Health, Education and Development

Preliminary draft

Yin-Chi Wang*

Washington University in St. Louis

February 2011

Abstract: Why are some poor countries able to take off while others are stuck in the poverty trap? Motivated by the observation that with similar or higher levels of educational attainment, trapped countries tend to have poorer health conditions compared to the initially poor later take-off countries, we argue that health and education interact with each other to determine the productivity of labor and the development pattern of a country. To capture this idea, we develop an overlapping-generations model with human capital separable in health and knowledge. Health investment not only determines the quality of life for the agent, but also serves as an important input to constitute the health capital of a country. The health capital and the knowledge capital then together form the human capital and determine the labor productivity. We calibrate the model to fit the U.S. economy, discuss the circumstances in which the poverty trap is more likely to emerge, and provide fiscal policy to keep countries away from poverty. To understand the factors distinguishing middle-income-low countries and trapped countries, we calibrate our model to fit a group of middle-income-low countries and trapped countries. We will examine the causes of poverty of each country, that is, whether poverty is caused by deficiency in health or education investment, or intrinsic factors specific to the country. We will also discuss the forces leading the middle-income-countries to take off. Finally, we will investigate whether a trapped country could be dragged out of poverty if its government subsidizes health and education investment.

JEL classification: O4, D5, I1, I2

Keywords: Growth, Overlapping Generations, Health, Education

*E-mail: yin-chi.wang@go.wustl.edu. I am grateful for valuable comments and suggestions from Costas Azariadis, Alejandro Badel, James Bullard, Jang-Ting Guo, Rody Manuelli, Juan Pantano, and Yongs Shin, and I am indebted to my advisor Ping Wang. I also thank Wung Lik Ng for support.

1 Introduction

Large and persistent differences in income level have existed across countries. Economists have sought to learn the causes of these differences, and in particular, to understand why some countries are able to pull out of the poverty trap while other countries remain mired in it. In addition to physical capital, education and health conditions are generally considered the crucial factors among important factors determining the development patterns. However, in the literature, there is no systematic, integrated micro-founded framework to investigate these two important factors. Such an omission could harm the prediction power of the model, especially when health and education affect the development of a country in very different ways. The intent of the current paper is to fill this gap in the literature and to explore the relationship of health and education together.[1]

Table 1 illustrates the economic performance (per capita GDP growth rate and per capita GDP relative to the U.S.), health conditions (life expectancy and under 5 mortality rates) as well as education attainment (years of schooling) in the past decades (1970-2007) for selected countries. These countries are divided into 4 groups, namely, high income, middle-income-high, middle-income-low and low income (trapped) countries.[2] It can be noted that both high income and middle-income-high countries have similar length of life expectancy and years of schooling during 1990-2005.[3] For middle-income-low countries, the life expectancy and the years of schooling in 2000 are slightly shorter than those in high and middle-income-high countries.[4] However, trapped countries are characterized by low income level, low growth rate, fewer years of schooling and much shorter life expectancy. Noticing this, the World Health Organization (WHO) and the United Nations (UN) have made efforts to improve the health conditions and education attainment in poor countries in the past 40 years to help poor countries and to reduce the inequality across nations. Indeed, the health conditions and the education attainment for countries trapped in poverty have improved significantly since 1970, as shown in Table 1.[5]

As we have mentioned, we are interested in understanding why some originally poor (now middle-

[1] Hereafter we will use education and knowledge interchangeably.

[2] The categorizing criteria are: countries with relative incomes of 0.65 or higher in year 2000 are grouped as high income countries; countries with relative incomes of 0.2 - 0.65 in 2000 are classified as middle-income-high countries; countries with relative incomes of 0.1 - 0.2 in 2000, or with an average GDP per capita growth rate of 2% or higher during 1970-2007 are classified as middle-income-low countries; and finally, countries with relative incomes of lower than 0.1, or with an average GDP per capita growth rate lower than 2% during 1970-2007 are classified as trapped countries.

[3] Brazil is an exception, since Brazil is between middle-income-high and middle-income-low.

[4] India is an exception. India is categorized as middle-income-low country because of her slow but steady growing in GDP per capita relative to the U.S.

[5] In 1970, the life expectancy in Malawi was less than 40 years while other trapped countries (Bangladesh, Cambodia, Ghana, Kenya and Zambia) also had a life expectancy of about 40-45 years. As time progressed to year 2000-2008, life expectancy in the trapped countries (except Zambia) had increased to 53-65 years. Mortality rate under 5 was

income-low) countries could take off (Egypt, China and India) while other poor countries remain trapped in poverty. Examining Table 1 more carefully, we find that there is a notable difference between the initially poor but later take-off countries and the trapped countries: *with similar years of schooling, the trapped countries tend to have lower life expectancy.* For example, in 1970, Egypt had 1.31 years of schooling and a life expectancy of 45.93 years. Bangladesh and Malawi had similarly low but slightly longer years of schooling than Egypt, but both countries had a much lower life expectancy compared to Egypt. Now let us compare China to the trapped countries. In 1980, years of schooling and life expectancy in China were 4.75 and 65.97 years respectively. However, with similar or much longer years of schooling, up to year 2008, none of the trapped countries have reached a life expectancy of 65.97 years. A similar case is found when we compare India to Malawi or Zambia: with comparable or shorter years of schooling, India tends to have longer life expectancy than Malawi or Zambia.

Two questions arise naturally after seeing the facts described above: (1) why could health play a role different than education? In what channel does health affect the economy? (2) How does health interact with education to lead countries into different development paths so that some countries are able to take off while others are stuck in the poverty trap? Apparently, a low life expectancy reveals poor health and public health conditions as well as inability of the people in the country to protect themselves from disease, even if the people have the knowledge to take necessary measures. Based on the observed facts, it seems to us that there exists an "optimal proportion" of health and education (or an optimal ratio between health and knowledge stock) for countries to be able to leap out from the trap. Education itself is not enough, and "imbalanced" accumulation of health and knowledge will not help much either.

Therefore, we feel the need to investigate both health and education at the same time and to see how health and education interact to affect the developmental outcomes of a country. To do this, it is crucial to decompose intangible human capital into two components: conventional knowledge capital, which is related to an individual's education and mental capability, and health capital, which is related to an individual's physical condition. With higher health capital, people enjoy a longer lifespan as well as a better quality of life. Moreover, the quality of labor improves when workers are healthier. Both channels imply that an individual with higher health capital is more likely to invest in knowledge capital because the reward from knowledge investment (education or learning) is higher due to greater productivity and greater enjoyment of the fruit of production. With higher knowledge capital, investment in health can promote health conditions and longevity more effectively. Moreover, labor productivity also increases when workers are more knowledgeable. Both channels give rise to a higher return on health investment, from productivity as well as utility

also reduced by about half during 1970 to 2000. In the same time, years of schooling in these trapped countries also increased substantially since 1970.

obtained from a better life. However, even with fair education attainment, if the environment is not favorable for health investment and the health investment for the society as a whole falls short of overcoming the adverse circumstances in health, a country could possibly remain mired in the poverty trap. This is easy to understand: when a person is sick, it is impossible for him to fully utilize his knowledge and ability to perform work well. When this happens, countries will underutilize the knowledge capital they have and will produce less output. This is exactly what we observe on trapped countries from Table 1.

More specifically, we develop a three-period overlapping-generations model with human capital decomposed into health and knowledge capital. Since parental investment in both health and education are crucial for children to have a better position for a better life in later stages,[6] we assume that all investments in health and knowledge are made by parents. Parents have limited altruism toward their offspring: they only care about their children's human capital. Health investment takes resources and determines the quality of life and the level of health capital of children, while education investment takes parental time[7] and determines the level of knowledge capital. Health and knowledge capital together compose children's human capital, which determines the productivity of labor of children.

We find that it is possible to have multiple equilibria: a high stable equilibrium, featured by positive investments in health and education, a middle unstable equilibrium, and a degenerate equilibrium (the poverty trap), characterized by no health investment. We then calibrate the high equilibrium to match the U.S. economy and perform comparative statics numerically. We find that as parents become more altruistic, they invest more in knowledge compared to health, and knowledge capital increases more relative to health capital as a result. Pure improvement in the quality of life when old yields the opposite effect. We also examine the cases in which the poverty trap is more likely to emerge. In addition, we show that by improving public health conditions and eliminating barriers to investment in health, the government is able to prevent the country from falling into the poverty trap. Since our ultimate goal is to understand the factors distinguishing middle-income-low countries and trapped countries as well as the causes leading to different developing paths, we calibrate our model to fit the middle-income-low countries and the trapped countries. We will examine the causes of poverty of each country under investigation, namely, whether poverty is caused by deficiency in health or education investment, or the intrinsic factors such as environmental or institutional barriers of the country. We will also discuss the forces leading the middle-income-countries to take off. Finally, we will investigate whether a country could be pushed out of the poverty trap with a fiscal policy subsidizing health and education investment.

[6] There are studies to investigate the importance of early childhood development. See See Leibowitz (1977), and the recent works on cognitive and uncognitive works by Cunha and Heckman (2006).

[7] Despite technology progress in home production and childcare, the parental care time per child has increased. See Hill and Stafford (1980) and Bryant and Zick (1996).

Related literature

This paper is related to the literature on health, education and development. The research on health was pioneered by Grossman (1972), and since then there has been a small, but growing literature devoted to studying the demand for health within an individual optimization framework in which better health promotes longevity.[8] The investigation on how the improvement in health (mortality rate reduction and life expectancy increases) leads to lower fertility and higher level of education (the quality-quantity tradeoff) includes Kalemli-Ozcan (2002), Becker et al (2005), and Soares (2005). Jayachandran and Lleras-Muney (2009) provide evidence from Sri Lanka, showing that increases in life expectancy resulting from the decline in maternal mortality rates did lead to higher education for girls in Sri Lanka. Our work is more related to Morand (2004), Chakraborty (2004), Manuelli and Seshadri (2009) and Manuelli et al (2010). Morand (2004) uses a neoclassical growth framework in which health and consumption are complements in utility to explore the relationship between health and epidemiological transition. Chakraborty (2004) considers health capital augmented by public investment; Manuelli and Seshadri (2009) and Manuelli et al (2010) consider private health and education investment. Manuelli and Seshadri (2009) focuses on explaining the differences in the fertility rates across countries, and Manuelli et al (2010) examines the timing of retirement. However, the focuses of these two papers are different from the focus of this current paper. Empirical works investigating the contribution of health to economic development include Acemoglu and Johnson (2007), Weil (2007), Ashraf et al (2008), and Lorentzen et al (2008) and others. However, researchers have not reached a consensus on the magnitude of the effects of health improvement on economic growth and development yet.

2 The model

Since intergenerational altruism and transmission are essential to the issues considered in the present paper, we conduct our analysis within the overlapping-generations framework. In addition to modeling the childhood, we study both the young adulthood and the old adulthood after retirement, where the incorporation of the latter enables us to understand the implication of health investment on the quality of life of the elderly. To abstract from marriage issues, we assume there is only single sex (female) in an economy with a fixed population (normalized to one). Agents have perfect foresight, who are completely identical, both *ex ante* and *ex post*. There are two sectors of economic activities: a goods sector and a human capital sector. Human capital constitutes of two separate components: health capital (related to an individual's physical condition) and knowledge capital (related to an individual's mental capability). Aside from an institutional sunk cost (related to

[8] See Ehrlich and Chuma (1990), Grossman (1998), Ehrlich and Yin (2005), Sanso and Aisa (2006), Benitez-Silva and Ni (2008).

public health environments and investment barriers), goods produced can be used for consumption as well as physical capital and health investments.

2.1 Production

Index time by t. The single good in the economy is produced with the beginning-of-period stock of physical capital (K_t) and effective labor (L_t). To a young adult who was born in $t-1$, her human capital (measured at the beginning of the period) is given by H_t. Denote ℓ_t^d as the labor demand for a young adult with human capital H_t and hence the effective labor is measured by $L_t = H_t \ell_t^d$. Goods production takes a Cobb-Douglas form:

$$Y_t = BK_t^{\eta} L_t^{1-\eta}, \tag{1}$$

where $B > 0$ is the technology scaling factor and $\eta \in (0, 1)$ denotes the capital income share. For the sake of simplicity, it is assumed that physical capital fully depreciates over a period of individual's lifetime.

The first component of human capital – the key ingredient of our model economy – is health capital (denoted P_t). It measures the general health condition of a person (therefore embodied), both physically and psychologically. The second component of human capital – as conventionally modeled – is knowledge capital (denoted M_t). It measures all embodied skills related to the ability of performing the job in producing the single good. In reality, either type of capital may be enhanced by an agent's own effort or by her parent's investment. Given that early childhood development is found essential in both health and skills as stressed by more recent studies pioneered by Heckman (2007) and further elaborated by Cuhan and Heckman (2007) and Manuelli and Seshadri (2009), we highlight the role played by the parent in enhancing a child's health and knowledge capital. To avoid unnecessary modeling complexity, we shall abstract from the consideration of child's own effort devoted to accumulating health and knowledge capital.

For simplicity, we assume that the health and knowledge shares of human capital are constant. Thus, the stock of human capital in period t is given by:

$$H_t = AP_t^{\alpha} M_t^{1-\alpha}, \tag{2}$$

where $A > 0$ is human capital scaling factor and $\alpha \in (0, 1)$ is the share of health capital. The representative woman born in period $t-1$ decides on investment in her child's health (h_t) and time devoted to educating her child (e_t) in period t, which determines the health and knowledge capital for her child in period $t+1$. In addition to her investment in her child's health, the representative woman's own health may also have a direct effect on her child's health capital due to genetic influences. Thus, the accumulation of the health capital of her child (born in period t) is specified as follows:

$$P_{t+1} = \xi h_t^{\varepsilon} P_t^{1-\varepsilon} + \phi P_t, \tag{3}$$

where $\xi > 0$ measures the productivity of parental investment in child's health with the share of parental investment given by $\varepsilon \in (0,1)$, and $\phi \in (0,1)$ indicates the strength of the direct parental influence on child's health capital. Similarly, besides parental time investment in child's education, a child's knowledge capital can also be enhanced by her innate ability directly related to her parent's knowledge capital:

$$M_{t+1} = \mu (e_t + \psi) M_t, \tag{4}$$

where $\mu > 0$ is knowledge scaling factor and $\psi > 0$ measures the strength of intergenerational transmission of the innate ability.

2.2 Household

Each agent lives for three periods: childhood, young adulthood and retirement. A woman in her childhood is entirely passive. In her young adulthood, she gives birth to one child at the beginning of this period and forms health and knowledge capital based on her parent's investment and time effort. As a young adult, she is endowed with one unit of time, which can be devoted to working (ℓ_t) or educating her child (e_t). That is, her time constraint in her young adulthood is given by,

$$\ell_t + e_t = 1. \tag{5}$$

In addition to health investment on her child (h_t), her wage earning ($w_t \ell_t$) can be used for consumption during her young adulthood (c_t^y) or savings for her old age (s_t):

$$c_t^y + s_t + h_t = w_t H_t \ell_t.$$

As she steps into her old adulthood, she retires and consumes what she has saved by leaving no pecuniary bequest to her child:

$$c_{t+1}^o = R_{t+1} s_t, \tag{6}$$

where R_{t+1} measures the gross interest rate prevailed in period $t+1$. By using (5), the two periodic budget constraints can be combined to yield the intertemporal budget constraint below:

$$c_t^y + \frac{c_{t+1}^o}{R_{t+1}} + h_t = w_t H_t (1 - e_t). \tag{7}$$

A representative woman born in period $t-1$ derives utility from consumption, both in young adulthood and retirement, denoted by c_t^y and c_{t+1}^o respectively. The utility derived from old age consumption is discounted not only by the time preference rate $\rho > 0$ but also by a factor $\pi(h_{t-1}) \in (0,1)$ that is based on the health condition given in her childhood:

$$\pi(h) = \begin{cases} \pi_H & if \quad h \geq h_c \\ \pi_L & if \quad h < h_c \end{cases}, \ \pi_H > \pi_L .$$

Thus, $\pi(\cdot)$ captures the quality of life when old and $h_c > 0$ is the threshold of health investment – when parental investment in child's health exceeds h_c, the child will enjoy a better quality of life in her old adulthood. In addition to her own consumption, the representative woman is altruistic, valuing her child's embodied human capital measured by H_{t+1} with an intergenerational discounting factor $\gamma \in (0, 1)$ (which will be referred to as the altruistic factor hereafter). Assuming the preferences take a log-linear form, we can now specify the representative woman's optimization problem as follows:

$$\begin{aligned} V_{t-1} = \quad & \max_{c_t^y, c_t^o, h_t, e_t} \ln c_t^y + \tfrac{\pi(h_{t-1})}{1+\rho} \ln c_{t+1}^o + \gamma \ln H_{t+1} \\ & \text{s.t.} \quad (2),\ (3),\ (4) \text{ and } (7) \end{aligned} \tag{P}$$

where, needless to say, both the effective wage rate w and the (gross) interest rate R are taken as given.

