

Productivity and Misallocation During a Crisis*

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Abstract

Measured total factor productivity often declines sharply during financial crises. In 1982, the Chilean manufacturing sector suffered a severe contraction in output, most of which can be accounted for a falling Solow residual. Using establishment data from the Chilean manufacturing census, I assess the contribution of misallocation to the drop in output. Using several measures, I find that the cross-sectional allocation of resources deteriorated during the crisis. To quantify the impact of this misallocation on aggregate measured TFP I develop a decomposition along the lines of [Hsieh and Klenow \(2009\)](#). The analysis allows for specifications that span two extremes: (i) all plants have identical factor intensities (ii) plants differ in factor intensities within each sector. While this raises difficult aggregation issues, I show the connection between changes in the extent of misallocation and changes in an aggregate Solow residual. Although the preliminary results are sensitive to the exact empirical specification, I find that increased misallocation had a substantial impact on aggregate total factor productivity during the crisis.

*Preliminary and Incomplete, please do not quote without permission of the author. I appreciate the comments of Amanda Agan, Fernando Alvarez, Robert Lucas, Devesh Raval, Rob Shimer, Nancy Stokey, Nico Trachter, and Andy Zuppann, as well as seminar participants at the University of Chicago and the Federal Reserve Bank of Chicago. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Chicago or the Federal Reserve System. All mistakes are my own.

Output and measured total factor productivity often decline sharply during financial crises. [Calvo et al. \(2006\)](#) analyze 22 severe crises in emerging markets and report that output and TFP typically decline by 10% and 9.5% respectively. As a prime example, Chile suffered a classic sudden stop in 1982. Output declined, unemployment rose, and capital inflows came to a halt. The contraction of the manufacturing sector was particularly severe, with output shrinking more than twenty percent and measured total factor productivity falling by more than ten percent.

Among the many factors that played a role, I assess the impact of misallocation of resources on the decline in measured TFP during the Chilean crisis. Accounting for the changes at both the macro and micro level can help distinguish between various theories of the crisis. There are a number reasons to think that misallocation played an important role during the crisis. High levels of debt and skyrocketing bank lending rates made external financing more difficult, potentially introducing a wedge in the cost of investment between constrained and unconstrained plants. Wages indexed to inflation and substantial severance pay requirements meant that reallocation of labor was difficult.

This study uses establishment data from a comprehensive census of the Chilean manufacturing sector to investigate two questions: (i) Did misallocation increase during the crisis? (ii) If so, can this account for the large movements in TFP during the crisis? In short, yes and yes: I document that the allocation of resources deteriorated during the crisis and accounted for a substantial drop in measured TFP.

Misallocation can be inferred from differences across plants in marginal revenue products of various factors of production, although there is no way to measure these directly. This study follows [Hsieh and Klenow \(2009\)](#) in using average revenue products as a proxy for marginal revenue products. Using several different measures, I find that cross-sectional dispersion in average products of capital increased during the crisis in a pattern that broadly matches movements in aggregate

TFP. By some measures the cross-sectional dispersion of average products of labor rose as well during the crisis, although the evidence is mixed. Another simple measure of the efficiency of the allocation of factor inputs is the cross-sectional correlation of revenue and capital. There is a clear decline in this correlation, suggesting again that misallocation increased during the crisis.

These findings are consistent with the hypothesis that the allocation of resources deteriorated during the crisis. To quantify the impact of these changes on measured TFP, I develop a decomposition of aggregate TFP using wedge accounting in the spirit of [Hsieh and Klenow \(2009\)](#). The procedure measures the wedge between the actual marginal product of a firm and the theoretical marginal product in a world with no optimization frictions. Given a theory of demand, one can compute how much aggregate TFP would change if all of these wedges were eliminated.

The decomposition has nothing to say about the source of these wedges; they could stem from adjustment costs, financing frictions, information issues, measurement error, or a host of other factors. Nevertheless, the time series of the distribution of the wedges can give clues about the source of aggregate fluctuations.

The two most serious confounding issues in assessing the the extent and impact of misallocation are measurement error and specification error. There are several aspects to the measurement issues. Most prominently, capital is notoriously difficult to measure.¹ If capital is poorly measured, dispersion in measured average products will overstate the role of misallocation in lowering TFP.

This issue is especially thorny when interpreting the level of TFP losses from misallocation. Fortunately the objects of interest in this study are changes in the extent of misallocation across time, particularly during the crisis. If measurement error has roughly the same effect in every year, then changes in measured misallocation during the crisis are more likely to reflect actual changes in

¹There are also aggregation issues; not all capital is created equal.

allocative efficiency. While the assumption that measurement error has the same effect each year is still strong, it is considerably weaker than that needed to compare misallocation across countries that use different surveys.²

Specification of production functions at the micro level leads to several difficult issues. A notable feature of the plant level data is that there are large persistent differences in capital intensity across plants even within narrowly defined industries. It is possible that these differences stem from persistent distortions (e.g., some plants have a long-term source of cheap capital), but it is also possible that the plants simply use different production technologies. This matters because if the differing capital intensity reflects differing technologies there is little to gain from reallocating resources across plants. I will consider two empirical specifications allowing for each of these extremes. The truth likely lies somewhere in between

The benefit of assuming plants differ in factor intensities is that relaxes a restrictive assumption could lead to overstatement of the losses from misallocation. The cost is that it brings up aggregation issues, making it difficult to interpret a representative firm with aggregate a Cobb-Douglas production function. Nevertheless, a central goal is to make the decomposition compatible with an aggregate production framework. I show how changes in the extent of misallocation will impact the aggregate Solow residual, even when the Solow residual does not reflect only technology.

This is done for two reasons. First, as shown by [Basu and Fernald \(2002\)](#) and [Basu et al. \(2010\)](#) the (appropriately modified) Solow residual is the relevant summary statistic for welfare even when it does not properly measure technology. Second, by focusing on the Solow residual, this study can make contact with the large macro literature that has studied sudden stops with the assumption of a representative firm. In this sense, the decomposition bridges the gap between [Hsieh and Klenow](#)

²[Hsieh and Klenow \(2009\)](#) argue convincingly their results are not driven by measurement error alone.

(2009) and Chari et al. (2007).

Another appealing property of the decomposition is that only the nominal values of revenue and payment to the various factors of production are needed.³ In a high inflation environment, deflators may be poorly measured, and can introduce additional noise into measurement. Further, using expenditures on inputs helps to control for the quality those inputs. For example, the wage bill may be a better measure of the efficiency units of labor than the number of workers if the workers are heterogeneous.

For the most part this study focuses on gross output production functions rather than value added. This is done for several reasons. First, many plants have negative value added (but positive revenue), particularly during the crisis. In principle these plants could be dropped from the analysis, but this would add significant bias to the results, as negative value added may be a meaningful indicator of misallocation. Second, the allocation of intermediate inputs across firms may be distorted during the crisis, as large changes in the exchange rate and financial distress could have disrupted some supply chains. Lastly, given these considerations, it will be informative to disentangle changes in input prices from changes in technology.

Methodologically, this study is related to Basu and Fernald (2002); Petrin and Levinsohn (2010); Basu et al. (2010); and to Petrin and Sivadasan (2010), who study output losses from allocative inefficiency in Chile from 1982-1994 with a focus on distortions in the labor market. In contrast this study focuses on interpreting the changes that occur during the large crisis.

The paper is also related to the long and growing literature explores the link between the allocation of resources across plants and measured TFP.⁴ propose plant-level misallocation as an

³Revenue, payments to labor, and expenditures on intermediate inputs are directly reported in the census. One must impute payments to capital.

⁴For example, Hopenhayn and Rogerson (1993), Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Buera and Shin (2010), and Midrigan and Xu (2010)

explanation for cross country income differences. This literature has mainly focused on cross-country differences. This paper extends this empirical work by studying fluctuations in output at business cycle frequencies.

In [Section 1](#), I discuss the economic environment and the impact of the recession. [Section 2](#) describes the plant level data gives aggregate statistics and time series. [Section 3](#) presents some reduced form evidence of misallocation. In [Section 4](#) the decomposition is developed and applied to the plant level data. [Section 5](#) concludes.

1 Reform and Recession in Chile

1.1 Reform

In the 1970's and early 1980's Chile's economic landscape underwent drastic changes. Pinochet took power in 1973 and, along with the Chicago Boys, instituted a number of reforms. From 1973 through 1979, nominal wages for some workers were periodically adjusted for increases in the CPI through legislation. In 1979, this was extended to all workers. Any new wage offers had a floor of the previous wage multiplied by the change in CPI since the previous wage offer. Another reform in 1978 attempted to reduce hiring frictions by eliminating the "just cause" doctrine and capping severance payments at five months. Hiring frictions remained sizable, as severance was equal to one month for every year worked (up to this five month cap).

In 1974 the Chilean government privatized the nation's banks. Several were subsequently bailed out after poor performance in 1976-77, but for the most part the banks were in private hands until the crisis in the early 1980's, when several banks had to be bailed out again. To tackle runaway inflation, the government instituted a series of planned currency devaluations relative to the dollar

in 1978 that culminated with a fixed nominal exchange rate with the dollar in 1979. The fixed rate remained in place until June 1982, in the middle of the recession. In 1973, tariffs were large and varied widely, averaging 105%. There were also many non-tariff trade barriers. From 1973-1979, these trade barriers were gradually reduced to a uniform tariff of 10%. Some argue that this led to a high exit rate among small plants (see [Pavcnik \(2002\)](#)).

