Credit Lines*

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Abstract

This paper develops a new theory of long term unsecured credit contracts based on costly contracting that matches the data in a variety of dimensions. Credit lines are long term relations between lending firms and households that pre-specify a credit limit and interest rate in each period. Households can unilaterally default in as in the U.S. Bankruptcy code, and can unilaterally switch credit lines. Lending firms can set a new credit limit at any time, but must commit to the interest rate or not depending on the regulatory setting. We solve and characterize the equilibria, finding the resulting set of contracts as well as the distribution of households over interest rates, credit limits and wealth. We find that this model replicates the main properties of typical lending contracts. We use the theory to study the new regulatory rules in the U.S. credit card market which require a stronger commitment from lending firms not to raise interest rates discretionally. This results in tighter limits but lower interest rates, reduced indebtedness and lower default. Typically, but not for all households, the new policy improves welfare.

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1 Introduction

Lending contracts in the U.S. do not live in a vacuum but are shaped by a set of regulations of which bankruptcy provisions and Regulation Z are the central ingredients. The regulations specify the conditions under which debt can be discharged and also the degree of commitment that lending institutions have to abide to. In this paper we take these regulations to heart together with the notion that contracting is costly both to borrowers and lenders, and develop a theory of unsecured credit where borrowing contracts take the form of credit lines that specify credit limits and interest rates. As in the U.S., some of the terms of a contract may be changed by the lending firms and households can switch lenders. Our theory reproduces the main features of the U.S. unsecured lending environment, including the patterns of switching, bankruptcy, interest rates, credit limits and debt levels across households.

We use the theory that we develop to study the effects of the new regulatory rules in the U.S. credit card market. The Federal Reserve has recently approved fundamental changes in the industry with the aim of curbing so called 'deceptive practices' on the part of issuers.[1] Prominent among these changes is the banning of interest rate hikes on existing credit card balances.[2] The Fed and consumer groups welcome this shift as they expect it to benefit consumers by protecting them against surprise and abusive changes in conditions. The banking industry have expressed concerns that this will hurt consumers by reducing competition and access to credit. The unexplored question that this paper addresses is how will consumers be affected in the end by the restrictions on pricing introduced?

In order to address this question effectively, we first develop a suitable analytical framework capable of generating loan contracts between lending firms and households that extend over many periods under the current U.S. legal system and the limited commitment of the parties that it entails. With such a model in place we are in a position to assess the enacted policy change (or an abstraction that approximates it) and its welfare implications.

We interpret the issue of commitment or lack there of by the banks as one where they cannot commit. This is not the same as interpreting that the current law allows the banks to choose a contract with or without commitment to future terms, but that with the previous legislation the bank can always ex-post change the terms of the contract even if they had promised not to do so. A rationale for this is that it can always be expressed as a function of a change in the circumstances

[1] The Board voted the new regulation on December 18, 2008. These rules were amended and renamed to the Credit Card Accountability Responsibility and Disclosure Act of 2009 or the Credit CARD Act of 2009. President Obama signed the legislation on May 22, 2009, making it part of the Truth in Lending Act. These rules are in effect already (June 2010).

[2] Except under limited circumstances associated to lack of payments.

of either the economy or the borrower, and that these are too complicated to prespecify fully in a given contract.

We find that BLANK BLANK

The analytical framework that we develop uses the basic equilibrium model of consumer idiosyncratic risk and incomplete markets used in much quantitative macroeconomics (e.g., Bewley (1986), Diaz, Pijoan-Mas, and Ríos-Rull (2003) İmrohoroğlu (1989), Huggett (1993), Aiyagari (1994)). The model introduces three fundamental assumptions. First, borrowing is not restricted to have to be conducted by means of one period debt contracts only, thus allowing for lasting credit lines. Second, within a given credit line, a lender can change the debt limit at any time but may or may not deviate from the interest rate initially signed depending on the regulatory setting. A form of partial commitment intends to capture the shift in the U.S. consumer protection law. Third, indebted households are not committed to honour their debts and can exercise the option to declare bankruptcy in a way that encompasses the main provisions of Chapter 7 of the U.S. bankruptcy code.

In the benchmark version of the model, there is no commitment and lending firms can alter the terms of the contract – both the interest rate and the limit – at the beginning of any period. In this setting, a household holding one credit card knows its associated current limit and interest rate, but can only make a (rational) guess about how these terms will evolve in subsequent periods. Switching to a different credit card is possible but costly for the household. Any of the credit lines available for a consumer to switch to specifies the consumer's current class type and – assuming initial full information – the size of the initial loan. Subsequently, the household can borrow any amount up to the debt limit set by the bank, switch to another line or declare default.[3] Note that, given the switching cost, consumers may in general wish to hold on to the present credit card even if their personal circumstances change. Credit lines are issued by competitive financial intermediaries or lending firms, the second group of actors in the economy. A bank launching a line offers the initial terms of the contract. In any subsequent period, and as long as the line survives, the bank sets the credit limit and interest rate at will under incomplete information about the card holder's current earnings.

There is a large set of potential types of credit lines. The characteristics of the credit lines that are effectively traded must be determined in equilibrium by the requirements that the lending firms' rules for debt limit and interest rates are time-consistent and that there is no profitable entry of other types of contracts. This calls for studying the behavior of households and existing

[3]Notice that the properties of the contracts of our model agree with the typical credit card contracts. The only issue is exclusivity, but this is implicit in the existing arrangements. Lending firms know the asset position of its customers every few months and rearrange the terms of the contract accordingly.

lending firms to deviations in the credit limit and interest rate. The stationary distribution of the model delivers profiles of consumption, wealth/debt, credit card limits and interest rates, default and switching across households.

We study first a quantitative version of this no-commitment benchmark economy. The parameters of the model are calibrated to match U.S. aggregate statistics for the distributions of wealth and earnings, the default and write-off rates on unsecured debt, and the credit limits and interest rates by income groups. Within this setting, lending firms do often raise interest rates after the initial period and credit limits respond to changes in the borrower's observable characteristics. High-income households enjoy disproportionately larger limits, borrow less frequently but, when they do, incur higher debts. Many households revolve credit repeatedly within a given contract, but about two thirds of borrowers are switching to a new contract, typically one with a looser limit but a higher interest rate.

In order to study the regulatory changes in the U.S., we also consider a version of the model where lending firms are not allowed to change the interest rate at their discretion. There is commitment to stick to the initially agreed price. In this case, compared to the no-commitment benchmark, there is even a larger set of potential types of contracts.In an equilibrium with free-entry, there must be no profitable entry of other types of contracts. In the quantitative setting, the adoption of such price commitment causes changes in the credit market. Generally, lending firms tighten debt limits but charge lower interest rates. This changes is sharper among high-income households. In terms of aggregates, borrowing decreases and wealth increases, the risk of default and the frequency of contract switching both decline, and average consumption rises.

There is an emerging literature analyzing bankruptcy and credit in quantitative general equilibrium models. In Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) there is no switching cost and credit conditions adjust instantaneously. So it considers one-period contracts, where the interest adjusts with the size of the loan, and there is no room for pre-approved credit limits. Livshits, MacGee, and Tertilt (2007b) and Livshits, MacGee, and Tertilt (2007a) and Athreya and Simpson (2006) share similar features. In contrast, the present paper has the realistic feature that, due to the cost of switching, households can keep the same credit conditions for loans of varying size and over changing personal circumstances. Mateos-Planas (2007) studies the determination of an endogenous credit limit that is binding for some households in a model where lending firms confront an adverse selection problem since the information about the characteristics of individuals is limited. It has one-period contracts with credit limits where the interest does not depend on the amount borrowed but, for tractability reasons, constraints the range of contracts that can arise. It cannot be used to study variability in these conditions. The present paper instead delivers long-lasting contracts and a non-trivial distribution of credit limits and interest rates. A new strand of the literature (Chatterjee, Corbae, and Rios-Rull (2005)) attempts to provide a theory of credit without exogenous punishment. In independent work, Drozd and Nosal (2007) have posed a model with credit lines and default in an environment with search type frictions. They also have multiperiod contracts, but full commitment and constant conditions within any given contract. Notice that none of these existing models can be used to address changes that affect the level of commitment in the lenders' policies.

[[There is also an important literature on endogenous borrowing constraints and contracts, which includes Kehoe and Levine (1993), Wang (1995), Kocherlakota (1996), Cole and Kocherlakota (2001), and Hopenhayn and Werning (2008). The main differences with the present paper are ...]]

Section 2 discusses both the regulatory environment and properties of the existing contracts that we are relevant for our study. Section 3 puts forward the model for the households and the financial intermediaries and describes the decisions of these agents. Section 4 defines the equilibrium and discusses issues related to its characterization and computation. Section **??** maps the theory (without commitment) to the data to obtain the benchmark quantitative model, and assesses its implications against evidence on debt, credit limits and interest rates across households. Section **??** analyzes the bank's behavior and the properties of the contracts. Section **??** studies how the main properties of the model economy vary under alternative calibration targets. Section **??** explores and evaluates quantitatively the effects of the proposed policy change, both for observable variables and for welfare. Section 8 concludes.

2 Institutions and credit card facts

3 The model

We now turn to describe each of the elements of the model, starting with households in Section 3.1, and the problem that they solve which takes as given the set of available contracts. We then continue with the financial intermediaries in Section 3.4.

3.1 Households

There are many infinitely-lived households with standard preferences over consumption streams and for whom certain actions will yield utility loses.

Endowments. Each period, households have a stochastic endowment of labor earnings ε in $\mathcal{E}$, a bounded set. The distribution of earnings is controlled by $F^{\varepsilon}_{e}(\varepsilon)$ where $e \in E$ is income class and it follows a Markov chain with matrix $\Gamma_{e,e'}$.[4] Income class is publicly observable while actual income may be privately observable. To separate these two features and to avoid cumbersome notation, we write the publicly observable income also as e.

Assets. Households are also associated to a publicly observed asset position $y \in Y$. If this asset is positive then they obtain a risk free (inverse of the) rate of return q^*. If they have an active contract with a lending firm they may have a negative asset position with an associated interest rate and debt limit that is specified by the terms of the contract. For mathematical and computational simplicity the set of possible asset levels is discrete. Such choice does not restrict the model in any relevant manner.

Contracts. A newly specified contract θ specifies the amount that the household borrows in this period, $y_0'^{\theta} \leq 0$, and the interest rate at which it does so q_0^{θ}. It is indexed by the additional observable characteristics of the customer at the time of signing which may be useful in making predictions about the future behavior of the household. In this model the only relevant such variable is the income class e. We write q_0^{θ}, e_0^{θ} and $y_0'^{\theta}$ to refer to the initial terms of a contract.

After the initial signing period, the contract becomes a credit line. This is, it specifies a credit limit and an interest rate for every possible asset level and income class of the borrower, $b^{\theta}(e, y)$ and $q^{\theta}(e, y)$. The dependence on these two variables arises from the fact that they are the only ones publicly observable, and therefore, the lending firm can discriminate households on this regard. The reason why the beginning of period asset position matters. It has to do with the fact that different assets held affect the decision to default or to switch or even of how much to save.

Credit history, bankruptcy filing and contract switching. Households have a credit history flag that we denote with h. Such a flag can take the value of 0 or the value of 1. If $h = 0$ the household is free to engage in contracting, and if $h = 1$ it is banned from doing so.[5] A household with $h = 0$ switches to $h = 1$ only upon engaging in the process of defaulting, or more precisely, filing for bankruptcy. A household with $h = 1$ switches to $h = 0$ with constant probability δ to reflect a certain expected duration of the bad credit history.

Filing for bankruptcy entails a utility cost χ_d. Engaging in contracting or recontracting (switching in our own jargon) also carries a utility cost, χ_s. Both utility costs are independent identically distributed random variables with continuous and bounded support and distribution $F^s(\chi_s)$ and $F^d(\chi_d)$. Both utility costs are privately observed. We sometimes write for compactness of notation

[4] This allows for simultaneously having a large range of possible values of income and a role for persistence without excessive computational burden.