2.3 Institutional Barriers

As we have discussed, even if people have the knowledge to take actions to fight against diseases, they could fail to do so if the environment is adverse to their efforts. For example, washing hands before preparing food for the family is the key to prevent children under 5 from having diarrhea (which is the most common source of morbidity in developing countries, and may result in death if the patient is not treated well). Yet the unwillingness of the family members to accommodate with this "new measure" due to customs or beliefs, or the lack of clean water, would frustrate the primary carers. To overcome such problem, it calls for large investment in the water and sanitation system, as well as the comprehensive instillation of correct health measures to eradicate "incorrect" beliefs and customs. All these kinds of "solutions" take large initial investment as well as the sequential maintenance cost. However, for trapped countries, their efforts and health investment would simply not suffice to overcome the unfavorable environment.[9]

We use an institutional sunk cost to measure investment barriers attributed to resources required to assure satisfactory public health environments as described above as well as corruption or unstable political situations. The magnitude of this institutional sunk cost is assumed to be proportional to the average health investment in the economy, $\delta\bar{h}$, where $\delta > 0$ measures the severity of the instiitutional barriers (i.e., higher δ indicates greater waste of resources). Thus, the goods market clearing condition in period t can be written as:

$$c_t^o + c_t^y + h_t + \delta\bar{h}_t + K_{t+1} = Y_t, \tag{8}$$

[9] Trapped countries usually have much lower health expenditure per capita than other countries. For example, in year 2003, the health expenditure per capita in Bangladesh, Kenya, Malawi and Zambia are \$10.3, \$19.7, \$17.4 and \$25.7 respectively, while the health expenditure per capita in China, Egypt, Colombia and Turkey are \$59.4, \$60.6, \$145.7 and \$258.8. Source: World Bank dataset, unit is current U.S. dollars.

where loanable funds market clearing simply implies:

$$K_{t+1} = s_t - \delta \bar{h}_t. \tag{9}$$

The labor market clearing condition is

$$\ell_t^d = \ell_t. \tag{10}$$

3 Optimization and Equilibrium

In this section, we will focus only on the benchmark case with positive investment and time devoted to child's health and education, respectively. We will relegate the discussion on the emergence of the poverty trap to Section 4 after we have calibrated the model to fit the U.S. economy. We will then illustrate in Section 5 how the government may act to prevent the economy from falling into a poverty trap.

3.1 Optimization

We solve generation-t representative woman's problem by deriving the first-order conditions (FOCs). Substituting c_{t+1}^o away using (7), we obtain the following FOCs with respect to c_t^y, h_t and e_t, respectively,

$$\frac{1}{c_t^y} = \frac{\pi (h_{t-1})}{(1+\rho)\left[w_t H_t (1-e_t) - c_t^y - h_t\right]}, \tag{11}$$

$$\frac{\alpha\gamma\xi\varepsilon h_t^{\varepsilon-1} P_t^{1-\varepsilon}}{\xi h_t^{\varepsilon} P_t^{1-\varepsilon} + \phi P_t} = \frac{\pi (h_{t-1})}{(1+\rho)\left[w_t H_t (1-e_t) - c_t^y - h_t\right]}, \tag{12}$$

$$\frac{\gamma (1-\alpha)}{e_t + \psi} = \frac{\pi (h_{t-1}) w_t H_t}{(1+\rho)\left[w_t H_t (1-e_t) - c_t^y - h_t\right]}. \tag{13}$$

While (11) equates the marginal utility and the marginal cost of consumption when young; (13) equates the marginal utility and cost of educating children. In (12), the marginal benefit of spending on health and the marginal cost of health spending, measured by the margianl utility of consumption when young, are equalized.

Standard factor demand conditions are given as follows:

$$B\eta (K_t/L_t)^{-(1-\eta)} = R_t, \tag{14}$$

$$B(1-\eta)(K_t/L_t)^{\eta} = w_t. \tag{15}$$

3.2 Equilibrium

Since the economy exhibits perpetual growth, we are focusing on the balanced growth path equilibrium.

Definition 1: *A **dynamic general equilibirum** in this economy is a tuple of quantities* $\{H_t, P_t, M_t, c_t^o, c_t^y, s_t, h_t, K_t, Y_t\}_{t=0}^{\infty}$ *together with a pair of prices* $\{w_t, R_t\}_{t=0}^{\infty}$ such that:

(i) *each woman optimizes, i.e.,* (2), (3), (4), (5), (6), (7), (11); (13), *and* (12) *all hold;*

(ii) *production efficiency is met, i.e.,* (1), (14) *and* (15) *hold;*

(iii) *good, loanable funds and labor markets all clear, i.e.,* (8), (9) *and* (10) *are met.*

It is not difficult to verify that one of (6), (7), (8), and (9) is redundant, i.e., the Walras' law holds true.

Definition 2: *A **balanced growth equilibirum** (BGP) is a dynamic general equilibirum along which all perpetually growing variables* $\{H_t, P_t, M_t, c_t^o, c_t^y, s_t, h_t, K_t, Y_t\}_{t=0}^{\infty}$ *grow at constant rates and all other endogenous variables are constant. A balanced growth equilibirum is called nondegenerate if these constant growth rates are strictly positive.*

It is straightforward to show that along a nondegenerate BGP, all perpetually growing variables grow at the common rate $g > 0$.

Along a nondegenerate BGP, h_t grows at a positive rate g and hence $h_t > h_c$ must hold $\forall t \geq t_c$ where $\pi(h_t) = \pi_H$. We regard this nondegenerate BGP as the benchmark and transform all perpetually growing quantities into stationary ratios by defining $z \equiv \frac{c_t^y}{H_t}$, $x = \frac{h_t}{H_t}$ and $k = \frac{K_t}{L_t} = \frac{K_t}{H_t \ell_t}$. From (4), $1 + g = \mu(e + \psi)$, which can then be substituted into (3) to derive

$$1 + g = \phi + \xi \left(\frac{H}{P} x\right)^{\varepsilon} \tag{16}$$

That is, both a higher human-physical capital ratio and a higher health-human capital ratio enhance the long-run economic growth. The above expression, together with (2), yields the health to knowledge capital ratio:

$$\frac{P}{M} = \left\{ \left[\frac{\mu(e+\psi) - \phi}{\xi}\right]^{\frac{1}{\varepsilon}} \frac{1}{Ax} \right\}^{\frac{-1}{1-\alpha}}, \tag{17}$$

which depends only on e and x. The effective wage rate and interest rate are given by $w(k) = (1-\eta)Bk^{\eta}$ and $R(k) = \eta B k^{\eta - 1}$, which are functions of the effective capital labor ratio k alone.

We are left with four equations with four unknowns of z, x, k, and e. The system can be further reduced to 3×3 in (e, x, k) where e can be determined in a recursive manner (see the Appendix). This enables us to characterize the BGP simply in (x, k) space. Specifically, we define transformed wage and consumption variables as follows:

$$\tilde{w}(e) = \frac{1}{1-e}\left[1 + \frac{1+\rho+\pi_H}{1+\rho}\tilde{z}(e)\right], \tag{18}$$

$$\tilde{z}(e) = \frac{1}{\alpha\gamma\varepsilon}\left[1 + \frac{\phi}{\mu(e+\psi) - \phi}\right]. \tag{19}$$

Since a higher time invested in child's education implies less time devoted to working, it is expected that effective consumption z and hence the transformed consumption variable ($\tilde{z} = \frac{z}{x}$) is decreasing in e. Thus, in response to an increase in time investment in child's education, a parent consumes less when old and requires less wage earnings to support the reduced level consumption. This is shown in (18) where $\tilde{z}$ enters $\tilde{w}(e)$ positively and hence e can generate a negative effect on the transformed wage ($\tilde{w} = \frac{w}{x}$). We will refer to this as the *intergenerational substitution effect* between the parent's own consumption and her time devtoed to educating her child. Yet, by investing more time in her child's education, the parent will have less work time. By diminishing returns, this translates into a higher marginal product of labor and hence a positive effect on the transformed wage. We will refer to this latter effect as the *diminishing returns effect*. As a result of the two opposing effects, the net impact of e on $\tilde{w}$ is ambiguous. Then, the reduced system is expressed as:

$$MB(e) = \frac{1}{\tilde{z}(e)}, \tag{20}$$

$$x = \frac{(1-\eta)Bk^{\eta}}{\tilde{w}(e)}, \tag{21}$$

$$x = \frac{k\mu(e+\psi)}{\frac{1}{1-\eta}\tilde{w}(e) - \frac{(1+\delta)+\left[1+\frac{\pi_H \eta B k^{\eta-1}}{(1+\rho)\mu(e+\psi)}\right]\tilde{z}(e)}{1-e}}. \tag{22}$$

where $MB_e \equiv \frac{\gamma(1-\alpha)(1-e)}{e+\psi} - \left(1+\frac{\pi_H}{1+\rho}\right)$ is the marginal benefit of parental time investment on child's education.

More specifically, (20) equates the marginal benefit of parental time investment in child's education MB_e with its marginal opportunity cost measured by the marginal benefit of parental pecuniary investment in child's health, denoted by MC_e.[10] Thus, this expression captures the parent's trade-off between two forms of investment in child. It can be easily shown that while the MB_e locus is downward sloping, starting from $\frac{\gamma(1-\alpha)}{\psi} - \left(1+\frac{\pi_H}{1+\rho}\right)$ with a horizontal intercept $e_{\max} \equiv \frac{\gamma(1-\alpha)-\psi\left(1+\frac{\pi_H}{1+\rho}\right)}{\gamma(1-\alpha)+1+\frac{\pi_H}{1+\rho}} < 1$, the MC_e locus is upward sloping with a horizontal intercept $e_{\min} \equiv \frac{\phi}{\mu} - \psi$. We impose a regularity condition: $\psi < \frac{\phi}{\mu} < e_{\max} + \psi$ under which (20) determines the unique parental investment in child's education on the BGP $e \in (e_{\min}, e_{\max})$. Figure 1 provides a graphical illustration of the the determination of e.

Next we turn to examine the equations determining the BGP values of health-human capital ratio (x) and the effective capital-labor ratio (k), which are jointly pinned down by (21) and (22). For illustrative purposes, we shall refer to (21) as the *intergenerational transfer* (IT) locus and (22) as the *market equilibrium* (ME) locus. Intuitively, the IT locus, derived from the FOCs with respect to c_t^y, h_t and e_t, captures the intergenerational transfer in the form of health investment. The ME locus, on the other hand, is derived from the material balance condition. We first examine how $\tilde{w}$ affect the

[10] This can be seen easily by dividing (12) over (13) with the use of (3) and (4).

IT and the ME loci. Carefully examining the IT locus reveals that for given x, if the transformed wage $\tilde{w}$ increases, $(1-e)$ (labor time) will decrease, and by factor subsitution, k must increase. This implies that for given x the IT locus will shift to the right when $\tilde{w}$ increases. Now we examine the ME locus. The $\frac{1}{1-\eta}\tilde{w}$ in the denominator is an re-expression of $\frac{y}{x} = \frac{Y/H\ell}{x}$ $\left(y = Bk^{\eta} = \frac{\tilde{w}x}{1-\eta}\right)$, and the $\left[1 + \frac{\pi_H \eta B k^{\eta-1}}{(1+\rho)\mu(e+\psi)}\right]\tilde{z}(e)$ in the denominator is the consumption expenditures for the young and the old generations. For given x, if the consumption expenditures remain unchanged, as the transformed wage $\tilde{w}$ increases, $\frac{y}{x}$ must go up, and $\frac{k}{x}$ in the next period must rise as well. Therefore, for given x, the ME locus will shift to the right as the transformed wage $\tilde{w}$ increases. Furthermore, it is easily seen from (21) and (22) that, for any given BGP value of e, the IT locus is strictly increasing and strictly concave in (k, x) space whereas the ME locus is U-shaped (strictly convex).

Figure 2 depicts the IT and ME loci. Notably, it is possible that the IT and ME loci may not intersect with each other – in this case, there is no interior solution ($x = 0$ and hence health invesment $h_t = 0$) and the economy is said to be in the poverty trap. In the benchmark case, the IT and ME loci intersect twice at E_1 and E_2.[11] The remaining task is the selection of equilibrium. It is noted that the dynamical system can be best reduced to 6×6 in $(e_{t-1}, e_t, e_{t+1}, x_{t-1}, x_t, x_{t+1})$, which is too complicated to verify the property of stability. Alternatively, we adopt the notion of stability proposed by Samuelson (1947), namely, the Samuelson corresponding principle. This is done by perturbing the BGP, say, with an increase in δ (the severity of the instiitutional barriers). With such a perturbation, the IT locus remains unchanged, but the ME locus shifts upward. Given H_t, larger barriers imply less resources available for capital accumulation and health investment. Thus, it is expected that x and k must fall in response. From Figure 2, only E_1 is consistent with this prediction, which is our selection of the benchmark equilibrium.

Additionally, Figure 2 shows that if the institutional factor δ is too big (when $\delta > \delta_c$), the economy will end up staying in the equilibrium E_0, which is the corner solution with $x = 0$, capturing the poverty trap. We shall perform comparative statics and propose policy recommendation based upon the stable equilibrium E_1, to which we now turn.

3.3 Comparative Statics

In order to characterize the stable equilibrium E_1, we examine analytically the comparative-static properties concerning changes in structural preference (γ and π_H) and technology (ϕ, μ, ψ, ε and α) parameters on three key endogenous variables (e, x and k). As discussed in the previous subsection, we can elaborate on the responses of parental time investment in child's education (e) recursively and then illustrate the responses of the health-human capital and effective capital-labor ratios (x and k) diagrammatically.

[11] It can be shown that the IT locus is strictly concave whereas the ME locus is strictly convex. Thus, the two loci cannot intersect with each other more than twice.

Let us focus on conducting comparative-static analysis on parental time investment in child's education using Figure 1. Consider to begin the case when parent becomes more altruistic (γ increases). In response, both the marginal utilities of investing in child's education and health (MB_e and MC_e, respectively) must go up, implying upward shifts in both MB_e and MC_e loci. It can be shown mathematically that the shift in the MB_e locus is always larger than that in the MC_e locus. As a result, the BGP value of e must rise unambiguously. Consider next an increase in parent's quality of life when old (higher π_H), which raises the marginal utility of parent's own consumption when old and thus reduces her incentive to invest in child's education (lower MB_e). Hence, the MB_e locus shifts downward while the MC_e locus remains unchanged, leading to a lower BGP value of e. Turning now to the strength of the intergenerational transmission of health (ϕ) and the knowledge scaling factor (μ), we can see that either an increase in ϕ or a decrease in μ raises the marginal utility of investing in child's health and hence shifts the MC_e locus downward. Thus, in response to either change, the BGP value of e increases. At last, we examine what happens if the strength of intergenerational transmission of the innate ability (ψ) or the share of health capital in human capital (α) changes. An increase in either ψ or α implies it is less value in child's education and more valuable in child's health, thereby shifting the MB_e locus downward and the MC_e locus upward. These shifts reinforce each other, resulting in a lower BGP value of e unambigously. The responses of e to these structural parameter shifts are summarized in Table 2.

We now proceed to perform the comparative statics with regard to x and k using the IT and the ME loci depicted in Figure 2. The structural parameters may affect the IT and the ME loci directly as well as influence both loci indirectly through e. We first discuss how the change in e affect the IT and the ME loci.

