De-regulating Markets for Financial Information
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Abstract

In October 2009, the house financial services committee voted to study the effects of removing ratings requirements for credit products. Eliminating such requirements would allow the issuers of credit products to decide whether or not to pay a ratings agency to rate their asset. If such a rating was not provided by the asset issuer, investors themselves might purchase a rating. This paper studies the circumstances under which free markets for information will provide information, in the absence of government mandates and the efficiency properties of each regime. Although government regulation requires too much information for some assets and too little for others, private markets also suffer from inefficiencies stemming from the non-concave nature of information production. The results inform the debate about how and when to require information provision for a wide range of financial and non-financial products.

In October 2009, the house financial services committee voted to study the effects of removing ratings requirements for credit products. Currently, some investors (pension funds for example) can only purchase credit products that achieve a minimum level of credit-worthiness, as determined by a nationally-recognized ratings agency. Eliminating such requirements would allow the issuers of credit products to decide for themselves whether or not to pay a ratings agency to rate their asset. If such a rating was not provided by the asset issuer, investors themselves might want to purchase a rating, or some equivalent summary statistic about the quality of a credit product that they consider buying. This paper examines the effect of such a policy change on credit asset prices, on the allocation of productive capital, and on macroeconomic efficiency.

In many models of financial markets, information acquisition or disclosure is bad for welfare because is it costly to produce, does not increase the total amount of resources available to agents and distorts the incentives to share risk evenly.[1] Yet, in reality, many market participants and

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[1]See, for example, Diamond (1985).

policy makers are concerned about whether repealing ratings requirements would compromise the informational efficiency of asset prices. To reflect this concern, we begin by constructing a model where financial asset prices guide real investment decisions. In such a setting, informative prices facilitate efficient real investment and maximize output and consumption. Thus, one contribution of this paper is to show how to evaluate welfare in a way that allows information to have a potentially positive effect on welfare.

Whether regulation is welfare-enhancing or not is not obvious. On the one hand, a free market for information might provide information in the right situations: when information's value exceeds its cost and not when the cost exceeds the value. A government regulation could miss this nuance and result in information over-provision. On the other hand, the market might not provide an efficient amount of information. One reason is that the production function for information is not strictly concave. There is a fixed cost to producing information, but it can be freely replicated. Thus, if information is discovered, it should be shared with all. Yet, it will typically not be purchased by all investors in equilibrium. Another reason for inefficiency is that some investors can free-ride by using equilibrium asset prices to partially infer what others have learned. An optimal regulation strategy balances these competing inefficiencies.

The repeal of ratings will also have different effects on the prices of various assets. Our results characterize these price effects. To do this, we abstract from the potential biases or conflicts of interest in the ratings process and simply model ratings as noisy signals about the future value of a risky asset. Such signals affect asset prices in two ways: First, a positive signal will push the price of the asset up, while a lower-than-expected signal will reduce the price investors are willing to pay for an asset. In expectation, signals are neutral. On average, the positive and negative effects of the signal cancel out. The second effect is that the rating makes the asset's payoff less uncertain. In doing so, it makes the asset less risky. Lowering risk lowers the equilibrium return and systematically raises the asset's price.

Given that information raises asset prices by lowering risk, the question of which assets would be most affected by the repeal of ratings requirements would appear to be straightforward: The assets for which the ratings convey the most information or those which investors have the least prior information about, these are the assets whose prices will be most affected by the policy change. That intuition turns out to be incorrect. Our analysis shows that when prior beliefs are very uncertain or signals are very informative, either asset issuers or investors will opt to purchase information, even if such information is not required by law. It is also true that if signals are uninformative or prior beliefs are very precise, the disappearance of ratings has little price impact. Thus, the assets mostly likely to be affected by the policy change are neither the highest- nor the lowest-information securities, but the ones in between.

We also investigate whether the repeal of ratings requirements would affect assets with smaller or larger investor base more. Again, we find that assets in the middle are most affected, but the reason is different from before. When the investor base is large, there are many potential purchasers of information. Since information is a good with a high fixed cost of discovery and a low marginal cost of replication, high-demand information can be produced and sold at a low (per-copy) price. If the information is cheap, many investors buy it. Thus, information about assets with a large investor base is abundant, even when the provision of such information is not required. When the investor base is small, each investor must hold many shares of the asset, in order for the market to clear. Since they are bearing lots of risk from this asset, the price they are willing to pay for the asset is very sensitive to changes in risk. Since providing information reduces the risk the asset poses to an informed investor, the incentive for an asset issuer to acquire a rating and provide it to all investors is high because providing that rating will greatly increase the price the asset sells for. Thus, for both large- and small-investor-base assets the market will produce information on its own, without a ratings requirement. For assets with medium-sized investor bases, this may not be the case. Following the ratings repeal, such assets may see their prices fall, on average, as information becomes more scarce.

The model is an equilibrium model of portfolio choice and costly information acquisition. Our analysis is related to work on costly information acquisition, such as Grossman and Stiglitz (1980), Verrecchia (1982), Peress (2010) and Peress (2004). But it extends this work by considering the trade-offs between issuer- and investor-purchased information. If the issuer does not provide the signal, investors themselves can choose to purchase the information from an information market. We model the market for information in a richer way than most of the previous literature. Instead of choosing from signals with a fixed price, agents in our model can pay a fee to acquire a signal that they can sell to other investors at a competitive price. As in Veldkamp (2006), this price depends on how many other investors buy the signal as well. This allows us to consider whether in the absence of ratings regulation, either issuer-provided or investor-purchased information markets will fill in the void. Finally, the model connects financial information choices to real investment choices, output and welfare.

The financial market interacts with the real economy in the following way. At time 1, an entrepreneur can choose how much to invest in a given technology. He has a fixed marginal cost of investment and his payoff depends on what price he can sell the asset for in the time-2 financial market. If financial asset prices are very sensitive to changes in the value of the capital stock (they are informationally efficient), then the entrepreneur has a strong incentive to invest the optimal amount. If investors are not well-informed, then additional capital investment will not have a big effect on the price they are willing to pay for the firm. That reduces the entrepreneur's incentive to pay now to invest in a large capital stock.[2]

Previous models of ratings provision (e.g. Sangiorgi, Sokobin, and Spatt (2008), Bolton, Freixas, and Shapiro (2007), Bolton, Freixas, and Shapiro (2008), Skreta and Veldkamp (2009), Damiano, Li, and Suen (2008), and Becker and Milbourn (2008)) consider ratings inflation and conflicts of interest in the ratings system. This paper abstracts from these incentive issues and instead focuses on how many investors will get any information, in the absence of ratings mandates. Finally, this work is related to a microeconomics literature on welfare and information disclosure (e.g. Shavell (1994), Diamond (1985) and Jovanovic (1982)). Our model differs in a few respects. First, it features a continuum of investors who can participate in an information market that has an equilibrium price. Second, those investors can learn some of what others observe because it is reflected in the market price of the risky asset. Third, informed trade in asset markets results in a more efficient allocation of productive capital and therefore more output. All three of these features make the efficiency properties of the problem substantially different from preceding work.

