

A Dynamic Politico-Economic Model of Intergenerational Contracts*

Francesco Lancia[†]

Alessia Russo[‡]

Abstract

This paper investigates the conditions for the emergence of implicit intergenerational contracts without assuming reputation mechanisms, commitment technology and altruism. We present a tractable dynamic politico-economic model in OLG environment where politicians play Markovian strategies in a probabilistic voting environment, setting multidimensional political agenda. Both backward and forward intergenerational transfers, respectively in the form of pension benefits and higher education investments, are simultaneously considered in an endogenous human capital setting with labor income taxation. On one hand, social security sustains investment in public education; on the other hand investment in education creates a dynamic linkage across periods through both human and physical capital driving the economy toward different Welfare State Regimes. Embedding a repeated-voting setup of electoral competition, we find that in a dynamic efficient economy both forward and backward intergenerational transfers simultaneously arise. The equilibrium allocation is education *efficient*, but, due to political overrepresentation of elderly agents, the electoral competition process induces overtaxation compared with a Benevolent Government solution with balanced welfare weights.

JEL Classification: C61, D71, E62, H11

Keywords: aging, Benevolent Government allocation, intergenerational redistribution, Markovian equilibria, repeated voting.

*We are indebted to Graziella Bertocchi and Vincenzo Denicolò for their constant advice and mentorship. Marco Bassetto, Marco Battaglini, Giuseppe Bertola, Michele Boldrin, Giacomo Calzolari, Gianluca Femminis, Timothy Kehoe, David K. Levine, Nicola Pavoni and Gilles Saint Paul provided valuable comments. We also thanks participants at 2010 NSF/NBER/CEME Conference in New York, XV Workshop on "Macroeconomic Dynamics" in Vigo, 2nd Conference on "Recent Development in Macroeconomics" in Mannheim, 24th ESPE Conference in Essen and 8th Workshop on "Macroeconomic Theory" in Pavia, 3rd Workshop in "Economics and Policy Analysis" in Turin, as well as the seminar participants at the Universities of Bologna, Modena and Milan for the useful discussions. All errors are our own.

[†]francesco.lancia@unicatt.it, Catholic University of Milan, Via Necchi 5, 20123 Milano, Italy.

[‡]alessia.russo3@unibo.it, University of Bologna, Strada Maggiore 45, 40125, Bologna and University of Modena and Reggio-Emilia, Viale Berengario 51, 41121, Modena, Italy.

“Why should I care about future generations? What have they done for me?”
(Groucho Marx)

1 Introduction

The implementation and the sustainability of intergenerational redistributive programs are crucial issues in the current political debate. On the one hand, demographic changes alter the economic nature underlying the enforcement of redistributive welfare programs; on the other hand public policies can be manipulated for political purposes, turning out to be more responsive to political pressures than to changes in economic environment. For this reasons, it becomes critical to explore the conditions under which intergenerational transfers, as outcome of a political voting game, can be implemented and why the welfare system developed so far has become a stable institution of modern society.

In most developed democracies the political balance between different age-cohorts has shifted in favor of the elderly. Between 1975 and 1990, the average OECD age of median voter has increased three times faster than in the previous 30 years (OECD, 2007). One of the main implications of this trend is that the conflicts over the public resources allocation arise not only on the economic characteristics and political affiliation but also on the evolution of socio-demographic aspects. Starting from this stylized fact, we focus on the difference in age as the relevant source of heterogeneity to analyze the implementation of intergenerational redistributive programs in democratic societies. The conflicts between age-classes arise not only because of different life-time span but also for the difference in ownership of productive factors as well as in sources of income. While elderly are mainly endowed with physical capital, working-age adults own human capital accumulated when young. As a consequence, the conflict between age-classes is likely to arise on a broad set of fiscal instruments generating different configurations of welfare programs in which economic and political factors may interact as complement or substitute.

Given the special focus of our analyses on the age-class heterogeneity and the inter-classes political conflicts, among all the redistributive programs, we point out the relevant role played by two critical age-target policies: public higher education spending and PAYG social security. These intergenerational redistributive programs, strongly interrelated each other, have deep redistributive impact and have recently even more experienced strong political support in modern democracies. Following the terminology adopted by Rangel (2003), we refer to public higher education spending as forward (i.e. productive) intergenerational transfers and to unfunded pension as backward (i.e. pork barrel and log-rolling) intergenerational transfers. The former are transfers going forward in time generating a cost for the present generation and a benefit for the future one, being crucial for future productive capacity through human capital production. By contrast, the latter are transfers going backward generating a cost for the present generation and a benefit for the past one, giving adults a claim on the future productivity of their young. This different timing of exchange generates different incentive problems. Furthermore, the aging of population plays a relevant role in stressing even more the timing of the intergenerational

bargaining from both a demographic and a political point of view. On one hand, demographic aging has a direct economic impact on the financial solvency of the public system, since the fraction of recipients – the retirees – tends to increase, while the share of contributors - the workers - tends to decrease. On the other hand, the political ideology influence of the elderly age-class in the electoral competition process (i.e. political aging) has an indirect economic impact through the electoral vote. As the population ages, so do the voters. In democratic society population aging leads to an increase in the political representation of the elderly agents, who gather a larger share of voters¹. As politicians seek re-election, they will try to address the needs of the crucial voting group - the old - with generous social security policies.

Empirical motivation relies on recognizing that since the Second World War the developed countries have experienced dynamically efficient growth path, i.e. the economic growth rate falls below the interest rate (Abel et al., 1989). Under such economic scenario, exacerbated by the recent demographic transition, there would be no elements in the previous literature to justify the implementation of PAYG social security programs², which would depress savings further and, consequently, intertemporal consumption and, in turns, economic growth. Furthermore, even in the case of dynamic inefficiency scenario, a PAYG social security scheme is a dynamically inconsistent agreement between successive generations. Adult generations would be better off discontinuing the PAYG scheme and setting up a new one. However, quite surprising, the share of per-capita GDP used to finance higher education and social security following retributive schemes remains substantial³. For this reasons, the existence of unfunded pension schemes seems puzzling. Hence the question arises of why PAYG schemes survive.

Departing from previous literature, we support the existence of pension system also in an economy experiencing dynamically efficient path, conditionally on the existence of public investment in human capital. The following idea is defended: selfish adults buy insurance for their future old age by paying productive education transfers to their children to raise the labor productivity of the next period. When old they partially grab the bigger output in the form of PAYG transfers by exerting political power in a probabilistic voting environment. Obviously the redistributive scheme works only if the cost of providing a productive transfer is low with respect to the value of receiving a pork-barrel transfer when old. Therefore, if a PAYG pension scheme is introduced⁴, its future beneficiaries may become supportive of higher funding in

¹The political influence of the old is magnified by their homogenous preferences over economic policies. According to Mulligan and Xala-i-Martin (1999) old agents are *single-minded*.

²There are many explanations in the literature on why pay-as-you-go (PAYG) social security has been introduced and then expanded (see Galasso and Profeta, 2002). The classical solution on the puzzle is that, if the economy is on dynamically inefficient path, then the introduction of a PAYG social security system is Pareto improving since it reduces the capital deepening. Among others, see Azariadis and Galasso (2002).

³OECD data show how public tertiary education and social security transfers become increasingly important and strategic among the main components of public expenditure in modern welfare countries. Focusing on European Union members, in 2007 public expenditures on higher education took on average of 1.46 percent of GDP (OECD, 2008) and pension transfers were on average 7.8 percent of GDP (OECD, 2008).

⁴PAYG pension schemes in which pensions are financed by contributions from current workers have often been criticized as detrimental to growth. According to Feldstein (1974) such pension schemes have a negative effect on capital accumulation since they discourage private saving and, unlike in the case of a funded pension system, the payments into the PAYG scheme do not contribute to the national saving. Moreover, the *implicit rate of return* on contributions to a PAYG scheme typically falls short of the interest rate. Therefore according to

public education via taxes. In other words, the existence of a retributive social security system gives incentives to invest optimally in human capital and, as a consequence, it becomes growth enhancing for the economic system. Thus, the two age-specific redistributive programs may self-sustain reaching optimality.

The aim of this article is to provide a tractable dynamic politico-economic theory to analyze how intergenerational conflicts affect, through the political mechanisms in the form of democratic vote, the size and composition of public expenditure in a context of population aging. Focusing on target-specific transfers, our main objective concerns the determination of the economic and institutional conditions which may induce the emergence of a decentralized implicit intergenerational contract based on side payments in the form of PAYG and public education transfers. The economy we study is characterized by overlapping generations living three periods: Young, Adult and Old. Besides their private consumption, both adults and old value the public transfers; the presence of a political system is justified by the need to finance the provision of the public spending under credit market constraint. In our environment there are two types of selfish agents: the private players choose their individual saving and vote for their political representatives and the elected "public player" decides on public policies. The electoral competition takes place in a majoritarian probabilistic environment, where political representatives compete proposing multidimensional fiscal platforms, concerning both the income tax level and the provision of intergenerational transfers in the benefit formula, subject to intra-period balanced budget. We assume away the provision of public goods - a key element in the political economy of fiscal policy⁵ - in order to bring out more clearly the impact of political institutions on intergenerational transfers.

Technically, this paper highlights two main features concerning fiscal policies. First, several political choices have to be set at the same time, so the political space cannot be reduced to a mere unidimensional problem. Second, political decisions and private intertemporal choices are mutually affected over time, then selfish perfect forward-looking agents will internalize how current political choices will influence the evolution of the economy and the implementation of future policies.

The focal point of this paper is the characterization of public policies in a multidimensional dynamic political setting. Following Maskin and Tirole (2001), we embody the "*minor causes should have minor effect*" principle to implement differentiable Stationary Markov subgame Perfect Equilibria (*SMPE*), where the size of income tax rate and the amount of intergenerational transfers are conditioned on the two payoff-relevant asset variables: physical and human capital. We determine the policy rules as equilibrium outcome in a finite horizon environment when the time goes to infinity. As a result we are able to overcome the main limit of the previous literature related to the adoption of trigger strategies equilibria, which are not robust to

such analysis, PAYG pension systems reduce per capita income. This standard argument is focused on physical capital accumulation and fails to take notice of the effect of PAYG pension systems have on the accumulation of human capital, particularly through public education. Primary and secondary education is now overwhelmingly publicly financed in all OECD countries, and universities also receive substantial funding from public sources.

⁵This issue is yet well investigated by Tabellini (1991), Lizzeri and Persico (2001), Hassler et al. (2005) and Bassetto (2008).

such refinement. We finally compare the political equilibrium outcome with the infinite horizon Benevolent Government (*Gvt*) solution in order to point out normative predictions.

Ruling out commitment devices and reputation mechanisms, we reach the following results: 1) the dynamic efficiency condition is the necessary requirement for the simultaneous existence of public education and PAYG programs; 2) the equilibrium political decisions are education *efficient*, while due to politicians' opportunistic behavior, strategic *persistence* underlies the determination of the income tax rate; 3) three different Welfare State Regimes (*WSR*) arise depending on institutional variables, i.e. the adults and old relative bargaining power, and on economic variables, i.e. the endogenous level of physical capital; 4) demographic aging increases the equilibrium per-capita level in public education spending and depending on the *WSR* has an ambiguous effect on the size of government; 5) political competition induces overtaxation and consequently more generous pension benefits compared with the *Gvt* solution.

The paper is organized as follows. Section 2 reviews the literature. In section 3 we present the model characterizing the economic and political environment. Section 4 describes the politico-economic equilibrium in the perfect forward-looking scenario. We provide a complete characterization of both the transition dynamics and the long run economy. In section 5 we discuss the main politico-economic results. Section 6 introduces the *Gvt* problem without commitment, comparing the results with the decentralized one. Section 7 concludes. All proofs are contained in the Appendix.

2 Literature Review

This paper relies on the dynamic political economy literature that incorporates forward-looking decision makers in a multidimensional policy space. In particular it supports and gives new theoretic fundamentals to the existing literature on social security sustainability, which recognizes the link between productive and redistributive public spending. Among the existing theoretical contributions, from a pure economic point of view Boldrin and Montes (2005) formalize public education and PAYG system as two parts of an intergenerational contract where public pension is the return on the investment into the human capital of the next generation. The authors show how an interconnected pension and public education system can replicate the allocation achieved by complete market. Allowing issue-by-issue voting, Rangel (2003) studies in a three-period OLG model the ability of non-market institutions to optimally invest in "forward intergenerational goods" and "backward intergenerational goods". Bellettini and Berti Ceroni (1999) incorporate politics in an OLG model to analyze how societies might sustain public investments (e.g. education) even if the interests of those benefitting from the investment are not represented in the political process. Restricting voting to a binary choice of tax rate and education, the authors study whether a given system can be maintained but do not determine the level of investments in education or social security. As main shortcoming the previous studies have assumed voters play trigger strategies. Although trigger strategies may be analytical convenient, they lead to multiplicity of equilibria. Furthermore, they require coordination among agents and costly enforcement of a punishment technology which may not work when agents

are not patient enough. Finally, they are not robust to refinement such as backward induction in a finite horizon economy when time tends to infinity. Unlike the previous literature, rather than emphasizing complementarity between education and pension payments purely sustained because of reputation mechanisms, our model adopts a different perspective. It focuses on the resolution of the intergenerational conflicts over the determination of the amount of the two public spending components by adopting Markovian strategies.

Many recent studies have identified the public good provision as the critical variable that allows the emergence of an intergenerational redistributive scheme. Bassetto (2008) studies how intergenerational conflicts shape government policies when taxes, transfers and public good are jointly determined. Without public good provision the government would be running a pure redistribution scheme, to which any household that is a net loser would object: hence, the only possible equilibrium would entail no taxes and no transfers. In a simpler underlying economic environment of majority voting Hassler et al. (2007) develop an OLG model of welfare state where tax revenues are used to finance public goods and age-dependent transfers. Studying a linear quadratic economy, they provide analytical solution but the voting strategies equilibrium turns out to be either constant or independent of fundamentals. Unlike these models we exclude public good provision. As a different device to bring out incentives to pay tax we allow for productive and long-lasting impact transfers to the future generation of workers. The dynamic intertemporal linkage occurs by affecting the expected benefits of adults, which when old grab future output by exerting political power in a probabilistic voting environment.

Finally, there are some studies that analyze the interaction between education and social security by adopting an altruistic motive. Kaganovich and Zilcha (1999) study a model where altruistically-motivated parents invest in the human capital of their children. When parents retire, the labor income of their children is taxed to finance their social security benefits. The link between the human capital of children and the parents' retirement benefits is disregarded in each parental educational decision, but it is captured by the government's social optimization. They find that for some parameter values the optimal policy entails not only subsidizing education but also to tax labor income to finance retirement benefits. In Ehrlich and Lui (1998) children provide support to parents in old age, so that parents have an interest in the education of their children over and above any altruistic motives. Despite the usefulness of these studies, they adopt a centralized point of view to justify the simultaneous investment in both redistributive programs. Differently from these studies we incorporate the voting institution to implement the intergenerational contract.

3 The Model

Consider a discrete-time OLG economy populated by an infinite number of overlapping generations of ideologically heterogeneous agents, living up to three-periods: Young, Adult and Old. Every agent born at time t survives with probability one until old age. Population grows at a constant rate $n > -1$, thus the mass of a generation born at time ι and living at time t is equal to $N_t^\iota = N_0 (1 + n)^\iota$.

3.1 Households

Agents born at time $t - 1$ evaluate consumption according to the following intertemporal non altruistic utility function:

$$U_{t-1} = u(c_{1,t}) + \beta u(c_{2,t+1}) \quad (1)$$

where $\beta \in (0, 1)$ is the individual discount rate. $c_{1,t}$ represents the consumption at time t when adult and $c_{2,t+1}$ is the consumption at time $t + 1$ when old. In the first period of life (i.e. childhood), the individual doesn't consume. The function $u(\cdot)$ is concave, twice continuously differentiable and satisfies the usual Inada condition, i.e. $\lim_{c_t \rightarrow 0} u_c(c_t) = +\infty$. Let us assume preferences exhibit logarithmic form, i.e. $u(\cdot) = \log(\cdot)$.

When young, agents spend all their time endowment in acquiring skills if education transfers, e_t , are publicly provided without having access to private credit markets⁶. When adult, the individuals work and consume their labor income, w_t , net of the proportional labor income taxes, τ_t , and individual savings, s_t , while when old, the individuals retire and consume their total income, equal to the sum of pension benefits that their children pass to them in the form of PAYG transfers, p_t , and the capitalized savings at a fixed gross rental price R . Then, the individual budget constraints for adults and old, respectively, are as follows:

$$c_{1,t} \leq C_{1,t}(\tau_t, w_t, s_t) \equiv w_t(1 - \tau_t) - s_t \quad (2)$$

$$c_{2,t+1} \leq C_{2,t+1}(p_{t+1}, s_t) \equiv R s_t + p_{t+1} \quad (3)$$

The net present value at time t of the lifetime wealth of an agent born at time $t - 1$ is given by:

$$I_t = w_t(1 - \tau_t) + \frac{p_{t+1}}{R} \quad (4)$$

3.2 Production

At each time t the homogenous private good, Y_t , is produced by a linear technology that uses labor, L_t , supplied by the adults and physical capital, K_t , supplied by the old. The linearity of the production function can be derived as an equilibrium outcome in a context of perfect international capital mobility and factor price equalization in the presence of goods trade. Furthermore, it emphasizes the intergenerational conflicts due to divergent economic interests between the two productive classes: workers (adults) and capitalists (old). Then, the production function at time t is given by:

$$Y_t = w_t L_t + R K_t \quad (5)$$

⁶For a recent discussion on the economic reasons of missing credit markets financing education, see Kehoe and Levine (2000).

where the wage rate, $w_t = \omega(1 + h_t)$, and the gross rental price to capital, R , are determined by the marginal productivity condition for factor price. At any time t , each adult supplies inelastically one unit of labor, $L_t = N_t^{t-1}$, with productivity equal to ω augmented by the level of human capital acquired the period before, h_t . Without loss of generality we normalize $\omega = 1$.

