Foreign Firms and the Diffusion of Entrepreneurial Skills*

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Abstract

Foreign firms may enhance a developing country's formation of know-how by exposing or directly transferring local entrepreneurs the productive ideas of developed countries. However, foreign firms may also reduce the domestic entrepreneurs' incentive to accumulate know-how by increasing their competition and reducing the returns to entrepreneurial skills. It is shown that if externalities drive the formation of skills, after openness, initial conditions determine if a country converges to one of two steady states or to exhibit non'monotone dynamics. If instead, the costs and benefits of skill formation are fully internalized, openness gradually removes the pre-existing sector, generates a new sector of domestic firms, and the country catches up with developed countries. In both models, convergence requires the destruction of pre-existent firms. The implications for empirical work are also discussed.

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1. Introduction

Would hosting foreign firms lead a developing country to catch up with developed countries? Can this form of openness instead lead the country to lag further behind? Does the presence of foreign firms enhance or impair the accumulation of productive know-how by domestic firms? What is the impact of this openness on the overall welfare of a country? Should the governments of developing nations promote the entry of foreign firms? In this paper I use simple general equilibrium growth models to answer these questions.

Entrepreneurial and managerial know-how can be the limiting factor in a country's aggregate productivity. They determine the technologies and market opportunities that local firms can efficiently operate and access.[1] Broadly defined, these "skills" can be as simple as knowing key individuals and conventions of a particular market and as sophisticated as the scientific and technological training to coordinate the development, selection and marketing of a new slow-release drug. Like other skills, entrepreneurial know-how is self-productive. Countries with an initial poor supply may never, by themselves, accumulate the amounts required to access the world's technological frontier.

A country can import entrepreneurial and managerial skills from abroad by allowing foreign firms to operate in the country. Foreign firms can combine their control of domestic labor with know-how that is only available elsewhere in the world. Indeed, this form international trade of skills seems to have gained importance with the increasing multinational activity and foreign direct investment of recent years. Burstein and Monge-Naranjo (2009) quantify significant output and consumption gains for developing countries that host firms from developed countries.[2] Moreover, Antras, Garicano and Rossi-Hansberg (2006) show that the distributional impact of foreign skills go well beyond those implied by standard factor endowment models because they alter the organization of production and the within occupations distribution of income. Yet, by taking skills as fixed endowments, these papers are silent about their accumulation over time and across countries.

Aside from static output gains, the presence of foreign firms can impact the acquisition of the country's own entrepreneurial and managerial know-how. Diffusion of productive know-how can take place via externalities, as the exposure to new and possibly more advanced ideas from abroad facilitates domestic entrepreneurs in their accumulation of know-how. Diffusion of productive ideas can also take place via implicit or explicit fully internalized transactions as a foreign expert trains

[1] There is an extensive literature that links the productivity of firms to the quality of their management. See, for example, Kaldor (1934), Lucas (1978), Oi (1983), Prescott and Visscher (1980), and Rosen (1982).

[2] Ramondo (2008) and McGrattan and Prescott (2008) show that the gains would be even larger if, instead of skills, productivity is driven by non-rival factors, i.e. that can be used simultaneously in multiple locations.

a local future firm manager. However, the presence of foreign firms increase the competition faced by local managers and reduce their total and marginal returns to entrepreneurial skills. This paper studies the aggregate dynamics and welfare consequences for a developing country that opens up to foreign firms.

In the model, production is carried out by teams of workers led by an entrepreneur.[3] The skills of the manager determines the productivity of the team. The model is an OLG economy in which some of the young individuals invest in skills to become the manager –and residual claimant– of a firm when old. Over time, the equilibrium skill formation of young future managers is determined by the set of productive ideas currently implemented by active managers operating in the country. Workers are fixed in their country of origin but managers can move across countries.[4] In the context of the model, a "closed" country is one in which only national managers can lead firms while an "open" country allows free entry of foreign firm leaders.[5] Entry of foreign managers impacts they country by increasing the domestic price of labor, reducing the marginal return to entrepreneurial skills and increasing the set of productive ideas upon which the young can form their future skills.

The presence of externalities is a standard assumption in models of human capital formation, e.g. Romer (1986), Lucas (1988) and Stokey (1991). I use a variant of Stokey (1991) in which the aggregate skills level of old managers impacts the cost of accumulating skills of young potential managers. In this variant of the model, the set of ideas that surround individuals during their formative years is a national public good shaped by all skills effectively used within the country. There is an externality because firm leaders only receive returns for their production activities, and not for their contribution to the ideas circulating in the country. Equally, the presence of foreign firms has an externality that impacts the national public good in an open economy. Hence, the framework integrates quite naturally the mechanism in Findlay (1978), in which foreign firms have positive technology spillovers on domestic firms.

Regardless of initial conditions, closed economies always follow a balanced-growth-path (BGP). A small open economy exhibits a significantly more complex dynamics. Openness to foreign firms has countervailing forces as it reduces both the marginal costs and the marginal returns for the accumulation of skills of domestic entrepreneurs. On one hand, the current entry of foreign firms

[3] In the model entrepreneurial and managerial know-how are equivalent. For models in which they are different see Holmes and Schmitz (1991) and Chari, Golosov, and Tsyvinski (2004).

[4] See Klein and Ventura (2004) for an analysis of cross country labor mobility.

[5] The emphasis on the cross-border reallocation of management conforms with the observation that multinational firms heavily rely on home expatriates –and home trained individuals– to manage their operations, specially in developing countries (see Chapters 5 and 6 of UNCTAD 1994). It also conforms with the emphasis of the existent literature on firm specific intangible assets for multinational activity (e.g. Barba-Navarretti 2004 and Markusen 2004).

enhances the set of productive ideas surrounding the forming crop of young managers. On the other hand, the foreseen future entry of foreign firms bids up the domestic cost of labor and squeezes total and marginal returns to entrepreneurial skills. As a result, open economies exhibit a form of predatory-prey dynamics because entry is an inverse function of the relative level of domestic entrepreneurial skills. Indeed, an open country can exhibit non-monotone dynamics and there might be two different steady states to which the country can converge depending on initial conditions. In one steady state, the country converges to the skill levels of developed countries (and foreign firms no longer enter). In the other steady state, the country remains forever behind (and foreign firms are always present). It is shown that, regardless of the path followed by the country, it catches up with developed countries if and only if for one period domestic firms are shut-down and the entire production of the country is led by foreign firms. The model also predicts leapfrogging, as the most backward countries not only are more likely to converge to the high steady state, but also to do it more quickly. Finally, it is shown that the extent in which a country gains with openness depends on initial conditions.

Another leading hypothesis for the formation of know-how is that it results from transactions that fully internalize the costs and benefits of all parties involved, e.g. Boyd and Prescott (1987a,b), Chari and Hopenhayn (1991) and Jovanovic and Nyarko (1995).[6] I consider a variant of the Boyd-Prescott model in which the skill formation of a young manager depends on the skills and actions of the leader for whom he works. Both trainers and apprentices must be purposely involved and a well functioning market for skill formation operates.[7] No externalities are present, since the skill formation of each young future manager depends solely on the skills of his manager/trainer. Instead of an externality, foreign firms disseminate skills to the host country by directly training the workers under their control.

The internalization of the returns of skills in future production of skills can easily lead to dynamic increasing returns and degenerate solutions.[8] I provide sufficient conditions for existence, uniqueness and efficiency of a BGP in a closed economy.[9] In an open economy, the entry of more advanced foreign firms introduces heterogeneity in the population of trainers, which in turn creates heterogeneity of domestic firms. Specifically, openness leads to the emergence of a sector of new-domestic entrepreneurs all of which have the same skill levels of foreign managers. This new sector grows over time and eventually overtakes the entire economy. Along the transition, the progeny of the pre-existing domestic managers shrinks in size and level of skills and then disappears.

[6] Agarwal, R. et al (2004) and Filson and Franco (2006) extend the Chari-Hopenhayn model.

[7] Abstracting from private information frictions that lead to inefficiency as.in Jovanovic and Nyarko (1995).

[8] Recall that the profit function in a Lucas (1978) is strictly convex. The self-productivity of skills adds to the convexity of the value function.

[9] The conditions in Boyd and Prescott (1987) and Prescott and Boyd (1987) are not enough for this result.

Regardless of its initial relative backwardness, an open country fully catches up with developed countries a finite number of periods after opening. Also, regardless of initial conditions, openness is always welfare improving.

The implications of both models complement the negative empirical results on the positive effect of foreign firms on the productivity of existing domestic firms. As in the model with externalities, with internalized diffusion a developing country catches up with developed countries if and only when domestic pre-existent firms (or their progeny) are entirely removed and replaced by a new sector of domestic firms. In both models, the country as a whole can catch up, even if pre-existing firms respond *reducing*, not increasing, the investments that drive their productivity levels. This implication is in line with ample evidence (e.g. Aitken and Harrison [1999]) on the absence of positive effects of foreign firms on the productivity of existing domestic firms.[10] But even if spillovers drive aggregate productivity, it is shown that it is quite likely that the negative impact of foreign competition overdoes the positive effect of spillovers.

The remarkable differences in the implications of the two models highlight the importance of distinguishing between externalities and fully internalized transfers of know-how. Some authors (e.g. Javorcik [2004] and Kugler [2005]) have argued for the existence of inter-industry spillovers, specifically, from foreign firms to local suppliers. However, productivity gains might be driven by internalized transfers, not spillovers, since as Javorcik herself reports, foreign firms in her sample were directly involved providing training, equipment and know-how to the local suppliers. At the level of domestic industries, skill formation at the interior of the firm seems to be a major mechanism for aggregate skill formation and dissemination, as indicated by the empirical evidence that links the characteristics and the outcomes of parent firms with their spin-offs.[11] Unfortunately, with the exception of the recent work of Malchow-Møller et. al (2007) and Poole (2006) on multinational firms in Denmark and Brazil, respectively, there has been little empirical work on internalized skill formation by multinational firms.[12]

The paper proceeds as follows. Section II lays out the model and equilibrium preliminaries. Sections III considers the equilibrium when geography defines the scope of learning and Section IV when the case of intrafirm learning. Section V developes the general case. Section VI concludes.

[10] See Xu (2000) and Alfaro et al. (2006) for a discussion of the empirical findings. While there is some evidence of positive spillovers for developed countries (e.g. Griffith et al 2002), for developing countries most authors come back empty-handed when trying to revert the negative results of Aitken and Harrison (1999).

[11] For the U.S. car industry, Keppler (2001, 2002, 2006) documents that the genesis of the most successful car makers can be traced to former employees of other car makers. Agarwal et al (2004), Filson and Franco (2006) and Franco (2005) show the same for the rigid disk drive industry.

[12] There is however, ample anecdotal. The best known case may be the emergence of a textile sector in Bangladesh after the seed planted by a Korean firm (see Easterly [2001]). Also, some multinationals spend significant resources in training their workers (see UNCTAD [1994]).

2. The Model

Consider a discrete time, infinite horizon OLG economy with a single consumption good and individuals that live for two periods. The utility of an individual born at time t that consumes c_t^t and c_{t+1}^t in periods t and $t+1$ is

$$U^t = c_t^t + \beta c_{t+1}^t,$$

where $0 < \beta < 1$.

The size (measure) of all generations is equal to one. In each period of their lives individuals have a unitary time endowment. When young, this time endowment is supplied as labor; when old, individuals have the option of using the time endowment to become "entrepreneurs", i.e. set up and control a firm. Entrepreneurship is just an option as old individuals can opt to remain workers. When young, individuals forsee the value of these options and decide whether and how much to invest in acquiring entrepreneurial skills.

As in Lucas (1978), the consumption good is produced by "firms", teams of one entrepreneur (or manager) and a group of workers. The entrepreneur is the residual claimant of the single firm he sets up and manages.[13] The (person-specific) skills or knowledge z of the entrepreneur determines the productivity of the firm under his control. With entrepreneurial skills z and n units of labor, a firm produces

$$y = zn^{\alpha},$$

units of the consumption good. The span-of-control parameter $\alpha \in (0,1)$ is the degree of decreasing returns to the amount of labor n.[14]

The focus of this paper is on the accumulation of the entrepreneurial skills over time. In contrast to much of the existing span-of-control models that take them as exogenously given, I assume that entrepreneurial skills are generated by net lifetime income maximizing investments. The returns to skills are in the form of the next period net-profits of a firm. The cost of investing in skills is determined by the ideas that the young person is exposed to while young. I now proceed to explain these elements.

The exposure to ideas of a young person, denoted $z^E \geq 0$, subsumes the contributions of two sources: (i) the productive know-how z that runs the firm where the individual is a worker, and (ii) an average Z^O of the know-how of all the entrepreneurs actively operating inside the country. To

[13] This formulation of equilibrium is equivalent to one in which firms with constant returns to scale (and zero-profits in equilibrium) are the ones hiring "managerial" services from the entrepreneurs. For a model that distinguishes between the economic functions of entrepreneurs and managers, see Holmes and Schmitz (1994).

[14] As Lucas (1978), I call these teams "firms" even if they can equally be seen as parts of a conglomerate of teams within the boundaries of the same firm. However, see Garicano (2000), Oi (1983) and Rosen (1982) for related issues.

this end, I assume that $z^E = F\left(z, Z^O\right)$,where $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ is a positive and linearly homogeneous function that is increasing and twice differentiable in both arguments. For much of the analysis, I restrict attention to the Cobb-Douglas case

$$z^E = (z)^\gamma \left(Z^O\right)^{1-\gamma}, \tag{2.1}$$

where $0 \leq \gamma \leq 1$ is a single parameter that determines how much a person learns inside his own job and how much he learns from outside.

The average Z^O of ideas outside each individual's firm is a national "public good", a non-rival factor to which everyone in the country can access freely.[15] Let μ_z be the probability measure (to be determined endogenously) that indicates the allocation of the country's total labor across the firms with different know-how levels. That is, for any Borel set $B \subset \mathbb{R}_+$, $\mu_z(B)$ indicates the share of the labor in control of entrepreneurs with know-how levels in B. Then, Z^O is given by a generalized (or Hölder) weighted mean of all the active firms:

$$Z^O = \left[\int_{\mathbb{R}_+} (z)^\rho \mu_z(dz)\right]^{\frac{1}{\rho}}, \tag{2.2}$$

where the parameter ρ can assume any value in the extended real numbers. This formulation encompasses many different ways of averaging the the know-how levels of active firms. If $\rho \to -\infty$, Z^O is the minimum value (Leontieff function); if $\rho \to \infty$, it is the maximum value. The arithmetic, geometric and harmonic means take place when, respectively, $\rho = 1, 0, -1$.