1 Model

This is a static model where an entrepreneur is endowed with a production technology and can choose how much to invest. The project's final output is unknown to all. The entrepreneur's payoff comes from selling the project on a financial market in the following period. The payoff to the financial assets is the project's output. The entrepreneur also chooses whether to purchase and disclose a rating. The rating is simply an informative signal about what the project's output will be. If the entrepreneur chooses not to provide a rating, investors can decide whether to buy the rating themselves from a market for information.

Entrepreneur's problem Entrepreneur chooses $k \geq 0$, how much real capital to invest in period 0. Each unit of capital has a price of 1. The entrepreneur must sell the investment for a price of p in period 1. In period 2, the investment will produce output

$$y = f(k) + u$$

[2]Work by Goldstein, Ozdenoren, and Yuan (2011), Albagli, Hellwig, and Tsyvinski (2009) and Angeletos, Lorenzoni, and Pavan (2010) also models an interaction between financial markets and the real economy. But these are papers where financial investors can affect real investment through their asset purchasing decisions. This feedback creates complementarities in demand among investors and the potential for multiple equilibria. Our model shuts down these strategic interactions by having real investment take place first. Furthermore, these models have binary actions and outcomes. By keeping outcomes normally distributed in our model, we can solve for information choices analytically.

where $f(k)$ is a concave production function, $f(0) = 0$ and $u \sim N\left(0, \frac{1}{h_u}\right)$. The units of output and investment are per-share of the project. If there are $\bar{x}$ shares of the project, total output is $\bar{x}y$ and the price of capital investment is $\bar{x}k$.

The entrepreneur can choose to get a rating or can wait for the market to do so. If the entrepreneur pays a fixed, per-capita cost χ, he can disclose the rating to all investors. Let $D = 1$ if the entrepreneur chooses to rate the asset and $D = 0$ otherwise.

The entrepreneur's expected utility is

$$(\mathbb{E}\left(p|k\right) - k)\bar{x} - \chi D. \tag{1}$$

Investors cannot observe k but will have rational expectations about the equilibrium level of k^* that the entrepreneur will choose. As is standard, the entrepreneur takes the market's expectation of k^* as given and then we impose as an equilibrium condition $k = k^*$.

Financial Assets There are two assets: The 'safe' asset offers riskless return r, and the risky asset pays y, which is the output of the entrepreneur's project. The price of the riskless asset is 1. The price of the risky asset is p, which is endogenous.

Investors' problem There is a continuum of ex-ante identical investors with measure Q. Each has utility

$$U = -e^{-\rho(m_i r + qu - d_i c)}, \tag{2}$$

where ρ is the coefficient of absolute risk aversion, q_i and m_i are the number of risky and riskless asset shares investor i ends up with, $d_i = 1$ if the investor purchases information and $d_i = 0$ otherwise and c is the price of information. Each agent is endowed with m_i^0 units of the riskless asset, but can borrow and lend that asset freely at the riskless rate r. Hence, each investor's budget constraint is

$$m_i + pq_i = m_i^0. \tag{3}$$

The Asset Auction The price of the risky asset is determined in an auction. Each investor submits a bidding function $b(q|\mathcal{I}_i)$ that specifies the maximum amount that he is willing to pay for q units of the risky asset as a function of his information. These bid functions determine the aggregate demand. The auctioneer specifies a market-clearing price p that equates aggregate demand and supply, and each trader pays this price for each unit purchased (uniform-price auction).

Public information After the entrepreneur invests and before financial portfolios are chosen, all agents observe a public signal about output. The signal is $w = y + \omega$ where the signal noise

$\omega \sim N(0, 1/h_w)$ is independent of all other shocks in the model.

Rating Agencies Credit-rating agencies produce ratings θ, which are noisy, unbiased signals about the risky asset payoff y: $\theta = y + \eta$ where $\eta \sim N(0, \sigma_\theta^2)$.

The market for ratings has three important features. First, ratings are produced according to a fixed-cost technology. θ can be discovered at a per-capita fixed cost χ. This can be interpreted as the cost of hiring a journalist to interview people and find primary sources of information. The information, once discovered, can be distributed to other traders at zero marginal cost. Second, reselling purchased information is forbidden. The realistic counterpart to this assumption is intellectual property law that prohibits copying a publication and re-distributing it for profit. Third, there is free entry. Any agent can discover information at any time. That information markets are competitive is crucial. The exact market structure is not. Veldkamp (2006) analyzes a Cournot and a monopolistic competition market as well. All three markets produce information prices that decrease in demand.

Profits from information discovery depend on the price charged and demand for information, given the pricing strategies of other agents. The market is perfectly contestable. One way to ensure that the market is contestable is to force agents to choose prices in a first stage and choose entry in a second stage.

Recall that $d_i = 1$ if investor i decides to discover information in period t and $d_i = 0$ otherwise. Let per capita demand for information with price c_i, given all other posted prices c_{-i}, be $I(\cdot, \cdot)$. Then the objective of the information producer is to maximize profit:

$$\max_{d_i, c_i} d_i(c_i I(c_i, c_{-i}) - \chi). \tag{4}$$

Asset Issuer The entrepreneur has $\bar{x}$ shares of the risky asset to sell. Since we have a continuum of investors and each investor should hold a measurable quantity of risky assets, the asset supply is measured on a per-capita basis. The entrepreneur's objective is to maximize expected profit. The issuer's expected profit is the price times quantity of the asset sold, minus the cost of the rating:

$$\Pi = p\bar{x} - s\chi, \tag{5}$$

where $s = 1$ if the issuer purchases a rating and $s = 0$ otherwise. Note that since asset supply and information costs are in per-capita units, profit is as well.

