

Trading Dynamics in Decentralized Markets with Adverse Selection

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February 15, 2011

Preliminary – Please Do Not Circulate

Abstract

We study a dynamic, decentralized market environment with asymmetric information and interdependent values between buyers and sellers, and characterize the complete set of non-stationary equilibria. For a given fraction of low-quality assets, or “lemons,” the model describes how prices, the volume of trade, and the composition of assets will evolve over time. Comparing economies in which the initial fraction of lemons varies, the model delivers a stark relationship between the severity of the lemons problem and *market liquidity*. We use this framework to understand how asymmetric information has contributed to the “frozen” credit market at the core of the current financial crisis, and to evaluate the efficacy of one of the policies that was implemented in attempt to restore liquidity.

*Camargo gratefully acknowledges financial support from CNPq.

†Lester gratefully acknowledges financial support from the Social Sciences and Humanities Research Council of Canada, as well as the Bank of Montreal professorship. The usual disclaimer applies.

1 Introduction

A central problem in the current financial crisis has been the inability of financial institutions to sell illiquid assets on their balance sheets. More specifically, banks holding large amounts of structured asset-backed securities, such as collateralized debt obligations and credit default swaps, have been mostly unable to find buyers for these assets. This “frozen” market has posed perhaps the greatest risk to the economy as a whole; if financial institutions can not acquire liquid assets (e.g. cash) in exchange for these illiquid assets, they can not make loans. As a result, consumers have more difficulty buying cars and homes, and businesses cannot acquire the financing they need for new investment. This, in turn, can lead to a further decrease in asset prices and a decline in economic growth. Given the danger associated with this downward spiral, the task of identifying the underlying frictions in this market, and understanding the inefficiencies introduced by these frictions, is of crucial importance. Without such an understanding, market participants remain unsure of how this market will behave in the future, and policymakers remain unsure of the optimal form of intervention.

While one could point to a number of potential reasons that trade in this market has broken down, many believe that the primary cause was *asymmetric information*. The story is simple: at the onset of the financial crisis, it became apparent that many assets being held by financial institutions were worth considerably less than had been previously claimed; they were of low quality or, in the language of Akerlof (1970), they were “lemons.” Of course, these financial institutions also held assets of higher quality, whose fundamental value (though difficult to discern) was likely at or near pre-crisis evaluations. However, as these assets tend to be relatively complex, it was quite difficult to differentiate high quality from low. This was especially true for buyers; the financial institutions that were selling these assets often had a team of analysts that had purchased the underlying assets (e.g. mortgages), studied their properties, and worked closely with the rating agencies to bundle them together in such a way as to maximize profits.¹ Thus the market had many of the

¹An extreme example of this asymmetric information is the “Abacus” deal, in which Goldman Sachs created and sold collateralized debt obligations to investors, while simultaneously betting against them. In general, there are many reasons to believe that financial institutions often had better information about the

basic ingredients of Akerlof’s classic “market for lemons”: sellers possessed assets that were heterogeneous in quality, and they were more informed about the quality of their assets than potential buyers. The most basic theory would predict that, in this type of environment, trade can break down completely.

However, there are several important features of this particular market that are not consistent with the assumptions typically embedded in existing models of markets with asymmetric information. For one, the market is *decentralized*; in contrast to the standard competitive paradigm, where the law of one price prevails, buyers and sellers in this market typically negotiate bilaterally. Therefore, a model of this market must allow the exchange of different quality assets to take place at potentially different prices. Moreover, the market is inherently *dynamic* and *non-stationary*; any serious analysis has to consider the manner in which the composition of assets in the market evolves over time, and how this affects both prices and the incentive of market participants to delay trade. Simply put, the notion that a buyer or seller is waiting for market conditions to improve – which is a common feature of these “frozen” markets – simply cannot be analyzed in a stationary environment, where market conditions are constant.

As we discuss below, there is not an existing theoretical model that incorporates all of these basic features into a single, coherent framework. As a result, though the problems within the markets for asset-backed securities and the optimal form of intervention have garnered a lot of attention, the majority of this discussion has had to take place outside the realm of formal economic analysis. The purpose of this paper is to develop a rigorous economic model that captures the important features of the market discussed above, and to use this model as a laboratory for understanding the effects of informational asymmetries on the patterns of trade in dynamic, decentralized market settings.

To be more specific, we consider a discrete time, infinite horizon, one-time entry model with an equal measure of buyers and sellers. Sellers each possess a single good of hetero-

quality of their assets than potential buyers, perhaps because they learn about the asset while they own it (as argued by Bolton et al. (2011)), or because they conduct research about the asset in anticipation of selling it (as argued by Guerrieri and Shimer (2010)). By now, there is a large literature on the role of asymmetric information in the financial crisis; see Gorton (2009), Heider et al. (2008), and the references therein.

geneous quality (high or low), and this quality is private information. At $t = 0$, a fraction q_0 of sellers possess a high quality asset, and the remainder possess a low quality asset. In each period $t = 0, 1, 2, \dots$, buyers and sellers are randomly matched, and buyers make a price offer chosen from an exogenously specified set of prices. The parameters are such that there are strictly positive gains from trade in every match. If a seller accepts the buyer's offer, trade ensues and the pair exits; if the seller rejects, the pair remain in the market and are randomly matched again the following period. Finally, we assume that agents are subject to stochastic discount factor shocks in each period, which we interpret as liquidity shocks across agents and over time.²

Within this environment, we completely characterize the set of equilibrium for all $q_0 \in (0, 1)$, and use this characterization to study the effects of asymmetric information on the dynamics of trade. First, given any q_0 , we show that all assets will be bought and sold (i.e. the *market will clear*) in a finite number of periods. The model delivers sharp predictions about the path of price offers, the volume of trade, and the composition of high and low quality assets over time; more specifically, we show that both average price offers and the average quality of assets in the market increase over time. Eventually the average quality is sufficiently high that all remaining buyers offer a price that is acceptable to sellers with high-quality assets, all remaining sellers accept, and the market clears.

However, the amount of time it takes until the market clears depends crucially on q_0 , as does the expected amount of time it takes for a seller with a high-quality asset to trade. We derive this latter statistic, and interpret it as the *liquidity* of the market for high-quality assets; a liquid market is one where sellers can quickly find a buyer to purchase their high-quality asset (at an acceptable price), whereas an illiquid market is one where this process takes a long time. We show that there is a monotonically decreasing relationship between this measure of liquidity and q_0 : as the lemons problem becomes less severe, the market for high-quality assets becomes more liquid. Thus our model provides a novel, micro-founded theory of liquidity that varies systematically with the composition of assets in the economy.

We also show that there exist multiple equilibria for some values of q_0 , and that these

²This is technically convenient, as it allows us to focus on pure strategy equilibrium.

equilibria have different liquidity properties. The driving force behind this multiplicity is that there are complementarities between buyers' actions. For example, when other buyers are offering high prices, both high- and low-quality sellers accept these offers in equal proportion, so that the future composition of high- and low-quality sellers in the market is the same as in the present. This provides buyers less incentive to wait for future periods to trade, and more incentive to offer a high price now. Hence an equilibrium emerges in which markets clear immediately. However, if other buyers are offering low prices, the proportion of low-quality sellers who accept this offer is greater than that of high-quality sellers, and consequently average quality increases in future periods. This causes future payoffs to rise, and thus provides buyers less incentive to offer a high price and trade immediately. As a result, an equilibrium emerges in which markets do not clear immediately. The existence of multiple equilibria for a given q_0 suggests that there is scope for coordination failures in dynamic, decentralized markets with asymmetric information, which provides an additional source of potential illiquidity in these markets.

In addition to providing a set of positive predictions about the patterns of trade in these types of markets, our model also provides a parsimonious framework to formally analyze how different forms of intervention could affect trading dynamics, and market liquidity in particular. To illustrate this, we consider one specific program introduced by the treasury department in order to restore liquidity in the market for asset-backed securities: the Public-Private Investment Program for Legacy Assets. We introduce a stylized version of this program into our model, and show that its efficacy depends crucially on the underlying state of the world. In particular, we show that this policy can improve liquidity if q_0 is sufficiently large, but it will have no effect – and could even *decrease* market liquidity – for smaller values of q_0 . This result underscores the need for rigorous models to formally evaluate the usefulness of policy intervention in attempt to correct these types of market failures.

1.1 Related Literature

Our work builds on the literature that studies dynamic, decentralized markets with asymmetric information and interdependent values. The primary focus of this literature has been

to determine what happens to equilibria in a decentralized environment as market frictions vanish.³ See Inderst (2005) and Moreno and Wooders (2010) for a steady-state analysis of this issue, and Moreno and Wooders (2002) and Blouin (2003) for analysis of this issue in a one-time entry model. Janssen and Roy (2002) also study a dynamic environment with asymmetric information and interdependent values; however, they assume trade takes place in a sequence of Walrasian markets. Though the framework we develop shares certain features in common with several of these papers, the focus will be quite different. We are interested in studying the relationship between information frictions and market liquidity, and the manner in which both market participants and policymakers can respond to overcome these frictions.

Several recent papers share a similar focus. Perhaps most relevant is Chiu and Koeppl (2009), who introduce asymmetric information into the random-matching framework of Duffie et al. (2005) and characterize regions of the parameter space in which trade shuts down (in a steady-state equilibrium). They, too, analyze the effect of intervention on trading dynamics, and show that a direct purchase of low-quality assets by the government can help to restore liquidity. We highlight the crucial differences between this model and our own Section 6. Other recent papers studying the effects of asymmetric information on asset market liquidity and policy interventions include Guerrieri and Shimer (2010), who utilize a competitive search framework, and Heider et al. (2008), who set their analysis within the maturity mismatch framework of Diamond and Dybvig (1983).

More generally, this paper adds to the class of models that provide a theory of endogenous market liquidity based on asymmetrically informed counterparties. Rocheteau (2009) provides an excellent survey of search-based models in which information frictions interfere with exchange, and thus decrease liquidity. In a separate strand of this literature, which assumes that trade takes place in a centralized competitive market, the pooling equilibrium price decreases as the number of lemons increases, thus decreasing a seller's ability to ex-

³Note that this was an exercise first conducted in a perfect information setting by Rubinstein and Wolinsky (1985) and Gale (1986a, 1986b). A parallel literature has emerged that studies dynamic, decentralized markets with imperfect information and *private values*; see, for example, Satterthwaite and Shneyerov (2007) and the references therein.

change their good for cash. In this sense, a lemons problem reduces liquidity; see Eisfeldt (2004) for an early contribution in this literature, and Kurlat (2009) for a more recent application. Finally, the dominant theory of liquidity in the finance literature, pioneered by Glosten and Milgrom (1985) and Kyle (1985), also uses informational asymmetries to generate differences in liquidity by focusing on the problem of a market-maker, and treating the size of the bid-ask spread as a measure of liquidity.

Finally, our paper is also closely related to the literature that studies sequential bargaining between a single seller and a single buyer in the presence of asymmetric information.⁴ Most relevant to the current project is the work of Vincent (1989), Evans (1989), and Deneckere and Liang (2006), who study the dynamic bargaining game in which a seller has private information about the quality of her good, a buyer makes offers in each period, and the buyer's valuation of the good is correlated with the seller's valuation. Equilibria in this environment tend to have the property that buyers use time to screen the different types of sellers: initially buyers will make low offers that only very low type sellers would accept.⁵ If the seller rejects such an offer, the buyer learns that the seller is not a very low type, and updates his posterior accordingly. In the following period, his offer increases, and so on. This notion of using price dispersion over time to overcome the problem of adverse selection is central to our work, as well as the majority of papers cited above.⁶ What is different about the market setting we consider, as opposed to the single buyer/seller setting considered in much of the bargaining literature, is that complementarities can arise in a market setting between e.g. a buyer and other buyers. We discuss this point in depth later in the paper.

2 The Model

Time is discrete, and begins in period $t = 0$. There is an equal measure of infinitely lived buyers and sellers, which we normalize to one. At $t = 0$, each seller possesses a single,

⁴Seminal contributions in this literature include Fudenberg et al. (1985) and Gul et al. (1986), among others.

⁵A "low type seller" is a seller with a good of very low quality.

⁶This basic idea goes back to, at least, Wilson (1980).

indivisible asset, which is either of high (H) quality or low (L) quality. The fraction of sellers with a high quality asset at $t = 0$ is denoted by $q_0 \in (0, 1)$. We describe below the payoffs to a buyer and a seller from each type of asset.

In each period, each agent's discount factor δ is drawn from a continuous and strictly increasing c.d.f. F with support $[0, \bar{\delta}]$, where $\bar{\delta} < 1$. These draws are i.i.d. across both agents and time. This is meant to capture the idea that buyers and sellers have different needs at different times. At a given time, some sellers may need to sell their asset urgently, while others may be more patient. Likewise, at a given time some buyers may desire consumption urgently, while others may be more patient. Across time, each individual agent may be more or less patient in any given period.⁷ The assumption that F is strictly increasing rules out mass points in the distribution of discount factors.

An asset of quality $j \in \{L, H\}$ yields flow utility y_j to a seller in each period that he holds the asset. It will be convenient to denote the present discounted lifetime value of a type $j \in \{L, H\}$ asset to a seller, computed before the seller draws his discount factor, by c_j , where

$$c_j = \frac{y_j}{1 - \mathbb{E}[\delta]}, \quad (1)$$

with $\mathbb{E}[\delta] = \int \delta dF(\delta) < 1$. We normalize y_L to zero, so that $c_L = 0$. When a buyer purchases the asset, we assume that he receives instantaneous utility u_j , and that⁸

$$u_H > y_H + \bar{\delta}c_H > u_L > c_L = 0. \quad (2)$$

The assumptions that $u_H > y_H + \bar{\delta}c_H$ and $u_L > 0$ assure us that there are gains from trade in every match.⁹ The assumption that $y_H + \bar{\delta}c_H > u_L$ generates the lemons problem, as

⁷Note that all types of agents draw their discount factors from the same distribution F . Though this is non-essential, we think that it is reasonable. For a deeper look at the use of random discount factors, see Higashi et al. (2009).

⁸Since (as we describe below) buyers exit the market upon trading, it is easiest to model this as an instantaneous payoff. One could define u_j as the expected discounted lifetime value of flow payoffs to a buyer, but this would make the analysis more cumbersome without providing any additional insights. More precisely, if buyers receive flow payoffs $y_j^B > y_j$ and we define $u_j = y_j^B + \delta y_j^B / (1 - \mathbb{E}[\delta])$, since buyers have heterogeneous discount factors, they would be heterogeneous with respect to u_j as well. The current formulation allows for sellers to receive flow payments while they own the asset, without introducing any additional heterogeneity in the buyers' payoffs.

⁹As in Duffie et al. (2005), our preference specification is such that buyers and sellers receive different

the price that buyers are willing to pay for a low quality asset would not be accepted by a sufficiently patient high quality seller.

In every period, after the agents draw their discount factors, buyers and sellers are randomly and anonymously matched in pairs. Discount factors and the quality of the seller's asset are private information. Once matched, the buyer can offer one of two prices, which are fixed exogenously: a high price $p_h \in (y_H + \bar{\delta}c_H, u_H)$ or a low price $p_\ell \in (0, u_L)$.¹⁰ The seller can accept or reject. If a seller accepts, trade ensues and the pair exits the market; there is no entry by additional buyers and sellers. If a seller rejects, no trade occurs and the pair remains in the market. This ensures that there is always an equal measure of buyers and sellers.

