A Pitfall with DSGE–Based, Estimated, Government Spending Multipliers

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Abstract

In this paper, we study issues related to the estimation of long–run government spending multiplier (GSM) in a Dynamic Stochastic General Equilibrium (DSGE) context. We stress a potential source of bias in the GSM arising from the combination of (i) Edgeworth complementarity between private consumption and government expenditures and (ii) countercyclical government expenditures. We find that the degree of Edgeworth complementarity and the cyclicality of policy interact through cross–equation restrictions, paving the way for potential biases. It turns out that the GSM increases with the degree of Edgeworth complementarity between private consumption and government expenditures. Thus, any bias in the degree of Edgeworth complementarity translates into a biased GSM.

Keywords: DSGE models, Edgeworth complementarity, Government spending rules, Maximum likelihood.

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1 Introduction

In the current crisis context, there has been a renewed academic and policy interest in studying the effects of government activity. A key quantity that has attracted considereable attention is the long–run government spending multiplier (GSM), i.e. the long–run increase in output consecutive to a one percent permanent increase in government spending.[1]

In this paper, we study issues related to the estimation of this multiplier in a Dynamic Stochastic General Equilibrium (DSGE) context. We stress a potential source of bias in the GSM arising from the combination of (i) Edgeworth complementarity between private consumption and government expenditures and (ii) countercyclical government expenditures. We find that the degree of Edgeworth complementarity and the cyclicality of policy interact through cross–equation restrictions, paving the way for potential biases. At the same time, we show that the GSM is an increasing function of the degree of Edgeworth complementarity between private consumption and government expenditures. Thus, any bias in the degree of Edgeworth complementarity translates into a biased GSM. The underlying mechanics are the following.

Many recent papers have documented that government spending policy is countercyclical.[2] This raises a severe challenge for Neoclassical models. In those setups, following any shock such that both output and consumption decline, countercyclical policy triggers an increase in public spending. This in turn increases output but reduces consumption even more (crowding out effect), finally making private consumption even more negatively correlated with public expenditures than under an exogenous spending policy. This seems to be at odds with the data. Allowing for Edgeworth complementarity helps mitigate this problem. With such a mechanism, a rise in public expenditures would make people want to consume more, thus counteracting the crowding-out effect. As a consequence, given a certain unconditional correlation between private consumption and government expenditures that we seek to match, allowing for a very countercyclical policy will require a high degree of Edgeworth complementarity. This will mechanically translate into a large GSM. Conversely, omitting countercyclical policy will imply a small degree of complementarity, thus yielding a downward–biased GSM.

To establish these results, we first work out a simple model with only limited dynamic features. The model is simple enough to provide us with an analytical characterization of the bias that would arise from omitting a countercyclical component to government spending policy. We use this simple framework to identify configurations in which this bias would be likely. We show that in this simple setup, omitting the countercyclical policy rule at the estimation stage would always yield a downward–biased estimate of the GSM. Because countercyclical policy and Edgeworth complementarity work in opposite directions in terms of generating a certain pattern of correlation between consumption and government expenditures, we can reinterpret this bias as a simultaneous equation bias. As a matter of fact, the simple model allows us to derive a formula for the bias that closely resembles those appearing

[1] See, among others, Christiano, Eichenbaum, and Rebelo (2009); Cogan, Cwik, Taylor, and Wieland (2010); Fernandez–Villaverde (2010); Uhlig (2010).

[2] See, e.g., Jones (2002); Leeper, Plante, and Traum (2010).

in standard econometrics textbooks in a demand–supply framework. By analogy with this celebrated framework, an econometrician omitting the countercyclical spending rule risks recovering the policy rule parameter when trying to estimate the private response to public spending. In all likelihood, this will happen when shocks to government spending account for a small portion of fluctuations and/or the feedback effect in the policy rule is strong.

In a second step, using post–war US data, we estimate a quantitative model version via maximum likelihood techniques. We show that the same sort of bias is present when the econometrician omits the countercyclical component of government policy. This, in turn, translates into significant differences in the estimated long–run government spending multiplier. In our benchmark specification with Edgeworth complementarity and countercyclical policy, the implied long–run multiplier amounts to 1.31. Using the same model and imposing an exogenous policy rule, we obtain a multiplier significantly smaller by slightly more than 0.30 point. Such a difference is clearly not neutral if the model is used to assess recovery plans of the same size as those recently enacted in the US.

To complement these results, we conduct several robustness analyses. First, we use our preferred model version as a data generating process to perform several simulation exercises in finite sample. We obtain quantitative results that echo our analytical formula derived in the simplified model version. Second, we modify the model specification allowing for (i) habits in consumption, (ii) dynamic adjustment costs, (iii) alternative policy rules, and (iv) news shocks in policy rule. We also investigate the robustness of our results to subsamples. We find that none of our conclusions is affected by these perturbations to the benchmark setup: we always find that omitting the endogenous component of government spending policy results in a smaller degree of Edgeworth complementarity between private consumption and government spending, thus implying a smaller GSM.

The rest of the paper is organized as follows. In section 2, we expound the simple model and illustrate the trade–off between Edgeworth complementarity and countercyclical policy in terms of matching the observed correlation between output and government expenditures. We then characterize the bias that would result from omitting countercyclical policy. Section 3 develops a quantitative version of this model that we take to post–war US data. We then explore the quantitative implications of policy rule mispecification. In section 4, we investigate the robustness of our results, using both actual and simulated data. The last section briefly concludes.

2 A Simple Illustrative Example

In this section, we work out an equilibrium model simple enough to obtain closed–form formulas illustrating how the long–run government spending multiplier is biased when the econometrician omits the endogenous component of public policy.

2.1 The Model

Consider a discrete time economy populated with a large number of infinitely–lived, identical agents. The representative household seeks to maximize

$$\log\left(c_t + \alpha_g g_t\right) - \frac{\eta}{1+\nu} n_t^{1+\nu} \tag{1}$$

subject to the time t budget constraint

$$c_t \leq w_t n_t - T_t, \tag{2}$$

where c_t is private consumption, g_t denotes public expenditures, n_t is the labor supply, w_t is the real wage rate, and T_t denotes lump–sum taxes. The Frisch elasticity of labor supply is $1/\nu$ and $\eta > 0$ is a scale parameter.

The parameter α_g, in turn, accounts for the complementarity/substitutability between private consumption c_t and public spending g_t. [3] If $\alpha_g \geq 0$, government spending substitutes for private consumption, with perfect substitution if $\alpha_g = 1$, as in Christiano and Eichenbaum (1992). In this case, a permanent increase in government spending has no effect on output and hours but reduces private consumption, through a perfect crowding–out effect. In the special case $\alpha_g = 0$, we recover the standard business cycle model, with government spending operating through a negative income effect on labor supply (see Aiyagari, Christiano, and Eichenbaum, 1992; Baxter and King, 1993). When the parameter $\alpha_g < 0$, government spending complements private consumption. Then, it can be the case (depending on the labor supply elasticity) that private consumption will react positively to an unexpected increase in government spending (see Bouakez and Rebei, 2007).

The representative firm produces a homogeneous final good y_t using labor as the sole input, according to the constant returns–to–scale technology

$$y_t = \mathrm{e}^{z_t} n_t.$$

Here, z_t is a shock to total factor productivity, assumed to be iid with $z_t \sim N(0, \sigma_z^2)$. Profit maximization implies that the marginal productivity of labor equals the real wage w_t.

Government purchases are entirely financed by taxes,

$$T_t = g_t.$$

As in the recent literature emphasizing the relevance of stabilizing government spending rules (see, among others, Leeper et al., 2010), we specify a feedback rule of the following form

$$g_t = \left(\frac{y_t}{y_{t-1}}\right)^{-\varphi_g} \mathrm{e}^{u_t} \tag{3}$$

[3] Such a specification has now become standard, following the seminal work by Aschauer (1985); Bailey (1971); Barro (1981); Braun (1994); Christiano and Eichenbaum (1992); Finn (1998); McGrattan (1994). Alternatively, CES specifications of utility have been considered (see Bouakez and Rebei, 2007; McGrattan, Rogerson, and Wright, 1997). This yields a reduced–form similar to our setup when the model is log–linearized.

where $\varphi_g \geq 0$, *i.e.* government spending stabilizes aggregate activity, i.e. increases when output growth is below its average value. With this restriction we make sure that the reduced–form model displays second–order stationarity. The random term u_t represents the discretionary part of policy and is assumed to be *iid* with $u_t \sim N(0, \sigma_u^2)$.[4]

Finally, the market clearing condition on the goods market writes

$$y_t = c_t + g_t.$$

Log–linearizing the equilibrium conditions around the deterministic steady state yields the dynamic system

$$\hat{y}_t = \alpha \hat{g}_t + z_t \tag{4}$$

$$\hat{g}_t = -\varphi_g(\hat{y}_t - \hat{y}_{t-1}) + u_t \tag{5}$$

where a letter with a hat denotes the logdeviation (with respect to steady–state value) of the associated variable and the composite parameter α is defined as

$$\alpha \equiv \frac{\tau_g(1-\alpha_g)}{1+\nu[1-\tau_g(1-\alpha_g)]},$$

where $\tau_g \in [0,1)$ is the steady–state public spending–output ratio. To ensure positiveness of the marginal utility of consumption, we henceforth impose the restriction $\alpha_g > (\tau_g - 1)/\tau_g$. For ν and τ_g set at given values, the value of α summarizes the complementarity/substitutability between private and public consumption. This composite parameter and the long–run government spending multiplier are tightly linked, as stated in the following proposition.

Proposition 1. *Under the preceeding assumptions, the long–run government spending multiplier* $\Delta y/\Delta g$ *is*

$$\frac{\Delta y}{\Delta g} = \frac{\alpha}{\tau_g} = \frac{1-\alpha_g}{1+\nu[1-\tau_g(1-\alpha_g)]}.$$

The multiplier is a decreasing function of α_g.

The proof is straightforward. ■

The degree of counter–cyclicality of government spending, φ_g, does not appear in the GSM formula. Thus, if this parameter is to affect empirically the GSM, it must be indirectly, through interactions between two potentially conflicting economic forces: Edgeworth complementarity and countercyclical policy. Too see this, notice that: *(i)* Edgeworth complementarity between private consumption and government spending (i.e. $\alpha_g < 0$) tends to increase the correlation between y and g, since under such a configuration an increase in government expenditures would induce people to consume more; *(ii)* at the same time, a countercyclical policy rule reduces this correlation. This yields a trade–off: given an

[4] A simpler rule would have government spending react to current output only. A problem with such a specification within our simplified model is that it would compromise identification of the policy parameter. Anticipating on the next section, we also notice that such a dynamic rule is favored by the data when we estimate a quantitative model version.

observed correlation between output and government spending, a highly countercyclical policy must be compensated by a high degree of Edgeworth complementarity. Conversely, if policy is exogenous, a lower degree of Edgeworth complementarity will suffice to match the observed pattern of correlation between output and government expenditures. This is illustrated in figure 1.