Notably, if the intergenerational substitution effect of parental investment in child's education e on the transformed wage dominates the diminishing returns effect, it is expected that $\tilde{w}(e)$ is decreasing in e. In this case, (21) implies that an increase in e will shift the IT locus to the left. Moreover, from (22), the negative dependence of $\tilde{w}$ on e also generates a force to shift the ME locus to the left. In addition to these forces, e also affects the ME locus through other channels. On the one hand, e shifts the ME to the left because of the *direct knowledge capital accumulation effect* (via $e + \psi$ in the numerator) and the diminishing returns effect (via $1 - e$ in the denominator). On the other hand, it shifts the ME locus to the right as a result of resource reallocation away from parent's consumption both when young ($\tilde{z}(e)$ in the denominator) and when old ($e + \psi$ in the denominator). If these negative *resource reallocation effects* on consumption are not too strong, then one would expect a leftward shift in the ME locus in response to an increase in e. On the contrary, if the diminishing returns effect on the transformed wage dominates the intergenerational substitution effect, it is expected that $\tilde{w}(e)$ is increasing in e. In this case, (21) implies that an increase in e will shift the IT locus to the right. From (22), the positive dependency of $\tilde{w}$ on e

together with the resource reallocation effects generate a force to shift the ME locus to the right. If the leftward force generated from direct knowledge capital accumulation effect and the diminishing returns effect (via $1-e$ in the denominator) is not too strong, one will expect that an increase in e will shift the ME locus to the right.

In summary, with a relatively weak diminishing returns effect on the transformed wage and resource reallocation effects on consumption, an increase in e is expected to shift both the IT and the ME loci leftward. Contrarily, if the diminishing returns effect on the transformed wage is relatively large, the IT and the ME loci will both shift rightward in response to a higher e Even one knows the direction of shifts in the IT and the ME loci, whether the BGP values of x and k will rise or fall still depends on the magnitudes of the shifts in the two loci. In the case where $\tilde{w}$ depends negatively on e, when both the shifts in the IT and the ME loci are moderate in response to an increase in e, the health-human capital ratio will rise but the effective capital-labor ratio will fall. In the case where $\tilde{w}$ depends positively on e, if the shifts in both loci are moderate in response of a higher e, x will decrease while the k will increase. If the shift in the IT locus is relatively larger than the ME locus as e increases, both x and k will fall. Finally, if the shift in the ME locus is relatively larger than the IT locus in response to an increase in e, both x and k will rise.

Now we are ready to examine how the structural parameters affect the IT and the ME loci. For parameters γ, ϕ, ε and α, they affect the IT and the ME loci indirectly through their effects on the BGP value of e, while π_H, ψ and μ have direct effects on the ME locus.From (22), π_H directly influences the ME locus through the resource reallocation affect. Given x, a higher π_H means a higher consumption expenditure, and k will decrease as a result, implying that the ME locus will shift to the left in response to a higher π_H. For μ and ψ, both of them directly generate a leftward force on the ME locus through the direct knowledge capital accumulation effect (the $\mu(e+\psi)$ in the numerator) and a rightward force via the resource reallocation effect (the $\mu(e+\psi)$ in the denominator). Hence, the net impacts of μ and ψ on the ME locus are in general ambiguous. In the follwing paragraphs, we discuss the impacts of the changes in these sturctural parameters on the BGP equilibrium, and disentangle the various forces influencing the IT and the ME loci directly or indirectly through e.

Dweiling upon the stable benchmark equilibrium, Figure 3 plots how the IT and the ME loci shift in response to changes in the parameters. Consider first, the changes in the parameters γ and π_H on the preference side. An increase in γ or a decrease in π_H raises the BGP value of e. Since the changes are on the prefernce side, one would expect that the intergenerational substitution effect would dominate the diminishing returns effect, and $\tilde{w}$ depends negatively on e. The IT locus shifts to the left as a result. For the ME locus, the negative dependency of $\tilde{w}$ on e and the knowledge capital accumulation effect bring the ME locus to the left, while the resource reallocation affect brings the ME locus to the right. If the leftward force is bigger, the ME locus will shift left in response to an increase in γ. Moreover, as long as the shifts in both loci are moderate, one would expect a higher BGP health-human capital ratio accompanied by a lower effective capital-labor ratio. Different from the case of γ, a decrease in π_H generates a direct effect on the ME locus through the resource reallocation effect, which shifts the ME locus to the right. As long as this direct resource reallocation effect is small, a decrease in π_H yields a similar outcome to that of an increase in γ.

Consider next, on the human capital technology side, an inrease in ϕ or a decrease in ε and α. The BGP value of e is higher as a result. One would expect that the changes in the technology would have direct impacts on the level of human capital, and hence the diminishing returns effect dominates the intergenerational substitution effect since. As a result, $\tilde{w}$ depends positively on e, and the IT locus shifts to the right in response to a higher e. For the ME locus, the positive dependency of $\tilde{w}$ on e, and the resource reallocation effect generate rightward forces, while the knowledge capital accumulation effect inserts a leftward force on the ME locus. As long as the rightward force is bigger, one would expect that the ME locus shift to the right as ϕ increases or ε and α decrease. Furthermore, if knowledge capital accumulation effect is sizable, the IT locus will shift more relatively to the ME locus, resulting in lower health-human capital ratio and effective capital-labor ratio.

Finally, we consider the impacts of increases in ψ and μ on the BGP equilibrium. When ψ or μ increases, the BGP value of e is lower. Since ψ and μ are parameters of technology in the knowledge sector, we expect that the diminishing returns effect dominates the intergenerational effect, and the IT locus shifts to the left when e decreases. Indirectly through the decrease in e, the resource reallocation effect generates a force to shift the ME locus to the left, and the knowledge capital accumulation effect pushes the ME locus to the right. Moreover, ψ and μ directly generate a resource reallocation effect (through $\mu(e+\psi)$ in the denominator), pushing the ME locus to the right, and a knowledge capital accumulation effect (through $\mu(e+\psi)$ on the numerator), pushing the ME locus to the left. Since the direct and the indirect resource reallocation effects and the knowledge capital accumulation effects are opposing to each other, if the net effect of these effects is to shift the ME locus to the left, or if the net effect is outweighed by the diminishing returns effect, one will see that the ME locus shifts leftward as ψ or μ increases. The BGP value of k decreases as a result, but the BGP value of x can either increase or decrease, depending on the relative magnitudes in the shifts of the IT and the ME loci.

We shall perform the comparative statics numerically in the next section in the calibrated benchmark economy, not only to verify our discussions above but also to generate unambiguous comparative-static outcomes quantitatively.

4 Numerical Analysis

In this section, we calibrate the benchmark model to match the U.S. economy. Numerical comparative statics as well as sensitivity analysis to check the robustness of benchmark parametrization are then performed. We illustrate how an increase in the institutional sunk cost could cause an economy to fall into the poverty trap, even if the economy is endowed with the same preferences and technologies as the U.S. We also show that government can alleviate institutional barriers to improve health environment via public health investments, and the country is prevented from falling into the poverty trap as a result.

4.1 Calibration

We want to calibrate our model such that the BGP implications of the parameters match the observations for the US economy. The model has 13 parameters: (i) preference parameters – time preference (ρ), quality of life when old (π_H) and altruistic factor (γ); (ii) goods production technology parameters – goods technology scaling factor (B) and income share of physical capital (η); (iii) human capital technology parameters – human capital scaling factor (A) and share of health capital (α); (iv) health capital technology parameters – health capital technology scaling factor (ξ), share of parental health investment (ε) and innate health (ϕ); (v) knowledge capital technology parameters – knowledge capital scaling factor (μ) and innate knowledge (ψ); and (iv) the institiution parameter – the severity of the institiutional barriers (δ). In the benchmark calibration, we normalize $\delta = 0$.

To begin with our calibration, first we set one model period equal to 25 years. Therefore, the representative woman lives for 3 periods – 75 years in total.[12] Since we have two sectors, it is without loss of generality to normalize one of the technology scaling parameters, say, the human capital scaling factor to one, i.e., $A = 1$. We choose $\pi_H = 0.8$, implying a 20% discount over the old age consumption which seems reasonable. The 25-year time preference rate is set at $\rho = 1.0938$, which corresponds to an annual time preference rate of 3%, as commonly chosen in the macroeconomics literature. Based on the NIPA data over the period $1960 - 2005$, we set the physical capital share of income as $\eta = 0.33$ and the 25-year economic growth rate (of per capita real GDP) as $g = 64.06\%$ (yielding a 2% annual economic growth rate).

To calibrate the effective capital labor ratio ($k = \frac{K}{H\ell}$), we must know the physical to human capital ratio ($\frac{K}{H}$) as well as the fraction of time people devote to work (ℓ). Based on the estimates by Kendrick (1976), the stock of human capital is about the same level as physical capital for the U.S., and hence we set the physical capital to human capital ratio equal to one, i.e., $\frac{K}{H} = 1$. Since

[12]The life expectancy at birth in the US has reached 75 years in 1989, 76 years in 1996 and 77 years in 2000. Source: U.S. Census Bureau, Statistical Abstract of the United States: 2003.

parental time investment in child's education is crucial in our paper, we cannot simply compute ℓ from labor-market work hours data. Instead, we use the $2003-2008$ American Time Use Survey: the average hours per day for people who engaged in the activities related to children's education and health are 2.1766 hours and the average sleeping hours is 8.59 hours per day. Therefore, the fraction of time parents devoted to children's education is $e = 14.0\%$ and the fraction of time to work is $\ell = 86.0\%$. This together with $\frac{K}{H} = 1$ yields an effective capital labor ratio of $k = 1.1628$.

We now turn to pinning down the technology parameters health and knowledge capital accumulation. Without loss of generality, we set the health to knowledge capital ratio to one ($\frac{P}{M} = 1$). In the benchmark case, we assume that health and knowledge contribute equally to the formation of human capital, so α is set to be 0.5. From (2) and the normalization of A, we obatin $\frac{H}{P} = 1$ and hence, by the common growth property of the BGP, $x = \frac{h}{H} = \frac{h}{P}$. In spite of lacking precise measurement, it is reasonable to assume that both the innate ability in knowledge and the direct parental influence on child's health capital account for 20% of the total formation of the child's knowledge capital and health capital, respectively. That is, $\frac{\psi}{e+\psi} = 0.2$ and $\frac{\phi}{\xi x^{\varepsilon}+\phi} = 0.2$. The first expression, in conjunction with the value of e, immediately yields $\psi = 0.035$, which can then be used with (4) and the value of g to get $\mu = \frac{1+g}{e+\psi} = 9.375$. Using the second expression and (16), we can calculate: $\phi = 0.2 \cdot (\xi x^{\varepsilon} + \phi) = 0.2 \cdot (1 + g) = 0.3281$.

Next, recall that the transformed consumption, $\tilde{z}(e) = \frac{z}{x}$, is equal to parent's consumption to children's health investment ratio ($\frac{c^y}{h}$). From the NIPA data on consumer expenditure over $2003-2008$, the average health expenditure to household consumption share falls slightly below 20%. It is reasonable to assume that half of the health expenditure is allocated to children and hence we set $\frac{c^y}{h} = 10$. From (20), we obtain $\gamma = 0.6032$ immediately, which together with (19) yields $\varepsilon = 0.4145$. Substituting these results into the two fundamental equations, (21) and (22), we solve jointly $B = 5.5002$ and $x = 0.2247$. The latter can then be plugged into (16) to calibrate $\xi = 2.4367$. Finally, we can obtain the calibrated value of the gross real interest rate from the BGP version of (14), $R = 1.6406$, which gives an annual real interest rate of about 2%.

The calibration rsults for the benchmark case are summarized in Table 3.

4.2 Comparitive Statics

We are ready to compute numerically the responses of (x), (k) and (e) to exogenous shifts in the following parameters: the altruistic factor (γ), the quality of life index when old (π_H), the strength of the direct parental influence on child's health (ϕ), the knowledge capital scaling factor (μ), the innate knowledge factor (ψ), the share of health capital (α), the share of parental health investment (ε), and the goods technology scaling factor (B). In addition to the three endogenous variables in the fundamental BGP equations, we are also interested in the responses of the economic growth rate (g) and four important "great ratios" – the consumption-health spending ratio $(\frac{c^y}{h})$, the health-knowledge capital ratio ($\frac{P}{M}$), the health-human capital ratio ($\frac{P}{H}$), and the knowledge-human capital ratio ($\frac{M}{H}$). The results are summarized in Table 4, where the numbers reported are in elasticities.

To begin, we shall verify the relative magnitudes of various effects discussed in the theoretical section above. In our benchmark economy, in response to an increase in parental investment in child's education (e), if the intergenerational substitution effect dominates the diminishing returns effect, the IT locus will shift to the left; if further that the resource reallocation effect is small, the ME locus will shift to the left. On the contrary, if the diminishing returns effect dominates the intergenerational substitution effect, the IT locus will shift to the left; together with a small knowledge capital accumulation effect, the ME locus will shift to the right as well in response to a higher e. Both these two cases are established in our quantitative analysis. Our numerical results suggest that the intergenerational substitution effect is stronger in response to preference shifts (γ or π_H), and the shifts in both loci are moderate such that a higher e will increase the BGP value of x and reduce the BGP value of k. When it comes to the changes in human capital technology parameters (ϕ, ε and α), our numerical results show that the diminishing returns effect is always stronger, and both the loci shift to the right in response to a higher BGP value of e. Furthermore, both the shifts are moderate. As a consequence, the BGP values of x and k both fall when e increases.

We are now ready to conduct quantitative comparative statics, where we are particularly interested in how the changes in γ, π_H, ϕ, ψ and μ affect the BGP equilibrium. First we look at the effects of changes in the altruistic factor γ. Table 4 shows that as γ increases by 1%, parental investment on child's education e increases by 0.94%, the health-human capital ratio x rises by 0.57%, and the effective capital-labor ratio k falls by 1.08%. As discussed in Section 3.3, when parents become more altruistic, the marginal benefit of parental time investment in child's education always dominates the marginal benefit of parental pecuniary investment in child's health and e increases as a result. Moreover, the increase in the marginal benefit of investing in child's health encourages parents to shift resources away from own consumption toward pecuniary investment in child's health, thereby leading to a lower consumption-health spending ratio – $\frac{c^y}{h}$ ratio falls by 1.18%. Furthermore, since parents spend more time on educating children, the rate of accumulation of the knowledge capital is higher, which translates into a higher economic growth rate – g increases by 0.76%. Due to the dominating effects on knowledge capital, the health-knowledge capital and the health-human capital ratios drop by 3.07% and 1.60%, respectively, while the knowledge-human capital ratio increases by 1.74%. In short, our numerical results indicate that more altruistic parents tend to invest more in child's knowledge than health capital.

Next we look at the effects of an increase in the quality of life when old π_H. As opposed to a higher altruistic factor, a higher quality of life when old raises a parent's marginal utility of consumption when old, encouraging a shift in her resources from investing in her child to her own enjoyment. As a result, a 1% increase in π_H suppresses e and x by 0.26% and 0.16%, respectively, while raising $\frac{c^y}{h}$ by 0.05%. Since both the IT and the ME loci shifts rightward moderately, k rises by 0.32%. Thus, parents respond to an increase in their quality of life when old by cutting more heavily on education than health investment in children, causing the BGP value of health-knowledge capital ratio to rise by 0.97%, the health-human capital ratio to increase by 0.48%, and the knowledge-human capital ratio to decrease by 0.47%. Relative to the altruistic factor, the parental decision variables as well as these great ratios are less responsive to changes in the quality of life parameter.

We now turn to examine the effects of the changes in the human capital technology parameters, where diminishing returns effect is relatively larger than the intergenerational substitution effect. First we investigate the effects of increases in the strength of the direct parental influence on child's health capital ϕ. An increase in ϕ cuts down the marginal cost of investing in child's education as well as generates a direct human capital accumulation effect. As a result, a 1% increase in ϕ boosts e and g up by 0.02% and 0.01% respectively. Since the diminishing returns effect is the dominant force, the IT and the ME loci both shift rightward, with the ME locus shifts to the right relatively smaller due to the direct knowledge capital accumulation effect. Therefore, the BGP values of x and k fall down by 0.24% and 0.02%. Moreover, with better transmission of parents' health capital, parents can consume more, resulting in a higher consumption-health spending ratio – $\frac{c^y}{h}$ increases by 0.24%. The associated direct knowledge capital accumulation effects resulting from better direct parental influence on child's health capital then cause the BGP value of health-knowledge capital ratio to rise by 0.66%, the health-human capital ratio to increase by 0.33%, and the knowledge-human capital ratio to decrease by 0.32%.