The human capital of a member of generation t in his working age, h_{t+1} , is an increasing function of the government real expenditure on education and the parental education⁷. Public education transfer is supplied in an egalitarian way, consequently, to each individual is given the same level of it. The following Cobb-Douglas human capital technology is adopted:

$$h_{t+1} = \left(\frac{\alpha h_t + (1 - \alpha) \bar{h}}{1 + n} \right)^\theta e_t^{1-\theta} \quad (6)$$

where $\theta \in (0, 1)$. h_t is the dynasty's human capital at time t and $\bar{h}$ is the constant society endowment of human capital.

Physical capital fully depreciates each period. Consequently, the level of saving determines the dynamics of per-capita physical capital accumulation. The capital market clears when:

$$(1 + n) k_{t+1} = s_t \quad (7)$$

3.3 Fiscal Constitution

In order to provide the intergenerational transfers, the agents in the economy have to devised a politician. In each period, the politician raises revenues through labor income taxes and uses the proceeds to purchase consumption good to pay transfers to the young and old generation. We assume for simplicity that the politician is prevented from borrowing: the public balance must hold in every period. This implies that in each period total benefits paid to old and young equalize total contributions collected from working generations. Expliciting τ_t the balanced budget constraint condition can be written as:

$$(1 + h_t) \tau_t N_t^{t-1} = e_t N_t^t + p_t N_t^{t-2} \quad (8)$$

Ceteris paribus, the more the population ages, the higher the aggregate pension benefits for old agents are and the lower the aggregate education transfers for young people are.

The condition above allows us to reduce the multidimensionality of political platform, $f_t \equiv (e_t, \tau_t, p(e_t, \tau_t))$. Let $\hat{e}_t$ be the maximum feasible value of education transfer at each time t . Then, the balanced budget constraint implies $\tau_t \in (0, 1)$ and $e_t \in (0, \hat{e}_t)$ at each time t .

3.4 Political System

The previous subsection describes the instruments that are available to the politician. We report here how people interact to chose a particular policy. Public policies are chosen through

⁷The importance and the empirical relevance of both the public spending in schooling inputs and the parental education input in the formation of the human capital of the young people has been explored theoretically as well as empirically. For a comprehensive survey of the related literature see Becker and Tomes (1986).

repeated voting system according to a majoritarian rule without commitment where elections take place at the beginning of each period. Young have no political power⁸. To describe the politicians' behavior we consider a probabilistic voting setting⁹. Suppose there are two parties, $l \in \{A, B\}$, which compete to detain political power, with no ability to extract individual rent from election. As a consequence their objective is simply the maximization of the probability of winning elections at each time in order to implement the proposed policy, f_t , with no ability to commit to future one. The electorate is heterogeneous in its own political ideology. Then, to each individual j belonging to the cohort $i \in \{1, 2\}$ is assigned the ideological values σ_i^j . We assume σ_i^j is a random variable extracted from the distribution function G_i and it represents the ideological bias towards party B .

The timing of the political bargaining game played at the beginning of each period is then characterized by the following three steps:

- i. each party proposes a political platform, f_t^l , in order to maximize its probability of winning the election;
- ii. the ideological bias is realized among voters;
- iii. fully rational and forward-looking voters take their voting choice.

At each time, first each party proposes the political platforms, second individual j from group i votes for party A if and only if the following inequality holds:

$$W_{i,t}^j(f_t^A) > W_{i,t}^j(f_t^B) + \sigma_i^j$$

where $W_{i,t}^j(\cdot)$ is the individual indirect utility function. Then, given the equilibrium policy choice of party B , f_t^B , the ex-ante political maximization problem for party A turns out to be equivalent to:

$$\max_{f_t^A \in \Pi(h_t, k_t)} (1+n) \left(G_1 \left(W_{1,t}^j(f_t^A) - W_{1,t}^j(f_t^B) \right) \right) + G_2 \left(W_{2,t}^j(f_t^A) - W_{2,t}^j(f_t^B) \right) \quad (9)$$

⁸We replicate the stylized facts that young people show a much lower turnout rate at elections with respect to adults and old. As Galasso and Profeta (2004) report in some countries elderly people have a higher rate at elections than the young. In the U.S. turnout rates among those aged 60-69 years in twice as high as among the young (19 – 29 years). Again in France it is almost 50% higher.

⁹Due to the multidimensionality of the political platform Condorcet winner generally fails to exist. Consequently the median voter theorem doesn't hold (Plot, 1967). In the literature there are three main influential approaches for making predictions when the policy space is multi-dimensional. The first is the implementation of structure-induced equilibria. By following Shepsle (1979), agents vote simultaneously, yet separately (i.e. issue by issue), on the issues at stake. Votes are then aggregated over each issue by the median voter. See Condez-Ruiz and Galasso (2005) for a more detailed discussion of this approach. The second is the legislative bargaining approach, which stems from the seminal work of Baron and Ferejohn (1989) and develops from Battaglini and Coate (2006). This approach applies when legislators' first loyalty is to their constituents and legislative coalitions are fluid across time and issues. The last approach, which will be exploited in this paper, concerns the adoption of probabilistic voting rule. While it dates back to the 1970s, its resurgence in popularity stems from Lindbeck and Weibull (1987). It applies to political environments where party discipline is strong and the winning political party simply implements its platform. See Persson and Tabellini (2000) for a survey of this framework.

where $\Pi(\cdot)$ is a continuous convex correspondence. By symmetry party B solves an equivalent problem. In the Nash equilibrium of the electoral competition game both candidates proposed the same policy platform, implementing the utilitarian optimum with respect to current voters. Summarizing, individual electoral choices depend on the proposed fiscal platform, on the impact of political program on agent's private behavior, and on ideology.

3.5 Timing

Within each period t , the timing of moves is as follows:

- i. a new generation of young people is born;
- ii. political bargaining takes place to select the policy to be implemented in period t , as described in the previous subsection;
- iii. the firms hire works and rent capital;
- iv. the adults receive the proceeds from labor, pay taxes and consume; the old receive the capitalized saving and transfer and consume; the young receive productive transfers;
- v. the old generation dies; the young and adult generation age and become adult and old in the next period, respectively.

4 Maximization and Equilibrium

As in Krusell et al. (1997), we describe the equilibrium of our economy as a dynamic politico-economic equilibrium. Due to the sequential nature of the timing of the game and solving backward, first, the agents determine the optimal level of saving given the fiscal stance (*Competitive Economic Equilibrium*) and, second, short-lived office-seeking-politicians determine both the level of taxation and the amount of backward and forward transfer in order to maximize the probability of winning elections (*Politico-Economic Equilibrium*).

4.1 Competitive Economic Equilibrium

In a competitive equilibrium, each adults chooses their lifetime consumption taking fiscal and redistributive policies as given. Maximizing Eq. (1) subject to the individual budget constraints (2) and (3), and feasibility constraints $c_{1,t} > 0$ and $c_{2,t+1} > 0$, the following first order condition for interior solutions must hold:

$$0 = \eta(\cdot) \equiv u_{c_{1,t}}(C_{1,t}(\cdot)) - R\beta u_{c_{2,t+1}}(C_{2,t+1}(\cdot)) \quad (10)$$

In equilibrium by implicit function theorem there exists a unique saving function, $s_t = K((1+h_t)(1-\tau_t), p(e_{t+1}, \tau_{t+1}))$, which satisfies the condition (10). Under Eq. (7), the equilibrium choices at time t of an adult agent born at time $t-1$ are given by:

$$(1+n)k_{t+1} = K((1+h_t)(1-\tau_t), p(e_{t+1}, \tau_{t+1})) \quad (11)$$

$$c_{1,t} = C_{1,t}(\tau_t, h_t, k_{t+1}) \quad (12)$$

$$c_{2,t+1} = C_{2,t+1}(p(e_{t+1}, \tau_{t+1}), k_{t+1}) \quad (13)$$

Given any separable additive intertemporal utility, Eq. (11), in terms of life-cycle after tax endowment, emphasizes the income and substitution effects due to a variation of the implemented policies on the individual saving choice.

Definition 1 (Competitive Economic Equilibrium) *Given the initial conditions (h_0, k_0) and the sequence of policies $\{f_t\}_{t=0}^\infty$, a Competitive Economic Equilibrium is defined as a sequence of allocations $\{c_{1,t}, c_{2,t+1}, h_{t+1}, k_{t+1}\}_{t=0}^\infty$ such that individual choices, Eq. (12) and Eq. (13), are consistent with the law of motion of the economy described in Eq. (6) and Eq. (11), and markets clear at any point in time.*

At time t the indirect utility, $W_{1,t}(\cdot)$, of an adult born a time $t-1$, is then equal to:

$$W_{1,t} \equiv \max_{s_t} \{U_{t-1} \mid I_t\} = u(C_{1,t}(\cdot)) + \beta u(C_{2,t+1}(\cdot)) \quad (14)$$

For an old individual born a time $t-2$ the indirect utility, $W_{2,t}(\cdot)$, at time t is as follows:

$$W_{2,t} \equiv u(C_{2,t}(\cdot)) \quad (15)$$

We call *autarky* indirect utility, $W_{1,t}^a$, the lifetime utility of an adult born at time $t-1$, when no taxation and public spending are considered:

$$W_{1,t}^a \equiv \max_{s_t} \{U_{t-1} \mid I_t = 1\} \quad (16)$$

Suppose there is no government that has the authority to levy taxes. As consequence, adults keep the entirety of their labor income to purchase final good and to save. Capital earns a gross return of R , used by old to buy consumption goods. Clearly, the economy converges to the unique steady state in at most one period, where $h^a = 0$, $k^a = \frac{\beta}{(1+\beta)(1+n)}$, $c_1^a = \frac{1}{1+\beta}$, $c_2^a = \frac{\beta}{1+\beta}R$ and $w^a = 1$.

Definition 2 (Feasible Allocation) *A feasible allocation is a sequence of individual choices $\{c_{1,t}, c_{2,t+1}, h_{t+1}, k_{t+1}\}_{t=0}^\infty$ and policies $\{f_t\}_{t=0}^\infty$ that satisfies the implementability constraint, Eq. (11), the balanced budget constraint, Eq. (8), and the fiscal feasibility conditions $\tau_t \in (0, 1)$ and $e_t \in (0, \hat{e}_t)$ at each time t .*

4.2 Politico-Economic Equilibrium

In this section we consider a government of politically-motivated but short-lived representatives that have the authority to levy labor income tax and to transfer income across generations. We

have already described how the political process takes place in each period. To characterize the politico-economic equilibrium, we need to consider the dynamic aspects of the political process that takes place in the economy.

As in Krusell et al. (1997), we restrict the notion of politico-economic equilibrium to the differentiable political SMPE concept as equilibrium refinement of subgame perfect equilibria¹⁰. The payoff-relevant state variables of our economy are the assets holding by adults and old. i.e. human capital and physical capital that were determined in the previous period but which go into effect in the current one¹¹.

At each time voting over a political platform generates dynamic linkages of policies across periods. The standard logic of competitive models, where agents optimize taking future equilibrium outcome as given breaks down when political choices are considered. Due to the non-negligible impact of current political actions on future equilibria, rational agents internalize these dynamic feedbacks. In our framework dynamic linkages generated by physical and human capital arises both directly, affecting asset accumulation decision (*direct dynamic feedbacks*), and indirectly, affecting future political choices (*indirect dynamic feedbacks*). Focusing on Markov strategies, in which the players' actions depend on the level of the fundamental state variables only, agents are able to fully internalize the overall direct and indirect impact of taxation and redistribution through the evolution of assets.

In a perfect forward-looking environment, in which parties play Markov strategies, the following definition of equilibrium is adopted:

Definition 3 (Politico-Economic Equilibrium) *A perfect foresight politico-economic SMPE is defined as the sequence of feasible individual choices $\{c_{1,t}, c_{2,t+1}, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ and policies $\{f_t\}_{t=0}^{\infty}$, such that the functional vector of differentiable policy decision rules, $F = (T, E)$, where $T : R \times R \rightarrow (0, 1)$ and $E : R \times R \rightarrow (0, \hat{e}_t)$ are respectively the taxation policy rule, $\tau_t = T(h_t, k_t)$, and public higher education policy rule, $e_t = E(h_t, k_t)$, satisfies the following points:*

i. Each party solves the maximization program (9) subject to the following set of constraints:

$$k_{t+1} = K(f_t, F(h_{t+1}, k_{t+1}), h_t)$$

$$h_{t+1} = H(e_t, h_t)$$

where $H(\cdot)$ and $K(\cdot)$ are defined in Eq. (6) and Eq. (11).

¹⁰The Markov-perfect concept implies that outcomes are history-dependent only in the fundamental state variables. The stationary part is introduced to focus only on the current value of the payoff relevant state variable. Consequently the vector of equilibrium policy decision rules is not indexed by time, i.e. the structural relation among payoff-relevant state variables and political controls is not time variant. The differentiable part is a convenient requirement to avoid multiplicity of equilibrium outcomes and in order to give clear positive political predictions.

¹¹The Markov restriction rules out equilibria in which the political outcome depends directly on the past outcomes, as in "reputation" equilibria. This results to be plausible in our environment, where periods are very long and bargaining takes place among different agents at each point in time.

ii. The fixed point condition holds, i.e. the policies are fixed points of the mappings E into $E^{ex}(h_t, k_t)$ and T into $T^{ex}(h_t, k_t)$, where the apex ex stand for expected.

The first equilibrium condition requires the political control variables, f_t , have to be chosen in order to maximize the party's objective function, taking into account that future redistribution and taxation depend on the current policy choices via both the equilibrium private decisions and future equilibrium policy rules. The second condition requires that, if the equilibrium exists, it must satisfy the fixed point requirement. From a technical point of view, we are looking for two differentiable policies which obey the recursive rules given by the vector of functions $f_t = F(h_t, k_t)$, where F is an infinite dimensional object and the key endogenous variables of the problem. The second fundamental element we are looking for is a function which describes the private sector response to a one-shot deviation of the government, when agents expect future policies to be set by politicians according to F as a function of current state and political control variables, $k_{t+1} = \tilde{K}(f_t, h_t)$ ¹².

Before recursively solve the equilibrium policy rule F , we investigate the marginal impact of e_t and τ_t on the parties' objective function. Maximizing Eq. (9) with respect to both policies and applying the envelope theorem, we obtain the following system of first order conditions for each $l \in \{A, B\}$:

$$\begin{cases} (1+n) \frac{dW_{1,t}}{d\tau_t^l} + \frac{g_2}{g_1} \frac{dW_{2,t}}{d\tau_t^l} = 0 \\ (1+n) \frac{dW_{1,t}}{de_t^l} + \frac{g_2}{g_1} \frac{dW_{2,t}}{de_t^l} = 0 \end{cases}$$

where g_i is the density function of G_i . Let us denote with $\phi \equiv \frac{g_2}{g_1}$ a synthetic measure of the ideological bias among voters which also represents the relative political weight of the old voters cohort¹³. If $0 < \phi < 1$ then on average, the old cohort cares less about ideology and has more swing-voters than the adults one. For $\phi > 1$ the opposite holds, where the preferences of old in the political debate represent the political majority. Finally, when $\phi = 1$, all voters are equally represented. Using the indirect utility functions, Eq. (14) and (15), the following first order conditions are attained for τ_t and e_t , respectively¹⁴:

¹²The function $\tilde{K}$ is known only conditioning on the existence of F . To derive $\tilde{K}$ start from Eq. (11):

$$k_{t+1} = K(f_t, f_{t+1}, h_t)$$

Function K describes the equilibrium behavior of private agents as a function of current state and both current and future policies. If there exists a differentiable function F , which describes the policy behavior followed by politicians in equilibrium, this rule can be internalized by fully rational private agents. It follows that:

$$k_{t+1} = K(f_t, F(h_{t+1}, k_{t+1}), h_t)$$

Plugging the Eq. $h_{t+1} = H(e_t, h_t)$ into the above equation and rearranging the terms we get:

$$k_{t+1} = \tilde{K}(f_t, h_t)$$

Due to the full depreciation of physical capital, $\tilde{K}$ is not a function of current level of physical capital, which strongly simplifies the analyses.

¹³In other terms, ϕ is a measure of how strongly the old generation pursues her own interest.