This figure shows two iso–correlation loci in the (φ_g, α_g) plane, depending on the relative sizes of the structural disturbances. Each point in these loci gives a particular (φ_g, α_g) combination resulting in the same correlation between output and government spending. When $\sigma_z > \sigma_u$, as is likely in practice, the iso-correlation locus is decreasing with φ_g, with a steep slope. This means that, as the degree of countercyclicality on policy increases, it takes more and more complementarity to match the observed correlation. When $\sigma_u > \sigma_z$, the iso–correlation curve is much flatter.

This trade-off paves the way for a potential bias in the estimated degree of Edgeworth complementarity (and, by virtue of the above proposition, in the estimated multiplier). Suppose that an econometrician seeks to estimate α but uses a misspecified model in which φ_g is set to zero while actually $\varphi_g > 0$. The above reasononing suggests that this would result in a downward–biased estimate of α, immediately translating into a downward–biased estimated multiplier. The next section formally establishes this.

2.2 The Effect of Omitting Endogenous Policy

Direct calculations yield the model's reduced–form

$$\hat{y}_t = \frac{\alpha\varphi_g}{1+\alpha\varphi_g}\hat{y}_{t-1} + \frac{\alpha}{1+\alpha\varphi_g}u_t + \frac{1}{1+\alpha\varphi_g}z_t \tag{6}$$

$$\hat{g}_t = \frac{\varphi_g}{1+\alpha\varphi_g}\hat{y}_{t-1} + \frac{1}{1+\alpha\varphi_g}u_t - \frac{\varphi_g}{1+\alpha\varphi_g}z_t \tag{7}$$

From this reduced form, the structural parameters $(\alpha, \varphi_g, \sigma_u, \sigma_z)$ can be recovered using the $p\lim$ of the maximum likelihood estimation or an instrumental variable technique (with a relevant choice of instrumental variables). An easy way to obtain a consistent estimator of α relies on indirect estimation using the following representation of the reduced form

$$\hat{y}_t = \pi_1\hat{y}_{t-1} + \epsilon_{1,t} \tag{8}$$

$$\hat{g}_t = \pi_2\hat{y}_{t-1} + \epsilon_{2,t} \tag{9}$$

The $p\lim$ estimators of π_1 and π_2 are given by

$$\hat{\pi}_1 = \frac{\mathrm{E}\{\hat{y}_t\hat{y}_{t-1}\}}{\mathrm{E}\{\hat{y}_t^2\}} \quad \text{and} \quad \hat{\pi}_2 = \frac{\mathrm{E}\{\hat{g}_t\hat{y}_{t-1}\}}{\mathrm{E}\{\hat{y}_t^2\}}$$

from which we deduce

$$\hat{\alpha} = \frac{\hat{\pi}_1}{\hat{\pi}_2} = \frac{\mathrm{E}\{\hat{y}_t\hat{y}_{t-1}\}}{\mathrm{E}\{\hat{g}_t\hat{y}_{t-1}\}}$$

From (6)–(7), we obtain:

$$\mathrm{E}\{\hat{y}_t\hat{y}_{t-1}\} = \frac{\alpha\varphi_g}{1+\alpha\varphi_g}\mathrm{E}\{\hat{y}_t^2\} \quad \text{and} \quad \mathrm{E}\{\hat{g}_t\hat{y}_{t-1}\} = \frac{\varphi_g}{1+\alpha\varphi_g}\mathrm{E}\{\hat{y}_t^2\}$$

The indirect estimator $\hat{\alpha}$ of α is thus consistent. Similarly, $\hat{\varphi}_g$ is also a consistent estimator of φ_g.

Now, imagine the econometrician ignores the feedback rule and seeks to estimate the parameter α from data $(\hat{y}_t, \hat{g}_t)$ generated by the model (6)–(7). The model considered by this econometrician is thus of the form

$$\hat{y}_t = \tilde{\alpha}\tilde{u}_t + \tilde{z}_t \tag{10}$$

$$\hat{g}_t = \tilde{u}_t. \tag{11}$$

By ignoring the parameter φ_g, the econometrician is implicitly estimating the government spending effects on output through a single–equation approach in a simultaneous–equation setup. As is well known from standard econometrics textbooks, she potentially faces a severe *simultaneous–equation* bias (see Greene, 1997; Hamilton, 1994).

To see this clearly, the ML estimator $\hat{\tilde{\alpha}}$ of $\tilde{\alpha}$ would be

$$\hat{\tilde{\alpha}} = \frac{\mathrm{E}\{\hat{y}_t \hat{g}_t\}}{\mathrm{E}\{\hat{g}_t^2\}}$$

which simply corresponds to the OLS estimator. While φ_g exterts no influence on the long–run multiplier (see proposition 1), the next proposition establishes that this parameter corrupts the estimated composite parameter $\tilde{\alpha}$ when policy is assumed to be exogenous.

Proposition 2. *Under the previous hypotheses, the ML estimator $\hat{\tilde{\alpha}}$ of $\tilde{\alpha}$ obeys*

$$\hat{\tilde{\alpha}} = \frac{\alpha(1+\alpha\varphi_g)\sigma_u^2 - \varphi_g\sigma_z^2}{(1+\alpha\varphi_g)\sigma_u^2 + 2\varphi_g^2\sigma_z^2}. \tag{12}$$

The proof is in appendix. ■

The estimated value of $\hat{\tilde{\alpha}}$ is thus corrupted, in a non–linear way, by φ_g, σ_u and σ_z. The OLS regression does not pin down the effects of $\hat{g}$ on $\hat{y}$ but an average of private behavior and public policy, with weights that depend on the relative size of the shocks' variances. Notice that this is more or less the formula displayed in standard econometrics textbooks in a demand–supply setup (e.g., see Hamilton, 1994, chap. 9).

In our underlying data generating process (DGP), we have assumed that $\varphi_g \geq 0$. This restriction has direct consequences on the sign of the bias. In particular, we have $\hat{\tilde{\alpha}} \leq \alpha$ whenever $\sigma_z > 0$ and the bias may increase with φ_g, depending on the particular parameter values.[5] This means that when the econometrician wrongly omits the feedback rule on government spending, thus ignoring the endogeneity of government spending, she underestimates the true value α. In this case, the long–run government spending multiplier is systematically downward–biased.[6]

[5] More precisely, depending on α, σ_u and σ_z, there exists a threshold positive value $\bar{\varphi}_g$ for φ_g such that $\partial\hat{\tilde{\alpha}}/\partial\varphi_g < 0$ when $\varphi_g < \bar{\varphi}_g$ and positive elsewhere. However, the bias never reverts back to zero, since $\lim_{\varphi_g \to \infty} \hat{\tilde{\alpha}} = 0$.

[6] When $\varphi_g = 0$, the bias is obviously zero. Indeed, in this case the model is well specified.

Depending on the relative size of the two shocks, the estimated parameter $\hat{\tilde{\alpha}}$ will take on different values. Suppose first that $\sigma_z > 0$ and $\sigma_u \to 0$. This would correspond to the case when government spending shocks do not contribute much to the variance of $\hat{y}$. In this case, we obtain $\hat{\tilde{\alpha}} \to -1/(2\varphi_g)$. Thus, omitting the endogeneity of government spending would lead us to estimate a negative value of $\tilde{\alpha}$. This is because endogenous public spending is negatively related to the shock z which shifts aggregate output. The covariance between $\hat{y}_t$ and $\hat{g}_t$ is negative and thus the estimated effect of public spending on output is negative. We are in the case when the output equation moves along the policy rule equation (which is truly downward slopping). In this case, the econometrician almost recovers the reverse government policy rule. Assume now that $\sigma_z \to 0$ and $\sigma_u > 0$. This would correspond to a situation when the bulk of fluctuations in $\hat{y}$ are accounted for by government spending shocks. In this case, the endogeneity bias vanishes since $\hat{\tilde{\alpha}} = \alpha$. Endogenous public spending is positively related to the shock u and the (inverse) government policy shifts along the output equation.

To sum up, we have shown analytically in a tractable model that omitting the endogenous component of government spending can result in a downward–biased estimate of the long–run government spending multiplier. In this simple setup, the downward bias is a mix of a simultaneous–equation bias and an omitted–variable bias. It is the result of two conflicting economic forces, one that magnifies the correlation between output and government spending (Edgeworth complementarity) and the other that reduces it (countercyclical government spending rule).

In the following section, we consider a quantitative DSGE model which we estimate on US data via maximum likelihood techniques. While the model is too complicated to get such a sharp bias characterization, it proves a useful tool to investigate whether omitting the endogenous component of government spending actually results in a quantitatively significant bias in the estimated long–run government spending multiplier.

3 A Quantitative Model

We now work out a quantitative extension of the previous model that we formally take to the data. We extend the previous setup by allowing for capital accumulation, habit formation in leisure decisions, and mutliple shocks. While the model is arguably very stylized, it turns out to deliver a good fit. For simplicity, the model abstracts from Keynesian features such as sticky prices or wages. Such mechanisms, however, would reinforce our conclusion. Indeed such Keynesian features would partly kill the negative wealth effect associated with public expenditures, thus calling for an even higher degree of Edgeworth complementarity, all the more so as policy is countercyclical.

We now briefly describe the augmented framework, then document our estimation strategy, and finally comment our empirical results.

3.1 The Model

The representative household's intertemporal expected utility is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \mathrm{e}^{a_t} \log(c_{t+i} + \alpha_g g_{t+i}) - \mathrm{e}^{b_t} \frac{\eta}{1+\nu} \left(\frac{n_{t+i}}{n_{t+i-1}^{\phi}} \right)^{1+\nu} \right\} \tag{13}$$

where $\mathrm{E}_t\{\cdot\}$ denotes the expectation operator conditional on the information set at period t and $\beta \in (0,1)$ is the subjective discount factor. As in the previous section, the parameter α_g governs the substitutability/complementarity between private consumption and public expenditures.