Given a value of z^E, the cost (in terms of current consumption) for a young individual of acquiring a next-period level of skills $z' \geq 0$ is equal to $z^E \phi\left(z'/z^E\right)$, where $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ is a non-negative, continuously differentiable and strictly convex function with $\lim_{x\to 0} \phi(x) = \phi'(x) = 0$ and $\lim_{x\to\infty} \phi(x) = \phi'(x) = \infty$. Therefore, the total and marginal cost of investing are strictly increasing in z' and strictly decreasing in z^E. It is convenient to focus on the special case

$$\phi\left(\frac{z'}{z^E}\right) = \frac{v_0}{1+v}\left(\frac{z'}{z^E}\right)^{1+v}, \tag{2.3}$$

where v_0, $v > 0$. For some of the formulas below, it is convenient to keep $\phi(\cdot)$ as a shorthand.

The parameters ρ, v and γ are key for the diffusion of know-how. The *curvature* parameter v determines the impact of z^E on the costs of acquiring z'; therefore, it will play an important role for cross-countries and cross-firms differences in know-how. The *diffusion* parameter ρ determines how easily superior ideas diffuse inside a country. The higher the value of ρ, the higher the impact of superior ideas on the common pool Z^O. In the extreme, if $\rho = +\infty$, only the very best of all

[15] Skills are rival factors but ideas are non-rival. since their use by one young entrepreneur in forming his own skills does not crowd out the use of others. precluse another from using them. This interplay between the formation of skills and the diffusion of ideas harmonizes two seemingly incompatible notions of "knowledge" in the literature.

the ideas are considered in Z^O. In the opposite extreme, a value $\rho = -\infty$, implies that only the lowest idea is understood by everyone.

Most interestingly, however, is the *internalization* parameter γ. By allowing any $0 \leq \gamma \leq 1$, the model encompasses two widely used –but conflicting– views of the accumulation and diffusion of knowledge. On one hand, if $\gamma = 0$, then a common value $z^E = Z^O$ holds for everyone and externalities are the only engine of accumulation and diffusion. Such assumption has a dominant presence in the literature on growth (e.g. Romer 1986, and Lucas 1988), the impact of openness to trade on growth (e.g. Stokey 1991) and the impact of openness to multinational firms (e.g. Findlay 1978). On the other hand, if $\gamma = 1$, then the exposure to ideas –and hence, the ability to accumulate skills– are uniquely determined by one's own firm, giving rise to a richer relationship between young and old entrepreneurs, one that fully internalize the costs and benefits of accumulating skills. Such is the view in Boyd and Prescott (1987a,b), and work on diffusion as in Chari and Hopenhayn (1991), Jovanovic and Nyarko (1995), and others.[16] By allowing any $0 \leq \gamma \leq 1$, the model here combines the impact of externalities with labor markets that internalize the learning opportunities offered by firms with different skill levels, which, as I will show, is critical dimension for the diffusion of foreign know-how.

3. The Formation and Diffusion of Skills

In this section I construct some objects needed to define a competitive equilibrium, explain the notions of "closed" and "open" economies, and characterize the equilibrium skill accumulation in both.

3.1. Equilibrium preliminaries

I consider perfect foresight competitive equilibria. In a closed economy, only domestic entrepreneurs can set up firms and hire local labor. In an open economy, foreign entrepreneurs are allowed free entry. In either case, the discount factor β pins down the interest rate (with which all future payoffs are discounted) and the key component of the price system is a sequence of wage functions $\{w_t : \mathbb{R}_+ \to \mathbb{R}_+\}_{t=0}^{\infty}$. The wage $w_t(z)$ indicates the price that an entrepreneur with skills z must pay for a unit of labor at time t. The dependence of wages on the skills of the entrepreneur is driven by the learning opportunities offered by the firm under his control.

The economic decisions of an old person are first, whether to remain a worker or become an entrepreneur, and, second, if an entrepreneur, how much labor to hire. As in Lucas (1978), this occupation choice is made on the basis of the individual's own skills z and the wage $w(z)$

[16] See also the extensions of the Chari-Hopenhayn model by Agarwal, R. et al (2004) and Filson and Franco (2006) as well as Dasgupta (2010).

associated with his skills z. Should he become an active entrepreneur, his net rents would be

$$\begin{aligned}\pi\left[z, w\left(z\right)\right] &\equiv \max_{\{n\}}\left\{zn^{\alpha} - w\left(z\right)n\right\} \\ &= \theta z^{\frac{1}{1-\alpha}}\left[w\left(z\right)\right]^{\frac{-\alpha}{1-\alpha}}, \end{aligned} \tag{3.1}$$

where $\theta \equiv \left(1-\alpha\right)\alpha^{\frac{\alpha}{1-\alpha}} > 0$. Given $w\left(z\right)$, $\pi\left[z, w\left(z\right)\right]$ is strictly increasing and convex in z, and strictly decreasing in $w\left(z\right)$. His optimal demand for labor would also be increasing in z and decreasing in $w\left(z\right)$:

$$n^{*}\left[z, w\left(z\right)\right] = \left[\frac{\alpha z}{w\left(z\right)}\right]^{\frac{1}{1-\alpha}}. \tag{3.2}$$

Therefore, the individual would become an active entrepreneur if and only if the rents $\pi\left[z, w\left(z\right)\right]$ dominate the entire schedule of wages available to workers, which is determined by the function $w\left(\cdot\right)$ and the set of active firms operating in the country.

The economic decisions of a young person are first, selecting the firm for which to work, and second, whether and how much to invest in entrepreneurial skills. With respect to the second decision, given the exposure of ideas z^{E} and the next period's cost of labor $w_{t+1}\left(\cdot\right)$, the optimal investment in entrepreneurial skills z' solves

$$V\left[z^{E}, w_{t+1}\left(\cdot\right)\right] \equiv \max_{z'}\left\{\beta\pi\left[z', w_{t+1}\left(z'\right)\right] - z^{E}\phi\left(z'/z^{E}\right)\right\}. \tag{3.3}$$

The two key variables for the optimal z' are z^{E} and $w_{t+1}\left(\cdot\right)$. Under the conditions laid out in Proposition 1 below, optimal investments in skills are determined by the condition

$$\beta\left[\pi_{1}\left(z', w_{t+1}\left(z'\right)\right) + \pi_{2}\left(z', w_{t+1}\left(z'\right)\right)\frac{\partial w_{t+1}\left(z'\right)}{\partial z'}\right] = \phi'\left(\frac{z'}{z^{E}}\right), \tag{3.4}$$

where $\pi_{1}\left(\cdot\right)$ and $\pi_{2}\left(\cdot\right)$ stand for, respectively, the first derivative of π with respect to the the skill z or the manager and the wage $w_{t+1}\left(z\right)$ he will have to pay for labor. A better exposure of ideas (i.e. a higher z^{E}) reduces directly the total and marginal costs of investment and lead to a higher z'. The function $w_{t+1}\left(\cdot\right)$ impacts the optimal choice z' in two ways. The first is the direct impact on entrepreneurial rents as higher values of $w_{t+1}\left(\cdot\right)$ reduce those rents for all levels of z'. Second, a negatively-sloped $w'\left(\cdot\right)$ reduce the cost of labor for the more skilled entrepreneurs and increase their ex-post rents. As I discuss below, if z^{E} is low, a corner solution of zero investments might be optimal.

Finally, since all young individuals are identical, in equilibrium, all workers must be indifferent to work for the different active firms as wages must compensate for differences in learning opportunities. Two active firms with arbitrary know-how levels $z_{0} < z_{1}$ pay wages that satisfy:

$$w_{t}\left(z_{0}\right) - w_{t}\left(z_{1}\right) = V\left[z_{1}^{E}, w_{t+1}\left(\cdot\right)\right] - V\left[z_{0}^{E}, w_{t+1}\left(\cdot\right)\right], \tag{3.5}$$

where $z_0^E = (z_0)^\gamma \left(Z^O\right)^{1-\gamma} < z_1^E = (z_1)^\gamma \left(Z^O\right)^{1-\gamma}$. Clearly, less skilled managers must pay higher wages as the right-hand-side of this equation is positive.

Occupation choice, i.e. the option of old individuals to foresake entrepreneurship and remain workers can completely reshape the set of possible steady states (balance-growth-path equilibria) and/or the transition dynamics of open and closed economies. However, for clarity of exposition, it is convenient to defer their analysis to a separate section (Section 4). Therefore, this section proceeds under the assumption that all the individuals are entrepreneurs when old and all of them invest in skills when young following (3.4).

3.2. Skill formation in closed economies

Consider a closed economy. Since, only the domestic old can supply the entrepreneurial skills and operate firms in the country, they are the sole source of entrepreneurial ideas for the domestic young generation. Let the probability measure ϕ_z^t describe the distribution of skills of the old generation in period t. Assume that ϕ_z^t has a bounded, strictly positive support $[z_L^t, z_H^t] \subset \mathbb{R}$. Given wages $w_t(z)$, the amount of labor $n_t^*(z)$ hired by an entrepreneur with skill level z is given by (3.2), and the distribution of labor employed across skill levels is given by $\mu_z^t(B) = \left(\int_B n_t^*(z)\,\phi_z^t(dz)\right) / \int_{z_L^t}^{z_H^t} n_t^*(z)\,\phi_z^t(dz)$ for any Borel set $B \subset [z_L^t, z_H^t]$. With μ_z^t thus determined, the external exposure to external ideas, Z_t^O, is determined as in expression (2.2), and the total exposure z_t^E of ideas of a young working in firm with know-how z is $(z)^\gamma \left(Z_t^O\right)^{1-\gamma}$.

To examine the accumulation of skills, deriving expression (3.1), $\pi_1 = [\alpha z / w(z)]^{\frac{\alpha}{1-\alpha}}$, and $\pi_2 = -[\alpha z / w(z)]^{\frac{1}{1-\alpha}}$ and plug them in (3.4). Then, from equilibrium condition (3.5), it follows that $\frac{\partial w_{t+1}(z')}{\partial z'} = -V_1\left[\left(z^E\right)', w_{t+2}(z'')\right]\frac{\partial\left(z^E\right)'}{\partial z'}$, where V_1 is the derivative of V with respect to the first argument, the exposure to ideas. Here $\left(z^E\right)'$ and (z'') are, respectively, the the next period's exposure of ideas and the investments for the workers of the forming entrepreneur. Finally, the envelope condition of (3.3) implies that

$$\begin{aligned} V_1\left[\left(z^E\right)', w_{t+2}(z'')\right] &= -\phi\left(\frac{z''}{\left(z^E\right)'}\right) + \left(\frac{z''}{\left(z^E\right)'}\right)\phi'\left(\frac{z''}{\left(z^E\right)'}\right), \\ &= -\frac{v v_0}{1+v}\left(\frac{z''}{\left(z^E\right)'}\right)^{1+v}, \end{aligned}$$

where the latter equality uses the functional form assumed for $\phi(\cdot)$. With all of this, after simplifying, equation (**??**) becomes

$$\beta\left(\frac{\alpha z'}{w'(z')}\right)^{\frac{\alpha}{1-\alpha}}\left[1 + \frac{\alpha v v_0 \gamma}{(1+v)}\frac{(z')^\gamma \left(Z^{O'}\right)^{1-\gamma}}{w'(z')}\left(\frac{z''}{\left(z^E\right)'}\right)^{1+v}\right] = v_0\left(\frac{z'}{z^E}\right)^v. \tag{3.6}$$

On the basis of this condition, the appendix contains the proof of the following result:

Lemma 1. *The equilibrium wage function $w_t(z')$, if it exists, is non-increasing. For positive values Z^O, $\left(Z^O\right)'$ assume that a function $z'(z)$ indicates the optimal investments in skills z' given current z. Then, if $v > \alpha/(1-\alpha)$,[17] the function $z'(\cdot)$ is strictly increasing. Additionally, if $\gamma > 1 - \frac{\alpha}{(1-\alpha)v}$, then $z'(z_1)/z'(z_0) > z_1/z_0$ for any $z_1 > z_0$.*

Albeit limited, this simple result has important implications for the limiting behavior of the skill distribution. Most interestingly, if $\gamma > 1 - \alpha/[(1-\alpha)v]$, i.e. one's own manager is a leading source of ideas, then pre-existing differences in the exposure to ideas will lead to widening gaps in skill formation. But precisely because of this, as explained below, asymptotically the economy converges to a homogeneous pool of entrepreneurs**.**

Regardless of the value of γ, if the initial generation of entrepreneurs is homogenous, so will be all the subsequent generations. To see this, assume that all old individuals posses the same skill level level $Z > 0$. Then, in equilibrium, all the young individuals must receive the same wage w and are exposed to the same level of ideas $z^E = Z$, and will invest the same amount in skills, Z'. In the next period, their own workers will be exposed to the same $\left(z^E\right)' = Z'$, and will receive the same wage w'. And so on. In every period, all firms will hire the same amount of labor $n^* = 1$, market-clearing wages and earnings for entrepreneurs are $w_t = \alpha Z_t$ and $\pi_t = (1-\alpha) Z_t$. Imposing these conditions, equation (3.6) implies that the growth in skills $G_t \equiv Z_{t+1}/Z_t$ must satisfy the difference equation

$$\beta\left[1 + \frac{\gamma v v_0}{(1+v)}\left(G_{t+1}\right)^{1+v}\right] = v_0\left(G_t\right)^v. \tag{3.7}$$

A balance growth path (BGP) is an equilibrium in which skills grow at a constant rate, i.e. $G_t = G > 0$. For this, G must be a root of the equation (3.7) when it is restricted to $G_{t+1} = G_t = G$. For the existence of a BGP, the cost of acquiring skills must be high and convex enough for at least one root to exist. Otherwise, the marginal revenue is always above the marginal cost of investing and skill accumulation would be degenerate. As detailed in the appendix, there are at most two roots; if so, the highest root is not relevant because (3.7) is an optimality condition; hence, the left-hand-side (the marginal revenue) must cross the right-hand-side (the marginal cost) from above. Finally, self-fulfilling (extrinsic) fluctuations are ruled out precisely because the marginal revenue must cross the marginal cost from above.

Therefore, the existence of an equilibrium BGP with homogeneous firms can be established. These statements are summarized in the following proposition which is proved in the appendix.

