

International recessions*

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Abstract

The 2008-2009 US crisis is characterized by an unprecedented degree of international synchronization, as all other G7 countries experienced large contractions. Another feature of the crisis is the sharp fall in US employment associated with an increase in productivity. These two features—international synchronization and productivity increase—are not present in many of the previous US contractions. We study a two-country model with financial markets frictions and show that the features of the recent recession are consistent with ‘credit shocks’ playing a more prominent role as a source of business cycle fluctuations, in an environment with international mobility of capital.

1 Introduction

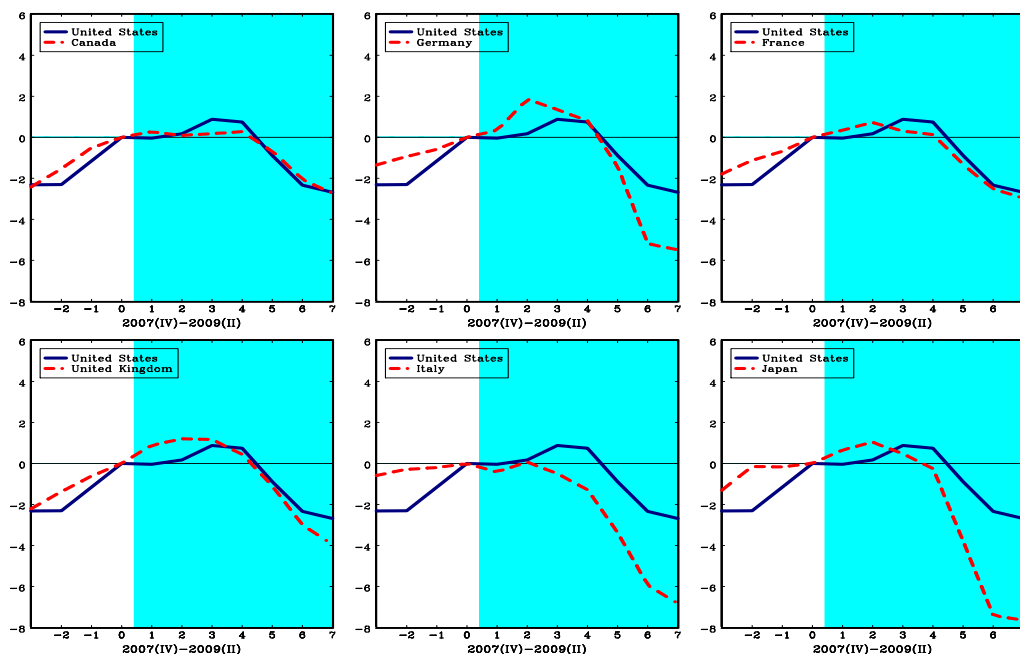
This paper is motivated by two observations about the US 2008-2009 crisis. The first is that the crisis has been characterized by a high degree of international synchronization as most developed countries have experienced large macroeconomic contractions at around the same time. The second observation is that, although employment in US has fallen dramatically, productivity in US has actually increased. As we will document below, these two features of the recent crisis differentiate the recent recession from many of the previous recessions experienced by the US economy.

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1.1 International co-movement

Figure 1 plots the US GDP against the GDP of the other G7 countries during the recent recession, up to the second quarter of 2009. The numbers are percent deviations from the level of GDP in the quarter preceding the beginning of the recession identified by the NBER Business Cycle Dating Committee (fourth quarter of 2007). Four quarters before the official recession are also plotted. The figure reveals the strong co-movement in macroeconomic activity among the G7 countries.

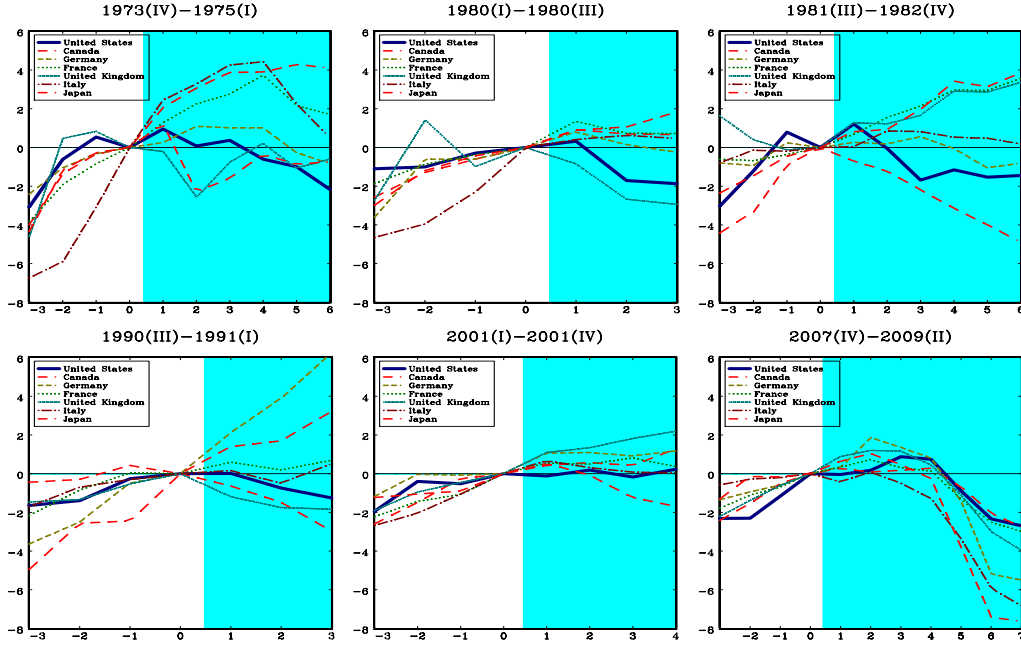
Figure 1: The dynamics of GDP during the 2008 recession: US vs. other G7 countries.



To examine whether the international synchronization of the recent recession differs from previous contractions, Figure 2 plots the GDP dynamics for the G7 countries in six of the most recent US recessionary episodes: one recession experienced in the first half of the 1970s, two in the first half of 1980s, one in the early 1990s and two in the 2000s. A quick glance at the figure shows that the macroeconomic synchronization of the US with other G7 countries has been significantly stronger in the recent recession. While the G7 countries experienced very different GDP dynamics during the previous US recessions, in the most recent contraction all countries moved in the same direction.

The higher cross-country synchronization of the recent recession can also be seen in Figure 3

Figure 2: The dynamics of GDP during the six most recent recessions in the G7 countries.