We make the following assumptions:

$$u_H - p_h > u_L - p_\ell \quad (3)$$

$$y_H + \bar{\delta}p_h \leq p_h \quad (4)$$

$$(u_L - p_\ell)/(u_H - p_h) \geq \bar{\delta}, \quad (5)$$

The first assumption implies that a buyer with perfect information would prefer a H quality asset to a L quality asset, which seems to be the most natural case. The second assumption, (4), implies that all sellers accept an offer of p_h regardless of their discount factor; though this assumption is non-essential, it simplifies the analysis. The final assumption ensures that buyers are sufficiently impatient that they would never prefer to simply not make an offer at all. This is important: since we have constrained buyers to offer either p_ℓ or p_h , we are not allowing them to simply wait for the next period. In section ?, we discuss this assumption at greater length, and illustrate how it could be relaxed without changing the structure of

levels of utility from holding a particular asset. This can arise for a multitude of reasons: agents can have different levels of risk aversion, financing costs, regulatory requirements, or hedging needs. In addition, the correlation of endowments with asset returns may differ across agents. The current formulation is a reduced-form representation of such differences; see Duffie et al. (2007), Vayanos and Weill (2008), and Gârleanu (2009).

¹⁰Exogenous prices in these types of models have been used extensively; see, for example, Wolinsky (1990) and Blouin and Serrano (2001). We provide a full discussion of this assumption in Section ?. As we show, allowing for fully flexible price offers does not appear to alter the structure of equilibria nor the patterns of trade. It does, however, significantly complicate the analysis.

equilibrium.

The history for a buyer is the set of all of his past discount factors and (rejected) price offers. However, a buyer has no reason to condition behavior on his past history: this history is private information, discount factors are i.i.d., and the probability that he meets his current trading partner in the future is zero, as there is a continuum of agents. Moreover, since there is no aggregate uncertainty, the buyer's history of past offers is not helpful in learning any information about the aggregate state. Thus, a pure strategy for a buyer is a sequence $\mathbf{p} = \{p_t\}_{t=0}^\infty$, with $p_t : [0, \delta] \rightarrow \{p_\ell, p_h\}$ measurable for all $t \geq 0$, such that $p_t(\delta)$ is the buyer's offer in period t , conditional on still being in the market and drawing discount factor δ .

A history for a seller is the set of all of his past discount factors and all price offers that he has rejected. The same argument as above implies that a seller has no reason to condition behavior on his past history. Thus, a pure strategy for a type j seller (i.e. a seller with a type $j \in \{L, H\}$ asset) is a sequence $\mathbf{a}_j = \{a_t^j\}_{t=0}^\infty$, with $a_t^j : [0, \bar{\delta}] \times \{p_\ell, p_h\} \rightarrow \{0, 1\}$ measurable for all $t \geq 0$, such that $a_t^j(\delta, p)$ is the seller's acceptance decision in period t as a function of his discount factor and the price offer he receives. We let $a_t^j(\delta, p) = 0$ denote the seller's decision to reject and $a_t^j(\delta, p) = 1$ denote the seller's decision to accept.

We consider symmetric pure-strategy equilibria. A strategy profile can then be described by a list $\sigma = (\mathbf{p}, \mathbf{a}_L, \mathbf{a}_H)$. In order to define equilibria, of course, we must describe the payoffs at each t under any strategy σ . Though this is a somewhat standard calculation for all t in which there is a strictly positive measure of agents remaining in the market, we must also specify what happens when there is a zero measure of agents remaining in the market. More specifically, when all remaining agents trade and exit the market in the current period, we must specify the (expected) payoff to an individual should he choose a strategy that results in *not* trading.

In order to avoid imposing ad hoc assumptions, we adopt the following procedure for computing payoffs. Consider the slightly more general version of our benchmark model in which, in each period t , agents get the opportunity to trade with probability $\alpha \in (0, 1]$,

where α is independent of an agent's type and history. Thus, in every period t , a fraction α of the buyers and sellers in the market are matched in pairs, and the remainder do not get the opportunity to trade. The definition of strategies when $\alpha \in (0, 1)$ is the same as in the special case we consider of $\alpha = 1$.¹¹ However, when $\alpha \in (0, 1)$, in every period t there is a strictly positive mass of agents remaining in the market, and thus payoffs are *always* well-defined; in particular, future payoffs are well-defined when all buyers and sellers trade in the current period. Therefore, we will define payoffs at $\alpha = 1$ as the limit of these well-defined payoffs as $\alpha \rightarrow 1$.¹²

Let us denote by $V_t^j(\mathbf{a}|\sigma, \alpha)$ the expected payoff to a seller of type $j \in \{L, H\}$ who is in the market in period t following strategy $\mathbf{a}$, given the strategy profile σ for all other agents. The payoff V_t^j is computed before the seller gets the draw for his discount factor and learns whether he can trade or not. For $\alpha \in (0, 1)$, V_t^j is well-defined for all $t \geq 0$, and satisfies the following recursion:

$$V_t^j(\mathbf{a}|\sigma, \alpha) = (1 - \alpha) \int [y_j + \delta V_{t+1}^j(\mathbf{a}|\sigma, \alpha)] dF(\delta) + \alpha \sum_{i \in \{\ell, h\}} \xi_t(p_i) \int \{a_t^j(\delta, p_i) p_i + [1 - a_t^j(\delta, p_i)] [y_j + \delta V_{t+1}^j(\mathbf{a}|\sigma, \alpha)]\} dF(\delta), \quad (6)$$

where $\xi_t(p)$ is the fraction of buyers who offer $p \in \{p_\ell, p_h\}$ in period t . Note that $\xi_t(p)$ is the probability that a buyer who can trade draws a discount factor δ with $p_t(\delta) = p$. In words, with probability $1 - \alpha$ a seller is not matched in period t , enjoys flow utility y_j , and proceeds to period $t + 1$. With probability α the seller is matched, in which case he either accepts the buyer's offer and exits the market, or rejects the offer and stays in the market.

Similarly, we denote by $V_t^B(\mathbf{p}|\sigma, \alpha)$ the expected payoff to a buyer who is in the market in period t following strategy $\mathbf{p}$, given the strategy profile σ for all other agents. The payoff V_t^B is also computed before the buyer draws his discount factor and learns whether he can

¹¹Now a player's strategy at time t would obviously be conditional on being matched. Moreover, a history for a player would also include the periods in which he was able to trade; however, for the same reasons given above, a player has no incentive to condition his behaviour on this information.

¹²Indeed, one interesting extension of our model would be to study more carefully the case of $\alpha \in (0, 1)$, and analyze the effects of matching frictions on the patterns of trade. As this is not central to the focus of this paper, we leave it for future work.

trade or not. Again, for $\alpha \in (0, 1)$, V_t^B is well-defined for all $t \geq 0$, and satisfies the following recursion:

$$\begin{aligned}
V_t^B(\mathbf{p}|\sigma, \alpha) &= (1 - \alpha)\delta V_{t+1}^B(\mathbf{p}|\sigma, \alpha) + \alpha \sum_{i \in \{\ell, h\}} \xi_t(p_i) \{ q_t A_t^H(p_i)[u_H - p_i] + (1 - q_t)A_t^L(p_i)[u_L - p_i] \\
&\quad + [1 - q_t A_t^H(p_i) - (1 - q_t)A_t^L(p_i)]\delta V_{t+1}^B(\mathbf{p}|\sigma, \alpha) \} \\
&= \delta V_{t+1}^B(\mathbf{p}|\sigma, \alpha) + \alpha \sum_{i \in \{\ell, h\}} \xi_t(p_i) \{ q_t A_t^H(p_i) [u_H - p_i - \delta V_{t+1}^B(\mathbf{p}|\sigma, \alpha)] \\
&\quad + (1 - q_t)A_t^L(p_i) [u_L - p_i - \delta V_{t+1}^B(\mathbf{p}|\sigma, \alpha)] \}, \tag{7}
\end{aligned}$$

where q_t is the fraction of H sellers in the market in period t and $A_t^j(p)$ is the likelihood that a seller of type $j \in \{L, H\}$ in the market in period t accepts an offer $p \in \{p_\ell, p_h\}$, i.e.

$$A_t^j(p) = \int a_t^j(\delta, p) dF(\delta).$$

In words, with probability $1 - \alpha$ a buyer is not matched in period t , enjoys no utility, and proceeds to period $t + 1$. With probability α a buyer is matched, in which case his partner either accepts his offer (and the buyer exits the market) or rejects his offer (and the buyer stays in the market).

Standard dynamic programming arguments show that for each σ , $\mathbf{a}$, $\mathbf{p}$, and $t \geq 0$, the payoffs $V_t^j(\mathbf{a}|\sigma, \alpha)$ and $V_t^B(\mathbf{p}|\sigma, \alpha)$ are continuous functions of α in the interval $(0, 1)$. Hence, the limits of both $V_t^j(\mathbf{a}|\sigma, \alpha)$ and $V_t^B(\mathbf{p}|\sigma, \alpha)$ are well-defined as α converges to one. Given this, for any strategy profile σ , let us define the payoff to a buyer who is in the market in period t following the strategy $\mathbf{p}$ by

$$V_t^B(\mathbf{p}|\sigma) = \lim_{\alpha \rightarrow 1} V_t^B(\mathbf{p}|\sigma, \alpha),$$

and the payoff to a seller of type $j \in \{L, H\}$ who is in the market in period t following the strategy $\mathbf{a}$ by

$$V_t^j(\mathbf{a}|\sigma) = \lim_{\alpha \rightarrow 1} V_t^j(\mathbf{a}|\sigma, \alpha).$$

We can now define equilibria in our environment, where $\alpha = 1$.

Definition 1. *The strategy profile $\sigma^* = (\mathbf{p}^* = \{p_t^*\}, \mathbf{a}_L^* = \{a_t^{L*}\}, \mathbf{a}_H^* = \{a_t^{H*}\})$ is an equilibrium if for each $t \geq 0$ and $j \in \{L, H\}$, we have that:*

(i) $p_t^*(\delta)$ maximizes

$$q_t A_t^H(p) [u_H - p - \delta V_{t+1}^B(\sigma^*)] + (1 - q_t) A_t^L(p) [u_L - p - \delta V_{t+1}^B(\sigma^*)]$$

for all $\delta \in [0, \bar{\delta}]$, where $V_t^B(\sigma^*) = V_t^B(\mathbf{p}^*|\sigma^*)$;

(ii) For each $p \in \{p_\ell, p_h\}$, $a_t^{j*}(\delta, p) = 1$ if, and only if,

$$p \geq y_j + \delta V_{t+1}^j(\sigma^*), \tag{8}$$

where $V_t^j(\sigma^*) = V_t^j(\mathbf{a}_j^*|\sigma^*)$.

Note that (8) implies that in equilibrium a seller accepts any offer that he is indifferent between accepting and rejecting. This is without loss since F has no mass points, and so the probability that a seller is ever indifferent between accepting and rejecting is zero.

3 Properties of Equilibria

For a given strategy profile, we say that the market “clears” in period t if all sellers remaining in the market accept the price offer made by the buyers. In this section, we establish that the market clears in finite time in every equilibrium and that, in every period before the market clears, the fraction of type H sellers in the population strictly increases. The necessary proofs have been relegated to the appendix.

Lemma 1. *Suppose the market has not cleared before period t , and that $q_t \in (0, 1]$. The market clears in period t if, and only if, all buyers in the market offer p_h .*

Intuitively, if a positive fraction of buyers offer p_ℓ , some of them will be matched with H quality sellers given our assumption of random matching. Moreover, of these H quality sellers, those with $\delta \geq \underline{\delta} \equiv (u_L - y_H)/c_H$ will reject an offer of p_ℓ . Since $\underline{\delta} < \bar{\delta}$ by (2), there is always a strictly positive mass of such sellers, and so the market will not clear in period t if a positive mass of buyers offers p_ℓ .

Therefore, in any equilibrium σ^* , the market clears in the first period in which all remaining buyers offer p_h , which we denote by $T = T(\sigma^*)$; we set $T = \infty$ if a positive mass of buyers offers p_ℓ in every period t . For all $t < T$, a positive mass of buyers offer p_ℓ , and the fraction of type j sellers who accept p_ℓ in t is $F[(p_\ell - y_j)/V_{t+1}^j(\sigma^*)]$. Since all sellers who receive an offer of p_h accept the offer and exit the market, we then have that

$$q_{t+1} = \frac{q_t \left[1 - F\left(\frac{p_\ell - y_H}{V_{t+1}^H(\sigma^*)}\right) \right]}{q_t \left[1 - F\left(\frac{p_\ell - y_H}{V_{t+1}^H(\sigma^*)}\right) \right] + (1 - q_t) \left[1 - F\left(\frac{p_\ell}{V_{t+1}^L(\sigma^*)}\right) \right]}. \quad (9)$$

Now notice that all type H sellers have the option to replicate the behavior of a type L seller. Since $y_H > y_L$, we then have that $V_t^H(\sigma^*) \geq V_t^L(\sigma^*)$ for all $t \geq 0$.¹³ Hence, since $F[(p_\ell - y_H)/V_{t+1}^H(\sigma^*)] \leq F(\underline{\delta}) < 1$ and F is strictly increasing in its support,

$$F\left(\frac{p_\ell - y_H}{V_{t+1}^H(\sigma^*)}\right) < F\left(\frac{p_\ell}{V_{t+1}^L(\sigma^*)}\right) \quad (10)$$

for all $t \geq 0$; that is, whenever buyers offer p_ℓ , the fraction of type L sellers who accept this offer is strictly larger than the fraction of type H sellers who accept the same offer. Looking at the law of motion for $\{q_t\}_{t=0}^T$, equation (9), the following result follows immediately.

Result 1. *For any equilibrium σ^* in which the market clears in period T , $q_{t+1} > q_t$ for all $t = 0, 1, \dots, T - 1$.*

This result is a common feature of dynamic models with adverse selection: since the opportunity cost (the foregone dividends) of selling a H quality asset is larger than that of selling a L quality asset, those sellers with H quality assets are de facto more patient and remain in the market, on average, longer than L quality sellers. As a result, over time the average quality in the market increases. As we now show, this implies that the market eventually clears in every equilibrium.

Result 2. *Let $q_0 \in (0, 1)$. In any equilibrium, the market clears in finite time.*

¹³Indeed, $y_H > y_L$ implies that $V_t^H(\mathbf{a}_L|\sigma^*, \alpha) > V_t^L(\mathbf{a}_L|\sigma^*, \alpha)$ for all $\alpha \in (0, 1)$ and $t \geq 0$. Taking the limit as α converges to one implies the desired result.

The intuition for this result is as follows (the formal proof is in the Appendix). Suppose, by contradiction, that there exists an equilibrium in which the mass of buyers who offer p_ℓ is strictly positive in every period t . We know the sequence $\{q_t\}_{t=0}^\infty$ is strictly increasing, and thus convergent (since it is bounded from above). Let us denote the limit of this sequence by q_∞ . First, it must be that $q_\infty < 1$; since buyers discount the future ($\bar{\delta} < 1$), it is straightforward to show that there exists a $q^* < 1$ such that all buyers will offer p_h for any $q > q^*$, so it must be $q_\infty \leq q^* < 1$. Second, if $q_t \rightarrow q_\infty < 1$, it must be that the “jumps” in quality between period t and $t + 1$ are getting arbitrarily small. From (9), this requires that the fraction of H sellers who accept p_ℓ is getting arbitrarily close to the fraction of L sellers who accept the same offer; roughly speaking, as $t \rightarrow \infty$, all sellers with discount rate $\delta \in [0, \bar{\delta}]$ behave the same independently of the quality of their asset. However, this is not possible given (10): given the differences in L and H quality assets, the owners of these assets always behave distinctly differently.