Asset supply noise There is a set of agents who are subject to random shocks that force them to buy or sell the asset, at any current price. The per-capita demand of this group of agents is normally distributed with mean zero: $\xi \sim N(0, \sigma_x)$. Let x denote the per-capita net supply of the asset, after accounting for the noise trader demand: $x \equiv \bar{x} - \xi$. Thus, $x \sim N(\bar{x}, \sigma_x)$.

Order of Events

1. The entrepreneur chooses capital investment k. All agents have rational expectations about this level of investment.

2. Asset issuers choose whether or not to acquire information at cost χ. If the issuer acquires information, it is revealed to all.

3. Investors decide whether to discover information about output, and what price to set for that information. After observing posted prices, other agents can re-optimize and post new prices. Information discovery decisions are reversible.

4. Investors choose whether or not to purchase information. Informed agents observe θ.

5. Investors submit menus of prices and quantities of assets they are willing to purchase at each price.

6. Asset auction takes place.

7. All payoffs are received.

Equilibrium An equilibrium is a capital choice k and a ratings acquisition choice D that maximize the entrepreneur's expected utility (1); a ratings production and pricing decision c_i by each agent that maximizes (4); an information purchasing decision d_i and asset demands q_i that maximize investors' utility (2); and prices for risky assets that clear the market: $\int q_i di = x$.

2 Solving the model

To solve the model, we start with the second-period financial market equilibrium and work backwards to determine the optimal first-period real investment, both with fixed information sets. Finally, we add back in information decisions by investors and entrepreneurs.

2.1 Equilibrium asset prices

We being by deriving the investors' optimal bid function for risky assets and verifying that it constitutes an equilibrium. Bids depend on each investor's information set $\mathcal{I}_i$, which includes information inferred from b being the price paid per unit.

$$b(q|\mathcal{I}_i) = \frac{E(u|\mathcal{I}_i) - q\rho V(u|\mathcal{I}_i)}{r}, \tag{6}$$

where $E(u|\mathcal{I}_i)$ and $V(u|\mathcal{I}_i)$ are the mean and variance of the risky asset's return, conditional on the investor's information. The price paid per unit is exogenous from each investor's perspective because he is infinitesimal compared to the rest of the market, implying that the price he faces is determined by other investor's bid functions, together with the aggregate supply. Therefore, a realized price b reveals information about others' bids, which, in turn, is partially informative about what they know.

Each bidder is infinitesimal, which implies that he takes the market-clearing price as given. Thus, the bidding function (6) is the inverse demand function of a trader who seeks to maximize (2) subject to (3), taking p as given. It can be easily verified that the objective function of this constrained maximization problem is concave in q, so that the first-order condition describes the optimal portfolio:

$$q_i = \frac{1}{\rho} V[y|\mathcal{I}_i]^{-1}(E[y|\mathcal{I}_i] - pr). \tag{7}$$

Because (6) is an inverse of (7), it is a best response given everyone else's bid function.

The expectation and variance in (7) are conditional on an information set that includes the public signal w. It also includes prior beliefs that come from rational expectations about the level of investment k and knowledge of the distribution of u. Thus, prior to observing any signals $E[y] = f(k^*)$ For investors who have observed the rating, it also includes θ. For these informed investors,

$$E\left[y|\theta, w\right] = \frac{f(k^*)h_u + \theta h_\theta + w h_w}{h_u + h_\theta + h_w} \tag{8}$$

$$Var\left[y|\theta, w\right] = \frac{1}{h_u + h_\theta + h_w}. \tag{9}$$

Thus, informed trader demand is

$$\begin{aligned} q_I &= \frac{\frac{f(k^*)h_u + \theta h_\theta + w h_w}{h_u + h_\theta + h_w} - rp}{\rho \frac{1}{h_u + h_\theta + h_w}} \\ &= \frac{f(k^*)h_u + \theta h_\theta + w h_w - rp\left(h_u + h_\theta + h_w\right)}{\rho} \end{aligned}$$

For investors who have not observed the rating, the equilibrium price of the risky asset partially reveals the rating that others have observed. Since the price depends on asset demand and demand depends on information in the price, there is a fixed point problem. We solve by guessing a linear price rule

$$p = \alpha + \beta x + \gamma\theta + \phi w,$$

and solving for the coefficients α, β and γ. The following linear transformation of the price is an unbiased signal about the project output y:

$$z \equiv \frac{p - \alpha - \beta\bar{x} - \phi w}{\gamma} = \theta + \frac{\beta}{\gamma}(x - \bar{x}).$$

Let h_z denote the precision of this signal. It is a measure of the informativeness of prices.

The posteriors of the uninformed investors will be:

$$E[y|p,w] = \frac{f(k^*)h_u + zh_z + wh_w}{h_u + h_z + h_w} \tag{10}$$

$$Var[y|p,w] = \frac{1}{h_u + h_z + h_w} \tag{11}$$

Therefore, demand by each uninformed trader will be:

$$q_U = \frac{f(k^*)h_u + zh_z + wh_w - rp(h_u + h_z + h_w)}{\rho}$$

If a fraction λ of traders choose to become informed, total demand will be $Q(\lambda q^I + (1-\lambda)q^U)$. Equating this total demand to asset supply x yields coefficients for the linear price rule and confirms the conjecture of a linear price. The following coefficients are derived in appendix A.1:

$$\alpha = \gamma Q \frac{f(k^*)h_u + wh_w - (Q-\lambda)\frac{\beta\bar{x}}{\gamma}h_z}{\lambda h_\theta + (1-\lambda)h_z} \tag{12}$$

$$\beta = \frac{-\rho}{\lambda h_\theta}\frac{\lambda h_\theta + (Q-\lambda)h_z}{r[h_u + h_w + \lambda h_\theta + (Q-\lambda)h_z]} \tag{13}$$

$$\gamma = \frac{\lambda h_\theta + (Q-\lambda)h_z}{r[h_u + h_w + \lambda h_\theta + (Q-\lambda)h_z]}, \tag{14}$$

$$\phi = \frac{Qh_w}{r[h_u + h_w + \lambda h_\theta + (Q-\lambda)h_z]} \tag{15}$$

where

$$h_z = \frac{1}{\frac{1}{h_\theta} + \left(\frac{\rho}{\lambda h_\theta}\right)^2 \frac{1}{h_x}}. \tag{16}$$

2.2 Real investment without a publicly announced rating

Suppose that the entrepreneur does not purchase a rating, but some fraction $\lambda\epsilon[0,1]$ of agents purchase the information themselves.