[5] See the discussion in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for a rationale for summarizing the credit scores this way.

$F_e(\varepsilon, \chi)$ to refer to the distribution of all idiosyncratic, privately observable i.i.d. shocks.

Before continuing, a word for making the case for this seemingly arbitrary and large set of unobservable, idiosyncratic variables. Their purpose is to introduce continuity in the profit functions of the lending firms. As this firms change the terms of their contracts profits change smoothly because so does the fraction of households taking each of their actions which have discrete support.

States. At the beginning of a period, the household is characterized by both public and privately observed variables. The publicly observed variables are the public content of income (which includes the information needed to forecast future income), e, its asset position, y, its credit history history h, and its credit line or contract $\theta \in \Theta$, with $\theta = 0$ meaning no contract. The set Θ of available contracts is an equilibrium object to be determined below. The privately observed variables are the private component of income, ε, and the utility costs for defaulting and switching $\chi = \{\chi_d, \chi_s\}$. To summarize we denote by z the state, and the public component of it by z^p.

Households Choices. The household makes three types of interrelated choices. These are the bankruptcy (or default) decision $d \in \{0, 1\}$ which only makes sense when the household has a contract and is in debt. Whether to choose a new contract (switch credit line) $s \in \{0, 1\}$, a choice that is is available only if the household currently has a good credit record $h = 0$. Is the household switches, then there is the choice to which contract to switch to, a choice that automatically involves its borrowing as this is determined by a new contract $\theta' \in \Theta$. If a household is not switching (and is not defaulting in which case U.S. law restricts savings to the exempt level), than it chooses how much to save, a choice restricted by the contract it has, $y' \in \mathcal{Y}^{\theta'}$. Note again that the main complication is the determination of the equilibrium set of contracts Θ.

Budget constraint and preferences. With what we have said, and the fact that all saving can be done at the risk free rate of return q^*, we can write the budget constraint in a compact if cumbersome way as

$$c + \left\{ q^* 1_{y' \geq 0} + \left[s\ q_0^{\theta'}\ 1_{y' = y_0'^{\theta'}} + (1 - s)\, q^{\theta}(y, e) \right] 1_{y' < 0} \right\} y'(1 - d) = (1 - d)\ y + \varepsilon, \qquad (1)$$

where the three terms that multiply the assets to be held the following period include the relevant interest rates when the household is saving, switching, or borrowing via an existing credit line. The right hand side is cash in hand (sum of assets from the past and (publicly and privately observed) income. When the household files for bankruptcy both the savings for next period and the influence of the debt in cash in hand are zero.

Households have standard preferences over streams of consumption, with per period utility function $u(c)$ and discount rate β. Consequently, we write the per period total felicity taking into account the possible utility costs associated with either defaults or switches as

$$u(c) - \chi_s \, s - \chi_d \, d. \tag{2}$$

3.2 The decision problem

It is convenient to split the decision making problem of the household in three different stages. The first stage determines the determination of credit status h', the second stage determines the type of contract θ', and the third stage determines the asset position y'. We proceed to solve for the value function, $v(z) = v(e, y, \theta, h, \varepsilon, \chi_d, \chi_s)$ backwards.

Third stage: the asset choice. The household's situation depends on choices and events of previous stages that determined a new credit record h' and a new credit line θ'. The various possibilities are

- If the household has chosen to default $d = 1$, its consumption will be its endowment, $c = \varepsilon$, and $h' = 1$, $\theta' = 0$, and $y' = 0$. Its corresponding value in stage 3, $v^{3,d}$, is a function only of the current endowment and income class and we write

  $$v^{3,d}(e, \varepsilon, \chi_d) = u(e + \varepsilon) - \chi_d + \beta \; E\left\{v(e', 0, 0, 1, \varepsilon', \chi')|e\right\} \tag{3}$$

  where the expectation is over the exogenous states ε', e' and χ' although in this case χ' is actually irrelevant as neither switching nor defaulting will be available options the next period and those costs are iid.

- If the household has not defaulted and had switched line it will repay all debts, keep the good credit record, and borrow as specified by the new contract for a new customer. Given that the endowment ε, is transitory, the stage 3 value for a switcher $v^{3,s}$ is a function of cash in hand, income class, and the new type of contract

  $$v^{3,s}(e, y, \theta', \varepsilon, \chi_s) = u\left[y + e + \varepsilon - q_0^{\theta'} y_0^{\prime\theta'}\right] - \chi_s + \beta \; E\left\{v\left(e', y'^{\theta'}, \theta', 0, \varepsilon', \chi'\right)|e\right\}. \tag{4}$$

- If the household has not defaulted nor switched, then it can choose how much to borrow subject to the debt limit and keep the same line. The value for a non-switcher $v^{3,n}$ is then a function of the initial contract and the updated credit record:

$$v^{3,n}(e,y,\theta,h',\varepsilon) = \max_{y' \geq b^{\theta}(y,e)} \left\{ u \left(y + e + \varepsilon - \left[q^* \, 1_{y' \geq 0} + q^{\theta}\,(e,y)\ 1_{y'<0} \right] \, y' \right) + \right.$$
$$\left. \beta \; E \left\{ v\left(e', y', \theta, h', \varepsilon', \chi'\right) | e \right\} \right\}. \quad (5)$$

We write the solution as $y'^{n}(e,y,\theta,h'\varepsilon)$. This notation covers the circumstance of a household with an initial bad record that did not cleared, $h = h' = 1$, in which case it is necessarily the case that they have the zero line $\theta = 0$ and cannot borrow because $b^0(y,e) = 0$.

Second stage: the contract choice. Only households with good credit standing because they had not defaulted can choose or switch contracts. Those with bad credit because they defaulted in the first stage have the value already determined in the third stage, $v^{2,d}(e,\chi_d,\varepsilon) = v^{3,d}(e,\chi_d,\varepsilon)$. Those that owe there bad credit to not having been forgiven by nature also have the value determined in the third stage, in this case $v^{2,n}(e,y,1,0,\varepsilon) = v^{3,n}(e,y,1,0,\varepsilon)$. Finally, a household with good credit $h' = 0$ can decide whether and where to switch and its value is

$$v^{2,n}(e,y,\theta,0,\varepsilon,\chi_s) = \max_{s\in\{0,1\}} \left\{ s \max_{\theta' \in \Theta^e} \; v^{3,s}(e,y,\theta',\varepsilon,\chi_s) + (1-s)\, v^{3,n}(e,y,\theta,0,\varepsilon) \right\} \quad (6)$$

Where Θ^e denotes the set of contracts available to a type e household. The decisions whether to switch and which line to choose if switching that solve the problem are $s(e,y,\varepsilon,\chi_s,\theta)$ and $\theta'^{s}(e,y,\varepsilon)$ respectively (notice that the decision of where to switch to does not depend on the utility cost of switching).

First stage: the default choice. Only households with good credit can default, otherwise nature decides whether to forgive or not. The corresponding value is represented as:

$$v(e,y,\theta,h,\varepsilon,\chi_d,\chi_s) = \begin{cases} \max_{d\in\{0,1\}} \left\{ (1-d)\ v^{2,n}(e,y,\theta,0,\varepsilon,\chi_s) + d\ v^3_d(e,\varepsilon,\chi_d) \ \right\} \\ \\ \delta\ v^{2,n}(e,y,0,0,\varepsilon,\chi_s) + (1-\delta)\ v^{2n}(e,y,0,1,\varepsilon,\chi_s) \end{cases} \quad (7)$$

With solution for the first row problem given by $d(e,y,\theta,\varepsilon,\chi)$. We refer to decision rules $\{y'^{n}, s, \theta'^{s}, d\}$ as *effective* decision rules because they operate along the equilibrium path.

The lack of commitment of lending firms requires that to characterize equilibria, we have to understand one period deviations from equilibrium by various agents.[6] We analyze the behavior

[6]This type of analysis is typical in the analysis of policy without commitment. See for example, Klein, Krusell, and Ríos-Rull (2008).

of the household under one period deviation of credit limits and interest rates in Section 3.3.1 and under the possible introduction of contracts that do not belong to the equilibrium set Θ in Section 3.3.2.

3.3 Deviations in debt limits and prices of a contract and in the set of contracts.

The behavior of the household that we have described is not sufficient to characterize equilibrium. Lending institutions have to assess what *would* households do if the terms of the contract, the credit limit and the interest rate, were different this period (Section 3.3.1). The reason being that that the lending firms can change the terms at will due to their lack of commitment. Consequently, lending firms evaluate their profits for all possible values of credit limits and interest rates. We also have to assess what would the response of existing contracts be to prospective contracts that are *not* in the equilibrium set of contracts (Section 3.3.2).

3.3.1 Deviations in debt limits and prices of an existing contract

Households if they were to face this arbitrary lending terms would have decisions to make that can be partitioned again into three stages. The analysis requires the value function generated in the previous stage, v, that summarizes what households get after the deviation from the lender is over.

Third stage: the asset choice. This stage is only relevant for a non–defaulter, non–switcher, and when is offered an arbitrary pair of credit limit b and interest rate q solves (note that we use red and tildes to denote this potential situation)

$$\widehat{v}^{3,n}(e, y, \theta, 0, \varepsilon, b, q) = \max_{y' \geq b} \{u(y + e + \varepsilon - [q \; 1_{y' \geq 0} + q \; 1_{y'<0}] \; y') + \beta \; E \{v(e', y', \theta, 0, \varepsilon', \chi')|e\}\} \quad (8)$$

with solution $\widetilde{y}'^{n}(e, y, \theta, 0, \varepsilon, b, q)$.

Second stage: the contract choice. The value of non defaulting is

$$\widetilde{v}^{2,n}(e, y, \theta, 0, \varepsilon, \chi_s, b, q) = \max_{s \in \{0,1\}} \left\{ s \max_{\theta' \in \Theta_e} v^{3,s}(e, y, \theta', \varepsilon, \chi_s), \;\; (1-s) \; \widetilde{v}^{3,n}(e, y, \theta, 0, \varepsilon, b, q) \right\} \quad (9)$$

with solution $\widetilde{s}(e, y, \theta, \varepsilon, \chi_s, b, q)$ and $\widetilde{\theta}'^{s}(e, y, \varepsilon)$.

First stage: the contract choice. The decision of whether to default or not comes from

$$\widetilde{d}(e, y, \theta, \varepsilon, \chi, b, q) = \underset{d\in\{0,1\}}{\text{argmax}} \left\{(1-d)\ \widetilde{v}^{2,n}(e, y, \theta, 0, \varepsilon, \chi_s, b, q) + d\ v^3_d(e, \varepsilon, \chi_d)\ \right\} \tag{10}$$

We refer to the red–tilde decision rules $\{\widetilde{y}, \widetilde{s}, \widetilde{\theta}'^s, \widetilde{d}\}$ as *within contract deviation decision rules*.

Characterizing the consumer behaviour given contracts amounts to solving functional equations (3- 7) that solve the problem of the household as well as functional equations (8- 10) that solve the household problem under deviations. Notice that the behaviour of the household as implied by equations (3- 7) is the same as that in (8- 10) when we impose that the credit limit and interest rate satisfy $b = b^\theta(y, e)$ and $q = q^\theta(y, e)$.

3.3.2 Deviation in contracts

In order to understand the free entry condition, we have to pose what would happen if an additional contract $\widehat{\theta} \notin \Theta^e$ were offered. Such contract, which needs only be offered one period, is what lending firms considering to issue contract $\widehat{\theta}$ need to know to assess the rewards to entry.

Note that the value function of those that have not chosen contract $\widehat{\theta}$ remains unchanged as well as the other objects determined as of the third stage: functions $\{v^{3,d}, v^{3,s}, v^{3,n}\}$ determined in equations (3-5). To denote the new objects that arise under this thought experiment we use green and hats. To reduce the total number of functions to deal with, we podr the problem of the household as we did in Section 3.3.1 (were we used red-tilde functions) directly analyzing all possible interest rates and credit limits instead of only the ones dictated by contract $\widehat{\theta}$ and then analyzing one period devitaions.