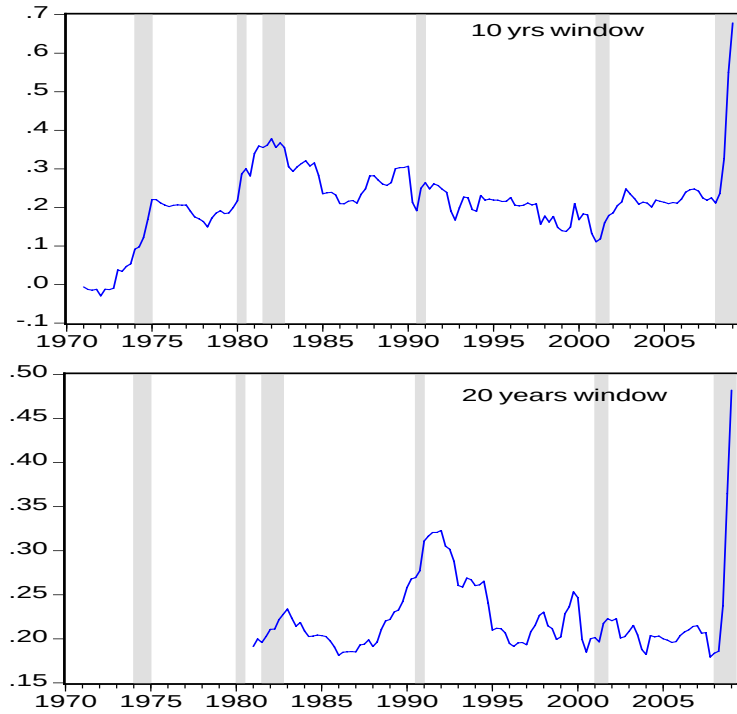


which plots the average correlation of US GDP with the GDP of each of the other G7 countries. The correlations are computed on rolling windows of 10 and 20 years. The dates in the graph correspond to the end points of the window used to compute the correlation. Although the figure shows that the increase in correlation can also be seen in previous recessions, the current contraction stands out as the one that marks an increase in correlation larger than in earlier periods. For a similar point see also Imbs (2010).

The dramatic increase in co-movement is also observed in other variables, in particular asset prices and employment. The top two panels of Figure 4 plot the growth rate of stock prices in the 1990s and in the 2000s.¹ The bottom two panels plot the growth rate of employment in the US and in the G6 during the the same two subperiods. The figure shows quite clearly how the last recession (and more in general the entire last decade) represents a period of high international synchronization between US and the rest of the developed world.

¹The stock prices in US are the MSCI BARRA US stock market index, while stock prices in the G6 are computed using the MSCI BARRA EAFE + Canada index which is an average of stock prices in advanced economies except the US.

Figure 3: Average rolling correlations of US GDP with other G7 countries.

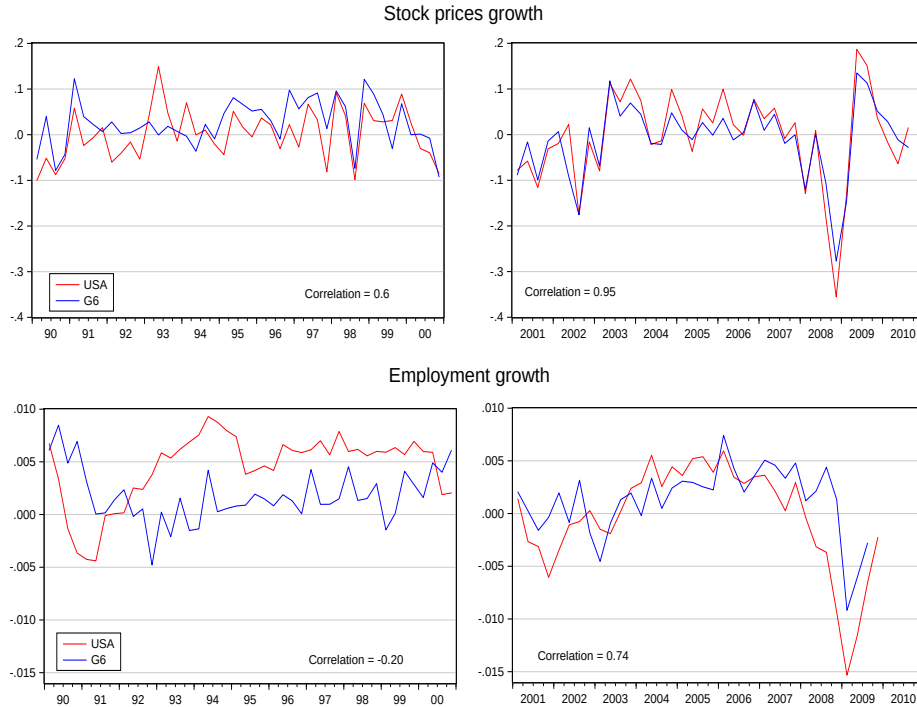


1.2 Labor input and labor productivity

Figure 5 plots working hours and labor productivity (output per hour) in the private non farm sector of the US economy for the six most recent recessions. The last panel shows that in the recent recession labor productivity has actually increased for most of the period. This pattern can also be seen in the 2001 recession. By contrast, in the first four recessions, labor productivity has slowed down markedly and its level at the end of the recession was not higher than before the recession.

The different behavior of productivity and labor during the two most recent recessions reflects a more general pattern for which the association between productivity and labor has declined sharply in the US economy. As a first visual measure of this decline Figure 6 reports the Hodrick-Prescott filtered series of output per hour and hours in the private non-farm business sector in the period 1947-1989 (the left panel) and the same two series in the period 1990-2010 (the right panel). Notice that in the first panel the two series exhibit positive co-movement, with productivity generally leading employment, while in the second the co-

Figure 4: International co-movement in stock prices and employment



movement is sharply negative and no clear leading pattern is discernible.

Figure 7 plots rolling correlations of productivity growth (growth in output per hour or output per employees in the private non-farm business sector) and labor growth (growth in hours worked or number of employees in the private nonfarm business sector) computed on 10 years rolling windows. The figure shows a drastic drop in the correlation between productivity and labor starting in the 1990s and it has become negative in the most recent decade. This pattern is also documented in Gali and Gambetti (2009).

Is the negative correlation between labor productivity and labor also a feature of other countries? Since comparable data on hours are not available for all G6 countries we use simply GDP per worker as a measure of productivity and employment as a measure of labor. Figure 8 plots the 10 years rolling correlation of growth in GDP per worker and growth in employment for the US and for the remaining G6 countries. Notice that even for the G6 countries the correlation between labor input and labor productivity in recent years is negative.