Proposition 1. (Closed economy BGP) *For a closed economy:* ***(a)*** *An equilibrium BGP with homogeneous skills exists if either (i) $\gamma > 0$ and $\left[v_0/(1+v)\right]^{\frac{1}{1+v}}/\gamma < \beta \leq \left(v_0/\left[\gamma^v(1+v)\right]\right)^{\frac{1}{1+v}}$ or*

[17] This assumption is needed for (3.3) to have a non-zero solution. Given $w(z)$, the function π is strictly convex in z with an elasticity of $1/(1-\alpha)$. Given z^E, the elasticity of ϕ wrt to z' is $1+v$; the condition $v > \alpha/(1-\alpha)$ ensures that the latter is 'more convex' which is needed for an interior solution.

(ii) $\gamma = 0$ *and* $\beta < [v_0(1+v)]^{\frac{1}{1+v}}$; ***(b)*** *if an equilibrium BGP exists it is unique ;* ***(c)*** *the economy exhibits sustained growth, i.e.* $G > 1$ *if* $\beta > v_0(1+v)/(1+v+vv_0\gamma)$*;* ***(d)*** *if either condition in (a) holds and initially the economy is populated by homogeneous entrepreneurs, then the only equilibrium is the BGP; other non-explosive fluctuations in* G_t *are ruled out;* ***(e)*** *if either* $\gamma > 1 - \alpha/[(1-\alpha)v]$ *or* $\gamma = 0$*, then, from any initial distribution with bounded support the economy will asymptotically converge to a BGP; if* $\gamma = 0$ *the convergence is after just one period.*

As advanced previously, part (e) suggests that heterogeneity may not be sustained in a closed economy. Most obviously, if $\gamma = 0$, regardless of how disperse the initial distribution of skills is, all the young are exposed to exactly the same $z^E = Z^O$, and after just one period, any pre-existing heterogeneity will disappear and the economy will be forever after in the homogeneous skills BGP. More interestingly, along the lines of Lemma 1, if $\gamma > 1 - \alpha/[(1-\alpha)v]$ the economy converges to a homogeneous entrepreneurs precisely it progressively widens the productivity gaps across the different entrepreneurs. Because of this, the top end of the distribution reproduces at a faster pace than the lower end, and in the limit, all the remaining entrepreneurs would be the offsprings of the initially highest entrepreneur(s).

For the remainder of the analysis, I will assume the parameter restrictions in Proposition 1 (b), (c) and (e), and use the homogeneous skills closed economy BGP both, as the initial condition at the time a country opens and also as the benchmark to assess the gains from openness.

3.3. The diffusion of foreign know-how in open economies

Consider now a country that freely allows old foreign entrepreneurs to enter the country, set up firms and hire domestic labor.[18] I use, respectively, the names "home" and "foreign", and the indexes h and f, for the host country and for the rest of the world. I assume that initially both home and foreign are in the BGP as described in Proposition 1 and that at time $t = 0$ openness takes place unexpectedly and permanently. In addition, I assume that home is *less developed*, i.e. at time $t = 0$, $Z_h < Z_f$. Finally, I assume home is *small,* i.e. it does not affect the equilibrium path of the foreign country.

The entry of foreign entrepreneurs impacts the accumulation of skills at home in three ways. First, foreign firms enhance the exposure to ideas of the domestic young directly working for them. Second, foreign firms have spillovers on the country's value of Z^O which benefit all the local young, including those working for domestic firms. While these two effects are positive a third one is detrimental: foreign entrepreneurs bid up future cost of labor $w_{t+1}(\cdot)$ in the country.

[18] I will assume that individuals from the home country cannot move. This is without loss of generality for workers and old entrepreneurs, since, in equilibrium they will be indifferent between moving to foreign or remaining in home. However, ruling out the possibility for domestic young potential entrepreneurs to move and "grow up" in the developed country is crucial. I will discuss further below the factual and analytical relevance of this assumption.

A free entry condition endogenously determines the mass of foreign know-how in the country. Foreign entrepreneurs would enter until they are indifferent between operating at home or remaining in the foreign country. Since in the foreign country they earn $\pi_f = (1-\alpha) Z_f$, and there are no mobility frictions, such a indifference can only happen when the (effective) cost of labor is the same in both countries.[19] Therefore, with openness, the domestic market-clearing wages for foreign firms must be

$$w_h = w_f = \alpha Z_f. \tag{3.8}$$

Facing the same effective wages, each foreign firm hires the same number of labor units, $n_f^* = 1$, as if they had remained in the foreign country.[20][21]

The entry of foreign skills can give rise to an interesting vintage structure for the population of domestic firms. To see this, Table 1 illustrates the diffusion of foreign skills into a country that opens up at time $t = 0$. Because skills grow at a rate $G > 1$, a BGP would exhibits a constant distribution of ratios of domestic-to-foreign skills ratios, $R \equiv z/Z_f$. I will carry out this discussion in terms of the ratios denoting by $j = 0, 1, 2, ...$ the different generations of domestic offspring of foreign entrepreneurs. That is, $j = 0$ indicates the foreign firm itself, $j = 1$ indicates the generation of domestic entrepreneurs that worked for a foreign firm when young, $j = 2$ the domestic entrepreneurs that worked for a foreign-trained domestic entrepreneur when young, $j+1$ those who were trained by a member of generation j, etc. The ratio R_j^t indicates the skills, relative to skills in the foreign country, Z_f^t, of a generation j of domestic firms at time t.

For every period $t \geq 0$, a mass of foreign firms enters, each one carrying skills Z_f^t, the same skills as those firms that remained in the developed. Then, by definition, $R_0^t = 1$ and since $w_h = w_f$, each of the foreign firms hires $n_f^t = 1$ domestic workers. Each of these domestic young is exposed to ideas in the ratio $R_0^{E,t} = (1)^{\gamma} \left(R^{O,t}\right)^{1-\gamma} \leq 1$ and will acquire skills in the ratio R_1^{t+1}. In the subsequent period $t+1$, they will expose each of their own n_1^{t+1} young workers to ideas in the ratio $R_2^{E,t+1} = \left(R_1^{t+1}\right)^{\gamma} \left(R^{O,t+1}\right)^{1-\gamma}$, and they will form skills in the ratio R_2^{t+2}. Each of them will go on at period $t+2$ to spawn n_2^{t+2} entrepreneurs with skills R_2^{t+3} that will be active at $t+3$. And so on, as indicated by the diagonal arrows in Table 1. Clearly, with the passage of every period, an additional 'vintage' of domestic firms is added.

[19] The key is that the effective unit of labor is the same in the two countries. For clarity, we assume that both countries have the same ratio of effective-to-physical units of labor. In the languaje of Burstein and Monge-Naranjo (2009), I am abstracting from differences in country embedded productivities. Adding those differences just require a bit of extra notation but add no substance to the results in this paper.

[20] Again, the equality is in terms of effective units of labor. If workers in the home country have fewer units of effective labor per unit of physical labor, foreign firms will be larger than the domestic firms in the foreign country. Moreover, since $Z_f > Z_h$, foreign firms are also larger than domestic firms at home.

[21] Since the cost of labor is the same in both countries, neither domestic workers nor old domestic entrepreneurs have the incentive to move from home to foreign.

period\generation	0	1	2	3	$\cdots$	j	$\cdots$	∞
0	$1_{\searrow}$	-	-	-	$\cdots$	-	$\cdots$	-
1	$1_{\searrow}$	$R_1^1{}_{\searrow}$	-	-	$\cdots$	-	$\cdots$	-
2	$1_{\searrow}$	$R_1^2{}_{\searrow}$	$R_2^2{}_{\searrow}$	-	$\cdots$	-	$\cdots$	-
3	1	$R_1^3{}_{\searrow}$	$R_2^3{}_{\searrow}$	$R_3^3{}_{\searrow}$	$\cdots$	-	$\cdots$	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$1_{\searrow}$	$R_1^t{}_{\searrow}$	$R_2^t{}_{\searrow}$	$R_3^t{}_{\searrow}$	$\cdots$	$R_j^t{}_{\searrow}$	$\cdots$	-
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
∞	$1_{\rightarrow}$	$R_1^\infty{}_{\rightarrow}$	$R_2^\infty{}_{\rightarrow}$	$R_3^\infty{}_{\rightarrow}$	$\cdots$	$R_j^\infty{}_{\rightarrow}$	$\cdots$	R_∞^∞

Table 1: Relative skills R_j^t of the j-th generation of a foreign entrepreneur at period $t \geq 0$.

Asymptotically there can be an infinite number of vintages of domestic firms that have, directly or indirectly, built up their relative skills with foreign ideas.[22] In a steady or balanced growth path, all the ratios R_j^t –and the average $R^{O,t}$– would have converged to an array of constants $\left\{R_j^\infty\right\}_{j=0}^\infty$, and the absolute skills Z_j^t of all vintages grow over time at the gross rate G. As indicated by the horizontal arrows in the table, in the BGP, each vintage j exposes their workers to ideas in the constant ratio $R_j^E = (R_j)^\gamma \left(R^O\right)^{1-\gamma}$, these workers form skills in the ratio $R_{j+1} \leq R_j$.

For simplicity, and without the risk of confusion, in the following discussion of the BGP I will ignore the superindex ∞.

3.3.1. Balance Growth Paths

In a BGP of an open economy, a constant mass $m_f \geq 0$ of foreign firms enter the country and a constant distribution of skill levels R (the relative gaps with respect to the foreign productivity Z_f) characterize the population of domestic firms. Obviously, the external exposure inside the country relative to that in the foreign country, $R^O = Z^O/Z_f$, would also be constant along a BGP.

It is convenient to separate the more general case $0 < \gamma < 1$ from the two special cases $\gamma = 0$ and $\gamma = 1$.

$0 < \gamma < 1$**: Partial Internalization with Externalities.** When $0 < \gamma < 1$, there are also two BGP. First, *'full-convergence'*, $1 = R^O = R_j$ all j, is a BGP. In such a BGP, the (net) entry of foreign firms is zero, $m_f = 0$, all domestic firms are homogeneous and identical to foreign firms in terms of skills, wages and learning same learning opportunities. If such BGP is reached, the equilibrium conditions are the same as in the closed economy BGP.

A second, *'interior'* or *'partial convergence'* BGP exists in which the skills and ideas circulating in the home country are inferior to those in the developed foreign countries, i.e. $R^O = Z^O/Z_f < 1$, and $R_j \leq 1$ all j with the inequality strict for at least some $j < \infty$. To lay out the equilibrium conditions for this interior BGP, let w_j denote the wage that a domestic firm of vintage j must

[22] Once again, when the entrepreneurial choices are introduced in Section 4, the number of vintages may be finite.

pay their workers. Because all young individuals are ex-ante identical and all have the option to work for any firm, whenever $R_j < 1$ the wage w_j must be higher than $w_f = \alpha Z_f$ to compensate the inferior learning opportunities. Define $d_j \geq 0$ to be the relative compensating differential in terms of Z_f, i.e. $w_j = (\alpha + d_j) Z_f$. In a BGP the compensating differentials must be an array of non-negative constants $\{d_j\}_{j=0}^{\infty}$. By construction, $R_0 = 1$, $d_0 = 0$, and $n_0 = 1$, and $R_0^E = \left(R^O\right)^{1-\gamma}$. Since the aggregate mass of labor is normalized to 1, and $n_f = 1$, then we can set $m_0 = m_f$.

An equilibrium BGP of an open economy is a domestic-to-foreign ratio of external exposure of ideas $0 < R^O \leq 1$, and an array of non-negative employment shares, firm sizes, relative skills, compensating differentials and exposures to know-how $\left\{m_j, n_j, R_j, d_j, R_j^E\right\}_{j=0}^{\infty}$ such that, $\forall j = 0, 1, 2, ...$

(a) old entrepreneurs of vintage j maximize profits, hiring workers in the amount

$$n_j = \left[\frac{\alpha}{\alpha + d_j} R_j\right]^{\frac{1}{1-\alpha}}, \tag{3.9}$$

(b) the employment shares for each j are

$$m_j = m_0 \prod_{i=0}^{j} n_i, \tag{3.10}$$

(c) and the entry of foreign firms ($m_f = m_0$) insures that the country's labor market clears

$$m_0 = \frac{1}{\sum_{k=0}^{\infty} \prod_{j=0}^{k} n_j}. \tag{3.11}$$

(d) The home country's relative external exposure of ideas is the weighted mean of all the skills implemented

$$R^O = \left[\sum_{j=0}^{\infty} m_j \left(R_j\right)^{\rho}\right]^{\frac{1}{\rho}}; \tag{3.12}$$

(e) and for each firm of vintage j, the relative exposure to ideas is

$$R_j^E = \left(R_j\right)^{\gamma} \left(R^O\right)^{1-\gamma}. \tag{3.13}$$

(f) Given exposure R_{j-1}^E and future wages $(\alpha + d_j) G Z_f$, skill investments R_j of workers in vintage $j-1$ are optimal:

$$\beta \left(\frac{\alpha R_j}{\alpha + d_j}\right)^{\frac{\alpha}{1-\alpha}} \left[1 + \frac{\alpha R_j^E}{\alpha + d_j} \frac{\gamma v v_0}{1+v} \left(G \frac{R_{j+1}}{R_j^E}\right)^{1+v}\right] = v_0 \left(G \frac{R_j}{R_{j-1}^E}\right)^{v}; \tag{3.14}$$

(g) and young individuals are indifferent to work in the different active firms $j \geq 1$

$$d_j = \beta G \left[\pi\left(R_1, \alpha + d_1\right) - \pi\left(R_{j+1}, \alpha + d_{j+1}\right)\right] + R_j^E \phi\left(G \frac{R_{j+1}}{R_j^E}\right) - \phi\left(G \frac{R_1}{R_0^E}\right), \tag{3.15}$$