After the only period that new contract $\widehat{\theta}$ was available to sign, the problem faced by holders of that contract in stage 3 is

$$\widehat{v}^{3,n}(e, y, \widehat{\theta}, 0, \varepsilon, b, q) = \max_{y'\geq b} \{u\,(y + e + \varepsilon - [q^*\ 1_{y'\geq 0} + q\ 1_{y'<0}]\,y') + \\ \beta\ E\ \left\{\widehat{v}\left(e', y', \widehat{\theta}, 0, \varepsilon', \chi'\right)\right\}\right\}. \tag{11}$$

with policy rule $\widehat{y}'^n(e, y, \widehat{\theta}, 0, \varepsilon, b, q)$ and with yet to be determined $\widehat{v}$. In the second stage such household faces

$$\widehat{v}^{2,n}(e, y, \widehat{\theta}, 0, \varepsilon, \chi_s, b, q) = \max\left\{\widehat{v}^{3,n}(e, y, \widehat{\theta}, 0, \varepsilon, b, q), \max_{\theta'\in\Theta^e} v^{3,s}(e, y, \theta', \varepsilon, \chi_s)\right\} \tag{12}$$

with solution $\widehat{s}(e, y, \widehat{\theta}, 0, \varepsilon, \chi_s, b, q)$. In the first stage, the problem is

$$\widehat{d}(e, y, \widehat{\theta}, 0, \varepsilon, \chi, b, q) = \underset{d \in \{0,1\}}{\text{argmax}} \left\{ (1-d)\ \widehat{v}^{2,n}(e, y, \widehat{\theta}, 0, \varepsilon, \chi_s, b, q) + d\ v^{3,d}(e, \varepsilon, \chi_d) \right\}. \quad (13)$$

Finally we obtain the hat function by using the contract continuation values that are obtained from the associated maximization problem solved by the managers of contract $\widehat{\theta}$ of the lending firm.

$$\widehat{v}(e, y, \widehat{\theta}, 0, \varepsilon, \chi) = \left\{ [1 - \widehat{d}(e, y, \widehat{\theta}, 0, \varepsilon, \chi, b^{\widehat{\theta}}(e, y), q^{\widehat{\theta}}(e, y))]\ \widehat{v}^{2,n} \left[e, y, \widehat{\theta}, 0, \varepsilon, \chi_s, b^{\widehat{\theta}}(e, y), q^{\widehat{\theta}}(e, y) \right] \right.$$
$$\left. + \widehat{d} \left[e, y, \widehat{\theta}, 0, \varepsilon, \chi, b^{\widehat{\theta}}(e, y), q^{\widehat{\theta}}(e, y) \right]\ v^{3,d}(\varepsilon, e, \chi_d) \right\}. \quad (14)$$

Equations (11- 14) yield the value of holding a new contract $\widehat{\theta}$ the periods after it has been introduced. We now turn to its value the period is newly offered. It is given by

$$\widehat{v}^{3,s}(e, y, \widehat{\theta}, \varepsilon, \chi_s) = u\left(y + e + \varepsilon - q_0^{\widehat{\theta}} y_0'^{\widehat{\theta}}\right) - \chi_s + \beta\ E\left\{ \widehat{v}\left(e', y'^{\widehat{\theta}}, \widehat{\theta}, 0, \varepsilon', \chi'\right) | e \right\}. \quad (15)$$

We now turn to construct functions that describe the context where holders of other constracts face the additional contract $\widehat{\theta}$. In the second stage the problem is

$$\widehat{v}^{2,n}(e, y, \theta, 0, \varepsilon, \chi_s, \widehat{\theta}) = \max \left\{ v^{3,n}(e, y, \theta, 0, \varepsilon), \max_{\theta' \in \Theta_e} v^{3,s}(e, y, \theta', \varepsilon, \chi_s), \widehat{v}^{3,s}(e, y, \widehat{\theta}, \varepsilon, \chi_s) \right\}. \quad (16)$$

Let $\widehat{s}^{\widehat{\theta}}(e, y, \theta, 0, \varepsilon, \chi_s) = 1$ denote whether a type $(e, y, \theta, 0, \varepsilon, \chi_s)$ would switch to contract $\widehat{\theta}$. In the first stage we have

$$\widehat{v}(e, y, \theta, 0, \varepsilon, \chi, \widehat{\theta}) = \max_{d \in \{0,1\}} \left\{ (1-d)\ \widehat{v}^{2,n}(e, y, \theta, 0, \varepsilon, \chi_s, \widehat{\theta}) + d\ v^{3,d}(e, \varepsilon, \chi_d) \right\}. \quad (17)$$

We refer to the green-hat decision rules $\{\widehat{y}'^{n}, \widehat{s}, \widehat{\theta}'^{s}, \widehat{d}, \widehat{s}^{\widehat{\theta}},\}$ as the decision rules associated with alternative contract $\widehat{\theta}$.

3.4 Lending Firms

Lending firms, or financial intermediaries, issue credit lines to households at a fixed initial cost π. The set of possible such credit lines θ consists of all possible initial conditions, income class e_0^{θ}, loan $y_0'^{\theta}$, and interest rate q_0^{θ}, and continuation terms for the credit limit and the interest rate b^{θ} and q^{θ} which for each contract are functions of the income class and asset position. The space of all potential available contracts Θ^P in general is thus the set:

$$\left\{\theta : \left[e_0^\theta, y_0'^\theta, q_0^\theta, b^\theta(e,y), q^\theta(e,y)\right] \in E \times \mathcal{Y}^- \times [0,1] \times \mathcal{C}_b \times \mathcal{C}_q\right\} \tag{18}$$

where $\mathcal{C}_b$ and $\mathcal{C}_q$ are the class of mappings $E \times \mathcal{Y} \to \mathcal{Y}^-$, and $E \times \mathcal{Y} \to [0,1]$, (sets of possible values for credit limits and interest rates, respectively). This is a very large set but in equilibrium only some such contracts will be operative. To characterize the set of equilibrium contracts, we start by describing the value of a given contract, without any concern with regard to whether the lending institution will issue it or whether its continuation terms are actually offered in Section 3.4.1. We then turn to look at the problem of the absence of commitment to more than one period ahead interest rates and credit limits in Section 3.4.2. It is at this stage that we the features of Regulation Z appear —that the bank can change the interest rates as well as the credit limit for the following period provided the latter is not smaller than the current level of debt. We then go on to describe commitment to a rate of return but not to the credit limit, which reflects the new legal environment in Section **??**. We leave the full characterization of equilibrium contracts for Section 4.

3.4.1 The value of a contract

Define the cash flow m for a bank of a household with observable type $\{e, y, \theta, 0\}$. Depending on the household's draw of the earnings, ε, and utility costs, χ, shocks, it can either default or not, and then it can switch or borrow different amounts. To simplify notation let $g(e, y, \theta, \varepsilon, \chi) = [1 - d(e, y, \theta, \varepsilon, \chi)]\,[1 - s(e, y, \theta, \varepsilon, \chi)]$, this an indicator function that denotes the survival of the contract because the household neither defaults nor switches. Consequently, the value for the bank is (taking into account the household's decision rules)

$$\begin{aligned}\Psi(e,y,\theta) = &-y\ 1_{y<0} \int_{\varepsilon,\chi} [1 - d(e,y,\theta,\varepsilon,\chi)]\ dF_e(\varepsilon,\chi) + \\ &\int_{\varepsilon,\chi} g(e,y,\theta,\varepsilon,\chi)\ 1_{y'^n(e,y,\theta,0,\varepsilon)<0}\quad q^\theta(e,y)\ y'^n(e,y,\theta,0,\varepsilon)\ \ dF_e(\varepsilon,\chi) \\ &+ q^* \int_{\varepsilon,\chi} g(e,y,\theta,\varepsilon,\chi)\ \left[\sum_{e'} \Gamma_{e,e'}\ \Psi\left[e', y'^n(e,y,\theta,0,\varepsilon), \theta\right]\right]\ dF_e(\varepsilon,\chi)\end{aligned} \tag{19}$$

where the first term is the expected payment of the loan, which takes into account that the household may default, the second term is the additional borrowing that the household may engage that requires the contract survives and the third term is the continuation value of the contract discounted at the risk free rate of return.

At the time of issuing the contract its value, denoted by $\Psi_0(\theta)$, is

$$\Psi_0(\theta) = q_0^\theta \;\; y_0^{\prime\theta} + q^* \int_{e'} \Gamma_{e^\theta, e'} \; \Psi(e', y_0^{\prime\theta}, \theta) \tag{20}$$

The absence of commitment implies that a bank is free to choose certain things each period regardless of what the contract specifies and we turn now to consider the problem of the bank under such circumstances. As with households, we refer to functions Ψ and Ψ_0 as *effective* contract values. As the reader may suspect, the lack of commitment prevents non-time consistent contracts from being offered.

3.4.2 The continuation problem of a bank

Recall that we are interpreting Regulation Z as an environment where the lending firm cannot commit to the continuing terms of a contract beyond one period and it chooses the terms before the household makes its decisions regarding default, switch and borrowing. The choices of the bank are therefore contingent in the observables at the beginning of the period, this is, the income class e, the asset position y and, of course the credit history h. To assess how the lending firm fares under the different choices of credit limits and interest rates we use the red-tilde functions derived in Section 3.3.

A lending firm at the beginning of a period sees a costumer of type $\{e, y, \theta\}$ and is considering which credit limit and interest rate to set. It has to both forecast what the consumer would do for each possible pair of credit limit and interest rate and it also has to forecast what future lending firms would do. The former is an out of equilibrium deviation that we have already analyzed in Section 3.3.1. From the household's problem in that Section, the bank obtains the tilde functions that will use in its decision making. Its problem is now

$$\begin{aligned}\widetilde{\Psi}(e, y, \theta, b, q) = -y \; 1_{y<0} \sum_{\varepsilon,\chi} F_e(\varepsilon, \chi)[1 - \widetilde{d}(e, y, \theta, \varepsilon, \chi, b, q)] + \\ \sum_{\varepsilon} F_e(\varepsilon, \chi) \; \widetilde{g}(e, y, \theta, \varepsilon, \chi, b, q) \; q \; \widetilde{y}'^n(e, y, \varepsilon, w, 0, b, q) \; 1_{\widetilde{y}'^n(e,y,\varepsilon,w,0,b,q)<0} \\ + q^* \sum_{\varepsilon} F_e(\varepsilon, \chi) \; \widetilde{g}(e, y, \theta, \varepsilon, \chi, b, q) \sum_{e'} \Gamma_{e,e'} \; \Psi(e', \widetilde{y}'^n(e, y, \varepsilon, w, 0, b, q), \theta)\end{aligned} \tag{21}$$

Alternative credit conditions for this period as described by the pair (b, q) affects all margins. The first line incorporates the default choice which is affected by future interest rates and the imposed credit limit. The second line shows how not only that the policy of the lending firms affects the possible amounts borrowed by the household, but also contracts that survive as described by function g (contracts that go on, that is where there was neither default nor switch). Finally, the third row incorporates the fact that the continuation value of the contract is again affected by the lending firms choices. Note that the nature of the one period deviation on the part of the bank is incorporated by having tildes only in the current function, but not in the function that shows the continuation values.

The bank faces several trade-offs in this problem. Choosing terms that are too stringent (low limit, high interest rate) may induce some borrowers to default since the remaining within the contract may not be so attractive, other households may choose to switch to better terms, while others may stay paying more interest but perhaps on smaller loans. Choosing terms that are too loose may reduce current default but attract more borrowing that may prove costly in the future when households have an opportunity to default on them. It is a careful assessment of all this margin what determines the choice of lending firms.