Figure 5: Labor productivity (output per hour) and hours in recessions

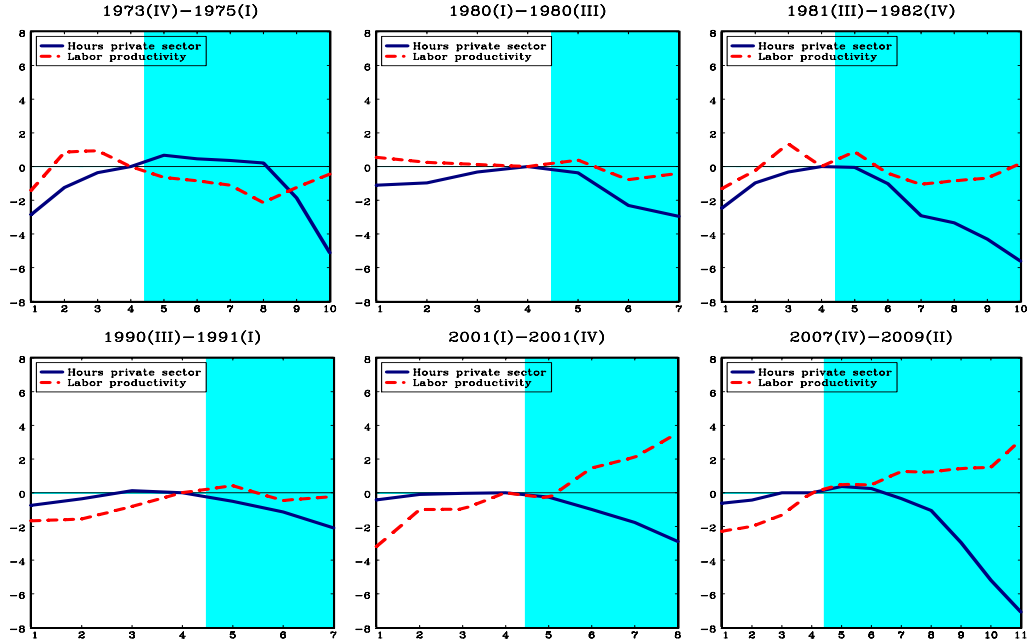


Figure 6: Hp filtered labor productivity and hours

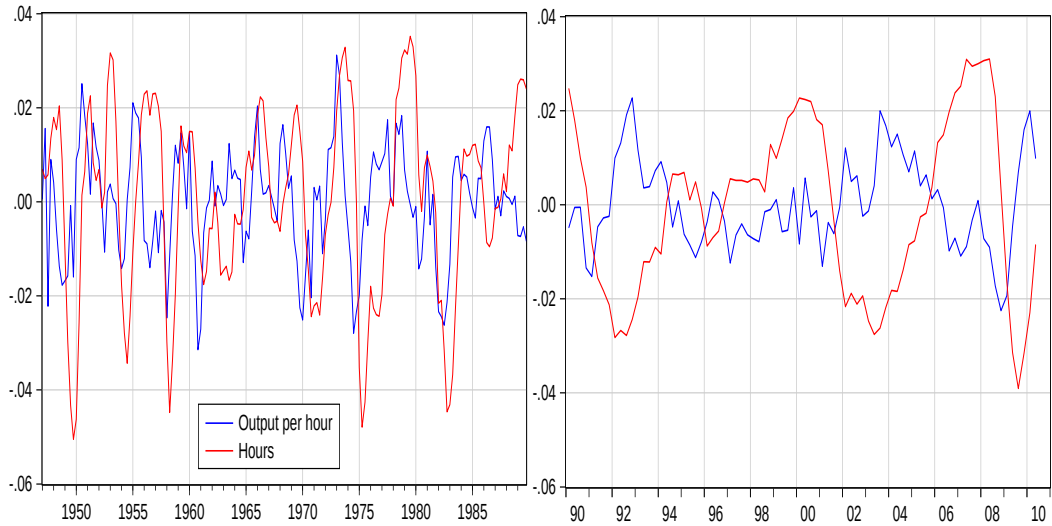
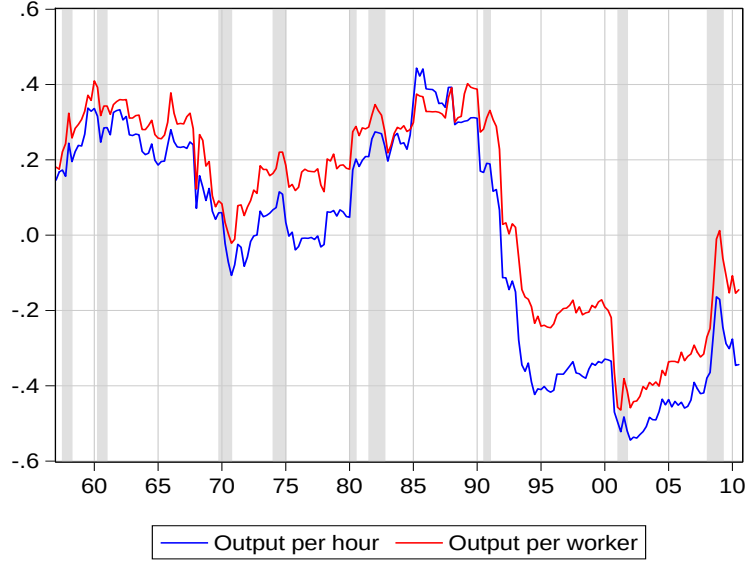


Figure 7: US: Rolling correlations of productivity and labor input



1.3 Hints from the data

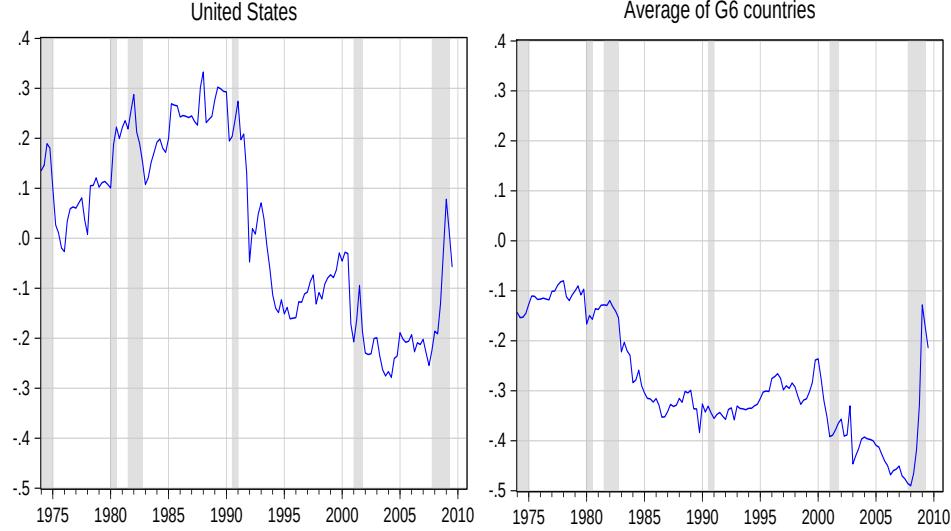
The evidence discussed above points to two distinguishing features of the US business cycle over the last decade and in particular during the recent crisis:

1. Historically high international synchronization with other developed economies
2. Historically low association between labor productivity and labor

Both findings suggest that in the recent decade shocks different from technological disturbances may have played a more prominent role in generating business cycle fluctuations. In particular, the observation that labor productivity is negatively associated with labor input casts doubts on the relevance of productivity shocks as the major source of macroeconomic fluctuations.

The cross-country synchronization in a period of high capital market integration is also difficult to reconcile with technology shocks. When countries are financially integrated, the standard international RBC model, such as the one studied by Backus, Kehoe and Kydland (1992), predicts that country-specific technology shocks generate divergent macroeconomic responses, unless the productivity shocks are internationally correlated. See, for example, Heathcote and Perri (2004). However, if productivity shocks that are internationally correlated were

Figure 8: US and G6: Rolling correlations of GDP per worker and employment growth



the main source of business cycle fluctuations, we should observe a higher correlation between productivity and labor. It is then difficult to reconcile the hypothesis of productivity driven recessions with the fact that productivity kept growing during the most recent contractions.