In what follows, we will present the case where the lemons problems is most severe by assuming that $p_\ell < y_H$, so that *no* type H seller accepts p_ℓ , i.e. $F[(p_\ell - y_H)/V_{t+1}^H(\sigma^*)] = 0$. Relaxing this assumption does not substantively change any of our results.

4 Characterizing Equilibria

In this section, we provide a complete characterization of the set of equilibria. The results from the previous section are instructive: we know that, in equilibrium, the fraction of H quality assets q increases over time until some finite period T , at which time the market clears. Using these results, we are able to construct the set of equilibria recursively, starting with the last period of trade. As a first step, we characterize the values of q such that there exists an equilibrium in which all buyers offer p_h (so that the market clears). Given this characterization, we can calculate the payoffs to buyers and type j sellers in the final period of trade. Naturally, if q_0 is in this region of the parameter space, there exists an equilibrium in which the market clears at $t = 0$. We refer to such equilibria as “0-step” equilibria.

Next, we characterize the values of q such that there exists an equilibrium in which (i)

some buyers have incentive to offer p_ℓ (so that the market does *not* clear immediately); and (ii) the implied value of q in the subsequent period, calculated using the law of motion (9) and the payoffs in the final period of trade, lies within the region of 0-step equilibrium. We refer to such equilibria as “1-step” equilibria. Given the region of 1-step equilibria, along with equilibrium payoffs when the economy is 1 period from market clearing, we can characterize the set of 2-step equilibrium, and the recursion continues.

0-step equilibria

Denote by $\pi_i^B(q, \delta, v_L, v_H, v_B)$ the payoff to a buyer who offers p_i , with $i \in \{\ell, h\}$, when: (i) the fraction of type H sellers in the market is $q \in (0, 1)$; (ii) the buyer’s discount factor is δ ; (iii) the continuation payoff to a seller of type j who chooses not to trade is $v_j \geq c_j$; and (iv) the continuation payoff to the buyer should he not trade is $v_B \in (0, u_H - p_h]$.¹⁴ We know that sellers always accept an offer of p_h , so that

$$\pi_h^B(q, \delta, v_L, v_H, v_B) \equiv \pi_h^B(q) = q(u_H - p_h) + (1 - q)(u_L - p_h).$$

We also know that a type H seller always rejects an offer of p_ℓ . Therefore,

$$\begin{aligned} \pi_\ell^B(q, \delta, v_L, v_H, v_B) &\equiv \pi_\ell^B(q, \delta, v_L, v_B) \\ &= (1 - q)F\left(\frac{p_\ell}{v_L}\right)[u_L - p_\ell] + \left\{q + (1 - q)\left[1 - F\left(\frac{p_\ell}{v_L}\right)\right]\right\}\delta v_B, \end{aligned} \quad (11)$$

where $F(p_\ell/v_L)$ is the fraction of type L sellers who accept p_ℓ . Note that $\pi_\ell^B(q, \delta, v_L, v_B)$ is strictly increasing in v_B . Since $v_B > 0$, $\pi_\ell^B(q, \delta, v_L, v_B)$ is also positive and strictly increasing in δ . Moreover, since $v_B \leq u_H - p_h$, (5) implies that $\delta v_B \leq u_L - p_\ell$, and so $\pi_\ell^B(q, \delta, v_L, v_B)$ is non-increasing in v_L .

Consider now the candidate 0-step equilibrium σ^0 in which, in every $t \geq 0$, $p_t(\delta) = p_h$ for all $\delta \in [0, \bar{\delta}]$, and type j sellers accept an offer p if, and only if, $\delta \leq (p - y_j)/p_h$. It is immediate to see that for all $t \geq 0$,

$$V_t^B(\sigma^0) = \pi_h^B(q_0) \equiv v_B^0(q_0) \quad \text{and} \quad V_t^j(\sigma^0) = p_h \equiv v_j^0.$$

¹⁴Note that in any equilibrium σ^* , it must be that $V_t^B(\sigma^*) > 0$ for all $t \geq 0$. The reason is that a buyer always has the option to offer p_ℓ as long as there is a positive mass of type L sellers in the market, and the expected payoff from doing so is strictly positive: since $V_t^L(\sigma^*) \leq p_h$, the probability that a type L seller accepts p_ℓ is at least $F(p_\ell/p_h) > 0$, in which case the buyer’s payoff is $u_L - p_\ell > 0$.

Note that we have introduced the following notation: in a 0-step equilibrium, the expected payoff to a buyer given q_0 is $v_B^0(q_0)$ and the expected payoff to a type j seller is v_j^0 , which is independent of q_0 .¹⁵ In what follows, we will denote by $v_B^k(\cdot)$ and $v_j^k(\cdot)$ the ex-ante expected payoffs to a buyer and type j seller, respectively, in period $t = 0$ of a k -step equilibrium (before agents draw discount factors).¹⁶

The strategy profile σ^0 is an equilibrium only if $v_B^0(q_0) > 0$ (for otherwise $V_0^B(\sigma^0) \leq 0$, which cannot happen in equilibrium) and in every period t all buyers find it optimal to offer p_h , which is true as long as

$$\pi_h^B(q_0) \geq \pi_\ell^B(q_0, \delta, v_L^0, v_B^0(q_0)) \quad (12)$$

for all $\delta \in [0, \bar{\delta}]$. Since $v_B^0(q_0) > 0$ implies that $\pi_\ell^B(q_0, \delta, v_L^0, v_B^0(q_0))$ is strictly increasing in δ , a necessary and sufficient condition for (12) is that

$$\pi_h^B(q_0) \geq \pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0(q_0)). \quad (13)$$

Condition (13) is slightly more subtle than it may appear. The left side is clearly the payoff to a buyer from offering p_h . The right side is the payoff to a buyer from offering p_ℓ in the current period and p_h in the ensuing period, conditional on all other buyers offering $p_t = p_h$ for all $t \geq 0$. There are two things to notice. First, when all other buyers offer p_h and exit the market, the payoff to a buyer who remains in the market and offers p_h in the next period is $v_B^0(q_0)$. This comes directly from our refinement for computing payoffs when the mass of agents in the market is zero. Indeed, under σ^0 , when the fraction of buyers and sellers who are matched in each period is $\alpha < 1$, all buyers who get the opportunity to trade exit the market, and so the fraction of type H sellers among the sellers who remain in the market stays the same. Second, (13) is the loosest possible constraint on q_0 that ensures that a buyer finds it optimal to offer p_h at $t = 0$ when he believes that all other buyers in

¹⁵In general, we will adopt the convention that a numerical subscript refers to a particular time period, while a numerical superscript refers to the number of periods it takes for the market to clear in equilibrium. In addition, we will use lower case v to denote equilibrium payoffs.

¹⁶Note that $v_B^k(\cdot)$ and $v_j^k(\cdot)$ are also the ex-ante expected payoffs to a buyer and type j seller, respectively, in period $t > 0$ of a $(k+t)$ -step equilibrium; what's important is that this payoff is calculated k steps before the market clears, and before the agent draws his period t discount factor.

the market offer p_h as well. This is ensured by our choice of σ^0 ; as we show in the proof of Proposition 1 (in the Appendix), there is no other strategy profile $\tilde{\sigma}^0$ – with the necessary property that $p_0(\delta) = p_h$ for all $\delta \in [0, \bar{\delta}]$ – that can support a 0-step equilibrium for $q_0 < \underline{q}^0$.

Proposition 1. *Let $\underline{q}^0 \in (0, 1)$ denote the unique value of q satisfying (13) with equality. There exists a 0-step equilibrium if, and only if, $q_0 \geq \underline{q}^0$.*

Notice that $q_0 u_H + (1 - q_0) u_L \geq p_h > y_H + \bar{\delta} c_H$ for any q_0 in the interval $[\underline{q}^0, 1)$, since a buyer is only willing to offer p_h if his payoff from doing so is non-negative. Hence, p_h corresponds to a market-clearing price in a Walrasian equilibrium. Thus, when the lemons problem is relatively small, i.e., when the fraction of type H sellers is sufficiently large, the equilibrium outcome in this dynamic, decentralized market coincides with that of a static, frictionless market; trade occurs instantaneously at a single market-clearing price. We will now show, however, that as the lemons problem becomes more severe, equilibrium outcomes no longer resemble those of a centralized Walrasian market; instead, these outcomes appear more consistent with models of decentralized trade with search frictions, in the sense that it takes time for buyers and sellers to trade, and they do so at potentially different prices.

1-step equilibria

Given the full characterization of 0-step equilibria above, we will now proceed to characterize the set of 1-step equilibrium. In doing so, the following convention will be useful: for any strategy profile σ , let σ_+ be the strategy profile such that for all $t \geq 0$, the agents' behavior in period t is given by their behavior in period $t + 1$ under σ . In addition, let

$$q^+(q, v_L) = \frac{q}{q + (1 - q)[1 - F(p_\ell/v_L)]}, \quad (14)$$

with $q \in (0, 1)$. By construction, $q^+(q, v_L)$ is the fraction of type H sellers in the market in the next period if this fraction is q in the current period, a positive mass of buyers offer p_ℓ , and the continuation payoff to a type L seller in case he rejects a price offer is v_L . Since $v_L \leq p_h$, $F(p_\ell/v_L) \geq F(p_\ell/p_h) > 0$, and so $q^+(q, v_L) > q$ for all $q \in (0, 1)$. Also notice that $q^+(q, v_L)$ is strictly increasing in q if $p_\ell/v_L < \bar{\delta}$ and that $q^+(q, v_L) \equiv 1$ if $p_\ell/v_L \geq \bar{\delta}$.

Consider a strategy profile σ^1 such that a positive mass of buyers offer p_ℓ in $t = 0$ and all buyers offer p_h in $t = 1$. In order for σ^1 to be an equilibrium, it must be that (i) σ_+^1 is a 0-step equilibrium when the initial fraction of type H sellers is $q' = q^+(q_0, v_L^0)$, and (ii) a positive mass of buyers find it optimal to offer p_ℓ in $t = 0$ when the market clears in $t = 1$.¹⁷ Formally, the following conditions are necessary and sufficient for σ^1 to be an equilibrium:

$$q^+(q_0, v_L^0) = q' \quad (15)$$

$$q' \geq \underline{q}^0 \quad (16)$$

$$\pi_h^B(q_0) < \pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0(q')) . \quad (17)$$

Condition (15) is simply the law of motion for q_t from $t = 0$ to $t = 1$; notice that q' is strictly increasing in q_0 . Condition (16) follows from Proposition 1. It ensures that the fraction of type H sellers in $t = 1$ is high enough for market clearing in this period to be an equilibrium outcome. Since $q' \geq \underline{q}^0$ implies that $v_B^0(q') > 0$, $\pi_\ell^B(q_0, \delta, v_L^0, v_B^0(q'))$ is strictly increasing in δ . Thus, the incentive of a buyer to offer p_ℓ in $t = 0$ when the market clears in $t = 1$ increases with the buyer's patience. Condition (17) then ensures that a positive mass of buyers indeed find it optimal to offer p_ℓ in $t = 0$ when the strategy profile under play is σ^1 ; if it is optimal for the most patient buyer to offer p_h in $t = 0$, then every other buyer prefers to offer p_h as well. As it turns out, combining (15) and (16) provides a lower bound on the set of 1-step equilibria, and (17) provides an upper bound. We formalize this below in Proposition 2; the proof is in the Appendix.

Proposition 2. *Let $\bar{q}^1$ denote the unique value of q_0 satisfying (17) with equality, and define $\underline{q}^1$ to be such that $q^+(\underline{q}^1, v_L^0) = \underline{q}^0$ if $p_\ell/v_L^0 < \bar{\delta}$ and $\underline{q}^1 = 0$ otherwise. Then $\underline{q}^1 < \underline{q}^0 < \bar{q}^1 < 1$ and there exists a 1-step equilibrium if, and only if, $q_0 \in [\underline{q}^1, \bar{q}^1) \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^1, \bar{q}^1) \cap (0, 1)$, there is a unique $q' \in [\underline{q}^0, 1]$ such that q' is the value of q_1 in a 1-step equilibrium when the initial fraction of type H sellers is q_0 .*

In words, if $q_0 = \bar{q}^1$ then the most patient buyer is exactly indifferent between offering p_ℓ and p_h when a positive mass of the other buyers are offering p_ℓ ; for any $q_0 > \bar{q}^1$ the payoff to

¹⁷It must also be the case that the type j sellers accept an offer of p in $t = 0$ if, and only if, $\delta \leq (p - y_j)/p_h$. This optimal behavior of sellers will be implicitly assumed throughout the analysis.

such a buyer from immediately trading at price p_h is greater than the payoff from offering p_ℓ and not trading with positive probability, in which case the buyer trades at price p_h in the ensuing period (when the fraction of type H sellers in the market, $q' = q^+(\bar{q}^1, v_L^0)$, is larger). When $p_\ell/v_L^0 < \bar{\delta}$, $\underline{q}^1$ is the unique value of q_0 such that if a positive mass of buyers offer p_ℓ , then the fraction of high quality sellers in the next period is exactly $\underline{q}^0$, the minimum value required for market clearing; notice that $\underline{q}^1 > 0$ in this case. If even the most patient type L seller would rather accept an offer of p_ℓ today than wait one period for an offer of p_h , i.e., if $p_\ell/v_L^0 \geq \bar{\delta}$, then we have $\underline{q}^1 = 0$.

The fact that $\underline{q}^0 < \bar{q}^1$ implies that there are multiple equilibria when $q_0 \in [\underline{q}^0, \bar{q}^1)$. In this region, if all other buyers are offering p_h , $q_1 = q_0$ and it is optimal for an individual buyer to offer p_h . However, if a positive mass of other buyers are offering p_ℓ , the market does not clear at $t = 0$ and the payoff to trading at $t = 1$ increases (since $q_1 > q_0$), rendering it optimal for patient buyers to offer p_ℓ and incur a chance that they trade only in the next period.¹⁸

The payoff to a buyer in a 1-step equilibrium is

$$v_B^1(q_0) = \int \max \{ \pi_h^B(q_0), \pi_\ell^B(q_0, \delta, v_L^0, v_B^0 [q^+(q_0, v_L^0)]) \} dF(\delta) \geq \pi_h^B(q_0).$$

We denote the fraction of buyers that offer p_h at $t = 0$ in a 1-step equilibrium by

$$\xi^1(q_0) = \int \mathbb{I}\{ \pi_h^B(q) \geq \pi_\ell^B(q, \delta, v_L^0, v_B^0 [q^+(q, v_L^0)]) \} dF(\delta),$$

where $\mathbb{I}$ represents the indicator function. Thus, the payoff to a type L seller is¹⁹

$$v_L^1(q_0) = \xi^1(q_0)p_h + (1 - \xi^1(q_0)) \int \max \{ p_\ell, \delta v_L^0 \} dF(\delta) \leq v_L^0. \quad (18)$$

¹⁸Note that the complementarity between agents' actions here – and the resulting multiplicity – requires (at least) two ingredients: that there is more than one agent on at least one side of the market, and that these agents are forward-looking. Identifying these two ingredients is helpful in understanding why multiple equilibria of this sort do *not* typically arise in similar models with only one seller and one buyer (see e.g. Vincent (1989), Evans (1989), and Deneckere and Liang (2006)), or one seller and a sequence of myopic buyers (see Hörner and Vieille (2009) and the references therein).