¹⁴Since in equilibrium the parties A and B face the same maximization problem and choose an identical political platform, we remove the apex l .

$$0 = \underbrace{\phi(1+h_t)u_{C_{2,t}}}_{\text{old's direct benefit}} - \underbrace{(1+h_t)u_{C_{1,t}}}_{\text{adults' direct cost}} + \underbrace{\beta(1+n)u_{C_{2,t+1}} \left((1+h_{t+1})\frac{d\tau_{t+1}}{d\tau_t} - (1+n)\frac{de_{t+1}}{de_t} \right)}_{\text{adults' exp. cost/benefit}} \quad (17)$$

$$0 = \underbrace{-\phi u_{C_{2,t}}}_{\text{old's direct cost}} + \underbrace{\beta u_{C_{2,t+1}} \left(\frac{dh_{t+1}}{de_t} \tau_{t+1} + (1+h_{t+1})\frac{d\tau_{t+1}}{de_t} - (1+n)\frac{de_{t+1}}{de_t} \right)}_{\text{adults' expected direct/indirect cost/benefit}} \quad (18)$$

Let us first refer to Eq. (17). At each time an interior solution for the income tax rate is simply determined as the outcome of a weighted bargaining between current old and adults, which get benefits and sustain costs by a variation in tax level. The first term in Eq. (17) represents the old's marginal benefits in terms of PAYG social security due to the increase in income tax rate. Since tax levying on labor income makes adults sustain the whole tax burden, the second term captures the adults' marginal cost caused by a positive variation on the fiscal dimension. Finally the third term measures the expected marginal impact of current variation in the tax rate on the utility of next-period old. Similarly, redistributive choices are taken as the outcome of a weighted bargaining between current old and future one. An increase in public higher education transfers is a double-edge sword. On one hand it makes current old sustain direct costs due to reduction in social security contributions, represented by the first part of Eq. (18). On the other hand future old enjoy direct benefits from expected return of productive investment in human capital, whose effects are captured by the second part of Eq. (18).

Note that the FOCs (17) and (18) internalize the strategic effects, capturing how politicians can affect future policies through their current choices of f_t . If $\frac{d\tau_{t+1}}{d\tau_t} > 0$ (< 0) and $\frac{d\tau_{t+1}}{de_t} > 0$ (< 0) agents know that a higher income tax rate and larger education transfers lead to a higher (lower) tax rate in the future. Thus, representatives may strategically increase (reduce) τ_t and e_t in order to distort the tax rate outcome of tomorrow. The same idea holds for e_{t+1} .

4.2.1 Political SMPE with Perfect Foresight

Due to the non-linearity and bidimensionality in the political space, the system of partial differential equations (17) and (18) cannot be easily solved using integration methods¹⁵. We start solving simultaneously for the maximization of the decisive voter with respect to the income tax rate and the level of public higher education transfers. As reported in Klein et al. (2008), the equilibrium is obtained as the limit of a finite-horizon equilibrium, whose characteristics do not significantly depend on the time horizon, as long as the time horizon is long enough. Consequently our resolution strategy consists in a constructive approach (*induction method*). We compute the FOCs defining the feasible equilibrium policy rules in a finite-horizon environment via backward induction. We start at a final round $t < \infty$ and we re-compute the equilibrium policy rules, $F_t = (E_t, T_t)$, as long as all the direct dynamic feedbacks, induced by political

¹⁵See for example Grossman-Helpman (1996) and Azariadis-Galasso (2002) frameworks in which by applying the envelope theorem the differential equation becomes linear and solution results straightforward to determine.

choices on private one, have been internalized. In particular, due to two-periods lag impact of e_t on private saving choice, we will perform recursive maximization until period $t - 2$. At each time political objective function, described in Eq. (9), has to be simultaneously maximized with respect to its arguments, i.e. the pair (e_t, τ_t) , subject to the balanced budget constraint, Eq. (8), the Euler condition of the economic optimization problem, Eq. (11), and the equilibrium policy rules of the following periods, computed via backward procedure. Once a recursive structure is identifiable, making the time horizon goes to infinity for all the time-variant coefficients determined so far, we obtain the equilibrium policy rules as fixed point of the recursive problem in multidimensional environment.

For notational purposes let us denote with Ω_i the relative political bargaining power for $i \in \{1, 2\}$, which is defined as follows:

$$\Omega_1 \equiv \frac{(1+n)(1+\beta)}{\phi + (1+n)(1+\beta)} \quad \text{and} \quad \Omega_2 \equiv \frac{\phi}{\phi + (1+n)(1+\beta)} \quad (19)$$

Remark 1 *The more population ages (i.e. n decreases and ϕ increases), the smaller is the relative political weight of adults (Ω_1) and larger is the relative political weight of old (Ω_2).*

Fixing $\theta = \frac{1}{2}$, we analytically determine a fundamental equilibrium capturing the effects that are inherent in the dynamic game itself, which turns to be unique. Let Υ^{Pol} be the state-space in which interior policy rules are simultaneously obtained and $\bar{R} \equiv \frac{1+n}{R-(1+n)}$ be an index measuring the economy dynamic efficiency. The following Theorem characterizes the equilibrium outcomes of public choices in a fully rational environment when Markov strategies are implemented:

Theorem 1 *Let $\psi^* = \frac{1}{\alpha} \left(\frac{2R}{\alpha} (R - \sqrt{R^2 - \alpha}) - 1 \right)$. Under dynamic efficiency condition, for any $(h_t, k_t) \in \Upsilon^{Pol}$ the set of feasible rational policies, $f_t \equiv (e_t, \tau_t)$, which can be supported by a perfect foresight political SMPE, has the following functional form:*

i.

$$E(h_t) = a_1 h_t + a_0 \quad (20)$$

where $a_1 \equiv \frac{\alpha}{1+n} \psi^*$ and $a_0 \equiv \frac{1-\alpha}{1+n} \bar{h} \psi^*$;

ii.

$$T(k_t, h_t) = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0 \quad (21)$$

where $b_3 \equiv R\Omega_1$, $b_2 \equiv \alpha\psi^*(\Omega_1 + 2\Omega_2)$, $b_1 \equiv \bar{h} \frac{1-\alpha}{\alpha} b_2 + \bar{R} (1 + \bar{h} (1 - \alpha) \psi^*) \Omega_2$ and $b_0 \equiv \Omega_2$.

Otherwise, for any $(h_t, k_t) \notin \Upsilon^{Pol}$ corner solutions result in at least one of the two dimensions.

Proof. (See Appendix). ■

From a structural point of view, while the policy rule associated to education transfers is linear in human capital production, the fiscal policy rule is a linear function in physical capital but not in the human capital level. The equilibrium conditions predict the simultaneous existence of both sides of the redistributive program for $(h_t, k_t) \in \Upsilon^{Pol}$.

4.3 Dynamics and Steady States

We now discuss the transition dynamics of the economy during the adjustment towards the steady state.

Definition 4 (Law of Motion) *The laws of motion of the collection $\{e_t, \tau_t, h_t, k_t\}_{t=0}^{\infty}$ are defined as the mappings:*

$$\begin{aligned} h_{t+1} &= H(E(h_t), h_t), & k_{t+1} &= \tilde{K}(E(h_t), T(h_t, k_t), h_t), \\ e_{t+1} &= E(H(E(h_t), h_t)), & \tau_{t+1} &= T(\tilde{K}(E(h_t), \tau_t, h_t), H(E(h_t), h_t)). \end{aligned}$$

The economy dynamics is basically driven by the human capital evolution which affects both the education transfers' law of motion and the transition dynamics of taxation policy. While the former is directly influenced only by human capital, the latter is affected by human capital both directly and indirectly through physical capital. This implies that convergence conditions in the state-space are also sufficient for the stable convergence of the policy rules evolution. The following Lemma states the conditions for economy's convergence stability.

Lemma 1 *Let $\underline{\phi} \equiv \beta(R - (1 + n))$ and $\underline{n} \equiv \sqrt{2R(R - \sqrt{R^2 - \alpha})} - \alpha - 1$. Given any feasible initial condition (h_0, k_0) , if $\phi > \underline{\phi}$ and $n > \underline{n}$, then the sequence $\{e_t, \tau_t, h_t, k_t\}_{t=0}^{\infty}$ is characterized by stable monotonic convergence. The speed of convergence for τ_t crucially depends on the initial condition and the human capital society endowment.*

Proof. (See Appendix). ■

Given the differentiability of the policy functions, the interior solution conditions and Lemma 1, the following proposition holds:

Proposition 1 *The feasible steady state (e^*, τ^*, h^*, k^*) exists and is unique.*

Proof. (See Appendix). ■

Thus, depending on the initial condition, (h_0, k_0) , and the level of the human capital society endowment, $\bar{h}$, the control and the state variables converge monotonically to the unique feasible steady state¹⁶.

5 Discussion

The dynamic efficiency requirement, $R > 1 + n$, is a necessary condition for the simultaneous existence of PAYG and public education programs. In our economy, during the transition path, the *implicit net return* to pensions is determined by both the population growth rate and the marginal increase in the taxable income due to human capital investment net of the current resources devoted to education. As long as the implicit net return is higher than the

¹⁶ According to the particular emerging Welfare State regime (see Section 5.1) different speeds of convergence and amounts of intergenerational transfers characterize the economy.

capital rental price, there will emerge incentives in investing simultaneously in both sides of the redistribution programs. By contradiction, suppose that the population growth rate exceeds the net rental price to physical capital, then it is straightforward to prove that b_1 tends to infinity¹⁷ and, consequently, the human capital will affect negatively the size of government. Thus, according to Eq. (6) and (21), an increase in education spending would determine a positive variation in the stock of human capital and, in turn, a decrease in tax rate. Consequently, the increase in physical capital will induce further reduction in the future tax level. This cannot be an equilibrium since, given $R < 1 + n$, agents have always an incentive to deviate choosing higher level of income tax rate in order to depress private saving and guarantee a higher future level in pension contributions even without investment in education. As long as the economy is in a dynamic inefficient scenario the simultaneous existence of both forward and backward transfers is excluded. We depart from traditional literature on redistributive policies, where no endogenous human capital formation is modelled, which states social security survives just in an economy characterized by a population growth rate higher than the rental price.

As depicted in Figure 1, for any non-zero level of income tax rate, the larger the human capital is, the more political support the education program receives, i.e. $\frac{de_t}{dh_t} = a_1 > 0$. Two different configurations may arise depending on the level of society human capital endowment. As shown in Panel (a), as long as $\bar{h} < \frac{1}{(1-\alpha)\psi^*}$, $E(h_t)$ lies within the feasibility boundaries, $(0, \hat{e}_t)$, for any level of human capital. Instead, as reported in Panel (b), if $\bar{h} \geq \frac{1}{(1-\alpha)\psi^*}$, there exists a threshold value of parental human capital, $\tilde{h} \equiv \frac{1-\bar{h}(1-\alpha)\psi^*}{\alpha\psi^*-1}$, such that for any level of h_t lower than $\tilde{h}$ boundary solution is attained, i.e. $E(h_t) = \hat{e}_t$. In other terms, due to complementarity between the inputs employed in the skill technology, the whole tax revenue is devoted to investment in public education and no social security program is implemented. Otherwise, if h_t is higher than $\tilde{h}$, the larger the stock of human capital is, the lower is the variation in education transfers and consequently the flatter is the equilibrium policy function. Indeed, due to the decreasing returns in parental human capital, in equilibrium politicians set positive transfers both for education and social security¹⁸.

Remark 2 $E(h_t)$ doesn't depend on strategic political components embedded in the parameter ϕ . For the determination of the transfers' level, only the mass effect component, n , matters.

As reported in Theorem 1, in equilibrium the amount of education transfers has to be equal to the highest feasible value of forward spending which maximizes the net implicit rate of future pensions. In other terms $E(h_t)$ maximizes the intertemporal utility of current adults

¹⁷See Proof of Theorem 1 in Appendix A for the derivation of b_1 .

¹⁸Note that the scenario characterized by the whole tax revenue devoted to public higher education investments, i.e. no current pension benefits, is an equilibrium outcome only as long as one-period future pension transfers are allocated to current adults. In other terms, when $\bar{h} \geq \frac{1}{(1-\alpha)\psi^*}$ and $h_t < \tilde{h}$, there exists an initial condition $\tilde{h}_0$ such that for any $h_0 > \tilde{h}_0$, due to public investments in higher education, future human capital level exceeds the threshold level $\tilde{h}$, i.e. $h_{t+1} \geq \tilde{h}$. In this case adults have incentive in taxing their income because of the future expected benefits in terms of PAYG social security. Thus there emerges a one-period-equilibrium characterized by an intergenerational contract with current backward transfers equal to zero. Otherwise, if $h_0 < \tilde{h}_0$, then no future pensions will be set for current adults and no incentive to implement an intergenerational contracts may emerge.

without considering the political distortions due to old's bargaining power. This result sounds to be counterintuitive, because, as shown in Eq. (18), old have actually incentives in reducing the education amount at the minimal level. This in turn, under dynamic efficiency, would remove the adults' incentives in substituting private saving with public one. As final result the autarky would be established. It cannot be an equilibrium for the setting of an intergenerational contract and, as a consequence, the emergence of a public education program not distorted by the political bias is justified.

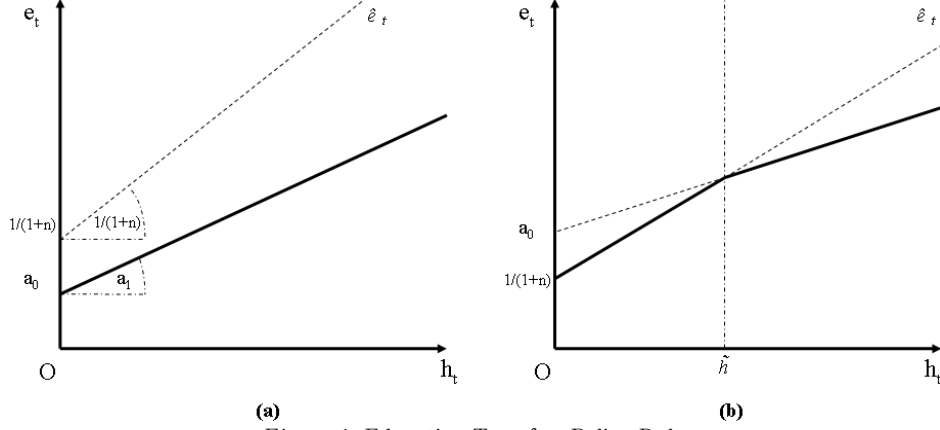


Figure 1: Education Transfers Policy Rule

Figure 2 reports the equilibrium fiscal policy rule described in Eq. (21). For illustrative purpose, it is useful to disentangle the effects of the two asset variables on $T(h_t, k_t)$ at each time t . Panel (a) describes the structural relation between the equilibrium tax rate and the level of h_t where the intercept, $T(k_t, 0)$, is a decreasing function in physical capital. As long as $k_t < \tilde{k}$ where $\tilde{k} \equiv \frac{b_1 - b_2}{b_3}$, the larger the human capital is, the higher is the opportunity cost to tax levy, i.e. $\frac{d\tau_t}{dh_t} < 0$. If instead $k_t \geq \tilde{k}$, incentives to increase simultaneously the taxable income and the income tax rate arise, i.e. $\frac{d\tau_t}{dh_t} \geq 0$. Panel (b) illustrates the structural relation between the equilibrium tax rate and the level of k_t . The equilibrium predicts for any value of k_t the higher the physical capital is, the lower is the income tax rate, consistently with previous literature¹⁹. The intuition for the fiscal policy function to be non-increasing in the capital stock is the following. By contradiction, if $T(h_t, k_t)$ were increasing in k_t , current adult would have incentive to save in order to provide the next generation with a higher level of capital and therefore receive a higher pension. This cannot be an equilibrium, since the higher amount of backward transfer reduces the level of saving that workers are willing to make.

Remark 3 $T(h_t, k_t)$ crucially depends on both the strategic political components embedded in the parameter ϕ and the demographic component, n , for the determination of the size of government.

¹⁹See among others Grossman and Helpman (1998), Azriadis and Galasso (2002), Forni (2005) and Bassetto (2008).