The entrepreneur chooses capital investment to maximize his objective function. His objective depends on the expected price, which is

$$\begin{aligned}\mathbb{E}\left(p|k\right) &= \alpha+\beta Ex+\gamma E\left(\theta|k\right)+\phi E\left(w|k\right) \\ &= \alpha+\beta Ex+(\gamma+\phi)f(k)\end{aligned}$$

where the last two expressions follow because the rating and public signal are both output, plus noise: $f(k)+u+\eta$ and $f(k)+u+\omega$.

The capital investment that maximizes the entrepreneur's objective is the k^* that satisfies

$$\frac{\partial E[p|k]}{\partial k}=1 \tag{17}$$

$$(\gamma+\phi)f'(k^*)=1. \tag{18}$$

$$f'(k^*)=1+\frac{rh_u}{h_w+\lambda h_\theta+(1-\lambda)\,h_z} \tag{19}$$

Let this level of capital investment be denoted k_l^*, where the l is for low-information.

2.3 Real investment with a publicly announced rating

If the entrepreneur purchases a rating and discloses it to all investors, the asset price is given by (17) setting the fraction of informed agents λ to 1. Substituting in (12), (13), (14) and (15) for α, β, γ and ϕ yields

$$p=\frac{f(k^*)h_u+\theta h_\theta+wh_w-\rho x}{r(h_u+h_\theta+h_w)}. \tag{20}$$

But the public signal and the rating both depend on the real investment the entrepreneur has chosen: $w=f(k)+u+\omega$, and $\theta=f(k)+u+\eta$. And the prior depends on the expected equilibrium capital investment: $f(k^*)=f(k^*)$. Thus, the price as a function of the capital investment decision is

$$p=\frac{f(k^*)h_u+h_\theta(f(k)+u+\eta)+h_w(f(k)+u+\omega)-\rho x}{r(h_u+h_\theta+h_w)}. \tag{21}$$

The price the entrepreneur expects to get for his asset, as a function of k is

$$E[p|k]=\frac{f(k^*)h_u+f(k)(h_\theta+h_w)-\rho x}{r(h_u+h_\theta+h_w)}. \tag{22}$$

The capital investment that maximizes the entrepreneur's objective is the k^* that satisfies

$$\frac{\partial E[p|k]}{\partial k}=1 \tag{23}$$

$$\frac{f'(k^*)(h_\theta + h_w)}{r(h_u + h_\theta + h_w)} = 1. \tag{24}$$

$$f'(k^*) = \frac{r(h_u + h_\theta + h_w)}{h_\theta + h_w}. \tag{25}$$

Let this level of capital investment be denoted k_h^*, where the h is for high-information. To determine the issuer's ratings decision, we need to compare his payoffs when he pays for a rating and invests k_h^* to his payoffs without the rating and with capital investment k_l^*. But in order to know what investment k_l^* is, we need to know how many investors will buy the rating when the issuer does not provide it. That is what we solve for next.

2.4 Investors' Rating Decision

Suppose that the entrepreneur decides not to provide a rating for the market. Then, the investors must choose for themselves whether or not to acquire the rating at a market price c. Since investors are ex-ante identical, they will only make different choices when those choices yield identical expected utility. Appendix computes the expected utility of an informed investor and the expected utility of an uninformed investor when a measure λ of the population of investors is informed. If there exists a $\lambda \epsilon [0, 1]$ that equates the two expected utilities, then this is an equilibrium. Appendix A.2 shows that the equilibrium fraction of informed investors is

$$\lambda = \frac{\rho}{h_\theta \sqrt{h_x \left[\frac{1}{(h_u + h_\theta)\exp(-2\rho c) - h_u} - \frac{1}{h_\theta}\right]}} \tag{26}$$

If this λ is not between 0 and 1, then there is a corner solution. If expected utility for uninformed is higher, the corner solution is $\lambda = 0$, otherwise, the solution is $\lambda = 1$. If the right side produces an imaginary number, it signifies that there is no positive measure of informed investors that equates expected utility for the informed and uninformed. In these instances, the only solution is for all investors to remain uninformed (unless the issuer acquires information and reveals it to them).

Proposition 1 *If*

$$h_\theta \exp\left(\frac{-2\rho\chi}{Q}\right) - h_u \left(1 - \exp\left(\frac{-2\rho\chi}{Q}\right)\right) < 0 \tag{27}$$

investors will not buy a rating

Proof in appendix A.3. Condition (27) ensures that the square root term in (40) is positive so that a real, positive λ exists that satisfies equation (40). This is a sufficient but not necessary condition for $\lambda = 0$. The reason is that the information cost c is endogenous and depends on λ. The measure of informed investors is decreasing in the information cost and the minimum information cost is χ/Q, which is the per-capita cost of information when all investors buy it. Thus, substituting χ/Q for c yields a sufficient condition for positive information demand.

Taking partial derivatives of (27) yields the following predictions about investor information demand.

Corollary 1 *Investors will not buy ratings if:*

1. *Signal precision h_θ is sufficiently low*

2. *The discovery cost of information χ is sufficiently high*

3. *Prior belief precision h_u is sufficiently high*

4. *The measure of total investors Q is sufficiently low*

5. *Risk aversion ρ is sufficiently high*

2.5 Entrepreneur's rating decision

The entrepreneur will provide a rating iff expected payoffs net of the information cost χ excess expected payoffs without information:

$$-k_h^* + \alpha + \beta Ex + (\gamma + \phi) f(k_h^*) - \chi > -k_l^* + \alpha + \beta Ex + (\gamma + \phi) f(k_l^*)$$

$$(\gamma + \phi - 1) f(k_h^*) - \chi > (\gamma + \phi - 1) f(k_l^*).$$

If $\gamma + \phi < 1$, then the entrepreneur optimally chooses zero investment because the expected payoff of the project when it is sold in the second period is less than the cost of the initial capital investment. If there is no investment, there is no reason to provide a rating. Suppose $\gamma + \phi > 1$.

Proposition 2

1. If
$$\frac{\rho h_\theta}{rQ\,(h_\theta + h_u)}\bar{x} > \chi, \tag{28}$$
then either the issuer will provide a rating or at least *some* investors will buy it

2. If condition (28) does not hold, the issuer will not provide a rating

Proof in appendix A.4. Taking partial derivatives of (28) reveals the following conditions under which an issuer will not provide a rating.