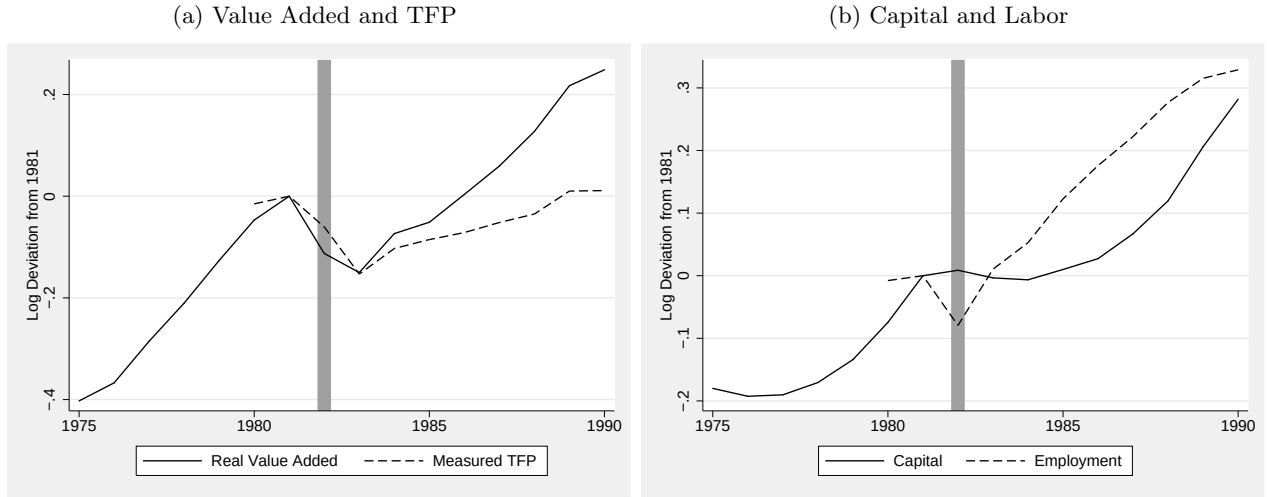
1.2 Recession

Following these reforms, the Chilean economy experienced rapid growth in the late 1970s, but suffered a large contraction in 1982. This section will describe the changes mostly in pictures using data from the World Bank's World Development Index, supplemented with data from the IMF's International Financial Statistics. [Figure 1a](#) shows the large decline in value added and TFP for the Chilean economy as a whole with data from the World Bank. Unemployment rose in 1982 while the capital stock, to a first approximation, remained constant.

[Figure 2](#) shows that the contraction of the manufacturing sector was particularly severe. Although manufacturing accounted for roughly a fifth of Chilean value added, it accounted for roughly two fifths (41%) of the decline of total real value added between 1981 to 1982. Manufacturing began to recover earlier than other sectors as well. In 1983 manufacturing began to expand while services and agriculture continued to contract.

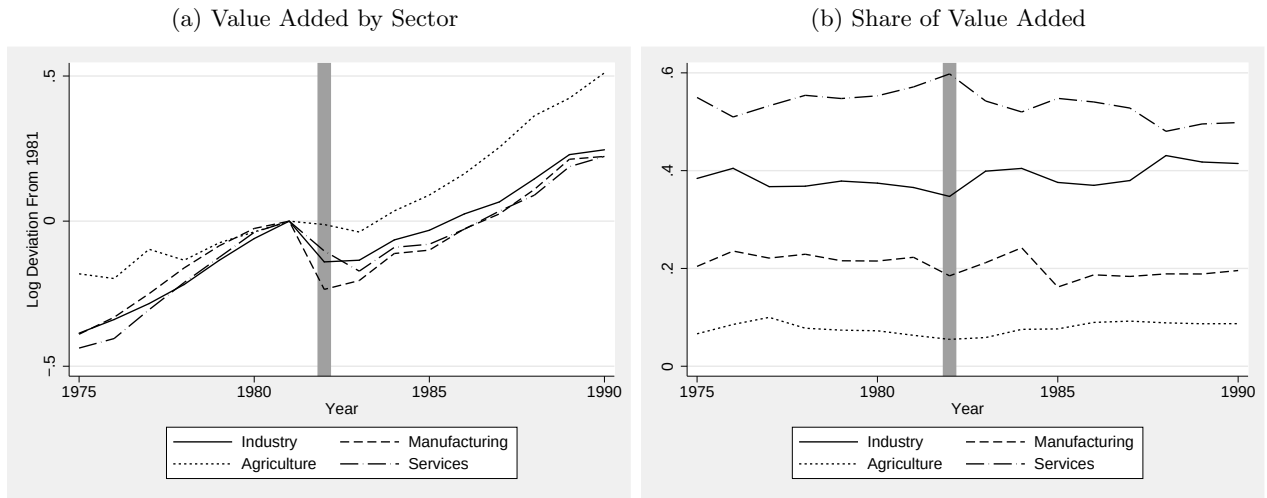
Inflation, in the triple digits during much the 1970's, began to decline in the late 1970's and early 80's. [Figure 3](#) shows that real interest rates spiked in 1981 and remained high in 1982. While there is no direct measure of the real interest rate, one can net out an estimate of expected inflation from the reported nominal rate. I compute two estimates of expected inflation: (i) actual ex post inflation during the period and (ii) a simple forecast using lagged inflation. Actual expectations of

Figure 1: Aggregate Production



Measured TFP is $\frac{VA}{K^\alpha L^{1-\alpha}}$ with $\alpha = 0.35$. Capital is constructed using perpetual inventory method assuming 10% depreciation. The initial capital stock in 1960 is constructed by assuming investment prior to 1960 grew each year at the average rate of 1960-1964. The employment series is available beginning in 1980. Source: WDI and author's calculations.

Figure 2: Real Value Added by Sector

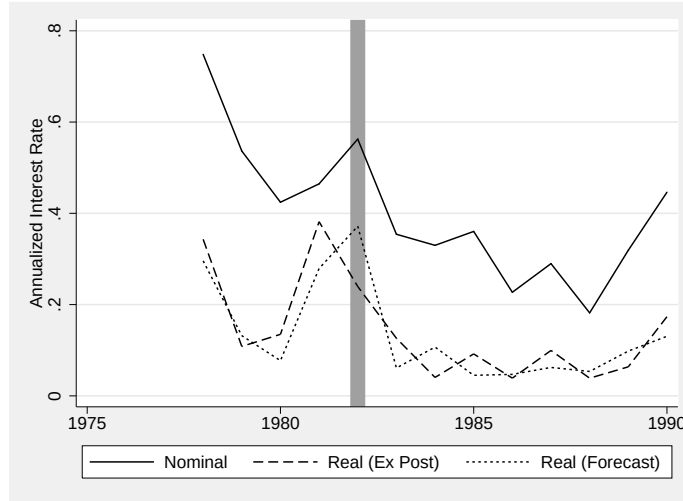


Manufacturing is a subset of Industry. Source: WDI and author's calculations.

inflation were most likely in between the two, and for the remainder of the paper I use a simple average of these measures.

Real wages also spiked during the recession. Because wages were indexed to previous inflation,

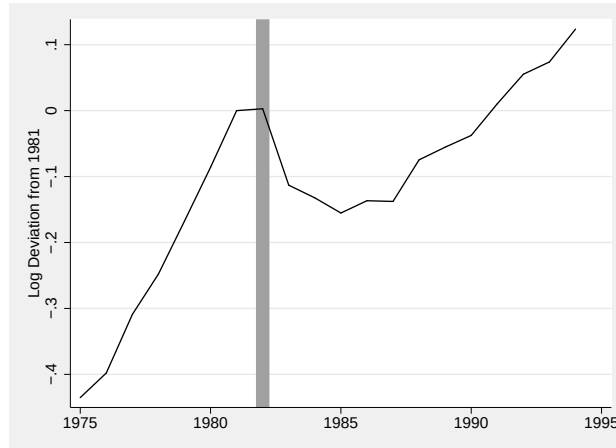
Figure 3: Real Interest Rates



The nominal interest rate is the average of the lending rate ("the bank rate that usually meets the short and medium term financing needs of the private sector") and the deposit rate, as reported by the IMF. For each month, the ex-post real interest rate is $(1 + R)/(1 + \pi) - 1$, where R is the nominal interest rate and π is (annualized) expected inflation over the next six months. Ex-post uses the actual inflation over the next six months, while the forecast uses actual inflation over the previous six months. The annual values of real interest rates are the average of these monthly real interest rates. Source: IMF and author's calculations.

the declining inflation led to an increasing real wage rate. [Figure 4](#) shows the large rise in real wages, which was especially pronounced in the manufacturing sector.

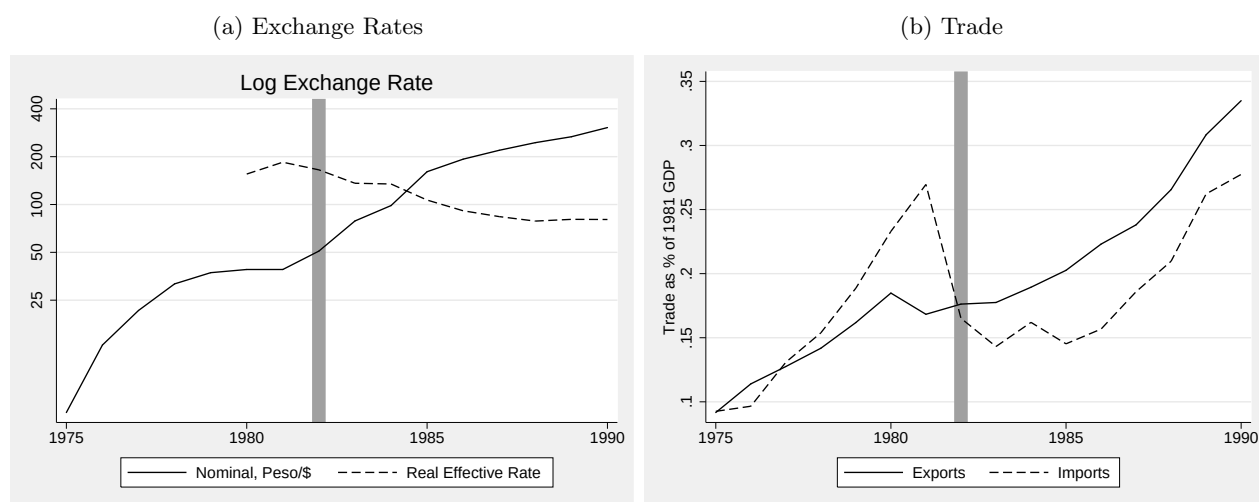
Figure 4: Real Wages



Source: [Cortazar \(1997\)](#).