Since productivity shocks cannot be the major force underlying macroeconomic fluctuations in the recent decade, what other shocks can reconcile the two facts outlined above? In this paper we argue that ‘credit shocks’ are a plausible candidate. In particular, we show that credit shocks can generate greater international synchronization and lower correlation between productivity and labor in an environment with international mobility of capital.

The empirical relevance of credit shocks has also been explored in Jermann and Quadrini (2009) but in closed economies. In this paper we show that these shocks are also important for understanding the macroeconomic dynamics of economies that are financially integrated as these shocks can generate significant cross-country co-movements in macroeconomic variables and asset prices. Besides the consideration of an open-economy framework and studying the international spillover of credit shocks, the current paper differs from Jermann and Quadrini in other dimensions. In particular, we study equilibria in which producers display significant precautionary behavior and the enforcement constraints are only occasionally binding. Although this makes the solution of the model more challenging, we are also able to generate interesting dynamics that can be related to the dynamics of liquidity observed in the data and to the

asymmetry between expansions and recessions.

1.4 The theoretical framework

We consider a model in which firms have an incentive to borrow but the debt is constrained by credit frictions resulting from the limited enforceability of debt contracts. The ability to borrow is subject to random disturbances that we call ‘credit shocks’. Good (credit) times are periods in which borrowers have lower incentives to default and, as a result, lenders are willing to provide more credit. In bad (credit) times the incentive to default is higher and lenders cut on lending. Following a credit cut, borrowers are forced to restructure their financial position by increasing equity. Because raising equity quickly is costly, the equity holders ask for a higher return which increases the financial cost for the firm. Since the financial cost contributes to the cost of hiring workers and acquiring investments, the demands for labor and investment decline.

In this environment a credit contraction in one country spills over other countries even if foreign borrowers are not forced to cut their borrowing. To better illustrate the mechanism, consider a world composed of two countries: country A and country B . A credit contraction in country A requires a substitution between debt and equity for firms operating in this country. In a closed economy, the increase in equity must be provided by investors of country A . At the same time, the market for loans clears locally without any spillover to country B . Thus, when economies are not financially integrated, a credit contraction in country A does not affect country B .

Let’s now consider the case in which the two countries are financially integrated. In this case firms located in country A can raise equity not only from investors in country A but also from investors in country B . Having access to a larger pool of suppliers, the cost of raising funds increases less, and therefore, the macroeconomic impact on country A is smaller. Essentially, financial integration makes the supply of funds to the producers of one country more elastic. Although the increase in the cost of equity in country A is smaller, the financing cost increases also for firms located in country B since now there is a single worldwide market (law of one price). Through the higher worldwide cost of financing, the credit contraction in country A affects also country B .

The above description clarifies why a credit shock in country A spills to country B , generating a recession in both countries. What happens to the productivity of labor? Because TFP does not change and the share of labor in production is smaller than one, a reduction in em-

ployment increases the productivity of labor. Thus, the model generates a negative correlation between productivity and hours.

Our paper is related to two recent contributions: Dedola & Lombardo (2010) and Devereux & Yetman (2010). Both studies investigate the international transmission of shocks in models with financial market frictions. They also show that shocks to the financial system can generate cross-country spillovers in macroeconomic variables. Also related is the study of Enders, Kollmann & Muller (2010). This paper introduces a banking sector in an international model and shows that shocks to this sector could have important effects on the global economy. The theoretical findings of these papers are consistent with the empirical results of Helbling, Huidrom, Kose & Otrok (2010) according to which credit market shocks matter in explaining global business cycles, especially during the 2009 global recession.

1.5 Outline of the paper

The remaining of this paper is organized in three main sections. In Section 2 we present first a simpler version of the model without capital accumulation. This allows us to derive some results analytically, providing simple intuitions for the quantitative results obtained with the more general model. Section 3 extends the model by adding capital accumulation and Section 4 presents the quantitative exercise.

2 The model without capital accumulation

There are two sectors populated by agents with different investment opportunities. In the first sector there is a continuum of risk-averse investors who are the shareholders of firms and discount the future at rate β . In the second sector there is a continuum of risk-averse workers with discount factor $\delta > \beta$. The different discounting between the owners of firms (investors) and workers implies that firms borrow from workers subject to the enforcement constraints we will describe below. This result is based on the assumption that the market for the ownership of firms is segmented, that is, only investors have access to this market while workers can only save in the form of bonds.

The assumption that agents are risk-averse implies that the effective discount rates for investors and workers are not constant in equilibrium but fluctuate in response to aggregate shocks. As we will see, fluctuations in the effective discount rates play a central role in the analysis of this paper. To facilitate the presentation, we first describe the closed-economy

version of the model. Once we have characterized the autarkic equilibrium, it will be trivial to extend it to the environment with international mobility of capital.

2.1 Investors and firms

Investors have lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. They are the owners of firms and derive income only from dividends. Denoting by d_t the dividends paid by firms, the effective discount factor for investors is $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$. This is also the discount factor used by firms since they maximize shareholders' wealth.

Firms operate the production function $F(z_t, h_t) = z_t \bar{k} h_t^\nu$, where $\bar{k}$ is a fixed input of capital, h_t is the variable input of labor, and z_t is a stochastic variable affecting the technology of all firms (total factor productivity). The parameter ν is smaller than 1 implying decreasing returns to scale in the variable input. The input of capital is fixed and does not depreciate. Therefore, in this version of the model we can think of $\bar{k}$ as a normalization constant. We will make the accumulation of capital endogenous in the next section when we present the general model.

Firms start the period with intertemporal debt b_t . Before producing they choose the labor input h_t , the dividends d_t , and the next period debt b_{t+1} . The budget constraint is

$$b_t + w_t h_t + d_t = F(z_t, h_t) + \frac{b_{t+1}}{R_t},$$

where R_t is the gross interest rate.

The payments of wages, $w_t h_t$, dividends, d_t , and current debt net of the new issue, $b_t - b_{t+1}/R_t$, are made before the realization of revenues. This implies that the firm faces a cash flow mismatch during the period. The cash needed at the beginning of the period is $w_t h_t + d_t + b_t - b_{t+1}/R_t$. From the budget constraint we can verify that this is equal to the cash revenue $F(z_t, h_t)$. To cover the cash flow mismatch, the firm contracts an intra-period loan which is equal to the liquidity need $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t$. This loan is repaid at the end of the period, after the realization of revenues.

Debt contracts are not perfectly enforceable. At the end of the period the firm can divert the liquidity l_t and default. Default gives the lender the right to liquidate the firm's capital. Suppose that the liquidation value is $\xi_t \bar{k}$, where ξ_t is a stochastic variable that depends on market conditions. This is the value that guarantees the firm's liabilities. Since default arises at the end of the period, the total liabilities of the firm are $l_t + b_{t+1}/R_t$. To ensure that the

firm does not default, the total debt is subject to the enforcement constraint²

$$\xi_t \bar{k} \geq l_t + \frac{b_{t+1}}{R_t}.$$

Since fluctuations in ξ_t affect the ability to borrow, we will call it ‘credit shock’. It can also be interpreted as an asset price shock because it affects the value of selling the firm’s assets. The asset price shock, however, is purely exogenous in this framework.³

To illustrate the role played by the stochastic liquidation value ξ_t , consider a pre-shock equilibrium in which the enforcement constraint is binding. Starting from this equilibrium, suppose that ξ_t decreases. We now show that in response to the decline in ξ_t the firm is forced to reduce either the dividends and/or the input of labor.