The parameter ϕ governs the habit persistence in labor supply and $\eta \geq 0$ is a scale parameter. When the parameter $\phi \neq 0$, labor supply decisions are subject to time non–separabilities. If $\phi < 0$, labor supply displays inter–temporal substitutability, whereas $\phi > 0$ implies inter–temporal complementarity. Eichenbaum, Hansen, and Singleton (1988) showed that a specification with intertemporal complementarities is favored by the data. More recently, this specification has proven to be empirically relevant, as it translates habit persistence in leisure choices into aggregate output persistence (see Bouakez and Kano, 2006; Dupaigne, Fève, and Matheron, 2007; Wen, 1998). While other specifications that allow to capture the persistence in hours have been considered in the literature (e.g. adjustment costs on labor input, as in Chang, Doh, and Schorfheide, 2007, or learning–by–doing, as in Chang, Gomes, and Schorfheide, 2002), it turns out that the implied reduced–form are almost identical to that resulting from our specification.

Utility derived from consumption is altered by a preference shock a_t, which obeys

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_{a,t}$$

where $|\rho_a| < 1$, $\sigma_a > 0$ and $\epsilon_{a,t} \sim N(0,1)$. Labor disutility is subject to a preference shock b_t, which obeys

$$b_t = \rho_b b_{t-1} + \sigma_b \epsilon_{b,t}$$

where $|\rho_b| < 1$, $\sigma_b > 0$ and $\epsilon_{b,t} \sim N(0,1)$. As noted by Galí (2005), this shock accounts for a sizeable portion of aggregate fluctuations. Moreover, it allows us to capture various distortions on the labor market, labeled *labor wedge* in Chari, Kehoe, and McGrattan (2007).

The representative household supplies hours n_t and capital k_t to firms, and pays a lump–sum tax T_t to the government. Accordingly, the representative household's budget constraint in every period t is

$$c_t + x_t \leq w_t n_t + r_t k_t - T_t \tag{14}$$

where w_t is the real wage, r_t is the rental rate of capital, and x_t denotes investment. The capital stock evolves according to

$$k_{t+1} = (1-\delta) k_t + x_t \tag{15}$$

where $\delta \in (0,1)$ is the constant depreciation rate. The representative household thus maximizes (13) subject to the sequence of constraints (14) and (15), $t \geq 0$.

The representative firm produces a homogeneous final good y_t through the constant returns–to–scale technology

$$y_t = k_t^{\theta} \left(\mathrm{e}^{z_t} n_t\right)^{1-\theta}$$

where k_t and n_t denote the inputs of capital and labor, respectively, $\theta \in (0,1)$ is the elasticity of output with respect to capital, and z_t is a shock to total factor productivity, which follows a random walk process with drift of the form

$$z_t = \log(\gamma_z) + z_{t-1} + \sigma_z \epsilon_{z,t}$$

where $\sigma_z > 0$ and $\epsilon_{z,t} \sim N(0,1)$. The constant term $\gamma_z > 1$ is the drift term and accounts for the deterministic component of the growth process. Profit maximization equalizes the marginal productivity of each input factor to their price, so that $r_t = \theta y_t / k_t$ and $w_t = (1-\theta) y_t / n_t$.

The government spending are entirely financed by taxes,

$$T_t = g_t$$

Notice that Ricardian equivalence holds in our setup, so that introducing government debt is unnecessary. Detrended government spending are given by the policy rule

$$g_t \mathrm{e}^{-z_t} = \bar{g} \tilde{g}_t \mathrm{e}^{g_t^{\star}},$$

where the endogenous policy component $\tilde{g}_t$ obeys

$$\log(\tilde{g}_t) = -\varphi_g \left(\Delta \log(y_t) - \log(\gamma_z)\right),$$

and the stochastic (discretionary) component is assumed to follow an autoregressive process of the form:

$$g_t^{\star} = \rho_g g_{t-1}^{\star} + \sigma_g \epsilon_{g,t}$$

where $|\rho_g| < 1$, $\sigma_g > 0$ and $\epsilon_{g,t} \sim N(0,1)$. Here, Δ stands for the first–difference operator. The parameter φ_g is the policy rule parameter that links the stationary component of government policy to demeaned output growth. Provided $\varphi_g > 0$, the policy rule features a countercyclical component that triggers an increase in government expenditures whenever output growth is below its average value.

The homogeneous good can be used for private consumption c_t, government consumption g_t, and investment x_t. The market clearing condition on the good market accordingly writes

$$y_t = c_t + x_t + g_t.$$

We induce stationarity by getting rid of the stochastic trend component z_t and we log–linearize the resulting system in the neighborhood of the deterministic steady state. The log–linear solution is of the form

$$\hat{s}_t = F(\psi)\hat{s}_{t-1} + G(\psi) \begin{pmatrix} \epsilon_{z,t} \\ \epsilon_{a,t} \\ \epsilon_{b,t} \\ \epsilon_{g,t} \end{pmatrix}, \tag{16}$$

where ψ is the vector of model's parameters and $\hat{s}_t$ is a vector collecting the loglinear model variables. The system matrices $F(\psi)$ and $G(\psi)$ are complicated functions of the model's parameters.

3.2 Data and Estimation

The data used for estimation come from the Federal Reserve Bank of St. Louis' FRED II database and from the Bureau of Labor Statistics website. They consist of government consumption expenditures and gross investment (GCE), private investment and private consumption, all deflated by the implicit GDP deflator (GDPDEF). Private investment is defined as the sum of gross private domestic investment (GPDI) and personal consumption expenditures on durable goods (PCDG). Private consumption is measured as the sum of personal consumption expenditures on non–durable goods (PCND) and services (PCESV). Output is then defined as the sum of private investment, private consumption and government expenditures. Hours are borrowed from Francis and Ramey (2009). These hours data refer to the total economy and are adjusted for low-frequency movements due to changes in demographics, thus displaying less low-frequency behavior than unadjusted data. All the series are converted to per–capita terms by dividing them by the civilian population, age 16 and over (CNP16OV). All the series are seasonally adjusted except for population. Our sample runs from 1960:1 to 2007:4.

We use as observable variables in estimation the logs of output, consumption, hours worked, and government expenditures. The measurement equation is

$$\begin{pmatrix} \Delta \log(y_t) \\ \log(n_t) \\ \Delta \log(c_t) \\ \Delta \log(g_t) \end{pmatrix} = \begin{pmatrix} \gamma_z - 1 \\ m_n(\psi) \\ \gamma_z - 1 \\ \gamma_z - 1 \end{pmatrix} + H\hat{s}_t. \tag{17}$$

Here, $m_n(\psi)$ is a function that gives average log hours as a function of ψ and H is a selection matrix. For a given ψ, using equations (16) and (17), the log–likelihood is evaluated via standard Kalman filter techniques. The estimated parameters are then obtained by maximizing the log–likelihood.[7]

The vector of parameters ψ is split in two subvectors ψ_1 and ψ_2. The first one, $\psi_1 = (\beta, \delta, \nu, \theta, \tau_g)$, contains parameters calibrated prior to estimation. Typically, these are parameters difficult to estimate in our framework. The subjective discount factor, β, is set to 0.9951, yielding a real annual interest rate of 3.75%. The depreciation rate, δ, is set to 0.0153, which implies an annual depreciation rate of slightly more than 6%. The parameter ν is set to 4 so that the long–run labor supply elasticity $\mu \equiv (1+\nu)(1-\phi)-1$ is close to 2 in the benchmark model, in accordance with previous studies (Smets and Wouters, 2007). Finally we set $\theta = 0.30$, so that the labor income share in output is 70%, and $\tau_g = 0.2$, so as to reproduce the average ratio of government expenditures to output in our sample.

The remaining parameters, contained in $\psi_2 = (\phi, \alpha_g, \gamma_z, \varphi_g, \rho_g, \rho_a, \rho_b, \sigma_z, \sigma_g, \sigma_a, \sigma_b)$, are estimated.[8] Estimation results are reported in table 1. We consider four model restrictions, according to whether α_g and φ_g are constrained. Finally, the table reports the log–likelihood $\mathscr{L}$ for each model specification, which we use naturally as our selection criterion. The restrictions are summarized below

[7] We used different measurement equations, using the the logged private consumption–output and logged government expenditures–output ratios instead of consumption growth and government expenditures growth. Estimation results were almost identical.

[8] The parameter vector ψ_2 also contains $\bar{n}$, which is the average level of log hours. In our setup, estimating $\bar{n}$ is equivalent to estimating η. This parameter does not play any role in our analysis and is not reported.

- Model (1): $\alpha_g = 0$, $\varphi_g = 0$, so that g has no direct effect on the marginal utility of private consumption and is exogenous
- Model (2): $\alpha_g = 0$, $\varphi_g \neq 0$, so that g has no direct effect on the marginal utility of private consumption and is endogenous
- Model (3): $\alpha_g \neq 0$, $\varphi_g = 0$, so that g has a direct effect on the marginal utility of private consumption and is exogenous
- Model (4): $\alpha_g \neq 0$, $\varphi_g \neq 0$, so that g has a direct effect on the marginal utility of private consumption and is endogenous

Overall, the model specifications yield precisely estimated parameters.[9] Several general comments can be made. First, except for ρ_g and σ_g, most parameters are pretty invariant to the restrictions imposed on α_g or φ_g. Second, all the shocks display sizable serial correlation, as is common in this kind of models, irrespective of the constraints imposed on the structural parameters.

The log–likelihood comparison suggests the following comments. First, comparing specifications (1) with (3) or (2) with (4), one can clearly see that the restrictions $\alpha_g = 0$ is strongly rejected by the data. The P–values of the associated likelihood ratio tests are almost zero. In specifications (3) and (4), the parameter α_g is negative and significantly different from zero, consistent with the above likelihood ratio tests. This result is not an artifact of allowing for an endogenous government policy. This suggests that private and public consumption are complements. This result echoes findings by Karras (1994), or by Bouakez and Rebei (2007) in a slightly different specification for the interaction of public and private consumption.

Second, the restriction $\varphi_g = 0$ is clearly rejected by the data. To see this, compare specifications (1) and (2) or (3) and (4). In both cases, the associated P–values are almost zero. These results strongly supports the view that government policy comprises an endogenous component. To sum up, specification (4) is our preferred model. This specification thus features (i) a positive effect of government spending on the marginal utility of private consumption and (ii) a countercyclical feedback effect in government spending.[10]

Even though all second order moments are given the same weight in the likelihood, comparing unconditional moments from the models to their empirical counterparts is useful to get some intuition why model (4) is preferred. Results are reported in table 2. We consider moments documenting the volatility, persistence, and co–movement of key variables. All the model versions perform equally well in terms of fitting standard errors of key aggregate variables.