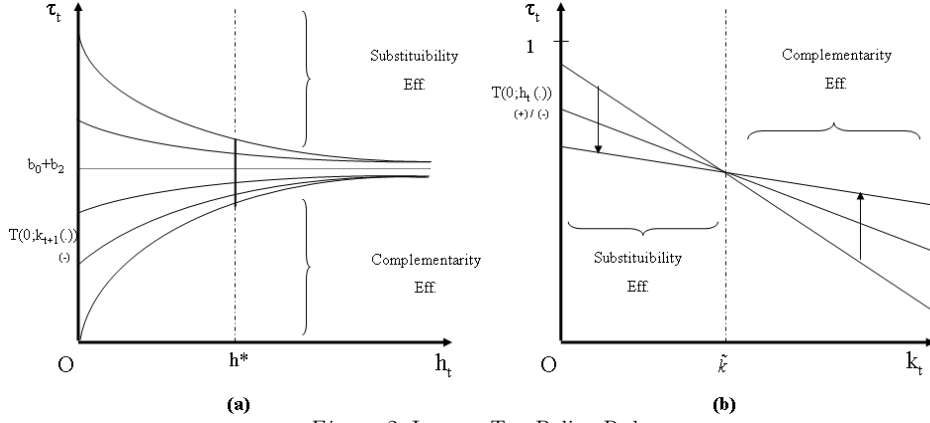


Figure 2: Income Tax Policy Rule

Due to the politicians' opportunistic behavior, strategic persistency criterion drives the setting of the income tax rate. In our environment human capital plays a crucial role in two different ways. On one hand it mitigates the politicians' strategic behavior. Precisely, the higher the level of human capital is, the flatter is the equilibrium policy function and the lower is the elasticity of $T(h_t, k_t)$ with respect to physical capital. The lower responsiveness of taxation policy decisions on the level of private savings weakens the strategic channel through which politicians can increase the probability to win election. On the other hand human capital perturbs the political choice concerning the size of government. Depending on the political bargaining intensity between adults and old embedded in the coefficients b_1 and b_2 of Eq. (21), the marginal impact of human capital on taxation decisions can be either positive or negative, as already pointed out in the above analyses of equilibrium tax structure. Formally, let us define $\bar{\Omega}_2 \equiv \frac{(\alpha - (1-\alpha)\bar{h})\psi^*}{(2(1-\alpha)\bar{h} + \alpha)\psi^* + \bar{R}}$, the following relation holds:

$$\begin{aligned} b_1 > b_2 & \quad \text{iff} \quad \Omega_2 > \bar{\Omega}_2 \\ b_1 \leq b_2 & \quad \text{iff} \quad \Omega_2 \leq \bar{\Omega}_2 \end{aligned} \quad (22)$$

The relation states that an economy where $\Omega_2 \leq \bar{\Omega}_2$ experiences a political competition characterized by a weak old bargaining power and $b_1 \leq b_2$. If $\Omega_2 > \bar{\Omega}_2$, then old exert a strong bargaining power and $b_1 > b_2$.

To summarize, a complete description of the recursive Markovian structure including both the economic environment and the political scenario is represented in Figure 3.

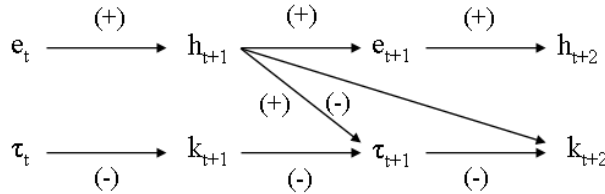


Figure 3: Markovian Structure

The picture points out the strategic relations which provide the necessary incentives to selfish agents to sustain simultaneously backward redistributive policies and forward one, i.e. (e_t, p_t) , as described above.

5.1 Welfare State Regimes

Figure 2 points out the strategic structural relation between income tax rate and human capital in the Markovian environment, which drives the economy towards different WSR. If pure political factors matter in splitting the public spending, then a Politico WSR will emerge. If the economic factors are also relevant, then a Politico-Economic WSR will arise. The following Corollary fully characterizes the conditions for the identification of the different regime configurations:

Corollary 1 *Given the stationary equilibrium policy rules $T(h_t, k_t)$ and $E(h_t)$:*

- i. *if $b_1 \leq b_2$, then the Politico Complementarity Welfare State Regime, PCR, arises, i.e. $\frac{d\tau_t}{dh_t} \geq 0$;*
- ii. *if $b_1 > b_2$ and $k_t \geq \tilde{k}$, then the Politico-Economic Complementarity Welfare State Regime, PECR, arises, i.e. $\frac{d\tau_t}{dh_t} \geq 0$;*
- iii. *if $b_1 > b_2$ and $k_t < \tilde{k}$, then the Politico-Economic Substitutability Welfare State Regime, PESR, arises, i.e. $\frac{d\tau_t}{dh_t} < 0$.*

Proof. (See Appendix). ■

While the economic factors driving the system into different WSR are endogenously determined by the capital asset accumulation through the saving choices, the political factors depend on the relative bargaining power of voters. An economy characterized by a weak level of old bargaining power in the political process, i.e. $b_1 \leq b_2$, will experience a PCR, for any level of k_t . Contrarily, an economy with a strong level of old's bargaining power in the political arena, i.e. $b_1 > b_2$, will experience a PECR if the system is high-capitalized, i.e. $k_t > \tilde{k}$, otherwise a PESR will emerge if the economy is low-capitalized, i.e. $k_t < \tilde{k}$.

Intuitively, as already pointed out, in equilibrium a higher level of current income tax rate will determine a decrease of future physical capital stock and, consequently, an increase of future tax rate. In the PCR, adults anticipate that, if they invest in education today, an increase in future human capital will determine a further positive variation in the level of income tax rate tomorrow. Given the increase in both the future tax rate and taxable income, i.e. gross future pension benefits, which maximize adult intertemporal utility, PCR emerges as the only sustainable WSR when adult bargaining power prevails.

To fully characterized the public spending process, based on the WSR criterion, we move the analyses to the equilibrium characterization for pension benefits.

Corollary 2 *Under decreasing return in education, in equilibrium the impact of education spending on the social security transfers is always positive, i.e. $\frac{dp_{t+1}}{de_t} > 0$.*

Proof. (See Appendix). ■

Remark 4 *The existence of a PAYG social security program supports public investment in higher education even in absence of altruism .*

Independently from the WSR characterizing the economy, an increase in public education transfers induces an higher pension benefits in the future, creating the incentive for adults in supporting the education program. *Ceteris paribus*, by supporting an higher education cost today, the adults internalize that it will generate an higher taxable income of tomorrow, guaranteeing a higher level of pension benefits when they will be old, for any level of $T(\cdot)$.

The interaction between political and economic institutions determine the amount and the dynamic evolution of pension system.

Corollary 3 *At each time t , for any given level of human capital, in PESR pension benefits are lower then the PCR and larger then the PECR, i.e. $p_t^{PECR} < p_t^{PESR} < p_t^{PCR}$.*

Proof. (See Appendix). ■

When the adult's bargaining power is sufficiently strong, i.e. $b_1 \leq b_2$ and PCR arises, the equilibrium pension benefits reach the highest feasible level. Otherwise, when old prevail in the political debate, depending on the physical capital stock, the pension benefits are lower in a high-capitalized economy then in a low-capitalized one.

To resume graphically, in Figure 4 we plot on the state-space (h_t, k_t) as illustrative case the WSR configurations which arises under definite parameters' conditions when $\bar{h} > \frac{1}{(1-\alpha)\psi^*}$ and $h_0 > \tilde{h}_0$ ²⁰. Panel (a) shows the case in which a weak level of adult bargaining power characterizes the political scenario. Contrarily, Panel (b) allows for a strong bargaining power of the adults.

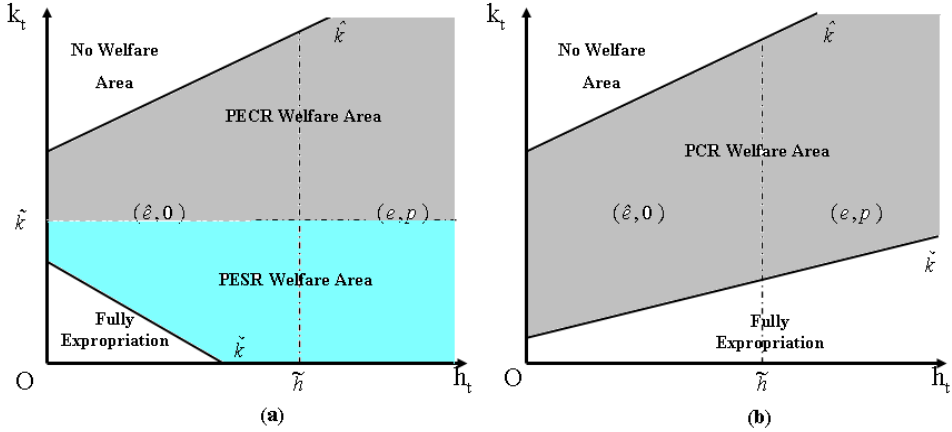


Figure 4: Panel (a) shows the case for $b_2 < b_1$, Panel (b) shows the case for $b_2 > b_1$.

As long as $k_t < \tilde{k}_t$ in both cases full expropriation occurs. The tax rate, equal to 100% of labor income, is assigned either to finance only public education program if $h_t < \tilde{h}$ or to support both redistributive social programs if $h_t \geq \tilde{h}$. Differently, as long as $k_t \geq \hat{k}_t$ autarky economy takes place. The panel (a) reports the politico-economic parameters' configurations which makes PECR and PESR arise, i.e. $b_2 + b_0 < 1$ and $b_1 + b_0 > 1$. Whereas the panel (b) shows the emergence of PCR due the pure political factors, i.e. $b_2 > b_1$.

²⁰It should be note that if $\bar{h} \leq \frac{1}{(1-\alpha)\psi^*}$, then the human capital doesn't play any role in splitting the public spending between education and retirement transfers. In other terms, it avoids the interesting case with pension benefits seated to zero.

5.2 Aging

Quantitatively, one of the most severe challenges concerning the intergenerational transfer system in the developed economies regards the impact of population aging both in demographic (n) and political (ϕ) terms. Demographic aging, which represents the quantitative component of the aging phenomenon, decreases partially the returns from a PAYG system in our economy characterized by endogenous human capital formation. Political aging, which represents the qualitative component of aging phenomenon, gives retirees stronger claim for pension benefits even on constant demographic terms. Based on the characterization of the political equilibrium, we now consider how aging affects the policy decisions of representatives who face electoral constraints in the form of both the size of welfare state, represented by the tax rate $T(\cdot)$, and the amount of intergenerational transfers, $E(\cdot)$ and $P(\cdot)$. Focusing on political aging the following Corollary holds:

Corollary 4 *The political aging, i.e. the increase in ϕ , has no quantitative impact on the education transfers, $\frac{dE}{d\phi} = 0$, and induces increase in the income tax rate, $\frac{dT}{d\phi} > 0$. It follows, for any level of $\bar{h}$, $\frac{dP}{d\phi} > 0$.*

Proof. (See Appendix). ■

The political effect is captured by a decrease in the political weight for the adult, that is, an increase in the political weight for the old. A stronger old ideological pressure in the political debate implies an higher income tax rate. This in turn determines a larger social security system supported by voting. Given the efficiency criterion driving the implementation of public education policy, the overall effect of political aging doesn't distort $E(\cdot)$.

Corollary 5 *The demographic aging, i.e. the decrease in n , induces an increase in education transfers, $\frac{dE}{dn} < 0$, and has an ambiguous impact on the income tax rate, $\frac{dT}{dn} \gtrless 0$. It follows $\frac{dP}{dn} \gtrless 0$.*

Proof. (See Appendix). ■

Departing from previous literature suggesting the size of social security to be increasing in population growth, our model predicts under which parametric condition also the inverse relation appears. Specifically, demographic aging has an ambiguous impact on the amount of pension transfers in per-capita terms. A first interesting case arises when the margin $R - (1 + n)$ is sufficiently small, which in turns implies, even without considering the human capital return, the implicit return to pensions to be close to the gross return to private saving. It gives incentives in a younger society to opting for higher pension benefits due to their larger demographic return, i.e. $\frac{dP}{dn} > 0$. A second illustrative case emerges when the relative adults political weight is larger than $\bar{R}$ and $\bar{h}$ is sufficiently high. In this scenario, even if population ages and, in turns, the demographic pension returns decrease, adults have incentives in depressing the current level of savings in order to compensate the smaller number of future tax payers with higher tax rate level tomorrow, i.e. $\frac{dP}{dn} < 0$.

6 Benevolent Government Allocation

In Section 4 we proved the existence of a bidimensional fiscal plane when public policy choices are taken through a repeated voting system. The politico-economic SMPE was also characterized in closed-form as a finite-horizon equilibrium when the time goes to infinity. We now implement as normative benchmark the infinite-horizon Gvt allocation under zero-cost enforceability constraint.

As in the political game, we exclude private agents' default on the implemented fiscal plane within the period. Furthermore, under balanced budget constraint government platform is characterized by the vector $f_t^g \equiv (e_t^g, \tau_t^g, p(e_t^g, \tau_t^g))$, where the apex g stands for Gvt. Given the initial conditions (h_0, k_0) , we define the Gvt optimization program in sequential version, as follows:

$$\max_{\{f_t^g\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t ((1+n) \delta u(C_{1,t}(\cdot)) + \beta u(C_{2,t}(\cdot))) \quad (23)$$

subject to the technological constraints given by Eq. (6) and (11).

The Gvt does not take into account the ideological bias, ϕ , but she assigns a welfare weight, δ , to each dynasty. Let us consider the restriction $\delta < \bar{\delta} \equiv \frac{1}{1+n}$, which induces weak deterrence power.

Remark 5 *In the infinite-horizon Gvt environment the relative welfare weights are:*

$$\Omega_R^g \equiv \frac{\delta(1+n)(1+\beta)}{\beta + \delta(1+n)} \quad \text{and} \quad \Omega_O^g \equiv \frac{\beta(1 - \delta(1+n))}{\beta + \delta(1+n)} \quad (24)$$

Differently from the relative political bargaining power of adults and old, Eq. (19), in the infinite-horizon game the Gvt takes into account both the relative welfare weight of the representative agent, Ω_R^g , and the old's relative welfare weight gap between current and future pensioners, Ω_O^g .

Remark 6 *The more population ages (i.e. n decreases), the smaller is relative welfare weight of the representative agent (Ω_R^g), the larger is the old's relative welfare weight gap (Ω_O^g).*

As in Klein et al. (2008)²¹, let us rewrite in recursive way the sequential Gvt program in order to derive the Gvt Generalized Euler Equations (*GEEs*)²², which capture the Gvt optimal trade-offs between taxation and redistribution wedges over time. Due to stationarity, we omit

²¹Recent studies extend the dynamic politico-economic modelling to the infinite-horizon Gvt problem as in Klein et al. (2008) and Azzimonti et al. (2009). These models differ from ours in that the policy space is one-dimensional and the dynamic linkages are not long-run persistent due to the full depreciation of the relevant-payoff state variables. Departing from past literature we find analytical solutions in a multi-dimensional state space where the equilibrium policies become non trivially dependent on fundamental asset variables.

²²The *GEE* is the *FOC* of the government maximization program. It is obtained deriving the Bellman equation with respect to the political control variables, f^g . *GEE* can be equivalently derived by using Bellman's principle to identify a Markov equilibrium with the solution of the sequential version of the Government program. The Euler equation of this sequential problem is exactly the *GEE*.

the time subscript, denoting by the prime symbols next-period values. The economic first order condition, Eq. (10), requires $\eta(f^g, f^{g'}, h, h', k') = 0$. In equilibrium, by implicit function theorem, there exists a unique $k' = K(f^g, f^{g'}, h, h')$ satisfying $\eta(f^g, f^{g'}, h, h', K(\cdot)) = 0$. If there exists a policy rule $F^g(h, k)$ which solves the Gvt optimization program, then under the transformation function of human capital, $h' = H(e^g, h)$, we derive the recursive formulation of $K(\cdot)$, whose functional form is then equal to $k' = \tilde{K}(f^g, h)$ ²³. The recursive economic first order condition becomes $\eta(f^g, h, \tilde{K}(f^g, h)) = 0$. Derivating the function $\eta(\cdot)$ with respect to its arguments we obtain $\tilde{K}_{f^g} = -\frac{\eta_{f^g}}{\eta_{k'}}$ and $\tilde{K}_h = -\frac{\eta_h}{\eta_{k'}}$, which give a measure of the variation in the amount of savings due to a change in either policies or human capital.

After some manipulations²⁴, Eq. (23) can be reformulated in terms of Bellman equation, as follows:

$$V^g(h, k) = \max_{\{f^g, h', k'\}} [(1+n)\delta u(C_1(\cdot)) + \beta u(C_2(\cdot))] + (1+n)\delta V^g(h', k') \quad (25)$$

We now provide the formal definition of the Gvt equilibrium allocation without commitment.

Definition 5 (Benevolent Government Allocation) *A perfect foresight SMPE of the Benevolent Government problem is defined as the sequence of feasible individual choices $\{c_1, c_2, h', k'\}$ and policies f^g , such that, given the Bellman Eq. (25), the functional vector of differentiable policy decision rules, $F^g = (T^g, E^g)$, where $T^g : R \times R \rightarrow (0, 1)$ and $E^g : R \times R \rightarrow (0, \hat{e})$ are respectively the taxation policy rule, $\tau^g = T^g(h, k)$, and public higher education policy rule, $e^g = E^g(h, k)$, satisfies the following conditions:*

i.

$$F^g(h, k) = \arg \max_{f^g} [(1+n)\delta u(C_1(\cdot)) + \beta u(C_2(\cdot))] + (1+n)\delta V^g(h', k')$$

subject to the following constraints:

$$k' = K(f^g, F^g(h', k'), h)$$

$$h' = H(e^g, h)$$

where $H(\cdot)$ and $K(\cdot)$ are defined in Eq. (6) and Eq. (11).

ii.

$$V^g(h, k) = M(V^g)(h, k)$$

where the functional $M : \mathbb{C}^\infty(R^2) \rightarrow \mathbb{C}^\infty(R^2)$ is defined as follows:

$$M(V^g)(h, k) := \max_{f^g} [(1+n)\delta u(C_1(\cdot)) + \beta u(C_2(\cdot))] + (1+n)\delta V^g(H(\cdot), \tilde{K}(f^g, h))$$

²³For derivation details see note 13.