Let’s start considering the case in which the firm is unwilling to change the input of labor. This implies that the intra-period loan $l_t = F(z_t, h_t)$ also does not change. Thus, the only way to satisfy the enforcement constraint is by reducing the intertemporal debt b_{t+1} . We can then see from the budget constraint, $w_t h_t + d_t + b_t = b_{t+1}/R_t + F(z_t, h_t)$, that the reduction in b_{t+1} requires a reduction in dividends. Thus, the firm is forced to substitute debt with equities.

Alternatively the firm could keep the dividend payments unchanged but reduce the input of labor. Since the reduction in h_t reduces the intra-period loan, $l_t = F(z_t, h_t)$, this will also ensure that the enforcement constraint is satisfied. Therefore, after a negative shock to ξ_t , the firm faces a trade-off: paying lower dividends or cutting employment. As we will see, the optimal choice will depend on the relative cost of changing these two variables, which depends on the stochastic discount factor for investors $m_{t+1} = \beta u_c(d_{t+1})/u_c(d_t)$.

²Here we adopt a similar approach to Hart and Moore. After defaulting the firm bargains the repaying with the lender. Under the assumption that the firm has all the bargaining power, the lender would recover only the threat value $\xi_t \bar{k}$. In anticipation of this, the lender will never lend more than $\xi_t \bar{k}$.

³We can also think of ξ_t as a liquidity shock along the lines of Kiyotaki and Moore (2008).

Firm's problem: The optimization problem of the firm can be written recursively as follows:

$$V(\mathbf{s}; b) = \max_{d, h, b'} \left\{ d + Em'V(\mathbf{s}'; b') \right\} \quad (1)$$

subject to:

$$b + d = F(z, h) - wh + \frac{b'}{R} \quad (2)$$

$$\xi \bar{k} \geq F(z, h) + \frac{b_{t+1}}{R_t}, \quad (3)$$

where $\mathbf{s}$ are the aggregate states, including the shocks z and ξ , and the prime denotes the next period variable. The enforcement constraint takes into account that the intra-period loan is equal to the firm's output, that is, $l_t = w_t h_t + d_t + b_t - b_{t+1}/R_t = F(z_t, h_t)$.

In solving this problem the firm takes as given all prices and the first order conditions are

$$F_l(z, h) = \frac{w}{1 - \mu}, \quad (4)$$

$$REm' = 1 - \mu, \quad (5)$$

where μ is the Lagrange multiplier for the enforcement constraint. These conditions are derived under the assumption that dividends are always positive, which will be the case if the investors' utility satisfies $u_c(0) = \infty$. The detailed derivation is in Appendix A.

We can see from condition (4) that limited enforcement imposes a wedge in the demand for labor. This derives from the fact that the labor input needs to be financed and, because of the agency problem, part of the financing has to come from equity (through lower payment of dividends). As long as the cost of equity ($1/Em'$) is greater than the cost of debt (the interest rate R), expanding the input of labor is costly in the margin because the firm needs to substitute debt with equity. It is then the equity premium $1/Em' - R$ that determines the labor wedge as can be seen from condition (5).⁴ This wedge is strictly increasing in μ and disappears when $\mu = 0$, that is, when the enforcement constraint is not binding. In this case

⁴Notice that we are using the term 'equity premium' to denote the differential between the expected shareholders' return and the interest rate on bonds. Since shareholders and bondholders are different agents, the equity premium is not only determined by the cost of risk (risk premium).

the equity premium becomes zero.

Some (partial equilibrium) properties The characterization of the firm's problem in partial equilibrium provides helpful insights about the property of the model once extended to a general equilibrium set-up. For partial equilibrium we mean the allocation achieved when the interest rate and the wage rate are both exogenously given and constant.

Under these conditions, equation (5) shows that μ decreases with the expected discount factor Em' . A decrease in ξ , that is, a negative credit shock, makes the enforcement constraint tighter. Because firms reduce the payment of dividends, the investors's consumption has to decrease. This induces a decline in the discount factor $m' = \beta u_c(d')/u_c(d)$ and an increase in the multiplier μ (condition (5)). Condition (4) then shows that the demand for labor declines.

Intuitively, when the credit conditions become tighter, firms need to rely more on equity financing and less on debt. However, it is costly to increase equity in the short-term since investors must cut consumption and their utility is concave. Because of this, the firm does not find optimal in the short-term to raise enough equity to keep the pre-shock production scale and it cuts employment. If investors' utility were linear (risk-neutrality), the discount factor would be equal to $Em' = \beta$ and the credit shock would not affect employment. This also requires that the interest rate does not change, which is the case in the partial equilibrium considered here. In the general equilibrium, of course, prices also change. In particular, movements in the demand of credit and labor affect the interest rate R and the wage rate w . To derive the aggregate effects we need to close the model and characterize the general equilibrium.

2.2 Closing the model and general equilibrium

There is a representative households/worker with lifetime utility $E_0 \sum_{t=0}^{\infty} \delta^t U(c_t, h_t)$, where c_t is consumption, h_t is labor and δ is the intertemporal discount factor. For the later analysis of the general model, it will be convenient to assume that the period-utility takes the form

$$U(c_t, h_t) = \log(c_t) - \alpha \frac{h_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}.$$

Workers have a higher discount factor than entrepreneurs, that is, $\delta > \beta$. This condition ensures that the enforcement constraint is 'occasionally' binding. Another key assumption is that there is market segmentation, that is, workers hold bonds issued by firms but they cannot

buy shares of firms. The budget constraint is

$$w_t h_t + b_t = c_t + \frac{b_{t+1}}{R_t},$$

and the first order conditions for labor, h_t , and next period bonds, b_{t+1} , are

$$U_h(c_t, h_t) + w_t U_c(c_t, h_t) = 0, \quad (6)$$

$$\delta R_t E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \right\} = 1. \quad (7)$$

General equilibrium: We can now define a competitive equilibrium. The sufficient set of aggregate states $\mathbf{s}$ are given by the level of productivity, z , the credit conditions, ξ , and the aggregate stock of bonds, B .

Definition 2.1 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for (i) workers' policies $h(\mathbf{s})$, $c(\mathbf{s})$, $b(\mathbf{s})$; (ii) firms' policies $h(\mathbf{s}; b)$, $d(\mathbf{s}; b)$ and $b(\mathbf{s}; b)$; (iii) firms' value $V(\mathbf{s}; b)$; (iv) aggregate prices $w(\mathbf{s})$, $R(\mathbf{s})$ and $m(\mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\mathbf{s})$. Such that: (i) household's policies satisfy the optimality conditions (6)-(7); (ii) firms' policies are optimal and $V(\mathbf{s}; b)$ satisfies the Bellman's equation (1); (iii) the wage and interest rates are the equilibrium clearing prices in the markets for labor and bonds, and the discount factor for firms is $m(\mathbf{s}') = \beta u_c(d_{t+1})/u_c(d_t)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic processes for z and ξ .*

To illustrate the main properties of the model, we look at some special cases. Consider first the economy without shocks. In this economy the enforcement constraint binds in the steady state equilibrium. To see this, consider the first order condition for the bond, equation (7), which in a steady state becomes $\delta R = 1$. Using this condition to eliminate R in (5) and taking into account that in a steady state $Em' = \beta$, we get $\beta/\delta = 1 - \mu$. Because $\delta > \beta$ by assumption, the lagrange multiplier μ is greater than zero. Firms want to borrow as much as possible because the cost of borrowing—the interest rate—is smaller than their discount rate.