[9] We numerically checked the estimation convergence by shocking initial conditions on parameter values in the likelihood maximization step. Upon convergence, we also plotted slices of the likelihood function around each estimated parameter value to check local identification.

[10] In section 4, we discuss the robustness of our estimation results, allowing successively for habits in consumption, dynamic adjustment costs on investment, alternative feedback policy rules, and news shocks on government spending rule.

More interstingly, we see that whether or not $\alpha_g = 0$ is imposed, the correlation between changes in private consumption and changes in government spending is smaller when $\varphi_g > 0$. To see this, compare the results for specifications (1) against (2) or (3) against (4). This illustrates our claim that in a standard neoclassical growth model, allowing for a countercyclical government spending policy works toward reducing the correlation between consumption growth and government spending shocks. Similarly, comparing the same specifications, we see that relaxing the constraint $\varphi_g = 0$ decreases the correlation between output growth and government expenditures growth. Conversely, whether or not $\varphi_g = 0$, relaxing the constraint $\alpha_g = 0$ increases the correlation between consumption growth and government spending growth. To see this, compare the results for specifications (1) against (3) or (2) against (4). This once again illustrates how Edgeworth complementarity and countercyclical policy interact in our model. Finally, we also see that, irrespective of constraints imposed on α_g, the restriction $\varphi_g = 0$ deteriorates the model's ability to capture the persistence of changes in government expenditures.

We complement the above results by performing specification tests for the innovation of each variables used for estimation in equation (17), *i.e.* $\Delta \log(y_t)$ output growth, $\log(n_t)$ the log of hours, $\Delta \log(c_t)$ private consumption growth, and $\Delta \log(g_t)$ government consumption growth. The innovations are obtained as the difference between the observed variables and their predicted value at convergence of the estimation stage. The specification tests, reported in table 3, are conducted for the four model's specifications. The first column reports the Shapiro and Wilk (1965) test statistic. The null hypothesis being tested is that the innovation of the variables listed on the left is normally distributed. A small value of the test statistic indicates a rejection of the null, whereas a value close to unity favors the normality assumptions. On the right, we report the P–value (in %) of the test statistic. Except for consumption growth, normality is rejected in all cases. However, rejection is essentially driven by a few outliers. Given the parametric parsimony of the model, such a rejection is hard to interpret. More interestingly, table 3 also includes serial correlation tests. We report the least–squares coefficient obtained by projecting each innovation on its own lag. For each coefficient, we report the associated 95% confidence interval. We find that omitting the feedback rule deteriorate the results. Indeed, comparing specification (3) and (4) shows that consumption and government spending innovations display less serial correlation when the policy rule coefficient is not constrained to zero.

Having explored the empirical properties of model (4), we now use this version to investigate the quantitative effects of omitting the government feedback rule.

3.3 Quantitative Results

Upon inspecting models (3) and (4), we see that imposing $\varphi_g = 0$ strongly affects the estimated value of α_g. When φ_g is freely estimated, we obtain $\alpha_g = -0.95$ whereas we get $\alpha_g = -0.63$ when we impose $\varphi_g = 0$. Importantly, these parameter estimates are significantly different from each other, according to a standard Wald test.

What does this imply for long–run government spending multiplier? To answer this question, we first characterize how α_g and the GSM are linked together. This relation is stated in the following proposition.

Proposition 3. *Under the preceeding assumptions, the long–run government spending multiplier* $\Delta y/\Delta g$ *is*

$$\frac{\Delta y}{\Delta g} \equiv \frac{1-\alpha_g}{1-s_x+\mu\left(1-s_x-\tau_g(1-\alpha_g)\right)},$$

where

$$\mu \equiv (1+\nu)(1-\phi)-1, \qquad s_x \equiv \frac{(\gamma_z-1+\delta)\theta\beta}{\gamma_z-\beta(1-\delta)}.$$

The multiplier $\Delta y/\Delta g$ *decreases with* α_g*.*

The proof is straightforward. ■

Notice that when α_g and ϕ are set to zero, the above formula collapses to those reported in Baxter and King (1993) and Aiyagari et al. (1992).

Clearly, φ_g does not show up in this formula. Thus, if φ_g is to affect the multiplier, it must be indirectly, through its empirical relation with α_g. We now use the above formula to assess the impact of φ_g on the GSM.

The estimated multipliers are reported in table 4, together with their standard errors. When α_g is restricted to zero, the estimated long–run multiplier is typically less than one, as obtains in standard models. Concretely, depending on the restrictions imposed on φ_g, we obtain values roughly comprised between 0.53 and 0.55. This is typically the range of available estimates derived from estimated DSGE models in the literature. Importantly, comparing specifications (1) and (2) in table 4, one can clearly see that φ_g has almost no discernible effect on the long–run multiplier. The reason why is simple: the parameters β, δ, and ν are restricted prior to estimation. In addition, the estimation results show that ϕ is relatively insensitive to the different specifications (see table 1).

In our framework, omitting the feedback effect in the policy rule can impact on the long–run output multiplier only when the parameter α_g is freely estimated. This is the novel feature of our model, because the link between the policy feedback parameter (φ_g) and the degree of Edgeworth complementarity between public and private consumptions (α_g) could obviously not be studied in frameworks imposing $\alpha_g = 0$. The columns associated with specifications (3) and (4) in table 4 thus give new results. More precisely, in model (3), the output multiplier is 0.97 while it reaches 1.31 in model (4). These values are significantly different from each other at conventional levels. As explained above, the higher multiplier in (4) derives from a smaller α_g than in (3).[11]

To complement on these results, we consider the following exercise. We set the feedback parameters φ_g to values on a grid between $\varphi_g = 0$ and the estimated value obtained in specification (4), i.e.

[11] Similar results obtain for the multiplier on private investment. This is not surprising since this multiplier is strictly proportional to $\Delta y/\Delta g$ and the proportionality factor does not depend on α_g. Interestingly, the multiplier on private consumption is positive in specification (4), though not significantly.

$\varphi_g = 0.6117$. For each value, all the remaining parameters in ψ_2 are re–estimated. The results are reported on figure 2. The upper left panel reports the log–likelihood as a function of φ_g. The grey area corresponds to restrictions on φ_g that are not rejected at the 5% level according to a likelihood ratio test. This grey area is also reported in each of the other panels in figure 2. The upper right panel reports the estimated value of α_g as a function of φ_g. The bottom panels report the long–run multipliers on output and consumption. The figure makes clear that even loose restrictions on φ_g (i.e. restrictions not too far from the estimated value) are easily rejected and rapidly translate into higher α_g and much lower multipliers. Importantly, the continuous and decreasing mapping from φ_g to α_g (and thus on long–run multipliers) echoes the analytical findings obtained in the simple model explored in the first section.

To sum up, there exists a strong interaction between the estimated values of φ_g and α_g that have potentially dramatic implications for the quantitative assessment of the long–run government spending multiplier that cannot be ignored if the model is to be used to assess recovery plans of the same size as those recently enacted in the US.

4 Robustness

In this section, we offer a robustness analysis. We first resort to simulations to investigate small sample issues and to check whether our results are an artifact of our particular sample. Second, we consider extensions of our benchmark specification (4), allowing successively for habits and dynamic adjustment costs on investment, alternative feedback policy rules, and news shocks on government spending rule. All these additional modeling elements have received considerable attention in the recent literature and are thus worth considering. Finally, we reestimate the model on subsamples to check whether the relation between α_g and φ_g still holds.

4.1 Results from Simulated Data

At this stage, we suspect that the greater α_g obtained under model (3) is the outcome of a mispecification bias. Indeed, we previously saw that omitting φ_g always increases the estimated value of α_g. In the simple model considered in the first section, we were able to formally show the existence of such a bias. In our DSGE framework, no such analytical results is available, though the same economic forces seem to be at play. To make our point, we thus resort to simulation techniques. Indeed, actual data are just one draw from an unknown DGP. Hence one cannot exclude that the negative link between α_g and φ_g is idiosyncratic to our sample. In addition, resorting to simulation enables us to investigate whether φ_g can be estimated to non–zero values even in a world where no such mechanism exists (an exercise that we can hardly perform on actual data).

To investigate this, we develop a controlled experiment in which we use model (4) as our DGP, using the estimated values reported in table 1. More specifically, using model (4) as our DGP we first want

to make sure that (i) estimating specification (4) on simulated data delivers consistent estimates and (ii) estimating specification (3) on the exact same simulated data yields severely biased estimates of α_g.[12] To complement on this, we also run the symmetric estimations in which we use model (3) as our DGP and successively estimate specifications (3) and (4) on simulated data. In this case, the crucial point is to check whether our estimation procedure is able to properly reject a policy feedback rule when no such rule exists in simulated data.

To begin with, table 5 reports the simulation results when using either specifications (4) or (3) as DGP and/or estimated model. Figure 3 reports the empirical density of parameter estimates obtained from a Gaussian kernel. The dark line corresponds to the parameter distribution obtained by estimating model (4) on data simulated from model (4). The grey line corresponds to the parameter distribution obtained by estimating model (3) on data simulated from model (4). The red vertical line denotes the true value used for simulation. The dark vertical line is the average value obtained by estimating model (4) on data simulated from model (4). Finally, the grey vertical line is the average value obtained by estimating model (3) on data simulated from model (4). Figure 4 reports analog densities obtained when using model (3) as the DGP.

We first check whether estimating model (4) on data simulated from model (4) yields consistent parameter estimates. It turns out that this is the case. Indeed, we see from figure 3 and table 5 that the average parameters estimates almost coincide with the true ones. Now, consider what happens when estimating model (3) on data simulated from model (4). In this case, all the parameters linked to government policy (α_g, ρ_g, σ_g) turn out to be biased. This is particularly striking when it comes to α_g, the average value of which is almost twice as small (in absolute term) as the true one. Figure 3 once again offers a visual illustration of this. Interestingly, the average estimated value of α_g from our simulation experiment is very similar to what obtains from actual data when estimating model (3). This suggests that using specification (4) as an approximation to the true DGP is indeed legitimate.

Consider now what happens when using specification (3) as our DGP. Once again, the results are reported in table 5 and figure 4. As before, the first thing to check is whether estimating specification (3) on data simulated from model (3) yields consistent estimates. Once again, this turns out to be the case. Now, consider what happens when estimating model (4) on data simulated from model (3). Basically, this procedure is able to recover the true parameters on average. This is particularly striking when it comes to the feedback parameter φ_g, which is zero on average. Recall that the latter does not exist in model (3), the DGP used for this simulation experiment, and appears only in model (4). This implies that a significant φ_g on actual data does not seem to be an artifact of our particular sample.