¹⁹The payoffs to a H quality seller is simply $v_H^1(q_0) = \xi^1(q_0)p_h + (1 - \xi^1(q_0)) \int \delta p_h dF(\delta)$. Since the behavior of type H sellers is trivial, we will not explicitly derive their payoffs in what follows; however, given our characterization of v_L^k below, it is easy to do so.

In Lemma 3 in the Appendix we show that $\xi^1(q_0)$ is continuous and increasing in q_0 , and it converges to one as q_0 increases to $\bar{q}^1$. Therefore, the average price, $\xi^1(q_0)p_h + [1 - \xi^1(q_0)]p_\ell$, is increasing in q_0 in the region of 1-step equilibria, and converges to p_h as q_0 converges to $\bar{q}^1$. Moreover, since $q^+(q_0, v_L^0)$ is continuous in q_0 , it is easy to see that both v_B^1 and v_L^1 are continuous and increasing in q_0 , and they converge to $v_B^0(\bar{q}^1)$ and v_L^0 , respectively, as q_0 increases to $\bar{q}^1$. In what follows, we write $v_L^1(\bar{q}^1)$ to denote the limit of $v_L^1(q_0)$ as q_0 increases to $\bar{q}^1$.

2-step equilibria

We now provide a complete characterization of 2-step equilibria. As it turns out, the process of characterizing k -step equilibria is nearly identical for all $k \geq 2$. Thus, the methodology developed here will allow for a complete characterization of equilibria in the next section. Since, by Proposition 2, there are only 0-step and 1-step equilibria when $p_\ell/p_h \geq \bar{\delta}$, we assume that $p_\ell/p_h < \bar{\delta}$ in what follows.

Consider a strategy profile σ^2 such that a positive mass of buyers offer p_ℓ in $t = 0$ and $t = 1$, and then all buyers offer p_h in $t = 2$. In order for σ^2 to be an equilibrium, it must satisfy the following three necessary and sufficient conditions:

$$q^+(q_0, v_L^1(q')) = q' \tag{19}$$

$$q' \in [\underline{q}^1, \bar{q}^1) \tag{20}$$

$$\pi_h^B(q_0) < \pi_\ell^B(q_0, \bar{\delta}, v_L^1(q'), v_B^1(q')). \tag{21}$$

The first condition, (19), is the analog of (15); it is the law of motion for q_t from $t = 0$ to $t = 1$, conditional on a 1-step equilibrium beginning at $t = 1$. Note that q' is no longer defined by a simple function, as in (15), but rather q' is the solution to a fixed point problem: if the type L sellers expect their continuation payoff to be that of a 1-step equilibrium where the initial fraction of type H sellers is q' , then the fraction of type L sellers who accept an offer of p_ℓ in $t = 0$ must be such that this conjecture is correct.²⁰ The second condition, (20), ensures that indeed there exists a 1-step equilibrium at $t = 1$ given an initial fraction

²⁰Note that the reason (15) is a simple function is that $v_L^0(q)$ is independent of q .

q' of H quality assets. The final condition, (21), ensures that a positive mass of buyers find it optimal to offer p_ℓ in $t = 0$ when behavior from $t = 1$ is given by σ_+^2 . As we show in the Appendix, (19) and (20) imply (21). Intuitively, we show that the incentive of the most patient buyer to choose p_ℓ in $t = 0$ is even greater than his incentive to choose p_ℓ in $t = 1$, when the fraction of type H sellers in the market is $q' > q_0$. Therefore, if the most patient buyer strictly prefers to choose p_ℓ in $t = 1$, which is guaranteed by (20), then he will also strictly prefer to offer p_ℓ at $t = 0$ and (21) will be satisfied. Hence, (19) and (20) are necessary and sufficient conditions for a 2-step equilibrium.

Let $Q_+^2 : q_0 \mapsto q'$ denote the map defined by (19); in words, $Q_+^2(q_0)$ is the value of q_1 in a 2-step equilibrium, given initial q_0 . In the Appendix, we show that $Q_+^2(q_0)$ is a well-defined function that is both continuous and strictly increasing in q_0 . These properties greatly simplify the characterization of 2-step equilibria: the necessary and sufficient conditions are simply $Q_+^2(q_0) \geq \underline{q}^1$ and $Q_+^2(q_0) \leq \bar{q}^1$. Hence, the lower bound of the set of 2-step equilibria satisfies the first of these conditions with equality, and the upper bound satisfies the second with equality. Moreover, given that the map is strictly increasing, it follows immediately that there can only be a single 2-step equilibrium for any q_0 that satisfies these conditions. We formalize this in the proof below.

Proposition 3. *Suppose that $\bar{\delta} > p_\ell/p_h$. Let $\bar{q}^2$ denote the unique value satisfying $q^+(\bar{q}^2, v_L^1(\bar{q}^1)) = \bar{q}^1$, and define $\underline{q}^2$ to be such that $q^+(\underline{q}^2, v_L^1(\underline{q}^1)) = \underline{q}^1$ if $p_\ell/v_L^1(\underline{q}^1) < \bar{\delta}$ and $\underline{q}^2 = 0$ otherwise. Then $\underline{q}^2 < \underline{q}^1 < \bar{q}^2 < \bar{q}^1$ and there exists a 2-step equilibrium if, and only if, $q_0 \in [\underline{q}^2, \bar{q}^2) \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^2, \bar{q}^2) \cap (0, 1)$, there is a unique $q' \in [\underline{q}^1, \bar{q}^1)$ such that q' is the value of q_1 in a 2-step equilibrium when the initial fraction of type H sellers is q_0 .*

Figure 4 below provides some intuition for the characterization of equilibrium so far. After deriving $\underline{q}^0$ and $\bar{q}^1$, we identified $\underline{q}^1$ as the value of q_0 that would “land” exactly on $\underline{q}^0$ at $t = 1$ given the law of motion $q^+(q_0, v_L^0)$. Since this law of motion is continuous and strictly increasing in q_0 (for $\delta < p_\ell/v_L^0$), we are assured that any $q_0 > \underline{q}^1$ will “land” at $q' > \underline{q}^0$ in a candidate 1-step equilibrium. Moving backwards, we then identified $\underline{q}^2$ and $\bar{q}^2$ as the

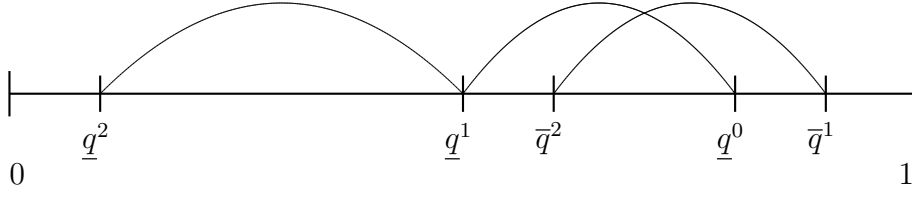


Figure 1: Deriving Bounds on Equilibrium Regions

values of q_0 that would “land” exactly on $\underline{q}^1$ and $\bar{q}^1$, respectively, given the law of motion $Q_+^2(q_0)$. Though this law of motion is slightly more complicated, the fact that it remains continuous and strictly increasing assures us that any $q_0 \in [\underline{q}^2, \bar{q}^2)$ will “land” within the region of 1-step equilibrium. Finally, since $v_L^1(\bar{q}^1) = v_L^0$, $\bar{q}^1 > \underline{q}^0$, and

$$q^+(\bar{q}^2, v_L^0) = \bar{q}^1 > \underline{q}^0 = q^+(\underline{q}^1, v_L^0),$$

the fact that $\bar{q}^2 > \underline{q}^1$ follows immediately from the fact that $q^+(q_0, v_L)$ is strictly increasing in q_0 for any v_L such that $p_\ell/v_L < \bar{\delta}$.

To complete the characterization, the payoff to a buyer in a 2-step equilibrium is

$$v_B^2(q_0) = \int \max \{ \pi_h^B(q_0), \pi_\ell^B(q_0, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0))) \} dF(\delta) \geq \pi_h^B(q_0). \quad (22)$$

If we denote the fraction of buyers that offer p_h at $t = 0$ in a 2-step equilibrium by

$$\xi^2(q_0) = \int \mathbb{I} \{ \pi_h^B(q) \geq \pi_\ell^B(q, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0))) \} dF(\delta), \quad (23)$$

then the payoff to a type L seller is given by

$$v_L^2(q_0) = \xi^2(q_0)p_h + (1 - \xi^2(q_0)) \int \max \{ p_\ell, \delta v_L^1(Q_+^2(q_0)) \} dF(\delta) \leq v_L^0. \quad (24)$$

In Lemma 3 in the Appendix, we show that v_B^2 is continuous in q_0 and converges to $v_B^1(\bar{q}^2)$ as q_0 increases to $\bar{q}^2$, while v_L^2 is also continuous and increasing in q_0 , and it converges to $v_L^1(\bar{q}_2)$ as q_0 increases to $\bar{q}^2$. This completes the characterization of 2-step equilibria.

A Full Characterization

We proceed by induction. Consider an arbitrary integer $k \geq 3$, and suppose that there exist cutoffs $0 \leq \underline{q}^{k-1} < \bar{q}^{k-1}$ and payoff functions $v_B^{k-1}(q)$ and $v_L^{k-1}(q)$ satisfying the following two conditions:

(A1) A $(k-1)$ -step equilibrium exists if, and only if, $q_0 \in [\underline{q}^{k-1}, \bar{q}^{k-1}] \cap (0, 1)$.

(A2) The functions $v_B^{k-1}(q)$ and $v_L^{k-1}(q)$ are uniquely defined, continuous, and increasing in q . Moreover, $\lim_{q \rightarrow \bar{q}^{k-1}} v_L^{k-1}(q) = v_L^{k-2}(\bar{q}^{k-1})$.

We have shown these conditions to be true for $k = 3$. Following the logic above, the following conditions are necessary and sufficient for a k -step equilibrium to exist:

$$q^+ (q_0, v_L^{k-1}(q')) = q' \quad (25)$$

$$q' \in [\underline{q}^{k-1}, \bar{q}^{k-1}) \quad (26)$$

$$\pi_h^B(q_0) < \pi_\ell^B(q_0, \bar{\delta}, v_L^{k-1}(q'), v_B^{k-1}(q')) . \quad (27)$$

As we show in the Appendix, given (A1) and (A2), several crucial features of 2-step equilibria are true for k . First, (25) and (26) imply (27), so the necessary and sufficient conditions are given by (25) and (26). Second, if we define $Q_+^k : q_0 \mapsto q'$ as the solution to (25), then Q_+^k is continuous and strictly increasing. This leads to the following simple characterization of k -step equilibria.

Proposition 4. *Suppose that (A1) and (A2) are true, and that $\bar{\delta} > p_\ell / v_L^{k-1}(\bar{q}^{k-1})$. Let $\bar{q}^k$ denote the unique value satisfying $q^+(\bar{q}^k, v_L^{k-1}(\bar{q}^{k-1})) = \bar{q}^{k-1}$, and define $\underline{q}^k$ to be such that $q^+(\underline{q}^k, v_L^{k-1}(\underline{q}^{k-1})) = \underline{q}^{k-1}$ if $p_\ell / v_L^{k-1}(\underline{q}^{k-1}) < \bar{\delta}$ and $\underline{q}^k = 0$ otherwise. Then $\underline{q}^k \leq \underline{q}^{k-1} < \bar{q}^k < \bar{q}^{k-1}$ and there exists a k -step equilibrium if, and only if, $q_0 \in [\underline{q}^k, \bar{q}^k] \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^k, \bar{q}^k] \cap (0, 1)$, there is a unique $q' \in [\underline{q}^{k-1}, \bar{q}^{k-1})$ such that q' is the value of q_1 in a k -step equilibrium when the initial fraction of type H sellers is q_0 .*

The payoffs in a k -step equilibrium are then given by

$$v_B^k(q_0) = \int \max \{ \pi_h^B(q_0), \pi_\ell^B(q_0, \delta, v_L^{k-1}(Q_+^k(q_0)), v_B^{k-1}(Q_+^k(q_0))) \} dF(\delta) \quad (28)$$

$$v_L^k(q_0) = \xi^k(q_0)p_h + (1 - \xi^k(q_0)) \int \max \{ p_\ell, \delta v_L^{k-1}(Q_+^k(q_0)) \} dF(\delta), \quad (29)$$

where

$$\xi^k(q_0) = \int \mathbb{I} \{ \pi_h^B(q) \geq \pi_\ell^B(q, \delta, v_L^{k-1}(Q_+^k(q_0)), v_B^{k-1}(Q_+^k(q_0))) \} dF(\delta) \quad (30)$$

is the fraction of buyers that offer p_h at $t = 0$ in a k -step equilibrium. The last crucial feature of 2-step equilibria that is true for all $k \geq 3$ is that v_L^k and v_B^k are continuous and increasing in q_0 , and that $\lim_{q_0 \rightarrow \bar{q}^k} v_L^k(q_0) = v_L^{k-1}(\bar{q}^k)$.

The inductive process described above continues as long as $p_\ell/v_L^{k-1}(\bar{q}^{k-1}) < \bar{\delta}$, which ensures $\bar{q}^k > 0$ and thus the existence of a k -step equilibrium. We have thus established the following result, which provides a complete characterization of the equilibrium set.

Theorem 1. *There exists a $K \geq 1$ and sequences $\{\underline{q}^k\}_{k=0}^K$ and $\{\bar{q}^k\}_{k=0}^K$, with $\bar{q}^0 = 1$ and $0 \leq \underline{q}^k \leq \underline{q}^{k-1} < \bar{q}^k < 1$ for all $k \in \{1, \dots, K\}$, such that a k -step equilibrium exists if, and only if, $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$. Moreover, for each $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$, there is a unique $q' \in [\underline{q}^{k-1}, \bar{q}^{k-1})$ such that q' is the value of q_1 in a k -step equilibrium when the initial fraction of type H sellers is q_0 , and the map $Q_+^k : q_0 \mapsto q'$ defined by (25) is continuous and strictly increasing.*

The payoffs for buyers and type L sellers are uniquely defined in every equilibrium and are determined recursively as follows: (i) $v_B^0(q_0) = q_0(u_H - p_h) + (1 - q_0)(u_L - p_h)$ and $v_L^0(q_0) \equiv p_h$; (ii) for all $k \in \{1, \dots, K\}$, v_B^k and v_L^k are given by (28) and (29), respectively.

The cutoffs $\{\underline{q}^k\}_{k=0}^{K-1}$ and $\{\bar{q}^k\}_{k=1}^K$ are defined recursively as follows: (i) $\underline{q}^0$ is the unique q such that $\pi_h^B(q) = \pi_\ell^B(q, \bar{\delta}, v_L^0, v_B^0(q))$ and, for $k = 1, \dots, K$, the cutoff $\underline{q}^k$ satisfies $q^+(\underline{q}^k, v_L^{k-1}(\underline{q}^{k-1})) = \underline{q}^{k-1}$ if $p_\ell/v_L^{k-1}(\underline{q}^{k-1}) < \bar{\delta}$ and $\underline{q}^k = 0$ otherwise; (ii) the cutoff $\bar{q}^1$ is the unique q such that $\pi_h^B(q_0) = \pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0[q^+(q_0, v_L^0)])$ and $\bar{q}^k$ satisfies $q^+(\bar{q}^k, v_L^{k-1}(\bar{q}^{k-1})) = \bar{q}^{k-1}$ for $k = 2, \dots, K$. Finally, $K = \max\{k : p_\ell/v_L^{k-1}(\bar{q}^{k-1}) < \bar{\delta}\}$.