Next we look at the effects of changes in the strength of intergenerational transmission of the innate ability ψ. As ψ increases, it is more valuable to invest in child's health and less value to invest in child's education. As a result, a 1% increase in ψ brings e down by 0.21%, shifts the IT and the ME loci leftward slightly, and brings x and k down by 0.03% and 0.05%. Parents now allocate a slightly larger portion of resource to children's health – $\frac{c^y}{h}$ decreases by 0.01%. The direct knowledge capital accumulation effect associated with the increase in ψ also raises the economic growth rate g by 0.03%, lowers the BGP value of health-knowledge capital ratio and the health-human capital ratio by 0.15% and 0.08%, and increases the knowledge-human capital ratio by 0.08%.

Finally we investigate the effects of increases in the knowledge scaling factor μ. The effects of changes in μ on the BGP equlibrium is very similar to ψ: an increase in μ depresses the value of investing in child's health, the BGP value of e.decreases as a result, and the associated diminishing returns effect is larger compared to the case of an increase in ψ. An 1% increase in μ brings e down by 0.02%, and shifts both loci leftward, with a relatively larger shift in the ME locus. Hence, x and k drop by 0.25% and 1.39% respectively. The associated direct human capital accumulation effects boost the economic growth rate g up by 0.99%, lower the BGP value of health-knowledge capital ratio and the health-human capital ratio by 5.38% and 2.91%, and increase the knowledge-human capital ratio by 3.4%.

To summarize, our numerical results suggest that changes in the structural parameters affect the BGP equilibrium in the following way: if the changes are in the preference side, the diminishing returns effect is always dominated by the intergenerational substitution effect, and the assocaited resource reallocation is small. Both the IT and the ME loci shift to the left moderately in response to an increase in the BGP value of e, resulting in a higher health-human capital ratio and a lower effective capital-labor ratio. If the changes in parameters are from the human capital technology side, the diminishing returns effect is the dominant force, both the IT and the ME loci move to the right in response to a higher BGP value of e, and the values of x and k depend on the magnitudes of the shifts in the IT and the ME loci.

4.3 Sensitivity Analysis

There are neither estimates on the stock of health capital, knowledge capital, or the share of health capital on human capital formation, nor good measures on the contribution of innate ability on knowledge capital formation or parental health transmission on children's health. Though our benchmark calibration is rested upon reasonably chosen assumptions, one may still cast doubt on our calibration criteria and the robustness of our calibration results. Therefore, we conduct a sensitivity analysis to examine the qualitative and the quantitative implications under alternative calibration criteria. The alternative calibration criteria include the assumptions lack of empirical knowledge as well as our chosen calibration targets. In particualr, we consider the following assumptions and calibration alternatives:

- The parental education time investment e is allowed to take the values of $\{0.07, 0.21\}$.
- We allow the quality of life when old π_H to take the values of $\{0.7, 0.9\}$.
- The consumption-health spending ratio ($\frac{c^y}{h}$) is allowed to take the values of $\{5, 20\}$.
- Share of health capital in the human capital (α) is allowed to take the values of $\{0.4, 0.6\}$.
- Since there is no measure on the health and knowledge capital, we allow the health-knowledge capital ratio ($\frac{P}{M}$) to take the values of $\{2, 0.5\}$.
- The account of the direct parental transmition on child's health capital ($\frac{\phi}{\xi x^{\varepsilon}+\phi}$, denoted as P_b in Table 4) is allowed to take the values of $\{0.3, 0.1\}$.
- The account of the innate ability on child's knowledge capital ($\frac{\psi}{e+\psi}$, denoted as M_b in Table 4) is allowed to take the values of $\{0.3, 0.1\}$.

- We allow the physical capital to human capital ratio ($\frac{K}{H}$) to take the values of $\{2, 0.5\}$.

- We allow the capital income share (η) to take the values $\{0.28, 0.38\}$.

Table 5 (a) reports the calibrated values of the parameters under these alternative assumptions. Table 5 (b) reports the result of the comparative statics under these re-calibrated parameters. Our sensitivity analysis suggests that most of the changes in the equilibrium outcomes are nonessential. The only exceptions are the results of the changes in the health-knowledge capital ratio, the health-human capital ratio and the knowledge-human capital ratio in response to the changes in parental health investment share (ε) under the assumption $\frac{c^y}{h} = 5$ and $\frac{P}{M} = 0.5$. In fact, the effects of increases in ε is not easy to see because an increase in ε affects the marginal product of parental health investment in an ambiguous way ($MP_h = \varepsilon\xi \left(\frac{h}{P}\right)^{\varepsilon-1}$). As we examine (17) and rewrite (17) as $\frac{P}{M} = \left[\frac{\xi}{\mu(e+\psi)-\phi}\right]^{\frac{1}{\varepsilon(1-\alpha)}} Ax^{\frac{1}{1-\alpha}}$, we find that though an increase in ε decreases the BGP value of e, the increase in ε can bring $\frac{P}{M}$ ratio down directly (through the power term $\frac{1}{\varepsilon(1-\alpha)}$). Therefore, under different parameter values of ξ and ε, $\frac{P}{M}$ ratio could either rise or fall, depending on the values of e and x.

In summary, our model is quite robust. For the parameters we are interested in (γ, π_H, ϕ, ψ and μ), the changes in the equilibrium outcomes are negligible under different calibration targets and assumptions.

4.4 Poverty Trap

Up to now, we assume that the severity of the institutional barriers for the U.S. economy equals to zero; namely, there is no institutional sunk cost for the U.S. economy. What happens if there exist institutional barriers that seriously hamper the health conditions of the economy? What makes an economy less likly to fall into the poverty trap? If government can run projects financed by wage income tax to reduce institutional barriers, what tax rate should government aim at? Is this tax rate affordable for poor countries? These are the questions we want to answer in this subsection.

4.4.1 The emergence of poverty trap

In Section 3.2, we have pointed out that if the existence of the institutional barriers creates severely detrimental effects on health conditions, the economy will end up trapping in the equilibrium with no health investment and low quality of life when old. Based on our calibrated parameters, for an economy endowed with the same preference and technologies as those of the U.S. economy, poverty traps can emerge when the severity of the institutional barriers exceeds 0.5582. We denote this threshold value of the institutional barrier δ_c. Under this critical value of δ_c, the total wasted resource resulting from the institutional barriers amounts to 2.52% of total output, and near 56%

of private health investment is futile. If the severity of the institutional barriers δ exceeds δ_c, the economy will fall into the degenerate equilibrium, i.e. the economy will get stuck in the poverty trap where health investment is futile and agents enjoy low quality of life when old.

Note that the threshold value δ_c differs across countries, since the environment a country rested upon – the preferences, the technologies, the natural environment and other exogenous factors – is unique to the specific country, and shapes the development pattern of the country in its own way. Holding those natural and environment-specific factors constant, our model is able to investigate the "threat" of the institutional barriers on the economy.[13] Though δ_c is an indicator of the severity of the institutional barriers, what matters for the development of a country is the magnitude of the total resource waste resulting from the institutional barriers.[14] To examine the threat of the insitutional barriers, we calculate the corresponding δ_c and the resource waste as percentage of total output in response to changes in the structural parameters. The latent threat of the institutional barriers on the economy is defined in the following sense: the latent threat is bigger if the corresponding institutional sunk cost to output ratio is lower. Put it in another way: if a relatively small portion of waste to total output could make a country to fall into the poverty trap, the threat of the institutional barriers faced by the economy must be larger. That is, it is "easier" for the country to reach to threshold of falling into the poverty trap.

Based on our benchmark calibration, Table 6 summarizes the corresponding value of δ_c and the associated resource waste as the percentage of total output when the structural parameters changes.[15] From Table 6, we see that with more altruistic parents (higher γ), the economy is less likely to fall into the trap; with higher knowledge scaling factor μ, the threat of falling into the trap is lower. Improvement in the quality of when old π_H leads to a higher possibility for the economy to fall into the trap. The result is not hard to understand. First, the representative woman relies on her saving in her old adulthood. Hence, as the quality of life when old is improved, she cares more about her own consumption in the old age and responds by working more and investing less on children's education and health. Second, a higher quality of life when old does not mean a larger stock of health capital of the representative woman or a higher labor productivity. That is to say, to help a country move out of the trap or to assist a country when she is facing the threat of the trap, it is more effective to enhance the health capital (for young people) and to improve the labor productivity. On the contrary, the aid can bring negative effects to the country if it is aimed at helping old people instead of helping young people.

[13] Though the severity of the institutional barriers is exogenous to the economy, it is usually determined by the preferences, the technologies and other unobserved and observed factors of the economy, and hence δ_c differs across countries. The following discussion is based upon the assumption that all other unobserved and observed factors are the same across countries.

[14] The resource waste directly influence the stock of physical capital in next period.

[15] Here we consider a 5% increase in the value of the structural parameters.

4.4.2 Policy to alleviate institutional barriers

Suppose that the government can take fiscal policies to fight against the institutional barriers. For example, the government can construct basic infrastructure, such as water supply system and septic system, to improve the sanitary condition and to alleviate the negative impact of the adverse environment. To capture this idea, assume that the severity of the institutional factor is decreasing in the government's spending relative to total output. Let G denotes total government spending on fighting against the institutional barriers. The severity of the institutional barriers is assumed to take the following form:

$$\delta(\frac{G}{Y}) = \bar{\delta}\left(1 - \nu(\frac{G}{Y})\right),$$

where $\bar{\delta} > 0$ is the institutional barriers specific to the country. If the country is on the verge of the poverty trap, then $\bar{\delta} = \delta_c$. To capture the administrative efficiency of the government, we use $\nu > 0$ to measure the effectiveness of the government in utilizing the public expenditure or investment: the bigger the ν is, the more effective the government is to fight against institutional barriers.

The government can finance the spending by imposing different taxes schemes. Here we consider wage income tax.[16] Suppose that the government imposes a tax on the wage earnings at a tax rate of τ_w. The representative woman's intertemporal budget constraint becomes:

$$c_t^y + \frac{c_{t+1}^o}{R_{t+1}} + h_t = (1 - \tau_w)\, w_t H_t \,(1 - e_t)\,. \tag{23}$$

Government balanced budget implies $G_t = \tau_w w_t H_t \ell_t = \tau_w w_t H_t (1 - e_t)$. The goods market clearing condition is thus

$$c_t^o + c_t^y + h_t + \bar{\delta}\left(1 - \nu\frac{\tau_w w_t H_t (1 - e_t)}{Y_t}\right)\bar{h}_t + K_{t+1} + G_t = Y_t.$$

The equilibrium is solved when the representative woman optimizes, production efficiency is satisfied and good, loanable funds and labor markets all clear. Applying the same procedure as discussed in Section 3. Under the wage income tax, the equation governing the equilibrium e (20) does not change. The new IT and the new ME loci are:

$$x = \frac{(1 - \tau_w)(1 - \eta) B k^\eta}{\tilde{w}(e)}, \tag{24}$$

$$x = \frac{k\mu(e + \psi)}{\frac{1-\tau_w(1-\eta)}{(1-\tau_w)(1-\eta)}\tilde{w}(e) - \frac{\left[1+\bar{\delta}\left(1-\frac{\nu\tau_w\tilde{w}(e)(1-e)}{1-\tau_w}\right)\right]+\left[1+\frac{\pi_H \eta B k^{\eta-1}}{(1+\rho)\mu(e+\psi)}\right]\tilde{z}(e)}{1-e}}. \tag{25}$$

The new IT locus depends only on the wage income tax rate (τ_w), while the new ME locus depends both on the magnitude of the rate of wage income (τ_w) and the government's administrative efficiency (ν) in fighting against the institutional barriers. It is apparent that the new IT locus

[16] The result of general income tax and consumption tax is similar to wage income tax.

shifts downward unambiguously in response to a bigger rate of wage income tax (τ_w). Carefully examining (25), the denominator of (25) is always increasing in response to a positive increment in τ_w and ν. This indicates that the new ME locus shifts to the right as the government implements the policy. Moreover, the more effective the government's eliminating the barriers is, the bigger the right shift of the ME locus is. We also know that if the magnitude of the right shift of the ME locus is relative larger than the IT locus, the economy will end up with higher x and k on the new BGP equilibrium. Figure 4 plots the case where the ME locus shifts to the right more than the IT locus under the wage income tax. Both the new BGP values of x and k – x^* and k^*, are higher than x_c and k_c – the values of x and k when the government takes no measure to fight against barriers and the economy is on the verge of the poverty trap.

Analytically we know that the BGP values of x and k may increase in response to the against-institutional-barriers policy. We are interested in learning the optimal wage income tax rate – the wage income tax rate that brings the highest level of welfare for the agents. How is this optimal wage income tax rate compared to the prevalent wage income tax rates across countries? Before answering this question and calculating the optimal wage income tax numerically, we need to compute the "lifetime welfare" perceived by the government along the BGP first.[17]

Suppose that the government discounts each generation by a factor of ρ_G. By setting $H_0 = 1$, using the relationship of consumption when young and when old $c_t^o = R_t \pi_H c_{t-1}^y / (1+\rho)$, and the BGP economic growth $H_{t+1}/H_t = \mu(e+\psi)$, we can obtain the total lifetime welfare for all generations perceived by the government as

$$\Omega = \frac{1+\rho_G}{\rho_G}\left\{\frac{1+\rho+\pi_H}{1+\rho}\ln z + \frac{1+\frac{\pi_H}{1+\rho}+(1+\rho_G)\gamma}{\rho_G}\ln\mu(e+\psi) + \frac{\pi_H}{1+\rho}\ln\frac{R}{1+\rho} + \frac{\pi_H}{1+\rho}\ln\pi_H\right\}.$$

Assume that the government discounts the new generation in the same way as the representative woman discounts their old age consumption, i.e. we set $\rho_G = \frac{1+\rho}{\pi_H} - 1$.[18]. Figure 5 plots the "lifetime welfare" perceived by the government with different administrative efficiency and under different tax rates.[19] Under our calibrated parameter values, the lifetime welfare for the economy on the verge of the poverty trap is around 2.86 (the red line in Figure 5). The representative woman and her offsprings always get worse off if the government imposes a wage income tax but does nothing to against the institutional barriers ($\nu = 0$), as shown by the blue line in Figure 5 (the blue line is always lower than the red line). Figure 5 also shows that, people are better off when the government is more effective, and the needed wage income tax rate to bring the maximum welfare is higher when

[17] The exercise carrying out here assumes that the economy is under the threat of the threhold institutional barriers δ_c.

[18] The tax rates needed to optimize the lifetime welfare calculated below is not changed much if we set the quality of time when old equal to 1 or 0.6.

[19] The economy under the investigation is on the verge of the poverty trap and the representative woman is poor before the policy comes into effect. Therefore, we only look at the wage tax rates below 25%.

the government has better administrative efficiency. When $\nu = 2.5$, the tax rate needed to maximize the lifetime welare falls into the range of 2.75%˜4.125%; when ν increases to 5, the tax rate needed rises to around 6%˜7.5%. A wage income tax rate of lower than 7.5% is considered to be quite low.[20] This numerical experiment shows that in our calibrated economy, the needed wage income tax rate to enhance lifetime welfare is acceptable, which implies that the cost of fighting against barriers should be affordable by the trapped countries.

5 Policy Analysis

As we are interested in learning the causes of poverty and the forces that push countries out of the poverty trap, in this section we turn to examine the real world cases. Does a country remain mired in poverty because of large institutional barriers, insufficient investment in health, or underinvestment in education? We will compare several middle-income-low and trapped countries and identify the sources of poverty for each country. We provide policy analysis of government subsidy to health and education. Since different country resides in distinct situation, we will discuss how government should allocate resources across the health and the knowledge sector differently depending on the condition of the country, namely, the optimal subsidy rates to health and education investment will be provided for different countries .

5.1 Calibrate Countries of Different Income Groups

We consider two groups of countries: middle-income-low and trapped countries. The countries under investigation include Egypt, China, and India for middle-income-low group, and Cambodia, Ghana, Kenya, Malawi and Zambia for trapped group.

