that $\Sigma'(0) < \Gamma'(0)$ if and only if $V < 1/2$. ■

>From lemma 4 we see that $\theta \geq x/(z+x)$ implies $\Omega_{WDLT} \neq \emptyset$. This is a sufficient condition for the existence of Ω_{WDLT} . We want to show now that this is also a necessary condition. We proceed by contradiction.

Let us assume that there exists $\hat{\theta} < x/(z+x)$ such that $\Omega_{WDLT} \neq \emptyset$. Hence there exists at least one couple $(\bar{r}, \bar{\beta}) \in \mathbb{R} \times [0, 1]$ such that $(r_0 = \bar{r}, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta}) \in \Omega_{WDLT}$. Let us consider $r \in (0, \bar{r})$. We define $C(r, \bar{\beta})$ as the set of equilibrium paths $[r_t, \beta_t^+, \beta_t^-]_0^\infty$ starting with the initial condition $(r_0 = r, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta})$. We compare it to $C(\bar{r}, \bar{\beta})$, the set of equilibrium paths $[\bar{r}_t, \bar{\beta}_t^+, \bar{\beta}_t^-]_0^\infty$ starting with the initial condition $(r_0 = \bar{r}, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta})$. From Section B.8.1 we know that $\frac{\partial n_b^+}{\partial r} \Big|_{\beta^+, \beta^-} > 0$ and $\frac{\partial n_b^-}{\partial r} \Big|_{\beta^+, \beta^-} > 0$. Given that $r < \bar{r}$ and $\beta_0^+ = \beta_0^- = \bar{\beta}_0^+ = \bar{\beta}_0^- = \bar{\beta}$ a forward iteration on the laws of motion (14) implies that for all path in $C(r, \bar{\beta})$, at each period t , there exists a path in $C(\bar{r}, \bar{\beta})$

such that $n_{bt}^+ \leq \bar{n}_{bt}^+$. As a consequence $\forall t, R^+(n_{bt}^+) \leq R^+(\bar{n}_{bt}^+)$. Following the trade surplus definition (2) this implies $S^{\max}(r, \beta) < S^{\max}(\bar{r}, \bar{\beta})$ where $S^{\max}(r, \beta) = \arg \max_{C(r, \beta)} S^+$. From definition (7) we know

that $S^{\max}(\bar{r}, \bar{\beta}) < V$. This in turn leads to $S^{\max}(r, \bar{\beta}) < V$ and so $(r_0 = r, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta}) \in \Omega_{WDLT}$.

Any continuation path $[r_t, \beta_t^+, \beta_t^-]_0^\infty$ of $(r_0 = r, \beta_0^+ = \bar{\beta}, \beta_0^- = \bar{\beta})$ is almost surely in Ω_{WDLT} : In particular this must be the case for the subset of continuation paths characterized by a non interrupted series of war shocks over the period $\{1, \dots, T\}$. Given that this sequence has a positive probability $(\lambda_W)^T$, this implies that $(r_T, \beta_T^+, \beta_T^-) \in \Omega_{WDLT}$: From definition (7) this means $r_T = r_t$. Along such a sequence of wars, trade is fully disrupted and the share of informed agents is depleted as memory loss takes place at a pace θ . We have: $(\beta_T^-, \beta_T^+) = \theta^T (\bar{\beta}, \bar{\beta})$. As a consequence we get that $\forall T < \infty, (r, \theta^T \bar{\beta}, \theta^T \bar{\beta}) \in \Omega_{WDLT}$.

For $\varepsilon, \eta > 0$, let us define the set $\Omega(\varepsilon, \eta) \equiv \{(r, \beta_0^+, \beta_0^-) \in \mathbb{R} \times [0, 1]^2 \mid 0 < r < \varepsilon, 0 \leq \beta_0^+ = \beta_0^- < \eta\}$. From the previous discussion we see that $\Omega_{WDLT} \neq \emptyset$ implies $\exists \varepsilon, \eta > 0$ such that $\Omega(\varepsilon, \eta) \subset \Omega_{WDLT}$. The interpretation is clear: if the WDLT is non empty, it must include the cases where beliefs are extremely pessimistic and the initial share of informed agents is very low. Hence there exists at least one $(\hat{r}, \hat{\beta}) \in \Omega(\varepsilon, \eta)$ such that $\Sigma(\hat{r}) < \Gamma(\hat{r})$ and $\hat{\beta} < \beta_\infty^+(\hat{\theta})$. Moreover the same line of reasoning as above implies that $(r_0 = \hat{r}, \beta_0^+ = \hat{\beta}, \beta_0^- = \hat{\beta}) \in \Omega_{WDLT}$ for $\theta = \hat{\theta} \Rightarrow (r_0 = \hat{r}, \beta_0^+ = \hat{\beta}, \beta_0^- = \hat{\beta}) \in \Omega_{WDLT}$ for $\theta > \hat{\theta}$.