A bank that is subject to no commitment, except to not restrict the existing debt, will solve at the beginning of each period

$$\underset{q,\ b\geq y}{\operatorname{argmax}}\ \widetilde{\Psi}(e, y, \theta, b, q) \tag{22}$$

$$\text{s.t.} \qquad b \geq y \tag{23}$$

In an economy with commitment to the interest rate, an additional restriction to the problem of the bank shows up and we would have

$$\begin{aligned} \underset{q,\ b\geq y}{\operatorname{argmax}} \qquad & \widetilde{\Psi}(e, y, \theta, b, q) && (24)\\ \text{s.t.} \qquad & b \geq y && (25)\\ & q \geq q_0^{\omega} && (26) \end{aligned}$$

A solution to any of these problem is a pair of functions $\widetilde{b}(e, y, \theta)$ and $\widetilde{q}(e, y, \theta)$ both of which are computed taking into account the continuation values for credit limits and interest rates specified by contract θ, objects that we have denoted $b^{\theta}(e, y)$ and $q^{\theta}(e, y)$. Clearly, a time consistent contract θ is one where

$$\widetilde{b}(e, y, \theta) = b^{\theta}(e, y) \qquad \text{and} \qquad \widetilde{q}(e, y, \theta) = q^{\theta}(e, y, \theta). \tag{27}$$

Only time consistent contracts can be equilibrium contracts as it will be formalized below. So we restrict our attention to them.

Responses to off-equilibrium contracts offerings In the same fashion that we characterized the problem of households if other contracts were ofered in Section 3.3.2, we also explore what would be the interest rate and credit limit responses of a bank that offers equilibrium contract $\theta \in \Theta$ when there is an off-equilibrium contract $\widehat{\theta} \notin \Theta$ offered. First, note that the behavior of a bank is not affected after $\widehat{\theta}$ is introduced. The reason is that such contract is no longer available so none of its customers will take it into consideration. Consequently, we only have to derive the response of interest rates and credit limits in the period that the new (off-equilibrium) contract is introduced. The extent to which existing lending firms respond to the possible entry of contracts depends crucially on timing issues. If a new contract enters after the lending firms have set their interest rates and credit limits policies for the period, the lending firms have no capacity to respond. If, however, new contracts are offered at the same time that existing lending firms set their period policies then they can respond to the entrant. Such as response can be written as functions as $\widehat{\widehat{b}}, \widehat{\widehat{q}}$ with arguments $(e, y, \theta, \widehat{\theta})$ that result from solving problem (21) under alternative circumstances where we use functions $\{\widehat{\widehat{d}}^{\theta}, \widehat{\widehat{g}}^{\theta}\}$. We write such problems compactly (ignoring arguments of functions) as

$$\widehat{\widehat{b}}, \widehat{\widehat{q}} \in \underset{b,q}{\text{Argmax}} \sum_{\varepsilon,\chi} F_e(\varepsilon, \chi) \left[-y\left(1 - \widehat{\widehat{d}}^{\theta}\right) + q\, \widetilde{y}'^{n}\, \widehat{\widehat{g}}^{\theta}\right] 1_{-y>0} + q^* \, \mathbb{E}\{\Psi\} \tag{28}$$

We now turn to the problem faced by potential intermediaries that could offer contracts that are different from the existing ones.

3.4.3 Potential Intermediaries offering new contracts

Some possible off-equilibrium contracts will not be taken up by households and hence we do not have to worry about them, but others would be taken if available. So in addition to characterizing the problem of existing contracts in the presence of off-equilibrium contracts, we also have to characterize the properties of off-equilibrium contracts themselves. Again, the functions $\{\widehat{d}, \widehat{g}, \widehat{y}'^{n}\}$ and $\{\widehat{\widehat{d}}, \widehat{\widehat{g}}\}$ that characterize the response of households to such offerings are crucial. For any possible alternative contract $\widehat{\theta}$ we just need to worry about two things, first to verify whether it is time consistent which amounts to verify (27), this is whether $\widetilde{b}(e, y, \widehat{\theta}) = b^{\widehat{\theta}}(e, y)$ and $\widetilde{q}(e, y, \widehat{\theta}) = q^{\widehat{\theta}}(e, y)$, and second to compute its value, this is to compute $\Phi_0(\widehat{\theta})$. Both calculations are identical to those described in Sections 3.4.1 and 3.4.2 and we avoid repeating the derivation.

There are three important additional considerations to make.

1. Only households in good credit standing, $h = 0$ can have an active contract. While a recent bankruptcy filing difficults access to credit (Musto (1999)), this is not necessarily the case in

theory. Within the context of our model two theoretical arguments can be made for such an exclusion. One is the existence of a coordinated equilibria in which people with bad credit default do not pay and lenders do not lend to such people (a dubious argument given the weight of U.S. law that prevents multiple filings in a short span of time). The other is the existence of regulations on the part of the Fed that difficult the extension of loans to household with bad credit histories. Interestingly current lenders would support such a policy because it enhances the value of existing contracts. A more careful study of the role of credit scores in shaping access to credit requires, we think, notions of reputation and multiple types as discussed in Chatterjee, Corbae, and Rios-Rull (2004) Chatterjee, Corbae, and Rios-Rull (2008).

2. The contract is not optimal. In principle a credit line is a restrictive type of contract as it specifies a constant interest rate and a credit limit instead of an interest rate for *each* credit level. The reason why actual contracts are like this is not clear. While the Fed regulators do not explicitly restrict the existence of complicated credit arrangements with level dependent interest rates, it is quite possible that the lending institutions fear that such contracts could be labeled abusive and do not use them. Alternatively, households or lenders or both could consider them too complicated to be worth it. Whatever the reason for the lack of complicated contracts (in any case, a typical feature of most actual contracts), we restrict our attention to contracst of this class.

3. At most one contract is possible at a time. Recently, Hatchondo and Martinez (2010) have looked at economies with credit risk in which borrowers do not have exclusive contracts with lenders, extending the analysis of Bizer and DeMarzo (1992). They find that the resulting contract takes the form of credit lines with history dependent terms. It may be very well possible that the combination of multiple simultaneous lenders and lack of commitment will prove to yield credit lines as optimal contracts.

4 Equilibrium: Definitions, Computation and Characterization

The definition of equilibrium in this economy is a little bit more contrived than in other more standard environments. In addition to the conditions of agents optimizing and market clearing, we have to characterize the set of equilibrium contracts Θ that emerges as the result of free entry in banking.

We have already defined in the set of potential contracts Θ^P as all possible combinations of initial class, initial loan, price and debt-limit rule. This set is very large. We have referred to Θ as the set of contracts available for the households to choose from. We start by defining equilibrium formally and then we discuss the conditions involved.

Definition 1. *An equilibrium consists of a set of available contracts $\Theta \in \Theta^P$, decision rules for households, $\{y'^n, s, \theta'^s, d\}$, and contract values for lending firms Ψ_0, and Ψ. In addition,* within contract deviation *functions, $\{\widetilde{y}'^n, \widetilde{s}, \widetilde{\theta}'^s, \widetilde{d},\}$, and alternative contract deviation functions $\{\widehat{y}'^n, \widehat{s}, \widehat{\theta}'^s, \widehat{d}, \widehat{\widehat{s}}^{\widehat{\theta}}, \widehat{\widehat{d}}^{\widehat{\theta}}\}$ for households as well as responses to $\{\widehat{\widehat{b}}^{\theta}, \widehat{\widehat{q}}^{\theta}\}$ and values of alternative contracts $\Phi_0(\widehat{\theta})$ for intermediaries such that:*

1. *Household decision rules solve their problem as defined in (3 -17).*
2. *Zero profits: Ψ_0 satisfies $\Psi_0(\theta) - \pi = 0, \qquad \forall \theta \in \Theta$.*
3. *Contracts are time consistent. For all $\theta \in \Theta$, $\{b^\theta(e, y), q^\theta(e, y)\}$ satisfy (27).*
4. *Unprofitable opening of other contracts. For all $\widehat{\theta} \in \Theta^P$, $\widehat{\theta} \notin \Theta$, at least one of the following conditions is not satisfied:*
   (a) Time consistency. That $\{b^{\widehat{\theta}}(e, y), q^{\widehat{\theta}}(e, y)\}$ satisfy (27) given the alternative contract deviation *decision rules.*
   (b) Entry possibilities: $\widehat{\Psi}_0(\widehat{\theta}) - \pi \geq 0$.
   (c) Some households choose it: $\widehat{\widehat{s}}(e, y, \varepsilon, \theta, \widehat{\theta}) = 1$ for some $(e, y, \varepsilon, \theta)$.

Remark 1. This definition uses a very loose definition of equilibrium contract set Θ. In the sense that it allows for contracts that are available but not used. So this set can be quite large. However, this is not a problem, define $\Theta^E \subset \Theta$ the set of effectively used contracts as those such that *i*) are time consistent, *ii*), yield zero profits, and *iii*) some households choose it. It is clear that if Θ decision rules and values is an equilibrium so is Θ^E with the same decision rules and values. Alternatively, for some equilibrium where $\Theta^E = \Theta$ is an equilibrium, we could enlarge Θ to include time consistent contracts that would give zero profits if somebody would take them, but nobody does. The type of contracts that we would enlarge Θ^E with include contracts that offer all possible levels of debt y'_0 for all types of households (e). The interest rate q_0 that this contract would offer could be very unattractive. This insight is very useful to find equilibria.

Remark 2. This definition allows for lending firms to respond immediately to the entry of new contracts. This feature may give lending firms a certain ability to keep higher profit margins as they can respond immediately to new contracts. Had we made the alternative assumption of lagged response of lending firms they would have to be more concerned about preempting entrance.

Remark 3. The issue of credit limit is easy to deal with. Note that we have defined the asset set as a compact set. In economies of this type $\beta < q^*$ guarantees the existence of an upper bound to asset holdings. So the upper bound is not a problem. The lower bound of assets is not a problem either. There is a maximum amount of debt that can be paid for with positive probability. This means that more debt than that is only consistent with $q = 0$. So we could use such a bound. Typically, though it is sufficient to pose a level of debt for which the rate of return is high but not infinity and that it prevents households from borrowing.

Remark 4. Note that we have not impose any condition on the distribution of agents. This is because the environment is essentially a small open economy (or storage economy) and the risk free rate is given. To complete characterize the economy, we need an initial condition, this is a measure of households over states, that we denote x and a Markovian process on the states to construct an updating operator for the distribution of agents. Moreover, in this economy standard conditions about the persistence properties for income classes Γ guarantee that there is a unique stationary distribution x^* and that such distribution is the limit of any initial one. These are all standard operations for our environment and we spare the reader the details. See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for example for details.

4.1 Computation and Characterization

Computation of equilibrium that is the actual finding of the equilibrium allocations is more than a mere numerical exercise. Our computational strategy has, not only to be able to find zero profit conditions and household optimization, but also to characterize the equilibrium contract set.

To simplify matters, we discretize the set of possible asset holdings. We do it for two important reasons. One is that the nonconvexity of the household problems prevents us from taking advantage of the FOC to characterize the solutions. Discreteness of the asset and choice set makes all maximization problems finite. Moreover, the equilibrium objects are functions of assets which means that we are also making finite the set of prices (interest rates and the like) that we have to find. Obviously, interest rates themselves are not discretized. Here is where the utility costs shocks play an important role by making profit functions continuous in interest rates.

An important step in the computation of equilibrium is to restrict the set of possible contracts to a manageable size, but not too much as to leave out contracts that may exist in equilibrium. We proceed by a guess and verify type strategy. A suitable candidate for the set of available contracts is to have one per each type of income class and loan size (e, y'). Lets denote the set of such pairs Θ with generic element θ. For each θ, we need to compute the initial q_0^θ, and continuation interest rate functions $q^\theta(e, y)$ and credit limits $b^\theta(e, y)$. We look for values that guarantee that households maximize, that lenders maximize and that yield zero profits. We call such an object a Θ-equilibrium, and it satisfies all equilibrium conditions except the last one, that all other possible contracts are unprofitable. Not all contracts in the Θ-equilibrium will be taken up by households, but if they were the would yield zero profits.