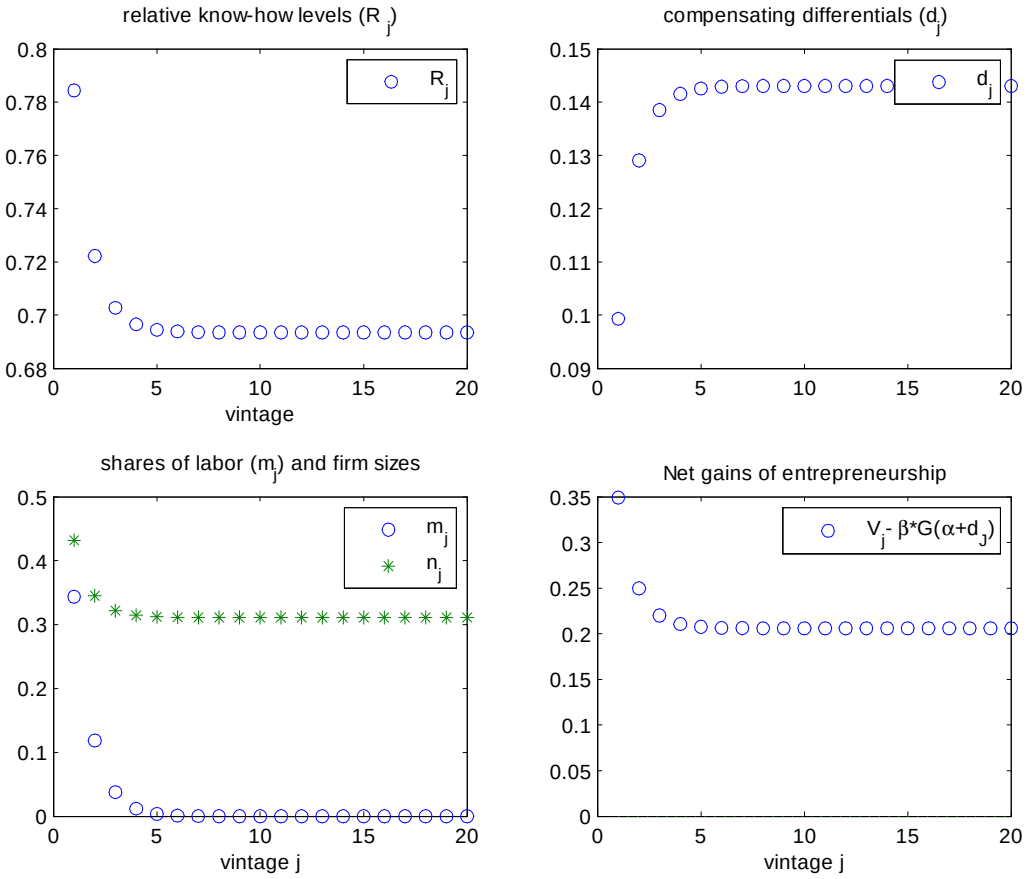

Figure 3.1: BGP of an economy with no binding occupation choices.

where the functions $\pi(\cdot,\cdot)$ and $\phi(\cdot)$ are as defined above and are used here to shorten the last expression.

Figure 1 illustrates the behavior of the different vintages of domestic firms in an interior BGP. Because $R^O < 1$, all the young domestic entrepreneurs are less exposed to productive ideas than their foreign counterparts. Even those working in foreign firms are only exposed to the ratio $\left(R^O\right)^{1-\gamma} < 1$. For those working in older vintages, the exposure to ideas decays consistently, $R_j^E \geq R_{j+1}^E$. As a result, as shown in the top-left panel, the formation of skills also decays consistently with the age of the vintage. The declining exposure to ideas also explain that compensating differentials d_j must increase with the vintage as shown in top-right panel of the figure. Both, the declining skills R_j and the increasing wages $\alpha + d_j$ explain the decline in the labor hired by each firm n_j and in the share m_j of the total labor force hired by the vintage j (bottom-left panel). In this example, the decline is so rapid that shares of all vintages above 5 or 6 are negligible.

The bottom-right panel of the figure reports for each j the value $v_j - \beta G(\alpha + d_j)$, which is the (discounted, normalized by Z_f) earnings of someone who, having worked in vintage j when

young, invests in skills and become an entrepreneur when old *minus* the value attainable by not investing in skills and returning as a worker when old. The fact that this value is strictly positive for all j indicates that, under the parameter values used in this example, entrepreneurship choices are not binding, i.e. all individuals want to become entrepreneurs when old; adding the option to remain workers would have no impact on the BGP.

The structure of equilibrium is simpler at the two extreme special cases of the internalization parameter.

$\gamma = 1$**: Full internalization.** In this case the diffusion of ideas is entirely within the firm. Regardless of how productive are the other firms in the country, the domestic workers in a foreign firm are exposed to exactly the same set of ideas as foreign workers in the foreign country. They will be able –and will find it optimal–to acquire the same level of skills as their foreign peers, i.e. $R_j^t = 1$. Once there, as entrepreneurs in the next period, they will expose their own workers to the same level of skills as the foreigners, which will accumulate the same level of skills, i.e. $R_{j+1}^{t+1} = 1$, and so on. Because of this, in any period t, and all vintages $j \leq t-1$, we have $R_j^t = R_{j+1}^t$. Then, to characterize the equilibrium, it suffices to keep track of the mass of foreign firms m_f^t, the total mass of 'new' domestic firms, denoted m_{new}^t, that have the same level of skills as the foreign firms (i.e. the cumulative mass of all the different generations j of foreign-trained domestic entrepreneurs) and the remaining share of labor controlled by the offspring of the pre-existing domestic firms at the time $t = 0$ when the country opened up.

As time passes by, the positive entry $m_f^t \geq 0$ of foreign firms builds up the mass of foreign-trained domestic entrepreneurs which will eventually overtake the entire labor force, i.e. $m_{\text{new}}^t \to 1$. In the BGP, these domestic entrepreneurs alone push the equilibrium domestic wage to the foreign level w_f, and foreign firms will no longer have gains of entering, i.e. $m_f = 0$.

$\gamma = 0$**: Only Externalities.** In this case the diffusion of foreign ideas is only through their spillover on Z^O. Domestic and foreign firms both offer the same learning opportunities to their workers and pay the same wages. Similar to the case with $\gamma = 1$, all domestic firms will be homogeneous; however, there can be persistent gaps in the set of ideas circulating in the home and in the foreign countries, i.e. $R^O < 1$ and in the relative skills of domestic firms, i.e. $R < 1$. In such interior BGP, the inflow of ideas emanating from the positive mass of foreign firms m_f reduces the marginal cost of investing skills in the same proportion as the reduction in the marginal return of investing in skills induced by the foreign competition. As I explain next (and prove in the appendix), when $\gamma = 0$ the interior BGP is globally stable. Therefore, the only way home converges to full convergence $R = R^O = 1$ is if the country starts there.

When $\gamma = 0$ the transition function for an open economy can be solved in closed form. Given Z_h^t and $w^t = \alpha Z_f^t$, each domestic entrepreneur hires $n_h^t = (R^t)^{\frac{1}{1-\alpha}}$ units of labor, where $R^t \equiv Z_h^t / Z_f^t$

is the homogeneous ratio of domestic-to-foreign entrepreneurial skills. [23] Since foreign firm hires $n_f^t = 1$, the clearing of the domestic labor market requires $m_f^t = 1 - \left(Z_h^t/Z_f^t\right)^{\frac{1}{1-\alpha}}$. Using the weights $1 - m_f^t$ and m_f^t to the homogeneous groups of domestic and foreign firms, the exposure to ideas for domestic youth relative to the foreigners' one is

$$R^{E,t} = R^{O,t} = \left[1 + \left(R^t\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^t\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}}. \tag{3.16}$$

Openness always improves the domestic exposure to ideas, i.e. $R^{E,t} > R^t$, because $R^{E,t}$ is an average of 1 and $R^t \leq 1$. However, $R^{E,t}$ might is not be monotone increasing in R^t. There are two countervailing forces. On the one hand, a higher R^t increases $R^{E,t}$ because domestic firms are a better source of ideas. On the other hand, a higher ratio R^t reduces the entry of foreign m_f^t and the country's exposure to to foreign ideas. When $\rho \leq -1/(1-\alpha)$, the first effect dominates and $R^{E,t}$ is always increasing in R^t because of a strong complementarity between the domestic and foreign sources of ideas. However, if $\rho > -1/(1-\alpha)$, the negative effect dominates for low values of R^t and the relative exposure $R^{E,t}$ declines with positive increments in R^t.

Over time, however, openness does not lead to full convergence; that is, even if $R^{E,t} > R^t$, the relative skills of domestic firms will remain below those of foreign firms, i.e. $R^{t+1} < 1$ because they will have to pay the same wages $w_{t+1} = \alpha G Z_f^t$ as foreign firms but they must build up their skills on the basis of inferior ideas, $R^{E,t} < 1$. Solving for the optimal accumulation of skills (3.14) for $\gamma = 0$ and using (3.16), it is straightforward to show

$$R^{t+1} = \left(R^{E,t}\right)^{\mu} = \left[1 + \left(R^t\right)^{\rho + \frac{1}{1-\alpha}} - \left(R^t\right)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}}, \tag{3.17}$$

where $\mu \equiv v/[v - \alpha/(1-\alpha)] > 1$. Thus, R^{t+1}, the next period's relative skills of domestic entrepreneurs, is a strictly convex function of the current period relative exposure $R^{E,t}$. An obvious fixed point in this mapping is when $R = R^E = 1$, i.e. if and when the country has reached a fixed point. However, as long as $-\infty < \rho < +\infty$, as shown by Figure 1, another unique and (stable) interior fixed point exists R^{int} in which $0 < R^{\text{int}} \leq R^{O,\text{int}} \leq R^{E,\text{int}} < 1$. In one limit, when $\rho = -\infty$, the interior BGP converges to $R^{\text{int}} = 0$ and all firms in the country will be controlled by foreign skills. On the other limit, when, $\rho = +\infty$ the interior BGP collapses to $R = 1$, and, after just one period after openness the home country will fully catch up with the foreign countries.

Appendix A contains the proof of the following result:

Proposition 2. (BGP open economy) *Assume that the parameter assumptions of Proposition* 1 *hold. Then:* ***(a)*** *Full convergence, i.e.* $R^O = 1$ *is always a BGP equilibrium;* ***(b)*** *if either* $\gamma = 1$

[23] The vintage structure is not needed as $R_j = R$ and $d_j = 0$ for all $j \geq 1$.

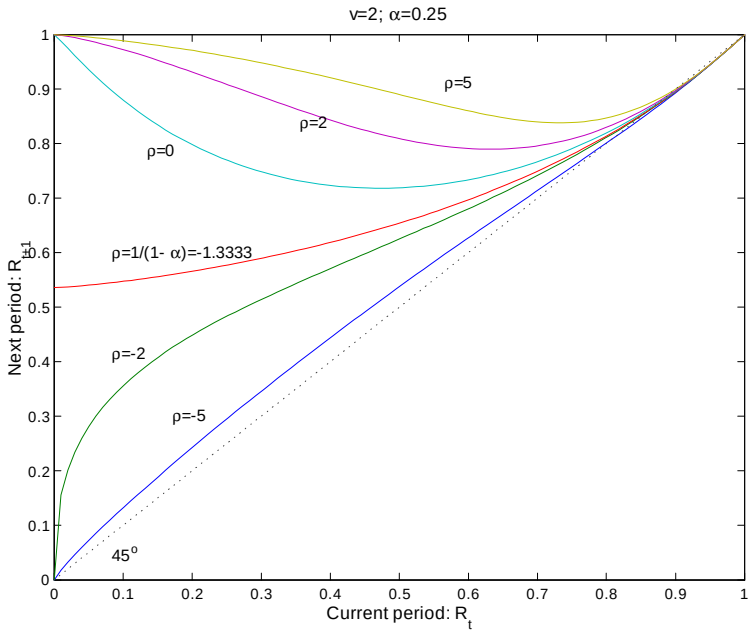

Figure 3.2: Transition Functions: Open Economy for the pure externality case $\gamma = 0$ with different values for ρ

or $\rho = \infty$*, then full convergence is the unique BGP and the country converges to it from any initial condition;* ***(c)*** *if* $\gamma = 0$*, and* $\rho < \infty$*, then there exists a unique interior equilibrium, i.e.* $0 < R^{O,int} < 1$ *and the country converges to it from any initial condition* $R^0 = Z_h^0/Z_f^0$*.*

Parts (a) and (b) include the case for $0 < \gamma < 1$, but, as discussed in Appendix A, it is difficult to formally prove the equivalent of results (b) or (c) for those cases. However, as also explained in that appendix, it is straightforward to compute the interior BGP and numerical results suggest that it always exists and that it is unique.

Add the following two corollaries:

Corollary 3. (Gains from openness) *Message: countries low gain more than the static gains. Countries close to the frontier actually lose.*

Corollary 4. (Leapfrogging 1) *If* $\rho > -1/(1-\alpha)$*Countries behind surpass those ahead.*

4. Occupation Choices and the Diffusion of Know-How

Entrepreneurship choices have a prominent presence in the development literature (e.g. Banerjee and Newman 1993). Sorting individuals between managerial and labor occupations can enhance the static gains of openness as shown by Antras, Garicano and Rossi-Hansberg (2006), Burstein and Monge-Naranjo (2009) and more forcefully by Eeckhout and Jovanovic (2009). In addition to these static gains, in this section I will argue that occupation choices can determine whether

and how quickly a developing country can catch up with the rest of the world. Specifically, I will show that occupation choices: (a) can change the form of either of the two BGPs; (b) can lead open economies away from the interior BGP and instead to fully catch up; (c) can accelerate the convergence.

Carrying a skill level z, an old person becomes an active entrepreneur only if his rents $\pi\left[z, w\left(z\right)\right]$ are above any of the wages $w\left(\cdot\right)$ available, i.e. if

$$\pi\left[z, w\left(z\right)\right] \geq \sup_{\zeta \in \text{ support}} w\left(\zeta\right), \tag{4.1}$$

where 'support' refers to the entire set of entrepreneurial skills –domestic or foreign– active in the country.

More interestingly, the future option of remaining a worker changes the investment decision of a young person. Regardless of his exposure to ideas z^E, a young person can always opt not to invest at all in skills, $z' = 0$, and attain a (discounted) earnings in the next period equal to $\beta \sup w_{t+1}\left(\cdot\right)$, where the supremum is taken with respect to the skills available in the country in the next period. This imposes a lower bound on the career value of all jobs:

$$V\left[z^E, w_{t+1}\left(\cdot\right)\right] \geq \beta \sup_{\zeta \in \text{ support } (t+1)} w_{t+1}\left(\zeta\right). \tag{4.2}$$

This bound can reduce the equilibrium gap between the wages paid by active entrepreneurs with different skills. Specifically, consider two entrepreneurs with skill levels $z_0 < z_1$. If the two of them fall below a certain threshold z_t^*, they will both pay the same wage. On the other hand, if the two are above the threshold, the wage difference will be given by the same equation (3.5) as in the previous section, reflecting the difference in the learning opportunities of the two jobs. Finally, if the two skill levels fall on different sides of the threshold, i.e. $z_0 < z_t^* < z_1$, their wages compare as follows:

$$w_t\left(z_1\right) = w_t\left(z_0\right) + \beta \sup_{\zeta \in \text{ support } (t+1)} w_{t+1}\left(\zeta\right) - V\left[z_1^E, w_{t+1}\left(\cdot\right)\right] < w_t\left(z_0\right) = w_t\left(z_t^*\right).$$

Obviously, $w_t\left(\cdot\right)$ is flat up to the threshold z_t^*, after which it becomes strictly decreasing.