Corollary 2 *The issuer will not provide a rating if*

1. h_θ is sufficiently low

2. χ is sufficiently high

3. h_u is sufficiently high

4. Q is sufficiently high

5. ρ is sufficiently low

6. $\bar{x}$ is sufficiently low

In summary, combining the results from corollaries (1) and (2) reveals when no ratings will be produced. Ratings will not be produced at all if

1. h_θ is sufficiently low

2. χ is sufficiently high

3. h_u is sufficiently high.

These are the instances where de-regulation will have the most dramatic effects on information provision. Instead of all investors being informed with credit rating mandates, the market will not provide information for anyone.

3 Effects of De-regulation on Asset Prices

Another question that financial market participants will want to answer is what will happen to the prices of assets after de-regulation. The following proposition shows that on average, the price of credit assets will fall as information becomes less abundant.

Proposition 3 *For an issuer that does not provide a rating, the average asset price is increasing in λ.*

Proof in appendix A.6. To see why this is true, note that ratings affect asset prices in two ways: First, a positive signal will push the price of the asset up, while a lower-than-expected signal will reduce the price investors are willing to pay for an asset. In expectation, signals are neutral. Thus on average, the positive and negative effects of the signal cancel out. The second effect is that the rating makes the asset's payoff less uncertain. In doing so, it makes the asset less risky. Lowering risk lowers the equilibrium return and systematically raises the asset's price.

The next result describes which assets are likely to be most affected by de-regulation. It shows that the assets mostly likely to be affected by the policy change are neither the highest- nor the lowest-information securities, but the ones in between. Likewise, for both large- and small-investor-base assets the market will produce information on its own, without a ratings requirement. For assets with medium-sized investor bases, this may not be the case. Following the ratings repeal, such assets may see their prices fall, on average, as information becomes more scarce.

To formalize these ideas, we consider the difference between the price of an asset with mandatory ratings and the price of an asset without mandatory ratings, but in an environment where either the entrepreneur or the investor can choose to purchase a rating.

Proposition 4 *Let $\bar{p}^M$ be the average price of the asset if ratings are mandatory and $\bar{p}^E$ be the average price of the asset if the ratings decision is an equilibrium outcome. Suppose condition (43) holds. Then*

1. *If h_u is sufficiently low, $\bar{p}^M = \bar{p}^E$*
2. $\lim_{h_u \to \infty} \left(\bar{p}^M - \bar{p}^E\right) = 0$
3. *There is an interval (a, b) such that $\bar{p}^M > \bar{p}^E$ for all $h_u \in (a, b)$,*

Proof in appendix A.7. This result considers what happens as prior beliefs become more or less precise. When priors are very imprecise, signals are valuable and will be acquired by issuers or investors. When priors are very precise, no information will be acquired, but the effect of the rating on the asset price will disappear in the limit. In between these extremes, there exists a region where not all investors are informed and where the asset price is strictly less than it would be under the mandatory ratings regime. The same result can be proven for the precision of the public signal as well.

4 Ratings Regulation and Welfare

The primary friction in the model is that investors' imperfect information about capital investment decisions of the firm reduces the entrepreneur's return to investing in capital. In other words, if investors don't know that the entrepreneur invested more, he won't be compensated for that investment when he sells his firm. Efficiency requires that the marginal return to investment be equal to its unit marginal cost: $f'(k) = 1$. Therefore if we somehow manage to ensure that that the private return to a marginal unit of investment is equal to its social return, $\frac{\partial \mathbb{E}(p|k)}{k} = f'(k)$, then investment will be efficient. With imperfect information, the left side is typically smaller than the right because prices can only respond to changes in k to the extent that investors know k. Providing information to financial markets helps to remedy this friction because it makes p more responsive to k. This section looks at whether government mandated information disclosure helps or hurts economic welfare.

There are a few different ways we might think about a policy maker's objective in this model. We examine each in turn.

4.1 Maximizing output

One possible objective a government might have is to simply maximize the production of real goods. This is obviously a simplification, but it makes for a good starting point. The relevant question becomes: Which ratings policies maximize output $f(k)$?

Since the production function is concave, a higher $f(k)$ corresponds to a lower marginal product of capital $f'(k)$. The marginal product that corresponds to the entrepreneur's investment choice is given by (19). The right hand side of that equation is $1 + \frac{rh_u}{h_w+\lambda h_\theta+(1-\lambda)h_z}$.

Inspecting equation (16) reveals that $h_\theta \geq h_z$. This makes sense because prices cannot reveal more information that what is contained in the signals they are revealing. If $h_\theta - h_z \geq 0$, then this expression is increasing (non-decreasing) in λ. If ratings are mandated by the government, $\lambda = 1$, which maximizes $f(k)$ over all feasible values ($\lambda \epsilon [0, 1]$). Thus, mandating ratings maximizes output of real economic goods. It provides the maximum possible information. Since information facilitates the efficient allocation of capital, mandatory information disclosure maximizes output.

4.2 Maximizing output net of costs

One obvious objection to the previously proposed social welfare objective is that it does not take into account the cost of investment or information production. In particular, it treats information as if it were free. More information might always be better. But if information is costly, it must be sufficiently valuable to justify its cost. Thus, another possible objective is to maximize $f(k)-k-\delta\chi$, where $\delta = 1$ if any agent (entrepreneur or investor) discovers information and $\delta = 0$ otherwise.

The first order condition tells us that $f(k) - k$ is maximized when $f'(k) = 1$. Note that since $rh_u > 0$, both $f'(k_h^*)$ and $f'(k_l^*)$ are both greater than one. The capital level that achieves a higher level of $f(k) - k$ is the greater of the two capital levels because by concavity of f, that one has a lower marginal product of capital, which achieves $f'(k)$ closer to 1. From the previous subsection, we know that $k_h^* > k_l^*$. Thus, k_h^* achieves a higher level of output, net of investment cost.

Next, note that since $f(k) - k$ is maximized when $\lambda = 1$, this means that if anyone incurs the cost χ to discover information, the output-maximizing outcome is for all investors to observe that information. Thus, any $\lambda \neq \{0, 1\}$ does not maximize output net of costs. The final question is: In what circumstances is the higher output associated with $\lambda = 1$ large enough to compensate for the cost of information? In other words, what are the parameters of the problem for which $f(k_h^*) - k_h^* - \chi > f(k_l^*) - k_l^*$? This remains to be worked out.