Finding a Θ-equilibrium is a large but manageable problem (if there are N_y asset positions and N_e income classes this amounts to $N_e^2 \times N_y^2 + N_e \times N_y$ interest rates and $N_e^2 \times N_y^2$ credit limits which is a fixed point problem that we have solved by backward iterations relatively easy. Appendix **??** describes our computational procedures to find a Θ-equilibrium in detail. Conditions for existence can be tricky in the sense that the fixed point in updating rules may not exist (there can be some sort of cycles). It can be proved *(we conjecture)* that an equilibrium exists for any finite economy. In all cases we have found a Θ-equilibrium without problem which amounts to a prove for an open set of economies.

Still a Θ-equilibrium is not an equilibrium until we verify the inexistence of alternative profitable contracts. Due to the extreme lack of commitment, once the contracts have started nothing remains from the initial conditions. Consequently, the possible contracts have to be specify different continuation policies. Such a possibility would include something like a contract with a tighter credit limit and a lower interest rate. The existence or not of these contracts is trivial to verify. They *never exist*. For each type of customer that a bank is attached to, the bank can choose a pair of interest rate and credit limits to maximize profits. There is (generically) a unique optimal interest rate per credit limit, and given this there will be a unique credit limit that maximizes profits. Once continuation policies are set there is a unique initial interest rate, essentially because of the law of one price.

For the environment where there is commitment not to raise the interest rate but not to the credit limit, two things have to be verified to ensure that a Θ-equilibrium is indeed an equilibrium. First, whether for the interest rate in the Θ-equilibrium there could be alternative credit line policies that could be time consistent optimal contracts for the bank. It is easy to verify that they are not. Second, whether for different interest rates, and different continuation credit line time consistent policies that yield non-negative profits there are consumers that are takers. This is also straightforward to verify via the construction of a fine grid of interest rates after the computation of a Θ-equilibrium. In all cases we found none.

The following propositions are very useful for characterizing the equilibrium contracts.

Proposition 1. *Consider the economy without commitment. The set of equilibrium contracts can be fully described by corresponding pairs of initial debt and income type. The bank's continuation decision rules are identical across contracts.*

Proposition 2. *In the economy with the commitment created by condition* (26)*, the set of equilibrium contracts can be described likewise if deviation contracts offering a higher initial interest rate* *can be ruled out. The bank's continuation decision rules are no longer identical across contracts.*

As discussed above, Proposition 1 is straightforward. To prove Proposition 2, we only need to check entry of contracts with a higher interest rate. Suppose we have one such equilibrium. For given initial debt and income type, there cannot be a successful deviation contract offering a lower interest. This contract would be more constrained than the existing one in the future while making a lower profit initially.

5 Quantitative Analysis I: Public Information

We now study in utmost detail a specific public information economy where banks cannot commit to their credit limits (although they cannot make consumers prepay so they are restricted to maintain any existing credit) or interest rates. This economy is designed to share some properties with the U.S. economy. These properties do not include the whole population, instead we are interested in the fate of the majority of the population that hold low assets and have a very small upside potential in terms of their income, in this sense, this is an unemployment economy like that posed by Krusell and Smith (1997, 1998) in their seminal work.

5.1 Generic Targets

We specify the model to be a year with a 3% risk free interest rate. Households that have defaulted are excluded from borrowing for 10 years. Households when unemployed have 11% of the earnings that they get when employed.[7] In this model economy the unemployement rate is 5% and, for individuals unemployment is *i.i.d.* which implies an average unemployment duration of a little under 13 months. We later explore non i.i.d. environments. Finally, we pose a slightly larger than log risk aversion.

5.2 Credit Related Targets

We target five main credit related variables.

V: **Needs serious rewrite** We seek to hit reasonable targets for measures of debt, default, switching and intermediation costs. Based on the 2001 SCF, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) suggest a default rate of 0.54 per cent and, conveniently measuring debt as negative net worth, a percentage in debt of 6.7 per cent and a debt-to-income ratio of 0.0067.

[7] There is a very active current discussion in macroeconomics about how to think of the value of employment versus unemployment (Shimer (2005), Hagedorn and Manovskii (2008), Costain and Reiter (2008), Nakajima (2007)) where the ratio is argued to be somewhere between 50% and 90%, which is due mostly to the value of leisure. Here we are only interested in consumption which is why we target such an extreme value.

Table 1: **Credit Related Targets**

Variable	Target	Actual value
Debt to income ratio	1.50%	1.50%
Percentage in debt	5.40%	5.36%
Default rate	.65%	.65%
Switch rate	2.0%	1.9%
Banking Value Added over Loans	4.0%	4.2%
Other Statistics		
Wealth to income ratio		15.0%

Implied Parameters

Parameter		Value	Parameter	Value
Intermediation cost	π	0.010		
Default cost	χ_d: Mean	0.125	Range	0.10
Switching cost	χ_s: Mean	0.040	Range	0.02

The standard estimates point at a intermediation costs of 4 or 5 per cent of bank loans. The 2006-2009 data on credit card usage from the Consumer Finance Monthly reported in Abdelrahman (2011) suggest a switching rate of 2 per cent among households with positive credit card balances. We specify the mean of parameter for the cost to defaulting χ^d and fix its standard deviation ξ^d to be 3/4 of that. This implies that about 10 per cent of agents experience a negative costs/positive benefit to defaulting. This could be interpreted as events not modeled here than tilt decision towards default.[8] Similarly, the standard deviation of the switching cost ξ^s is fixed at 1/2 of its average value χ^s. The chosen χ^d and χ^s imply a default rate of 0.65 per cent and a switching rate of 1.9 per cent, fairly close to the targets. With a 5.4 percentage in debt and debt-to-income ratio of 0.015, this setting achieves reasonable balance in the trade-off between the two debt targets. As for the intermediation cost, with the chosen π they are 4.2 per cent of bank loans values.

[8]This is an economical way of introducing factors like the non-discretionary expenditures in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007c). In the latter, these shocks occur with 10 a percent probability. In any event, this level of noise is required to impart the required smoothness in the policy rules.

Table 2: **Main Credit statistics**

Mass with a contract	80.5%
Mass no contract but clean history	12.3%
Loan size (all)	0.28
Interest Rate (average)	25%
Loan size (initial)	0.23
Interest Rate (initial)	-.3%
Loan size (continuing)	0.31
Loan price (continuing)	0.47
Debt limit (continuing)	0.35

Units are relative to per capita earnings.

5.3 Equilibrium properties

The stationary equilibrium allocation of this economy has many other properties beyond those that we have imposed. In particular, the economy generates a distribution of households by earnings, wealth, and contract type as well as a range of interest rates, loan sizes, and debt limits for each type of household. The most important of these statistics are listed in Table 2.

The implications of this benchmark for average interest rates, size of debt per loan, an debt limits are in Table 2. The interest rate is nearly zero for initial offers; on continuing contracts the interest rate is very high near 45 per cent, pointing to the very large power that banks command over households due to the large size of the switching costs. We see also that loans increase on average, although there are also reductions. The economy also has a very large number of households with contracts showing the recurrence of unemployment.

5.3.1 Wealth, choices and credit conditions

Wealth distribution and choices. According to their contract/bankruptcy status households can be in one of three distinct states:

1. *No initial contract and good credit* ($h = 0$, $\theta = 0$). The distribution of wealth for this households both those employed and those unemployed is depicted in the top two graphs of Figure 1. Households that remain employed have an absorbing state (or point to where they settle after enough periods) of a positive asset position of 0.20.[9] All unemployed, $e = 1$,

[9] Higher wealth positions are households who recently recovered their good standing and are slowly dissaving.

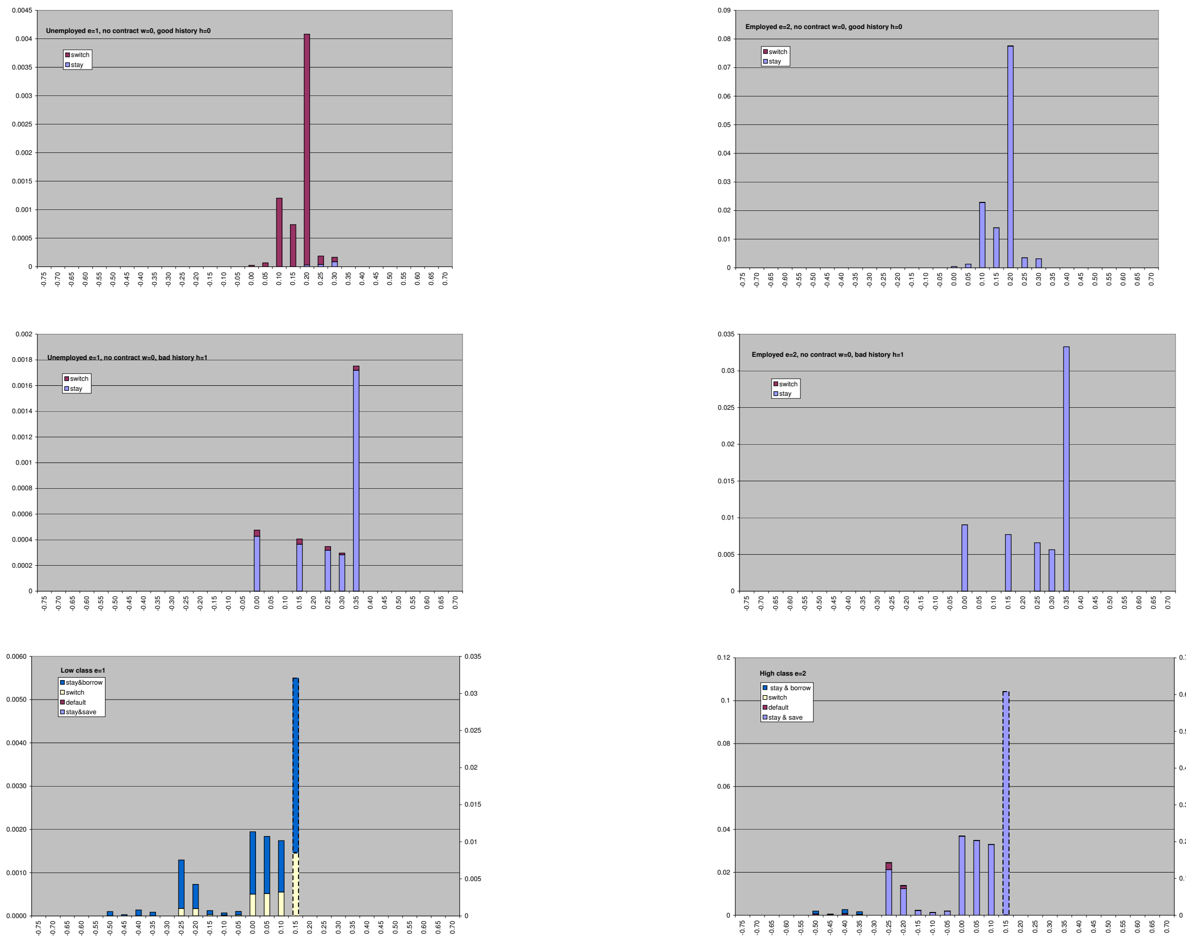

Figure 1: Distribution. $\theta = 0$, $h = 0$ top; $\theta = 0$, $h = 1$ middle; $\theta \neq 0$, $h = 0$ bottom.

borrow by acquiring a contract; all the employed, $e = 2$, stay without contract and save.

2. *No initial contract and bad credit* ($h = 0$, $\theta \neq 0$, middle of Figure 1). The absorbing state is a higher state position 0.35. Within those who regain access to credit, all the unemployed switch and borrow, and all employed stay and save.