[12] In practice our Monte Carlo simulation is run as follows: using model (4), we generate 1000 samples of observables $(\Delta \log(y_t), \log(n_t), \Delta \log(c_t), \Delta \log(x_t)$, with the same sample size as actual data, after having eliminated 800 initial observations, thus ensuring that initial conditions do not contaminate our estimation results. To do so, the four structural shocks innovations are drawn from independent Gaussian distributions with zero mean and unit variance. On each simulated sample, we estimate specifications (3) and (4) and thus generate a population of estimated parameters.

4.2 Additional Real Frictions

A central ingredient of our preferred specification is the presence of dynamic complementarities in labor supply. Importantly, output dynamics inherit the built–in persistence of hours worked generated by this mechanism.

The recent DSGE literature, however, has emphasized alternative real frictions capable of generating very strong aggregate persistence. Important such mechanisms are habits in consumption and dynamic investment adjustment costs, see Christiano, Eichenbaum, and Evans (2005). We considered versions of our preferred model augmented with these additional mechanisms.

When either of these are included, they do not significantly contribute to the model's fit. A standard likelihood ratio test would not reject the restriction of no habits in consumption and/or no dynamic adjustment costs. For example, in the case of specification (4) augmented with habits in consumption and dynamic adjustment costs, the log–likelihood is equal to 2702.22, to be compared to our reference specification (4) where the log–likelihood is equal to 2701.32. The habits in consumption parameter is equal to 0.11 (not significantly different from zero at conventional levels) and the adjustment cost parameter is almost zero. In addition, we redo the specification tests (normality and serial correlation). Including habits in consumption and dynamic adjustment costs does not improve upon the model performance: the normality test statistic is almost the same for each innovation and the serial correlation coefficients are very similar.

More importantly for our purpose, the empirical interaction between α_g and φ_g still holds under this more complete framework. In particular, when φ_g is constrained to zero, we obtain $\alpha_g = -0.21$, yielding a multiplier $\Delta y/\Delta g = 0.85$. In contrast, when φ_g is freely estimated, government policy turns out to be countercyclical ($\varphi_g = 0.60$) and the parameter $\alpha_g = -0.79$, implying a multiplier equal to 1.19. This confirms our main result.

4.3 Alternative Specifications of the Feedback Rule

We also experimented with alternative specifications for the government spending feedback rule. These alternative rules are specified as follows

$$
\begin{array}{lrcl}
\text{(A)} & \log(\tilde{g}_t) & = & -\varphi_g(\log(y_t) - z_t - \log(\bar{y})) \\
\text{(B)} & \log(\tilde{g}_t) & = & -\varphi_g(\log(y_{t-1}) - z_{t-1} - \log(\bar{y})) \\
\text{(C)} & \log(\tilde{g}_t) & = & -\varphi_z(\Delta z_t - \log(\gamma_z)) \\
\text{(D)} & \log(\tilde{g}_t) & = & -\varphi_z(\Delta z_t - \log(\gamma_z)) - \varphi_a \Delta a_t - \varphi_b \Delta b_t \\
\text{(E)} & \log(\tilde{g}_t) & = & -\varphi_g(\Delta \log(y_t) - \log(\gamma_z)) - \varphi_z(\Delta z_t - \log(\gamma_z)) \\
\text{(F)} & \log(\tilde{g}_t) & = & -\varphi_g(\Delta \log(y_t) - \log(\gamma_z)) - \varphi_z(\Delta z_t - \log(\gamma_z)) - \varphi_a \Delta a_t - \varphi_b \Delta b_t
\end{array}
$$

where $\bar{y}$ denotes the steady–state value of detrended output. Results are reported in table 6. For comparison purpose, we reproduce the results obtained with our preferred specification (4), referred to here as the benchmark specification. The other specifications are: (A) the stationary component of

government spending reacts to current deviations of output from its stochastic trend; (B) the stationary component of government spending reacts to the lagged deviations of output from its stochastic trend; (C) the stationary component of government spending reacts to changes in total factor productivity, resembling the specification used by Smets and Wouters (2007); (D) the stationary component of government spending reacts to changes in all the structural shocks; (E) combines our benchmark specification with (C); (F) combines the benchmark specification with (D). Table 6 reports the estimated values of α_g and the policy rule parameters, together with the implied long–run multipliers and the log–likelihood. In all these cases we estimate the parameters freely (in particular, we do not impose ex ante sign restrictions on φ_g). To ease comparison, we also report the estimation results obtained under an exogenous policy (i.e. specification (3)).

As is clear from table 6, our benchmark specification dominates the alternative specifcation from (A) to (D). Notice that these specifications are not nested with our benchmark. Since our benchmark implies a higher log–likelihood, the alternative rules are not a better description of the data. Moreover, the specification test results deteriorate (not reported), especially so in terms of serial correlation of innovations. In addition, the specifications that yield the lowest fit are (A) and (B). Specification (A) implies a procyclical government spending policy. In this case, the implied α_g is higher than under an exogenous policy (-0.58 in specification (A) and -0.63 under an exogenous policy). This mechanically translates into a smaller multiplier in (A), as expected from our previous analysis. In specification (B), the policy rule is almost acyclical and we obtain roughly similar multipliers than under an exogenous policy.

Specification (C) implies a countercyclical policy rule in response to technology shocks, since the estimated values of φ_z is positive. Once again, as expected, the estimated α_g is slightly lower than under an exogenous policy ($\alpha_g = -0.71$), resulting in a higher multiplier.

Specification (D) adds to the former case by allowing policy to respond to all the shocks. This specification implies a countercyclical policy rule, in the sense that the estimated values of φ_z and φ_a are positive while the estimated value of φ_b is negative. As expected, the Edgeworth complementarity parameter turns out to be lower than under an exogenous policy ($\alpha_g = -0.79$).

Specifications (E) and (F), which nest our benchmark, imply higher log–likelihoods, by construction. A likelihood ratio test would not reject our benchmark when compared to specification (E). In contrast, the log–likelihood is much higher in specification (F). However, specification tests outcomes do not improve much when compared to our benchmark.

4.4 News Shocks in the Government Spending Rule

As emphasized by Ramey (2009) and Schmitt–Grohé and Uribe (2008), the expected component in public expenditures constitutes an important element of government policy. We accordingly modify

our benchmark specification to allow for news shocks in the government spending rule, according to

$$g_t^\star = \rho_g g_{t-1}^\star + \sum_{i=0}^{q} \sigma_{g,q} \epsilon_{g,t-q},$$

where the $\epsilon_{g,t-q}$ are orthogonal with each other.

We first imposed $q = 4$. According to our estimation results, we obtain that lags $i = 1, 2, 3$ are not significant. This specification delivers a significantly better fit to the data than our preferred model (4), according to the likelihood ratio test (in this case, the log–likelihood is equal to 2717.39). However, the parameter estimates do not change too much compared to specification (4). In particular, the parameter α_g is now equal to -0.86, whereas the feedback rule parameter is equal to 0.59. In addition, allowing for news shocks in government spending does not improve upon the specification tests of our reference model (4).

Importantly, adding news shocks does not modify our main conclusion. When policy is exogenous ($\varphi_g = 0$), we obtain $\alpha_g = -0.21$, with an associated multiplier equal to 0.56. In contrast, when φ_g is freely estimated, α_g is smaller, resulting in a higher multiplier 1.11.

4.5 Subsample Analysis

Perotti (2004) showed that empirical measures of governement spending multipliers can prove sensitive to the particular sample selected. Importantly, he argues that multipliers over the sample starting from 1980:1–2001:4 are smaller than those found over the sample 1960:1–1979:4. We here investigate whether our results still hold if we re–estimate our model over the same subsamples.

Results are reported in table 7. Our previous conclusions are broadly confirmed. First, the restriction $\varphi_g = 0$ is rejected, suggesting that government spending policy is endogenous. Second, when this restriction is imposed, we obtain a higher α_g, resulting in a smaller multiplier. This holds over both subsamples. We also obtain a smaller GSM over the second subsample, confirming results in Perotti (2004).

5 Conclusion

This paper has proposed to assess quantitatively the consequences of misspecifying the government spending rule on the estimated long–run multplier within a DSGE framework. We first considered a simplified model version to show analytically that omitting the feedback rule at the estimation stage yields a downward–biased estimate of the government spending mutliplier. To establish this, we first showed that the mutliplier is an increasing function of the degree of Edgeworth complementarity. In turn, complementarity and countercyclical policy interact through cross–equation restrictions, paving the way for a potential bias. We then estimated on postwar US data a quantitative model version and obtained that omitting the endogeneity of government spending exerts a severe, downward impact on

the estimated long–run multiplier. Our results appear to be very robust to a series of perturbations to the benchmark specification. We also used simulation experiments and found that our results should not be interpreted as idiosyncratic to our sample and/or model.

In our framework, we have deliberately abstracted from relevant details in order to highlight, as transparently as possible, the empirical link between policy rule parameters and the degree of Edgeworth complementarity between private and public consumption. However, the recent litterature insists on other modeling isssues that might potentially affect our results. We mention three of them. First, other specifications of agent's preferences (Galí, López–Salido, and Valles, 2007; Monacelli and Perotti, 2008) have proven to be useful mechanisms for reproducing the aggregate effect of government spending shocks, thus competing with the approach retained in this paper. It will be useful to investigate how such alternative specifications interact with the policy feedback rule. Second, as put forth in Leeper et al. (2010), a more general specification of government spending rule, lump–sum transfers, and distotionary taxation is needed to properly fit the US data. This richer specification includes in addition to the automatic stabilizer component, a response to government debt and co–movement between tax rates. An important quantitative issue may be to assess which type of stabilization (automatic stabilization and/or debt stabilization) interacts with the estimated degree of Edgeworth complementarity. Third, Fiorito and Kollintzas (2004) have suggested that the degree of complementarity/substitutability between government and private consumptions is not homogeneous over types of public expenditures. This suggests to disagregate government spending and inspect how feedback rules affect the estimated degree of Edgeworth complementarity in this more general setup. These three issues will constitute the object of further researches.