4.3 Maximizing a weighted sum of utilities

This is the most commonly used social welfare criterion. In this setting, the objective this produces depends on how one weights the issuer (a single entity) versus the investors (a continuum of agents). The question of how one models the noise traders then also comes into play. Since we have no guidance on how to weight these various constituencies, we sill simply examine their utilities separately in order to answer the question of who gains and who loses from reform.

A simple revealed preference argument establishes that the asset issuer is always weakly better off without the ratings mandate. Without the mandate, the asset issuer can always choose to pay for and disclose the rating. But with the mandate, he cannot choose to forgo a rating.

Thus, the question becomes: How does the ratings mandate affect investors? On the one hand, information produces more efficient investment decisions that increase the total production and therefore the total payoffs to all risky assets. This makes investors better off. On the other hand, proposition 3 tells us that information increases the price investors must pay issuers for the asset, which makes them worse off.

Future work will compare these competing effects and determine in which circumstances investors prefer ratings mandates.

5 Conclusions

Coming soon.

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A Mathematical Appendix

A.1 Solving for the financial market equilibrium

This appendix solves for the equilibrium price in the risky asset market. It verifies the conjecture of the existence of a price that is linear in signals and asset supply and it derives the formulas for the linear weights.

Beginning with the market clearing condition (**??**), we replace z with its definition and solve for p:

$$\begin{aligned} f(k^*)h_u + wh_w + \lambda\left[\theta h_\theta - rp\left(h_u + h_\theta + h_w\right)\right] + (1-\lambda)\left[zh_z - rp\left(h_u + h_z + h_w\right)\right] &= \rho x \\ f(k^*)h_u + wh_w + \lambda\left[\theta h_\theta - rp\left(h_u + h_\theta + h_w\right)\right] + (1-\lambda)\left[\frac{p-\alpha-\beta\bar{x}}{\gamma}h_z - rp\left(h_u + h_z + h_w\right)\right] &= \rho x \\ f(k^*)h_u + wh_w + \lambda\theta h_\theta - (1-\lambda)\frac{\alpha+\beta\bar{x}}{\gamma}h_z + p\left[-\lambda r\left(h_u + h_\theta + h_w\right) - (1-\lambda)r\left(h_u + h_z + h_w\right) + (1-\lambda)\frac{h_z}{\gamma}\right] &= \rho x \end{aligned}$$

$$p = \frac{f(k^*)h_u + wh_w + \lambda\theta h_\theta - (1-\lambda)\frac{\alpha+\beta\bar{x}}{\gamma}h_z - \rho x}{\lambda r\left(h_u + h_\theta + h_w\right) + (1-\lambda)r\left(h_u + h_z + h_w\right) - (1-\lambda)\frac{h_z}{\gamma}} \tag{29}$$

which has a linear form as conjectured. Equating coefficients:

$$\begin{aligned} \alpha &= \frac{f(k^*)h_u + wh_w - (1-\lambda)\frac{\alpha+\beta\bar{x}}{\gamma}h_z}{\lambda r\left(h_u + h_\theta + h_w\right) + (1-\lambda)r\left(h_u + h_z + h_w\right) - (1-\lambda)\frac{h_z}{\gamma}} \\ \beta &= \frac{-\rho}{\lambda r\left(h_u + h_\theta + h_w\right) + (1-\lambda)r\left(h_u + h_z + h_w\right) - (1-\lambda)\frac{h_z}{\gamma}} \\ \gamma &= \frac{\lambda h_\theta}{\lambda r\left(h_u + h_\theta + h_w\right) + (1-\lambda)r\left(h_u + h_z + h_w\right) - (1-\lambda)\frac{h_z}{\gamma}} \end{aligned} \tag{30}$$

What we see is that in this setting, having a public signal is equivalent to having a more precise prior. Simply replace $f(k^*)h_u$ with $f(k^*)h_u + wh_w$ and h_u with $h_u + h_w$. The price informativeness h_z is unchanged.

Computing price informativeness yields

$$h_z = \frac{1}{\frac{1}{h_\theta} + \left(\frac{\beta}{\gamma}\right)^2\frac{1}{h_x}}. \tag{31}$$

Substituting in expressions for β and γ yields (16).

A.2 Solving for the equilibrium fraction of informed investors

Utility of the informed investor Recall the utility function:

$$V = -E\left[\exp\left\{-W\right\}\right]$$

where

$$W^I = r\left(w_0 - c\right) + q^I\left[u - rp\right]$$

Because of CARA-Normal, utility is

$$V = -\exp\left\{-\rho\left[E_I\left(W^I\right) - \frac{\rho}{2}Var_I\left(W^I\right)\right]\right\}$$

Use that $q^I = \frac{E[u|I_i]-rp}{\rho Var[u|I_i]}$ so that

$$W^I = r\left(w_0 - c\right) + \frac{E\left[u|I_i\right] - rp}{\rho Var\left[u|I_i\right]}\left[u - rp\right]$$

to conclude that

$$E_I\left(W^I\right) = r\left(w_0 - c\right) + \frac{\left[E_I\left(u\right) - rp\right]^2}{\rho Var_I\left(u\right)}$$

and

$$\begin{aligned} Var_I\left(W^I\right) &= \left[\frac{E_I(u)-rp}{\rho Var_I(u)}\right]^2 Var_I(u) \\ &= \frac{[E_I(u)-rp]^2}{\rho^2 Var_I(u)} \end{aligned}$$

Replacing these expressions:

$$\begin{aligned} V &= -\exp\left\{-\rho\left[r(w_0-c)+\frac{[E_I(u)-rp]^2}{\rho Var_I(u)}-\frac{\rho}{2}\frac{[E_I(u)-rp]^2}{\rho^2 Var_I(u)}\right]\right\} \\ &= -\exp(-\rho r(w_0-c))\exp\left\{-\frac{1}{2}\frac{[E_I(u)-rp]^2}{Var_I(u)}\right\} \end{aligned}$$

This expected utility is conditional on 4 random variables: θ, x, w and u. Here V is the expected utility conditional on realized values of θ and x such that the realized value of u is still unknown. The overall (ex-ante) expected utility must take into account uncertainty in the realization of θ, w, and x. More properly, write this as:

$$V(\theta,w,x) = -\exp(-\rho r(w_0-c))\exp\left\{-\frac{1}{2Var(u|\theta,w,x)}[E(u|\theta,w)-rp(\theta,w,x)]^2\right\} \tag{32}$$