In particular this is the case for any θ such that $\Sigma(\hat{r}) < \theta < \Gamma(\hat{r})$, a non empty range given L'Hospital rule and the fact that $\Sigma(0) = \Gamma(0)$ and $\Sigma'(0) < \Gamma'(0)$. For such a θ we have $S^+(r, \beta_t^+, \beta_t^-) < V$ for all continuation paths $[r_t, \beta_t^+, \beta_t^-]_0^\infty$, starting with initial condition $(r_0 = \hat{r}, \beta_0^+ = \beta_0^- < \beta_\infty^+(\theta))$. This means that condition (45) is verified; but this condition is equivalent to (52), namely $\theta \geq \Gamma(\hat{r})$. A contradiction.

B.8.3 Existence of Ω_{PDLT}

This proof follows the same line than the previous one. First for all $\theta \geq \theta_P$ with $\theta_P \equiv 1 - \frac{z^2 - (x + \sqrt{2V})}{z^3(1 - \sqrt{2V})}$ we are able to characterize a non empty subset of Ω_{PDLT} . Then for $\theta < \theta_P$ we show by contradiction that Ω_{PDLT} must be empty.

We first start with the specific case $z = 1$. Let consider $\theta \in [0, 1]$ and $(r, \beta) \in [1, +\infty) \times [0, 1]$. We want to provide a sufficient condition for $(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta) \in \Omega_{PDLT}$. From definition (7) $(r_0 = r, \beta_0^+ = \beta, \beta_0^- = \beta) \in \Omega_{PDLT}$ iff for all continuation paths $[r_t, \beta_t^+, \beta_t^-]_0^\infty$, $r_t = r$ and

$$S^-(r, \beta_t^+, \beta_t^-) > V \quad (59)$$

In all generality the stochastic dynamics of S^- is difficult to characterize except if r is larger than 1 and (β_t^+, β_t^-) are low enough. Hence we also impose the following additional sufficient condition which guaranties that all continuation paths $[r, \beta_t^+, \beta_t^-]_0^\infty$ evolve within regime A (see Section B.8.1)

$$\beta_t^- \leq 1 - x(1 + r)/r \text{ and } r \geq 1 \quad (60)$$

Within regime A the trade surplus is an increasing function of β_t^-

$$S^-(r, \beta_t^+, \beta_t^-) = (n_{Bt}^- - x)^2 / 2 = \left(1 - \frac{x(1 + r)}{(1 - \beta_t^-)r}\right)^2 / 2 \quad (61)$$

and the lom of β_t^- is derived from (14)

$$\beta_t^- / (1 - \theta) = (1 - I_{WAR,t-1}) [1 - x/r - x\beta_{t-1}^-] + I_{WAR,t-1}\beta_{t-1}^- \quad (62)$$

This is an oscillating dynamics upper bounded by $\beta_{\max}^-$ as long as $I_{WAR,t-1} = 0$ and condition (60) is satisfied. Hence we get that $\beta \leq \beta_{\max}^-$ implies that for all continuation paths we have $\beta_t^- \leq \beta_{\max}^-$ where

$$\beta_{\max}^-(r, \theta) = (1 - \theta)(1 - x/r) \quad (63)$$

Combining (60) and (63), we get that the sufficient condition (60) becomes

$$\theta \geq \Phi(r) \equiv \frac{x}{1 + x - x(1 + r)/r} \quad (64)$$

and the necessary and sufficient condition (59) becomes

$$\theta \geq \Delta(r) \equiv \frac{(1 + r)/(1 - \sqrt{2V}) - 1}{r/x - 1} \quad (65)$$

In the space $(r, \theta) \in [1, +\infty) \times [0, 1]$ the two functions $\Phi(r)$ and $\Delta(r)$ are decreasing in r with $\Phi(+\infty) = x$ and $\Delta(+\infty) = x/(1 - \sqrt{2V})$. Given that $\Phi(+\infty) < \Delta(+\infty)$ we infer that for all $\theta \geq \Delta(+\infty)$ there exists a threshold $\tilde{r}(\theta)$ such that $\forall r > \tilde{r}(\theta)$, the couple (r, θ) verifies conditions (64) and (65); this in turn implies that for $\beta \leq \beta_{\max}^-(r, \theta)$ we have $(r, \beta, \beta) \in \Omega_{PDLT}$.