In a model with uncertainty, however, the constraint may not be always binding. However, they will become binding after a sufficiently large and unexpected decline in ξ . In this case firms will be forced to cut dividends and this affects the discount factor Em' . Furthermore,

the change in the demand for credit impacts on the equilibrium interest rate. Using condition (5) we can see that these changes affect the multiplier μ , which in turn impacts on the demand for labor (see equation (4)). On the other hand, a sufficiently large increase in ξ may make the enforcement constraint non-binding. This implies that the response to a positive credit shock is bounded since the multiplier μ cannot be negative. Therefore, the responses of the economy to credit shocks could be asymmetric: negative shocks induce large falls in employment and output while the impacts of positive shocks is moderate.

The asymmetric responses caused by occasionally binding constraints is also a feature of the model studied in Mendoza (2010). This paper, however, does not consider credit shocks, which is the main focus of the current paper. Furthermore, the analysis is limited to the case of a small open economy, and therefore, it does not address one of the central issues studied in our paper, that is, international spillover and comovement.

2.3 Capital mobility

Let's consider now two countries with the same size, preferences and technology as described in the previous section. Although we characterize here only the case with two symmetric countries, the model can be easily extended to any number of countries and with different degrees of heterogeneity. The shocks z and ξ are country-specific and they follow a joint Markov process.

Investors/firms: We have to specify what agents can do in a financial market that is internationally integrated. For investors, the opening of the international market allows them to hold shares of foreign firms, in addition to their domestic holdings. Because firms are subject to country specific shocks, investors would gain from diversifying the cross-country ownership. Therefore, in a financially integrated economy, investors choose to own the world-wide portfolio of shares and we have a representative 'worldwide' investor. Having a common representative shareholder, firms in different countries will use the same discount factor $m_{t+1} = \beta u_c(d_{t+1} + d_{t+1}^*)/u_c(d_t + d_t^*)$, where investors' consumption is the sum of dividends paid by domestic firms, d_t , plus the dividends paid by foreign firms, d_t^* .⁵ From now on we will use the star superscript to denote variables pertaining to the foreign country.

Besides the common discount factor, firms continue to solve problem (1) and the first order

⁵Notice that this follows from the assumption that investors' utility depends only on consumption. If investors derived utility also from leisure, a perfect diversification of portfolio will not be necessarily optimal.

conditions are given by equations (4) and (5). Let's focus on condition (5), which we rewrite here for both countries:

$$\begin{aligned} R_t Em_{t+1} &= 1 - \mu_t, \\ R_t^* Em_{t+1}^* &= 1 - \mu_t^*. \end{aligned}$$

The first condition is for firms located in the domestic country and the second if for firms located in the foreign countries. Since the discount factor is common to domestic and foreign firms, that is, $Em_{t+1} = Em_{t+1}^*$, and the interest rate is equalized across countries, $R_t = R_t^*$, the above conditions imply that the lagrange multiplier will also be equalized, that is, $\mu_t = \mu_t^*$. Therefore, independently of which country is hit by a shock, if the enforcement constraint is binding in one country, it will also be binding in the other. This also implies that the domestic and foreign labor wedges $1/(1 - \mu_t)$ and $1/(1 - \mu_t^*)$ (see condition (4)) are equalized across countries. This property is crucial for understanding the cross-country impact of credit shocks.

Households/workers: We keep the assumption that financial markets are segmented and households/workers cannot hold shares of firms. With capital mobility, however, they can engage in international financial transactions with foreign workers. More precisely, in addition to holding bonds issued by domestic firms, domestic workers can buy state contingent claims from foreign workers. We still assume that firms borrow from domestic workers but they cannot sign state contingent contracts with workers. The assumption that firms borrow only from domestic workers is without loss of generality: whether they borrow domestically or abroad is irrelevant in an integrated capital market. The unavailability of state-contingent claims between firms and workers is essential to retain market incompleteness.

Denote by $n_{t+1}(\mathbf{s}_{t+1})$ the units of consumption goods received at time $t + 1$ by domestic workers if the aggregate states are $\mathbf{s}_{t+1}$. These are worldwide states, and therefore, they include aggregates states of both countries, as will be made precise below. Of course, in equilibrium, the consumption units received by workers in the domestic country must be equal to the consumption units paid by workers in the foreign country, that is, $n_{t+1}(\mathbf{s}_{t+1}) + n_{t+1}^*(\mathbf{s}_{t+1}) = 0$. This must be satisfied for any possible realization of the states $\mathbf{s}_{t+1}$.

The budget constraint of a worker in the domestic country is

$$w_t h_t + b_t + n_t = c_t + \frac{b_{t+1}}{R_t} + \int_{\mathbf{s}_{t+1}} n_{t+1}(\mathbf{s}_{t+1}) q(\mathbf{s}_{t+1}) / R_t,$$

where $q_t(\mathbf{s}_{t+1})/R_t$ is the unit price of the contingent claims.

Given the specification of the utility function, the first order conditions for the choice of labor, h_t , next period bonds, b_{t+1} , and foreign claims, $n_{t+1}(\mathbf{s}_{t+1})$, are

$$\alpha h_t^\gamma c_t = w_t, \quad (8)$$

$$\delta R_t E_t \left(\frac{c_t}{c_{t+1}} \right) = 1, \quad (9)$$

$$\delta R_t \left(\frac{c_t}{c_{t+1}(\mathbf{s}_{t+1})} \right) p(\mathbf{s}_{t+1}) = q(\mathbf{s}_{t+1}), \quad \text{for all } \mathbf{s}_{t+1}, \quad (10)$$

where $p(\mathbf{s}_{t+1})$ is the probability (or probability density) of the aggregate states in the next period for the world economy.

Since in equilibrium the prices and probabilities of the contingencies are the same for domestic and foreign workers, condition (10) implies that

$$\frac{c_t}{c_t^*} = \frac{c_{t+1}(\bar{\mathbf{s}}_{t+1})}{c_{t+1}^*(\bar{\mathbf{s}}_{t+1})} = \chi. \quad (11)$$

Therefore, the ratio of consumption of domestic and foreign workers remains constant over time. This is a well known property of environments with a full set of state-contingent claims. In our environment the constancy of the consumption ratio is among workers (and among investors) but not between workers and investors because of the assumption of market segmentation.

Before continuing we would like to clarify that the assumption of contingent claims among workers is not essential for the results of this paper. We could simply assume that workers can engage in international non-contingent lending and borrowing only. Or equivalently, that firms can engage in international borrowing. However, the availability of contingent claims greatly simplifies the characterization of the equilibrium because it allows us to reduce the number of sufficient state variables. This property is especially convenient once we add capital accumulation.