Continue to condition on the price and work on computing the expectation w.r.t. θ. Let

$$s^2 \equiv Var[E(u|\theta,w)-rp(\theta,w,x)|p,w] \tag{33}$$

$$Z(\theta,w,x) \equiv \frac{E(u|\theta,w)-rp(\theta,w,x)}{s} \tag{34}$$

Replacing (33) and (34) into (32):

$$V(\theta,w,x) = -\exp(-\rho r(w_0-c))\exp\left\{-\frac{s^2}{2Var(u|\theta,w,x)}Z^2\right\}$$

Note: Z is a random variable that depends on x and θ. s is a fixed number (thanks to the Gaussian distributions). Let us condition on a realized value of p and compute the expectation of Z.

$$\begin{aligned} \mu_z &\equiv E[Z|p,w] \\ &= E\left[\frac{E(u|\theta,w)-rp(\theta,w,x)}{s}|p,w\right] \\ &= \frac{E(u|p,w)-rp}{s} \end{aligned}$$

by the law of iterated expectations. Just to make clear, I am taking the expectation with respect to θ but taking p as given. Now if I want to compute expected utility conditional on p I have:

$$V(p) = -\exp(-\rho r(w_0-c))E\left[\exp\left\{-\frac{s^2}{2Var(u|\theta,w,x)}Z^2\right\}|p,w\right]$$

Conditional on p, Z follows a Normal distribution with mean μ_z and standard deviation 1. Therefore Z^2 follows a noncentral χ^2 distribution. The MGF is

$$M(t) = E\left[\exp\left\{-tZ^2\right\}|p,w\right] = \frac{\exp\left(-\frac{\mu_z^2 t}{1+2t}\right)}{\sqrt{1+2t}}$$

so, letting $t=\frac{s^2}{2Var(u|\theta,w,x)}$:

$$E\left[\exp\left\{-\frac{s^2}{2Var(u|\theta,w,x)}Z^2\right\}|p,w\right] = \frac{1}{\sqrt{1+2\frac{s^2}{2Var(u|\theta,w,x)}}}\exp\left(-\frac{\mu_z^2\frac{s^2}{2Var(u|\theta,w,x)}}{1+2\frac{s^2}{2Var(u|\theta,w,x)}}\right) \tag{35}$$

$$= \frac{1}{\sqrt{1+\frac{s^2}{Var(u|\theta,w,x)}}}\exp\left(-\frac{\left(\frac{E(u|p,w)-rp}{s}\right)^2\frac{s^2}{2Var(u|\theta,w,x)}}{1+2\frac{s^2}{2Var(u|\theta,w,x)}}\right)$$

$$= \frac{1}{\sqrt{1+\frac{s^2}{Var(u|\theta,w,x)}}}\exp\left(-\frac{(E(u|p,w)-rp)^2}{2(Var(u|\theta,w,x)+s^2)}\right) \tag{36}$$

By the law of total variance:

$$\begin{aligned} Var(u|p,w) &= Var(E(u|\theta,w)|p,w) + E[Var(u|\theta,w)|p|] \\ &= s^2 + Var(u|\theta,w,x) \end{aligned} \tag{37}$$

Using (37) in (36):

$$\begin{aligned} E\left[\exp\left\{-\frac{s^2}{2Var(u|\theta,w,x)}Z^2\right\}|p,w\right] &= \frac{1}{\sqrt{\frac{Var(u|\theta,w,x)+s^2}{Var(u|\theta,w,x)}}}\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}\right) \\ &= \sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}}\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}\right) \end{aligned}$$

so

$$V_I(p) = -\exp(-\rho r(w_0-c))\sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}}\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}.\right)$$

Utility of the uninformed investor

$$V(\theta,w,x) = -\exp(-\rho r w_0)\exp\left\{-\frac{1}{2Var(u|p,w)}[E(u|p,w)-rp(\theta,w,x)]^2\right\}$$

Conditional on p, this is not random, so we just have:

$$V_U(p) = -\exp(-\rho r w_0)\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}\right)$$

Utility comparison

$$\begin{aligned} V_I(p)-V_U(p) &= -\exp(-\rho r(w_0-c))\sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}}\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}\right) + \exp(-\rho r w_0)\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}\right) \\ &= \exp(-\rho r w_0)\exp\left(-\frac{(E(u|p,w)-rp)^2}{2Var(u|p,w)}\right)\left[1-\exp(\rho c)\sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}}\right] \\ &= \left[\exp(\rho c)\sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}}-1\right]V_U(p) \end{aligned}$$

Ex-ante indifference (i.e. taking expectations over p) requires:

$$\begin{aligned} V_I - V_U &= \left[\exp(\rho c)\sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}}-1\right]EV_U(p) = 0 \\ \exp(\rho c)\sqrt{\frac{Var(u|\theta,w)}{Var(u|p,w)}} &= 1 \end{aligned} \tag{38}$$

Solve (38) for λ:

$$\begin{aligned} \exp(\rho c)\sqrt{\frac{\frac{1}{h_u+h_w+h_\theta}}{\frac{1}{h_u+h_w+h_z}}} &= 1 \\ \exp(\rho c)\sqrt{\frac{h_u+h_w+h_z}{h_u+h_w+h_\theta}} &= 1 \end{aligned}$$

Use (16) to replace $h_z t$:

$$\exp(\rho c)\sqrt{\frac{h_u+h_w+\frac{1}{\frac{1}{h_\theta}+\left(\frac{\rho}{\lambda h_\theta}\right)^2\frac{1}{h_x}}}{h_u+h_w+h_\theta}} = 1 \tag{39}$$