Before calibrating our model to match these selected countries, we first describe the data. The data of life expectancy, fertility rate per woman, health expenditure per capita are obtained from World Bank, years of schooling is acquired from Barro-Lee (2010), and the risk free interest rates are gathered from the central bank of each country. We need the data of employee compensation of each country and total GDP to calculate labor income share and capital income share, and the sources of data of employee compensation and total GDP are United Nations database (UN) and Penn World Table (PWT) 6.3 respectively. The economic growth rate for real GDP per capita is obtained from PWT 6.3, and individual consumption expenditure is obtained from UN. Since our targeted countries do not have time use survey, we take the U.S. parental time investment adjusted by total fertility rate as parental time investment for each country.

[20] The individual income tax rate in Zambia falls into the range of 10% ˜30%. Therefore, 7.5% is considered as an acceptable tax rate.

The calibration procedures for middle-income-low countries and trapped countries are given as follows:

Middle-income-low countries

To calibrate the middle-income-low countries, we make similar assumptions on P/M ratio, K/H ratio, and intergenerational transmission of health and knowledge. After making these assumptions, we are left with parameters B, γ, ε, ξ and δ. The technology coefficient in goods sector B can be expressed in terms of interest rate. Rewrite (21) and (22) in term of interest rate and equate these two equations. The severity of institutional barriers δ can be solved once we plug in the value of interest rate. ξ can then be solved using (17). We then pin down γ and ε by solving (19) and (20) with the use of consumption to health expenditure ratio.

Trapped countries

In our model, the trapped countries are featured by "zero" health investment.[21] However, in reality all trapped countries have positive health investment despite their low income level. Therefore, we begin our calibration with the goods market clearing condition (8). We will shed the IT equation (21) since (12) is no longer satisfied.

Similar to the calibration for middle-income-low countries, we make assumptions on K/H and intergenerational transmission of innate health and knowledge. By doing this, we are left with parameters B, γ, and δ. The technology coefficient B is pinned down by interest rate and capital income share. To calibrate δ, we divide (8) by H and rewrite (8) in terms of z, x, k, e and interest rate R. Then we set the x value of the U.S. as our benchmark value of x. With the information of the years of schooling of the trapped countries and the health expenditures per capita of both the trapped countries and the U.S., we are able to calculate x for the trapped countries. Once we obtain x for the trapped countries, we can solve delta from (8). Finally, by combining (11) and (13) we are able to pin down the altruistic factor γ.

5.2 Subsidy to Health and Education

From the calibration results in the previous subsection, we will learn the causes of poverty, i.e. whether the institutional barriers, deficient investment in health or insufficient investment in education results in the poor performance of an economy. Now we turn to discuss whether government could implement policies to improve the performance of an economy and to haul a country out of the poverty trap.

Specifically, we consider government subsidy to health and education. It is a well known fact that both good health and education have positive externalities (both intergenerational and cross-sectional) and people may under invest in both. Hence, government intervention is called for to fix

[21] The poverty trap in our model is the corner solution where the representative woman makes no health investment

the problem. Since the resource is limited, we will analyze how the optimal targets of wage income tax rate as well as optimal subsidy rates in the health and education sector for the middle-income-low and trapped countries.

5.2.1 The tax and the subsidy scheme

Suppose now the government provides subsidies to the representative woman for her health and education investment on her child. The government finances the subsidies by a wage income tax scheme. Denote the wage income rate τ_w and the government subsidy rate for health investment and parental education time investment b_p and b_M respectively. The budget constraint for the representative woman now becomes

$$c_t^y + \frac{c_{t+1}^o}{R_{t+1}} + (1 - b_p)\, h_t = (1 - \tau_w)\, w_t H_t \left[1 - (1 - b_e)\, e_t\right]. \tag{26}$$

The government runs a balanced budget policy. Let T_t be the total taxes collected by the government and S_{pt} and S_{et} be the total amount of the subsidies to the health and the knowledge sector at period t. Government balanced budget implies the following equation holds:

$$S_{pt} + S_{et} = T_t.$$

The total tax revenue collected, the total government subsidies, and the subsidies in the two sectors satisfy

$$\begin{aligned} T_t &= \tau_w w_t H_t \left[1 - (1 - b_e)\, e_t\right], \\ b_p h_t &= S_{pt}, \\ (1 - \tau_w)\, w_t H_t\, (1 - b_e)\, e_t &= S_{et}. \end{aligned}$$

Since the government only tilts resources to health and education investment and does not consume resources at all, the market clearing condition (8) is unchanged.

Following the same procedure depicted in Section 3, the transformed wage (18) and consumption variables (19) becomes

$$\begin{aligned} \hat{w}(e) &= \frac{1}{(1 - \tau_w)\left[1 - (1 - b_e)\, e\right]} \left[1 - b_p + \frac{1 + \rho + \pi_H}{1 + \rho} \hat{z}(e)\right], \\ \hat{z}(e) &= \frac{1 - b_p}{\alpha\gamma\varepsilon} \left[1 + \frac{\phi}{\mu(e + \psi) - \phi}\right]. \end{aligned}$$

The reduced system expressed in e, x and k are

$$\begin{aligned}\widehat{MB}\left(e\right) &= \frac{1}{\hat{z}\left(e\right)}, \\ x &= \frac{(1-\eta)Bk^{\eta}}{\hat{w}\left(e\right)}, \\ x &= \frac{k\mu\left(e+\psi\right)}{\frac{1}{1-\eta}\hat{w}\left(e\right)-\frac{(1+\delta)+\left[1+\frac{\pi_{H}\eta Bk^{\eta-1}}{(1+\rho)\mu(e+\psi)}\right]\hat{z}(e)}{1-e}},\end{aligned}$$

where $\widehat{MB}\left(e\right) \equiv \frac{\gamma(1-\alpha)[1-(1-b_e)e]}{(1-b_e)(1-b_p)(e+\psi)} - \frac{1+\rho+\pi_H}{(1+\rho)(1-b_p)}$ is the marginal benefit of parental time investment on child's education.

5.2.2 Determine the tax rate and the subsidy rates

Now we are ready to analyze whether the trapped countries can be rescued from poverty and whether the performance of the middle-income-low countries can be enhanced by our tax and subsidy policy. Notably, government of trapped countries faces a three-fold problem here: (1) can the income tax and the health and education subsidy scheme bring dramatic changes to agents' behavior such that the economy escapes from the trap? (2) which wage income tax rate should the government choose? (3) given a certain wage income tax rate, how should the government set the subsidy rates for health and education investment?

To proceed on our analysis, the average income tax rates and the shares of government spending on health and education expenditure of our selected middle-income-low and trapped countries are collected. Based upon our calibration results, we will analyze whether the countries under investigation could perform better under different income tax rates and different allocation of expenditures on health and education.

6 Conclusion

In this paper, we develop a three-period overlapping generations model with human capital separable in health and knowledge capital. Agents make education and health investment optimally for their children. Health investment plays two important roles here: it determines the utility agents derive in the old age, and it is an input to the health capital, which is Pareto complements to the knowledge capital in producing human capital. We find that it is possible to have multiple equilibria – the trap, the middle unstable and the stable equilibrium. We show that the it is possible for an economy to escape from the poverty trap if the government is effective in using the collected tax revenue to fight against institutional barriers. We find that pure improvement in the quality of life when old without enhancing the health capital for the young agents is actually bad for the economy. This is because the agents rely solely on their savings for their retirement, and only the health improvment enhancing the labor productivity could augment output. Beside this, our model is able to address how health capital changes relatively to knowledge capital and human capital as technologies or preferences shift.

Our model provides a microfundation for parental decision on the investment in children's health and knowledge. We are able to disentagle different forces and discuss the associated equilibrium result as structural parameters shift. With the decomposition of human capital into health and knowledge capital, our framework enables us to examine whether the government does the things correctly – i.e., whether the government makes the best use of the tax revenue to improve the welfare or the economic growth rate. This feature is quite novel in the literature, and is of great importance since the public expenditure on health and education accounts to a large portion of government spending.

In our model, the institutional barriers are set as exogenous, while in reality, institutional barriers are often an endogenous outcome of the economic system. Our model captures the idea of how the institutional barriers influence the economy in a negative way, and we do partially endogenize the severity of the institutional barriers via introducing government policies. Still, endogenizing the institutional barriers is a direction for future research.

References

[1] Grossman, Michael (1972) "On the Concept of Health Capital and the Demand of Health," *Journal of Political Economy* 80, 223-255.

[2] Grossman, Michael (1998) "On Optimal Length of Life," *Journal of Health Economics* 17, 499-509.

[3] Heckman, James (2007) "The Economics, Technology and Neuroscience of Human Capability Formation," *Proceedings of the National Academy of Sciences* 104(33), 13250-13255

[4] Cunha, F. and James Heckman (2007) "The Technology of Skill Formation," *The American Economic Review* 97(2), 31-47.

[5] Fogel, Robert W. (1992) "Second Thoughts on the European Escape from Hunger: Famines, Price Elasticities, Entitlements, Chronic Malnutrition, and Mortality Rates," NBER Working Paper.

[6] Ehrilch, I. and H. Chuma (1990) "A Model of The Demand for Longevity and the Value of Life Extension", *Journal of Political Economy* 98, 761-782.

[7] Sanso, M. and R. M. Aisa (2006) "Endogenous Longevity, Biological Deterioration and Economic Growth," *Journal of Health Economics* 25, 555-578.

[8] Benitez-Silva, Hugo and Huan Ni (2008) "Health Status and Health Dynamics in an Empirical Model of Expected Longevity," *Journal of Health Economics* 27, 564-584.

[9] Lorentzen, P., J. McMillan and R. Wacziarg (2008) "Death and Development," *Journal of Economic Growth* 13, 81-124..

[10] Acemoglu, Daron and Simon Johnson (2007) "Disease and Development: The Effect of Life Expectancy on Economic Growth," *Journal of Political Economy* 115, 925–984.

[11] Ashraf, Quamrul H., Ashley Lester, and David N. Weil (2008) "When Does Improving Health Raise GDP?" *mimeo.*

[12] Chakraborty, Shankha (2004) "Endogenous Lifetime and Economic Growth," *Journal of Economic Theory* 116, 119–137.

[13] Castello-Climent, Amparo and Rafael Domenech (2008) "Human Capital Inequality, Life Expectancy and Economic Growth," *The Economic Journal* 118, 653–677.

[14] Doepke, Matthias (2004) "Accounting for Fertility Decline During the Transition to Growth," *Journal of Political Economy* 9, 347–383.

[15] Soares, Rodrigo R.(2005) "Mortality Reductions, Educational Attainment, and Fertility Choice," *The American Economic Review* 95(3), 580–601.

[16] Galor, Oded (2005) "The Demographic Transition and the Emergence of Sustained Economic Growth," *Journal of European Economic Association* 3(2-3), 494–504.

[17] Ljungqvist, Lars and Harald Uhlig (2000) "Tax Policy and Aggregate Demand Management under Catching up with the Joneses, *The American Economic Review* 90(3), 356–366.

[18] Manuelli, Rodolfo E. and Ananth Seshadri (2009) "Explaining International Fertility Differences," *The Quarterly Journal of Economics* 124(2), 771–807.

[19] Manuelli, Rodolfo E., Ananth Seshadri and Yongseok Shin (2010) "Lifetime Labor Supply and Human Capital Investments," Working Paper.

[20] Morand, Olivier F. (2004) "Economic Growth, Longevity and the Epidemiological Transition," *The European Journal of Health Economics* 5, 166–174.

[21] Palivos, Theodore (1995) "Endogenous Fertility, Multiple Growth Paths, and Economic Convergence," *Journal of Economic Dynamics & Control* 19, 1489–1510.

[22] Palivos, Theodore (2001) "Social Norms, Fertility and Economic Development," *Journal of Economic Dynamics & Control* 25, 1919–1934.

[23] Weil, David N. (2007) "Accounting for the Effect of Health on Economic Growth," *The Quarterly Journal of Economics* 122(3), 1265–1306.

[24] Barro, Robert J. and Gary S. Becker (1989) "Fertility Choice in a Model of Economic Growth," *Econometrica* 57(2), 481–501.

[25] Becker, Gary S., Kevin M. Murphy and Robert Tamura (1989) "Human Capital, Fertility, and Economic Growth," *The Journal of Political Economy* 98(5), S12–S37.

[26] Becker, Gary S. (1960) "An Economic Analysis of Fertility," *Demographic and Economic Change in Developed Countries*, NBER 225–256.

[27] Easterlin, Richard A (1968) Population, Labor Force, and Long Swings in Economic Growth. New York: Columbia University Press.

[28] Doepke, Matthias (2005) "Child mortality and fertility decline: Does the Barro-Becker model fit the facts?" *Journal of Population Economics* 18(2), 337–366.

[29] Ulimwengu, John (2009) "Farmers' health and agricultural productivity in rural Ethiopia," *African Journal of Agricultural and Resource Economics* 3(2), 83–100.

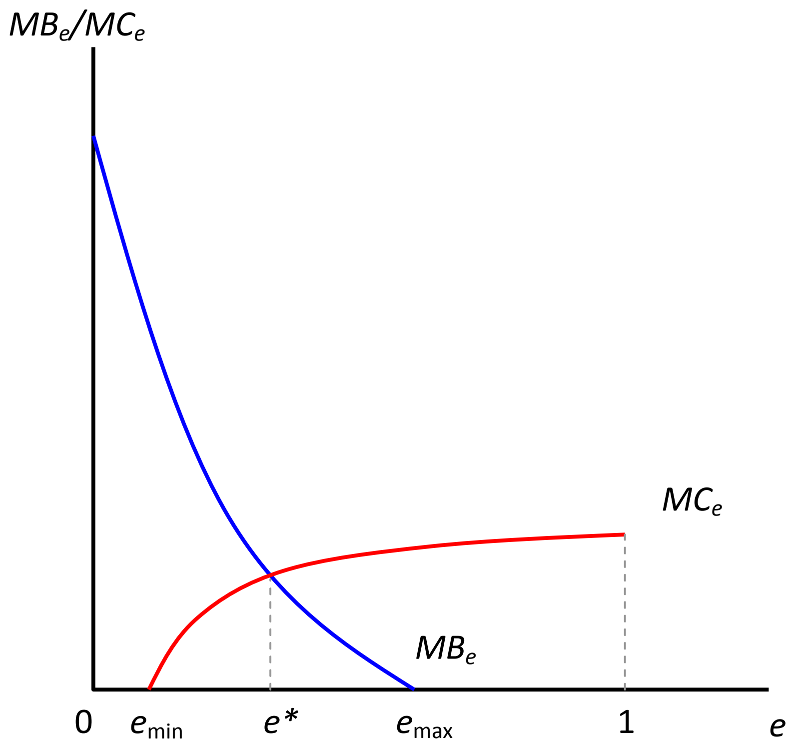

Figure 1: The system governing the equilibrium *e*.