5 Discussion

In this section, we illustrate how the theory developed above can provide insight into a number of important issues. First we conduct comparative statics across values of q_0 , paying particular attention to how the initial composition of assets in the market affects the *liquidity* of H quality assets. Then we study the dynamics of trade for a given value of q_0 , exploring the model’s implications for how prices, trading volume, average quality, and liquidity evolve over time in this type of environment. We argue that establishing such a benchmark is important, as it allows us not only to understand how “frozen” markets can “thaw” over time on their own, but also provides a framework to formally analyze the effects of various policies that have been implemented in attempt to “unfreeze” such markets; we study one such policy in Section 6. Finally, we discuss how the existence of multiple equilibria for a given value of q_0 suggests that there is scope for coordination failure in dynamic, decentralized markets with asymmetrically informed agents. Since such multiplicity does *not* arise in several closely related (and well-known) environments, we highlight those features of our framework that are crucial for generating these coordination failures.

5.1 Liquidity and Lemons

In this section, we study how the fraction of lemons in the market affects the liquidity of H quality assets.²¹ An asset typically considered liquid if it can be sold quickly and at little discount. In many models, trade is instantaneous by construction, and thus the only measure of liquidity is the difference between the actual market price and the price in some (theoretical) frictionless benchmark; in these models, time is a margin that simply cannot adjust.²² In the current model, the opposite is true: since p_h is the only price that type

²¹Focusing on the ability to sell H quality assets is standard in this literature, going back to Akerlof (1970). Of course, a seller can always sell a L quality asset instantaneously at price $p_\ell > c_L$.

²²In the finance literature, the typical measure of liquidity is the (inverse of) the bid-ask spread, which can be generated by exogenous transaction costs (see, e.g., Amihud and Mendelson (1986) and Constantinides (1986)), asymmetric information (see Kyle (1985) and Glosten and Milgrom (1985)), or search frictions (see Duffie et al. (2005)), among other things. An alternative definition of liquidity, which is more related to the current study, can be found in Eisfeldt (2004), and again more recently in Kurlat (2009); in these models, a lemons problem forces the (pooling equilibrium) price in a centralized market down. Therefore, sellers of

H sellers will accept, the appropriate measure of liquidity for these assets is the expected amount of time it takes to sell them. We will now derive this statistic and show that our model implies a systematic relationship between the severity of the lemons problem (i.e. q_0) and the liquidity of H quality assets.

Theorem 1 offers a complete characterization of the set of equilibria by specifying a sequence of cut-offs that partition the parameter space into regions in which there exists an equilibrium that takes $k = 0, 1, 2, \dots$ periods for the market to clear given some initial composition q_0 . Figure 2 below illustrates these cut-offs for a set of parameter values that imply $K = 4$; we will use this numerical example throughout this section to convey the intuition behind several of the model's key implications.²³ In addition to plotting these cut-offs, we have also highlighted the regions of the parameter space in which equilibrium take at most $k \in \{0, 1, 2, 3, 4\}$ and at a minimum $k \in \{0, 1, 2, 3\}$ periods before all goods are bought and sold.

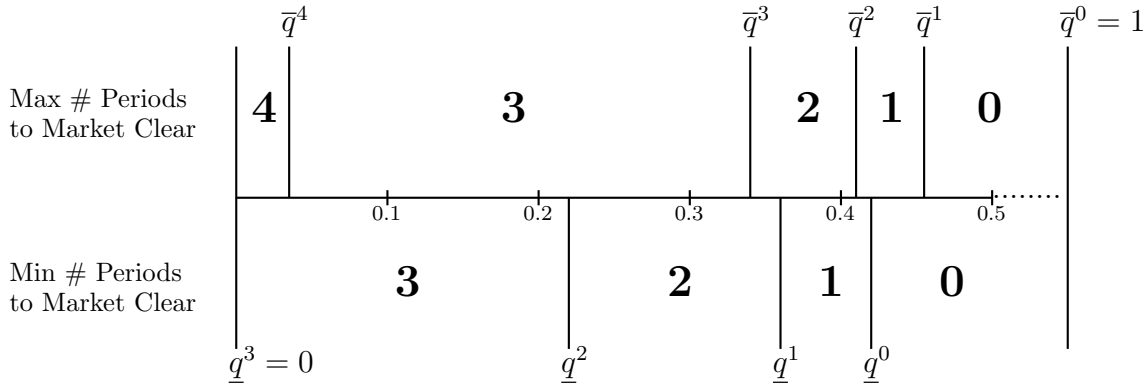


Figure 2: Equilibrium Cutoffs

Notice that there is a natural monotonicity to the equilibria: for any $0 < q_0 < \tilde{q}_0 < 1$, if there exists a k -step equilibrium beginning at q_0 , then there also exists a $\tilde{k}$ -step equilibrium beginning at $\tilde{q}_0$ such that $\tilde{k} \leq k$.

high quality assets sell at a discount which depends on the lemons problem, though again trade remains instantaneous.

²³For this example, we set $u_H = 1$, $p_h = 0.6$, $u_L = 0.42$, $p_\ell = 0.05$, and assume F is uniformly distributed over $[0, .5]$.

Now, consider a k -step equilibrium with an initial fraction $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$ of high quality assets. Let

$$\eta^s(q, \delta) = \pi_h^B(q) - \pi_\ell^B(q, \delta, v_L^{s-1}[Q_+^s(q)], v_B^{s-1}[Q_+^s(q)]) \quad (31)$$

and define $\delta^s(q)$ such that

$$\delta^s(q) = \begin{cases} 0 & \text{if } \eta^s(q, 0) \leq 0 \\ \eta^s(q, \delta^s(q)) = 0 & \text{if } \eta^s(q, 0) > 0 \end{cases}$$

for $s = 1, 2, \dots, k$. In words, when the market is s steps (or periods) away from clearing and the fraction of high quality assets is q , all buyers with $\delta < \delta^s(q)$ offer p_h and all buyers with $\delta \geq \delta^s(q)$ offer p_ℓ . By construction, $\delta^0(q) \equiv \bar{\delta}$, since all buyers offer p_h in the final period of trade. Given q_0 , and defining $q_1, q_2, \dots, q_k$ by the recursion

$$q_{t+1} = Q_+^{k-t}(q_t)$$

for $t = 0, 1, \dots, k-1$, we can define the expected number of periods it takes to sell an asset of quality H in a k -step equilibrium by

$$E_H^k(q_0) = \sum_{s=0}^{k-1} \left\{ \left[\prod_{t=0}^s \{1 - F[\delta^{k-t}(q_t)]\} \right] F[\delta^{k-s-1}(q_{s+1})] (s+1) \right\}. \quad (32)$$

As we established earlier, for some values of q_0 there exist multiple equilibria that take a different number of periods for the market to clear. This, of course, makes comparisons across values of q_0 difficult. However, within the set of k -step equilibria, $\delta^k(q_0)$ is increasing in q_0 , and hence the probability of a H quality seller receiving an offer of p_h and exiting the market is increasing in q_0 . Moreover, as q_0 increases, we systematically move from equilibria that take at most k periods before market-clearing to equilibria that take at most $k-1$ periods before market-clearing. Therefore, so long as we are consistent in our equilibrium selection device, the relationship between q_0 and our measure of market liquidity is straightforward. For the sake of illustration, let us denote by $\bar{k}(q_0)$ the maximum number of periods it could take for the market to clear in equilibrium:

$$\bar{k}(q_0) = \max\{k : \exists \text{ a } k\text{-step equilibrium given } q_0\}.$$

Clearly, then, $E_H^{\bar{k}}(q_0)$ is a decreasing function of q_0 ; as the fraction of lemons in the market decreases, so too does the expected amount of time it takes for a H quality seller to trade.²⁴ It follows immediately that the expected payoff to a type H buyer, $v_H^{\bar{k}}(q_0)$, is increasing in q_0 ; as markets become more liquid, sellers with H quality assets do not need to wait as long to receive an offer of p_h . Figure 3 below plots $E_H^{\bar{k}}(q_0)$ for the numerical example specified above. Given the properties of v_L^k and v_B^k specified in Proposition 4, one can easily show that $\lim_{q_0 \rightarrow \bar{q}^k} \delta^k(q_0) = \delta^{k-1}(\bar{q}^k)$, which explains why $E_H^{\bar{k}}(q_0)$ is a continuous function that converges to zero as $q_0 \rightarrow \bar{q}^1$.

INSERT FIGURE 3 HERE

5.2 The Dynamics of Trade

We now illustrate typical market dynamics for a given value of q_0 . Again, the numerical example described above is a convenient vehicle for conveying the intuition; we choose $q_0 = 0.1$, which falls within the set of 3-step equilibria. In figure 4 below, we plot q_t , as well as the average price p_t^{avg} , for $t = 0, \dots, 4$, where

$$p_t^{avg} = \xi^{k-t}(q_t)p_h + [1 - \xi^{k-t}(q_0)]p_\ell.$$

Intuitively, when q_t is sufficiently low, all buyers offer p_ℓ . All H quality sellers and patient L quality sellers reject this offer, but sufficiently impatient L quality sellers accept, causing the average quality to rise in the following period. If this increase is sufficiently large, some impatient buyers will then have incentive to offer p_h , while patient buyers will continue to offer p_ℓ and (perhaps) wait for market conditions to improve. Thus, average prices increase over time along with average quality. In this case, the price path exhibits an S -shape: prices are persistently low in early periods, and then quickly increase in the latter stages of trade. When q_t is sufficiently large, all remaining buyers offer p_h , all sellers accept, and the market clears.

INSERT FIGURE 4 HERE

²⁴Note that we have suppressed the argument of $\bar{k}$ in $E_H^{\bar{k}}(q_0)$ for notational convenience, though it should be understood that $\bar{k}$ depends on q_0 .

5.3 Multiplicity of Equilibria

The presence of multiple equilibria for some values of q_0 suggests that, in some cases, illiquidity can be caused by a coordination failure: if other buyers are offering low prices, the market will not clear immediately, quality will increase in future periods, and hence an individual buyer may also have incentive to offer a low price and (perhaps) trade at a later time. If, on the other hand, all buyers are offering a high price, the market composition will not improve over time, and thus an individual has greater incentive to offer a high price in the current period, thereby increasing market liquidity. Notice that the complementarity between agents' actions here – and the resulting multiplicity – requires (at least) two ingredients: that there is more than one agent on at least one side of the market, and that these agents are forward-looking.

Identifying these two ingredients is helpful in understanding why multiple equilibria of this sort do *not* typically arise in certain related environments. For example, in models of bargaining with asymmetric information in which there is only one buyer and one seller (see e.g. Vincent (1989), Evans (1989), and Deneckere and Liang (2006)), clearly there is no scope for complementarity between buyers' actions; as a result, there is typically a unique sequential equilibrium in these models. Alternatively, in similar frameworks in which a single seller with private information meets a sequence of buyers (see e.g. Hörner and Vieille (2009) and the references therein), the buyers are typically assumed to be myopic. As a result, there is no potential for buyers to coordinate their behaviour based on future payoffs, and again the type of multiplicity that we find here does not emerge.²⁵

²⁵Note that these two ingredients are not sufficient for multiplicity. For example, in Janssen and Roy (2002), there is a continuum of forward-looking buyers and sellers who trade in a sequence of centralized markets in the presence of asymmetric information. However, their equilibrium requires that buyers receive zero expected payoffs from trading at any date, thus precluding the possibility of the multiplicity we find in our model, where buyers coordinate their actions based on variations in future versus current payoffs.

6 Application: Policy Analysis

In this section, we will use the theory developed above to assess the theoretical implications of a policy recently employed in attempt to restore liquidity in the market for asset-backed securities. First, we will argue that our model shares many features of the market for asset-backed securities. For one, buyers and sellers in this market negotiate bilaterally, as opposed to trading against their budget constraint in a competitive, centralized market where the law of one price prevails. Moreover, the market is inherently dynamic and non-stationary: there is a relatively fixed stock of assets of a particular vintage, and the manner in which the composition of assets remaining in the market evolves over time affects both prices and the incentive of market participants to delay trade. Finally, many believe that the presence of asymmetric information led this market to become illiquid; the decline of housing prices in various parts of the country introduced considerable heterogeneity into the quality of residential mortgage-backed securities, and many of the usual buyers of these assets did not possess the expertise to comfortably value the assets that were being offered by financial institutions.

As a result, both the prices and the volume of these assets being sold quickly dropped.²⁶ Unfortunately, a lack of liquidity in this market posed a threat to the economy at large. The treasury department described the “the challenge of legacy assets” as follows:

One major reason [for the prolonged recession] is the problem of “legacy assets” - both real estate loans held directly on the books of banks (“legacy loans”) and securities backed by loan portfolios (“legacy securities”). These assets create uncertainty around the balance sheets of these financial institutions, compromising their ability to raise capital and their willingness to increase lending. As a result, a negative cycle has developed where declining asset prices have triggered further deleveraging, which has in turn led to further price declines. The excessive discounts embedded in some legacy asset prices are now straining the capital of U.S. financial institutions, limiting their ability to lend and increasing the cost

²⁶For a detailed analysis, see Krishnamurthy (2010).

of credit throughout the financial system.

Given the danger associated with this downward spiral, it is not surprising that restoring liquidity in this market moved to the forefront of economics policymakers' concerns and efforts. However, the optimal form of intervention was not obvious, in large part because the majority of existing theoretical models are inconsistent with the basic features of this market. As a result, despite the fact that the optimal form of intervention garnered an extraordinary amount of attention, the majority of this discussion had to take place outside the realm of formal economic analysis.

The current model provide a rigorous, yet parsimonious framework that allows for the analysis of a variety of policy proposals. In this section, we consider one specific program introduced by the treasury department in order to restore liquidity in this market: the Public-Private Investment Program for Legacy Assets. Under this program, the government issued non-recourse loans to private investors to assist in buying legacy assets, with a minimum fraction of the purchase price being financed by the private investor's own equity. This program essentially subsidizes the buyer's purchase and partially insures his down-side loss; if the asset turns out to be a lemon, the buyer can walk away and incur only a fraction of the total loss from the purchase (his equity investment). The treasury department described the "merits of this approach" as follows:

This approach is superior to the alternatives of either hoping for banks to gradually work these assets off their books or of the government purchasing the assets directly. Simply hoping for banks to work legacy assets off over time risks prolonging a financial crisis, as in the case of the Japanese experience. But if the government acts alone in directly purchasing legacy assets, taxpayers will take on all the risk of such purchases - along with the additional risk that taxpayers will overpay if government employees are setting the price for those assets.

In attempt to capture this policy response, suppose a buyer who pays price p for an asset must pay a fraction γ himself, but he can borrow the remaining $(1 - \gamma)p$ from the

government. For simplicity, assume the buyer observes the quality of the asset immediately after buying it, and then faces a choice: he can either pay back the loan to the government, or default on the loan and surrender the asset.