Let us consider now the general case $z < 1$. Condition (60) and (61) become

$$\beta_t^- \leq 1 - \frac{x(1 + r)}{z^2 r} \quad (66)$$

$$S^-(r, \beta_t^+, \beta_t^-) = \left[\frac{z^2 r(1 + x)(1 - \beta_t^-)/(1 + r) - z^2 x}{1 - z^2(1 - r(1 - \beta_t^-)/(1 + r))} - x \right]^2 / 2 > V \quad (67)$$

Moreover the law of motion (48) is given by

$$\beta_t^-/(1 - \theta) = (1 - I_{WAR,t-1}) \left[\frac{z(\frac{r}{1+r}(1 + x)(1 - \beta_{t-1}^-) - x)(1 - \beta_{t-1}^-)}{1 - z^2(1 - \frac{r}{1+r}(1 - \beta_{t-1}^-))} \right] + \beta_{t-1}^-$$

As a consequence we get

$$\beta_{\max}^-(r, \theta) = (1 - \theta)z \frac{r - x}{1 - z^2 + r} \quad (68)$$

Combining (68) with (66) and (67) we get the implicit definitions of $\Phi(r)$ and $\Delta(r)$ respectively

$$(1 - \theta)z \frac{r - x}{1 - z^2 + r} \equiv 1 - \frac{x(1 + r)}{z^2 r}$$

$$\left[\frac{z^2 r(1 + x)(1 - \beta_{\max}^-(r, \theta))/(1 + r) - z^2 x}{1 - z^2(1 - r(1 - \beta_{\max}^-(r, \theta))/(1 + r))} \right] = x + \sqrt{2V}$$

Taking the limit $r \rightarrow +\infty$ in the two previous equations leads to $\beta_{\max}^-(+\infty, \theta) = (1 - \theta)z$ and $\Phi(+\infty) = 1 - \frac{1}{z} + \frac{x}{z^3}$ and $\Delta(+\infty) = 1 - \frac{z^2 - (x + \sqrt{2V})}{z^3(1 - \sqrt{2V})}$. Hence we have $\Phi(+\infty) < \Delta(+\infty)$ iff $V < (x - z^2)^2 / 2 = S^-(1)$. Hence for all $\theta \geq \Delta(+\infty)$ there exists a threshold $\tilde{r}(\theta)$ such that $\forall r > \tilde{r}(\theta)$, the couple (r, θ) verifies conditions (66) and (67); this in turn implies that for $\beta \leq \beta_{\max}^-(r, \theta)$ we have $(r, \beta, \beta) \in \Omega_{PDLT}$.

Let us prove now that $\theta < \Delta(+\infty) \equiv \theta_P$ leads to $\Omega_{PDLT} = \emptyset$. We proceed by contradiction. Let assume that there exists $\hat{\theta} < \theta_P$ such that $\Omega_{PDLT} \neq \emptyset$. Following the same reasoning than in the previous section this implies $\exists \varepsilon, \eta > 0$, such that $\{(r, \beta_0^+, \beta_0^-) \in \mathbb{R} \times [0, 1]^2 \mid 1/\varepsilon < r, 0 \leq \beta_0^+ = \beta_0^- < \eta\} \subset \Omega_{PDLT}$. But this must imply that $\Phi(r) \geq \Delta(r)$ for all $r > 1/\varepsilon$. A contradiction given that $\Phi(+\infty) < \Delta(+\infty)$.

Finally, straightforward computations show that $\theta_P > \theta_W$.

B.9 Proof of Proposition 10

Let us consider a triplet $(r_0, \beta_0^+, \beta_0^-) \notin \Omega_{WDLT}$. We want to show that there is a non-zero measure subset of continuation paths $[\beta_t^-, \beta_t^+, r_t]_{t=0}^\infty$ which enter into Ω_{WDLT} in finite time. To this purpose we aim to exhibit a non-zero measure scenario over the period $0, \dots, T$ such that $(r_T, \beta_T^-, \beta_T^+) \in \Omega_{WDLT}$. The proof proceeds in two stages. First we show that with a strictly positive probability the equilibrium path belief can go in finite time below the cutoff $\underline{r}_0(\theta)$ as given by lemma 4: $\Pr\{\exists T_1 < \infty \mid r_{T_1} < \underline{r}_0(\theta)\} > 0$. Then we show that just after the threshold $\underline{r}_0(\theta)$ is reached, there is a non zero probability sequence of T_2 consecutive wars which takes place over the time range $[T_1, T_1 + T_2]$ such that $(r_{T_1+T_2}, \beta_{T_1+T_2}^+)$ verifies the sufficient condition of lemma 4. Setting $T = T_1 + T_2$ we get $(r_{T_1+T_2}, \beta_{T_1+T_2}^+, \beta_{T_1+T_2}^-) \in \Omega_{WDLT}$.