3. *Households with a contract* ($h = 0$, $\theta \neq 0$ bottom of Fig. 1). The absorbing asset state is a positive 0.15; most of the remaining mass is spread over the positive asset positions. Most debtors hold an intermediate level of debt -0.25 or -0.20, with the rest scattered among a few positions with lower and higher debts. The unemployed do not default, and switch when above a certain position, specially above zero. Employed households default when above an intermediate debt level.

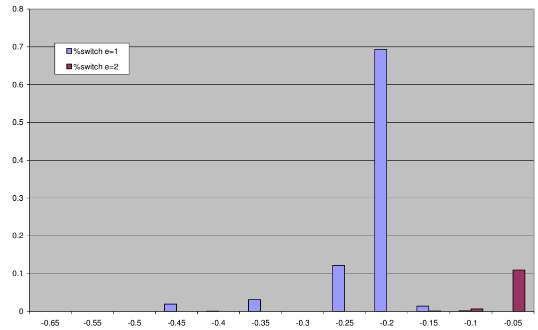

Figure 2: Distribution of contracts take up.

Distribution of new contracts. As Figure 2 shows, the majority of contracts are signed by households that are in the absorbing state of those without a contract. However, there is also substantial switching, about 12 percent of new contracts are switches to higher debt (-0.25), and another 11 percent are made by employed people that switch to a small loan with a small interest rate (-0.05). These employed households got a low switching cost realization and avoid the big hit that the bank is imposing on them.

The terms of new contracts. They are shown in Figure 3. The initial interest rate is increasing in the size of the starting loan, as we would expect. Note the teaser rates for small loans. Accordingly, the initial bank profits are smaller than the fixed cost of intermediation, implying banks will have to make profits in the future. This is not the case for very large initial loans (that in any case do not happen in equilibrium).

The terms of continuing contracts. We show the terms chosen by the bank, the implied household's borrowing and the bank's value as functions of the household's asset position in Figure 4. Only asset levels below .2 are relevant, as households with a contract do not hold more assets than .15. The bank extends credit to the unemployed, and they take it up to the limit. For large enough debt, the bank stops increasing the limit. For the employed, the credit limit is never binding. Interestingly, the interest rate is hump shaped in assets, with the biggest increase happening for relatively wealthy households who become unemployed. Banks make profits on the employed with an intermediate level of debt and, especially, on the unemployed with high assets. At high levels of debt, the bank's value of the contract becomes negative.

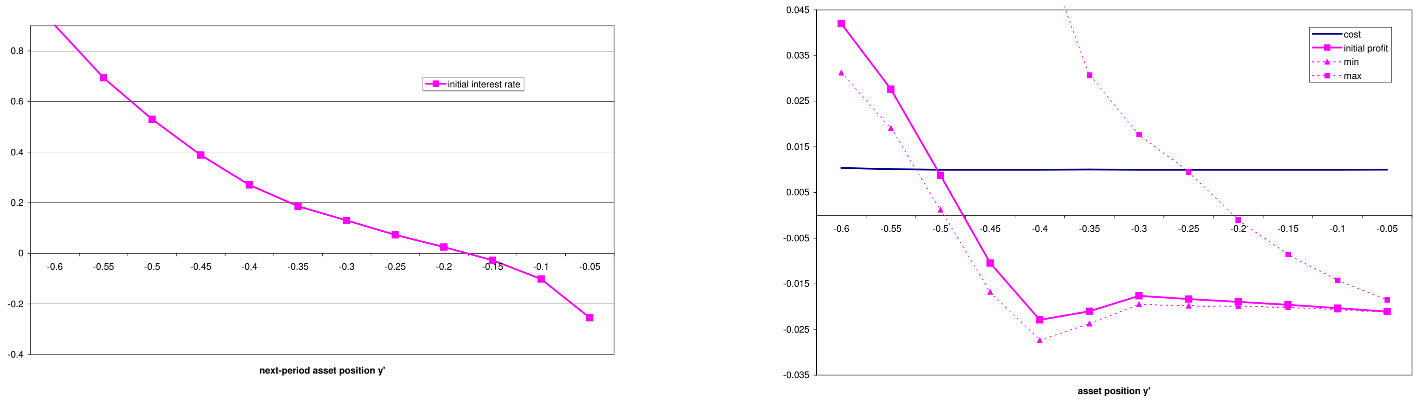

Figure 3: Initial interest rates and profits as function of initial debt.

Options faced by unemployed households: Switching or Staying. As it can be seen in Figure 5 for levels of assets above -0.30 the existing contract charges a higher interest rate than the best outside competing alternative, and in some cases, a lot higher. Interestingly, switchers and stayers will end up with similar debt levels but quite different interest rates. The larger the initial wealth of the household, the lower the amount of debt that it will end up taking which gives more power to the bank. For high levels of debt below 0.35 the existing contract charges a lower interest. This is associated with a negative value of the contract, and the bank is just trying to manage its loses. It wishes that the household left which will not do. For employed households at debt levels where such a household wishes to borrow the pattern in the interest and debt gap is similar, but they never exercise the switching option.

Banks make different choices attending to the household's reaction in terms of defaulting, switching and borrowing. With unemployed borrowers, the bank is mainly concerned about containing switching, except at very high debts where default is the driving concern (at those positions, switching is not an option for the household). The bank can raise its interest rate above the market's when switching is not too sensitive to that choice, which is only the case at low-debt/positive-assets states. With an employed borrower, the bank can manage a smooth response of switching and default and so is able to make money at some high levels of current debt, but not at the highest. Again, when debts are very high, the bank must reduce the interest rate to deal with all risks.

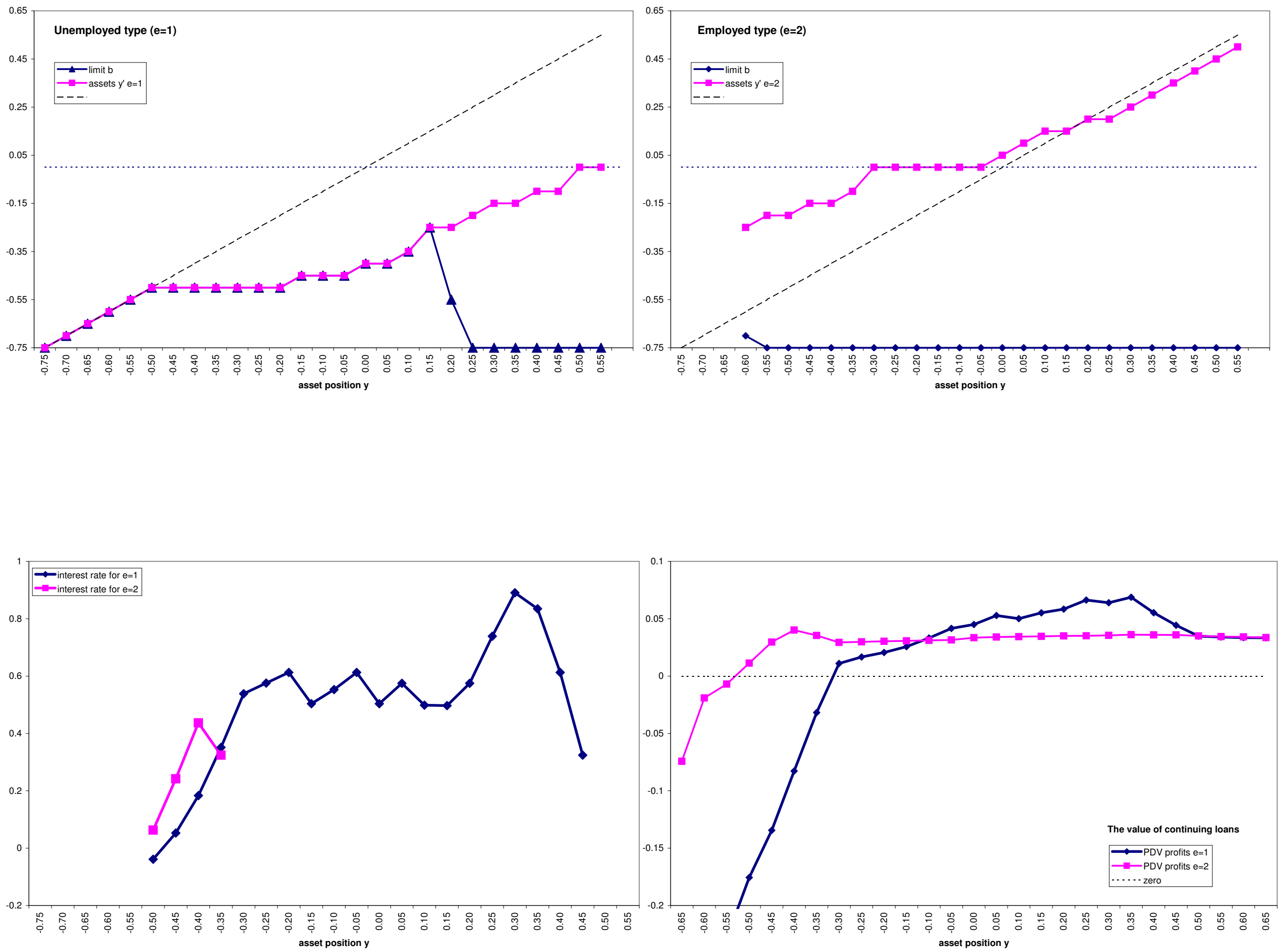

Figure 4: Continuation limit (top), interest rate (bottom left), and profits (bottom right).

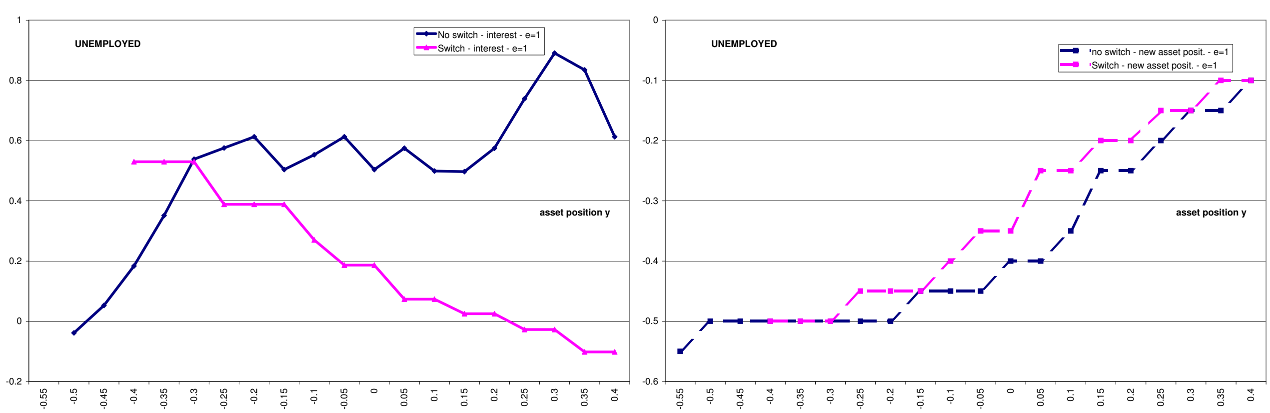

Figure 5: Interest and debt on existing and new contracts.

5.3.2 Household dynamics

Perhaps a good way on understanding the properties of the equilibrium allocation is to trace the fate of a household starting from the absorbing state, .2, of those that do not have a contract and get an unemployment shock. Dissaving alone would make its consumption quite low relative to recent experience. The household will borrow incurring in the utility costs of signing the contract. Figure 3 points to its options. The household chooses a debt size of -0.20 at a 2.5 percent interest rate, as shown in Figure 5. Note that, at this moderate debt size, this is one of the situations seen in Figure 3 where the initial default-adjusted return does not cover the administrative costs of managing the contract, the bank expects to recover these costs over time so the household will be an asset for the bank.