Figure 5a shows the evolution of exchange rates over time, demonstrating the decelerating depreciation against the Dollar in the late 1970's culminating with a fixed exchange rate before the crisis. The Chilean Peso underwent a large depreciation against the Dollar in June of 1982 (this is not immediately apparent from Figure 5a because annual average exchange rates are plotted), and this depreciation continued through the 1985. In contrast to the Peso/\$ nominal rate, Chile's real effective exchange rate appreciated during the crisis, as many of Chile's trading partners were undergoing similar crises and also depreciating against the dollar. Figure 5b shows that while imports cratered during the crisis, exports remained fairly flat.

Figure 5: Trade



The real effective exchange rate is a trade weighted average. Source: WDI.

For further description of the Chilean experience before, during, and after the recession see the excellent summaries in [Diaz-Alejandro \(1985\)](#) and [Bosworth et al., eds \(1994\)](#).

2 Plant Level Data

2.1 Data Description

Plant level data comes from a high quality annual census of the manufacturing sector, the Encuesta Nacional Industrial Anual (ENIA) conducted by the Instituto Nacional de Estadísticas spanning the years 1979 to 1996. The survey covers all manufacturing plants with at least 10 employees and has been used by numerous other studies.⁵ The dataset is an unbalanced panel with nearly 5000 unique observations each year. The survey collects data on revenue, blue and white collar employment and wages, and intermediate inputs. Plants report investment, sales, and depreciation of several types of capital. Entry and exit from the dataset does not correspond to actual entry and exit; a firm that shrinks to nine employees would not show up in the data. Indeed, some firms disappear in one year only to reappear a year or two later. The survey contains no information linking plants to firms, though the vast majority of plants belong to single plant firms.⁶

Table 2.1 gives information about the cross section of firms in 1980, summarized at the two digit industry level. Across the largest industries there are big differences in output per worker and in the average size of establishments.

The largest difficulty in this study is the construction of plant level capital variables. For the purposes of this study, we are interested in constructing two variables, capital services used and expenditure on capital. The survey collects information on investment and depreciation of several types of capital (machines, buildings, and vehicles) and rental payments, and these must

⁵Among the many, see Liu (1993), Roberts and Tybout (1996), Pavcnik (2002), Bergoeing et al. (2003), Hsieh and Parker (2007), Greenstreet (2007), and Petrin and Levinsohn (2010).

⁶Hsieh and Parker (2007), using information provided by the INE, report that most plants in the dataset are themselves firms. In 1984, approximately 350 plants belonged to multi-plant firms, under 10%.

Industry (2 Digit)	Number of Firms	Share of Output	Share of Employment
Food, Beverages and Tobacco	1699	27.3%	26.6%
Chemicals and Chemical, Petroleum, Coal, Rubber and Plastic Products	471	20.0%	10.2%
Basic Metal Industries	79	19.4%	6.3%
Fabricated Metal Products, Machinery and Equipment	803	10.7%	16.4%
Textile, Wearing Apparel and Leather Industries	1075	9.1%	20.3%
Paper and Paper Products, Printing and Publishing	294	6.7%	6.3%
Wood and Wood Products, Including Furniture	641	3.6%	9.4%
Non-Metallic Mineral Products, (except of Petroleum and Coal)	180	2.8%	3.8%
Other Manufacturing Industries	66	0.3%	0.7%

be aggregated.⁷

Real capital stocks are constructed for each of the three types of capital using the perpetual inventory method, assuming depreciation rates of 5%, 10%, and 20% for buildings, machines, and vehicles, respectively.⁸ The procedure uses reported nominal investment and capital price deflators for each type of capital from [Bergoeing et al. \(2003\)](#). In addition, initial capital stocks are backed out from reported depreciation and investment in the first year.⁹

Real rental rates are constructed for each type of capital using nominal interest rates¹⁰ along with the depreciation rates. These rental rates are then averaged over time. Real capital services are then calculated as the sum of the individual capital stocks weighted by the respective average

⁷In principle one could have each type of capital enter the production function separately, although this brings up other issues. Rental payments are not denominated by type of capital. In addition, many firms have a stock of zero for some types of capital in some years.

⁸See appendix D in [Greenstreet \(2007\)](#) for more information on the construction of these capital stocks.

⁹In some years, the survey collects information on the value of fixed assets of each type. [Greenstreet \(2007\)](#) reports that among plants that were in the survey in those years, the constructed capital stocks and the deflated fixed asset values match well.

¹⁰Lending and deposit rates from the IMF give virtually identical results.

rental rates¹¹ corrected for the portion of the year the plant was open, plus the real value of reported rental payments (deflated using the CPI).

To calculate the nominal cost of capital, each capital stock is converted to a nominal value using its respective deflator, which is then multiplied by the nominal rental rate for that type of capital. The cost of capital is the sum of this value across capital types plus nominal reported rental payments.¹²

In this study, I will use the word revenue for the value of gross output, which includes sales along with changes in the value of inventories and value of capital produced.

I drop all plant-year observation for which the entry for revenue, capital services, wages, or materials is either missing or zero. This reduces the sample by roughly 12%.

The plant exit rate¹³ is particularly high in the first several years of the panel, a fact attributed by Pavcnik (2002) to the increased openness toward the end of the 1970's. Several of the calculations are done using only plants that were in the sample for the first ten years of the survey, 1979-1988. The relation between this sub-panel and the universe of plants changes over time as age composition of the sub-panel is obviously not staying constant. Nevertheless, this sub-panel can be used to show that the effects found do not stem solely from entry and exit patterns.

¹¹Time-averaged rental rates are used so that a change in capital services can come only from changes in stocks of each type of capital, not changes in weights.

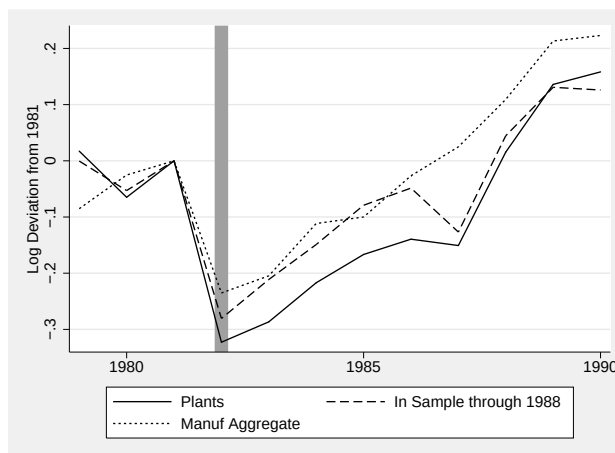
¹²Ideally these measures would also include the expected change in the price of capital, though I have no direct way to measure this. Capital price deflators show large swings in the prices of buildings, machinery, and vehicles. If these changes are expected, there would be massive and unrealistic swings in capital's share of revenue (for some years, the user cost of certain types of capital would be negative). While this is not a completely innocuous simplification, I assume that all changes in real capital prices are unexpected. Thus variation in the user cost of capital will mostly reflect changes in the real interest rate.

¹³This is exit from the survey, which could either mean that the plant shut down or that the plant went below the ten employee threshold.

2.2 Aggregating Plant Level Data

While the survey includes the universe of plants with at least ten employees, we first look to see how the plant level data compares to the information about the manufacturing sector from the WDI. **Figure 6** compares time series of aggregate real value added across three series: (1) real value added among all plants in the data set, (2) real value added among plants that were in the sample for all of the first ten years, 1979-1988, and (3) real value added for the manufacturing sector as reported in the WDI.

Figure 6: Value Added



Sources: Source: ENIA and author's calculations.

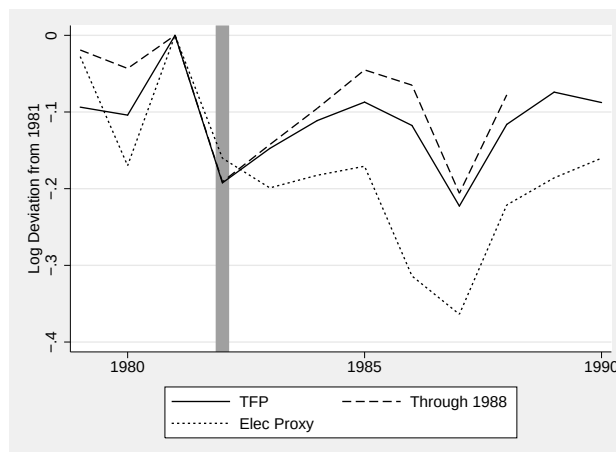
All three series show a qualitatively similar pattern: a large contraction in 1982 followed by an immediate but slow recovery. The plant level data shows a larger drop in 1982.

We can also use the establishment data to compute an aggregate Solow residual for the manufacturing sector. Capital services and labor can be aggregated across firms. **Figure 7** shows the Solow residual for the aggregate value added production function of $Y = K^\alpha L^{1-\alpha}$ with the parameterization $\alpha = .35$, for all firms and for the subsample of firms in the survey through 1988. For both series, the aggregate Solow residual matches the pattern of value added in the years around

the crisis. Most of the fall in value added can be accounted for by a falling Solow residual.

A prime candidate to explain the fall in the measured Solow residual during the crisis is that capital was underutilized, so that the actual capital stock overstates the quantity of capital services used. A common method to assessing utilization is to proxy for capital services using electricity consumed, and this can be done with the plant level data. Figure 7 shows a third series: the Solow residual for all plants when electricity is used as a proxy for capital services. We can see that only a small portion of the decline in measured TFP in 1982 can be explained by underutilization of capital.