Clearly a buyer who receives a quality H asset will always repay his loan, as will a buyer who has paid price p_ℓ for a L quality asset. However, a buyer who paid p_h for a L quality asset will default if $\gamma < 1 - u_L/p_h$. Therefore, this policy can be summarized by a transfer $\tau = (1 - \gamma)p_h - u_L \in [0, p_h - u_L]$ to those unlucky buyers who paid p_h for a L quality asset.

In a slight abuse of notation, let

$$\pi_h^B(q, \tau) \equiv v_B^0(q, \tau) = q(u_H - p_h) + (1 - q)(u_L - p_h + \tau)$$

denote the payoffs to a buyer from offering p_h given policy τ . Following the same logic as before, $\underline{q}^0(\tau)$ can then be defined as the value of q_0 such that

$$\pi_h^B(q_0, \tau) = \pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0(q_0, \tau)),$$

where π_ℓ^B and $v_L^0 = p_h$ are as defined above. Moreover, $\underline{q}^1(\tau)$ and $\bar{q}^1(\tau)$ can be defined as the values of q_0 satisfying the following two equations, respectively:

$$\begin{aligned} q^+(q_0, v_L^0) &= \underline{q}^0 \\ \pi_\ell^B[q_0, \bar{\delta}, v_L^0, v_B^0(q^+(q_0, v_L^0), \tau)] &= \pi_h^B(q_0, \tau). \end{aligned}$$

It is straightforward to show that $\underline{q}^0$, $\underline{q}^1(\tau)$ and $\bar{q}^1(\tau)$ are decreasing in τ . Therefore, in some regions of the parameter space, it is clear that an increase in τ can decrease the amount of time it takes for the market to clear, and thus increase market liquidity. For example, there exists a 0-step equilibrium given $q_0 \in (\underline{q}^0(\tau), \underline{q}^0(0))$ for $\tau \in (0, p_h - u_L)$, whereas the market would take at least one additional period to clear if $\tau = 0$. Intuitively, since the transfer τ increases the payoff from offering p_h , buyers are more willing to offer p_h given any fraction of lemons in the market.

However, this policy has a second, opposing effect as well. Since buyers are more willing to offer p_h when they are partially insured against buying a lemon, the average price sellers receive in the future increases as τ grows larger. *Ceteris paribus*, this makes sellers more

likely to reject offers of p_ℓ in early rounds of trade, opting instead to wait for larger payoffs later in the game. To see this, let

$$\xi^1(q, \tau) = \int \mathbb{I}\{\pi_h^B(q, \tau) \geq \pi_\ell^B(q, \delta, v_L^0, v_B^0[q^+(q, v_L^0), \tau])\} dF(\delta),$$

and let the payoff to a type L seller in a 1-step equilibrium be defined

$$v_L^1(q, \tau) = \xi^1(q, \tau)p_h + (1 - \xi^1(q, \tau)) \int \max\{p_\ell, \delta v_L^0\} dF(\delta) \leq v_L^0. \quad (33)$$

Since ξ^1 is increasing in τ , so too is $v_L^1(q, \tau)$, holding q fixed. Let $\underline{q}^2(\tau)$ satisfy

$$q^+(q_0, v_L^1(\underline{q}^1(\tau), \tau)) = \underline{q}^1(\tau). \quad (34)$$

As τ increases, these two opposing forces are at work. On the one hand, since $\underline{q}^1$ is decreasing in τ , this tends to decrease $\underline{q}^2$ as well; holding v_L^1 constant in (34), $\underline{q}^2$ is decreasing in $\underline{q}^1$. On the other hand, holding $\underline{q}^1$ fixed, v_L^1 is increasing in τ , which tends to make sellers more likely to reject an offer of p_ℓ at $t = 0$. This implies a smaller jump in quality between q_0 and q_1 , and hence a larger value of $\underline{q}^2$. This second effect is shut down when calculating $\underline{q}^1(\tau)$, since v_L^0 is constant in τ , which explains why $\underline{q}^1$ is unambiguously decreasing in τ . However, when the market is $s \geq 2$ periods away from market-clearing, this second effect is active, and can offset – and even dominate – the first effect. That is, this policy can increase the time required for market clearing, thus making the market *less liquid*.

Using the numerical example from above, Table 1 below summarizes the effect of a policy τ that is equal to 25% of the loss $p_h - u_L$ from purchasing a lemon at price p_h , relative to the benchmark of $\tau = 0$.²⁷

Policy	$\underline{q}^3$	$\bar{q}^4$	$\underline{q}^2$	$\bar{q}^3$	$\underline{q}^1$	$\bar{q}^2$	$\underline{q}^0$	$\bar{q}^1$
$\tau = 0$	0	.036	.206	.344	.379	.410	.422	.455
$\tau = .1(p_h - u_L)$	0	.049	.231	.301	.340	.369	.382	.412

Table 1: Policy Analysis

One can see immediately that this policy increases liquidity for intermediate values of q_0 , but has little effect on (and can even decrease) liquidity for small values of q_0 . In figure 5,

²⁷The remaining cut-offs were derived and calculated in exactly the same way as in Section 4.

we plot $E_H^{\bar{k}}(q_0)$ for $\tau = 0$ and $\tau = .1(p_h - u_L)$. The lesson is clear: even without considering the *cost* of this type of intervention, its efficacy depends crucially on the underlying severity of the lemons problem.

INSERT FIGURE 5 HERE

Appendix

Proof of Lemma 1

From (6) and (8), we have that for any equilibrium σ^* ,

$$V_t^j(\sigma^*) = \sum_{i \in \{\ell, h\}} \xi_t(p_i) \int \max \{p_i, y_j + \delta V_{t+1}^j(\sigma^*)\} dF(\delta)$$

for all $t \geq 0$. Given (4), it should be obvious that $V_t^j(\sigma^*) \leq p_h$ for all $t \geq 0$, so that all sellers in the market always accept an offer of p_h . Thus, the market clears in period t if all buyers offer p_h . Now observe that since a seller has the option of always rejecting any offer he receives, $V_t^j(\sigma^*) \geq c_j$ for all $t \geq 0$. Thus, letting $\underline{\delta} = (u_L - y_H)/c_H$, we have

$$y_H + \underline{\delta} V_{t+1}^H(\sigma^*) \geq y_H + \underline{\delta} c_H = u_L > p_\ell.$$

Therefore, a type H seller with discount factor $\delta \geq \underline{\delta}$ always rejects an offer of p_ℓ . Since $\underline{\delta} < \bar{\delta}$ by (2), there is always a strictly positive mass of such sellers. Hence, the market does not clear in period t if a positive mass of buyers offers p_ℓ .

6.1 Proof of Result 2

I shortened this proof a bit, but made no substantive changes. Let σ^* be an equilibrium and assume, towards a contradiction, that $T(\sigma^*) = \infty$. First notice that there is $q^* \in (0, 1)$ such that

$$q^*[u_H - p_h] + (1 - q^*)[u_L - p_h] = \max \{u_L - p_\ell, \bar{\delta}[u_H - p_h]\}. \quad (35)$$

Since the highest payoff possible for a buyer is $u_H - p_h$, the right-hand side of (35) is the highest payoff possible for a buyer who offers p_ℓ ; if the fraction of type H sellers in the market is above q^* , then all buyers offer p_h and the market clears. Thus, for all $t \geq 0$, q_t is bounded above by q^* , and so is the limit q_∞ of the sequence $\{q_t\}_{t=0}^\infty$. Now notice that the sequences $\{V_t^L(\sigma^*)\}$ and $\{V_t^H(\sigma^*)\}$ are bounded, and thus have convergent subsequences. Dropping subscripts if necessary, we can assume that both sequences converge. Denote their respective limits by V_∞^L and V_∞^H and note that $V_\infty^H \geq V_\infty^L$, given that $V_t^H(\sigma^*) \geq V_t^L(\sigma^*)$ for

all t . Since the c.d.f. F is continuous, the law of motion (9) for q_t implies that

$$q_\infty \left[1 - F \left(\frac{p_\ell - y_H}{V_\infty^H} \right) \right] + (1 - q_\infty) \left[1 - F \left(\frac{p_\ell}{V_\infty^L} \right) \right] = 1 - F \left(\frac{p_\ell - y_H}{V_\infty^H} \right). \quad (36)$$

Since $q_\infty < 1$, (36) implies that $F(p_\ell/V_\infty^L) = F[(p_\ell - y_H)/V_\infty^H]$, a contradiction since $(p_\ell - y_H)/V_\infty^H < p_\ell/V_\infty^L$. Thus, the market must clear in finite time.

Proof of Proposition 1

Let $\eta^0(q, \delta) = \pi_h^B(q) - \pi_\ell^B(q, \delta, v_L^0, v_B^0(q))$. Straightforward algebra shows that $\eta^0(q, \delta)$ is strictly increasing in q . Since η^0 is continuous in q , $\eta^0(0, \bar{\delta}) < 0$, and $\eta^0(1, \bar{\delta}) > 0$, there is a unique $q \in (0, 1)$, that we denote by $\underline{q}^0$, such that $\eta^0(\underline{q}^0, \bar{\delta}) \geq 0$ if, and only if $q \geq \underline{q}^0$. Since $v_B^0(q) \leq u_H - p_h$, (5) then implies that $\pi_\ell^B(q, \bar{\delta}, v_L^0, v_B^0(q)) > 0$, and so $v_B^H(\underline{q}^0) = \pi_h^B(\underline{q}^0) > 0$. This assures that the strategy profile σ^0 is an equilibrium if $q_0 \geq \underline{q}^0$.

As the next step, suppose that $q_0 < \underline{q}^0$ and consider a candidate 0-step equilibrium $\tilde{\sigma}^0$ with the necessary property that all buyers offer p_h in $t = 0$. One alternative for a buyer is to offer p_h in every period. Let $\tilde{\mathbf{p}}$ denote this strategy. If $\tilde{\sigma}^0$ is to be an equilibrium, then it must be that $V_t^B(\tilde{\sigma}^0) \geq V_t^B(\tilde{\mathbf{p}}|\tilde{\sigma}^0)$ for all $t \geq 0$. Now observe that when the probability that an agent can trade in a period is $\alpha \in (0, 1)$,

$$V_t^B(\tilde{\mathbf{p}}|\tilde{\sigma}^0, \alpha) = \sum_{\tau=1}^{\infty} \alpha(1-\alpha)^{\tau-1} (\mathbb{E}[\delta])^{\tau-1} v_B^0(q_{t+\tau-1}^\alpha),$$

where $q_{t+\tau-1}^\alpha$ is the fraction of type H sellers in the market in period $t + \tau - 1$. It is easy to see that

$$q_{t+1}^\alpha = \frac{q_t^\alpha [1 - \alpha + \alpha \xi_t(p_\ell)]}{q_t^\alpha [1 - \alpha + \alpha \xi_t(p_\ell)] + (1 - q_t^\alpha) \left\{ 1 - \alpha + \alpha \xi_t(p_\ell) \left[1 - F \left(p_\ell / V_{t+1}^L(\tilde{\sigma}^0|\alpha) \right) \right] \right\}},$$

where $\xi_t(p_\ell)$ is the probability that a buyer who gets the opportunity to trade in period t offers p_ℓ . Clearly the sequence $\{q_t^\alpha\}_{t=0}^\infty$ is non-decreasing. Hence,

$$V_t^B(\tilde{\mathbf{p}}|\tilde{\sigma}^0, \alpha) \geq \sum_{\tau=1}^{\infty} \alpha(1-\alpha)^{\tau-1} (\mathbb{E}[\delta])^{\tau-1} v_B^0(q_0),$$

which implies that $V_t^B(\tilde{\mathbf{p}}|\tilde{\sigma}^0) \geq v_B^0(q_0)$. Thus it must be that $V_1^B(\tilde{\sigma}^0) \geq v_B^0(q_0)$ if $\tilde{\sigma}^0$ is to be an equilibrium. Since $V_1^L(\tilde{\sigma}^0) \leq p_h$ and $q_0 < \underline{q}^0$ implies that $\pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0(q_0)) > \pi_h^B(q_0)$,

we then have

$$\pi_\ell^B(q_0, \bar{\delta}, V_1^L(\tilde{\sigma}^0), V_1^B(\tilde{\sigma}^0)) \geq \pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0(q_0)) > \pi_h^B(q_0)$$

for all $q_0 < \underline{q}^0$. Thus, there exists $\delta' < \bar{\delta}$ such that it can *not* be optimal for a buyer with discount factor in $(\delta', \bar{\delta}]$ to offer p_h at $t = 0$ if $q_0 < \underline{q}^0$, so that the market clearing immediately cannot be an equilibrium outcome.

6.2 Proof of Proposition 2

Recall that $q^+(q, v_L^0)$ is strictly increasing in q when $p_\ell/v_L^0 < \bar{\delta}$ and that $q^+(q, v_L^0) \equiv 1$ otherwise. From this it is immediate to see that there exists $\underline{q}^1 < \underline{q}^0$ such that $q^+(q_0, v_L^0) \geq \underline{q}^0$ if, and only if, $q_0 \in [\underline{q}^1, 1]$. Note that $\underline{q}^1 = 0$ if $p_\ell/v_L^0 \geq \bar{\delta}$ and $\underline{q}^1$ is such that $q^+(\underline{q}^1, v_L^0) = \underline{q}^0$ otherwise. Now, let $\eta^1(q, \delta) = \pi_h^B(q) - \pi_\ell^B(q, \delta, v_L^0, v_B^0[q^+(q, v_L^0)])$. It is easy to see that

$$\begin{aligned} \frac{\partial \eta^1}{\partial q}(q, \delta) &= F\left(\frac{p_\ell}{p_h}\right) \{u_L - p_\ell - \delta v_B^0[q^+(q, p_h)]\} \\ &\quad + (u_H - u_L) \left\{ 1 - \delta \left\{ q + (1 - q) \left[1 - F\left(\frac{p_\ell}{p_h}\right) \right] \right\} \frac{\partial q^+}{\partial q} \right\}. \end{aligned}$$

Thus, from (5) and the fact that

$$\{q + (1 - q) [1 - F(p_\ell/p_h)]\} \frac{\partial q^+}{\partial q} = 1 - \frac{qF(p_\ell/p_h)}{q + (1 - q) [1 - F(p_\ell/p_h)]} < 1,$$

we can conclude that $\partial \eta^1 / \partial q > 0$ regardless of the value of p_ℓ/p_h . Since $\eta^1(0, \bar{\delta}) < 0$ and $\eta^1(1, \bar{\delta}) > 0$, there exists $\bar{q}^1 \in (0, 1)$ such that $\eta^1(q, \bar{\delta}) < 0$ if, and only if $q_0 \in [0, \bar{q}^1)$. Hence, $\pi_h^B(q_0) < \pi_\ell^B(q_0, \bar{\delta}, v_L^0, v_B^0[q^+(q_0, v_L^0)])$ if, and only if $q_0 \in [0, \bar{q}^1)$. Next, observe that since $v_B^0[q^+(q, p_h)] > v_B^0(q)$ for all $q \in (0, 1)$,

$$\pi_\ell^B(\underline{q}^0, \bar{\delta}, v_L^0, v_B^0[q^+(\underline{q}^0, p_h)]) > \pi_\ell^B(\underline{q}^0, \bar{\delta}, v_L^0, v_B^0(\underline{q}^0)) = \pi_h^B(\underline{q}^0).$$

Thus, $\eta^1(\underline{q}^0, \bar{\delta}) < 0$, from which we obtain that $\bar{q}^1 > \underline{q}^0$. To finish, the fact that $q^+(q, v_L^0)$ is strictly increasing in q ensures that for each $q_0 \in [\underline{q}^1, \bar{q}^1) \cap (0, 1)$ there exists a single 1-step equilibrium.