²⁴See Appendix B for the derivation of both Bellman equation and GEEs.

The first condition requires the political variables, f^g , have to be chosen by the Gvt in order to maximize the utilitaristic social welfare, internalizing the equilibrium private saving decision and all the direct and indirect feedback effects. The second requirement is the fix point condition, given the mapping $M(V^g)$.

In terms of wedges, the *GEEs* of the sequential Gvt program with respect of e and τ are as follows²⁵:

$$0 = \Delta_e + \delta \frac{\eta_e}{\eta_{k'}} \Delta_s + \delta \Xi \Delta_{\tau'} \quad (26)$$

$$0 = (1 + h) \Delta_{\tau} + \delta(1 + n) \frac{\eta_{\tau}}{\eta_{k'}} \Delta_s \quad (27)$$

where $\Xi \equiv H_e \left(\tau' + (1 + h') \left(\frac{\tilde{K}'_{e'}}{\tilde{K}'_{\tau'}} \frac{H'_{h'}}{H'_{e'}} - \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}} \right) \right) = \frac{d((1+h')\tau')}{de}$ and the first best wedges Δ_x with $x \in \{s, \tau, e\}$ ²⁶ are equal to:

$$\begin{aligned} \Delta_s &\equiv u_{C_1} - \beta R u_{C'_2} && \text{savings/consumption wedge} \\ \Delta_{\tau} &\equiv (1 + h) (\beta u_{C_2} - \delta u_{C_1}) && \text{backward redistribution wedge} \\ \Delta_e &\equiv \delta H_e \left(\beta (1 + n) \frac{H'_{h'}}{H'_{e'}} u_{C'_2} + \delta u_{C'_1} \right) - \beta u_{C_2} && \text{forward redistribution wedge} \end{aligned}$$

Under differentiability condition of policy rules we are able to provide a non-trivial formulation of the government first order condition in the case of no commitment. The above wedges can be interpreted as *potential* deviations from the efficient intertemporal decisions and they acquire straightforward economic meaning in the recursive dynamic environment. Note that, although the infinite persistent impact of the current tax rate and public education investment, only the current and the subsequent period matter directly. Thus the marginal costs and benefits in equilibrium can be summarized by terms involving only two consecutive periods. As a consequence, the *GEEs* can also be viewed as resulting from a variational (two-periods) problem (Klein et al., 2008). In other words, let us think of our *variational* problem as follows: given the state variables (h, k) and (h'', k'') fixed, let us vary (h', k') through the controls (τ, τ') and (e, e') , in order to obtain the highest possible utility. Recalling that the *SMPE* in the political case has been obtained as the limit of a finite horizon economy, whose convergence has been attained after two periods, we may easily conjecture no structural differences between the two equilibrium policy rules. For this reason in the following paragraph we will use the guess of the political equilibrium to verify the *GEEs* and obtain the Gvt solution without commitment.

Before solving quantitatively the Gvt problem, let us interpret the *GEEs* rewritten in terms

²⁵ See appendix B for the detailed derivation.

²⁶ The wedges Δ_s, Δ_{τ} and Δ_e are derived as the marginal direct impact on the intertemporal agents' utility respectively of a variation in individual savings, taxation/pension contribution and education investments. For example, a marginal variation in the income tax rate determines a direct cost for current adults equal to $\delta(1 + n)(1 + h)u_{c_1}$ and a direct benefits for current old equal to $\beta(1 + n)(1 + h)u_{c_2}$. The intergenerational backward redistribution wedge becomes then $\Delta_{\tau} \equiv \beta(1 + n)(1 + h)u_{c_2} - \delta(1 + n)(1 + h)u_{c_1}$ which normalized by $(1 + n)(1 + h)$ is equal to $\Delta_{\tau} \equiv \beta u_{c_2} - \delta u_{c_1}$. The same characterization holds for Δ_s and Δ_e .

of a linear weighted combination of wedges. First consider Eq. (27). Due to a marginal increase in taxation, τ , the intertemporal savings wedge, Δ_s , is scaled by the reduction in household savings, $\tilde{K}_\tau < 0$. Furthermore an increase in income tax rate determines an increase in the gap between u_{C_2} and u_{C_1} which is captured by the intratemporal utility wedge, Δ_τ . Note that, due to full depreciation of physical capital k'' is equal to $\tilde{K}(f', h')$ and it is not a function of k' . Then a variation in the current tax rate does not affect next period's wedges through its effect on future level of physical capital. More cumbersome dynamic effects emerge instead from the equilibrium determination of public education transfers, Eq. (26). As before an increase in e makes private savings wedge scaled by the variation in household savings, $\tilde{K}_e < 0$, which is negative due to the substitution effects with public savings that are increased via the retributive pension scheme. The second component, Δ_e , represents the intertemporal utility variation due to an increase in education transfers today, which affects the utility of current old and simultaneously a variation of the next-period adults' and old's utility through h' . Finally, differently from τ , a variation in the current level of education transfers also affect next period's wedges, $\Delta'_{\tau'}$, through its effect on h' , which induces a variation of both k'' and h'' .

6.1 The Government SMPE

To solve the Government optimization problem, we guess a time consistent bidimensional policy structurally equivalent to Eq. (20) and (21), which verifies the conditions (26) and (27). Fixing $\theta = \frac{1}{2}$, let Υ^{Gvt} be the state-space in which interior policy rules are obtained. Then the next Proposition characterizes the optimal feasible time-consistent policy rules:

Proposition 2 *Under dynamic efficiency condition, for any $(h, k) \in \Upsilon^{Gvt}$ the set of individual feasible rational policies, $f^g \equiv (e^g, \tau^g)$, which can be supported by a Government SMPE with perfect foresight, has the following functional form:*

i.

$$E^g(h) = a_1^g h + a_0^g \quad (28)$$

where $a_1^g = a_1$ and $a_0^g = a_0$;

ii.

$$T^g(h, k) = -b_3^g \frac{k}{1+h} + b_2^g \frac{h}{1+h} + b_1^g \frac{1}{1+h} + b_0^g \quad (29)$$

where $b_3^g \equiv R\Omega_R^g$, $b_2^g \equiv \frac{\alpha\sqrt{\psi^*}}{R-\alpha\sqrt{\psi^*}}(\Omega_O^g + R\sqrt{\psi^*}\Omega_R^g - \alpha\psi^*)$, $b_1^g \equiv \bar{h}^{\frac{1-\alpha}{\alpha}} \frac{R}{R-(1+n)} b_2^g + \bar{R}(\Omega_O^g - \bar{h}(1-\alpha)\psi^*)$ and $b_0^g \equiv \Omega_O^g$.

For any $(h, k) \notin \Upsilon^{Gvt}$ corner solutions result in at least one of the two dimensions.

Proof. (See Appendix). ■

The two equilibrium concepts described in definition 3 and 5 lead to the implementation of the same education program. Specifically, in equilibrium both the Gvt and the office-seeking politicians set the same amount of forward transfers, inducing education-efficient political fiscal

planes, as already discussed in Paragraph 5, i.e. $E^g(h) = E(h)$ for any level of human capital. The main difference concerns their quantitative predictions on the taxation policy dimension, which are fully captured by the policy parameters. In the following subsection we discuss in details how a political equilibrium diverges from the Gvt optimal allocation.

6.2 Are the political choices on pensions and education optimal?

Both the politicians and the Gvt have incentives to provide intergenerational transfers in the environment introduced in section 3. Moreover, their equilibrium policies share similar structural properties. However the quantitative differences detected so far imply distinct predictions in terms of WSRs' identification and political behavior. For this reason we now examine how the politicians act relatively to the Gvt in terms of taxation design. In order to obtain clear predictions, we normalize the vector of welfare weights by δ assigning $\phi \equiv \frac{\beta}{\delta}$. Consequently we are able to write the relative welfare weights, Eq. (24), in terms of political weights, making the two solutions comparable²⁷. Let us introduce the following notation $\Lambda \equiv \{(\phi, n) \in (\underline{\phi}, \infty) \times (\underline{n}, \bar{n}) \mid b_2 \geq b_1\}$ and $\Lambda^c \equiv \{(\phi, n) \in (\underline{\phi}, \infty) \times (\underline{n}, \bar{n}) \mid b_2^g \geq b_1^g\}$. In other terms (Λ, Λ^g) delimits the parametric space in which PCR emerges respectively for the political and the Gvt cases. The following Corollary resumes the conditions for the Welfare regimes' comparison between the political and Gvt cases in the parametric space (ϕ, n) .

Corollary 6 *For any level of $\bar{h}$ and $n \in (\underline{n}, \bar{n})$ the following condition holds: $\Lambda \subset \Lambda^g$.*

Proof. (See Appendix). ■

The parametric space in which PCR emerges is always larger in the Gvt environment than in the political one. Furthermore let $\bar{\phi}$ be a sufficiently large value of the ideological bias²⁸, such that for any $\phi < \bar{\phi}$, the following Proposition is stated.

Proposition 3 *For $\delta < \bar{\delta}$ and for any $\phi < \bar{\phi}$, the political SMPE induces overtaking with respect to the time-consistent Government SMPE, i.e. $T(h_t, k_t) > T^g(h_t, k_t)$ for any $(h_t, k_t) \in \Upsilon^{Pol} \cap \Upsilon^{Gvt}$.*

Proof. (See Appendix). ■

According to the above proposition, if the Gvt adopts a politically equivalent system of welfare weights, for any value of human and physical capital the level of income tax rate is always lower than in the political case, i.e. $T(h_t, k_t) > T^g(h_t, k_t)$. We conclude that politicians involved in a Markov game among successive generations of players deliver the Gvt allocation if they reduce the political weight they assign to the old agents. Given the invariant level of education transfers achieved by both the politicians and the Gvt, high tax rate implies pension benefits too generous. These distortions come from the politicians' strategic behavior. In determining taxation rules, short-lived politicians take into account that future politicians will

²⁷In particular the relative Welfare weights rewritten in terms of political weights are equal to $\Omega_R^g \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)}$ and $\Omega_O^g \equiv \frac{\phi-\beta(1+n)}{\phi+(1+n)}$.

²⁸See proof of Proposition 1 for the exact determination of $\bar{\phi}$.

compensate the fiscal cost of current adults by paying the pensions in their old age. This stems from the fact that higher taxes on today environment lead to a lower private wealth in old age, i.e. to a lower state variable in the following period, thereby triggering more transfers by the future politicians. The policy response of the future politicians thus reduces the current (electoral) cost of transferring resources to the elderly and leads to overspending, unless the adult enjoy an unusually large political power. Consequently, by transferring too much resources to old age due to both the overrepresenting of current elderly agents and the policy response of the future politicians, the politicians fail to provide the optimal income tax rate policy.

7 Conclusions

In this paper we investigate the conditions for the emergence of implicit intergenerational contracts without assuming reputation mechanisms, commitment technology and altruism. We present a tractable dynamic politico-economic model in OLG environment where political representatives compete proposing multidimensional fiscal platforms. Both backward and forward intergenerational transfers, respectively in the form of pension benefits and higher education investments, are simultaneously considered in an endogenous human capital setting with income taxation when agents play Markovian strategies. The infinite horizon Gvt solution without commitment is used as benchmark to evaluate the efficiency of politically determined rules.

The dynamic mechanisms driving our results are intuitive: Social security system sustains investment in public education, that, in turns, creates a dynamic linkage across periods through both human and physical capital driving the economy towards different WSR.

We show that intergenerational contracts may be politically sustained uniquely as long as the economy is in dynamic efficiency, i.e. the rental gross price of capital is larger than the economic growth rate. Departing from the previous literature, our economic environment is in line with empirical findings on the dynamic efficiency status of most developed countries, especially after the demographic transition. By endogenizing human capital formation through public education investments, backward and forward redistributive programs may optimally self-sustain each other even in the absence of a benevolent Gvt. In equilibrium political decisions are education efficient.

Relatively to the prediction about the transition towards the steady state, we find three different WSR may emerge depending on both the relative political bargaining power between adults and old and the endogenous capital asset accumulation. The emergence of different regimes leads the economy towards different dynamic paths and persistence degrees of distortionary redistribution. In the regime supported by adults, the equilibrium pension benefits reach the highest feasible level.

Demographic aging increases the equilibrium per-capita level of forward transfers, i.e. public education spending. Due to the decreasing return in human capital accumulation aging does not always exacerbate the generous behavior of the politicians towards the elderly. Political aging has instead positive impact on taxation but no effects on the level of public education investments.

Finally, due to the distortions generated by repeated political competition process and to the political overrepresentation of elderly agents, political equilibrium is characterized by over-taxation compared with the Gvt solution.

Our analysis leaves some natural directions for future research. We have assumed only adults and old compete in the political debate. Using the developed methodology, a change in the voting rule, which enables also the young to vote, would generate different equilibrium allocations both in terms of education transfers and government size. Another direction for future research concerns the introduction of a dynamic electoral stage by endogenizing the probability of re-election, which would introduce a new source of distortion.

8 Technical Appendix A

Proof of Theorem (1).

Following Klein et al. (2008), our resolution strategy consists in computing the first order conditions starting from a time $t < \infty$ large enough and solving backward for each time $t - j$ with $j = 1, 2, \dots$, subject to: 1) the economic Euler condition, Eq. (10), 2) the balanced budget constraint, Eq. (8), and 3) the equilibrium policy rules of the following periods. We recursively determine the conditions for the existence of fixed points taking the limit for $j \rightarrow \infty$.

Suppose the economy ends at time $t < \infty$ and adults at that time have one period temporal-horizon. Thus, the political objective function is as follows:

$$W_t \equiv (1 + n) u(C_{1,t}) + \phi u(C_{2,t}) \quad (1A)$$

where $C_{1,t} \equiv (1 + h_t)(1 - \tau_t)$ and $C_{2,t} \equiv (1 + n)Rk_t + p_t$. At time t there are no incentives in investing in education, i.e. $e_t = 0$. Assuming interior solution, the fiscal dimension, τ_t , is determined according to the Euler condition, as follows:

$$\frac{u_{C_{2,t}}}{u_{C_{1,t}}} = \frac{1}{\phi} \quad (2A)$$

Under logarithmic utility, the functional form of the equilibrium fiscal policy rule at time t is $\tau_t = -R\Omega_{1,t}\frac{k_t}{1+h_t} + \Omega_{2,t}$ where $\Omega_{1,t} \equiv \frac{1+n}{1+n+\phi}$ and $\Omega_{2,t} \equiv \frac{\phi}{1+n+\phi}$. Consequently, the equilibrium policy rules, $F_t = (E_t, T_t)$, are equal to:

$$F_t : \begin{cases} T_t = -b_{1(0)}\frac{k_t}{1+h_t} + b_{0(0)} \\ E_t = 0 \end{cases} \quad (3A)$$

where $b_{1(0)} \equiv R\Omega_{1,t}$ and $b_{0(0)} \equiv \Omega_{2,t}$. The number in the brackets represents the number of iterations.

Next we consider period $t - 1$, in which adults born at time $t - 2$ live up three periods²⁹. The political objective function is now as follows:

$$W_{t-1} \equiv (1 + n) (u(C_{1,t-1}) + \beta u(C_{2,t})) + \phi u(C_{2,t-1}) \quad (4A)$$

where $C_{1,t-1} \equiv (1 + h_{t-1})(1 - \tau_{t-1}) - (1 + n)k_t$ and $C_{2,t-1} \equiv (1 + n)Rk_{t-1} + p_{t-1}$. After plugging the equilibrium policy rules of period t , Eq. (3A), into Eq. (4A), we maximize with respect to $f_{t-1} \equiv (e_{t-1}, \tau_{t-1})$. Applying envelope theorem, we get the following system of Euler equations:

$$\begin{cases} \frac{u_{C_{2,t-1}}}{u_{C_{1,t-1}}} = \frac{1+\beta}{\phi+\beta(\phi+1+n)} \\ \frac{u_{C_{2,t-1}}}{u_{C_{1,t-1}}} = \frac{1}{R} \left(\frac{1+\beta}{\phi+\beta(\phi+1+n)} \right) \frac{dh_t}{de_{t-1}} \end{cases} \quad (5A)$$

Equating the two conditions in (5A), we get the necessary condition for the determination of

²⁹Due to three-periods effects of the political variable e not all the intergenerational *direct dynamic feedbacks* are internalized at time $t - 1$ and further recursion is necessary.

the equilibrium level of e_{t-1} , i.e. $\frac{dh_t}{de_{t-1}} = R$. Recalling that at time t , $h_t = \left(\frac{\alpha h_{t-1} + (1-\alpha)\bar{h}}{1+n}\right)^\theta e_{t-1}^{1-\theta}$ and plugging $\frac{dh_t}{de_{t-1}}$ into the equilibrium condition, we derive the equilibrium public education transfers at time $t-1$. Let us denote $\psi_{(1)} \equiv \left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$ and $\gamma_{(1)} = \frac{1+n}{R}$. By solving the system (5A), the equilibrium policy rules are then equal to:

$$F_{t-1} : \begin{cases} T_{t-1} = -b_{4(1)} \frac{k_{t-1}}{1+h_{t-1}} + b_{3(1)} \frac{h_{t-1}}{1+h_{t-1}} + b_{2(1)} \frac{\bar{h}}{1+h_{t-1}} + b_{1(1)} \frac{1}{1+h_{t-1}} + b_{0(1)} \\ E_{t-1} = a_{1(1)} h_{t-1} + a_{0(1)} \end{cases} \quad (6A)$$

where $a_{0(1)} \equiv \frac{(1-\alpha)\bar{h}}{1+n} \psi_{(1)}$, $a_{1(1)} \equiv \frac{\alpha}{1+n} \psi_{(1)}$ and $b_{0(1)} \equiv \Omega_{2,t-1}$, $b_{1(1)} \equiv \gamma_{(1)} \Omega_{2,t-1}$, $b_{2(1)} \equiv (1-\alpha) \Omega_{1,t-1} \psi_{(1)}$, $b_{3(1)} \equiv \alpha \left(\Omega_{1,t-1} + \frac{1}{(1-\theta)} \Omega_{2,t-1} \right) \psi_{(1)}$ and $b_{4(1)} \equiv R \Omega_{1,t-1}$. Now $\Omega_{2,t-1} \equiv \Omega_2 \equiv \frac{\phi}{\phi+(1+n)(1+\beta)}$ and $\Omega_{1,t-1} \equiv \Omega_1 \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)(1+\beta)}$ are, respectively, the indexes of the relative old's and adults' political power in an economy that lasts more than one period.