Stage 1: Given the initial conditions $(r_0, \beta_0^+, \beta_0^-) \notin \Omega_{WDLT}$ there must be a strictly positive measure subset S_1 of continuation paths which violate definition (7). This implies that the first passage time $t_1 \equiv \arg \min_{S_1} \{t \mid S^-(t) < V < S^+(t)\}$ is finite. In particular let us consider the subset $\sigma_1 \subset S_1$ consisting of paths such that there is War at date t_1 . The subset σ_1 has a strictly positive measure. Moreover we have: $r_{t_1} = r_0$ and $\ln r_{t_1+1} = \ln r_{t_1} - \ln \frac{1-\lambda_P}{\lambda_W}$. There are two possibilities. Either $(r_{t_1+1}, \beta_{t_1+1}^+)$ verifies condition (4) and the proof is completed. Or there is a strictly positive measure subset $S_2 \subset \sigma_1$ of continuation paths which violate definition (7). This implies that the first passage time $t_2 \equiv \arg \min_{S_2} \{t \mid S^-(t) < V < S^+(t)\}$ is finite. In particular, let us consider the subset of $\sigma_2 \subset S_2$ consisting of paths such that there is War at date t_2 . The subset σ_2 has a positive measure and we have $r_{t_2} = r_{t_1} = r_0$ and $\ln r_{t_2+1} = \ln r_{t_2} - \ln \frac{1-\lambda_P}{\lambda_W} = \ln r_0 - 2 \ln \frac{1-\lambda_P}{\lambda_W}$. This line of reasoning is applied for a finite number of N steps corresponding to the date t_N such that $\ln r_{t_N} = \ln r_0 - N \ln \frac{1-\lambda_P}{\lambda_W}$ and $r_{t_N} < \underline{r}_0(\theta)$. Then we know that the subset σ_{t_N} has a strictly positive measure and we set $T_1 = t_N$.

Stage 2: The continuation paths starting at date T_1 are such that $r_{T_1} < \underline{r}_0(\theta)$ and $\beta_{T_1} \in [0, 1]$. Let us consider the subset of continuation paths with a sequence of T war shocks over the period $t = T_1, \dots, t = T_1 + T$. This subset has a measure $(\lambda_W)^T > 0$. Moreover no trade takes place during the sequence of war shocks. Thus, at date $T_1 + T$, and using (14), we get $\beta_{T_1+T}^+ = (1-\theta)^T \beta_{T_1}^+$ and $\underline{r}_0(\theta) > r_{T_1+T}$ and $\ln r_{T_1+T} \geq \ln \hat{r}_{T_1+T} \equiv \ln r_{T_1} - T \ln \frac{1-\lambda_P}{\lambda_W}$ where $\hat{r}_{T_1+T}$ corresponds to the posterior belief arising when all the stage equilibria are informative during the sequence of war shocks. By definition the cutoff $\beta_\infty^+(r, \theta)$ is increasing in r ; thus $\beta_\infty^+(r_{T_1+T}, \theta) \geq \beta_\infty^+(\hat{r}_{T_1+T}, \theta)$. We now want to characterize a finite time T such that $(r_{T_1+T}, \beta_{T_1+T}^+)$ verifies condition of lemma 4: $\beta_{T_1+T}^+ < \beta_\infty^+(r_{T_1+T}, \theta)$. A sufficient condition is $\beta_{T_1+T}^+ < \beta_\infty^+(\hat{r}_{T_1+T}, \theta)$ which is equivalent to $(1-\theta)^T \beta(T_1) < x \hat{r}_{T_1+T} / \left[\frac{\theta}{1-\theta} - x \right]$. Taking the log and using the definition of $\hat{r}_{T_1+T}$ this is equivalent to

$$T \times \log \frac{(1-\theta)(1-\lambda_P)}{\lambda_W} < \ln r_{T_1} - \log \beta(T_1) + \log x - \log \left(\frac{\theta}{1-\theta} - x \right)$$

Clearly, this condition is verified for a sufficiently large (but finite) T as soon as $\log \frac{(1-\theta)(1-\lambda_P)}{\lambda_W} < 0$. Setting $T_2 = T$ we get that $(r_{T_1+T_2}, \beta_{T_1+T_2}^+)$ verifies condition of lemma 4 and so belongs to Ω_{WDLT} .