Figure 7: TFP



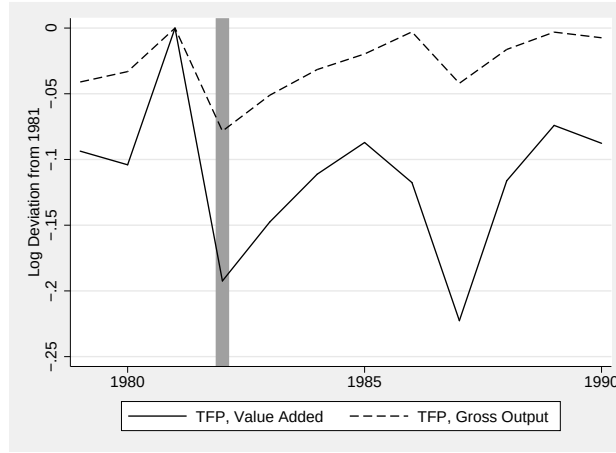
The TFP series is calculated for all plants from the sample and for the sub-panel of plants that were in the survey for the first ten years of the sample period, 1979-1988. The series Elec Proxy measures TFP using electricity to proxy for capital services. Source: ENIA and author's calculations.

This study will focus on a different measure of TFP, computed using a gross output production function instead of a value added production function. This is done for several reasons. First, many plants have negative value added, particularly during the crisis. In principle these plants could be dropped from the analysis, but this would add significant bias to the results, as negative value added may be a meaningful indicator of misallocation. Second, the allocation of intermediate

inputs across firms may be distorted further during the crisis, as large changes in the exchange rate and financial distress could have disrupted some supply chains. Lastly, given these considerations, it will be informative to disentangle changes in input prices from changes in technology.

Figure 8 shows TFP calculated for a value added production function with capital share 0.35 (as in Figure 7) and also for the gross output production $Y = TFPK^{\alpha_K}L^{\alpha_L}M^{\alpha_M}$ with $\alpha_K = 0.175$, $\alpha_L = 0.325$, $\alpha_M = 0.5$. These series follow the same pattern except that, as expected, changes in TFP from value added are magnified.¹⁴

Figure 8: TFP



The series TFP (VA) is measured TFP from a value added production function with a capital share of $\alpha_K = 0.35$, the same series that appears in Figure 7. The series TFP (Gross Output) is measured TFP from a gross output production function, with $\alpha_K = 0.175$, $\alpha_L = 0.325$, $\alpha_M = 0.5$.

¹⁴When production is Cobb-Douglas with constant returns to scale (and if input prices are unchanged) $\Delta \log TFP_{ValueAdded} = \frac{1}{\alpha_M} \Delta \log TFP_{GrossOutput}$ where $\alpha_M < 1$ is the cost share of materials.

3 Evidence of Misallocation

3.1 Average Revenue Products of Capital and Labor

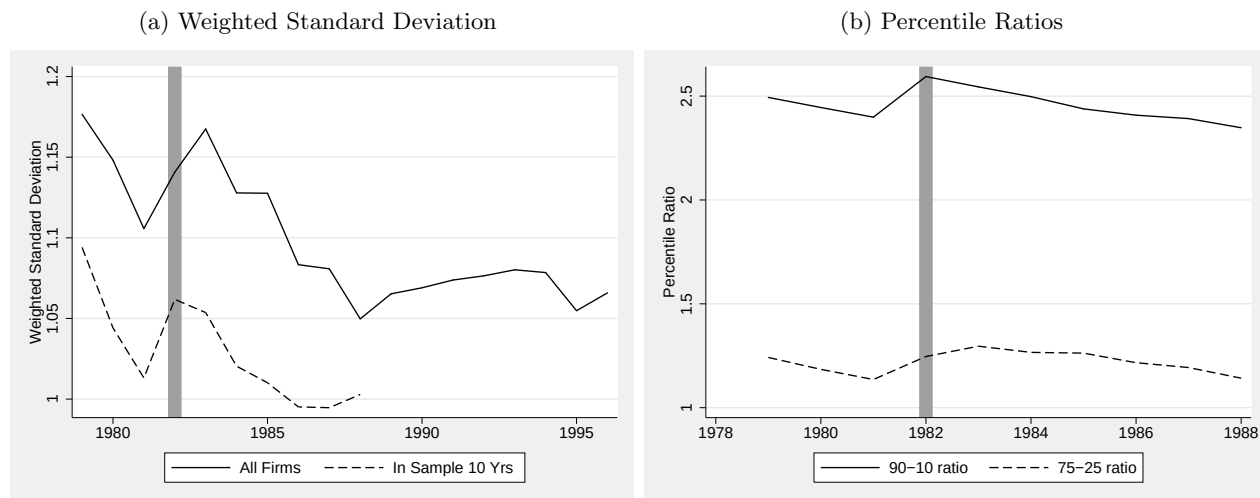
There are a number of ways to look for changes in the efficiency of the allocation of resources across plants. One way to measure the efficiency of the allocation of resources is to look at the dispersion of marginal products of various factor inputs. While we cannot directly measure marginal products, we follow [Hsieh and Klenow \(2009\)](#) and measure average products; if plants have Cobb-Douglas production functions with the same factor shares, the average products will be proportional to marginal products.

Some of the dispersion in average products may reflect differences in production functions across firms (e.g., production may be more capital intensive in some firms) or measurement error. Even in this case, *changes* in dispersion of average products will be informative, even if the levels are not.

The figures below show measures of dispersion of the log average product of various factors of production over time. [Figure 9](#) shows the evolution of the cross-sectional dispersion of average product of capital (APK) over time. Logs are used so that mismeasurement of the aggregate price levels have no effect: within each year these will cancel. [Figure 9a](#) shows the standard deviation of log average product of capital (log of nominal revenue divided by real capital services used) weighted in each year by log revenue. The calculation is done among all plants and among the balanced sub-panel of plants in the survey through 1988. [Figure 9b](#) shows the 90-10 and 75-25 ratios in the distribution of APKs within each year, a measure of dispersion that is less sensitive to outliers. Specifically, this is the log of the ratio APKs the 90th and 10th (or 75th and 25th)

percentiles in the distribution among firms among firms in the balanced sub-panel.

Figure 9: Dispersion in Log Average Product of Capital



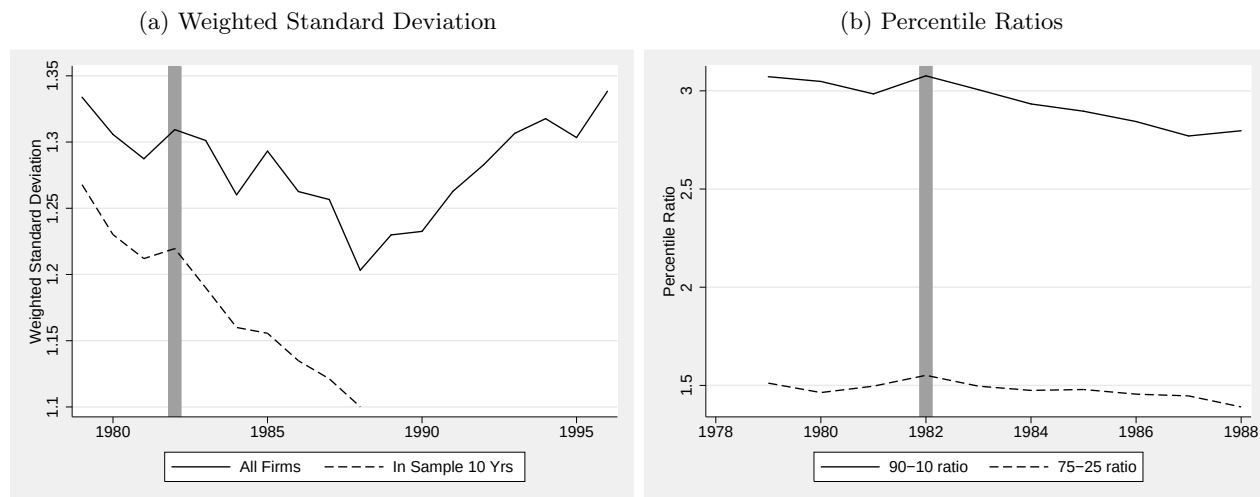
Standard deviation is weighted by log revenue. Percentiles are from the unweighted distribution among firms in the sample from 1978-1988.

There are several features to note. First, in both cuts of the data, there is a clear decrease in the standard deviation of APKs in 1981 followed by a sustained increase in 1982, qualitatively matching movements in those years of aggregate measured TFP (although the match in 1983 is less good). Second, there is a clear downward trend in the standard deviation of APKs over time. It is tempting to interpret this as a gradual improvement in the efficiency of the allocation of resources across firms. Another possible explanation is that measurement error for capital variables is particularly large early in the sample, a distinct possibility given how initial capital stocks are constructed. Importantly, even if this kind of measurement error could explain away the long run trend, it cannot explain the clear bump in dispersion during the crisis. Third, the standard deviation is larger among all plants than among plants remaining in the sample throughout the first 10 years.

As a robustness check, we can look at the average product of machinery, the type of capital most likely to be relevant for production. [Figure 10](#) shows that around the crisis, dispersion of

average product of machines followed a broadly similar, though less stark, pattern as the average product of capital.

Figure 10: Dispersion in Log Average Product of Machines

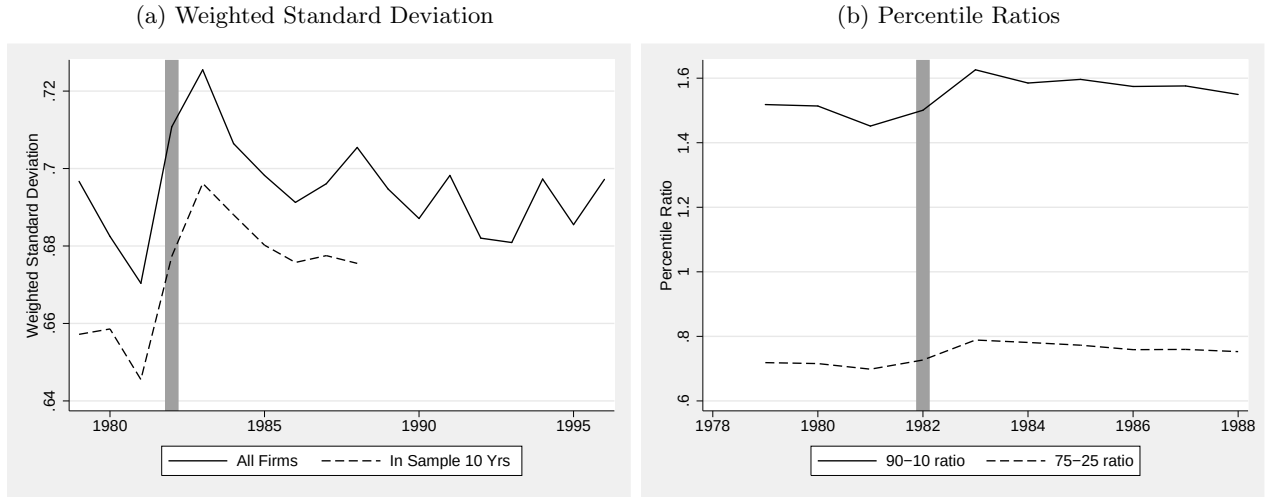


Standard deviation is weighted by log revenue. Percentiles are from the unweighted distribution among firms in the sample from 1978-1988.