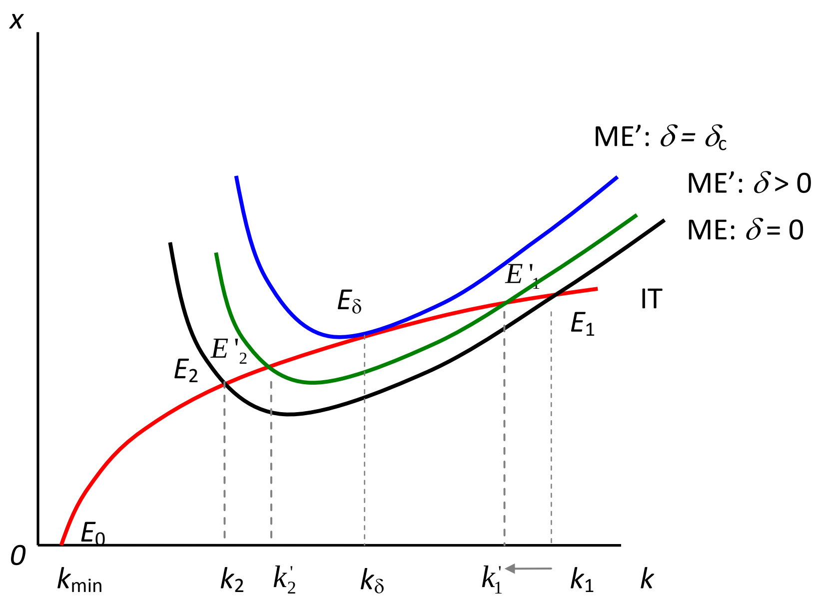

Figure 2: The system of (*k*, *x*) in the benchmark model

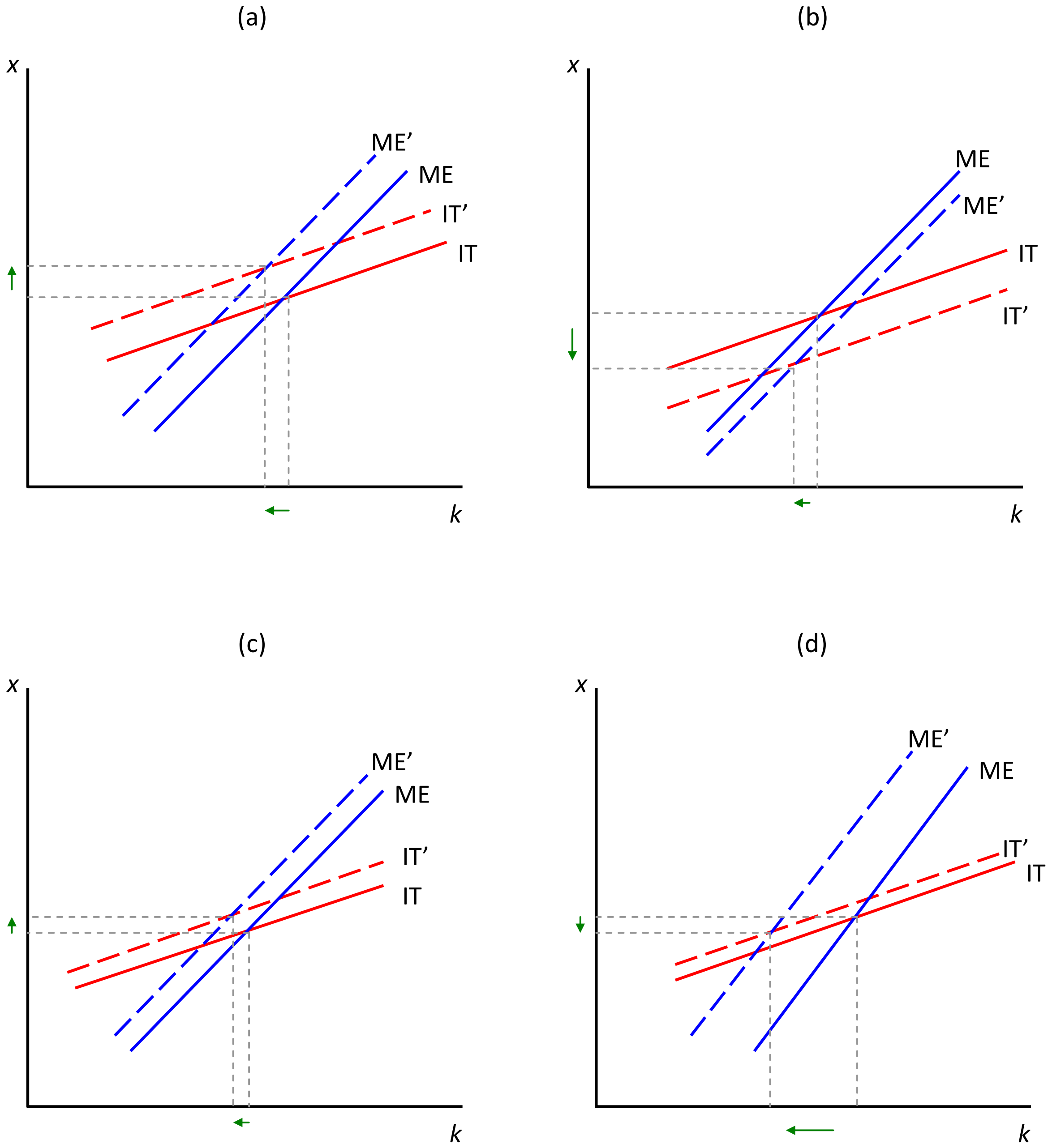

Figure 3. Comparative Statics

(a) γ increases or π_H decreases. (b) ϕ increases. (c) ψ increases. (d) μ increases.

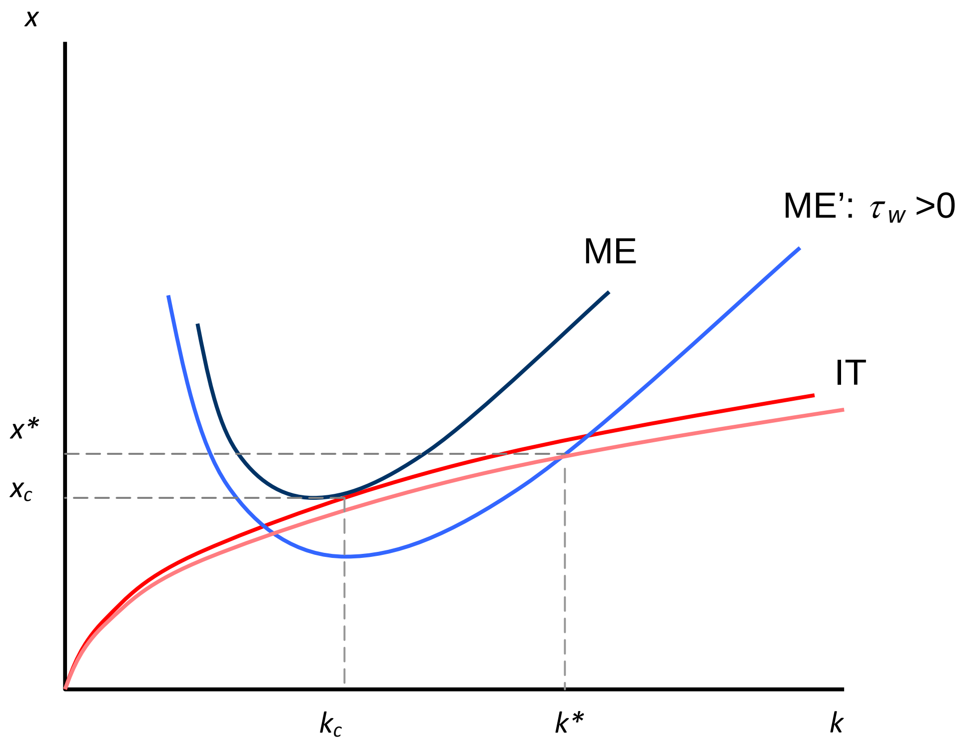

Figure 4: System of (*k*, *x*) under wage income tax

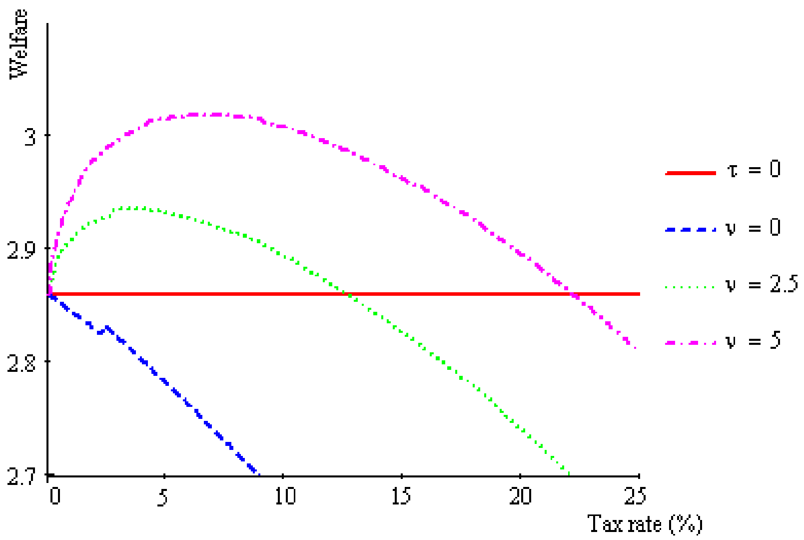

Figure 5: Lifetime welfare under wage income tax

Table 1 Comparative Economic Performance, Health Condition and Educational Attainment

	High Income				Middle Income High				Middle Income Low					Trap					
	USA	UK	Japan	Germany	Greece	Korea	Argentina	Brazil	Colombia	Egypt	Turkey	China	India	Bangladesh	Cambodia	Ghana	Kenya	Malawi	Zambia
Growth rate per capita, 1970-2007	2.13	2.27	2.14	1.92	2.26	5.74	0.92	1.93	1.87	3.19	2.24	7.62	3.11	1.12	1.10	0.76	0.42	1.92	-0.89
Per capita GDP/GDPUS																			
1970	1.00	0.70	0.70	0.76	0.57	0.14	0.57	0.24	0.19	0.11	0.17	0.03	0.06	0.09	0.09	0.10	0.10	0.04	0.14
1980	1.00	0.69	0.76	0.80	0.65	0.22	0.51	0.35	0.22	0.12	0.17	0.04	0.06	0.06	0.04	0.07	0.08	0.05	0.12
1990	1.00	0.69	0.86	0.79	0.52	0.40	0.29	0.26	0.19	0.12	0.17	0.06	0.07	0.05	0.04	0.05	0.06	0.03	0.10
2000	1.00	0.69	0.73	0.74	0.51	0.49	0.32	0.22	0.17	0.12	0.16	0.11	0.07	0.05	0.05	0.04	0.05	0.03	0.08
2005	1.00	0.72	0.71	0.71	0.61	0.53	0.32	0.21	0.17	0.12	0.17	0.15	0.08	0.05	0.06	0.04	0.05	0.03	0.04
2007	1.00	0.75	0.70	0.73	0.65	0.55	0.36	0.22	0.18	0.13	0.18	0.20	0.09	0.05	0.07	0.04	0.05	0.03	0.05
Years of schooling																			
1970	10.77	7.29	8.20	5.02	6.52	6.34	6.30	2.81	3.92	1.31	2.43	3.43	1.57	1.38	5.06	3.58	2.17	1.57	2.98
1980	12.03	7.71	9.25	5.61	7.10	8.29	7.30	2.77	4.90	2.65	3.55	4.75	2.34	2.25	5.31	5.05	3.79	2.33	4.13
1990	12.15	8.13	9.97	8.04	8.58	9.35	8.34	4.46	5.99	4.38	5.01	5.62	3.45	3.16	5.54	6.17	5.60	2.89	4.89
2000	12.71	8.81	10.92	9.95	8.89	11.06	8.73	6.41	6.91	5.91	6.08	7.11	4.20	4.48	5.79	7.12	6.62	3.48	6.13
2005	12.09	9.21	11.26	11.85	9.89	11.47	9.13	7.17	7.05	6.59	6.47	7.62	4.68	5.20	5.90	7.50	7.10	4.38	6.33
Life expectancy at birth																			
1960	69.77	71.13	67.67	69.54	68.85	54.15	65.22	54.50	56.72	45.93	50.26	46.60	42.43	40.26	42.40	45.86	46.33	37.77	45.08
1970	70.81	71.97	71.95	70.46	71.84	61.25	66.59	58.56	60.88	50.43	55.69	61.97	48.83	44.10	43.63	48.93	52.19	40.52	48.97
1980	73.66	73.68	76.09	72.63	74.36	65.80	69.51	62.49	65.48	56.56	60.33	65.97	55.12	47.74	39.97	53.09	57.69	44.77	51.90
1990	71.80	75.88	79.10	75.40	77.00	71.80	72.50	66.70	68.30	62.89	64.70	68.40	57.60	54.40	54.85	58.40	59.80	46.70	52.00
2000	76.90	77.86	81.30	78.30	78.20	76.00	74.60	70.30	70.99	68.23	69.90	71.20	60.90	61.20	56.88	58.30	51.30	47.30	43.10
2005	77.74	79.07	81.93	79.31	79.17	78.43	74.77	71.65	72.27	69.54	71.39	72.58	62.78	64.59	59.26	56.53	52.41	51.03	42.82
2008	78.30	79.90	82.80	80.20	80.10	79.80	75.50	73.30	72.98	70.14	74.30	73.80	64.30	64.70	60.97	61.90	54.30	52.90	48.20
Mortality under 5 (per 1000)																			
1960	30.00	26.10	40.90	40.00	46.00	137.50	71.70	177.70	143.60	303.60	217.50		240.10	243.20		212.30	201.40	360.40	212.70
1970	23.20	20.80	17.40	26.00	31.80	52.00	69.40	135.00	104.00	236.30	200.40	116.60	186.40	236.40		183.00	151.80	323.30	177.90
1980	14.90	14.20	9.90	15.30	19.80	19.60	43.40	90.40	51.90	176.30	136.50	59.10	148.90	203.50	153.40	149.20	110.20	253.50	159.80
1990	11.20	9.50	6.20	8.60	10.50	8.70	27.90	55.70	35.00	89.50	84.20	45.50	118.20	147.50	116.70	120.10	99.10	218.10	178.60
2000	8.40	6.60	4.40	5.30	6.50	6.40	20.70	34.00	26.10	46.60	41.60	36.00	92.70	89.60	106.40	105.80	104.70	164.40	165.70

Note:
1. Growth rate refers to the average growth rate of GDP per capita for the years from 1970 to 2007.
2. Source: Data for GDP per capita, Penn World Table; years of schooling, Barro-Lee (2010); life expectancy, fertility and others, World Bank.

Table 2. Summary of the effects of changes in parameters on the equilibrium parental education time e

Increase in	γ	π_H	ϕ	ψ	μ	α	ε
Effects on e	+	-	+	-	-	-	-

Table 3. Benchmark parameter values and calibration result

Parmeters and observed moments		Benchmark $e = 0.14$
per capita real economic growth rate (annual=2%)	g	0.6406
human capital scaling factor	A	1.0000
income share of physical capital	η	0.3300
time preference rate (annual ρ=0.03)	ρ	1.0938
quality of life when old	π_H	0.8000
share of physical strength	α	0.5000
physical capital to human capital ratio	$\frac{K}{H}$	1.0000
health capital to knowledge capital ratio	$\frac{P}{M}$	1.0000
consumption to health expenditure ratio	$\frac{c^y}{h}$	10
Calibration		
effective capital labor ratio	k	1.1628
health investment to human capital ratio	x	0.2247
consumption to human capital ratio	$\frac{c_t^y}{H_t}$	2.247
the critical level of the institutional factor	δ_c	0.5582
environmental cost to GDP ratio	$\frac{N\delta h}{Y}$	0.0252
critical k	k_c	0.7173
critical x	x_c	0.1916
technology coefficient in production	B	5.5002
interest rate (25 years)	R	1.641
technology coefficient in mental capability production	μ	9.375
basic physical strength inherited from parents	ϕ	0.3281
basic mental capability	ψ	0.0350
altruistic factor	γ	0.6032
share of parental investment in health	ε	0.4145
productivity of parental investment in child's health	ξ	2.4367

Table 4. Numerical Results - Comparative Statics

	x	k	e	g	$\frac{c^y}{h}$	$\frac{c^y}{H}$	$\frac{P}{M}$	$\frac{P}{H}$	$\frac{M}{H}$
Benchmark value	0.2247	1.1628	0.14	1.6406	10	2.2475	1	1	1
γ	0.5652	-1.0764	0.944	0.7552	-1.1803	-0.5907	-3.073	-1.6005	1.7398
π_H	-0.1589	0.3162	-0.2614	-0.2091	0.053	-0.1063	0.9734	0.4809	-0.4696
ϕ	-0.2394	-0.0205	0.0172	0.0138	0.2463	0.007	0.6587	0.3267	-0.3214
ψ	0.0253	-0.0496	-0.2084	0.0333	-0.0083	0.0169	-0.1497	-0.075	0.0753
μ	-0.2521	-1.3863	-0.0165	0.9862	-0.235	-0.4814	-5.3893	-2.9057	3.3996
α	1.3585	1.3878	-1.0989	-0.8791	-0.7675	0.5779	10.6036	5.0046	-3.6591
ε	0.9562	0.0826	-0.0689	-0.0551	-0.9862	-0.028	-0.6474	-0.3264	0.3318
B	1.5108	1.5108	0	0	0	1.5108	3.1359	1.5109	-1.4048

Note: Numbers reported (other than row 2) are the elasticities of e, x, k, $\frac{c^y}{H}$, $\frac{P}{M}$, $\frac{P}{H}$, and $\frac{M}{H}$ to exogenous shifts in parameters. The elasticities are calcuated at a 5% changes in parameters.