Identical results of as of Lemma 1 and Proposition 1 of the Section 3 hold even with a flat region of $w_t\left(\cdot\right)$. More interestingly, when $\gamma > 1 - \alpha/\left[\left(1-\alpha\right)v\right]$, a closed economy could converge faster to the BGP with homogeneous entrepreneurs because the lower tail of the skill distribution is being progressively eliminated and only the higher end of the distribution of existing firms spawn new entrepreneurs.

Next, occupation choices can change the BGP in a close economy. As detailed in the appendix:

Lemma 2. (Closed economy BGP, occupation choices) *Under the same parameter assumptions as in Proposition* 1*, there exists a unique BGP in a closed economy with occupation choices. If*

the G in the BGP without occupation choices satisfies $\beta(1-2\alpha) > v_0 G^v/(1+v)$, then it is also the BGP with occupation choices. If not, the unique BGP is described by a fraction ω of young individuals who invest in skills and an intergenerational growth rate G of skills, such that (i) young individuals are ex-ante indifferent between the two occupations:

$$\beta\left[\frac{2-\omega}{\omega}\right]^{\alpha}\left[\frac{2(1-\alpha)-\omega}{2-\omega}\right]=\frac{v_0}{1+v}G^v;$$

and (ii) those who invest do so optimally, i.e. G is the lower root of

$$\beta\left[\frac{2-\omega}{\omega}\right]^{\alpha}\left[1+\frac{\gamma v v_0}{(1+v)}G^{1+v}\right]=v_0 G^v.$$

The most interesting case to compare open and closed economies is the case $\beta(1-2\alpha) > v_0 G^v/(1+v)$ and all the young become entrepreneurs in a closed economy BGP and it will be the focus of the ensuing analysis.[24]

4.1. Open Economies

As in the previous section, it is convenient consider separately the different cases for the internalization parameter γ.

$\gamma = 1$**: Full internalization.** As before, full convergence is the only BGP for an open economy. Those who work for a foreign firm will mature next period into a domestic entrepreneur with exactly the same level of skills as their foreign peers. Overtime, this new group of 'world class' domestic entrepreneurs will fully take over the entire set of firms operating in the home country and foreign firms will cease to enter.

Occupation choices accelerate the transition to the full convergence steady state. To illustrate this, consider a country initially endowed with domestic entrepreneurs with very low skills. With occupation choices, these initial old entrepreneurs would supply their labor, and because of this the country will attain the fully convergence BGP after just one period. Without occupation choices, the initial old would have to remain active entrepreneurs, hiring and training some of the labor, reproducing over time a mass of lower productivity domestic firms. Without occupation choices the convergence is only asymptotic.

But even if the old generation of domestic entrepreneurs is skilled enough to remain active, occupation choices can accelerate the convergence to the BGP. For instance, their own workers may not find it optimal to invest in skills due to their relative inferior exposure to knowledge. If so, next period they would be workers, and the economy will converge after two periods. In general,

[24] The case of indifference $\beta(1-2\alpha) \leq v_0 G^v/(1+v)$ is a knife-edge for open economies. Upon openness all domestic entrepreneurial shuts down and the country will converge in the next period.

it may take an integer n of generations of progeny of the pre-existing domestic entrepreneurs until the n-th one finds it optimal not to invest.

In general, occupation choices change the convergence from asymptotic to convergence in finite time.

$\gamma = 0$**: Only Externalities.** Recall that without occupation choices, the interior BGP was globally stable. With occupation choices, however, the interior BGP may no longer exist and an open economy necessarily converges to $R = 1$.

There are two possibilities. The first is driven by the occupation choices of the old: with relative skills R^{int} the old may be better off as workers, not as entrepreneurs. This happens when the domestic profits $\pi_h = \theta Z_h^{\frac{1}{1-\alpha}} [w_f]^{\frac{-\alpha}{1-\alpha}}$ fall short of the wages $w_f = \alpha Z_f$, which boils down to

$$R^{\text{int}} < [\alpha/(1-\alpha)]^{1-\alpha}. \tag{4.3}$$

The second possibility is that the young, being exposed to $R^{O,\text{int}} = \left[1 + \left(R^{\text{int}}\right)^{\rho+\frac{1}{1-\alpha}} - \left(R^{\text{int}}\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\rho}}$, are better-off not investing and remaining workers.As a worker, the discounted payoff is $w_f^t = \beta G \alpha Z_f^t$; as an entrepreneur, given that the optimal investment is $Z_h^{t+1} = Z_f^t G \left(R^{O,\text{int}}\right)^{\mu}$, he attains a net discounted payoff equal to $Z_f \left\{\beta(1-\alpha) G \left(R^{O,\text{int}}\right)^{\frac{\mu}{1-\alpha}} - R^{O,\text{int}} \phi\left[G\left(R^{O,\text{int}}\right)^{\mu-1}\right]\right\}$. After some basic simplifications and using functional form of $\phi(\cdot)$, a young person would not become an entrepreneur if

$$R^{O,\text{int}} < \left[\frac{\beta G \alpha}{\beta(1-\alpha) G - \phi(G)}\right]^{\frac{1-\alpha}{\mu}}, \tag{4.4}$$

i.e. if his exposition to ideas is too low relative to that of the future foreign competition.

If either of these two conditions hold, an open country will always catch up with the developed countries after a finite number of periods. Regardless of initial conditions, the value of R^t must eventually get near the R^{int}, and at that point, after one or two periods, would jump to $R = 1$.

However, even if the interior BGP survives the two occupation choices, i.e. if neither (4.3) nor (4.4) hold, occupation choices can redirect the convergence of an open economy towards full convergence. Indeed, in any period $t \geq 0$ if R^t falls below the threshold $[\alpha/(1-\alpha)]^{1-\alpha}$, then all the old would opt to remain workers, clearing the way for the young to be exposed to only foreign ideas, i.e. $R^{O,t} = 1$, leading to $R^{t+1} = 1$. Likewise, the occupation choice of the young could trigger the convergence; if $[\alpha/(1-\alpha)]^{1-\alpha} < R_t < R^{O,t} < \left(\beta G \alpha/[\beta(1-\alpha) G - \phi(G)]\right)^{\frac{1-\alpha}{\mu}}$, the current old will become active entrepreneurs but the young are better off not investing, $R^{t+1} = 0$; then, all the entrepreneurial skills at $t+2$ will be foreign, $R^{O,t+1} = 1$, and the young at $t+2$ will invest at the rate R^{t+2}. Respectively, the country would have fully catched up at $t+2$ or at $t+3$.

In sum, instead of just accelerating the convergence to the BGP, when $\gamma = 0$ occupation choices completely change the direction of convergence.

$0 < \gamma < 1$**: Partial Internalization with Externalities.** With occupation choices, it can be the case that in the interior BGP only a finite number $J < \infty$ of vintages of domestic firms are active.This occurs if the relative skills R_j decay rapidly with j and the parameters of the economy are such that the earnings of entrepreneurs are not too far above the workers' wages. In such a case, the young workers in vintage $j = J$ in period t will not invest in skills because they return as old workers in the vintage $j = J$ in period $t + 1$.

Denote by d_J the compensating differential (in units of Z_f) received by workers of the last vintage J. Since entrepreneurs do not care who is providing the labor, both young and old receive wages equal to $w_J = (\alpha + d_J) Z_f$. For a young person to be indifferent between being a worker in both periods of life, the implied lifetime earnings $(\alpha + d_J)(1 + \beta G)$ must be equal to the lifetime earnings attained from any other option, in particular $j = 0$, the foreign firm. Equating the net-present values from these two options, the compensating differential d_J must be equal to

$$d_J = \frac{\beta G \left[\pi \left(R_1, \alpha + d_1\right) - \alpha\right] - R_0^E \phi \left(G \frac{R_1}{R_0^E}\right)}{1 + \beta G}. \tag{4.5}$$

In all the other vintages, $j = 0, ..., J-1$, d_j behaves as in the previous section, with labor fully provided by young individuals who invest in skills and become entrepreneurs when old. Since there is no population growth, it has to be the case that half of the workers in the last vintage J are young and half old.

Therefore, an interior BGP equilibrium (i.e. $0 < R^O < 1$) of an open economy with occupation choices is defined as a $J \in \mathbb{N}$ (possibly ∞ if occupation choices do not bind) and an array $\left\{m_j, n_j, R_j, d_j, R_j^E\right\}_{j=0}^{J}$ that satisfes conditions (a)-(b) and (d)-(g) of the previous section, but with the following modifications: (i) the summations run from $j = 0$ to $j = J$; (ii) d_J is given by (4.5); and (iii) while condition (c) holds, the absolute mass of foreign firms is $m_f = m_0 (1 + n_J/2)$ since the total labor force includes $n_J/2 > 0$ old persons supplying labor for the last vintage of domestic firms.

If occupation choices not bind, a number $J < \infty$ of vintages is determined by the following conditions: (1) workers in vintage $J-1$ find it optimal to invest at the rate R_J instead of remaining a worker:

$$\beta G \left[\pi \left(R_J, \alpha + d_J\right) - (\alpha + d_J)\right] - R_{J-1}^E \phi \left(G \frac{R_J}{R_{J-1}^E}\right) > 0;$$

and (2) workers in vintage R_J find it optimal to remain workers, given that, as entrepreneurs they would not spawn entrepreneurs, i.e.

$$\max_R \left\{\beta G \left[\pi \left(R, \alpha + d_J\right) - (\alpha + d_J)\right] - R_J^E \phi \left(G \frac{R}{R_J^E}\right)\right\} < 0.$$

The vintage structure is similar to that in Chari and Hopenhayn (1991). However, there are two important additional aspects for computing an equilibrium. First, the levels of R_j are endogenous.

Second, the level of R^O, the externality, is determined endogenous and could lead to multiple BGP equilibria.

In the appendix we show the following:

Proposition 3. *Consider an economy with occupation choices. Then, either (a) the only BGP equilibrium is one with full convergence, i.e.* $R^O = 1$ *and* $R_j = 1$ *for all* $j = 1, ...\infty$, *or (b) there is an interior BGP with* $R^{O,int} \geq R^{O,int}_{noc}$ *and* $R_j \geq R^{noc}_j$ *for* $j = 1, ..., J < \infty$,*where noc indicates the interior BGP with the same economy but forcing all the old to be entrepreneurs next period. If*

Comparing the interior BGPs, it can be seen that occupation choices have two different positive effects on the formation of domestic know-how. First, a '*selection effect*', as the older vintages are no longer active. Their labor is allocated to the younger, more productive vintages, and with this reallocation, the current young are exposed to a higher ratio R^O of ideas. Second, there is an *'investment effect'*: the young working in any of the active vintages $j = 0, ..., J-1$ invest more because of the lower compensating differentials d_j that result for the better career options in vintages $j \geq J$.

Figure 3 use parameter values **XXXX** to illustrate the interior BGPs of economies with occupation choices (dots) and without them (stars). As in the previous case (Figure 1) infinitely many vintages would be active if once individuals do not have the option to remain workers in both periods of life. In this example however, as shown by the bottom-right panel, occupation choices are binding because $v_j - \beta G(\alpha + d_j)$ becomes negative. Indeed, once occupation choices are introduced, only three vintages of domestic firms would remain active for high. As shown by the top-left panel, the relative productivity R_j of the active vintages are higher than without occupation choices, and as shown by the top-right panel, the compensating differentials d_j are lower; this explains why, as shown by the bottom-left panel that the units of labor n_j of each firm and the shares of labor m_j hired by the three active vintages are higher with occupation choices.

As in the case of $\gamma = 0$, the impact of occupation choices goes beyond changing the interior BGP. First, they can remove the interior BGP altogether. In such a case, regardless of initial conditions, openness would necessarily lead a country to full catch up with developed countries. Second, even if the interior BGP persists, occupation choices can drastically change the dynamics of an open country because, depending on initial conditions, the country would move towards full convergence instead of the interior BGP.

Corollary 4. (Destruction for convergence) *Open countries only catch up with developed countries if pre-existent, inferior domestic skills stop being reproduced at some point. If so, such destruction is gradual if* $\gamma > 0$ *and abrupt (in one period) if* $\gamma = 0$.

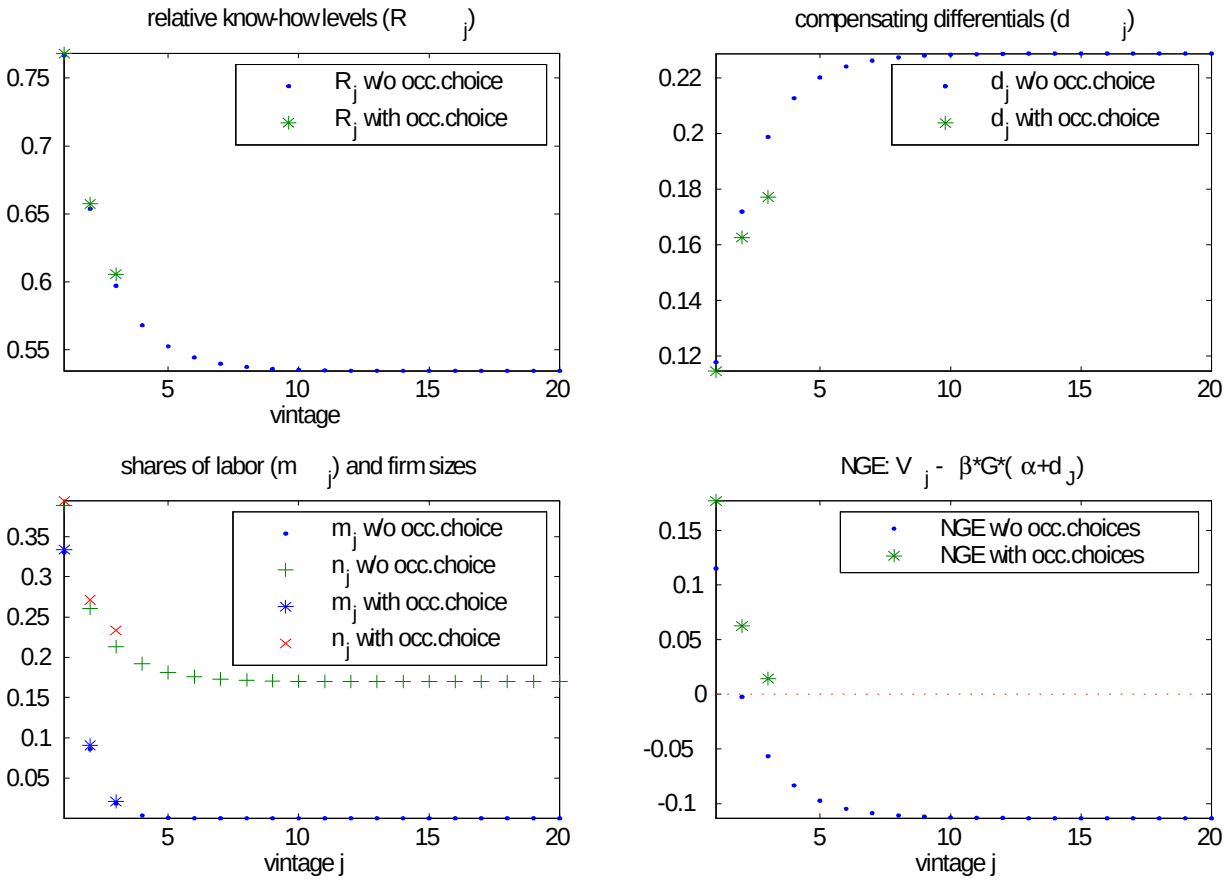

Figure 4.1: BGP open economy, binding OC; only 3 vintages of DF.