Aggregate states and equilibrium: We can now define the equilibrium for the open-economy version of the economy. The aggregate states $\mathbf{s}$ are given by the exogenous variables z , ξ , z^* , ξ^* , the financial liabilities of firms, B_t and B_t^* , and the net foreign asset position of domestic firms, N_t . Since in equilibrium the net foreign asset position of domestic firms is the

negative of the foreign position, once we know B_t , B_t^* and N_t we also know the total wealth of domestic workers, $B_t + N_t$, and foreign workers, $B_t^* - N_t$. Therefore, the claims purchased by households are contingent on $\mathbf{s}_t = (z, \xi, z^*, \xi^*, B_t, B_t^*, N_t)$.

Definition 2.2 (Recursive equilibrium) *A recursive competitive equilibrium is defined by a set of functions for: (i) households' policies $h(\mathbf{s})$, $c(\mathbf{s})$, $b(\mathbf{s})$, $n(\mathbf{s}; \mathbf{s}')$, $h^*(\mathbf{s})$, $c^*(\mathbf{s})$, $b^*(\mathbf{s})$, $n^*(\mathbf{s}; \mathbf{s}')$; (ii) firms' policies $h(\mathbf{s}; b)$, $d(\mathbf{s}; b)$, $b(\mathbf{s}; b)$, $h^*(\mathbf{s}; b)$, $d^*(\mathbf{s}; b)$, $b^*(\mathbf{s}; b)$; (iii) firms' values $V(\mathbf{s}; b)$ and $V^*(\mathbf{s}; b)$; (iv) aggregate prices $w(\mathbf{s})$, $w^*(\mathbf{s})$, $R(\mathbf{s})$, $m(\mathbf{s}, \mathbf{s}')$, $q(\mathbf{s}; \mathbf{s}')$; (v) law of motion for the aggregate states $\mathbf{s}' = \Psi(\bar{\mathbf{s}})$. Such that: (i) household's policies satisfy the optimality conditions (6)-(10); (ii) firms' policies are optimal and satisfy the Bellman's equation (1) for both countries; (iii) the wages clear the labor markets; the interest rates and the price for contingent claims clear the financial markets; the discount rate used by firms satisfies $\bar{m}(\mathbf{s}, \mathbf{s}') = \beta u_c(d_{t+1} + d_{t+1}^*)/u_c(d_t + d_t^*)$; (iv) the law of motion $\Psi(\mathbf{s})$ is consistent with the aggregation of individual decisions and the stochastic process for z , ξ , z^* , ξ^* .*

The only difference with respect to the equilibrium in the closed economy is that there is the additional market for foreign claims and the discount factor for firms is given by the worldwide representative investor. The market clearing condition for the foreign claims is $N(\mathbf{s}') + N^*(\mathbf{s}') = 0$. This is in addition to the clearing conditions for the domestic bond markets (lending to firms).

Although the general definition of the recursive equilibrium is based on the set of state variables $\mathbf{s}_t = (z, \xi, z^*, \xi^*, B_t, B_t^*, N_t)$, we can use some of the properties derived above and characterize the equilibrium using a smaller set of states. Let $W_t = B_t + B_t^*$ be the worldwide wealth of households/workers. This is the sum of bonds issued by domestic firms, B_t , and foreign firms, B_t^* . Then using the fact that the consumption ratio of domestic and foreign workers is constant at χ and the employment policy of firms does not depend on the individual debt, the recursive equilibrium can be characterized using the state variables $\mathbf{s}_t = (z, \xi, z^*, \xi^*, W_t)$. Essentially, the assumption of cross-country risk-sharing among workers and among investors (but not between workers and investors) allows us to reduce the number of 'endogenous' states to only one variable.

Intuitively, by knowing W_t , we know the worldwide liability of firms, but not the distribution between domestic and foreign firms. However, to characterize the firms' policies, we only need to know the worldwide debt, which is equal to W_t . Since investors own an internationally diversified portfolio of shares, effectively there is only one representative global investor. It is

as if there is a representative firm with two units: one unit located in the domestic country and the other in the foreign country. Since both units have a common owner, it does not matter how the debt is distributed between the two units. What matters from the perspective of the investor, is the total debt and the total payment of dividends. Total workers' wealth is also a sufficient statistics for the characterization of the workers' policies since the consumption ratio between domestic and foreign households remains constant at χ . Therefore, once we solve for the aggregate worldwide consumption, country-specific consumption can be determined by χ . This property limits the computational complexity of the model, making feasible the use non-linear approximation methods. We will come back to this point after the description of the general model with capital accumulation.

We are not ready to prove the following proposition about the impact of a financial shock.

Proposition 2.1 *A credit shock to the domestic country (change in ξ_t) has the same impact on employment and output of domestic and foreign countries.*

Proof 2.1 *We have already shown that the Lagrange multiplier μ_t is common for the firms of both countries. If the two firms have the same productivity and the wage ratio in the two countries does not change, the first order conditions for the firms imply that they all choose the same employment and investment. To complete the proof we have to show that the ratio of wages of the two countries stays constant. Because firms in both countries have the same demand for labor and the ratio of workers' consumption remains constant, the first order condition for the supply of labor implies that the wage ratio between the two countries does not change.*

Before turning to capital accumulation, we would like to emphasize another feature of the model. As we have seen, the credit shock of one country spills over other countries if the two economies are financially integrated. However, the impact on the originating country is smaller when capital markets are integrated.

To see this, consider the channel through which a credit shock affects employment. After a credit contraction the firm is forced to reduce the payment of dividends and this decreases the discount factor $m' = \beta u_c(d' + d^*)/u_c(d + d^*)$. From condition (5) we can see that this increases μ which in turn decreases the demand for labor (see condition (4)). The bigger the reduction in dividends, relatively to investors' consumption, the larger the impact on the discount factor, and therefore, on the demand of labor. In an economy that is financially integrated, the change

in dividends induced by the credit contraction in one country leads to a lower reduction in the consumption of investors since they are diversified. As a result, the decrease in the discount factor, and therefore, the impact on the demand of labor, are smaller. This can be proved analytically for the limiting case of a small open economy.

Proposition 2.2 *Consider a credit shock only to the domestic country. If the country is a small open economy and $\psi = 0$, the credit shock has not effect on domestic (and foreign) employment.*

Proof 2.2 *In the case of a small open economy, investors are perfectly diversified internationally and the reduction in the dividends paid in country 1 is negligible relatively to investors' consumption. Therefore, the discount factor does not change, which implies that the demand for labor in country 1 and elsewhere remains unchanged. At the same time, the reduction in the demand for debt is also negligible relative to the size of the international market. This implies that the interest rate does not change. Furthermore, the wealth effects on the supply of labor are negligible, leaving the wage rate unaltered.*

3 Model with capital accumulation

We now relax the assumption that the input of capital is fixed. This introduces additional state variables that increase the computational complexity of the model, unless we use local approximation techniques. However, when the enforcement constraint is only occasionally binding, we need to use global approximation techniques. Unfortunately, these techniques are computationally intensive and become quickly impractical when we have a large numbers of state variables. Therefore, in order to limit the number of state variables, we will make some special assumptions about the functional form for the production function.