Table 5 (a). Sensitivity and robustness test: calibration result

	k	x	γ	μ	ψ	ξ	ϕ	ε	B	R
Benchmark	1.1628	0.2247	0.6032	9.3749	0.035	2.4367	0.3281	0.4145	5.5002	1.6406
$\pi_H = 0.9$	1.1628	0.2177	0.6226	9.3749	0.035	2.4207	0.3281	0.4015	5.5002	1.6406
$\pi_H = 0.7$	1.1628	0.2322	0.5837	9.3749	0.035	2.4528	0.3281	0.4283	5.5002	1.6406
$e = 0.07$	1.0753	0.2247	0.2789	18.7498	0.018	5.0032	0.3281	0.8964	5.2192	1.6406
$e = 0.21$	1.2658	0.2247	0.9849	6.2499	0.053	1.9171	0.3281	0.2538	5.8221	1.6406
$\frac{c^y}{h} = 5$	1.1628	0.4211	0.6439	9.3749	0.035	2.5692	0.3281	0.7766	5.5002	1.6406
$\frac{c^y}{h} = 20$	1.1628	0.1163	0.5828	9.3749	0.035	2.0821	0.3281	0.2145	5.5002	1.6406
$\alpha = 0.4$	1.1628	0.2247	0.5026	9.3749	0.035	3.3201	0.3281	0.6217	5.5002	1.6406
$\alpha = 0.6$	1.1628	0.2247	0.754	9.3749	0.035	1.9826	0.3281	0.2763	5.5002	1.6406
$\frac{P}{M} = 2$	1.1628	0.2247	0.6032	9.3749	0.035	2.8131	0.3281	0.4145	5.5002	1.6406
$\frac{P}{M} = 0.5$	1.1628	0.2247	0.6032	9.3749	0.035	2.1107	0.3281	0.4145	5.5002	1.6406
$P_b = 0.3$	1.1628	0.2247	0.6032	9.3749	0.035	2.3291	0.4922	0.4737	5.5002	1.6406
$P_b = 0.1$	1.1628	0.2247	0.6032	9.3749	0.035	2.5592	0.1641	0.3684	5.5002	1.6406
$M_b = 0.3$	1.1628	0.2247	0.6893	8.203	0.06	2.2553	0.3281	0.3627	5.5002	1.6406
$M_b = 0.1$	1.1628	0.2247	0.5362	10.5468	0.016	2.6326	0.3281	0.4663	5.5002	1.6406
$\frac{K}{H} = 0.5$	0.5814	0.1124	0.6032	9.3749	0.035	3.2477	0.3281	0.4145	3.4569	1.6406
$\frac{K}{H} = 2$	2.3256	0.4495	0.6032	9.3749	0.035	1.8282	0.3281	0.4145	8.7512	1.6406
$\eta = 0.28$	1.1628	0.2846	0.6032	9.3749	0.035	2.2094	0.3281	0.4145	6.5314	1.6406
$\eta = 0.38$	1.1628	0.1806	0.6032	9.3749	0.035	2.6678	0.3281	0.4145	4.7406	1.6406

Table 4 (b). Sensitivity and robustness test: varying assumption of e

		γ	π_H	ϕ	ψ	μ	α	ε	B
Benchmark	x	0.5651	-0.1589	-0.2395	0.0253	-0.2521	1.3585	0.9561	1.5108
	k	-1.0764	0.3161	-0.0206	-0.0496	-1.3864	1.3877	0.0825	1.5108
	e	0.944	-0.2613	0.0173	-0.2084	-0.0165	-1.0988	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1803	0.053	0.2463	-0.0083	-0.235	-0.7675	-0.9862	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.5779	-0.028	1.5108
	$\frac{P}{M}$	-3.073	0.9734	0.6587	-0.1497	-5.3893	10.6036	-0.6474	3.1359
	$\frac{P}{H}$	-1.6005	0.4809	0.3267	-0.075	-2.9057	5.0046	-0.3264	1.5109
	$\frac{M}{H}$	1.7398	-0.4696	-0.3214	0.0753	3.3996	-3.6591	0.3318	-1.4048
e=0.07	x	0.6224	-0.1743	-0.2383	0.0129	-0.253	1.2876	0.9519	1.5109
	k	-1.181	0.3478	-0.022	-0.0248	-1.3843	1.5265	0.0911	1.5112
	e	1.0406	-0.2871	0.0184	-0.2292	-0.0183	-1.2021	-0.076	-0.0004
	g	0.8325	-0.2297	0.0147	0.0166	0.9846	-0.9617	-0.0608	-0.0003
	$\frac{c^y}{h}$	-1.1978	0.0583	0.2461	-0.0042	-0.2349	-0.7088	-0.9378	0.0001
	$\frac{c^y}{H}$	-0.5539	-0.1165	0.008	0.0088	-0.4819	0.5332	-0.0305	1.511
	$\frac{P}{M}$	-1.011	0.2973	0.0403	-0.0205	-2.9349	6.3815	-0.805	3.1368
	$\frac{P}{H}$	-0.512	0.1481	0.0202	-0.0103	-1.5257	2.8117	-0.4066	1.5113
	$\frac{M}{H}$	0.5255	-0.147	-0.0201	0.0103	1.6517	-2.7063	0.4151	-1.4051
e=0.21	x	0.51	-0.1438	-0.2405	0.0373	-0.2512	1.4277	0.9603	1.5108
	k	-0.9745	0.2859	-0.0186	-0.0733	-1.3881	1.2541	0.0746	1.5108
	e	0.8509	-0.2366	0.0156	-0.1884	-0.0149	-0.9982	-0.0623	0
	g	0.6807	-0.1893	0.0125	0.0493	0.9875	-0.7986	-0.0499	0
	$\frac{c^y}{h}$	-1.1078	0.0479	0.2498	-0.0123	-0.2325	-0.7523	-0.9405	0
	$\frac{c^y}{H}$	-0.6261	-0.0963	0.0063	0.025	-0.4808	0.6218	-0.0253	1.5108
	$\frac{P}{M}$	-4.8529	1.6513	1.4221	-0.4057	-7.835	15.3588	-0.4929	3.1356
	$\frac{P}{H}$	-2.5948	0.8093	0.6988	-0.2039	-4.4019	6.2166	-0.248	1.5108
	$\frac{M}{H}$	2.9816	-0.7778	-0.6752	0.206	5.6442	-5.1711	0.2511	-1.4047

Table 5 (b) continued. Sensitivity and robustness test: varying assumption of π_H

		γ	π_H	ϕ	ψ	μ	α	ε	B
$\pi_H = 0.9$	x	0.5667	-0.1731	-0.2398	0.0253	-0.2518	1.3596	0.9575	1.5108
	k	-1.0793	0.3448	-0.0199	-0.0496	-1.3869	1.3857	0.08	1.5108
	e	0.9467	-0.2847	0.0167	-0.2083	-0.016	-1.0973	-0.0668	0
	g	0.7573	-0.2278	0.0134	0.0333	0.9866	-0.8778	-0.0534	0
	$\frac{c^y}{h}$	-1.1245	0.0578	0.2496	-0.0083	-0.2323	-0.7312	-0.9396	0
	$\frac{c^y}{H}$	-0.5897	-0.1159	0.0068	0.0169	-0.4812	0.5786	-0.0271	1.5108
	$\frac{P}{M}$	-3.2019	1.1102	0.6984	-0.1562	-5.5259	10.8778	-0.7032	3.1359
	$\frac{P}{H}$	-1.6708	0.5476	0.3462	-0.0782	-2.9858	4.5823	-0.3547	1.5109
	$\frac{M}{H}$	1.823	-0.533	-0.3403	0.0785	3.5098	-4.0777	0.3611	-1.4048
$\pi_H = 0.7$	x	0.5635	-0.1437	-0.2391	0.0253	-0.2524	1.3573	0.9547	1.5108
	k	-1.0733	0.2857	-0.0212	-0.0496	-1.3857	1.39	0.0853	1.5108
	e	0.9412	-0.2364	0.0178	-0.2084	-0.017	-1.1005	-0.0712	0
	g	0.753	-0.1891	0.0142	0.0333	0.9857	-0.8804	-0.0569	0
	$\frac{c^y}{h}$	-1.1236	0.0479	0.2493	-0.0083	-0.2321	-0.7305	-0.9388	0
	$\frac{c^y}{H}$	-0.5918	-0.0962	0.0073	0.0169	-0.4816	0.5772	-0.0289	1.5108
	$\frac{P}{M}$	-2.9432	0.8394	0.6191	-0.1432	-5.2514	10.3316	-0.5898	3.1359
	$\frac{P}{H}$	-1.5301	0.4154	0.3072	-0.0717	-2.8252	4.3748	-0.2971	1.5109
	$\frac{M}{H}$	1.6569	-0.407	-0.3025	0.072	3.29	-3.9278	0.3016	-1.4048

Table 5 (b) continued. Sensitivity and robustness test: varying assumption of $\frac{c^y}{h}$

		γ	π_H	ϕ	ψ	μ	α	ε	B
$\frac{c^y}{h}=5$	x	0.5221	-0.1471	-0.2304	0.025	-0.2603	1.3286	0.919	1.5108
	k	-0.997	0.2926	-0.0381	-0.049	-1.3708	1.4458	0.1528	1.5108
	e	0.8714	-0.2421	0.0319	-0.2089	-0.0305	-1.1422	-0.1272	0
	g	0.6971	-0.1937	0.0255	0.0329	0.9744	-0.9138	-0.1017	0
	$\frac{c^y}{h}$	-1.1114	0.049	0.2463	-0.0082	-0.2296	-0.7216	-0.928	0
	$\frac{c^y}{H}$	-0.6183	-0.0985	0.013	0.0167	-0.487	0.559	-0.0516	1.5108
	$\frac{P}{M}$	-1.1324	0.3348	0.1014	-0.0558	-3.2688	6.8579	0.4679	3.1358
	$\frac{P}{H}$	-0.5745	0.1667	0.0507	-0.0279	-1.7073	3.0065	0.2326	1.5109
	$\frac{M}{H}$	0.5915	-0.1653	-0.0505	0.028	1.8666	-2.868	-0.2299	-1.4047
$\frac{c^y}{h}=20$	x	0.5894	-0.1655	-0.2445	0.0254	-0.2475	1.3754	0.9771	1.5108
	k	-1.121	0.3294	-0.0107	-0.05	-1.3951	1.3551	0.043	1.5108
	e	0.9851	-0.2722	0.0089	-0.2081	-0.0086	-1.0743	-0.036	0
	g	0.7881	-0.2177	0.0072	0.0335	0.9928	-0.8595	-0.0288	0
	$\frac{c^y}{h}$	-1.1312	0.0552	0.2512	-0.0084	-0.2337	-0.7361	-0.9455	0
	$\frac{c^y}{H}$	-0.5751	-0.1107	0.0037	0.0171	-0.4783	0.5887	-0.0146	1.5108
	$\frac{P}{M}$	-6.4625	2.3515	1.8502	-0.3368	-8.8721	19.5565	-1.7856	3.1361
	$\frac{P}{H}$	-3.5455	1.1431	0.9047	-0.1691	-5.0816	7.6515	-0.9137	1.511
	$\frac{M}{H}$	4.3095	-1.0813	-0.8655	0.1705	6.8125	-6.0192	0.9574	-1.4049

Table 4 (b) continued. Sensitivity and robustness test: varying assumption of α

		γ	π_H	ϕ	ψ	μ	α	ε	B
$\alpha = 0.4$	x	0.5652	-0.1589	-0.2394	0.0253	-0.2521	1.2232	0.9562	1.5108
	k	-1.0764	0.3162	-0.0205	-0.0496	-1.3863	0.9349	0.0826	1.5108
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-0.7536	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.6029	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.8032	-0.9392	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.3708	-0.028	1.5108
	$\frac{P}{M}$	-1.4859	0.4443	0.2275	-0.0693	-3.3193	4.6456	-0.6265	2.581
	$\frac{P}{H}$	-0.9052	0.2654	0.1362	-0.0416	-2.0634	2.5757	-0.3783	1.5109
	$\frac{M}{H}$	0.6272	-0.175	-0.0903	0.0278	1.5058	-1.6797	0.2563	-0.9478
$\alpha = 0.6$	x	0.5652	-0.1589	-0.2394	0.0253	-0.2521	1.5636	0.9562	1.5108
	k	-1.0764	0.3162	-0.0205	-0.0496	-1.3863	2.1037	0.0826	1.5108
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.6203	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-1.2963	-1.2963	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.6165	-0.9392	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.8989	-0.028	1.5108
	$\frac{P}{M}$	-5.8727	2.0829	1.5752	-0.3108	-8.7217	36.0122	-0.6147	3.994
	$\frac{P}{H}$	-2.5963	0.8085	0.6158	-0.1249	-4.0956	9.2761	-0.2482	1.5109
	$\frac{M}{H}$	4.6384	-1.1542	-0.8894	0.1888	8.2034	-9.5465	0.3782	-2.0698

Table 5 (b) continued. Sensitivity and robustness test: varying assumption of $\frac{P}{M}$

		γ	π_H	ϕ	ψ	μ	α	ε	B
$\frac{P}{M}=0.5$	x	0.5652	-0.1589	-0.2394	0.0253	-0.2521	1.3585	0.9562	1.5108
	k	-1.0764	0.3162	-0.0205	-0.0496	-1.3863	1.3878	0.0826	1.5108
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.0989	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.7309	-0.9392	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.5779	-0.028	1.5108
	$\frac{P}{M}$	-3.073	0.9734	0.6587	-0.1497	-5.3893	9.5073	0.002	3.1359
	$\frac{P}{H}$	-1.6005	0.4809	0.3267	-0.075	-2.9057	4.4784	0.001	1.5109
	$\frac{M}{H}$	1.7398	-0.4696	-0.3214	0.0753	3.3996	-3.4086	-0.001	-1.4048
$\frac{P}{M}=2$	x	0.5652	-0.1589	-0.2394	0.0253	-0.2521	1.3585	0.9562	1.5108
	k	-1.0764	0.3162	-0.0205	-0.0496	-1.3863	1.3878	0.0826	1.5108
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.0989	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.7309	-0.9392	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.5779	-0.028	1.5108
	$\frac{P}{M}$	-3.073	0.9734	0.6587	-0.1497	-5.3893	11.7407	-1.2758	3.1359
	$\frac{P}{H}$	-1.6005	0.4809	0.3267	-0.075	-2.9057	4.4784	-0.6484	1.5109
	$\frac{M}{H}$	1.7398	-0.4696	-0.3214	0.0753	3.3996	-4.576	0.6701	-1.4048

Table 5 (b) continued. Sensitivity and robustness test: varying assumption of P_b

		γ	π_H	ϕ	ψ	μ	α	ε	B
P_b= 0.1	x	0.4658	-0.1307	-0.1073	0.0208	-0.3751	1.4914	0.9639	1.5108
	k	-1.0847	0.3186	-0.0092	-0.0501	-1.3965	1.3997	0.0832	1.5108
	e	0.9516	-0.2633	0.0077	-0.208	-0.0074	-1.1077	-0.0694	0
	g	0.7613	-0.2106	0.0062	0.0336	0.9938	-0.8862	-0.0555	0
	$\frac{c^y}{h}$	-1.0297	0.0237	0.111	-0.0037	-0.1046	-0.8537	-0.9465	0
	$\frac{c^y}{H}$	-0.5878	-0.1072	0.0031	0.017	-0.4778	0.574	-0.0282	1.5108
	$\frac{P}{M}$	-3.275	1.042	0.3553	-0.1603	-5.6159	11.0511	-0.6309	3.1357
	$\frac{P}{H}$	-1.7107	0.5144	0.1769	-0.0803	-3.0388	4.6478	-0.318	1.5108
	$\frac{M}{H}$	1.8707	-0.5015	-0.1753	0.0807	3.5832	-4.1244	0.3231	-1.4047
P_b= 0.3	x	0.6907	-0.1943	-0.4062	0.031	-0.0965	1.1911	0.9465	1.5109
	k	-1.0657	0.3139	-0.0341	-0.0484	-1.3731	1.3744	0.0824	1.5115
	e	0.9342	-0.2595	0.0286	-0.2094	-0.0283	-1.0888	-0.0688	0
	g	0.7474	-0.2076	0.0229	0.0325	0.9762	-0.871	-0.055	0
	$\frac{c^y}{h}$	-1.242	0.0903	0.4272	-0.0139	-0.3911	-0.5733	-0.9298	0
	$\frac{c^y}{H}$	-0.5942	-0.1049	0.0123	0.0171	-0.4857	0.5837	-0.0273	1.5111
	$\frac{P}{M}$	-2.8149	0.8901	0.8867	-0.1334	-5.0985	10.0546	-0.6659	3.1391
	$\frac{P}{H}$	-1.4608	0.4402	0.4386	-0.0668	-2.7365	4.2688	-0.3358	1.5124
	$\frac{M}{H}$	1.5759	-0.4307	-0.4292	0.067	3.1702	-3.8502	0.3415	-1.406