Corollary 5. (Leapfrogging 2) *Assume the interior BGP exists (i.e.* $R^{int} > XXX$ *and* $R^{O,int} > XXX$*); and consider two open countries XX and XX such that; then country 1 will catch up with developed countries and country 2 will converge to the interior BGP, falling behind forever country 1.*

5. Quantitative Analysis

I now assess the gains that a developing country of attains from being open to foreign firms. For this exercise I consider a slightly richer model than the one used in the previous section. The extensions do not add nor subtract analytical results to match some key observations and to use parameter values that are standard in the literature. In particular, the extended model accounts for the observation that only a small fraction of the population is allocated to managerial occupations. With this aspect, the formation and international diffusion of entrepreneurial skills is restricted to only the fraction of individuals who provide (mid-level) managerial services when young, which, presumably, are the ones who, having direct contact with them, can learn from the decision-makers at the top the firm.

5.1. A Model with Middle-Managers and Workers

Consumption goods are produced with entrepreneurial or top managerial skills (z), mid-management services (n), and labor services (l):

$$y = zn^{\alpha}l^{\lambda}.$$

Each young cohort is composed of two groups: a fraction $\omega < 1/2$ of potential managers and a fraction $1-\omega$ of perennial wokers. Perennial workers provide labor services when young and old and do not accumulate skills. Potential managers, in either period, can be workers or middle-managers. Moreover, they can invest in skills when young and become top-managers (entrepreneurs) when old. As in the basic model, their exposure to ideas z^E is determined by both, the skills z of the top-manager for whom he works when young, and the average Z^O implemented in the country at that time.[25]

Under the parameter values used for these exercises (see the discussion below), in the closed economy BGP all the potential managers are middle-managers when young, invest in skills, and become active entrepreneurs when old. When all all the ω entrepreneurs have skills Z_t, the aggregate supply of labor is $2(1-\omega)$, the aggregate supply of mid-management services is, ω and the total supply of managerial skills is ωZ_t. Production teams are composed of one top manager,

[25] The weights μ_z in Z^O are the shares of the country's aggregate supply of middle-managers used by firms with the different levels z of skills. It is easy to show that those shares coincide with the shares of the country's aggregate labor used by the different zs.

one middle manager and $\varrho \equiv 2(1-\omega)/\omega > 1$ workers. The income of workers, mid-managers and entrepreneurs are, respectively, $w_t = \lambda \varrho^{\lambda-1} Z_t$, $w_t^m = \alpha \varrho^{\lambda} Z_t$, and $\pi_t = (1-\alpha-\lambda)\varrho^{\lambda} Z_t$. It is straightforward to verify that if $\lambda/\varrho < \alpha < (1-\lambda)/2$ this is a static equilibrium because (i) an old person with skills $z = Z_t$ is better-off being an entrepreneur instead of a mid-manager and (ii) a young manager is better-off providing managerial services instead of labor.

As detailed in Appendix B, along the closed economy BGP, the gross growth rate of skills G is given by

$$\beta \left[\varrho^{\lambda} + \frac{\gamma v v_0}{1+v} G^{1+v} \right] = v_0 G^v,$$

the same as expression (3.7) except for the term ϱ^{λ}. Since $\varrho^{\lambda} > 1$, top-managers have more workers under their control. Finally, for an equilibrium BGP it is required that potential managers opt to invest in skills instead of remaining mid-managers when old, i.e. $w_t^m + \beta w_{t+1}^m < w_t^m + \beta \pi_{t+1} - Z_t \phi(G)$. This inequality boils down to $\phi(G) < \beta(1-2\alpha-\lambda)\varrho^{\lambda} G$. Thus, under conditions very similar as for the basic model, there exists a unique BGP for a closed economy.

The analysis of open economies is very similar to that in basic model and it is further discussed in Appendix B.

5.2. Parameter Values

While the model is admitedly very stylized, it is straightforward to set up some of its parameters to make it consistent with some basic observations. The parameter values are presented in Table 2. I consider each period of life to represent 20 years; then, regardless of their occupations, all individuals are active for 40 years, which roughly coincides with labor market participations that start in the mid-20s and end in the mid-60s. The discount factor is set to $\beta = (1.04)^{20}$ so that the annual interest rate is equal to 4%, roughly the historical average fo the U.S. I set $\omega = 0.1$ so that 10% of the population is in managerial occupations, 5% as middle-managers and 5% as entrepreneurs. These values are in the ball-park of the numbers for related concepts in **(REFERENCES XXXX),** and variations within a reasonable range have no significant impact on the results. Finally, the span-of-control $\lambda + \alpha$ is set to 0.85, the most commonly used value in the literature. The individual values λ and α are not of particular interest as long as (i) the occupation choice conditions $\lambda/\varrho < \alpha < (1-\lambda)/2$ are satisfied, and (ii) the effect of ϱ^{λ} on G is undone with the calibration of the other parameters, which is what I do.

	Parameter / Definition	Value	Criterion/target
ω	fraction with managerial potential	0.1	% in managerial occupations
λ	output share, labor	0.775	see text
α	output share, mid-managers	0.075	see text
β	discount factor	$(1.04)^{20}$	annual risk free rate$\approx$ 4%
v_0	level of costs, skill acquisition	depending on (γ, ρ, v)	annual growth (BGP)$\approx$ 2%
γ, ρ, v	internalization, diffusion, curvature	see text	comparative statics

Table 2. Parameter values for the quantitative exercises

I set the value v_0 so that the equilibrium gross growth rate is $G = (1.02)^{20}$, i.e. a 2% implied net annual growth rate of output. Notice that aside from v_0 and ϱ^{λ} the growth rate depends on the internalization, diffusion and curvature parameters γ, ρ and v. These three parameters –in that order– are central for the analysis but unfortunately there is not much information about them. Therefore, I will explore the implications of openness for different specifications for (γ, ρ, v); for each one v_0 is reset so that the BGP growth rate is $G = (1.02)^{20}$.

5.3. The Gains from Openness

6. Concluding Remarks

TO BE EDITED

In this paper I used simple general equilibrium growth models to study the impact of multinational firms on the formation of skills and the long run behavior of output a small developing country. Within the context of a simple model environment, I examined three different growth models: (a) an exogenous growth model, (b) an endogenous growth model with an externality in the formation of skills and (c) an endogenous growth model in which skills are internally produced in the firm. The impact of multinational firms on the host country in models (a) and (b) is via spillovers. These two models are rather standard in the growth literature and the existence and measurement of spillovers have been the subject of a vast empirical literature. Spillovers are also the tenet underlying much debate and policy proposals and programs. I show that spillovers are not sufficient to propel the country to catch up with developed countries.

In model (c) there are no spillovers and any transfer of skills is the result of a market transaction. In a competitive environment, firms will optimally transfer a level of skills that gradually obliterate the existent sector and creates a new sector of domestic firms that are at par with the ones in developed countries. In finite time, the small country converges to the income level of developed countries. Spillovers are not necessary.

These results are very suggestive about the role of government policy. In model (a), a benevolent government would definitely want to subsidize foreign firms. In model (b) optimal policy can

be quite kinky. For instance, if local skills are very low, a subsidy would be pointless since the country will converge next period. For higher initial level of skills, the government may want to subsidize foreign firms fully obliterating the local firms for one period and converge to the developed country level in the next. The competition effect in this model can also introduce interesting issues of time consistency.

In model (c) the equilibrium is efficient and the government must not subsidize. However, if there are obstacles to the transmission of skills from foreign firms to local agents, a government may want to undo the obstacle by providing a subsidy. But it is necessary to go beyond these somewhat speculative arguments. Optimal government policy in models with obstacles to the flow of firms and/or the transmission of skills deserve a rigorous and comprehensive consideration.

A. Proofs and Additional Aspects of the Basic Model

Proof of Lemma 1 The value of V may be $+\infty$ as illustrated in the proof for Proposition 1 for the case of homogeneous firms. In such a case, the function $w(\cdot)$ is not well defined. However, if V is bounded, then the monotonicity of the function $w(\cdot)$ exists arises directly from the envelope condition on V. Next, the monotonicity of $z'(\cdot)$ arises from the fact that it is given by

$$\beta(\alpha z')^{\frac{\alpha}{1-\alpha}} g(z') = v_0 \left(\frac{z'}{z^E}\right)^v,$$

where $g(z') \equiv w'(z')^{\frac{-\alpha}{1-\alpha}} \left[1 + \frac{\alpha\gamma v v_0 \left(z''/\left(z^E\right)'\right)^{1+v} (z')^{\gamma} \left(Z^{O'}\right)^{1-\gamma}}{(1+v)w(z')}\right]$, which is non-decreasing in z'. For a maximization, the left-hand-side must cross the right-hand side from above, hence the need for the condition $v > \alpha/(1-\alpha)$. Thus, a z or higher z^E leads to a higher optimal value for z'. Finally, straightforward differentiation leads to

$$\frac{\partial \ln(z')}{\partial \ln(z^E)} = \frac{v}{\left[v - \frac{\alpha}{1-\alpha}\right] - \frac{\partial g(z')}{\partial z'} \frac{z'}{g(z')}} \geq \frac{v}{v - \frac{\alpha}{1-\alpha}} > 1,$$

where the first inequality holds because $\frac{\partial g(z')}{\partial z'} \frac{z'}{g(z')} \geq 0$ and the second because $v > \alpha/(1-\alpha)$. Since $z^E = z^{\gamma}\left(Z^O\right)^{1-\gamma}$, then,

$$\frac{\partial \ln(z')}{\partial \ln(z^E)} \geq \frac{\gamma v}{v - \frac{\alpha}{1-\alpha}},$$

which is greater than one if $\gamma > 1 - \frac{\alpha}{(1-\alpha)v}$, as stated.■

Proof of Proposition 1. Define $L(G) \equiv \beta\left[1 + \frac{v v_0 \gamma}{1+v}(G)^{1+v}\right]$ the marginal return to investing (the right- and lef-hand sides of the equation 3.7) and $R(G) \equiv v_0 (G)^v$ the marginal cost of investing (the right-hand-side). Notice that $L(0) > 0 = R(0)$ and that the curvatures of $L(\cdot)$ and $R(\cdot)$ are $1+v$ and v, respectively. Therefore, $L(\cdot)$ may lay above $R(\cdot)$ for all positive reals. In such a case, the optimal investment would be degenerate, $G = +\infty$, and the career value V for a young person would be unbounded, $V = +\infty$. However, $L(\cdot)$ may lay below $R(\cdot)$ for some range; if so, because of its higher curvature, it would cross $R(\cdot)$ twice, first from above and then from below. The second root, however, is not relevant because it is a local minimum. Therefore, (3.7) has at most one valid fixed point, which proves **(b).** To prove **(a)** first, if $\gamma = 0$, then $G = (\beta/v_0)^{1/v}$ is the unique solution; the inequality $\beta < [v_0(1+v)]^{\frac{1}{1+v}}$ insures that $\phi(G) < 1$ and therefore, the growth in skills is not so high so as to lead to negative aggregate consumption. For $\gamma > 0$, from the intermediate value theorem, a sufficient condition for $L(\cdot)$ to cross $R(\cdot)$ is to find a point $G > 0$ for which $L(G) < R(G)$. It is straightforward to show that the two curves are parallel to each other, i.e. $\partial L/\partial G = \partial R/\partial G$, only at the point $\bar{G} = (\beta\gamma)^{-1}$, and obviously, $\partial L/\partial G < (>)\, \partial R/\partial G$ for $G < (>)\, \bar{G}$. It is also straightforward to show that $L\left(\bar{G}\right) < R\left(\bar{G}\right)$ if $\beta < \left(v_0/[\gamma^v(1+v)]\right)^{\frac{1}{1+v}}$. The inequality

$[v_0/(1+v)]^{\frac{1}{1+v}}/\gamma < \beta$ is equivalent to $\phi\left(\bar{G}\right) < 1$, which insures that the first crossing G (the BGP) does not imply negative aggregate consumption. **(c)** The inequality $\beta > v_0(1+v)/(1+v+vv_0\gamma)$ is equivalent to $L(1) > R(1)$ which insures that the first crossing, if it exists, is higher than 1. **(d)** Solving equation (3.7) for G_{t+1}

$$G_{t+1} = \left[\frac{1+v}{\gamma v v_0}\left(\frac{v_0}{\beta}(G_t)^v - 1\right)\right]^{\frac{1}{1+v}},$$

which is a strictly concave function of G_t that first crosses the 45^0 line from below, precisely at the valid BGP G. The implied $\{G_{t+1}\}$ from any $0 < \left|G_0 - G^{BGP}\right| < \epsilon$ would diverge. Therefore, the only possibility is $G_t = G^{BGP}$ for all t. **(e)** If $\gamma = 0$, then $z^E = Z^O$ for all, regardless of the initial distribution. In this case, the economy converges to a uniform entrepreneurs BGP in which $z_{t+1} = (\beta/v_0)^{1/v} Z_t^O$. I now consider $\gamma > 1 - \alpha/[(1-\alpha)v]$, consider any $z_0 < z_1$ and denote by z_i^t the skills levels of the t generation of a dynasty of entrepreneurs starting with z_i for $i = 0, 1$. From Lemma 1, $\lim_{t\to\infty} z_0^t/z_0^t = 0$. Given an initial mass M_i^0 of entrepreneurs of each type i, the mass M_i^t at time t equals $M_0^i \Pi_{\tau=0}^{t-1} n_i^\tau$, where n_i^τ is the labor employed at time τ by the dynasty i. Since $w_\tau(\cdot)$ is non-increasing then $n_0^\tau/n_1^\tau \leq \left(z_\tau^0/z_\tau^1\right)^{\frac{1}{1-\alpha}}$, which is a decreasing sequence that converges to 0. With it $\lim_{t\to\infty} M_0^t/M_1^t = 0$. Since this holds for any pair $z_0 < z_1$, the only possibility is that the dynasty starting with the highest initial skill level takes over the entire population.■

HERE. DISCUSS THE GENERAL CASE. perhaps, construct an argument with iterates on R_1Parts (a) and (b) include the case for $0 < \gamma < 1$, but, as discussed in Appendix A, it is difficult to formally prove the equivalent of results (b) or (c) for those cases. However, as also explained in that appendix, it is straightforward to compute the interior BGP and numerical results suggest that it always exists and that it is unique.