We next turn to labor. [Figure 11](#) and [Figure 12](#) show the evolution of dispersion of the average products of labor (APL). If workers are heterogeneous, there may be skill differences across plants. [Figure 11](#) follows [Hsieh and Klenow \(2009\)](#) and uses wage as a proxy for a worker's skill. Consequently a firm's wage bill can proxy for units of human capital employed. Again, because logs are taken, the real or nominal wage has no effect on these measures of dispersion. [Figure 11](#) uses the count of employees as the measure of labor.

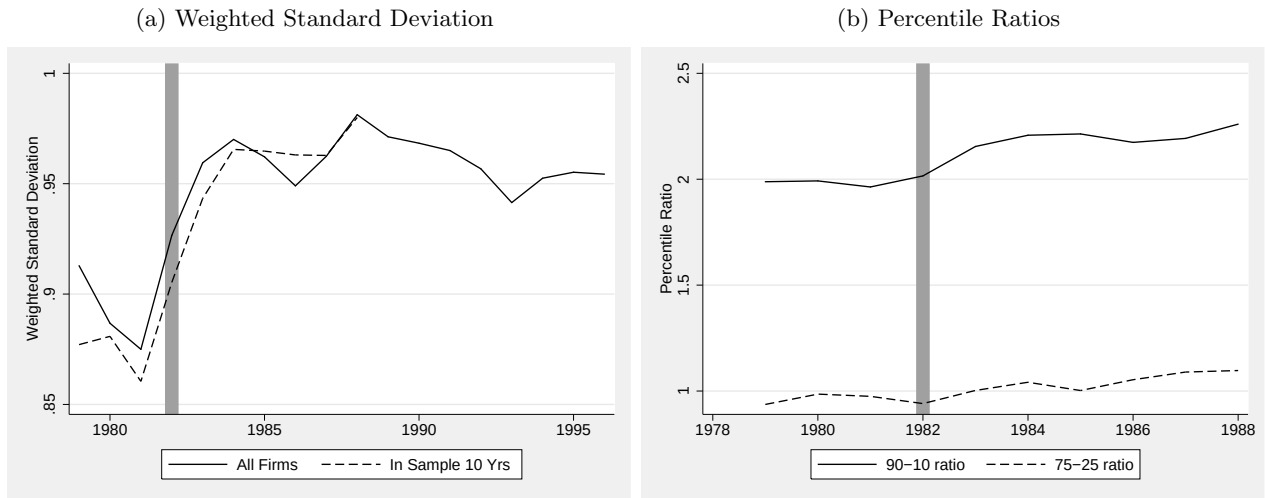
By both measures the standard deviation of APL rose during the crisis and continued to remain high. However, the percentile ratios do not indicate that dispersion in APLs was particularly high in 1982.

Figure 11: Dispersion in Log Average Product of Labor: Controlling for Skills



Standard deviation is weighted by log revenue. Percentiles are unweighted. Labor input is measured by the total wage bill.

Figure 12: Dispersion in Log Average Product of Labor



Standard deviation is weighted by log revenue. Percentiles are unweighted.

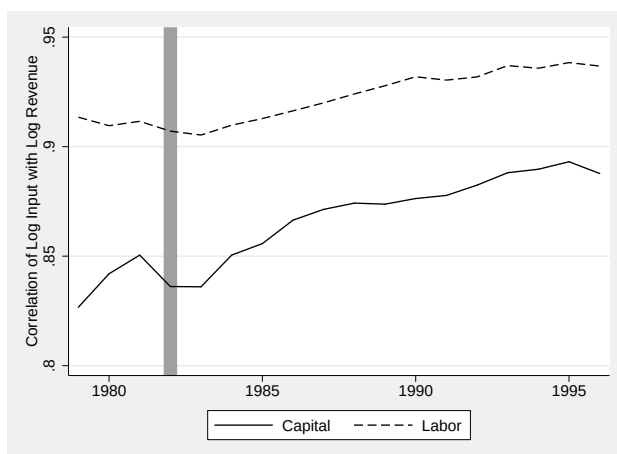
3.2 Size and Input Use

Another way to gauge the changes in the efficiency of the allocation of resources across plants is to ask: Do more productive plants use more resources? One measure of this is the cross-sectional

correlation of capital and revenue. [Bartelsman et al. \(2009\)](#) use a similar measure, the correlation of labor and firm level Solow residual. I prefer output to the Solow residual because it relies less on modeling assumptions and because using the Solow residual builds in artificial negative correlation if capital is poorly measured. [Petrin and Levinsohn \(2010\)](#) criticize the use of this type of correlation as a measure of the efficiency of the allocation of resources because it is not motivated by theory. The same critique applies here. The decomposition in [Section 4](#) takes a more structured approach to the efficiency of the allocation of resources.

[Figure 13](#) shows the correlation of log revenue with log capital expenditures and of log revenue with log labor expenditures. By this measure, there was a worsening of the allocation of capital across firms in 1982. The allocation of labor gets slightly worse, and continues to decline in 1983, though this change is relatively small.

Figure 13: Correlation



3.3 The Takeaway

Across the different metrics, there is a clear pattern of deteriorating allocation of resources across firms during the crisis. This is especially clear for capital, but there is also some evidence for that

the allocation of labor became less efficient as well.

While this points to efficiency losses from misallocation, it leaves open the question of how large these losses are. The next section will attempt to quantify the contribution of misallocation to changes in the Solow residual.

4 Business Cycle Accounting at the Micro Level

NOTE: RESULTS ARE PRELIMINARY

To quantify the impact of changes in the dispersion of average products on the decline in measured TFP, I develop an accounting procedure similar to [Hsieh and Klenow \(2009\)](#). At the plant level, each firm is assumed to produce using a Cobb-Douglas production function. For each plant, I compute deviations, or wedges, from theoretical frictionless first order conditions. Given a theory of demand for the plants' output, one can compute how much the Solow residual would change if these wedges were eliminated. This gives a measure of the loss of efficiency from misallocation.

These wedges may be due to a variety of frictions, not all of which represent inefficiencies. For example, if plants face investment adjustment costs, the allocation of resources will be constrained efficient even though frictionless first order conditions fail to hold.

There are large, persistent differences across plants in capital intensity. This could simply reflect the differences in technology across plants. Alternatively this may reflect persistent distortions faced by some plants. This matters because if the differing capital usage reflects differing technologies there is little to gain from reallocating resources across plants. I will consider two empirical specifications allowing for each of these extremes.

This raises aggregation issues and makes it difficult to interpret an aggregate production func-

tion. These issues are discussed in more detail in [Section 4.1.3](#).

The decomposition uses only nominal values of revenue and expenditures that are directly reported in the survey (except for expenditure on capital services). In an inflationary environment such as Chile in the early 80's, prices could be measured poorly, so using price deflators convert expenditures to quantities can introduce more noise into measurements. In addition, using price deflators that are common for an entire industry¹⁵ can mask differences in output and input prices facing different firms. These differences are precisely what the decomposition is attempting to uncover. Using expenditures on inputs also helps to control for differences in input quality. For example, the wage bill can be used to control for the quality of the workforce, whereas deflating by a common wage (or simply using the number employees) could conflate quantity and price differences.

4.1 The Decomposition

This description of the decomposition gives the general case in which there are plant specific factor intensities. An environment in which all plants share common factor intensities is obviously a special case.

Consider an economy composed of many sectors. Each sector is composed of plants that produce differentiated products that can be combined into a sector aggregate.¹⁶ These sector aggregates are then combined into a single aggregate good. Let Y_i be the output of firm i , $Y_s = \left(\sum_{i \in I_s} Y_i^{\frac{\sigma_s - 1}{\sigma_s}} \right)^{\frac{\sigma_s}{\sigma_s - 1}}$ be the quantity of sector aggregate for sector s , and $Y = \left(\sum_s Y_s^{\frac{\gamma - 1}{\gamma}} \right)^{\frac{\gamma}{\gamma - 1}}$ be the quantity of the manufacturing aggregate. If P_i is the price of the good produced by firm i , then $P_s = \left(\sum_{i \in I_s} P_i^{1 - \sigma_s} \right)^{\frac{1}{1 - \sigma_s}}$

¹⁵Several other decompositions use common price deflators for the industry.

¹⁶Some argue that declines in aggregate productivity are driven by reallocation across sectors. If this is the case, the decline should show up in increased dispersion of between sector wedges. It is useful to have an intermediate sector level so that dispersion in wedges between sectors is not conflated with dispersion in wedges within sectors.

and $P = \left(\sum_s P_s^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$ are ideal price indices for the sectoral good and aggregate good respectively.¹⁷ Consumers minimize costs so that that demand for good i is $Y_i = Y \left(\frac{P_s}{P} \right)^{-\gamma} \left(\frac{P_i}{P_s} \right)^{-\sigma_s}$.