This equation defines the equilibrium value of λ. You can also solve explicitly:

$$\begin{aligned}
\frac{h_u + h_w + \frac{1}{\frac{1}{h_\theta} + \left(\frac{\rho}{\lambda h_\theta}\right)^2 \frac{1}{h_x}}}{h_u + h_w + h_\theta} &= \exp(-2\rho c) \\
h_u + h_w + \frac{1}{\frac{1}{h_\theta} + \left(\frac{\rho}{\lambda h_\theta}\right)^2 \frac{1}{h_x}} &= (h_u + h_w + h_\theta)\exp(-2\rho c) \\
\frac{1}{\frac{1}{h_\theta} + \left(\frac{\rho}{\lambda h_\theta}\right)^2 \frac{1}{h_x}} &= (h_u + h_w + h_\theta)\exp(-2\rho c) - h_u - h_w \\
\frac{1}{h_\theta} + \left(\frac{\rho}{\lambda h_\theta}\right)^2 \frac{1}{h_x} &= \frac{1}{(h_u + h_w + h_\theta)\exp(-2\rho c) - h_u - h_w} \\
\frac{\rho}{\lambda h_\theta} &= \sqrt{h_x \left[\frac{1}{(h_u + h_w + h_\theta)\exp(-2\rho c) - h_u - h_w} - \frac{1}{h_\theta}\right]} \\
\lambda &= \frac{\rho}{h_\theta \sqrt{h_x \left[\frac{1}{(h_u + h_w + h_\theta)\exp(-2\rho c) - h_u - h_w} - \frac{1}{h_\theta}\right]}} \qquad (40)
\end{aligned}$$

A.3 Proof of proposition 1

From the expression for λ, a positive solution for λ requires

$$\frac{1}{h_\theta \exp(-2\rho c) - h_u (1 - \exp(-2\rho c))} - \frac{1}{h_\theta} > 0 \qquad (41)$$

which reduces to

$$h_\theta \exp(-2\rho c) - h_u (1 - \exp(-2\rho c)) > 0 \qquad (42)$$

Since the ratings agency must make nonnegative profits and at most Q investors purchase the rating, this means that $c \geq \frac{\chi}{Q}$. Therefore (42) cannot hold unless (27) holds.

A.4 Proof of proposition 2

1. Suppose to the contrary that the issuer does not provide information, and investors do not buy it either. Expected profits for the issuer will be:

   $$\Pi^0 = \frac{1}{r}\left[f(k^*) - \frac{\rho}{Q h_u}\bar{x}\right]$$

   If instead the issuer paid for a rating, expected profits would be:

   $$\Pi^I = \frac{1}{r}\left[f(k^*)\frac{h_u}{h_u + h_\theta} - \frac{\rho}{Q(h_\theta + h_u)}\bar{x} + f(k^*)\frac{h_\theta}{h_\theta + h_u}\right] - \chi$$

   Condition (28) implies $\Pi^I > \Pi^0$, which contradicts the assumption that the issuer does not provide information.
2. If condition (28) does not hold, then $\Pi^I \leq \Pi^0$, so an issuer will not provide a rating even if he expects investors not to buy it either. By Proposition **??**, this implies that the issuer will not provide a rating regardless of what he expects investors to do.

A.5 Additional comparative statics for λ

Proposition 5 *If condition (27) holds and h_x is sufficiently low $\lambda = Q$*

This is immediate from the formula for λ

Corollary 3 *If condition (27) holds and h_x is sufficiently low, the issuer will not provide a rating, but prices will be the same as if a rating were mandatory*

Proposition 6 *λ is decreasing in h_u*

Fixing c, this is immediate from the formula for λ. But the zero profit condition for the ratings agency $c = \frac{\chi}{\lambda}$ implies that a decrease in λ increases c, which implies a further decrease in λ

Consider condition

$$\frac{\rho}{\sqrt{h_x h_\theta \frac{1-\exp\left(\frac{-2\rho\chi}{Q}\right)}{\exp\left(\frac{-2\rho\chi}{Q}\right)}}} > Q \tag{43}$$

Proposition 7 *If condition (43) holds and h_u is sufficiently low, then $\lambda = Q$*

From the formula for λ,

$$\lim_{h_u \to 0} \lambda = \frac{\rho}{\sqrt{h_x h_\theta \frac{1-\exp(-2\rho c)}{\exp(-2\rho c)}}}$$

If condition (43) holds, then setting $c = \frac{\chi}{Q}$ we have $\lim_{h_u \to 0} \lambda = Q$, which makes $c = \frac{\chi}{Q}$ the equilibrium price for the rating.

Corollary 4 *If condition (43) holds and h_u is sufficiently low, the issuer will not provide a rating, but all investors will buy it*

A.6 Proof of proposition 3

Let $\bar{p}$ be the price that arises when $x = \bar{x}$ and $\theta = \bar{u}$. Due to linearity, it suffices to show that this is increasing in λ. Using the market clearing condition and the definition of z:

$$\lambda \frac{\bar{u}h_u + \bar{u}h_\theta - r\bar{p}\,(h_u + h_\theta)}{\rho} + (Q - \lambda)\,\frac{\bar{u}h_u + \bar{u}h_z - r\bar{p}\,(h_u + h_z)}{\rho} = \bar{x}$$

Solving for $\bar{p}$:

$$\bar{p} = \frac{1}{r}\left[\bar{u} - \frac{\rho\bar{x}}{Qh_u + \lambda h_\theta + (Q-\lambda)h_z}\right]$$

so

$$\frac{\partial \bar{p}}{\partial \lambda} = \frac{1}{r}\frac{h_\theta - h_z + (Q-\lambda)\frac{\partial h_z}{\partial \lambda}}{\left(Qh_u + \lambda h_\theta + (Q-\lambda)h_z\right)^2}\rho\bar{x} > 0$$

because $\frac{\partial h_z}{\partial \lambda} > 0$

A.7 Proof of proposition 4

1. This follows from Corollary (4)
2. This follows because as $h_u \to \infty$, $p \to \frac{\bar{u}}{r}$ no matter whether there is a rating or not
3. Let h_u^* be the maximum value of h_u such that $\lambda = Q$. This value must exist due to Proposition 7 and Corollary 1. Let Π^I be the issuer's expected profits if he provides a rating and Π^B be his profits if he lets investors buy the ratings themselves. Proposition (7) implies $\Pi^I - \Pi^B = -\chi$ for $h_u < h_u^*$. Furthermore, because both Π^I and Π^B are continuous in h_u and λ is continuous in h_u at h_u^*, $\Pi^I - \Pi^B$ is continuous in h_u at h_u^*. This implies there is a positive ε such that if $h_u \in (h_u^*, h_u^* + \varepsilon)$, the issuer prefers not to provide a rating even though $\lambda < Q$. Since a mandatory rating has the same effect on asset prices as $\lambda = Q$, Proposition (**??**) implies that for $h_u \in (h_u^*, h_u^* + \varepsilon)$, $\bar{p}^M > \bar{p}^E$