One period later, most households in this situation will find employment. Of those, the majority repay the loan and set their assets at zero. A few more periods of employment and this group of households will gradually approach the assets level of .15 again, see Figure 4 or 1, but now they will be holding a latent contract they can use to borrow without signing costs in case of need. This type of households, to which we will return later, will be of special interest. Not all the households regaining employment with debt pay their debts. A sizeable fraction of about 10 per cent choose to default after obtaining a good draw of the defaulting costs. They consume their labor earnings (recall that they cannot save this period) and are left with zero assets and a tarnished credit history. As long as they remain bankrupt, these households will build assets and settle to a higher asset position of 0.35, shown in Figure 1. When their credit record is cleared, if in need to borrow they will have a to acquire a new contract thus starting over the cycle.

Some of the households who lost their job will remain unemployed for yet one more period. The situation of these households is dire, they have low income and sizeable debts. If they stay in their contract, they have access to a new debt of .50 but at a high interest rate of 61 per cent. A proportion of 23 percent switch to a new contract involving debt 0.45 at an interest of 39 per cent (see Fig 5). So households borrow heavily at a sharply increased interest by using the existing contract or by switching to a new contract (those hit with a favorable switching cost). After the second period of unemployment there is still the possibility of a third period of unemployment. The bank in this case treats the household to a low interest of 5 per cent and a small rise in debt. If unemployment persisted further, a very unlikely event, the bank would subsidize the household with a negative interest and keep rolling over the debt outstanding. See Figures 4 and 1.

Once this highly indebted household finds a job, it either defaults or reduces its debt. The bank chooses an interest rate that balances its desire for a high return on a reduced debt and a low default rate. The outcome is an interest rate of 6.4 per cent, a high default rate and no switching, with households that remain in the contract choosing a debt level from where a few default and the rest pay off their debt. See Figure 4 and 1.

The value of the bank. One remark is in order about the bank's performance along these trajectories that follow from a spell of unemployment without preexisting contract. Referring back to Figure 4 and 5, these experiences correspond to situations where the bank commands a relatively modest or negative value and is charging an interest that does not differ too much from the market's. This is true even when there is a sharp increase in the interest rate in the second period of unemployment. Thus banks cannot extract much value from customers who do not initially own a contract and have had to acquire one when hit by unemployment. The big gains to the bank seen in Figure 4 must happen when unemployment hits a household that already holds a contract.

To see this, we return to the type of households who, after an unemployment spell, had rebuilt their asset position and hold a latent contract. For example, as discussed, the typical household within this group has a positive assets position 0.15. When it loses employment, a 26 per cent will borrow a level of debt of -0.20 by switching to a new contract at at 4 percent interest rate; the rest of households borrow -0.25 on the existing contract at a higher interest of 49 per cent. As seen previously with reference to Figure 4, 5, the contract's value here is very high. For another example, consider a household who, not having yet reached the absorbing state, has an asset position of 0.05. If employment is lost, a proportion of 28 per cent switch to a new contract with those staying in their current contract borrowing -0.40 and being charged a high 57 per cent. If employment is found in the next period, a 7 per cent do switch to a new contract and those staying will reduce debt but still borrow -0.15 at a 20 per cent interest. For these two consecutive periods, the bank enjoys an abnormally high value.

It is therefore during short unemployment spells on existing contract holders with no recent unemployment history when the switching costs allow banks to charge an interest that is considerably higher than for a new contract and so make abnormal profits. However, a sharp rise in interest rate above the initial one does not necessarily go with abnormally high returns to the lender, as when the bank deals with a new borrower experiencing a second consecutive period of unemployment.

6 Commitment and policy reform

The previous analysis reveals that, in the absence of commitment in the pricing policy, existing banks often use their ability to increase the interest on an existing credit line. The recent regulatory change in the U.S. restricts severely this type of practice. In this section we set out to explore this policy question.

The policy change is expressed as a commitment not to increase the interest rate on an existing contract. Two main effects could be anticipated. First, some contracts will see diminished their ability to make profits on bigger loans, and this would be specially true of contracts that start with small debt and, accordingly, a low interest rate. In order to balance the books, these contracts will have to make more profits on the initial loan by raising the starting interest rates. Second, bank managers will not be able to rise interest rates and will have to resort to using the debt limit in order to keep liabilities manageable at the permitted interest rates. At this level of generality, it is hard to speculate on how exactly these adjustments will impinge on general credit conditions and access, and welfare. This section pursues this question within the benchmark model economy.

6.1 Aggregates

Consider first aggregate steady-state outcomes from this policy change shown in Table 4. There is a reduction in the default rate and an increase in the switching rate among debt holders. But the total mass of switchers declines which means there is less switching to borrow among those without debt. The total value of debt decreases. The average size of loan liabilities has decreased too and the average interest rate has decreased, although starting rates have increased sharply. Average consumption is up by about 2.6 per cent.

Table 3: **Aggregate Effects of Commitment**

	Commitment	
	No	Yes
Mass with a contract	80.5%	79.5%
Mass no contract but clean h=0	12.3$	15.2%
Debt/Earnings	1.50%	1.35%
Default rate	0.65%	0.53%
Switch rate for debtors	1.9%	3.4%
Loan size (all)	0.28	0.26
Interest Rate (average)	25%	22.%
Loan size (initial)	0.23	0.35
Interest Rate (initial)	-.3%	22.%
Loan size (continuing)	0.31	.24
Loan price (continuing)	0.47	0.8226
Debt limit (continuing)	0.35	0.2433
Aver. Consumption/earnings	0.7482	0.7298

6.2 Wealth, choices and credit conditions with commitment

Distribution and choices. The distribution of wealth and choices with commitment is in Figure 6, to be compared with Figure 1 before the policy change. Households who do not own a contract and have a good credit record are now concentrated at two main asset positions. The first is 0.10, the resting point for agents without a contract who had regained credit with a relatively low stock of assets or have experienced some unemployment. There is now a second asset position 0.35 consisting of households who regained credit after having accumulated a large amount of assets and remained employed.[10] For households who own a contract, the absorbing states involve wealth 0.20 and 0.15 corresponding to the most used contracts with initial debt -0.35 and -0.45. For the unemployed, the proportion who switch becomes larger at low assets positions and smaller at high positions. For the employed, the default pattern is very similar. There is little to remark regarding the policy effect for households who do not have a contract.

Distribution of switching over contracts. Consider Figure 7, to be compared with Figure 2 before the policy. Most unemployed switch to a higher initial debt at -0.35, and now a large proportion also switch to an even higher debt -0.45. Therefore, with the new commitment policy, a larger proportion of switching occurs to borrow higher debt levels.

Conditions when acquiring a new contract. As anticipated, with the policy change the interest rate has increased for contracts in the low range of initial debt. Graphically, Figure 8 shows the menu of initial interest rates and bank profits before and after the policy change. At higher levels of initial debt the interest is lower with commitment. The reason is that these banks, given the higher interest of new small loans, will be later on able to also charge a higher interest to existing customers who may wish to borrow a moderate amount.

Conditions on continuing contracts. We have to note that, because of commitment, the initial conditions of a contract now matter in that the starting interest rate sets a ceiling rate for the duration of the contract. We consider the two most popular contracts, one starting with a debt of -0.35 and low interest 0.22 and the other with high debt -0.45 and a high interest 0.35. The policy cap is effectively constraining the chosen interest at many asset positions. Compared with the case without commitment before the policy, the level of interest rates is appreciably lower.

[10] Interestingly, the asset policy rule without a contract has two resting points. **Victor, does this make sense?**

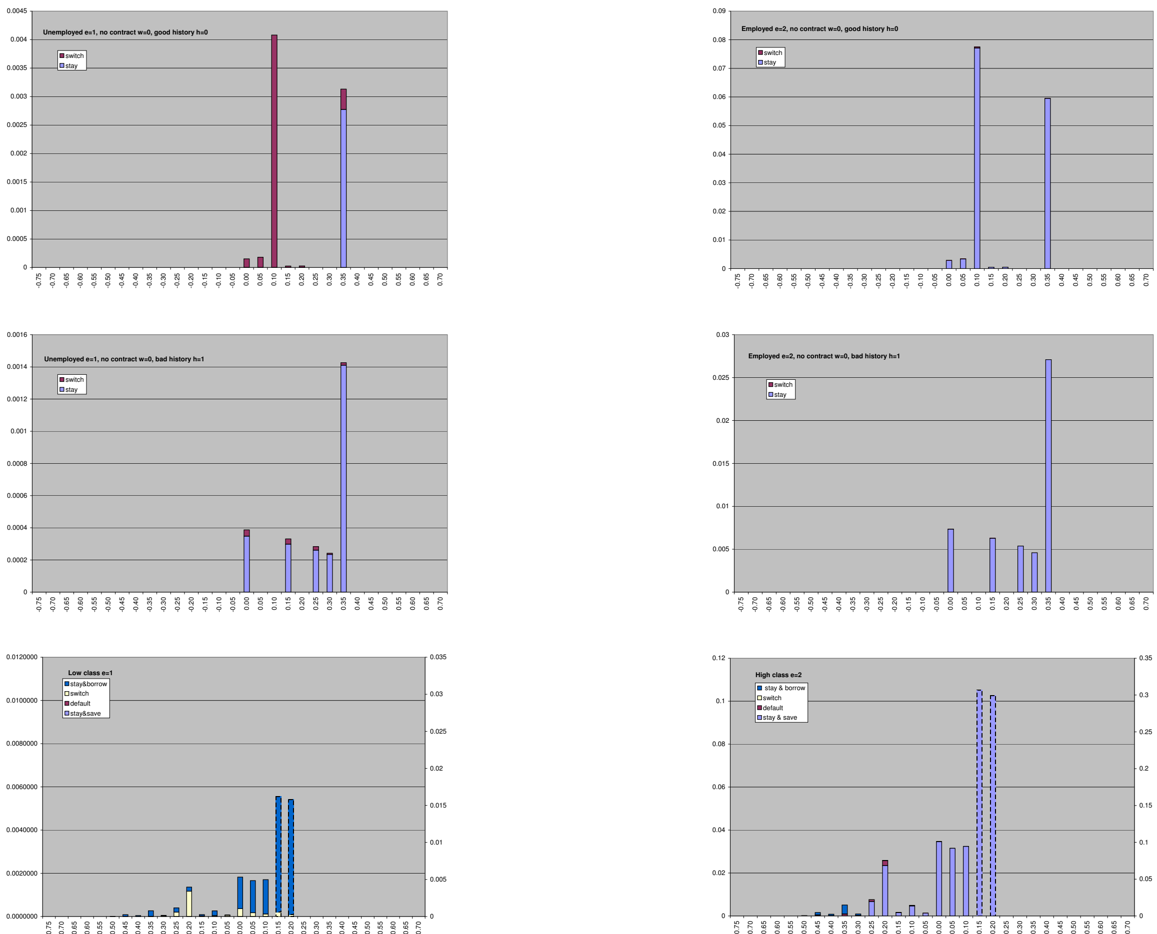

Figure 6: Distribution with commitment. $\theta = 0, h = 0$ top; $\theta = 0, h = 1$ middle; $\theta \neq 0, h = 0$ bottom.

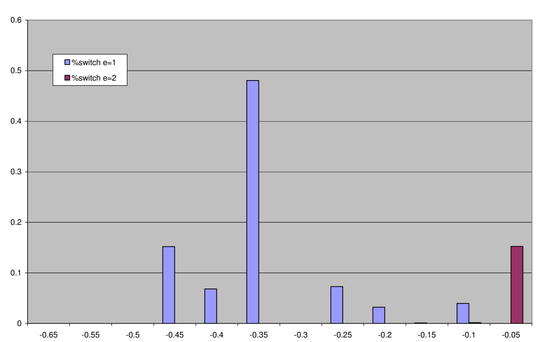

Figure 7: Distribution of contract take up with commitment.