Finally let us consider time $t-2$. At this point, all the direct dynamic feedbacks are internalized. The political objective function is equivalent to Eq. (4A). The recursive problem is now subject to the equilibrium policy rules of the next two periods, (3A) and (6A). Maximizing the political objective function with respect to $f_{t-2} \equiv (e_{t-2}, \tau_{t-2})$ the system of Euler conditions are:

$$\begin{cases} \frac{u_{C2,t-2}}{u_{C1,t-2}} = \frac{1}{\phi+(1+n)\beta} \\ \frac{u_{C2,t-2}}{u_{C1,t-2}} = \frac{1}{R(\phi+(1+n)\beta)} \left(1 + \frac{\alpha\theta}{1-\theta} \left(\frac{1-\theta}{R} \right)^{\frac{1}{\theta}} \right) \frac{dh_{t-1}}{de_{t-2}} \end{cases} \quad (7A)$$

Let us now denote with $\psi_{(2)} \equiv \left(\frac{\alpha\theta}{R} \left(\frac{1-\theta}{R} \right)^{\frac{1}{\theta}} + \frac{1-\theta}{R} \right)^{\frac{1}{\theta}}$ and $\gamma_{(2)} \equiv \frac{1+n}{R} + \left(\frac{1+n}{R} \right)^2$. Furthermore, let us introduce the following notation $g_{(2)} \equiv \frac{1+n}{R} \psi_{(1)} + \psi_{(2)}$. As before, solving the system (7A) we yield the following pair of equilibrium policy rules at time $t-2$:

$$F_{t-2} : \begin{cases} T_{t-2} = -b_{4(2)} \frac{k_{t-1}}{1+h_{t-1}} + b_{3(2)} \frac{h_{t-1}}{1+h_{t-1}} + b_{2(2)} \frac{\bar{h}}{1+h_{t-1}} + b_{1(2)} \frac{1}{1+h_{t-1}} + b_{0(2)} \\ E_{t-2} = a_{1(2)} h_{t-1} + a_{0(2)} \end{cases} \quad (8A)$$

where $b_{0(2)} \equiv b_{0(1)}$, $b_{1(2)} \equiv \gamma_{(2)} \Omega_2$, $b_{2(2)} \equiv (1-\alpha) \left(\left(\Omega_1 + \frac{1}{1-\theta} \Omega_2 \right) \psi_{(2)} + \frac{\theta}{1-\theta} \Omega_2 g_{(2)} \right)$, $b_{3(2)} \equiv \alpha \psi_{(2)} (\Omega_1 + \frac{1}{1-\theta} \Omega_2)$, $b_{4(2)} \equiv b_{4(1)}$ and $a_{0(2)} \equiv \frac{(1-\alpha)\bar{h}}{1+n} \psi_{(2)}$, $a_{1(2)} \equiv \frac{\alpha}{1+n} \psi_{(2)}$. It is straightforward to show that $\psi_{(2)}$ can be derived as a differentiable monotonic transformation of $\psi_{(1)}$, $m(\cdot)$, characterized by $m(0) > 0$, $m_\psi > 0$ and $m_{\psi\psi} > 0$. In particular $m\left(\psi_{(1)}\right) = \left(\frac{\alpha\theta}{R} \psi_{(1)} + \frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$. The argument can be repeated for each time $j > 0$ such that:

$$\psi_{(j+1)} = m\left(\psi_{(j)}\right) \quad (9A)$$

Furthermore for each j the following series can be derived:

$$\gamma_{(j)} \equiv \sum_{l=1}^j \left(\frac{1+n}{R} \right)^l \quad \text{and} \quad g_{(j)} \equiv \left(\frac{1+n}{R} \right)^{j-1} \psi_{(1)} + \left(\frac{1+n}{R} \right)^{j-2} \psi_{(2)} + \dots + \psi_{(j)}$$

Using the above notation, starting from $t = 3$ we can finally derive the recursive structure which characterizes the political problem:

$$F_{t-j} : \begin{cases} T_{t-j} = -b_{4(j)} \frac{k_{t-j}}{1+h_{t-j}} + b_{3(j)} \frac{h_{t-j}}{1+h_{t-j}} + b_{2(j)} \frac{\bar{h}}{1+h_{t-j}} + b_{1(j)} \frac{1}{1+h_{t-j}} + b_{0(j)} \\ E_{t-j} = a_{1(j)} h_{t-j} + a_{0(j)} \end{cases} \quad (10A)$$

where $a_{0(j)} \equiv \frac{(1-\alpha)\bar{h}}{1+n} \psi_{(j)}$, $a_{1(j)} \equiv \frac{\alpha}{1+n} \psi_{(j)}$ and $b_{0(j)} \equiv b_{0(1)}$, $b_{1(j)} \equiv \gamma_{(j)} \Omega_2$, $b_{2(j)} \equiv (1-\alpha) ((\Omega_1 + \frac{1}{1-\theta} \Omega_2) \psi_{(j)} + \frac{\theta}{1-\theta} \Omega_2 g_{(j)})$, $b_{3(j)} \equiv \alpha \psi_{(j)} \left(\Omega_1 + \frac{1}{(1-\theta)} \Omega_2 \right)$, $b_{4(j)} \equiv b_{4(1)}$.

If a political SMPE exists, then the limits for $j \rightarrow \infty$ of the set of time-variant parameters $\{a_{0(j)}, a_{1(j)}, b_{0(j)}, b_{1(j)}, b_{2(j)}, b_{3(j)}, b_{4(j)}\}$ exist and are finite. Note that the fixed points determination for the two stationary policy rules crucially depends on the existence of the fixed point of the policy e and, in final instance, on the determination of the limit for $\psi_{(j)}$. Thus, we start with the redistributive policy dimension. The computation consists in solving the non-linear difference equation (9A). The $\lim_{j \rightarrow \infty} \psi_{(j)}$ is equivalent to the solution(s), if any, of such difference equation given ψ_0 as initial condition. Let us denote with $\hat{\psi}_j$ the value of ψ_j such that $\left(\frac{dm(\psi_j)}{d\psi_j} \right)_{\psi_j = \hat{\psi}_j} = 1$. We yield respectively zero, one or two fixed points as solution of the difference equation iff $m(\hat{\psi}_j) \gtrless \hat{\psi}_j$. Thus, $\hat{\psi}_j$ is equal to:

$$\hat{\psi}_j = \frac{1}{\theta} \left(\frac{R}{\alpha} \right)^{\frac{1}{1-\theta}} - \frac{1-\theta}{\alpha\theta} \quad (11A)$$

Note that $R > \alpha^\theta$ in all the parameters' space. Such condition guarantees the existence of at least one stable fixed point. For analytical tractability we determine the solutions for quadratic form case. For $\theta = \frac{1}{2}$ under the above condition the two fixed points are:

$$\psi_{1,2}^* = \frac{1}{\alpha} \left(\frac{2R}{\alpha} \left(R \pm \sqrt{R^2 - \alpha} \right) - 1 \right)$$

We focus on the stable equilibrium, denoted by $\psi^* = \frac{1}{\alpha} \left(\frac{2R}{\alpha} \left(R - \sqrt{R^2 - \alpha} \right) - 1 \right)$ and we take $\psi_0 = \psi^*$ as initial condition. The solution of the difference equation (9A) is represented in Figure 6.

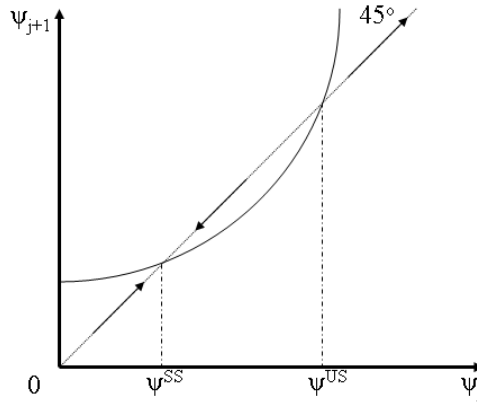


Figure 6: $\psi_{(j+1)} = m[\psi_{(j)}]$

Under the condition $R > (1+n)$ the $\lim_{j \rightarrow \infty} \gamma_{(j)} < \infty$ is equal to $\frac{1+n}{R-(1+n)} \equiv \bar{R}$. Consequently the $\lim_{j \rightarrow \infty} g_{(j)} = \lim_{j \rightarrow \infty} \psi^* \sum_{l=1}^j \left(\frac{1+n}{R}\right)^l < \infty$ is equal to $\frac{\bar{R}}{\alpha} \left(\frac{2R}{\alpha} \left(R - \sqrt{R^2 - \alpha}\right) - 1\right)$. Under such convergence conditions the fixed points are finally attained. Rearranging the terms we can reformulate the individual rational fiscal and redistribution policies as follows:

$$T(h_t, k_t) = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0 \quad (12A)$$

where $b_0 \equiv \Omega_2$, $b_1 \equiv \bar{h}(1-\alpha)\psi^*(\Omega_1 + (2+\bar{R})\Omega_2) + \bar{R}\Omega_2$, $b_2 \equiv \alpha\psi^*(\Omega_1 + 2\Omega_2)$ and $b_3 \equiv R\Omega_1$;

$$E(h_t) = a_1 h_t + a_0 \quad (13A)$$

where $a_0 \equiv \frac{1-\alpha}{1+n}\bar{h}\psi^*$ and $a_1 \equiv \frac{\alpha}{1+n}\psi^*$.

We denote with $(K_t^\tau, H_t^\tau) = \{(k_t, h_t) | \check{k}_t < k_t < \hat{k}_t\}$ where $\hat{k}_t \equiv \frac{b_2+b_0}{b_3}h_t + \frac{b_1+b_0}{b_3}$ and $\check{k}_t(h_t) \equiv \frac{b_2+b_0-1}{b_3}h_t + \frac{b_1+b_0-1}{b_3}$. While

$$H_t^e = \begin{cases} \{h_t | h_t \in (0, \infty)\} & \text{if } \bar{h} < \frac{1}{(1-\alpha)\psi^*} \\ \{h_t | h_t \in (\tilde{h}, \infty)\} & \text{if } \bar{h} \geq \frac{1}{(1-\alpha)\psi^*} \end{cases}$$

where $\tilde{h} \equiv \frac{1-\bar{h}(1-\alpha)\psi^*}{\alpha\psi^*-1}$. Jointly considering the above feasibility conditions for both fiscal and redistributive dimensions, non-degenerate policies, i.e. $\tau_t \in (0, 1)$ and $e_t \in (0, \hat{e}_t)$, are achieved at each time for any $(k_t, h_t) \in \Upsilon^{Pol} \equiv (K_t^\tau, H_t^\tau) \cap H_t^e$. ■

Proof of Lemma (1). Let us first consider the transition dynamics of h_t and e_t . Plugging the equilibrium education transfers, Eq. (20), into the human capital production, Eq. (6), we obtain the law of motion $h_{t+1} = H^d(h_t)$, which is equal to:

$$h_{t+1} = \lambda_1 h_t + \lambda_0 \quad (14A)$$

where $\lambda_0 \equiv \frac{(1-\alpha)\bar{h}}{1+n}\sqrt{\psi^*}$ and $\lambda_1 \equiv \frac{\alpha}{1+n}\sqrt{\psi^*}$. It should be noted the serial correlation between current and future level of human capital is always positive, i.e. $\lambda_1 \geq 0$. To determine the law of motion of the redistributive policy we plug Eq. (6) into the equilibrium education policy rule at time $t+1$. The law of motion $e_{t+1} = E^d(h_t)$ is then as follows:

$$e_{t+1} = \xi_1 h_t + \xi_0 \quad (15A)$$

where $\xi_0 \equiv a_0 \left(\frac{a_1}{\sqrt{\psi^*}} + 1\right)$ and $\xi_1 \equiv \frac{a_1^2}{\sqrt{\psi^*}}$. Note that, if the dynamics of h_t is characterized by stable convergence, i.e. $\lambda_1 < 1$, then also the dynamics of e_t is convergent toward the steady state. Thus, using the expression of λ_1 , the sufficient condition for the convergence stability of both h_t and e_t requires:

$$n > \underline{n} \quad (16A)$$

where $\underline{n} \equiv \sqrt{2R \left(R - \sqrt{R^2 - \alpha}\right) - \alpha} - 1$. Due to linearity, both h_t and e_t converge monotonically

toward the steady states.

Let us now analyze the transition dynamics of k_t and τ_t . First, consider the following recursive formulation for the equilibrium saving under log-utility, $k_{t+1} = \tilde{K}(e_t, \tau_t, h_t)$, which is obtained plugging the human capital production, Eq. (6), and the expected equilibrium policies e_{t+1} and τ_{t+1} according to Eq. (20) and (21). The saving function can then be rewritten as follows:

$$k_{t+1} = \frac{\beta R(1+h_t)(1-\tau_t)}{(R(1+\beta)-b_3)(1+n)} - \frac{(b_0+b_2-(1+n)a_1)H(e_t, h_t) + (b_0+b_1-(1+n)a_0)}{R(1+\beta)-b_3} \quad (17A)$$

Plugging the equilibrium policy rules, Eq. (20) and Eq. (21), into Eq. (17A), we obtain the law of motion $k_{t+1} = K^d(h_t, k_t)$:

$$k_{t+1} = \pi_2 k_t + \pi_1 h_t + \pi_0 \quad (18A)$$

where:

$$\begin{aligned} \pi_2 &\equiv \frac{R\beta b_3}{(1+n)(R(1+\beta)-b_3)} \\ \pi_1 &\equiv -\left(\frac{(b_0+b_2-a_1(1+n))\lambda_1}{(R(1+\beta)-b_3)} + \frac{R\beta(b_0+b_2-1)}{(1+n)(R(1+\beta)-b_3)}\right) \\ \pi_0 &\equiv -\left(\frac{b_0+b_1-a_0(1+n)}{(R(1+\beta)-b_3)} + \frac{(b_0+b_2-a_1(1+n))\lambda_0}{(R(1+\beta)-b_3)} + \frac{R\beta(b_0+b_1-1)}{(1+n)(R(1+\beta)-b_3)}\right) \end{aligned}$$

It should be noted that current and future level of physical capital are positively interrelated each other, $\pi_2 \geq 0$, on the contrary the way h_t perturbs k_{t+1} depends on the WSR's intensity embedded in π_1 .

Under condition (16A), the dynamics of physical capital is characterized by stable convergence if $\pi_2 < 1$, which requires:

$$\phi > \underline{\phi} \quad (19A)$$

where $\underline{\phi} \equiv \beta(R-(1+n))$. Let us denote by $Q_t^h \equiv \frac{1+h_t}{1+h_{t+1}}$. Plugging Eq. (14A) and (17A) into the equilibrium income tax policy at time $t+1$, after some manipulations, we attain the law of motion $\tau_{t+1} = T^d(\tau_t, h_t)$, as follows:

$$\tau_{t+1} = \sigma(h_t)\tau_t + \zeta(h_t) \quad (20A)$$

where:

$$\begin{aligned} \sigma(h_t) &\equiv \frac{R\beta b_3}{(1+n)(R(1+\beta)-b_3)} Q_t^h \\ \zeta(h_t) &\equiv \frac{R(1+\beta)(1+n)(b_1-b_2) + (1+n)^2(a_1-a_0)b_3}{(R(1+\beta)-b_3)(1+n)} \frac{1}{1+\lambda_1 h_t + \lambda_0} \\ &\quad - \frac{\beta R b_3}{(R(1+\beta)-b_3)(1+n)} \frac{1+h_t}{1+\lambda_1 h_t + \lambda_0} + \frac{R(1+\beta)(b_0+b_2)-(1+n)b_3 a_1}{R(1+\beta)-b_3} \end{aligned}$$

Note that, under Eq. (16A), the convergence condition for k_t , Eq. (19A), is also sufficient for the convergence of τ_t , i.e. $\sigma(h^*) < 1$. Furthermore the speed of convergence for τ_t basically depends on the WSR characterizing the economy jointly with the exogenous human capital society endowment. To show how such elements may affect the type of convergence let us take the derivative of $\Gamma(h_t)$ with respect to the human capital asset. We obtain:

$$\frac{d\zeta(h_t)}{dh_t} = \frac{-b_3 \left(\beta R(1+\lambda_0 - \lambda_1) + (1+n)^2 \lambda (a_1 - a_0) \right) - R(1+\beta)(1+n)(b_1 - b_2)\lambda_1}{(1+n)(R(1+\beta) - b_3)(1+\lambda_1 h_t + \lambda_0)^2}$$

It is straightforward to show how the sign of $\frac{d\zeta(h_t)}{dh_t}$ crucially depends on the differences $(a_1 - a_0)$ and $(b_1 - b_2)$ and in final instance on $\bar{h}$, and on the relative political power weights of adults and old embedded in the coefficients b_1 and b_2 . When $\frac{d\zeta(h_t)}{dh_t} \gtrless 0$ and $\tau_0 \gtrless \tau^*$ then the speed of convergence toward the steady state is lower (higher) than in the opposite case.