Proof of Proposition 2. Part (a) is straightforward: since $R = 1$, the equilibrium conditons for the closed economy BGP apply to the open economy with $m_f = 0$ because $w_h = w_f$. Part (b) is also straightforward. Assume that there is a BGP with $R_j \leq 1$ with the inequality strict for some $j < \infty$. Since the economy is open, $w_h = w_f$, and $m_f > 0$, otherwise, there would be excess supply of labor because $n_j \leq 1$ with strict inequality for the j s for which $R_j < 1$. Now, if $\gamma = 1$, and by definition $R_0 = 1$, then $R_0^E = 1$ and in equilibrium $R_1 = 1$. Similarly, if $\gamma < 1$ but $\rho = +\infty$, then $R^O = 1$ implying that $R_0^E = 1$, and then $R_1 = 1$. In either case, whenever $R_j = 1$, it is the case that $R_j^E = 1$ and $R_{j+1} = 1$. Therefore, by induction $R_j = 1$ for all $j = 1, 2, ...\infty$ and the result is established. For part (c) recall that the transition function is

$$R_{\text{next}} = F(R) \equiv \left[1 + (R)^{\rho + \frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}}.$$

a twice continously differentiable function. It is easy to verify that $F(1) = 1$ and that

$$F(0) = \begin{cases} 1 & \text{if } \rho > -1/(1-\alpha), \\ 2^{\frac{-\mu}{1-\alpha}} \in (0,1) & \text{if } \rho = -1/(1-\alpha), \\ 0 & \text{if } \rho < -1/(1-\alpha). \end{cases}$$

Moreover, the first derivative is

$$F'(R) = \left(\frac{\mu}{\rho}\right)\left[1 + (R)^{\rho+\frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}\right]^{\frac{\mu}{\rho}-1} \times \left[\left(\rho + \frac{1}{1-\alpha}\right) R^{\rho+\frac{\alpha}{1-\alpha}} - \left(\frac{1}{1-\alpha}\right) R^{\frac{\alpha}{1-\alpha}}\right],$$

which is obviously a continuous function, and moreover, regardless of the parameter values, $F'(1) = \mu > 1$. Therefore, there is an $\varepsilon_1 > 0$ (which may depend on the parameter values) such that $F(R) < R$ for all $R \in (1-\varepsilon_1, 1)$. Therefore, whenever $\rho \geq -1/(1-\alpha)$ the intermediate value theorem implies the existence of an interior BGP because there is an $R \in (0,1)$ such that $F(R) = R$. For the case of $-\infty < \rho < -1/(1-\alpha)$, using de l'Hôpital's rule, it is easy to verify that $\lim_{R\to 0} F'(R) = +\infty$. Then, there exist an $\varepsilon_0 > 0$ (which may depend on the parameter values) such that $F(R) > R$ for all $R \in (0, \varepsilon_0)$. Then, again from the intermediate value theorem the existence of the interior BGP is established for these set of parameters.

To show uniqueness, first define $H(R) \equiv 1 + (R)^{\rho+\frac{1}{1-\alpha}} - (R)^{\frac{1}{1-\alpha}}$. Obviously $F(R) = H(R)^{\frac{\mu}{\rho}}$, $F'(R) = \left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-1\right)} H'(R)$ and

$$F''(R) = \left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-2\right)} \left\{H(R) H''(R) + \left(\frac{\mu}{\rho} - 1\right)[H'(R)]^2\right\},$$

where I have taken the term $\left(\frac{\mu}{\rho}\right) H(R)^{\left(\frac{\mu}{\rho}-2\right)}$ as common factor and then simplified. This common term is positive for all $R>0$ hence $sign\{F''(R)\} = sign\left\{H(R)H''(R) + \left(\frac{\mu}{\rho}-1\right)[H'(R)]^2\right\}$. Since $[H'(R)]^2$ is obviously positive, the term $\left(\frac{\mu}{\rho}-1\right)[H'(R)]^2$ is always positive, zero or negative, depending on whether $\frac{\mu}{\rho} \gtreqless 1$. $H(R)$ is always positive can only change signs if $H''(\cdot)$ changes signs. The unique inflection point of $H(\cdot)$, where it changes from concave to convex or vice-versa is $R^I = \left\{\alpha/\left[(1-\alpha)\left(\rho^2(1-\alpha)+\alpha\rho\right)\right]\right\}^{\frac{1}{\rho}}$, which may or may not fall in the interval $[0,1]$. In any event, the transition function $F(\cdot)$ has at most one inflection point, which is (larger than) [smaller than] R^I if $\frac{\mu}{\rho}=1$ ($\frac{\mu}{\rho}<1$) [$\frac{\mu}{\rho}>1$]. Because $\mu>1$, if $F(\cdot)$ changes curvature it is necessarity from concave to convex and since $\lim_{R\to 1} F'(R) = \mu > 1$, then, $F(\cdot)$ crosses at most once the 45^0 line in the interval $[0,1]$. But we have established already that it crosses it at least once, hence the crossing exists and is unique.

Finally, to establish global stability of the interior BGP, recall that existence was established by showing $F(R)>R$ for some a lower sub-interval of $[0,1]$ and $F(R)<R$ for a higher sub-interval. Because of this, the continuity of F and the fact that its crossing is unique, then $F(\cdot)$ must cross the 45^0 from above, which implies the global estability of the BGP.■

Proof of Lemma 2. If $\beta(1-2\alpha) \geq v_0 G^v/(1+v)$, then when all the initial old have the same skill level Z: (a) all of the old are better off being active entrepreneurs and (b) all young individuals would be better off investing and become entrepreneurs in the next period. Therefore, optimal occupation choices are satisfied, and, by assuming all the parameter conditions of Proposition 1, then the allocations are an equilibrium. Alternatively, if $\beta(1-2\alpha) < v_0 G^v/(1+v)$,if all the young invest, then the net present value $w - Z\phi(G) + \beta G\pi$ would be strictly below the option $w+\beta G w$ of not investing and remaining a worker next period. If only ω of the young invest, then, in the BGP, the aggregate supply of labor will be $2-\omega$ (all the mass 1 of young individuals plus the mass $1-\omega$ of the old who did not invest) and the supply of entrepreneurs will be ω. With homogeneous entrepreneurs with skills Z, workers' wages are equal to $w = \alpha[(2-\omega)/\omega]^{\alpha-1} Z$ and entrepreneurs'earnings are $\pi = (1-\alpha)[(2-\omega)/\omega]^{\alpha} Z$. By construction, the condition

$$\beta\left(\frac{2-\omega}{\omega}\right)^{\alpha}\left(\frac{2(1-\alpha)-\omega}{2-\omega}\right) = \frac{v_0}{1+v}G^v,$$

implies that the fraction ω makes the a young individual to be indifferent between remaining a worker or investing at the rate G and becoming an entrepreneur; likewise, the first root of

$$\beta\left(\frac{2-\omega}{\omega}\right)^{\alpha}\left(1+\frac{\gamma v v_0}{(1+v)}G^{1+v}\right) = v_0 G^v.$$

implies that for those investing, the growth rate G is optimal, given than only ω of the current young are investing.■

Proof of Proposition 3. To be added.

B. Aspects of the Model with Middle-Managers and Labor

Given the wage of workers w, and the wage function $w_m(\cdot)$ for middle-managers, the earnings of an entrepreneur with skills z are

$$\pi\left[z; w_m(z), w\right] \equiv \max_{\{n,l\}} \left\{ z n^{\alpha} l^{\lambda} - w_m n - w l \right\} = (1-\alpha-\lambda)\, z^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{w_m(z,Z)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} \left(\frac{\lambda}{w} \right)^{\frac{\lambda}{1-\alpha-\lambda}}.$$

Consider first a closed economy in which all entrepreneurs have skills $Z_h = Z > 0$. From the market-clearing conditions, $n = 1$ and $l = \varrho$, the wage for workers is $w = \lambda \varrho^{\lambda-1} Z$ and an the wage function $w_m(z, Z)$ must satisfy $w_m(Z, Z) = \alpha Z \varrho^{\lambda}$. In this case, the earnings of entrepreneurs are $\pi = \pi(Z, Z) = (1-\alpha-\lambda) Z \varrho^{\lambda}$.

If instead the entrepreneur had skills z, given Z, Z_{t+1} $w_{m,t+1}(z', Z_{t+1})$, and w_{t+1}, he would have to pay his mid-managers wages defined by

$$w_m(z, Z) = w_m(Z, Z) + V\left[Z, w_{m,t+1}(\cdot), w_{t+1}\right] - V\left[z^{\gamma} Z^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1}\right] \tag{B.1}$$

to compesate for better or worse learning possibilities. Here, similar to the basic model, the learning or career value $V(\cdot)$ of a mid-management job with exposure z^E of ideas is

$$V\left[z^E, w_{m,t+1}(\cdot), w_{t+1}\right] \equiv \max_{z'} \left\{ \beta \pi\left[z', w_{m,t+1}(z', Z_{t+1}), w_{t+1}\right] - z^E \phi\left(z'/z^E\right) \right\}. \tag{B.2}$$

The optimal skill accumulation z' for the mid-manager in that job is

$$\beta \left[\pi_1\left[z', w_{m,t+1}(z', Z_{t+1}), w_{t+1}\right] + \pi_2\left[z', w_{m,t+1}(z', Z_{t+1}), w_{t+1}\right] \frac{\partial w_{m,t+1}(z', Z_{t+1})}{\partial z'} \right] = \phi'\left(\frac{z'}{z^E} \right), \tag{B.3}$$

where $\pi_1(\cdot)$ and $\pi_2(\cdot)$ stand for, respectively, the derivative of π with respect to the the manager's skill z and with respect to the wage $w_{m,t+1}(z, Z)$ that he must pay his mid-managers, which are given by

$$\pi_1(\cdot) = z^{\frac{\alpha+\lambda}{1-\alpha-\lambda}} \left(\frac{\alpha}{w_m(z,Z)} \right)^{\frac{\alpha}{1-\alpha-\lambda}} \left(\frac{\lambda}{w} \right)^{\frac{\lambda}{1-\alpha-\lambda}} \text{ and } \pi_2(\cdot) = -z^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{w_m(z,Z)} \right)^{\frac{1--\lambda}{1-\alpha-\lambda}} \left(\frac{\lambda}{w} \right)^{\frac{\lambda}{1-\alpha-\lambda}}.$$

Finally, from (B.1), $\frac{\partial w_{t+1}(z')}{\partial z'} = -V_1\left[\left(z^E\right)', w_{t+2}(z'') \right] \frac{\partial \left(z^E\right)'}{\partial z'}$, and applying the envelope condition on V one period ahead:

$$V_1\left[\left(z^E\right)^{t+1}, w_{m,t+2}(\cdot), w_{t+2} \right] = -\frac{v v_0}{1+v} \left(\frac{z^{t+2}}{\left(z^E\right)^{t+1}} \right)^{1+v},$$

where I have used the functional form assumed for $\phi(\cdot)$.

BGP closed economies. Impose $z_t = z_t^E = Z_t$, and $G = Z_{t+1}/Z_t$ for all periods t. Plugging $w_{t+1} = \lambda \varrho^{\lambda-1} Z_{t+1}$ and $w_{m,t+1}(Z, Z) = \alpha \varrho^{\lambda} Z_{t+1}$, we get, that theh derivatives boil down to $\pi_1 = \varrho^{\lambda}$ and $\pi_2 = -1$; we also get $V_1 = -\frac{v v_0}{1+v} (G)^{1+v}$, $\frac{\partial \left(z^E\right)'}{\partial z'} = \gamma$, and that $\phi'\left(\frac{z'}{z^E}\right) = v_0 G^v$. Plugging all of these in (B.3), the resulting expression is

$$\beta \left(\varrho^{\lambda} + \frac{\gamma v v_0}{1+v} G^{1+v} \right) = v_0 G^v,$$

which is exactly the condition in the text.

The reminder of equilibrium conditions (ex-ante and ex-post optimality of occupation choices) follow exactly the same lines as discussed above for the basic model (i.e. $\lambda = 0$ and $\omega = 1$).