A Proof of Proposition 2

Letting $\rho \equiv \alpha\varphi_g/(1+\alpha\varphi_g)$, the reduced–form equations (6) and (7) rewrite:

$$\hat{y}_t = \rho\hat{y}_{t-1} + \frac{\alpha}{1+\alpha\varphi_g}u_t + \frac{1}{1+\alpha\varphi_g}z_t \tag{A.1}$$

$$\hat{g}_t = \rho\hat{g}_{t-1} + \frac{1}{1+\alpha\varphi_g}u_t - \frac{\varphi_g}{1+\alpha\varphi_g}\Delta z_t, \tag{A.2}$$

Provided $\alpha_g < 1$, $\alpha > 0$. Now assuming that $\varphi_g \geq 0$, we have $\rho \in [0,1)$. Equations (A.1) and (A.2) are then second–order stationary and admit an MA(∞) representation

$$\hat{y}_t = \frac{1}{1+\alpha\varphi_g}\sum_{i=0}^{\infty}\rho^i L^i(\alpha u_t + z_t) \tag{A.3}$$

$$\hat{g}_t = \frac{1}{1+\alpha\varphi_g}\sum_{i=0}^{\infty}\rho^i L^i(u_t - \varphi_g\Delta z_t), \tag{A.4}$$

where $\Delta \equiv 1-L$ and L is the backshift operator.

Equation (A.4) rewrites

$$\hat{g}_t = \frac{1}{1+\alpha\varphi_g}\left(\sum_{i=0}^{\infty}\rho^i L^i u_t - \varphi_g\left(1+(\rho-1)\sum_{i=1}^{\infty}\rho^i L^i z_t\right)\right) \tag{A.5}$$

Since $\mathrm{E}\{\hat{y}_t\} = \mathrm{E}\{\hat{g}_t\} = 0$ and using equations (A.3) and (A.5), the covariance between $\hat{y}_t$ and $\hat{g}_t$ is given by

$$\begin{aligned}\mathrm{E}\{\hat{y}_t\hat{g}_t\} &= \left(\frac{1}{1+\alpha\varphi_g}\right)^2\left[\alpha\sigma_u^2\sum_{i=0}^{\infty}\rho^{2i} - \varphi_g\sigma_z^2\left(1+\rho(\rho-1)\sum_{i=0}^{\infty}\rho^{2i}\right)\right] \\ &= \left(\frac{1}{1+\alpha\varphi_g}\right)^2\left[\frac{\alpha\sigma_u^2}{1-\rho^2} - \frac{\varphi_g\sigma_z^2}{1+\rho}\right]\end{aligned}$$

Now, using the expression of ρ, one gets

$$\mathrm{E}\{\hat{y}_t\hat{g}_t\} = \frac{\alpha(1+\alpha\varphi_g)\sigma_u^2 - \varphi_g\sigma_z^2}{(1+\alpha\varphi_g)(1+2\alpha\varphi_g)} \tag{A.6}$$

From the MA(∞) representation of equation (A.5) and $\mathrm{E}\{\hat{g}_t\} = 0$, the variance of $\hat{g}_t$ is given by

$$\begin{aligned}\mathrm{E}\{\hat{g}_t^2\} &= \frac{\sigma_u^2}{(1+\alpha\varphi_g)^2}\sum_{i=0}^{\infty}\rho^{2i} + \frac{\varphi_g^2\sigma_z^2}{(1+\alpha\varphi_g)^2}\left(1+(\rho-1)^2\sum_{i=0}^{\infty}\rho^{2i}\right) \\ &= \frac{\sigma_u^2}{(1+\alpha\varphi_g)^2(1-\rho^2)} + \frac{\varphi_g(1-\rho)}{(1+\alpha\varphi_g)^2(1+\rho)}\end{aligned}$$

Using the expression of ρ, the above equation becomes

$$\mathrm{E}\{\hat{g}_t^2\} = \frac{(1+\alpha\varphi_g)\sigma_u^2 + 2\varphi_g^2\sigma_z^2}{(1+\alpha\varphi_g)(1+2\alpha\varphi_g)} \tag{A.7}$$

Now, substituting (A.6) and (A.7) into the ML estimator $\hat{\tilde{\alpha}}$ of $\tilde{\alpha}$ yields the result of the Proposition. This completes the proof.

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Table 1. Estimation Results

	Specification (1) $\alpha_g = 0, \varphi_g = 0$	Specification (2) $\alpha_g = 0, \varphi_g \neq 0$	Specification (3) $\alpha_g \neq 0, \varphi_g = 0$	Specification (4) $\alpha_g \neq 0, \varphi_g \neq 0$
α_g	–	–	−0.6340	−0.9452
			(0.1357)	(0.1410)
φ_g	–	0.5723	–	0.6117
		(0.0803)		(0.0660)
ϕ	0.4004	0.4138	0.3766	0.4110
	(0.0764)	(0.0730)	(0.0714)	(0.0638)
γ_z	1.0043	1.0043	1.0043	1.0044
	(0.0007)	(0.0007)	(0.0007)	(0.0007)
ρ_g	0.9469	0.9592	0.9535	0.9756
	(0.0087)	(0.0078)	(0.0079)	(0.0055)
ρ_a	0.9762	0.9775	0.9795	0.9834
	(0.0049)	(0.0047)	(0.0036)	(0.0031)
ρ_b	0.8346	0.8345	0.8366	0.8399
	(0.0424)	(0.0411)	(0.0366)	(0.0330)
σ_z	0.0107	0.0107	0.0107	0.0107
	(0.0005)	(0.0005)	(0.0005)	(0.0005)
σ_g	0.0130	0.0120	0.0129	0.0119
	(0.0007)	(0.0006)	(0.0007)	(0.0006)
σ_a	0.0103	0.0102	0.0123	0.0133
	(0.0009)	(0.0009)	(0.0011)	(0.0013)
σ_b	0.0270	0.0266	0.0273	0.0266
	(0.0014)	(0.0014)	(0.0015)	(0.0014)
$\mathscr{L}$	2655.3227	2679.4375	2665.9338	2701.3173

Notes: Sample period: 1960:1–2007:4. Standard errors in parentheses.

Table 2. Moments Comparison

	Data	Specification (1) $\alpha_g = 0, \varphi_g = 0$	Specification (2) $\alpha_g = 0, \varphi_g \neq 0$	Specification (3) $\alpha_g \neq 0, \varphi_g = 0$	Specification (4) $\alpha_g \neq 0, \varphi_g \neq 0$
$\sigma(\Delta y)$	0.0093	0.0093	0.0093	0.0095	0.0095
$\sigma(\Delta c)$	0.0050	0.0071	0.0071	0.0074	0.0065
$\sigma(\Delta x)$	0.0327	0.0255	0.0281	0.0255	0.0308
$\sigma(\Delta g)$	0.0110	0.0170	0.0145	0.0168	0.0140
$\sigma(\Delta n)$	0.0066	0.0069	0.0068	0.0070	0.0070
$\rho(\Delta y)$	0.3200	0.1165	0.1226	0.1116	0.1304
$\rho(\Delta c)$	0.2482	0.0217	0.0232	0.0176	0.1980
$\rho(\Delta x)$	0.2057	0.1599	0.0413	0.1425	-0.0681
$\rho(\Delta g)$	0.0948	-0.0161	0.1076	-0.0139	0.1337
$\rho(\Delta n)$	0.3886	0.3299	0.3437	0.3100	0.3511
$\text{corr}(\Delta y, \Delta c)$	0.5115	0.6336	0.6461	0.7282	0.6541
$\text{corr}(\Delta y, \Delta x)$	0.9043	0.8025	0.8567	0.7315	0.8246
$\text{corr}(\Delta y, \Delta g)$	0.2913	0.5616	0.3354	0.5873	0.3697
$\text{corr}(\Delta c, \Delta g)$	0.2368	0.1999	0.0204	0.4976	0.4001
$\text{corr}(\Delta x, \Delta g)$	-0.0288	0.1719	0.0209	0.0213	-0.1001

Notes: Sample period: 1960:1–2007:4.

Table 3. Specification Tests

Specification		Normality		Serial Correlation	
	Innovation in	Shapiro–Wilk Statistic	P–value (in %)	Coefficient	Confidence Interval (at 95%)
(1)	$\Delta \log(y_t)$	0.9657	0.0531	0.1883	[0.0482 , 0.3284]
	$\log(n_t)$	0.9737	0.3513	0.0055	[−0.1372 , 0.1482]
	$\Delta \log(c_t)$	0.9847	7.2771	0.3146	[0.1783 , 0.4508]
	$\Delta \log(g_t)$	0.9686	0.1019	0.1685	[0.0267 , 0.3103]
(2)	$\Delta \log(y_t)$	0.9668	0.0673	0.1821	[0.0419 , 0.3224]
	$\log(n_t)$	0.9738	0.3577	-0.0170	[−0.1597 , 0.1256]
	$\Delta \log(c_t)$	0.9850	7.8055	0.3125	[0.1762 , 0.4489]
	$\Delta \log(g_t)$	0.9672	0.0741	0.0393	[−0.1045 , 0.1831]
(3)	$\Delta \log(y_t)$	0.9655	0.0510	0.1948	[0.0549 , 0.3348]
	$\log(n_t)$	0.9746	0.4386	0.0354	[−0.1071 , 0.1778]
	$\Delta \log(c_t)$	0.9860	10.7253	0.2851	[0.1476 , 0.4225]
	$\Delta \log(g_t)$	0.9699	0.1405	0.1504	[0.0081 , 0.2926]
(4)	$\Delta \log(y_t)$	0.9681	0.0917	0.1787	[0.0384 , 0.3191]
	$\log(n_t)$	0.9744	0.4246	-0.0223	[−0.1647 , 0.1201]
	$\Delta \log(c_t)$	0.9865	12.3860	0.0924	[−0.0505 , 0.2352]
	$\Delta \log(g_t)$	0.9693	0.1211	-0.0008	[−0.1447 , 0.1431]

Notes: Sample period: 1960:1–2007:4. $\Delta \log(y_t)$ denotes output growth, $\log(n_t)$ hours, $\Delta \log(c_t)$ private consumption growth, and $\Delta \log(g_t)$ government consumption growth. Specification (1): $\alpha_g = \varphi_g = 0$, Specification (2): $\alpha_g = 0$, $\varphi_g \neq 0$, Specification (3): $\alpha_g \neq 0$, $\varphi_g = 0$, Specification (4): $\alpha_g \neq 0$, $\varphi_g \neq 0$. Coefficients are obtained by projecting each innovation on its own lag.