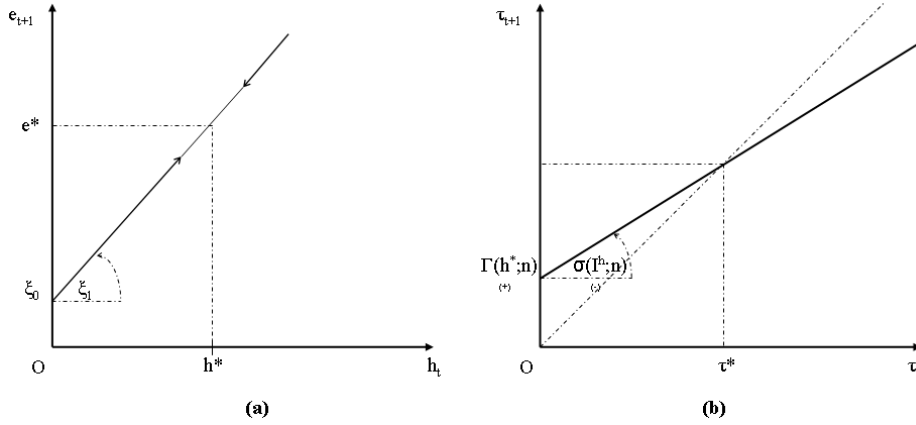


Figure 7: Panel (a) shows the law of motion of e_t , Panel (b) shows the law of motion of τ_t .

From a qualitative point of view the dynamics of e_t and τ_t are mirror image respectively to the dynamics of h_t and k_t . They mainly differ from an autoregressive component of infinite order in the past level of public education, which arises because of the infinite persistence of education spending on the future level of human capital through the parental transmission. The Figure 7 emphasizes the dynamics of the political variables. The Panel (a) shows that, once the human capital converges to the steady state also the education policy reaches its balanced growth path. Differently, the Panel (b) highlights how the convergence condition of h_t is necessary but not sufficient for the stable convergence of the fiscal policy rule, which also requires the dynamic stability of k_t . ■

Proof of Proposition (1). Under Lemma 1, due to linearity of the laws of motion, Eq. (14A), (15A), (18A) and (20A), there exists a unique steady state $\{e^*, \tau^*, h^*, k^*\}$. Equating $h_{t+1} = h_t = h^*$ in Eq. (14A) and $k_{t+1} = k_t = k^*$ in Eq. (18A), the following steady state levels for the state variables are obtained:

$$h^* = \frac{(1-\alpha)\bar{h}\sqrt{\psi^*}}{(1+n) - \alpha\sqrt{\psi^*}} \quad (21A)$$

$$k^* = \frac{\beta R (b_0 + b_2 - 1) + (1+n) (b_0 + b_2 - (1+n) a_1) \lambda_1}{b_3 ((1+n) + \beta R) - R (1+\beta) (1+n)} h^* \quad (22A)$$

$$+ \frac{((1+n) (1+\lambda_0) + \beta R) b_0 + ((1+n) + \beta R) b_1 + (1+n) b_2 \lambda_0 - (1+n)^2 (a_1 \lambda_0 + a_0) - \beta R}{b_3 ((1+n) + \beta R) - R (1+\beta) (1+n)}$$

Plugging Eq. (21A) and (22A) into the equilibrium policy rules described in Theorem 1, we obtain the following the steady states levels for the political control variables:

$$e^* = \frac{(1-\alpha) \bar{h} \psi^*}{(1+n) - \alpha \sqrt{\psi^*}} \quad (23A)$$

$$\tau^* = - \frac{(1+n) (R (1+\beta) (b_1 - b_2) + (1+n) (a_1 - a_0) b_3)}{b_3 ((1+n) + \beta R) - R (1+\beta) (1+n)} \frac{1}{1+h^*} \quad (24A)$$

$$+ \frac{\beta R b_3}{b_3 ((1+n) + \beta R) - R (1+\beta) (1+n)} - \frac{(1+n) (R (1+\beta) (b_0 + b_2) - (1+n) b_3 a_1)}{b_3 ((1+n) + \beta R) - R (1+\beta) (1+n)}$$

By balanced budget constraint the pension steady state level is:

$$p^* = (1+n) (1+h^*) \tau^* - (1+n)^2 e^*$$

■

Proof of Corollary (1). The proof is straightforward. The derivative of Eq. (21) with respect to h_t is equal to:

$$\frac{d\tau_t}{dh_t} = \frac{b_3 k_t + b_2 - b_1}{(1+h_t)^2} \quad (25A)$$

For any level of k_t , if $b_1 \leq b_2$, then $\frac{d\tau_t}{dh_t} \geq 0$. Otherwise, if $b_1 > b_2$, then the sign of Eq. (25A) depends on the value reached by k_t . When $k_t < \tilde{k}$ where $\tilde{k} \equiv \frac{b_1 - b_2}{b_3}$ the income tax rate is a decreasing function of h_t , i.e. $\frac{d\tau_t}{dh_t} < 0$. The opposite holds for $k_t \geq \tilde{k}$. ■

Proof of Corollary (2). Given the balanced budget constraint (8), let us denote with $P(h_t, k_t) \equiv (1+n) (1+h_t) T(h_t, k_t) - (1+n)^2 E(h_t)$ the equilibrium pension policy rule. Under the decreasing return in education and the equilibrium level of policy rules, Eq. (21) and Eq. (20), the total amount of pension contributions can be rewritten as follows:

$$p_{t+1} = P(h_{t+1}, k_{t+1}) \equiv (1+n) (-b_3 k_{t+1} + (b_2 + b_0 - (1+n) a_1) h_{t+1} + (b_1 + b_0 - (1+n) a_0)) \quad (26A)$$

The derivative of (26A) with respect to e_t is equal to:

$$\frac{dp_{t+1}}{de_t} = (1+n) \left(-b_3 \frac{dk_{t+1}}{de_t} + (b_2 + b_0 - (1+n) a_1) \frac{dh_{t+1}}{de_t} \right) \quad (27A)$$

where under log utility $\frac{dk_{t+1}}{de_t} = - \frac{(b_2 + b_0 - a_1(1+n))}{R(1+\beta)} \frac{dh_{t+1}}{de_t}$. After some algebra, the derivative (27A)

is as follows:

$$\frac{dp_{t+1}}{de_t} = \frac{R(1+\beta)(1+n)(b_2+b_0-a_1(1+n))}{R(1+\beta)-b_3} \frac{dh_{t+1}}{de_t} \quad (28A)$$

Noting that $(b_2+b_0-a_1(1+n)) > 0$ and $R(1+\beta)-b_3 > 0$, Eq. (28A) takes always positive values for any Welfare Regime and in the whole state space. ■

Proof of Corollary (3). Let us denote with $\rho = \frac{b_2+b_0}{b_1+b_0}$ a measure of the Welfare State Regimes' intensity. According to Eq. (22), the higher the adults' relative power is, the larger is the value of ρ . Normalizing the Eq. (26A) by the factor (b_1+b_0) , we obtain:

$$\hat{p}_t = (1+n) \left[-\hat{b}_3 k_t + (\rho - (1+n) \hat{a}_1) h_t + (1 - (1+n) \hat{a}_0) \right] \quad (29A)$$

where $\hat{p}_t \equiv \frac{p_t}{b_1+b_0}$, $\hat{b}_3 \equiv \frac{b_3}{b_1+b_0}$, $\hat{a}_0 \equiv \frac{a_0}{b_1+b_0}$ and $\hat{a}_1 \equiv \frac{a_1}{b_1+b_0}$. Taking the derivatives of Eq. (29A) with respect to ρ and k_t , the marginal impacts $\frac{d\hat{p}_t}{d\rho} = (1+n) h_t > 0$ and $\frac{d\hat{p}_t}{dk_t} = -(1+n) \hat{b}_3 < 0$ are attained. In other words, the higher the level of ρ and the lower the level of physical capital are, the larger is the amount of pension benefits. ■

Proof of Corollary (4). The equilibrium education transfer chosen by politicians is the linear policy rule $E(h_t) = a_1 h_t + a_0$, with a_1 and a_0 defined in Theorem 1. Political population aging, an increase in ϕ , does not affect at all the amount of equilibrium forward transfers, then $\frac{dE}{d\phi} = 0$. The equilibrium level of income tax rate is instead a linear function of k_t and non linear in h_t , $T(k_t, h_t) = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0$, where the coefficients are fully described in Proposition 2. A variation in the exogenous political ideological bias ϕ determines the following marginal changes in the structural parameters: $\frac{db_3}{d\phi} = -\frac{R(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$, $\frac{db_2}{d\phi} = \frac{\alpha\psi^*(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$, $\frac{db_1}{d\phi} = \frac{(1+n)(1+\beta)(1+n+(1-\alpha)\psi^*\bar{h}R)}{(R-(1+n)(\phi+(1+n)(1+\beta)))^2} > 0$ and $\frac{db_0}{d\phi} = \frac{(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$. Then, for any level of $\bar{h}$ $\frac{dT}{d\phi} > 0$, which implies positive correlation between the pension benefits and the ideological bias in favor of old agents. Finally, using the above results, the derivative of pensions transfers obtained by balanced budget constraint, $P(h_t, k_t) = (1+n)((1+h_t)T(h_t, k_t) - (1+n)E(h_t))$, with respect to the political aging parameter is $\frac{dP}{d\phi} = (1+n)\left((1+h_t)\frac{dT}{d\phi}\right) > 0$. ■

Proof of Corollary (5). To determine the effect of demographic population aging on the level of education transfers chosen by politicians, i.e. a decrease in n , note that $\frac{da_1}{dn} = -\frac{\alpha}{(1+n)^2}\psi^* < 0$ and $\frac{da_0}{dn} = -\frac{1-\alpha}{(1+n)^2}\bar{h}\psi^* < 0$. Then it follows $\frac{dE}{dn} < 0$. Concerning the impact of n on the political equilibrium level of income tax rate the following marginal changes in the structural parameters hold: $\frac{db_3}{dn} = \frac{\phi+R(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$, $\frac{db_2}{dn} = -\frac{\alpha\psi^*\phi(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$, $\frac{db_1}{dn} = D_0 + D_1 D_2 \gtrless 0$, where $D_0 \equiv \frac{\phi}{(R-(1+n)(\phi+(1+\beta)(1+n)))} > 0$, $D_1 \equiv \frac{\phi(1+n+\bar{h}\psi^*(1-\alpha)R)}{(R-(1+n)(\phi+(1+\beta)(1+n)))} > 0$ and $D_2 \equiv \left(\frac{1}{R-(1+n)} - \frac{1+\beta}{\phi+(1+\beta)(1+n)}\right) = \frac{1}{1+n}(\bar{R} - \Omega_A) \gtrless 0$ if $\bar{R} \gtrless \Omega_A$, finally $\frac{db_0}{dn} = -\frac{\phi(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$. Then it follows that $\frac{dT}{dn} \gtrless 0$ depending on the difference $(\bar{R} - \Omega_A)$ and on the level of $\bar{h}$. In particular a sufficient condition to yield $\frac{dT}{dn} < 0$ is $\bar{R} < \Omega_A$ and $\bar{h}$ high enough. Finally the marginal variation of pension benefits due to population growth is equal to $\frac{dP}{dn} = (1+n)((1+h_t)\frac{dT}{dn} - (1+n)\frac{dE}{dn}) \gtrless 0$. ■

9 Technical Appendix B

9.1 Derivation of recursive formulation and Generalized Euler Equation

We derive the recursive formulation of the Gvt program starting from its sequential version:

$$V_0^g(h_0, k_0) = \max_{\{f_t^g, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t U(f_t^g, h_t, k_t, k_{t+1}) \quad (1B)$$

where (h_0, k_0) are the initial conditions of the payoff-relevant state variables of the dynamic optimization program and $U(f_t^g, h_t, k_t, k_{t+1}) \equiv (1+n) \delta u(C_{1,t}(\tau_t^g, h_t, k_{t+1})) + \beta u(C_{2,t}(f_t^g, h_t, k_t))$. Equivalently we rewrite the above value function in the following terms:

$$V_0^g(h_0, k_0) = \max_{(f_0^g, k_1)} U(f_0^g, h_0, k_0, k_1) + (1+n) \delta \max_{\{f_t^g, h_{t+1}, k_{t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t U(f_t^g, h_t, k_t, k_{t+1}) \quad (2B)$$

By definition, we have:

$$V_1^g(h_1, k_1) = \max_{\{f_t^g, h_{t+1}, k_{t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t U(f_t^g, h_t, k_t, k_{t+1}) \quad (3B)$$

Due to stationarity condition the indirect utility function satisfies $V_0^g(\cdot) \equiv V_1^g(\cdot) \equiv \dots \equiv V_t^g(\cdot)$. We omit time indexes and denote by prime symbol next period variables. Plugging Eq. (3B) into Eq. (2B) we yield the following Bellman equation:

$$V^g(h, k) = \max_{(f^g, h', k')} U(f^g, h, k, k') + (1+n) \delta V^g(h', k')$$

subject to the constraints $k' = K(f^g, F^g(h', k'), h) = \tilde{K}(f^g, h)$ and $h' = H(e^g, h)$. Then the Bellman equation can be rewritten as follows:

$$V^g(h, k) = \max_{f^g} U(f^g, h, k, \tilde{K}(f^g, h)) + (1+n) \delta V^g(H(e^g, h), \tilde{K}(f^g, h)) \quad (4B)$$

The *GEE* are obtained as the first order condition of the Gvt optimization plan. The derivation below follows the method proposed by Klein et al. (2008) extending to the OLG case with two political controls in bidimensional state-space. In the following let us denote with $Y_x \equiv \frac{\partial Y}{\partial x}$ the partial derivative of Y with respect to x , while $\frac{dY}{dx}$ denotes total derivative. Furthermore, for simplicity of notation we will omit the apex g . The political first order conditions of Eq. (4B) with respect to $f \equiv (e, \tau)$ are equal to:

$$0 = U_e + U_{k'} \tilde{K}_e + (1+n) \delta (V_{h'} H_e + V_{k'} \tilde{K}_e) \quad (5B)$$

$$0 = U_\tau + U_{k'} \tilde{K}_\tau + (1+n) \delta V_{k'} \tilde{K}_\tau \quad (6B)$$

Using Benveniste-Scheinkman formula we obtain the following expression for V_h and V_k :

$$V_h = U_h + U_{k'} \tilde{K}_h + (1+n) \delta \left(V_{h'} H_h + V_{k'} \tilde{K}_h \right) \quad (7B)$$

$$V_k = U_k \quad (8B)$$

From Eq. (5B) and (6B) we obtain the expression for $V_{h'}$ and $V_{k'}$:

$$V_{h'} = \frac{1}{(1+n) \delta H_e} \left(\frac{U_\tau \tilde{K}_e - U_e \tilde{K}_\tau}{\tilde{K}_\tau} \right) \quad (9B)$$

$$V_{k'} = -\frac{U_\tau + U_{k'} \tilde{K}_\tau}{(1+n) \delta \tilde{K}_\tau} \quad (10B)$$

Plugging Eq. (9B) and (10B) into (7B) we get the final expression for V_h :

$$V_h = U_h + \frac{U_\tau \tilde{K}_e - U_e \tilde{K}_\tau}{\tilde{K}_\tau} \frac{H_h}{H_e} - U_\tau \frac{\tilde{K}_h}{\tilde{K}_\tau} \quad (11B)$$

Using stationarity condition and plugging Eq. (8B) and (11B) into (5B) and (6B), we obtain the GEEs of the Gvt problem respectively for e and τ :

$$0 = U_e + U_{k'} \tilde{K}_e + (1+n) \delta \left(\left(U_{h'} + \frac{U_{\tau'} \tilde{K}'_{e'} - U'_{e'} \tilde{K}'_{\tau'}}{\tilde{K}'_{\tau'}} \frac{H'_{h'}}{H'_{e'}} - U_{\tau'} \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}} \right) H_e + U'_{k'} \tilde{K}_e \right) \quad (12B)$$

$$0 = U_\tau + (U_{k'} + (1+n) \delta U'_{k'}) \tilde{K}_\tau \quad (13B)$$

From definition of $U(\cdot)$, we have: $U_e = -\beta(1+n)^2 u_{C_2}$, $U_\tau = (1+n)(1+h)(\beta u_{C_2} - \delta u_{C_1})$, $U_h = (1+n)(\beta \tau u_{C_2} + \delta(1-\tau) u_{C_1})$, $U_k = \beta R(1+n) u_{C_2}$ and $U_{k'} = -\delta(1+n)^2 u_{C_1}$. Using the above partial derivatives and rewriting $\tilde{K}_i$ where $i \in (f^g, h)$ in terms of $\eta(\cdot)$, we get the *GEEs* as a weighted combination of intergenerational wedges:

$$0 = \Delta_e + \delta \frac{\eta_e}{\eta_{k'}} \Delta_s + \delta \Xi \Delta'_{\tau'} \quad (14B)$$

$$0 = (1+h) \Delta_\tau + \delta(1+n) \frac{\eta_\tau}{\eta_{k'}} \Delta_s \quad (15B)$$

where $\Xi \equiv H_e \left(\tau' + (1+h') \left(\frac{\tilde{K}'_{e'}}{\tilde{K}'_{\tau'}} \frac{H'_{h'}}{H'_{e'}} - \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}} \right) \right) = \frac{d((1+h')\tau')}{de}$ and the first best wedges Δ_x with $x \in \{s, \tau, e\}$ are defined as $\Delta_\tau \equiv \beta u_{C_2} - \delta u_{C_1}$, $\Delta_s \equiv u_{C_1} - \beta R u_{C_2}'$ and $\Delta_e \equiv \delta H_e \left(\beta(1+n) \frac{H'_{h'}}{H'_{e'}} u_{C_2}' + \delta u_{C_1}' \right) - \beta u_{C_2}$.