Firm i has access to the production function $Y_i \leq A_i K_i^{\alpha_{Ki}} L_i^{\alpha_{Li}} M_i^{\alpha_{Mi}}$ and faces demand $Y_i \leq D_s P_i^{-\sigma_s}$. Let R , W , and Q denote the nominal costs of units of capital, labor, and intermediate materials respectively. Define the wedges τ_{Ki} , τ_{Li} , τ_{Mi} to satisfy the following equations¹⁸

$$\begin{aligned} e^{\tau_{Ki}} &= \frac{\sigma_s - 1}{\sigma_s} \alpha_{Ki} \frac{P_i Y_i}{R K_i} \\ e^{\tau_{Li}} &= \frac{\sigma_s - 1}{\sigma_s} \alpha_{Li} \frac{P_i Y_i}{W L_i} \\ e^{\tau_{Mi}} &= \frac{\sigma_s - 1}{\sigma_s} \alpha_{Mi} \frac{P_i Y_i}{Q M_i} \end{aligned}$$

These wedges measure deviations from optimization in a frictionless world. Whatever the reasons for the wedges, the firms actions will be observationally equivalent to a firm maximizing profits in a frictionless world subject to distortions:

$$\pi_i = \max_{P_i, Y_i, K_i, L_i, M_i} P_i Y_i - e^{\tau_{Ki}} R K_i - e^{\tau_{Li}} W L_i - e^{\tau_{Mi}} Q M_i$$

Given the production function and the demand curve, the price faced by the consumer will be

$$P_i = \frac{1}{\lambda_i A_i} e^{\tau_i}$$

¹⁷Throughout, variables with an i subscript are refer to firm i in sector s , variables with an s subscript refer to aggregates for sector s , and variables with no subscript refer to aggregates for all manufacturing plants. Time subscripts are omitted for ease of exposition.

¹⁸These can be written a number of ways. There are four observable quantities ($P_i Y_i$, $W L_i$, $R K_i$, $Q M_i$). The first order conditions are all linear combinations of the logs of these quantities, and there are only three linearly independent differences in these quantities. However, there is no unique way to assign meanings to the wedges - any linear combination will do just as well - so interpreting a particular wedge is perilous.

where $\tau_i \equiv \alpha_{Ki}\tau_{Ki} + \alpha_{Li}\tau_{Li} + \alpha_{Mi}\tau_{Mi}$ aggregates the effect of the three wedges and $\lambda_i \equiv \frac{\sigma_s - 1}{\sigma_s} \frac{\alpha_{Ki}^{\alpha_{Ki}} \alpha_{Li}^{\alpha_{Li}} \alpha_{Mi}^{\alpha_{Mi}}}{R^{\alpha_{Ki}} W^{\alpha_{Li}} Q^{\alpha_{Mi}}}$ reflects the impact of changes in input prices. Note that if firms share factor intensities, α_{Ki} , α_{Li} , and α_{Mi} , then they will also share a common λ_i ; changes in input prices will have the same impact on output prices (holding everything else constant).

4.1.1 Sector Level

At the sector level, an economist may define a representative firm with technology $Y_s = TFP_s \times K_s^{\alpha_{Ks}} L_s^{\alpha_{Ls}} M_s^{\alpha_{Ms}}$. One can define wedges at the sector level, τ_{Ks} , τ_{Ls} , τ_{Ms} to satisfy the equations

$$\begin{aligned} e^{\tau_{Ks}} &= \frac{\sigma_s - 1}{\sigma_s} \alpha_{Ks} \frac{P_s Y_s}{R K_s} \\ e^{\tau_{Ls}} &= \frac{\sigma_s - 1}{\sigma_s} \alpha_{Ls} \frac{P_s Y_s}{W L_s} \\ e^{\tau_{Ms}} &= \frac{\sigma_s - 1}{\sigma_s} \alpha_{Ms} \frac{P_s Y_s}{Q M_s} \end{aligned}$$

Again, this can be thought of as arising from the optimization problem of a firm facing demand $Y_s = DP_s^{-\sigma_s}$ and several distortions:

$$\pi_s = \max_{P_s, Y_s, K_s, L_s, M_s} P_s Y_s - e^{\tau_{Ks}} R K_s - e^{\tau_{Ls}} W L_s - e^{\tau_{Ms}} Q M_s$$

Again, given the production function and the demand curve, the facing the consumer will be

$$P_s = \frac{e^{\tau_s}}{\lambda_s TFP_s}$$

where $\tau_s \equiv \alpha_{Ks}\tau_{Ks} + \alpha_{Ls}\tau_{Ls} + \alpha_{Ms}\tau_{Ms}$ and $\lambda_s = \frac{\sigma_s - 1}{\sigma_s} \frac{\alpha_{Ks}^{\alpha_{Ks}} \alpha_{Ls}^{\alpha_{Ls}} \alpha_{Ms}^{\alpha_{Ms}}}{R^{\alpha_{Ks}} W^{\alpha_{Ls}} Q^{\alpha_{Ms}}}$ are defined in the same way as τ_i and λ_i . We can equate the two expressions for the price level and $P_s^{1-\sigma_s} = \sum_i P_i^{1-\sigma_s}$ to get

the following expression:

$$\lambda_s^{\sigma_s-1} TFP_s^{\sigma_s-1} = \sum_i (\lambda_i A_i)^{\sigma_s-1} e^{(\sigma_s-1)(\tau_s-\tau_i)}$$

We now define the quantity A_s to satisfy $\lambda_s A_s \equiv (\sum_i \lambda_i A_i^{\sigma-1})^{\frac{1}{\sigma-1}}$ to be what sector level TFP would be if all the wedges were zero. We can also define $B_i \equiv (\lambda_i A_i / \lambda_s A_s)^{\sigma_s-1}$ to be the revenue share of firm i if there were no wedges. With these definitions we can write a simple expression for a sector's TFP,

$$TFP_s = A_s \left(\sum_i B_i e^{(\sigma_s-1)(\tau_s-\tau_i)} \right)^{\frac{1}{\sigma_s-1}} \quad (1)$$

There are two components. The first depends only on firm level technology parameters (and, if factor intensities differ across firms, factor prices). The second term depends on the dispersion of wedges across firms. If each τ_i were equal to τ_s this term would be one. If there is a large amount of dispersion across firms in wedges, this term will be significantly below one and TFP in the industry will be smaller than it could be. In this sense, changes in the second term measure the contribution of dispersion to changes in sector level TFP.

Relative TFP $\frac{TFP_s}{A_s}$ is the ratio of measured TFP for the representative firm to what TFP would be if all wedges were eliminated. This will be given a more clear interpretation in [Section 4.1.3](#).

4.1.2 Aggregate Level

One can extend this procedure to aggregate across sectors, and procedure nests nicely. Consider a representative firm at the aggregate level with the production function $Y \leq TFP \times K^{\alpha_K} L^{\alpha_L} M^{\alpha_M}$ and facing demand $Y \leq \bar{D} P^{-\sigma}$. Defining the wedges τ_K, τ_L, τ_M in the same manner, the optimal price will be $P = \frac{e^\tau}{\lambda TFP}$, with $\lambda = \frac{\sigma-1}{\sigma} \frac{\alpha_K^{\alpha_K} \alpha_L^{\alpha_L} \alpha_M^{\alpha_M}}{R^{\alpha_K} W^{\alpha_L} Q^{\alpha_M}}$ and $\tau \equiv \alpha_K \tau_K + \alpha_L \tau_L + \alpha_M \tau_M$. Again, using

the relation $P^{1-\gamma} = \sum P_s^{1-\gamma}$ gives an expression for TFP at the aggregate level in terms of the relative total factor productivity of each of the sectors:

$$\left(\frac{TFP}{A}\right)^{\gamma-1} = \sum B_s \left(\frac{TFP_s}{A_s}\right)^{\gamma-1} e^{(\gamma-1)(\tau-\tau_s)}$$

As above, we can define A (satisfying $(\lambda A)^{\gamma-1} \equiv \sum_s (\lambda_s A_s)^{\gamma-1}$) to be aggregate technology in a wedgeless world, and $B_s \equiv (\lambda_s A_s / \lambda A)^{\gamma-1}$ to be the revenue share of sector s . We can write TFP in the industry as

$$TFP = A \left\{ \sum_s B_s e^{(\gamma-1)(\tau-\tau_s)} \left(\sum_i B_i e^{(\sigma_s-1)(\tau_s-\tau_i)} \right)^{\frac{\gamma-1}{\sigma-1}} \right\}^{1/(\gamma-1)}$$

Here, we can see that TFP at the aggregate level depends on an aggregate of firm level technology parameters and a term that depends on wedges within and between sectors.

4.1.3 Interpreting the Representative Firm

When firms differ in factor intensities, aggregation becomes difficult, stretching the interpretation of a representative firm with a Cobb-Douglas. For example, consider a sector with two firms: firm 1 is capital intensive while firm 2 is labor intensive. If firm 1 has a large share of the market, the sector will appear capital intensive (for the most efficient allocation) and vice versa. It is not clear what form the representative production function should take. More generally, individual firms differ in factor intensities, there is no obvious way to construct a representative firm that aggregates nicely.

In principle, the decomposition can be done for any representative production function. A desirable feature is that if all firms in an industry faced the same capital wedge τ_K^0 , then the

representative firm will face that same capital wedge τ_K^0 . An important special case of this is that if every firm had wedges of 0, TFP_s would be equal to A_s . To guarantee these features, one can set factor intensities for the representative production function as the average factor intensities of each of the firms, weighted by revenue, for example $\alpha_{Ks}^* = \sum_i \alpha_K \frac{P_i Y_i}{P_s Y_s}$ and $\alpha_K^* = \sum_s \alpha_{Ks} \frac{P_s Y_s}{P Y}$. Similarly, one must assume that the log markup at the aggregate level is a weighted average of the log markup of each sector, $\log \frac{\gamma^*}{\gamma^*-1} = \sum_s \log \frac{\sigma_s}{\sigma_s-1} \frac{P_s Y_s}{P Y}$. See [Section A](#) for details.