Table 4. Estimated Multipliers

	Specification (1)	Specification (2)	Specification (3)	Specification (4)
	$\alpha_g = 0, \varphi_g = 0$	$\alpha_g = 0, \varphi_g \neq 0$	$\alpha_g \neq 0, \varphi_g = 0$	$\alpha_g \neq 0, \varphi_g \neq 0$
$\Delta y/\Delta g$	0.5319	0.5429	0.9738	1.3128
	(0.0606)	(0.0603)	(0.1410)	(0.1803)
$\Delta c/\Delta g$	−0.5955	−0.5872	−0.2596	−0.0020
	(0.0461)	(0.0458)	(0.1072)	(0.1370)
$\Delta x/\Delta g$	0.1274	0.1301	0.2333	0.3148
	(0.0146)	(0.0145)	(0.0339)	(0.0433)

Notes: Sample period: 1960:1–2007:4. Standard errors in parentheses.

Table 5. Simulation Results

	DGP: Specification (4) $\alpha_g \neq 0, \varphi_g = 0$			DGP: Specification (3) $\alpha_g \neq 0, \varphi_g \neq 0$		
		Estimated Models			Estimated Models	
Parameters	True Value	(4)	(3)	True Value	(4)	(3)
α_g	-0.9452	-0.9095	-0.5945	-0.6340	-0.6100	-0.6134
φ_g	0.6117	0.6140	—	—	-0.0075	—
ϕ	0.4110	0.4052	0.3848	0.3766	0.3845	0.3845
γ_z	1.0044	1.0045	1.0043	1.0043	1.0044	1.0044
ρ_g	0.9756	0.9729	0.9181	0.9535	0.9455	0.9459
ρ_a	0.9834	0.9736	0.9572	0.9795	0.9698	0.9699
ρ_b	0.8399	0.8302	0.8093	0.8366	0.8213	0.8215
σ_z	0.0107	0.0107	0.0107	0.0107	0.0107	0.0107
σ_g	0.0119	0.0118	0.0137	0.0129	0.0128	0.0129
σ_a	0.0133	0.0123	0.0096	0.0123	0.0113	0.0113
σ_b	0.0266	0.0265	0.0277	0.0273	0.0272	0.0272

Notes: Simulation results obtained from 1000 replications. Model (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Model (3): $\alpha_g \neq 0$ and $\varphi_g = 0$. In each case, we report the average value of parameters across simulations.

Table 6. Alternative Government Spending Rules

Specification	α_g	φ_g	φ_z	φ_a	φ_b	ϕ	$\Delta y/\Delta g$	$\mathscr{L}$
Benchmark	-0.9452	0.6117	—	—	—	0.4110	1.3128	2701.3173
(A)	-0.5774	-0.4866	—	—	—	0.4072	0.9652	2669.6339
(B)	-0.6287	0.1029	—	—	—	0.3710	0.9623	2666.1522
(C)	-0.7091	—	0.3952	—	—	0.3995	1.0693	2691.5604
(D)	-0.7893	—	0.4254	0.0133	-0.0500	0.3769	1.1105	2694.4357
(E)	-1.0398	0.8176	-0.1660	—	—	0.4181	1.4244	2702.0942
(F)	-1.2220	2.5102	-1.4928	-0.1132	0.2264	0.3700	1.5455	2714.2522
Exogenous Policy	-0.6340	—	—	—	—	0.3766	0.9738	2665.9338

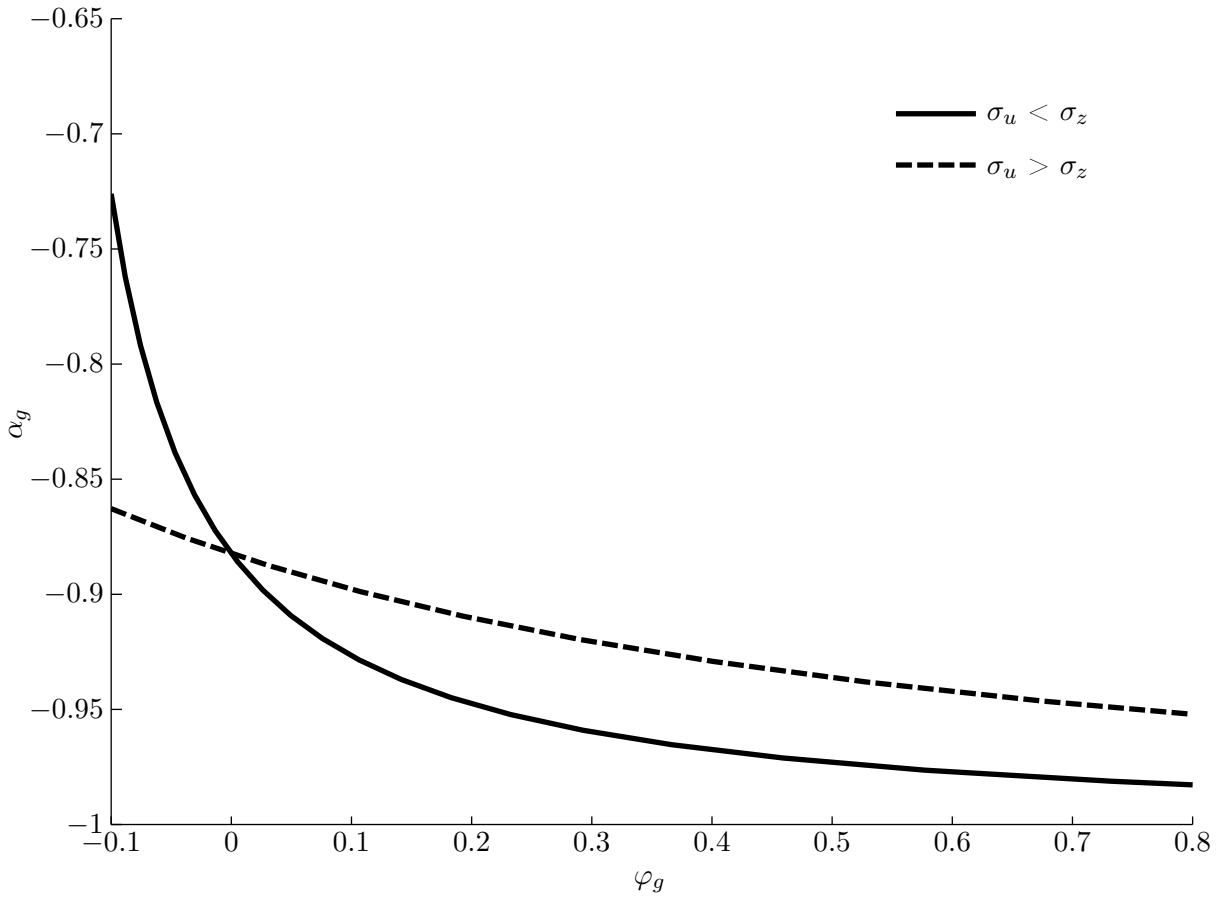
Notes: Sample period: 1960:1–2007:4. Benchmark is specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$. Exogenous policy is specification (3): $\alpha_g \neq 0$ and $\varphi_g = 0$.

Table 7. Subsample Robustness

Specification	α_g	φ_g	ϕ	$\Delta y/\Delta g$	$\mathscr{L}$
1960:1-1979:4					
(4)	-1.0167	0.6033	0.3755	1.3263	1101.3585
(3)	-0.7910	–	0.3896	1.1276	1088.1722
1980:1-2007:4					
(4)	-0.8068	0.5597	0.4146	1.1856	1614.9772
(3)	-0.6100	–	0.3777	0.9580	1599.7944

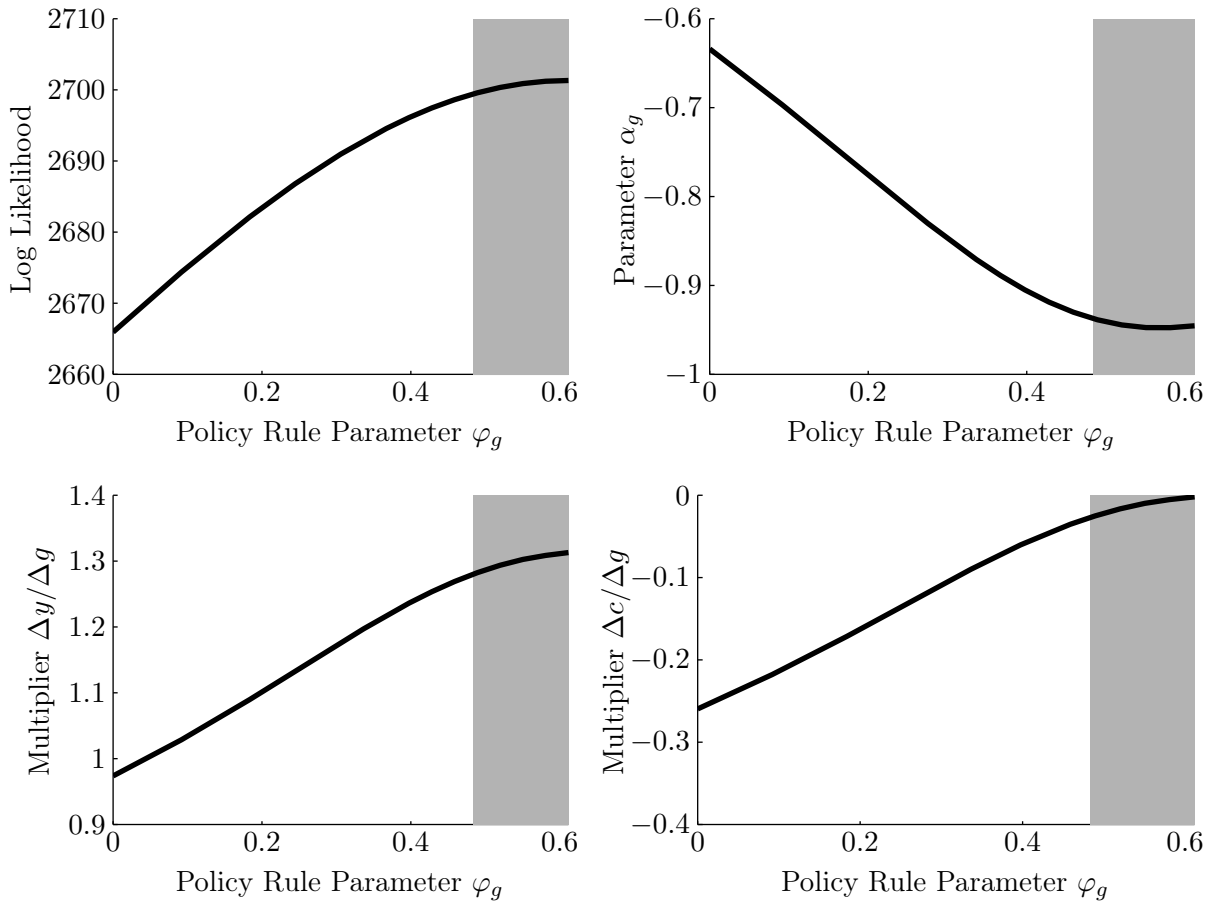
Notes: Specification (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Specification 3: $\alpha_g \neq 0$ and $\varphi_g = 0$.