6.3 Lemma 2 and Proof

Lemma 2. *The probability $\xi^1(q_0)$ that a buyer offers p_h in $t = 0$ in a 1-step equilibrium is continuous and increasing in q_0 , and it converges to one as q_0 increases to $\bar{q}^1$.*

Since $\pi_\ell^B(q_0, \delta, v_L^0, v_B^0[q^+(q_0, v_L^0)])$ is strictly increasing in δ , the function η^1 in the proof of Proposition 2 is strictly decreasing in δ . Let $\delta^1(q_0)$, with $q_0 \in [\underline{q}^1, \bar{q}^1) \cap (0, \bar{q}^1)$, be such that

$$\delta^1(q_0) = \begin{cases} 0 & \text{if } \eta^1(q_0, 0) \leq 0 \\ \eta^1(q_0, \delta^1(q_0)) = 0 & \text{if } \eta^1(q_0, 0) > 0. \end{cases}$$

Since $\eta^1(\bar{q}^1, \bar{\delta}) = 0$ and η^1 is strictly increasing in q by the proof of Proposition 2, $\delta^1(q_0)$ is uniquely defined. By construction, δ^1 is the cutoff discount factor below which a buyer finds it optimal to offer p_h in $t = 0$. Hence, the probability $\xi^1(q_0)$ that a buyer offers p_h in $t = 0$ is equal to $F(\delta^1(q_0))$. Since η^1 is jointly continuous, it is easy to see that δ^1 depends continuously on q_0 . Moreover, the cutoff $\delta^1(q_0)$ is strictly increasing in q_0 if $\eta^1(q_0, 0) > 0$, as η^1 is strictly increasing in q . The desired result follows from the fact that the c.d.f. F is continuous and strictly increasing and $\lim_{q_0 \rightarrow \bar{q}^1} \delta^1(\bar{q}^1) = \bar{\delta}$ (given that $\eta^1(\bar{q}^1, \bar{\delta}) = 0$).

6.4 Proof of Proposition 3

We first show that (19) and (20) imply (21), so that conditions (19) and (20) completely determine the range of initial values of q_0 for which there exists a 2-step equilibrium. Before we start, notice that

$$\left\{ q + (1 - q) \left[1 - F\left(\frac{p_\ell}{v_L}\right) \right] \right\} q^+(q, v_L) = q$$

for all $q \in (0, 1)$ and $v_L \geq p_\ell$. Hence,

$$\pi_\ell^B(q, \delta, v_L, \pi_h^B[q^+(q, v_L)]) = \delta \pi_h^B(q) + (1 - q) F\left(\frac{p_\ell}{v_L}\right) [u_L - p_\ell - \delta(u_L - p_h)] \quad (37)$$

for all $q \in (0, 1)$ and $\delta \in [0, \bar{\delta}]$.

Suppose that $q' \in [\underline{q}^1, \bar{q}^1)$. In order to prove that (21) is satisfied, it is sufficient to show

$$\pi_h^B(q') - \pi_h^B(q_0) \geq \pi_\ell^B(q', \delta, v_L^0, v_B^0[q^+(q', v_L^0)]) - \pi_\ell^B(q_0, \delta, v_L^1(q'), v_B^1(q')) \quad (38)$$

for all $\delta \in [0, \bar{\delta}]$. Condition (38) implies that the incentive of a buyer to choose p_ℓ in $t = 0$ is even greater than his incentive to choose p_ℓ in $t = 1$, when the fraction of type H sellers in the market is $q' > q_0$; in particular, this is true for the most patient buyer. First, note that

$$\begin{aligned}\pi_\ell^B(q', \bar{\delta}, v_L^0, v_B^0[q^+(q', v_L^0)]) &= \pi_\ell^B(q', \bar{\delta}, v_L^0, \pi_h^B[q^+(q', v_L^0)]) \\ &= \delta \pi_h^B(q') + (1 - q') F\left(\frac{p_\ell}{v_L^0}\right) [u_L - p_\ell - \bar{\delta}(u_L - p_h)].\end{aligned}$$

Second, since $v_B^1(q') \geq \pi_h^B(q')$, we have

$$\begin{aligned}\pi_\ell^B(q_0, \bar{\delta}, v_L^1(q'), v_B^1(q')) &\geq \pi_\ell^B(q_0, \bar{\delta}, v_L^1(q'), \pi_h^B(q')) \\ &= \delta \pi_h^B(q_0) + (1 - q_0) F\left(\frac{p_\ell}{v_L^1(q')}\right) [u_L - p_\ell - \bar{\delta}(u_L - p_h)];\end{aligned}$$

the second equality follows from (19) and (37). Therefore,

$$\begin{aligned}\pi_\ell^B(q', \bar{\delta}, v_L^0, v_B^0[q^+(q', v_L^0)]) - \pi_\ell^B(q_0, \bar{\delta}, v_L^1(q'), v_B^1(q')) \\ \leq \bar{\delta} [\pi_h^B(q') - \pi_h^B(q_0)] + \left\{ (1 - q') F\left(\frac{p_\ell}{v_L^0}\right) - (1 - q_0) F\left(\frac{p_\ell}{v_L^1(q')}\right) \right\} [u_L - p_\ell - \bar{\delta}(u_L - p_h)].\end{aligned}$$

Since $v_L^0 > v_L^1(q')$ for all $q' \in [\underline{q}^1, \bar{q}^1]$, $u_L < p_h$, and $q' > q_0$, the second term on the right-hand side of the above inequality is negative, which confirms (38).

We now show that there exists a 2-step equilibrium if, and only if, $q_0 \in [\underline{q}^2, \bar{q}^2] \cap (0, 1)$. Since $0 < \underline{q}^1 < \bar{q}^1 < 1$, (19) and (20) can be satisfied only if the denominator of

$$q^+(q_0, v_L^1(q_0)) = \frac{q_0}{q_0 + (1 - q_0)[1 - F(p_\ell/v_L^1(q'))]}$$

is greater than q_0 . In other words, for a given $q' \in [\underline{q}^1, \bar{q}^1]$, there is $q_0 \in (0, 1)$ such that (19) is satisfied only if $p_\ell/v_L^1(q') < \bar{\delta}$. Now observe that if $p_\ell/v_L^1(q') < \bar{\delta}$, then

$$q^-(q') = \frac{q' [1 - F(p_\ell/v_L^1(q'))]}{1 - q' F(p_\ell/v_L^1(q'))}$$

belongs to the interval $(0, 1)$ and is such that $q^+(q^-(q'), v_L^1(q')) = q'$. Thus, (19) is satisfied for $q' \in [\underline{q}^1, \bar{q}^1]$ if, and only if, $p_\ell/v_L^1(q') < \bar{\delta}$. Moreover, it is immediate to see that $q^-(q')$ is the only possible value of q_0 for which (19) and (20) can hold.

Since $v_L^1(q')$ is increasing in q' , $p_\ell/v_L^1(\tilde{q}) < \bar{\delta}$ implies that $p_\ell/v_L^1(q') < \bar{\delta}$ for all $q' > \tilde{q}$. Let then $\tilde{q}^1$ be such that $\tilde{q}^1 = 0$ if $p_\ell/v_L^1(\underline{q}^1) < \bar{\delta}$ and $\tilde{q}^1 = \sup\{q' \in [\underline{q}^1, \bar{q}^1] : p_\ell/v_L^1(q') = \bar{\delta}\}$

if $p_\ell/v_L^1(\underline{q}^1) \geq \bar{\delta}$; $\tilde{q}^1$ is well-defined since $p_\ell/v_L^1(\bar{q}^1) = p_\ell/v_L^0 < \bar{\delta}$. By construction, there is $q_0 \in (0, 1)$ such that (19) and (20) are satisfied if, and only if, $q' \in [\underline{q}^1, \bar{q}^1) \cap (\tilde{q}^1, 1)$, in which case $q_0 = q^-(q')$. Since F and v_L^1 are continuous in δ and q' , respectively, it is easy to see that q^- is continuous in q' . Moreover, since v_L^1 is increasing in q' , straightforward algebra shows that q^- is strictly increasing in q' . Thus, we have that: (i) when $p_\ell/v_L^1(\underline{q}^1) < \bar{\delta}$, there exists a 2-step equilibrium if, and only if, $q_0 \in [q^-(\underline{q}^1), q^-(\bar{q}^1))$; (ii) when $p_\ell/v_L^1(\underline{q}^1) \geq \bar{\delta}$, there exists a 2-step equilibrium if, and only if, $q_0 \in (q^-(\tilde{q}^1), q^-(\bar{q}^1))$. It is immediate to see that $q^-(\bar{q}^1) = \bar{q}^2$. We are done if we show that $q^-(\tilde{q}^1) = 0$ when $p_\ell/v_L^1(\tilde{q}) \geq \bar{\delta}$. This follows from the fact that $\lim_{q' \rightarrow \tilde{q}^1} F(p_\ell/v_L^1(q')) = 1$.

To finish the proof, notice that since $q^-(q')$ is continuous and strictly increasing in q' , it is invertible and its inverse is strictly increasing and continuous. Thus, the map Q_+^2 that takes an initial fraction q_0 of type H sellers into the value of q_1 in a 2-step equilibrium is continuous and strictly increasing.

6.5 Lemma 3 and Proof

Lemma 3. *Suppose that $\bar{\delta} > p_\ell/v_L^0$. Then: (i) v_B^2 is continuous in q_0 , and it converges to $v_B^1(\bar{q}^2)$ as q_0 increases to $\bar{q}^2$; (ii) v_L^2 is continuous and increasing in q_0 , and it converges to $v_L^1(\bar{q}_2)$ as q_0 increases to $\bar{q}^2$.*

We have shown that Q_+^2 is continuous, strictly increasing, and converges to $\bar{q}^1$ as q_0 increases to $\bar{q}^2$. Given these properties, along with the continuity of v_B^1 and v_L^1 , it follows immediately that v_B^2 is continuous as well. Since $v_L^1(\bar{q}^1) = v_L^0$ and $v_B^1(\bar{q}^1) = v_B^0(\bar{q}^1) = v_B^0[q^+(\bar{q}^2, v_L^0)]$, we then have that

$$\lim_{q_0 \uparrow \bar{q}^2} v_B^2(q_0) = \int_0^{\bar{\delta}} \max \{ \pi_h^B(\bar{q}^2), \pi_\ell^B(\bar{q}^2, \delta, v_L^0, v_B^0[q^+(\bar{q}^2, v_L^0)]) \} dF(\delta) = v_B^1(\bar{q}^2).$$

Let $\eta^2(q_0, \delta) = \pi_h^B(q_0) - \pi_\ell^B(q_0, \delta, v_L^1(Q_+^2(q_0)), v_B^1(Q_+^2(q_0)))$. An argument similar to the one used in the Proof of Proposition 2 shows that for each $q_0 \in [\underline{q}^2, \bar{q}^2)$ there is a unique $\delta^2 = \delta^2(q_0)$ in $[0, \bar{\delta})$, depending continuously on q_0 , such that $\eta^2(q_0, \delta) \geq 0$ if, and only if $\delta \leq \delta^2(q_0)$. Thus, $\xi^2(q_0) = F(\delta^2(q_0))$ is continuous and increasing in q_0 (strictly increasing

when $\delta^2(q_0) < \bar{\delta}$, from which we obtain that v_L^2 is continuous and increasing in q_0 . To finish, notice that

$$\begin{aligned}\lim_{q_0 \uparrow \bar{q}^2} \eta^2(q_0, \delta) &= \pi_h^B(\bar{q}^2) - \pi_\ell^B(\bar{q}^2, \delta, v_L^1(\bar{q}^1), v_B^1(\bar{q}^1)) \\ &= \pi_h^B(\bar{q}^2) - \pi_\ell^B(\bar{q}^2, \delta, v_L^0, v_B^0[q^+(\bar{q}^2, v_L^0)]) = \eta^1(\bar{q}^2, \delta),\end{aligned}$$

so that $\lim_{q_0 \uparrow \bar{q}^2} \xi^2(\bar{q}^2) = \xi^1(\bar{q}^2)$, from which we can conclude that

$$\lim_{q_0 \uparrow \bar{q}^2} v_L^2(q_0) = \xi^1(\bar{q}^2)p_h + (1 - \xi^1(\bar{q}^2)) \int_0^{\bar{\delta}} \max\{p_\ell, \delta v_L^0\} dF(\delta) = v_L^1(\bar{q}^1).$$

6.6 Proof of Proposition 4

We omit many of the details, as the proof follows that of Proposition 3 very closely. Suppose, by induction, that the following holds for all $s \in \{1, \dots, k-1\}$: (i) for all $q_0 \in [\underline{q}^s, \bar{q}^s)$ there is a unique $q' \in [\underline{q}^{s-1}, \bar{q}^{s-1})$ such that $q' = Q_+^s(q_0)$ is the value of q_1 in a s -step equilibrium when the initial fraction of type H sellers is q_0 ; (ii) if $q' = Q_+^s(q_0)$, then

$$\begin{aligned}\eta^{s-1}(q', \delta) &= \pi_h^B(q') - \pi_\ell^B(q', \delta, v_L^{s-1}(Q_+^s(q')), v_B^{s-1}(Q_+^s(q'))) \\ &\geq \eta^s(q_0, \delta) = \pi_h^B(q_0) - \pi_\ell^B(q_0, \delta, v_L^s(q'), v_B^s(q'))\end{aligned}\tag{39}$$

for all $q_0 \in [\underline{q}^s, \bar{q}^s)$ and $\delta \in [0, \bar{\delta}]$; (iii) v_L^s is continuous and increasing in $[\underline{q}^s, \bar{q}^s)$ and $v_L^s(q_0)$ converges to $v_L^{s-1}(\bar{q}^s)$ as q_0 increases to $\bar{q}^s$. Conditions (i) to (iii) are true when $k = 3$ by Propositions 2 and 3 and Lemmas 2 and 3.

First notice that $\underline{q}^k < \underline{q}^{k-1} \leq \bar{q}^k$. This fact follows from (iii) (and the definition of $\bar{q}^k$) and its proof is identical to the proof that $\underline{q}^2 < \underline{q}^1 \leq \bar{q}^2$; simply replace the superscripts “1” and “2” with “ $k-1$ ” and “ k ”, respectively.