One consequence is that this representative production function will change as the sector's composition shifts.¹⁹ If one used an aggregate production function that was constant over time, say, with parameters $\{\hat{\alpha}_K, \hat{\alpha}_L, \hat{\alpha}_M, \hat{\gamma}\}$ then the aggregate Solow residual $T\hat{F}P$ could be decomposed as follows

$$T\hat{F}P = A \times e^{\hat{\tau}-\tau^*} \times \left\{ \sum_s B_s e^{(\gamma-1)(\tau^*-\tau_s^*)} \left(\sum_i B_i e^{(\sigma_s-1)(\tau_s^*-\tau_i)} \right)^{\frac{\gamma-1}{\sigma-1}} \right\}^{1/(\gamma-1)} \quad (2)$$

where $\hat{\tau} \equiv \hat{\alpha}_K \hat{\tau}_K + \hat{\alpha}_L \hat{\tau}_L + \hat{\alpha}_M \hat{\tau}_M$ summarizes the wedges for the representative firm with constant factor intensities, and τ^* is analogously defined for the representative firm that has production parameters defined above (and change over time): $\{\alpha_K^*, \alpha_L^*, \alpha_M^*, \gamma^*\}$.

The aggregate Solow residual can therefore be decomposed into three terms. The first, A , depends only on production function parameters, firm level productivity terms, and input prices. The second term, $e^{\hat{\tau}-\tau^*}$, measures the gap between a the assumed representative production function and an aggregate production function that better reflects the state of underlying technology. The third term measures the efficiency loss from misallocation of resources. This third term is the focus of the analysis.

¹⁹For the special case where firms share common factor intensities, the representative production function will stay constant over time.

4.1.4 Measuring Relative TFP

To measure relative TFP, $\frac{TFP_s}{A_s}$, I need expressions for the wedges and for the frictionless shares, B_i . The wedges can be computed directly using the ratios of nominal revenue and payments to the factors of production. To compute the frictionless shares, note that we can use the demand system, $Y_i = D_s P_i^{-\sigma}$ to write output as $Y_i = D_s^{\frac{1}{1-\sigma}} (P_i Y_i)^{\frac{\sigma}{\sigma-1}}$. With this and the production function $A = Y / (K^{\alpha_{Ki}} L^{\alpha_{Li}} M^{\alpha_{Mi}})$, we can get an expression for firm level productivity $\lambda_i A_i$:

$$\lambda_i A_i \propto \alpha_{Ki}^{\alpha_{Ki}} \alpha_{Li}^{\alpha_{Li}} \alpha_{Mi}^{\alpha_{Mi}} \frac{(P_i Y_i)^{\frac{\sigma_s}{\sigma_s-1}}}{(RK_i)^{\alpha_{Ki}} (WL_i)^{\alpha_{Li}} (QM_i)^{\alpha_{Mi}}}$$

which consequently gives an expression for $B_i = \frac{(\lambda_i A_i)^{\sigma_s-1}}{\sum_i (\lambda_i A_i)^{\sigma_s-1}}$. Conveniently this expression depends only on nominal revenue and payments to the factors of production.

4.2 Application to the Chilean Data

I report here the results from the two extreme specifications. First I assume that plants differ in factor intensities even within industry. Second, that I assume that all plants use production function with the same factor intensities.

4.2.1 Firm Specific Factor Intensities

In this section I adhere to the view that all long run differences in factor intensities reflect differences in underlying technology. To this end, I compute the parameters of each firm's production function by using the average cost share of each input over the years that the plant is in the sample. Similarly I compute the elasticity of substitution for each sector averaging the markup implied revenue/cost margin in each year.

Figure 14: Impact of Misallocation: Plant Specific Factor Intensities

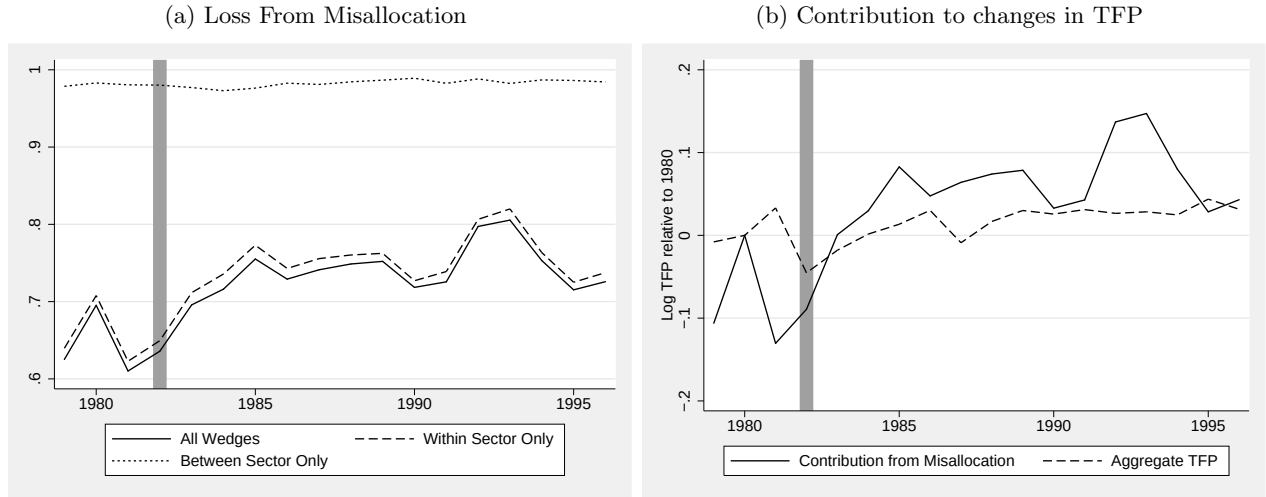


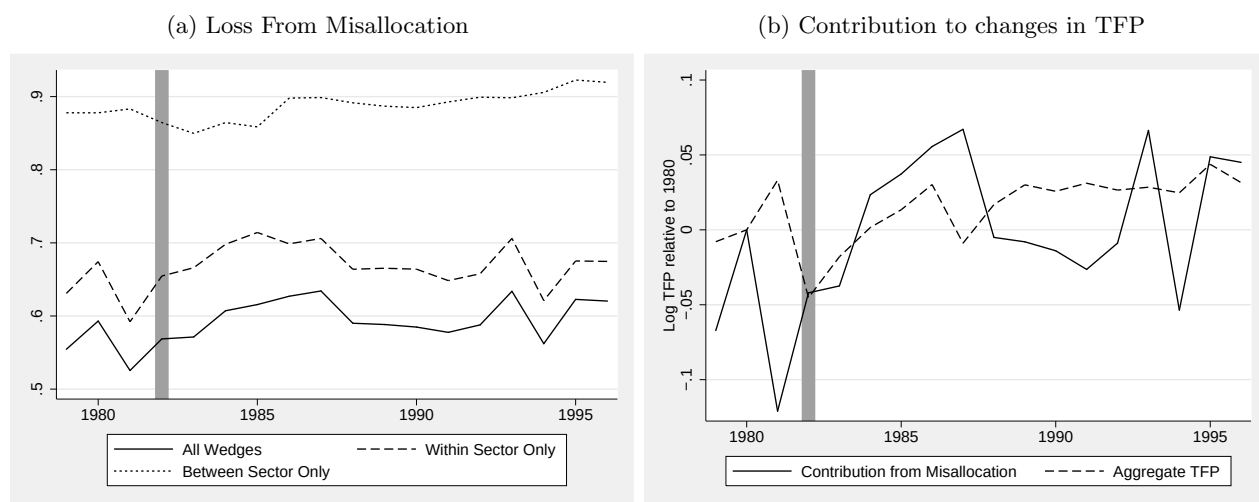
Figure 14 shows the impact of misallocation. Figure 14a shows the loss of TFP relative to a frictionless world (the third term in equation (2)). The solid line represents the loss in TFP from all distortions, both within and across sectors. The line labeled “within sector only” shows TFP losses from dispersion within sectors. The line labeled “between sector only” shows TFP losses if there were no within sector dispersion in wedges. The levels of these lines imply that roughly a third of TFP is lost due to misallocation, although much of this could be due to measurement error. The figure implies that virtually all of the efficiency losses come from within sector misallocation.

Figure 14b shows both the change in the aggregate Solow residual that can be attributed to changes in the extent of misallocation along with the actual changes in the Solow residual. The solid line here matches the solid line in Figure 14a. If changes in the wedges were the only determinants of changes in TFP, then these lines would track each other perfectly. 1981 appears to be poorly explained by changes in misallocation, and misallocation over-explains the drop in TFP from 1980-1982.

4.2.2 Common Factor Intensities

We can perform the same decomposition, keeping the factor intensities the same for every firm. This conforms with the view that all long term differences in factor usage across plants stem from persistent distortions. All firms are assumed to have the production parameters $\alpha_K = .15$, $\alpha_L = .35$, and $\alpha_M = .5$, with elasticity of demand $\sigma = 4$. This means the decomposition is cleaner, but comes with the caveat that the extent of misallocation is likely overstated.

Figure 15: Impact of Misallocation: Common Factor Intensities



Relative to [Figure 14](#), the decomposition assigns a larger role for misallocation between sectors. The contribution to changes in TFP is similar follows a similar but exaggerated pattern as with plant specific factor intensities. Again relative TFP drops from 1980 to 1982, accounting for the entire change in the Solow residual. But again, the misallocation implies a decline in TFP in 1981 that does not match the increase in Solow residual.

5 Conclusion

This study demonstrated that changes in the extent of misallocation played a large role in the Chilean crisis of 1982. It first documented that, by various measures, the allocation of resources deteriorated during the crisis. It then developed a decomposition to quantify the impact of these changes, and found that misallocation could account for substantial changes in the aggregate Solow residual (although these results are fairly preliminary).