Proof of Proposition (2). Let us guess as equilibrium policy functions for the Benevolent Government solution the following functional form respectively for e and τ :

$$e^g = a_1^g h + a_0^g \bar{h} \quad (16B)$$

$$\tau^g = b_3^g \frac{k}{1+h} + b_2^g \frac{h}{1+h} + b_1^g \frac{1}{1+h} + b_0^g \quad (17B)$$

which are structurally equivalent to the equilibrium policy rules in the political case. If Eq. (16B) and (17B) are the equilibrium of the Gvt problem, then they must satisfy simultaneously the *GEEs* given by conditions (12B) and (13B). Let us manipulate the *GEEs*, plugging the expressions for each partial derivative. We obtain for τ and e , respectively:

$$0 = -\beta u_{C_2} + \delta \left(\begin{aligned} & \left(\beta \tau' u_{C_2'} + \delta (1 - \tau') u_{C_1'} \right) \\ & + (1 + h') \left(\beta u_{C_2'} - \delta u_{C_1'} \right) \left(\frac{\tilde{K}_{\tau'}'}{K_{\tau'}'} \frac{H_{h'}'}{H_{e'}'} - \frac{\tilde{K}_{h'}'}{K_{\tau'}'} \right) \\ & + \beta (1 + n) u_{C_2'} \frac{H_{h'}'}{H_{e'}'} \end{aligned} \right) H_e \quad (18B)$$

$$0 = \beta u_{C_2} - \delta u_{C_1} \quad (19B)$$

Using the equation of $H(\cdot)$, the following expressions result:

$$H_e = \frac{\alpha h + (1 - \alpha) \bar{h}}{2(1 + n)h'} \quad (20B)$$

$$\frac{H_{h'}'}{H_{e'}'} = \frac{\alpha e'}{\alpha h' + (1 - \alpha) \bar{h}} \quad (21B)$$

Under logarithmic utility and linear production function, plugging the guess given by Eq. (16B) and (17B) into the saving function, we obtain the following recursive function for saving choice:

$$\begin{aligned} k' = \tilde{K}(e, \tau, h) &= \frac{\beta R}{(1 + n)(b_3^g + R(1 + \beta))} (1 + h)(1 - \tau) \\ &- \frac{b_2^g + b_0^g - (1 + n)a_1^g}{b_3^g + R(1 + \beta)} \sqrt{\frac{(\alpha h + (1 - \alpha) \bar{h}) e}{1 + n}} - \frac{b_1^g + b_0^g - (1 + n)a_0^g \bar{h}}{b_3^g + R(1 + \beta)} \end{aligned} \quad (22B)$$

Using Eq. (21B) and (22B) and simplifying, we get $\frac{\tilde{K}_{e'}'}{K_{\tau'}'} \frac{H_{h'}'}{H_{e'}'} - \frac{\tilde{K}_{h'}'}{K_{\tau'}'} = \frac{1 - \tau'}{1 + h'}$. Finally rearranging all the terms, Eq. (18B) becomes as follows:

$$0 = -u_{C_2} + \delta \left(1 + (1 + n) \frac{\alpha e'}{\alpha h' + (1 - \alpha) \bar{h}} \right) u_{C_2'} H_e \quad (23B)$$

Using the political Euler condition $\beta u_{C_2} - \delta u_{C_1} = 0$ and the economic one $u_{C_1} - R\beta u_{C_2'} = 0$, Eq. (23B) simplifies to:

$$1 = \left(1 + (1 + n) \frac{\alpha e'}{\alpha h' + (1 - \alpha) \bar{h}} \right) \frac{1}{R} H_e \quad (24B)$$

which is also equivalent to:

$$e = \left(\left(1 + (1 + n) \frac{\alpha e'}{\alpha h' + (1 - \alpha) \bar{h}} \right) \frac{1}{2R} \right)^2 \frac{\alpha h + (1 - \alpha) \bar{h}}{1 + n} \quad (25B)$$

Let us now make a further assumption on the guess on e , considering the following variant of Eq. (16B) :

$$e^g = a_1^g(\psi^g)h + a_0^g(\psi^g)\bar{h} \quad (26B)$$

such that $a_1^g(\psi^g) = \frac{\alpha}{1+n}\psi^g$ and $a_0^g(\psi^g) = \frac{1-\alpha}{1+n}\psi^g$, i.e. we guess the policy e as a linear convex combination between parental human capital h and human capital society endowment $\bar{h}$ scaled by a constant which has to be determined, ψ^g . Then Eq. (25B) can be rewritten as follows:

$$e^g = \frac{\alpha}{1+n}\tilde{\psi}^g h + \frac{1-\alpha}{1+n}\tilde{\psi}^g \bar{h} \quad (27B)$$

where $\tilde{\psi}^g \equiv \left(\left(1 + (1+n)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}} \right) \frac{1}{2R} \right)^2$. Plugging the guess of e^g given by Eq. (26B) into the expression of $\tilde{\psi}^g$ and simplifying we get:

$$\tilde{\psi}^g = (1 + \alpha\psi^g)^2 \left(\frac{1}{2R} \right)^2 \quad (28B)$$

By fixed-point condition $\tilde{\psi}^g = \psi^g$ which yield the following solutions $\psi_{1,2}^g = \frac{1}{\alpha} \left(\frac{2R}{\alpha} \left(R \pm \sqrt{R^2 - \alpha} \right) - 1 \right)$. Similar arguments as in Proof of Theorem 1 can be made. Then let us consider the stable root $\psi^* = \frac{1}{\alpha} \left(\frac{2R}{\alpha} \left(R - \sqrt{R^2 - \alpha} \right) - 1 \right)$ as feasible solution. It immediately follows that $a_1^g = \frac{\alpha}{1+n}\psi^*$ and $a_0^g = \frac{1-\alpha}{1+n}\psi^*$ are the solutions for the guess on e which turns out to be equivalent to the political outcome. After plugging the guesses, Eq. (16B) and Eq. (17B), and the recursive saving function, Eq. (22B), into Eq. (19B), the GEE for the policy τ is as follows:

$$\beta \frac{1}{(1+n)Rk + (1+n)(1+h)\tau - (1+n)^2 e^g} = \delta \frac{1}{(1+h)(1-\tau) - (1+n)\tilde{K}(e^g, \tau, h)} \quad (29B)$$

After some algebraic manipulations we obtain the following well-defined system:

$$\begin{cases} b_0^g = \frac{\beta(R+b_3^g)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g} \\ b_1^g = \frac{(1+n)\beta(b_0^g+b_1^g-(1-\alpha)\bar{h}\psi)-(1-\alpha)\bar{h}\sqrt{\psi}(-\beta(b_0^g+b_2^g)-\delta(1+n)(R(1+\beta)+b_3^g)\sqrt{\psi}-\alpha\beta\psi)}{(1+\bar{h})(R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g)} \\ b_2^g = \frac{\bar{h}\alpha\sqrt{\psi}(\beta(b_0^g+b_2^g)+\delta(1+n)(R(1+\beta)+b_3^g)\sqrt{\psi}-\alpha\beta\psi)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g} \\ b_3^g = -\frac{R(1+n)\delta(R(1+\beta)+b_3^g)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g} \end{cases}$$

Solving the system we obtain the following two solutions for τ :

$$\tau^{g1} = b_3^{g1} \frac{k}{1+h} + b_2^{g1} \frac{h}{1+h} + b_1^{g1} \frac{1}{1+h} + b_0^{g1} \quad (30B)$$

where, under $\Omega_R^g \equiv \frac{\delta(1+n)(1+\beta)}{\beta+\delta(1+n)}$ and $\Omega_O^g \equiv \frac{\beta(1-\delta(1+n))}{\beta+\delta(1+n)}$:

$$\begin{aligned} b_3^{g1} &= -R\Omega_R^g; & b_1^{g1} &= \bar{h} \frac{1-\alpha}{\alpha} \frac{R}{R-(1+n)} \mu_2 + \bar{R} (\Omega_O^g - \bar{h} (1-\alpha) \psi^*); \\ b_2^{g1} &= \frac{\alpha\sqrt{\psi^*}}{R-\alpha\sqrt{\psi^*}} (\Omega_O^g + R\sqrt{\psi^*}\Omega_R^g - \alpha\sqrt{\psi^*}); & b_0^{g1} &= \Omega_O^g; \end{aligned}$$

and

$$\tau^{g2} = b_3^{g2} \frac{k}{1+h} + b_2^{g2} \frac{h}{1+h} + b_1^{g2} \frac{1}{1+h} + b_0^{g2} \quad (31B)$$

where $b_3^{g2} = -R$, $b_1^{g1} = (1-\alpha) \psi^* \bar{h}$, $b_2^{g1} = \alpha \psi^*$ and $b_0^{g1} = 0$.

Note that the Eq. (30B) is equivalent to Eq. (31B) under the condition $\Omega_R^g = 1$ and $\Omega_O^g = 0$, which implies $\delta = \frac{1}{1+n}$. Recall that, for the existence of the fix point, the condition $\delta < \frac{1}{1+n}$, which induces Ω_O^g to be strictly greater than zero, is required. Consequently the Eq. (31B) is not feasible. ■

Proof of Proposition (3). Let us first consider the following normalization of the relative Welfare weights, Eq. (24), after assigning $\phi \equiv \frac{\beta}{\delta}$:

$$\Omega_R^g \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)} \quad \text{and} \quad \Omega_O^g \equiv \frac{\phi-\beta(1+n)}{\phi+(1+n)} \quad (32B)$$

Using the weights (32B) and comparing the parameters of the policy rules of Eq. (21) and Eq. (29) we obtain for any $\phi < \bar{\phi}$ where:

$$\bar{\phi} \equiv \frac{1}{2} \left(-1 - n - \frac{\sqrt{(1+n)^2 (-1 + 2R\sqrt{\psi} - \alpha\psi) \left(-(1+2\beta)^2 + 2R(1+2\beta(1+\beta)) \sqrt{\psi} - \alpha\psi \right)}}{-1 + 2R\sqrt{\psi} - \alpha\psi} \right)$$

the following inequalities must hold:

$$b_0^g < b_0, \quad b_1^g < b_1, \quad b_2^g < b_2, \quad b_3^g > b_3$$

Then we conclude $T(h_t, k_t) > T^g(h_t, k_t)$ for any $(h_t, k_t) \in \Upsilon^{Pol} \cap \Upsilon^{Gvt}$. ■

References

- [1] Abel, A. B., Mankiw, N. G., Summers, L. H. and Zeckhauser, R. J., 1989, "Assessing Dynamic Efficiency: Theory and Evidence", *Review of Economic Studies* 56 (1), 1-19.
- [2] Azariadis, C., and Galasso, V., 2002, "Fiscal Constitutions", *Journal of Economic Theory*, 103, 255-281.
- [3] Azzimonti Renzo, M., Sarte, P. D., and Soares, J., 2009, "Discretionary Taxes and Public Investment when Government Promises are not Enforceable", *Journal of Economic Dynamics and Control*, forthcoming.
- [4] Bassetto, M., 2008, "Political Economy of Taxation in an Overlapping-Generations Economy", *Review of Economic Dynamics*, 11, 18-43.
- [5] Baron, D., and Ferejohn, J., 1989, "Bargaining in Legislatures," *American Political Science Review*, 83, 1181-1206.
- [6] Battaglini, M., and Coate, S., 2007, "Inefficiency in Legislative Policy-Making: A Dynamic Analysis", *American Economic Review*, 97, 118-149.
- [7] Becker, G. S., and Tomes, N., 1986. "Human Capital and the Rise and Fall of Families", *Journal of Labor Economics*, 4, 1.
- [8] Bellettini, G., and Berti Ceroni, C., 1999. "Is Social Security Really Bad for Growth?," *Review of Economic Dynamics*, vol. 2 (4), 796-819,
- [9] Boldrin, M. and Montes, A., 2005, "The Intergenerational State Education and Pension", *Review of Economic Studies*, 72, 651-664.
- [10] Conde-Ruiz, J., and Galasso, V., 2005, "Positive Arithmetic of the Welfare State", *Journal of Public Economics*, 89, 933-955.
- [11] Ehrlich, I., and Lui, F., 1998, "Social Security, the Family, and Economic Growth", *Economic Enquiry*, 36, 390-409.
- [12] Feldstein, M., 1974, "Social Security, Induced Retirement and Aggregate Capital Accumulation", *Journal of Political Economy*, 82 (5).
- [13] Forni, L., 2005, "Social Security as Markov Equilibrium in OLG Models", *Review of Economic Dynamics*, 8, 178-194.
- [14] Galasso, V., and Profeta, P., 2002, "The Political Economy of Social Security: A Survey", *European Journal of Political Economy*, 18, 1-29.
- [15] Galasso, V., and Profeta, P., 2004, "Politics, Ageing and Pensions", *Economic Policy*, 19, 63-115.

- [16] Grossman, G. M., and Helpman, E., 1998, "Intergenerational Redistribution with Short-lived Government", *Economic Journal*, 108, 1299-1329.
- [17] Hassler, J., Krusell, P., Storesletten, K. and Zilibotti, F., 2005, "The Dynamics of Government", *Journal of Monetary Economics*, 52, 1331-1358.
- [18] Hassler, J., Storesletten, K. and Zilibotti, F., 2007, "Democratic Public Good Provision", *Journal of Economic Theory*, 133, 127-151.
- [19] Kaganovich, M., and Zilcha, I., 1999, "Education, Social Security, and Growth", *Journal of Public Economics*, 71, 289-309.
- [20] Kehoe, T. J., and Levine, D. K., 2000, "Liquidity Constrained Markets versus Debt Constrained Markets", *Econometrica* 69, 575-598.
- [21] Klein, P., Krusell, P., and Ríos-Rull, J., 2008. "Time-Consistent Public Policy," *Review of Economic Studies*, 75 (3), 789-808.
- [22] Krusell, P., Quadrini, V. and Ríos-Rull, J. V., 1997, "Politico-Economic Equilibrium and Economic Growth", *Journal of Economic Dynamics and Control*, 21, 243-272.
- [23] Lindbeck, A. and J. Weibull, 1987, "Balanced-budget Redistribution as the Outcome of Political Competition," *Public Choice*, 52, 273-297.
- [24] Lizzeri, A., Persico, N., 2001. "The Provision of Public Goods under Alternative Electoral Incentives", *American Economic Review*, 91 (1), 225-239.
- [25] Maskin, E., and Tirole, J., 2001, "Markov Perfect Equilibrium: I. Observable Actions", *Journal of Economic Theory*, 100, 191-219.
- [26] Mulligan, C. B. , and Xala-i-Martin, X., 1999. "Gerontocracy, Retirement, and Social Security," NBER Working Papers 7117.
- [27] OECD, 2007, "Social Expenditure Database 1980-2003", OECD Press.
- [28] OECD, 2008, "Education at a Glance - OECD Indicators 2008", OECD Press.
- [29] Persson, T. and Tabellini, G., 2000, "Political Economy - Explaining Economy Policy," MIT Press.
- [30] Plott, C., 1967, "A notion of equilibrium and its possibility under majority rule", *American Economic Review*, 57 (4), 787-806.
- [31] Rangel, A., 2003, "Forward and Backward Intergenerational Goods: Why is Social Security Good for Environment?", *American Economic Review*, 93, 813-834.
- [32] Shepsle, K. A., 1979, "Institutional Arrangements and Equilibrium in Multidimensional Voting Models", *American Journal of Political Science*, 23 (1), 27-59.

- [33] Tabellini, G., 1991, "The Politics of Intergenerational Redistribution," *Journal of Political Economy*, 99 (2), 335-57.