We now show that if $q' = q^+(q_0, v_L^{k-1}(q'))$ and $q' \in [\underline{q}^{k-1}, \bar{q}^{k-1})$, then $\eta^{k-1}(q', \delta) \geq \eta^k(q_0, \delta)$

for all $\delta \in [0, \bar{\delta}]$. For this, note that

$$\begin{aligned}
& \pi_\ell^B(q_0, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')) \\
&= (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)[u_L - p_\ell] + \delta \left\{ q_0 + (1 - q_0) \left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) \right] \right\} v_B^{k-1}(q') \\
&= (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)[u_L - p_\ell] + \delta \left\{ q_0 + (1 - q_0) \left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) \right] \right\} \pi_h^B(q') \\
&\quad + \delta \left\{ q_0 + (1 - q_0) \left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) \right] \right\} [v_B^{k-1}(q') - \pi_h^B(q')] \\
&= \delta \pi_h^B(q_0) + (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right)[u_L - p_\ell - \delta(u_L - p_h)] \\
&\quad + \delta \left\{ q_0 + (1 - q_0) \left[1 - F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) \right] \right\} [v_B^{k-1}(q') - \pi_h^B(q')]
\end{aligned}$$

where the last equality follows from (37). Similarly, one can show that

$$\begin{aligned}
& \pi_\ell^B(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q'')) \\
&= \delta \pi_h^B(q') + (1 - q')F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right)[u_L - p_\ell - \delta(u_L - p_h)] \\
&\quad + \delta \left\{ q' + (1 - q') \left[1 - F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right) \right] \right\} [v_B^{k-2}(q'') - \pi_h^B(q'')],
\end{aligned}$$

where $q'' = Q_+^{k-1}(q')$. By (39) and Lemma 4 below, we then have that

$$v_B^{k-1}(q') - \pi_h^B(q') \geq v_B^{k-2}(q'') - \pi_h^B(q''),$$

so that

$$\begin{aligned}
& \pi_\ell^B(q_0, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')) - \pi_\ell^B(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q'')) \leq \delta [\pi_h^B(q_0) - \pi_h^B(q')] \\
&\quad + \left\{ u_L - p_\ell - \delta(u_L - p_h - [v_B^{k-1}(q') - \pi_h^B(q')]) \right\} \left\{ (1 - q')F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right) \right. \\
&\quad \left. - (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) \right\}. \quad (40)
\end{aligned}$$

Since $q' \geq q_0$ and $v_L^{k-1}(q') < v_L^{k-2}(q'')$ by Lemma 5 below, we have that

$$(1 - q')F\left(\frac{p_\ell}{v_L^{k-2}(q'')}\right) - (1 - q_0)F\left(\frac{p_\ell}{v_L^{k-1}(q')}\right) < 0.$$

In addition, $u_L < p_h$ and $u_L - p_\ell \geq \delta(u_H - p_h) > \delta[v_B^{k-1}(q') - \pi_h^B(q')]$. Hence, (40) implies that

$$\pi_\ell^B(q, \delta, v_L^{k-1}(q'), v_B^{k-1}(q')) - \pi_\ell^B(q', \delta, v_L^{k-2}(q''), v_B^{k-2}(q'')) < \pi_h^B(q) - \pi_h^B(q'),$$

which is the desired result. Consequently, (ii) holds when $s = k$. Since $\eta^{k-1}(q', \bar{\delta}) < 0$ for all $q' \in [\underline{q}^{k-1}, \bar{q}^{k-1})$, as the most patient buyer must strictly prefer to offer p_ℓ in $t = 0$ in a $(k - 1)$ -step equilibrium when $k \geq 3$, we then have that (25) and (26) imply (27).

Now observe that the same argument used in the proof of Proposition 3 shows that there is a k -step equilibrium if, and only if, $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$, that for each $q_0 \in [\underline{q}^k, \bar{q}^k) \cap (0, 1)$, there is a unique $q' \in [\underline{q}^{k-1}, \bar{q}^{k-1})$ such that q' is the value of q_1 in a k -step equilibrium when the initial fraction of type H sellers is q_0 , and that the map $Q_+^k : q_0 \mapsto q'$ is continuous and strictly increasing; simply replace the superscripts “1” and “2” with “ $k - 1$ ” and “ k ”, respectively. In particular, (i) holds when $s = k$.

To finish, notice that the same argument used in the proof of Lemma 3 shows that (iii) holds when $s = k$; once more just replace the superscripts “1” and “2” with “ $k - 1$ ” and “ k ”, respectively.

6.7 Lemma 4 and Proof

Lemma 4. *Given $q' = q_+ [q, v_L^{k-1}(q')]$, $v_B^k(q) - \pi_h^B(q) \geq v_B^{k-1}(q') - \pi_h^B(q')$.*

Let $\tilde{\eta}^k(q, \delta) = -\eta^k(q, \delta)$. Moreover, let $\delta^k(q)$ denote the maximum of zero and the value of δ such that $\tilde{\eta}^k(q, \delta) = 0$. Then (39) implies that $\delta^{k-1}(q') \geq \delta^k(q)$. This fact, along with $\tilde{\eta}^k(q, \delta) \geq \tilde{\eta}^{k-1}(q', \delta)$ for all δ , implies

$$\begin{aligned} v_B^k(q) - \pi_h^B(q) &= \int_{\delta^k(q)}^{\bar{\delta}} \tilde{\eta}^k(q, \delta) dF(\delta) \\ &\geq \int_{\delta^k(q)}^{\bar{\delta}} \tilde{\eta}^{k-1}(q', \delta) dF(\delta) \\ &\geq \int_{\delta^{k-1}(q')}^{\bar{\delta}} \tilde{\eta}^{k-1}(q', \delta) dF(\delta) = v_B^{k-1}(q') - \pi_h^B(q'). \end{aligned}$$

6.8 Lemma 5 and Proof

Lemma 5. $v_L^{k-1}(q') < v_L^{k-2}(q'')$.

Proof to be added.

References

- G.A. Akerlof. The Market for “Lemons”: Quality Uncertainty and the Market Mechanism. *The Quarterly Journal of Economics*, 84(3):488–500, 1970.
- Y. Amihud and H. Mendelson. Asset pricing and the bid-ask spread. *Journal of Financial Economics*, 17(2):223–249, 1986.
- M.R. Blouin. Equilibrium in a decentralized market with adverse selection. *Economic Theory*, 22(2):245–262, 2003.
- M.R. Blouin and R. Serrano. A decentralized market with common values uncertainty: Non-steady states. *Review of Economic Studies*, pages 323–346, 2001.
- P. Bolton, T. Santos, and J. Scheinkman. Inside and outside liquidity. *Quarterly Journal of Economics*, *forthcoming*, 2011.
- J. Chiu and T.V. Koepl. Market making of last resort: Optimal intervention when financial markets are frozen. 2009.
- G.M. Constantinides. Capital market equilibrium with transaction costs. *The Journal of Political Economy*, 94(4):842–862, 1986.
- R. Deneckere and M.Y. Liang. Bargaining with interdependent values. *Econometrica*, 74(5):1309–1364, 2006.
- D.W. Diamond and P.H. Dybvig. Bank runs, deposit insurance, and liquidity. *The Journal of Political Economy*, 91(3):401–419, 1983. ISSN 0022-3808.
- D. Duffie, N. Garleanu, and L.H. Pedersen. Over-the-counter markets. *Econometrica*, pages 1815–1847, 2005.
- D. Duffie, N. Gârleanu, and L.H. Pedersen. Valuation in over-the-counter markets. *Review of Financial Studies*, 20(6):1865, 2007.

- A.L. Eisfeldt. Endogenous liquidity in asset markets. *The Journal of Finance*, 59(1):1–30, 2004.
- R. Evans. Sequential bargaining with correlated values. *The Review of Economic Studies*, pages 499–510, 1989.
- D. Fudenberg, D. Levine, and J. Tirole. Infinite-horizon models of bargaining with one-sided incomplete information. *Game theoretic models of Bargaining*, pages 73–98, 1985.
- D. Gale. Bargaining and Competition Part I: Characterization. *Econometrica*, 54(4):785–806, 1986a.
- D. Gale. Bargaining and Competition Part II: Existence. *Econometrica*, 54(4):807–818, 1986b.
- N. Gârleanu. Portfolio choice and pricing in illiquid markets. *Journal of Economic Theory*, 144(2):532–564, 2009.
- L. Glosten and P. Milgrom. Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Agents. *Journal of Financial Economics*, 14:71–100, 1985.
- G. Gorton. Information, Liquidity, and the (Ongoing) Panic of 2007. *The American Economic Review*, 99(2):567–572, 2009.
- V. Guerrieri and R. Shimer. Competitive Equilibrium in Asset Markets with Adverse Selection. 2010.
- F. Gul, H. Sonnenschein, and R. Wilson. Foundations of dynamic monopoly and the coase conjecture. *Journal of Economic Theory*, 39(1):155–190, 1986.
- F. Heider, M. Hoerova, and C. Holthausen. Information asymmetries in the interbank market: Theory and policy response. *mimeo, European Central Bank, Frankfurt*, 2008.
- Y. Higashi, K. Hyogo, and N. Takeoka. Subjective random discounting and intertemporal choice. *Journal of Economic Theory*, 144(3):1015–1053, 2009.

- J. Hörner and N. Vieille. Public vs. private offers in the market for lemons. *Econometrica*, 77(1):29–69, 2009.
- R. Inderst. Matching markets with adverse selection. *Journal of Economic Theory*, 121(2):145–166, 2005.
- M. Janssen and S. Roy. Dynamic Trading in a Durable Good Market with Asymmetric Information. *International Economic Review*, 43:257–282, 2002.
- A. Krishnamurthy. How Debt Markets Have Malfunctioned in the Crisis. *Journal of Economic Perspectives*, 24(1):3–28, 2010.
- P. Kurlat. Lemons, Market Shutdowns and Learning. *unpublished manuscript, MIT*, 2009.
- A.S. Kyle. Continuous Auctions and Insider Trading. *Econometrica*, 53(6):1315–1335, 1985.
- D. Moreno and J. Wooders. Prices, delay, and the dynamics of trade. *Journal of Economic Theory*, 104(2):304–339, 2002.
- D. Moreno and J. Wooders. Decentralized trade mitigates the lemons problem. *International Economic Review*, 51(2):383–399, 2010. ISSN 1468-2354.
- G. Rocheteau. A monetary approach to asset liquidity. *unpublished manuscript, University of California at Irvine*, 2009.
- A. Rubinstein and A. Wolinsky. Equilibrium in a Market with Sequential Bargaining. *Econometrica*, 53(5):1133–1150, 1985.
- M. Satterthwaite and A. Shneyerov. Dynamic Matching, Two-Sided Incomplete Information, and Participation Costs: Existence and Convergence to Perfect Competition. *Econometrica*, 75(1):155–200, 2007.
- D. Vayanos and P.O. Weill. A Search-Based Theory of the On-the-Run Phenomenon. *The Journal of Finance*, 63(3):1361–1398, 2008.

- D.R. Vincent. Bargaining with common values. *Journal of Economic Theory*, 48(1):47–62, 1989.
- C. Wilson. The nature of equilibrium in markets with adverse selection. *The Bell Journal of Economics*, 11(1):108–130, 1980.
- A. Wolinsky. Information revelation in a market with pairwise meetings. *Econometrica*, pages 1–23, 1990.

Figure 3: Liquidity and the Fraction of Lemons

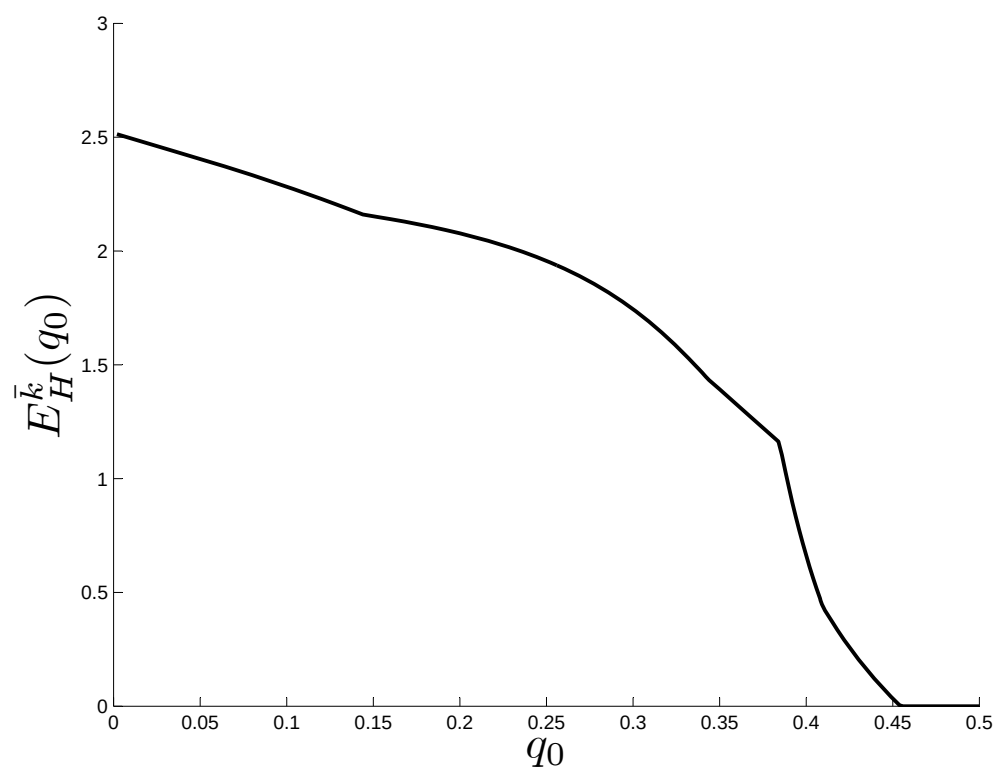


Figure 4: The Dynamics of Trade

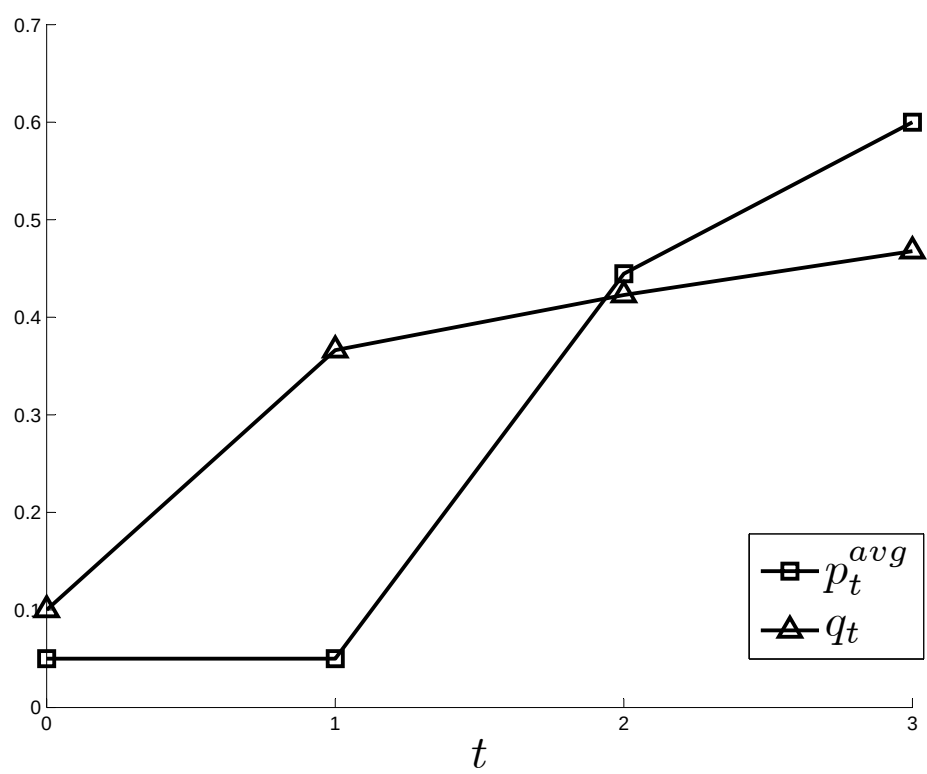


Figure 5: The Effect of Policy on Liquidity