Appendix

A Dispersion and Jensen's Inequality

Relative TFP can be written as $\frac{TFP_s}{A_s} = (\sum_i B_i e^{(\sigma_s-1)(\tau_s-\tau_i)})^{\frac{1}{\sigma_s-1}}$. If factor shares are defined to satisfy $\alpha_{Ks} = \sum \alpha_{Ki} \frac{P_i Y_i}{P_s Y_s}$ then $TFP_s \leq A_s$. In the course of the proof, I will make use of the fact that the revenue shares can be written as

$$\frac{P_i Y_i}{P_s Y_s} = \left(\frac{P_i}{P_s} \right)^{1-\sigma_s} = \frac{(\lambda_i A_i)^{\sigma_s-1} e^{(1-\sigma_s)\tau_i}}{\sum_i (\lambda_i A_i)^{\sigma_s-1} e^{(1-\sigma_s)\tau_i}} = \frac{B_i e^{-(\sigma_s-1)\tau_i}}{\sum_i B_i e^{-(\sigma_s-1)\tau_i}} \quad (3)$$

and similarly $\frac{P_s Y_s}{P Y} = \frac{B_s e^{-(\gamma-1)\tau_s}}{\sum_i B_i e^{-(\gamma-1)\tau_i}}$

The first order condition with respect to capital from the sector level problem

$$\begin{aligned} e^{-\tau_{Ks}} &= \frac{\sigma_s}{\sigma_s-1} \frac{1}{\alpha_{Ks}} \frac{RK_s}{P_s Y_s} \\ &= \frac{\sigma_s}{\sigma_s-1} \frac{1}{\alpha_{Ks}} \sum_i \frac{RK_i}{P_i Y_i} \frac{P_i Y_i}{P_s Y_s} \\ &= \frac{1}{\alpha_{Ks}} \sum_i \alpha_{Ki} e^{-\tau_{Ki}} \frac{P_i Y_i}{P_s Y_s} \end{aligned}$$

where the last equality use the first order condition from the firm's problem and the expression for revenue share (3). Let $\omega_{Ki} \equiv \frac{\alpha_{Ki} P_i Y_i}{\alpha_{Ks} P_s Y_s}$.

$$e^{-\tau_{Ks}} = \sum_i e^{-\tau_{Ki}} \omega_{Ki}$$

Under the assumption that $\alpha_{Ks} = \sum \alpha_{Ki} \frac{P_i Y_i}{P_s Y_s}$, we have $\sum \omega_{Ki} = 1$. Since e^x is convex, Jensen's

inequality gives $E[e^x] \geq e^{E(x)}$, so we can write

$$e^{-\tau_{Ks}} \geq \exp \left[\sum_i [-\tau_{Ki}] \omega_{Ki} \right]$$

Lastly we can write

$$e^{(\sigma_s-1)\alpha_{Ks}\tau_{Ks}} \leq \exp \left[\alpha_{Ks} (\sigma_s - 1) \sum_i \tau_{Ki} \omega_{Ki} \right]$$

We can derive similar expressions for labor and for materials. With these we can write

$$\begin{aligned} e^{(\sigma_s-1)\tau_s} &= e^{(\sigma_s-1)(\alpha_{Ks}\tau_{Ks} + \alpha_{Ls}\tau_{Ls} + \alpha_{Ms}\tau_{Ms})} \\ &\leq \exp \left[(\sigma_s - 1) \sum_i \alpha_{Ks}\tau_{Ki}\omega_{Ki} + \alpha_{Ls}\tau_{Li}\omega_{Li} + \alpha_{Ms}\tau_{Mi}\omega_{Mi} \right] \end{aligned}$$

We can substitute back in the expressions for ω_{Ki} , ω_{Li} , and ω_{Mi} , to get in to get

$$\begin{aligned} e^{(\sigma_s-1)\tau_s} &\leq \exp \left[(\sigma_s - 1) \sum_i (\alpha_{Ki}\tau_{Ki} + \alpha_{Li}\tau_{Li} + \alpha_{Mi}\tau_{Mi}) \frac{P_i Y_i}{P_s Y_s} \right] \\ &= \exp \left[\sum_i (\sigma_s - 1) \tau_i \frac{P_i Y_i}{P_s Y_s} \right] \end{aligned}$$

Using Jensen's inequality one more time, we have

$$e^{(\sigma_s-1)\tau_s} \leq \sum_i \left[e^{(\sigma_s-1)\tau_i} \frac{P_i Y_i}{P_s Y_s} \right]$$

Lastly, using the expression for the revenue share (3), we get

$$e^{(\sigma_s-1)\tau_s} \leq \sum_i \left[e^{(\sigma_s-1)\tau_i} \frac{B_i e^{(1-\sigma_s)\tau_i}}{\sum_i B_i e^{(1-\sigma_s)\tau_i}} \right] = \sum_i \left[\frac{B_i}{\sum_i B_i e^{(1-\sigma_s)\tau_i}} \right] = \frac{1}{\sum_i B_i e^{(1-\sigma_s)\tau_i}}$$

This implies

$$\sum_i B_i e^{(\sigma_s - 1)(\tau_s - \tau_i)} \leq 1$$

Aggregate Level

At the aggregate level we want to show

$$\left(\sum B_s \left(\frac{TFP_s}{A_s} \right)^{\gamma-1} e^{(\gamma-1)(\tau_s - \tau_s)} \right)^{\frac{1}{\gamma-1}} \leq \left(\sum_s B_s \left(\frac{TFP_s}{A_s} \right)^{\gamma-1} \right)^{\frac{1}{\gamma-1}}$$

This can be done if we define $\log \mu = \sum_s \log \mu_s \frac{P_s Y_s}{PY}$ where $\mu_s = \frac{\sigma_s}{\sigma_s - 1}$ and $\mu = \frac{\sigma}{\sigma - 1}$. The capital wedge for the aggregate level can be written as

$$\begin{aligned} e^{-\tau_K} &= \frac{\sigma}{\sigma - 1} \frac{1}{\alpha_K} \frac{RK}{PY} \\ &= \frac{\sigma}{\sigma - 1} \frac{1}{\alpha_K} \sum_i \frac{RK_s}{P_s Y_s} \frac{P_s Y_s}{PY} \\ &= \frac{1}{\alpha_K} \sum_i \alpha_{Ks} e^{-\tau_{Ks} + \log \frac{\mu}{\mu_s} \frac{P_s Y_s}{PY}} \\ &= \sum_i e^{-\tau_{Ks} + \log \frac{\mu}{\mu_s} \omega_{Ks}} \end{aligned}$$

where $\omega_{Ks} = \frac{\alpha_{Ks} P_s Y_s}{\alpha_K PY}$. With the assumptions that $\alpha_K = \sum_s \alpha_{Ks} \frac{P_s Y_s}{PY}$, we have $\sum_s \omega_{Ks} = 1$.

$$e^{-\tau_K} \geq \exp \left[\sum_s -\tau_{Ks} + \log \left(\frac{\mu}{\mu_s} \right) \omega_{Ks} \right]$$

Lastly we can write

$$e^{(\gamma-1)\alpha_K \tau_K} \leq \exp \left[\alpha_K (\gamma - 1) \sum_s \left(\tau_{Ks} + \log \left(\frac{\mu_s}{\mu} \right) \right) \omega_{Ks} \right]$$

We can derive similar expressions for labor and for materials. With these we can write

$$\begin{aligned}
e^{(\gamma-1)\tau} &= e^{(\gamma-1)(\alpha_K\tau_K + \alpha_L\tau_L + \alpha_M\tau_M)} \\
&\leq \exp \left[(\gamma-1) \sum_s \alpha_K\tau_{Ks}\omega_{Ks} + \alpha_L\tau_{Ls}\omega_{Ls} + \alpha_M\tau_{Ms}\omega_{Ms} + \log \left(\frac{\mu_s}{\mu} \right) (\alpha_K\omega_{Ks} + \alpha_L\omega_{Ls} + \alpha_M\omega_{Ms}) \right]
\end{aligned}$$

We can substitute back in the expressions for ω_{Ks} , ω_{Ls} , and ω_{Ms} , to get in to get

$$\begin{aligned}
e^{(\gamma-1)\tau} &\leq \exp \left[(\gamma-1) \sum_s (\alpha_{Ks}\tau_{Ks} + \alpha_{Ls}\tau_{Ls} + \alpha_{Ms}\tau_{Ms}) \frac{P_s Y_s}{PY} + \log \left(\frac{\mu_s}{\mu} \right) (\alpha_{Ks} + \alpha_{Ls} + \alpha_{Ms}) \frac{P_s Y_s}{PY} \right] \\
&= \exp \left[(\gamma-1) \sum_s \left(\tau_s + \log \left(\frac{\mu_s}{\mu} \right) \right) \frac{P_s Y_s}{PY} \right]
\end{aligned}$$

Also, under the assumption that $\log \mu = \sum_s \log \mu_s \frac{P_s Y_s}{PY}$, this becomes

$$e^{(\gamma-1)\tau} \leq \exp \left[\sum_s (\gamma-1) \tau_s \frac{P_s Y_s}{PY} \right]$$

Using Jensen's inequality one more time, we have

$$e^{(\gamma-1)\tau_s} \leq \sum_s \left[e^{(\gamma-1)\tau_s} \frac{P_s Y_s}{PY} \right]$$

Using $\frac{P_s Y_s}{PY} = \frac{X_s e^{-(\gamma-1)\tau_s}}{\sum X_s e^{-(\gamma-1)\tau_s}}$

$$\begin{aligned}
e^{(\gamma-1)\tau_s} &\leq \sum_s \left[e^{(\gamma-1)\tau_s} \frac{X_s e^{-(\gamma-1)\tau_s}}{\sum X_s e^{-(\gamma-1)\tau_s}} \right] \\
&= \frac{1}{\sum X_s e^{-(\gamma-1)\tau_s}}
\end{aligned}$$

Rearranging gives

$$\left(\sum X_s e^{(\gamma-1)(\tau-\tau_s)}\right)^{\frac{1}{\gamma-1}} \leq 1$$

$$X_s = \frac{B_s \left(\frac{TFP_s}{A_s}\right)^{\gamma-1}}{\sum_s B_s \left(\frac{TFP_s}{A_s}\right)^{\gamma-1}}$$

$$\left(\sum B_s \left(\frac{TFP_s}{A_s}\right)^{\gamma-1} e^{(\gamma-1)(\tau-\tau_s)}\right)^{\frac{1}{\gamma-1}} \leq \left(\sum_s B_s \left(\frac{TFP_s}{A_s}\right)^{\gamma-1}\right)^{\frac{1}{\gamma-1}}$$

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