BGP open economies. Home being small and open free entry and foreign being in a BGP, implies that at home the wage of workers is equal to $w = \lambda \varrho^{\lambda-1} Z_f$, and that foreign firms pay domestic mid-managers wages equal to $w_{m,\,t+1}(Z_f) = \alpha \varrho^{\lambda} Z_f$. Then, a domestic firm with skills z must pay a wage

$$w_m(z, Z_f) = w_m(Z_f) + V\left[(Z_f)^{\gamma} \left(Z^O\right)^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1} \right] - V\left[z^{\gamma} \left(Z^O\right)^{1-\gamma}, w_{m,t+1}(\cdot), w_{t+1} \right], \tag{B.4}$$

where Z^O is the outside set of ideas circulating in the country, which must be determined as part of the equilibrium. Let $R = z/Z_f$ and denote by $d(R)$ the compensating differential, relative to Z_f, of an entrepreneur with skills $z =$

RZ_f (to shorten the notation, I omit indicating the dependence on Z^O). That is, $w_m(z, Z_f) = \left[\alpha\varrho^\lambda + d(z/Z_f)\right] Z_f$. After simplifying, the derivatives π_1 and π_2 become

$$\pi_1 = \Phi R^{\frac{\alpha+\lambda}{1-\alpha-\lambda}} \left(\frac{\alpha}{\alpha\varrho^\lambda + d(R)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} \quad \text{and } \pi_2 = -\Phi R^{\frac{1}{1-\alpha-\lambda}} \left(\frac{\alpha}{\alpha\varrho^\lambda + d(R)}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}. \tag{B.5}$$

where $\Phi \equiv [\varrho]^{\frac{\lambda(1-\lambda)}{1-\alpha-\lambda}}$. Next, using the fact that $Z_{f,\,t+2}/Z_{f,\,t+1} = G$, defining $R^E \equiv z^\gamma \left(Z^O\right)^{1-\gamma}/Z_f = R^\gamma \left(R^O\right)^{1-\gamma}$ as we defined in the text for the basic model, then

$$V_1\left[\left(z^E\right)^{t+1}, w_{m,t+2}(\cdot), w_{t+2}\right] = -\frac{vv_0}{1+v}\left(G\frac{R_{t+2}}{R^E_{t+1}}\right)^{1+v}, \tag{B.6}$$

where R^E_{t+1} is the relative exposure to ideas that the entrepreneur will provide to his mid-managers in the next period and R_{t+2} is their relative accumulation of skills. It is also straightforward to compute,

$$\frac{\partial\left(z^E\right)^{t+1}}{\partial z^{t+1}} = \gamma\frac{R^E_{t+1}}{R_{t+1}} = \gamma\left(\frac{R^{O,t+1}}{R^{t+1}}\right)^{1-\gamma}. \tag{B.7}$$

Finally,

$$\phi'\left(\frac{z_{t+1}}{z^E_t}\right) = v_0\left[G\frac{R_{t+1}}{R^E_t}\right]^v. \tag{B.8}$$

Plugging conditions (B.5), (B.6), (B.7) and (B.8) into condition (B.3), implies that the optimal accumulation of skills R_{t+1} for a worker of a mid-manager of an entrepreneur with relative skills R_t

$$\beta\Phi R^{\frac{\alpha+\lambda}{1-\alpha-\lambda}}_{t+1}\left[\left(\frac{\alpha}{\alpha\varrho^\lambda + d(R_{t+1})}\right)^{\frac{\alpha}{1-\alpha-\lambda}} + \frac{\gamma vv_0 R^E_{t+1}}{1+v}\left(\frac{\alpha}{\alpha\varrho^\lambda + d(R_{t+1})}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}\left(G\frac{R_{t+2}}{R^E_{t+1}}\right)^{1+v}\right] = v_0\left[G\frac{R_{t+1}}{R^E_t}\right]^v,$$

where I have grouped and simplified terms involving R_{t+1}. Because R^E_{t+1} obviously depends on R_{t+1}, it is instructive to re-write this condition as

$$\beta\Phi R^{\frac{\alpha+\lambda}{1-\alpha-\lambda}}_{t+1}\left[\left(\frac{\alpha}{\alpha\varrho^\lambda + d(R_{t+1})}\right)^{\frac{\alpha}{1-\alpha-\lambda}} + \frac{\gamma vv_0}{1+v}\left(\frac{\alpha}{\alpha\varrho^\lambda + d(R_{t+1})}\right)^{\frac{1-\lambda}{1-\alpha-\lambda}}\frac{(GR_{t+2})^{1+v}}{(R_{t+1})^{\gamma v}\left(R^O_{t+1}\right)^{(1-\gamma)v}}\right] = v_0\left[G\frac{R_{t+1}}{R^E_t}\right]^v. \tag{B.9}$$

Notice that interpreting t as a 'vintage', this condition determines the time invariant $\{R_j\}_{j=1}^\infty$ for an open economy BGP. Also, notice that forcing $\lambda = 0$, the condition boils down to the the equation in the text for basic model.

Now, to compute the compensating differentials $d(R)$, first observe that the earnings of an entrepreneur with skills z can be written as

$$\pi(z, Z_t) = Z_f\left[\Phi(1-\alpha-\lambda)R^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d(R)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}\right].$$

Let let R^1_{t+1} denote the relative skills of domestic entrepreneur who was trained at time t in a foreign firm.Then, the career value for a mid-manager in a foreign firm is

$$V_{f,t} = Z_{f,t}\left[\beta G\Phi(1-\alpha-\lambda)\left(R^1_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R^1_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} - \left(R^O_t\right)^{1-\gamma}\frac{v_0}{1+v}\left(\frac{GR^1_{t+1}}{\left(R^O_t\right)^{1-\gamma}}\right)^{1+v}\right],$$

and his life-time utility is $\alpha\varrho^\lambda Z_{t+1} + V_{f,t.}$. On the other hand, working for a domestic firm with relative skills R, the mid-manager would attain

$$Z_{f,t}\left[\alpha\varrho^\lambda + d(R) + \beta G\Phi(1-\alpha-\lambda)\left(R^R_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^\lambda + d\left(R^R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}} - \frac{v_0}{1+v}\frac{\left(GR^R_{t+1}\right)^{1+v}}{\left(R^\gamma\left(R^O_t\right)^{1-\gamma}\right)^v}\right]$$

where R^R_{t+1} indicates the optimal skill accumulation –that solves (B.9)– when exposed to any level R. Equating this expression with the life-time utility attainable working for a foreign firm, then

$$\begin{aligned} d(R) &= \beta G\Phi(1-\alpha-\lambda)\left[\left(R^1_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R^1_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}-\left(R^R_{t+1}\right)^{\frac{1}{1-\alpha-\lambda}}\left(\frac{\alpha}{\alpha\varrho^{\lambda}+d\left(R^R_{t+1}\right)}\right)^{\frac{\alpha}{1-\alpha-\lambda}}\right] \\ &+\left(\frac{v_0}{1+v}\right)\frac{G^{1+v}}{\left(R^O_t\right)^{(1-\gamma)v}}\left[\frac{\left(R^R_{t+1}\right)^{1+v}}{R^{\gamma v}}-\left(R^1_{t+1}\right)^{1+v}\right]. \end{aligned}$$

Instead of solving for any arbitrary levels R, we can restrict this equation for the time-invariant vintage structure in a BGP for an open economy using j to index each vintage, i.e. $d_j \equiv d(R_j)$. Notice that this is a generalization of the expression in the text. First, it does not presume $\lambda = 0$; if it did, we recover the expression in the text for the basic model. Second, it allows the computation for time varying $\left\{R^t_j,\ d^t_j,\ R^{O,t} : j \geq 1,\ t \geq 0\right\}$. Indeed, on the basis of these equations, I use a simple shooting algorithm on $R^{O,t}$ to compute for the transition dynamics of a closed economy starting in a BGP to the final steady state after openness.

References

[1] Agarwal, R., Echambadi, R., Sarkar, M. and Franco, A. (2004)"Knowledge Transfer through Inheritance : Spin-out Generation, Growth, and Survival," Academy of Management Journal, Vol. 47 no. 4, 501-522.

[2] Aitken, A. and Harrison, B., (1999) "Do Domestic Firms Benefit from Direct Foreign Investment? Evidence from Venezuela", American Economic Review, Vol. 89, pp. 605-618.

[3] Alfaro, L., Chanda, A., Kalemli-Ozcan, S. and Sayek, S. (2006) "How Does Foreign Direct Investment Promote Economic Growth? Exploring the Effects of Financial Markets on Linkage", HBS, Working paper.

[4] Alvarez, F. and Stokey, N. (1998) "Dynamic Programming with Homogeneous Functions." Stokey; Journal of Economic Theory, Vol. 82, N.1, pp. 167-89.

[5] Antras, P., Garicano, L., and Rossi-Hansberg, E. (2005) "Offshoring in a Knowledge Economy," forthcoming Quarterly Journal of Economics.

[6] Barba-Navaretti, G. and Venables, A. (2004), "Multinational Firms in the World Economy," Princeton University Press.

[7] Boyd, J. and Prescott, E. (1987) "Dynamic Coalitions: Growth and the Firm," in Contractual Arrangements for Intertemporal Trade, in E. C. Prescott and N. Wallace, eds., University of Minnesota Press, pp. 146–160.

[8] Burstein, A. and Monge-Naranjo, A. (2009) "Foreign Know-How, Firm Control, and the Income of Developing Countries" The Quarterly Journal of Economics, Forthcoming, February 2009.

[9] Chari, V. and Hopenhayn, H. (1991) "Vintage Human Capital, Growth, and the Diffusion of New Technology" The Journal of Political Economy, Vol. 99, No. 6

[10] Easterly, W. (2001) The Elusive Quest for Growth: Economists' Adventures and Misadventures in the Tropics, MIT Press.

[11] Findlay, R. (1978) "Relative Backwardness, Direct Foreign Investment, and the Transfer of Technology: A simple dynamic model," Quarterly Journal of Economics, Vol. 92, pp. 1-16.

[12] Filson, D. and Franco, A. (2006) "Spin-outs: Knowledge Diffusion through Employee Mobility," Rand Journal of Economics, forthcoming.

[13] Franco, A. (2005) "Employee Entrepreneurship: Recent Research and Future Directions," Handbook of Entrepreneurship, volume 2.

[14] Griffith, R., Redding, S. and Simpson, H. "Productivity Convergence and Foreign Ownership at the Establishment Level", The Institute For Fiscal Studies, WP 02/22,

[15] Gourinchas, P. and Jeanne, O. (2003) "The Elusive Gains from International Financial Integration," NBER Working Papers 9684.

[16] Grossman, G. and Helpman, E. (1991) "Trade, Knowledge Spillovers and Growth", European Economic Review, Vol. 35, N. 3, pp. 517-526.

[17] Helpman, E. (1984) "A Simple theory of international trade with multinational corporations," Journal of Political Economy, Vol. 92, pp.451-471.

[18] Holmes, T. and Schmitz, J. (1990) "A Theory of Entrepreneurship and Its Application to the Study of Business Transfers" Journal of Political Economy, volume 98, page 265

[19] Howitt, P. (2000) "Endogenous Growth and Cross-Country Income Differences," American Economic Review, Vol. 90, pp. 829-46.

[20] Javorcik, B. (2004) "Does Foreign Direct Investment Increase the Productivity of Domestic Firms? In Search of Spillovers Throught Backward Linkages" American Economic Review, Vol. 94, N. 3, pp. 605-627.

[21] Jones, C. (2006) "Growth and Ideas" in Handbook of Economic Growth, Elsevier.

[22] Jovanovic, B. and Rob, R. (1989) "The Growth and Diffusion of Knowledge" Review of Economic Studies, Vol. 56, pp. 569-582.

[23] Jovanovic, B. and Nyarko, Y. (1995) "The Transfer of Human Capital." Journal of Economic Dynamics and Control, Vol. 19 (1995), pp. 1033-1064.

[24] Kaldor, N. (1934) "The Equilibrium of the Firm," Economic Journal, Vol. 44.

[25] Keller, W. (2004) "International Technology Diffusion", Journal of Economic Literature, Vol. XLII, pp. 752-782.

[26] Klepper, S. (2001) "Employee Startups in High-Tech Industries," Industrial and Corporate Change, Oxford University Press, vol. 10(3), pages 639-74, September.

[27] Keppler, S. (2002) "The capabilities of new firms and the evolution of the US automobile industry," Industrial and Corporate Change, Oxford University Press, vol. 11(4), pages 645-666, August.

[28] Keppler, S. (2006) "Strategic Disagreements, Spinoffs, and the Evolution of Detroit as the Capital of the U.S. Automobile Industry" Carnegie Mellon University, working paper.

[29] Klein, P. and Ventura, G. (2004) "Do Migration Restrictions Matter?" University of Western Ontario working paper.

[30] Klenow, P. (1998), "Ideas vs. Rival Human Capital: Industry Evidence on Growth Models," Journal of Monetary Economics, Vol. 42, pp. 3-24.

[31] Klenow, P. and Rodríguez-Clare, A. (1997) "The Neoclassical Growth Revival: Has it Gone too Far?" NBER Macroeconomics Annual 1997.

[32] Klenow, P. and Rodríguez-Clare, A. (2005) "Externalities and Growth," forthcoming, Handbook of Economic Growth.

[33] Kugler, M. (2005) "Spillovers from foreign direct investment: within or between industries?" forthcoming Journal of Development Economics.

[34] Lucas, R. (1978) "On the Size Distribution of Business Firms," Bell Journal of Economics, Vol. 9, pp. 508-523.

[35] Markusen, J. (2004) Multinational Firms and the Theory of International Trade, The MIT Press.

[36] Markusen, J., Venables, A., (1998) "Multinational firms and the new trade theory," Journal of International Economics, Vol. 46, pp.183-203

[37] Poole, J. (2006) "Multinational Spillovers throught Worker Turnover" UCSD working paper.

[38] Oi, W. (1983) "Heterogeneous Firms and the Organization of Production," Economic Inquiry, 21, pp. 147-171.

[39] Prescott, E. and Boyd, J. (1987) "Dynamic Coalitions: Engines of Growth" American Economic Review Papers and Proceedings, Vol. 77, No. 2, pp. 63-67

[40] Prescott, E. and Visscher, M. (1980) "Organization Capital," Journal of Political Economy, vol. 88(3), pp. 446-61.

[41] Ramondo, N. (2006) "Size, Geography, and Multinational Production", U of Texas working paper.

[42] Rodríguez-Clare, A. (1996) "Multinationals, Linkages and Economic Development," American Economic Review, Vol. 86, pp. 852-873.

[43] Rodríguez-Clare, A. (2006) "Trade, Diffusion and the Gains from Openness" PSU working paper.

[44] Romer, P. (1986) "Increasing Returns and Long-Run Growth" Journal of Political Economy, Vol. 94, pp. 1002-37.

[45] Rosen, S. (1982) "Authority, Control, and the Distribution of Earnings," Bell Journal of Economics, 13, pp. 311-323.

[46] Stokey, N. (1991) "Human Capital, Product Quality, and Growth" The Quarterly Journal of Economics, Vol. 106, No. 2. (May, 1991), pp. 587-616.

[47] Teece, D. (1977) "Technology Transfer by Multinational Firms: The Resource Cost of Transferring Technological Know-How", The Economic Journal, Vol. 87, pp. 242-261.

[48] UNCTAD (1994) World Investment Report 1994: Transnational Corporations, Employment and the Workplace, United Nations, New York and Geneva.

[49] Xu, B (2000) "Multinational enterprises, technology diffusion, and host country productivity growth" Journal of Development Economics, vol. 62, issue 2, pages 477-493