Table 5 (b). Sensitivity and robustness test: varying assumption of M_b

		γ	π_H	ϕ	ψ	μ	α	ε	B
M_b= 0.1	x	0.5761	-0.1619	-0.2392	0.0114	-0.2523	1.3448	0.9553	1.5108
	k	-1.0965	0.3221	-0.021	-0.0225	-1.386	1.4142	0.0841	1.5108
	e	0.8556	-0.2366	0.0156	-0.0944	-0.0149	-0.9943	-0.0624	0
	g	0.77	-0.213	0.0141	0.0151	0.9859	-0.8949	-0.0562	0
	$\frac{c^y}{h}$	-1.1273	0.054	0.2494	-0.0038	-0.2322	-0.7267	-0.939	0
	$\frac{c^y}{H}$	-0.5837	-0.1083	0.0072	0.0077	-0.4815	0.5693	-0.0285	1.5108
	$\frac{P}{M}$	-2.6964	0.8418	0.5283	-0.0578	-4.9125	9.7432	-0.678	3.1358
	$\frac{P}{H}$	-1.397	0.4165	0.2624	-0.0289	-2.629	4.149	-0.3419	1.5108
	$\frac{M}{H}$	1.5019	-0.408	-0.259	0.029	3.0269	-3.7616	0.3478	-1.4047
M_b= 0.3	x	0.5517	-0.1552	-0.2397	0.0423	-0.2519	1.3754	0.9572	1.5108
	k	-1.0516	0.3089	-0.02	-0.0829	-1.3867	1.3554	0.0807	1.5109
	e	1.0528	-0.2919	0.0192	-0.3489	-0.0184	-1.228	-0.077	0
	g	0.737	-0.2043	0.0134	0.0558	0.9864	-0.8596	-0.0539	0
	$\frac{c^y}{h}$	-1.1201	0.0517	0.2496	-0.0139	-0.2323	-0.7361	-0.9395	0
	$\frac{c^y}{H}$	-0.5993	-0.1038	0.0069	0.0284	-0.4812	0.5887	-0.0273	1.5108
	$\frac{P}{M}$	-3.5255	1.137	0.8282	-0.2969	-5.9804	11.7001	-0.6096	3.1362
	$\frac{P}{H}$	-1.8482	0.5606	0.4099	-0.149	-3.2551	4.8911	-0.3071	1.511
	$\frac{M}{H}$	2.0363	-0.5454	-0.4017	0.1501	3.8879	-4.2959	0.3119	-1.4049

Table 5 (b) continued. Sensitivity and robustness test: varying assumption of $\frac{K}{H}$

		γ	π_H	ϕ	ψ	μ	α	ε	B
$\frac{K}{H}=0.5$	x	0.5652	-0.1589	-0.2394	0.0253	-0.2521	1.3585	0.9562	1.5108
	k	-1.0754	0.3162	-0.0205	-0.0496	-1.3863	1.3878	0.0826	1.5108
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.0989	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.7309	-0.9392	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.5779	-0.028	1.5108
	$\frac{P}{M}$	-3.073	0.9734	0.6587	-0.1497	-5.3893	10.6036	-1.8837	3.1359
	$\frac{P}{H}$	-1.6005	0.4809	0.3267	-0.075	-2.9057	4.4784	-0.9651	1.5109
	$\frac{M}{H}$	1.7398	-0.4696	-0.3214	0.0753	3.3996	-4.0029	1.0141	-1.4048
$\frac{K}{H}=2$	x	0.5652	-0.1589	-0.2394	0.0253	-0.2521	1.3585	0.9562	1.5108
	k	-1.0764	0.3162	-0.0205	-0.0496	-1.3863	1.3878	0.0826	1.5108
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.0989	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.7309	-0.9392	0
	$\frac{c^y}{H}$	-0.5907	-0.1063	0.007	0.0169	-0.4814	0.5779	-0.028	1.5108
	$\frac{P}{M}$	-3.073	0.9734	0.6587	-0.1497	-5.3893	10.6036	0.6732	3.1359
	$\frac{P}{H}$	-1.6005	0.4809	0.3267	-0.075	-2.9057	4.4784	0.3338	1.5109
	$\frac{M}{H}$	1.7398	-0.4696	-0.3214	0.0753	3.3996	-4.0029	-0.3283	-1.4048

Table 5 (b) continued. Sensitivity and robustness test: varying assumption of η

		γ	π_H	ϕ	ψ	μ	α	ε	B
η=0.28	x	0.6443	-0.1805	-0.238	0.0287	-0.1533	1.2592	0.9502	1.4023
	k	-1.0035	0.2941	-0.0191	-0.0462	-1.2933	1.2884	0.0768	1.4023
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.0989	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.7309	-0.9392	0
	$\frac{c^y}{H}$	-0.516	-0.128	0.0085	0.0204	-0.3838	0.4823	-0.0337	1.4023
	$\frac{P}{M}$	-2.9424	0.9277	0.6616	-0.1428	-5.2428	10.3049	-0.2183	2.9031
	$\frac{P}{H}$	-1.5297	0.4586	0.3281	-0.0715	-2.8202	4.3646	-0.1095	1.4024
	$\frac{M}{H}$	1.6564	-0.4483	-0.3228	0.0718	3.2832	-3.9204	0.1101	-1.3105
η=0.38	x	0.4736	-0.1338	-0.2411	0.0213	-0.3662	1.4744	0.9631	1.6375
	k	-1.1606	0.3419	-0.0222	-0.0536	-1.4939	1.5038	0.0892	1.6375
	e	0.944	-0.2614	0.0172	-0.2084	-0.0165	-1.0989	-0.0689	0
	g	0.7552	-0.2091	0.0138	0.0333	0.9862	-0.8791	-0.0551	0
	$\frac{c^y}{h}$	-1.1241	0.053	0.2495	-0.0083	-0.2322	-0.7309	-0.9392	0
	$\frac{c^y}{H}$	-0.6771	-0.0811	0.0054	0.0129	-0.5941	0.6896	-0.0213	1.6375
	$\frac{P}{M}$	-3.2233	1.0265	0.6553	-0.1576	-5.5576	10.9542	-1.0336	3.4093
	$\frac{P}{H}$	-1.6824	0.5068	0.325	-0.079	-3.0045	4.6112	-0.5237	1.6376
	$\frac{M}{H}$	1.837	-0.4943	-0.3198	0.0793	3.5356	-4.0983	0.5378	-1.5136

Table 5. δ_c and the resource waste as a percentage of total output in response of a 5% increase in parameters

	δ_c	$\frac{\delta_c h}{Y}$	likelihood of trap
Benchmark	0.5582	0.02523	
B	0.5582	0.02523	0
π_H	0.51397	0.02288	+
α	0.54279	0.0254	-
γ	0.53449	0.0255	-
μ	0.55329	0.02529	-
ψ	0.55716	0.02524	-
ϕ	0.5635	0.02518	+
ε	0.53838	0.02545	-

Appendix

In the appendix, we provide detailed mathematical derivations.

A.1 Reduce the System to 3×3

After doing all transformations, we know that the BGP stationary variables are e, z, x, and k. We want to further reduce the system. Rewrite (12) using $(h_t/P_t)^{\varepsilon} = [\mu(e+\psi) - \phi]/\xi$, and substitute (11) into (12). We obtain the following equation:

$$\frac{c^y}{h} = \frac{1}{\alpha\gamma\varepsilon}\left[1 + \frac{\phi}{\mu(e+\psi) - \phi}\right].$$

Hence, $\frac{c^y}{h} = \frac{c^y}{H} / \frac{h}{H} = z/x$. Define $\tilde{z}(e) = \frac{c^y}{h}$ as the transformed consumption variable, and the BGP stationary ratio z can be expressed in terms of e and x. We can substitute z out in the system using $z = \tilde{z}(e)\,x$ so that the system only has three stationary BGP ratios e, x and k.

Expressing (11) in terms of the BGP stationary ratios $z = \tilde{z}(e)\,x$, x, and k, the (11) along the BGP is:

$$\tilde{z}(e)\,x = \frac{1+\rho}{1+\rho+\pi_H}\left[w(1-e) - x\right]. \tag{27}$$

Rearranging the above equation gives:

$$w = \frac{1}{1-e}\left[1 + \frac{1+\rho+\pi_H}{1+\rho}\tilde{z}(e)\right]x.$$

Define $\tilde{w}(e) = \frac{1}{1-e}\left[1 + \frac{1+\rho+\pi_H}{1+\rho}\tilde{z}(e)\right]$ as the transformed wage variable. Rewriting the above equation yields $x = w/\tilde{w}(e) = (1-\eta)Bk^{\eta}/\tilde{w}(e)$. This is the IT locus – the the first equation to determine the equilibrium x and k once the equilibrium e is determined. The equation independently determining the equilibrium e can be found by substituting (12) into (13) and expressing the equation in terms of e, x and k using $z = \tilde{z}(e)\,x$ and $w = \tilde{w}(e)\,x$.

New we want to find the ME locus. By combining (**??**) and (10), the market clearing condition becomes

$$\tilde{z}(e)\,x = \frac{1}{1 + \frac{\pi_H R}{(1+\rho)\mu(e+\psi)}}\{Bk^{\eta}(1-e) - k(1-e)\mu(e+\psi) - (1+\delta)x\}. \tag{28}$$

Rewriting $Bk^{\eta}(1-e) = \frac{w(k)(1-e)}{1-\eta} = \frac{\tilde{w}(e)x(1-e)}{1-\eta}$ and the equation above can be rearranged to arrive at the ME locus (22).

A.2 Analytical Comparative Statics

(Incomplete and need to be re-edited)

Recall that (20) determines the equilibrium e. The analytical comparative statics of e can be carried out by taking total differentiation to (20).

If we further simplify (20), we can find an expression for e as:

$$e = \frac{[\gamma(1-\alpha)(1+\rho)\mu + (1+\rho)\alpha\gamma\varepsilon(\phi - \mu\psi) - (1+\rho+\pi_H)\mu\psi]}{\mu[(1+\rho)[\gamma(1-\alpha) + \alpha\gamma\varepsilon] + 1 + \rho + \pi_H]}.$$

Note that $e \in (0,1)$ since the total time endowment is normalized to 1 and we only discuss the interior cases of e. Let $\Lambda = \mu\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right]$. The comparative statics of e with respect to $\gamma, \pi_H, \phi, \mu, \psi, \varepsilon, \alpha$, and B are

$$
\begin{aligned}
\frac{de}{d\gamma} &= \frac{1}{\Lambda^2}\left\{\begin{array}{c} \left[(1-\alpha)(1+\rho)\mu+(1+\rho)\alpha\varepsilon(\phi-\mu\psi)\right]\mu\left[(1+\rho)\alpha\gamma\varepsilon+1+\rho+\pi_H\right] \\ +(1+\rho+\pi_H)\mu^2\psi(1+\rho)(1-\alpha) \end{array}\right\} > 0, \\
\frac{de}{d\bar{\pi}} &= \frac{1}{\Lambda^2}\left\{\begin{array}{c} -\mu^2\psi\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right] \\ -\mu\left[\gamma(1-\alpha)(1+\rho)\mu+(1+\rho)\alpha\gamma\varepsilon(\phi-\mu\psi)-(1+\rho+\pi_H)\mu\psi\right] \end{array}\right\} < 0, \\
\frac{de}{d\phi} &= \frac{(1+\rho)\alpha\gamma\varepsilon}{\mu\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right]} > 0, \\
\frac{de}{d\mu} &= -(1+\rho)\alpha\gamma\varepsilon\phi\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right] < 0, \\
\frac{de}{d\psi} &= \frac{-(1+\rho)\alpha\gamma\varepsilon\mu-(1+\rho+\pi_H)\mu}{\mu\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right]} < 0, \\
\frac{de}{d\varepsilon} &= \frac{1}{\Lambda^2}\left\{\begin{array}{c} (1+\rho)\alpha\gamma(\phi-\mu\psi)\mu\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right] \\ -\left[\gamma(1-\alpha)(1+\rho)\mu+(1+\rho)\alpha\gamma\varepsilon(\phi-\mu\psi)-(1+\rho+\pi_H)\mu\psi\right]\mu(1+\rho)\alpha\gamma \end{array}\right\} \lesseqgtr 0, \\
\frac{de}{d\alpha} &= \frac{1}{\Lambda^2}\left\{\begin{array}{c} \mu^2\gamma(1+\rho)\left[\gamma\varepsilon(\phi-\mu\psi)-1\right]\left[(1+\rho)\left[\gamma(1-\alpha)+\alpha\gamma\varepsilon\right]+1+\rho+\pi_H\right] \\ +\left[\gamma(1-\alpha)(1+\rho)\mu+(1+\rho)\alpha\gamma\varepsilon(\phi-\mu\psi)-(1+\rho+\pi_H)\mu\psi\right]\mu(1+\rho)\gamma \end{array}\right\} \lesseqgtr 0, \\
\frac{de}{dB} &= 0.
\end{aligned}
$$

The direction of changes in x and k in response to shifts in parameters are hard to be checked analytically. To see why, we first examine the effect of changes in e on $\tilde{w}(e)$ and $\tilde{z}(e)$ as follows:

$$
\begin{aligned}
\frac{\partial\tilde{w}(e)}{\partial e} &= \frac{(1+\rho+\pi_H)}{(1-e)^2}\left\{\begin{array}{c} \frac{1}{1+\rho+\pi_H}+\frac{1}{(1+\rho)\alpha\gamma\varepsilon} \\ +\frac{\phi\mu}{(1+\rho)\alpha\gamma\varepsilon[\mu(e+\psi)-\phi]}\cdot\left[\frac{\mu(e+\psi)-\phi-\mu(1-e)}{\mu(e+\psi)-\phi}\right] \end{array}\right\} \lesseqgtr 0, \\
\frac{\partial\tilde{z}(e)}{\partial e} &= \frac{-\phi\mu}{\alpha\gamma\varepsilon\left[\mu(e+\psi)-\phi\right]^2} < 0.
\end{aligned}
$$

The sign of $\partial\tilde{w}(e)/\partial e$ is unclear. Changes in all the parameters other than the scaling factor in goods production B would change e, and hence $\tilde{w}(e)$ is affected as well. Since the IT locus (21) and the ME locus (22) both have the term $\tilde{w}(e)$, we cannot judge how the equilibrium x and k shift because both (21) and (22) shift as parameters vary. However, we know how the equilibrium x and k change as the scaling factor in goods production B increases. Taking differentiation to the IT locus (21) with respect to B gives us:

$$
\frac{dx}{dB} = \frac{(1-\eta)k^\eta}{\tilde{w}(e)} > 0.
$$

That is, the IT locus shifts up. Differentiating the ME locus (22) with respect to B gives us

$$
\frac{dx}{dB} = \frac{k(1-e)\mu(e+\psi)\eta k^{\eta-1}\tilde{c}(e)}{\left\{\frac{1-e}{1-\eta}\tilde{w}(e)-(1+\delta)-\left[\frac{\eta B k^{\eta-1}}{(1+\rho)\mu(e+\psi)}+1\right]\right\}^2(1+\rho)\mu(e+\psi)} > 0.
$$

That is, the ME locus shifts up in response to an increase in B as well. Since both IT and ME shift up, x increases for sure, but direction of k is unclear.