Figure 1: Iso–Correlation loci in the (φ_g, α_g) plane



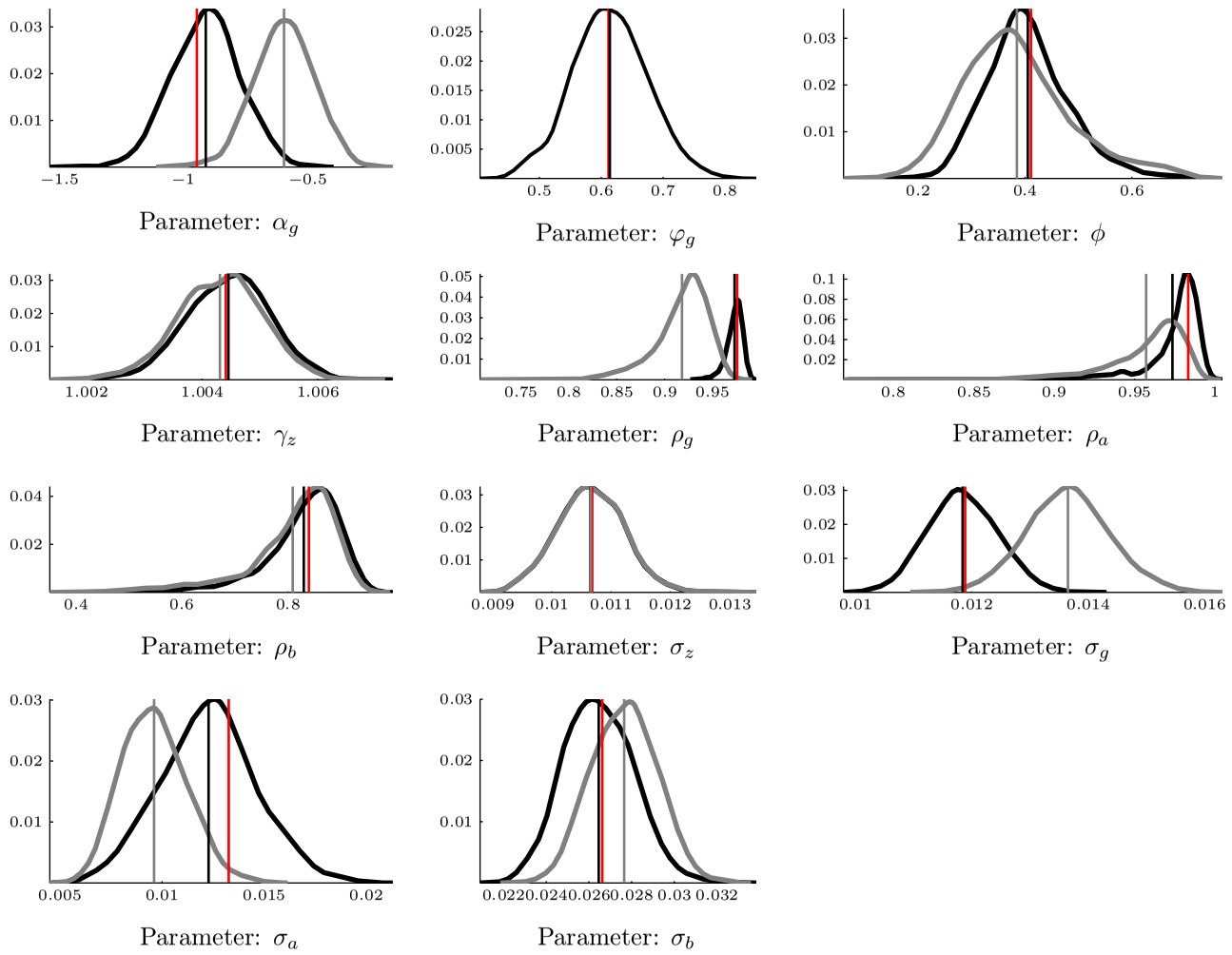
Notes: The iso-correlation loci are computed assuming, for illustrative pruposes, $\nu = 5$, $\tau_g = 0.2$, and $\text{Corr}(\hat{y}, \hat{g}) = 0.3$. In the $\sigma_u < \sigma_z$ case, we impose $\sigma_u/\sigma_z = 0.5$. In the $\sigma_u > \sigma_z$ case, we impose $\sigma_u/\sigma_z = 2$.

Figure 2: Sensitivity to constraints on policy rule parameter φ_g



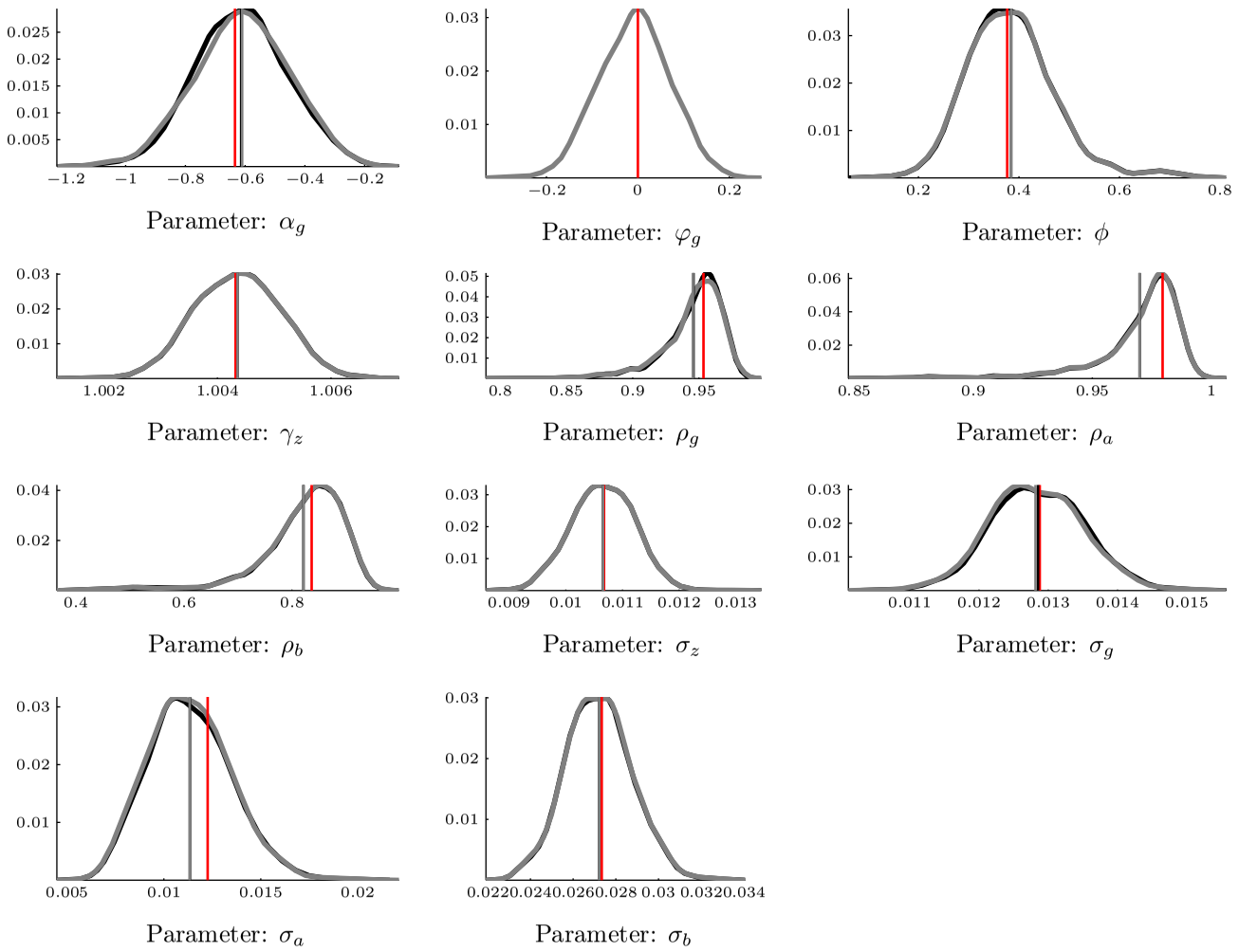
Notes: Sample period: 1960:1–2007:4. The feedback parameters φ_g takes values on a grid between 0 and 0.6117. For each value, all the remaining parameters in ψ_2 are re–estimated. The upper left panel reports the log–likelihood as a function of φ_g. The grey area corresponds to restrictions on φ_g that are not rejected at the 5% level according to a likelihood ratio test. This grey area is also reported in each of the other panels. The upper right panel reports the estimated value of α_g as a function of φ_g. The bottom panels report the long–run multipliers on output and consumption.

Figure 3: Simulation Results Under Model (4)



Notes: Simulation results obtained from 1000 replications. The empirical density of parameter estimates is obtained from a Gaussian kernel. The dark line corresponds to the parameter distribution obtained by estimating model (4) on data simulated from model (4). The grey line corresponds to the parameter distribution obtained by estimating model (3) on data simulated from model (4). The red vertical line denotes the true value used for simulation. The dark vertical line is the average value obtained by estimating model (4) on data simulated from model (4). Finally, the grey vertical line is the average value obtained by estimating model (3) on data simulated from model (4).

Figure 4: Simulation Results Under Model (3)



Notes: Simulation results obtained from 1000 replications. The empirical density of parameter estimates is obtained from a Gaussian kernel. The dark line corresponds to the parameter distribution obtained by estimating model (3) on data simulated from model (3). The grey line corresponds to the parameter distribution obtained by estimating model (4) on data simulated from model (3). The red vertical line denotes the true value used for simulation. The dark vertical line is the average value obtained by estimating model (3) on data simulated from model (3). Finally, the grey vertical line is the average value obtained by estimating model (4) on data simulated from model (3).