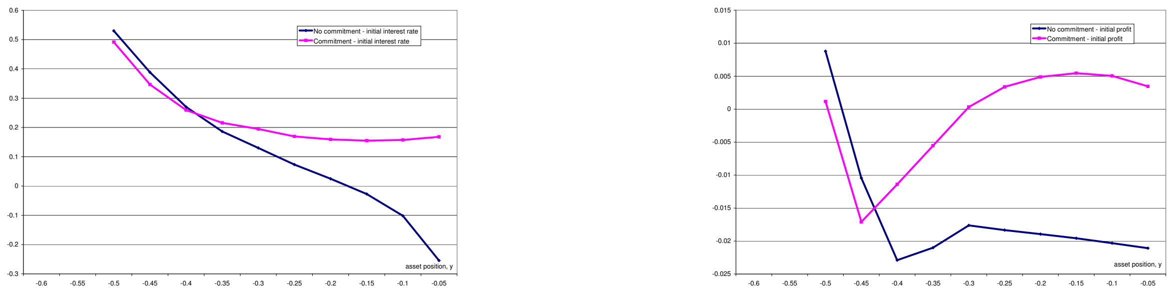

Figure 8: Initial interest rates and profits with and without commitment.

The unemployed seem to be more affected. Limits become generally stricter.

** Add figures? **

6.3 Household dynamics with commitment

** to be filled in **

The new policy thus seems will tend to make conditions more adverse for the unemployed not holding a contract and perhaps those who, having experienced a past long episode of unemployment, hold a contract with a high interest rate ceiling. These households pay higher interest rates or pay more costs of signing. Agents with only a short past unemployment experience will hold on to a contract with a low interest rate ceiling and enjoy better credit conditions. On the other hand, this discussion has left aside households at these asset positions but with different types of contracts as well as those on other asset positions. There is no substitute to a welfare analysis and this is what we turn to.

6.4 Welfare

We want to assess the merits of the policy change from a welfare point of view. To keep the analysis of the transitional effects simple, we will examine the impact of removing the commitment policy and returning to the economy without commitment.[11] We compare utility before and after this reverse change for the households that live in the distribution over wealth, income and contracts corresponding to the economy with commitment. As we have noted above, the consequences for different types of households are likely to be different so we will need to do the analysis at various

[11] Analyzing the reverse direction of change would require specific assumption about how commitment deals with the initial contract without commitment. A simple possibility we have explored for doing that assumes that an existing contract will be imposed an interest ceiling equal to the interest rate in the new equilibrium of a new contract with initial size equal to the initial size of the existing contract.

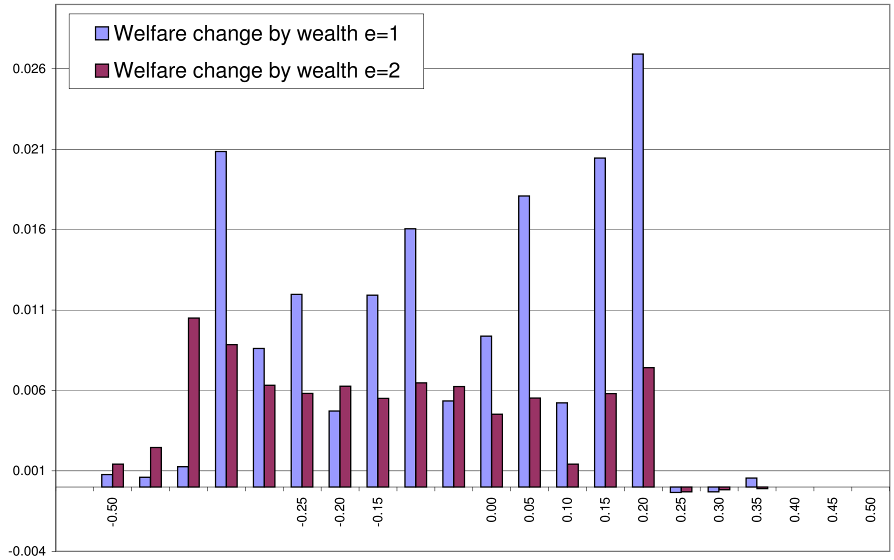

Figure 9: Distribution of welfare changes to not reversing the reform.

levels of aggregation. The measure of welfare change will be the utility gain to keeping the policy change in place. This will be expressed in units of equivalent consumption.

Commitment improves overall welfare by 0.59 per cent, with the unemployed households improving by a larger 1.71 per cent. Figure 9 shows the changes in welfare for the different wealth and income groups who belong in the initial distribution of Figure 6. Most types gain and some lose out. The largest gains, experienced by unemployed households, are between 2.0 and 2.6 per cent. The largest improvement occurs at the main asset position 0.20; the largest groups with debts see big increases in welfare too above 1 per cent. Loses are of a smaller order of magnitude and concentrated at the top asset positions.

The pattern of changes in Figure 9 is not monotonic in wealth, which suggests that there is no simple account of welfare effects based solely on the asset position. The mix of contracts held within each group must be important. For example, the picture seems to confirm the conjecture advanced earlier that the groups holding low-interest contracts benefit more from keeping the commitment, whereas households who end up holding high-interest contracts gain less with commitment. To understand the full pattern of changes we will need to dig deeper into the heterogeneity of households' credit conditions within each wealth and income group. (** to be completed

**).

Consider the unemployed households first. The group with the highest asset positions above 0.20 consist of households without a contract. When unemployed, they need to acquire a contract of moderate initial debt. As we know from Figure 8, they have to pay higher interest under commitment, hence the loss for them to keeping the policy reform in place.

In the main positive asset positions, the low interest ceiling dominates the contract mix, although its composition is diverse. The welfare gains reflect the fact that, having to borrow a moderate level of debt, the household benefits directly from the (binding) interest ceiling in the economy with commitment. It is true that the households with high-interest contracts do less well under commitment since they bear higher interest rates whether they switch or not. But they represent a small number. Some individuals hit the tighter debt limits for low-interest contracts under commitment. These households have to switch more and borrow less and benefit less from the policy change.

At the top end of debt, the conditions for the unemployed are practically the same in the two economies with only a small difference in welfare.

Turning now to the employed, note that they only need to borrow when highly indebted at position. They gain less from commitment because of the higher interest rates banks charge since outside loans are also more costly. In the remaining asset position, employed agents win substantially by keeping commitment. This reflects the predominant chance that they will end up in a situation where they will benefit if unemployed.

7 Extensions and sensibility

7.1 Private information

7.2 Income persistence

7.3 The role of switching costs

8 Conclusions

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Appendix

A Some Issues in Computation

The model economy without commitment to the interest rate is well behaved. Discrete choice models have a problem of continuity of decision rules and/or value functions. Often, continuity is ensured by posing i.i.d. shocks to the decision makers. This is what we did with the household decisions. The lending firms do not have this problem as their choice of the interest rate is a continuous one which ensures that the profit functions are continuous functions of the interest rates.

The model economy that implements the new policy does not have this feature because the choice of the lending firms towards existing contracts is a discrete one described by

$$\max_{b \geq y} \widetilde{\Psi}(e, y, \theta, b; q^{\theta}, b^{*}) = -y \; 1_{y<0} \sum_{\varepsilon,\chi} F_e(\varepsilon, \chi) \; [1 - \widetilde{d}(e, y, \theta, \varepsilon, \chi, b)] +$$
$$\sum_{\varepsilon} F_e(\varepsilon, \chi) \; \widetilde{g}(e, y, \theta, \varepsilon, \chi, b) \; q^{\theta} \; \widetilde{y}'^{n}(e, y, \varepsilon, w, 0, b) \; 1_{\widetilde{y}'^{n}(e,y,\varepsilon,w,0,b)<0}$$
$$+ \, q^{*} \sum_{\varepsilon} F_e(\varepsilon, \chi) \; \widetilde{g}(e, y, \theta, \varepsilon, \chi, b) \; \sum_{e'} \Gamma_{e,e'} \; \Psi(e', \widetilde{y}'^{n}(e, y, \varepsilon, w, 0, b), \theta) \quad (29)$$

where we have written the semicolon to indicate objects that matter, that the bank takes as given but that are constant which include the policy function b^{*} followed by next period's bank manager and the contracts interest rate q^{θ}. Notice also that we have dropped the interest rate out of the problem of the house since under this policy it is not relevant.

A time consistent policy for bank managers is a fixed point: the solution to this maximization problem is a function that sets the credit limit as a function of $\{e, y, \theta\}$ that should be equal to function b^{θ}. The discreteness of the choice of credit limit may induce the inexistence of such a fixed and successive iterations may, and sometimes do, generate an endless cycle.

To avoid this problem we introduce a small change in the problem of the bank. Such a problem is a minimal cost in getting the desired outcome. It will only be triggered when the problem enters a cycle which for now has been sufficient.

Let b_1 and b_2 be a pair of solutions such that when bank managers take as given one policy, then they choose the other and viceversa. Let the associated values be Ψ_1 and Ψ_2. Once we identify one such loop define

$$\overline{\Psi}(p, \Psi_1, \Psi_2) = p \; \Psi_1 + (1 - p) \; \Psi_2 \quad (30)$$

$$
\begin{aligned}
\overline{\Psi}_1(p, \Psi_1, \Psi_2) &= -y\ 1_{y<0} \sum_{\varepsilon,\chi} F_e(\varepsilon, \chi)\ [1 - \widetilde{d}(e, y, \theta, \varepsilon, \chi, b_1)] + \qquad (31)\\
&\sum_{\varepsilon} F_e(\varepsilon, \chi)\ \widetilde{g}(e, y, \theta, \varepsilon, \chi, b_1)\ q^{\theta}\ \widetilde{y}'^{n}(e, y, \varepsilon, w, 0, b_1)\ 1_{\widetilde{y}'^{n}(e,y,\varepsilon,w,0,b_1)<0}\\
&+q^{*} \sum_{\varepsilon} F_e(\varepsilon, \chi)\ \widetilde{g}(e, y, \theta, \varepsilon, \chi, b_1) \sum_{e'} \Gamma_{e,e'}\ \overline{\Psi}(p)(e', \widetilde{y}'^{n}(e, y, \varepsilon, w, 0, b_1), \theta)\\
\overline{\Psi}_2(p, \Psi_1\Psi_2) &= -y\ 1_{y<0} \sum_{\varepsilon,\chi} F_e(\varepsilon, \chi)\ [1 - \widetilde{d}(e, y, \theta, \varepsilon, \chi, b_2)] + \qquad (32)\\
&\sum_{\varepsilon} F_e(\varepsilon, \chi)\ \widetilde{g}(e, y, \theta, \varepsilon, \chi, b_2)\ q^{\theta}\ \widetilde{y}'^{n}(e, y, \varepsilon, w, 0, b_2)\ 1_{\widetilde{y}'^{n}(e,y,\varepsilon,w,0,b_2)<0}\\
&+q^{*} \sum_{\varepsilon} F_e(\varepsilon, \chi)\ \widetilde{g}(e, y, \theta, \varepsilon, \chi, b_2) \sum_{e'} \Gamma_{e,e'}\ \overline{\Psi}(p)(e', \widetilde{y}'^{n}(e, y, \varepsilon, w, 0, b_2), \theta)\\
\overline{\overline{\Psi}}(\overline{p}, p, \Psi_1, \Psi_2) &= \overline{p}\ \Psi_1 + (1 - \overline{p})\ \Psi_2) \qquad (33)
\end{aligned}
$$

Then we can solve the problem

Where we can interpret ζe^2 as the cost of exerting effort choice e for a suitably chosen ζ. We are satisfied when we obtain a fixed point this is a solution $\overline{p}$ that is equal to p. We think of this as a heuristic approach to solve the fixed point problem of the branch bank managers.

Remark 1 This case has only occurred less than *xxxx* percent of the time.

Remark 2 The solution, while not independent of ζ yields statistics that are essentially identic for all ζ. The case reported is valid for environments where the effort cost is *xxxx* percent of profits.

Remark 3 There is a residual slight inconsistency for the households that do not internalize the randomness in the continuation policies of the bank. Switching their beliefs between the two policies does not change the outcome.