Investors-firms: The production function takes the form:

$$y_t = z_t(K_t + K_t^*)^{1-\theta} k_t^\theta h_t^\nu \equiv F(z_t, K_t + K_t^*, k_t, h_t),$$

where K_t is the aggregate capital in the domestic country and K_t^* in the foreign country, k_t is the individual input of capital and h_t is the input of labor. We assume that $\theta + \nu < 1$.

The dependence of the production function from the worldwide stock of capital, $K_t + K_t^*$, captures positive externalities. The purpose of the externalities is to have constant returns in the reproducible factor (AK model), without losing the competitive structure of the model, that is, each individual producer runs a production function with decreasing returns. As we will see, the AK structure makes simplifies the numerical solution. This is the only motivation for using this particular structure of the production function.

Given i_t the flow of investment, the stock of capital evolves according to

$$k_{t+1} = (1 - \tau)k_t + \Upsilon\left(\frac{i_t}{k_t}\right)k_t,$$

where τ is the depreciation rate and the function $\Upsilon(\cdot)$ is strictly decreasing and concave, capturing adjustment costs in investment. The adjustment cost prevents an excessive volatility of investment when the economy is financially integrated. This is a common element of international macro models.

With capital accumulation the budget constraint of the firm becomes

$$b_t + d_t + i_t = F(z_t, K_t, k_t, h_t) - w_t h_t + \frac{b_{t+1}}{R_t},$$

and the enforcement constraint

$$\xi_t k_{t+1} \geq F(z_t, k_t, h_t) + \frac{b_{t+1}}{R_t}.$$

We will now take advantage of the AK structure of the production function and normalize the model by the worldwide stock of capital $K_t + K_t^*$. Using the tilde sign to denote normalized variables, we can rewrite the budget constraint, law of motion for capital and enforcement constraint as follows:

$$\tilde{b}_t + \tilde{d}_t + \tilde{i}_t = F(z_t, \tilde{k}_t, h_t) - \tilde{w}_t h_t + \frac{g_t \tilde{b}_{t+1}}{R_t}, \quad (12)$$

$$g_t \tilde{k}_{t+1} = (1 - \tau)\tilde{k}_t + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\tilde{k}_t, \quad (13)$$

$$\xi_t g_t \tilde{k}_{t+1} \geq F(z_t, \tilde{k}_t, h_t) + \frac{g_t \tilde{b}_{t+1}}{R_t}. \quad (14)$$

The variable $g_t = (K_{t+1} + K_{t+1}^*)/(K_t + K_t^*)$ is the gross growth rate of worldwide capital

and $\tilde{k}_t = k_t/(K_t + K_t^*)$ is the normalized individual capital. Denote by $s_t = K_t/(K_t + K_t^*)$ the aggregate share of capital owned by domestic firms ($s_t^* = 1 - s_t$ is the share of foreign firms). Since in equilibrium $k_t = K_t$, we also have that $\tilde{k}_t = s_t$.

As in the simpler model without capital accumulation, investors hold an internationally diversified portfolio of shares, and firms use the common discount factor $m_{t+1} = \beta[(d_{t+1} + d_{t+1}^*)/(d_t + d_t^*)]^{-\sigma}$. In terms of variables normalized by the worldwide capital, the discount factor can be rewritten as

$$m_{t+1} = g_t^{-\sigma} \beta \left(\frac{\tilde{d}_{t+1} + \tilde{d}_{t+1}^*}{\tilde{d}_t + \tilde{d}_t^*} \right)^{-\sigma} = g_t^{-\sigma} \tilde{m}_{t+1}.$$

Using normalized variables, the optimization problem solved by a firm is

$$\tilde{V}(\tilde{s}; \tilde{k}, \tilde{b}) = \max_{\tilde{d}, \tilde{h}, \tilde{i}, \tilde{b}'} \left\{ \tilde{d} + g^{1-\sigma} E \tilde{m}' \tilde{V}(\tilde{s}'; \tilde{k}', \tilde{b}') \right\} \quad (15)$$

subject to (12), (13), (14),

where $\tilde{V}$ is the firm's value normalized by aggregate worldwide capital $K + K^*$, and $\tilde{s}$ denotes the normalized aggregate states as specified below.

We can now see the analytical convenience of having the capital externality. Thanks to this assumption, we can write the firm's value function as $V_t = (K_t + K_t^*) \cdot \tilde{V}_t$ and rescale the problem of the firm by worldwide capital. By doing so, we do not need to keep track of the aggregate stock of capital as a state variable. Of course, because we are looking at a general equilibrium, we also need to make sure that the supply of labor does not growth over time. This will be the case with the worker's utility specified earlier.

Appendix B derives the first order conditions for the firm's problem. After imposing the

equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, the first order conditions can be written as

$$F_h(z_t, s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t}, \quad (16)$$

$$\bar{g}_t^{-\sigma} R_t E \tilde{m}_{t+1} = 1 - \mu_t, \quad (17)$$

$$Q_t \Upsilon'(\tilde{i}_t) = 1, \quad (18)$$

$$Q_t = \xi_t \mu_t + \bar{g}_t^{-\sigma} E \tilde{m}_{t+1} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \tilde{i}_{t+1} + \left[1 - \tau + \Upsilon(\tilde{i}_{t+1}) \right] Q_{t+1} \right\}. \quad (19)$$

Here μ_t is the Lagrange multiplier associated with the enforcement constraint and Q_t is the Lagrange multiplier associated with the law of motion for the stock of capital (Tobin's q). We can verify that there is no capital that enters these equations. This confirms that we can ignore the stock of capital when we solve for the normalized equilibrium.

Notice that the property established in the simpler model for which the Lagrange multiplier is common across domestic and foreign firms, also applies to this extended model. In fact, from condition (17) we can see that the common discount factor and the equalization of the interest rates across countries imply $\mu_t = \mu_t^*$. Therefore, if the enforcement constraint is binding in one country, it must also be binding in the other country. The labor wedge in the demand of labor, $1/(1 - \mu_t)$, is also equalized across countries.

Aggregate states and equilibrium: Denote by $\tilde{W}_t = \tilde{B}_t s_t + \tilde{B}_t^* (1 - s_t)$ the normalized worldwide wealth of households/workers. Thanks to the AK structure of the model and the normalization described above, we only need to keep track of two 'endogenous' state variables: $\tilde{W}_t$ and s_t . Therefore, compared to the simpler model considered earlier, the introduction of capital accumulation adds only one state variable, that is, the share of worldwide capital owned by domestic firms, s_t . This additional state is necessary because of the adjustment cost in investment. In absence of investment adjustment costs, we could ignore s_t .

With only two 'endogenous' states it becomes manageable to solve the model numerically using global approximation methods. In addition to the two endogenous states, we also have the four exogenous processes for z_t , ξ_t , z_t^* , ξ_t^* . However, the processes for the exogenous states

can be approximated with parsimonious discrete Markov chains. Appendix C reports for the full list of equilibrium conditions and describes the computational procedure.

We are not ready to prove the following proposition about the impact of a credit shock.

Proposition 3.1 *A credit shock to the domestic country (change in ξ_t) has the same impact on the employment and output of the foreign country in the ‘current period’. The impact on investment, however, is not the same.*

Proof 3.1 *We have already shown that the Lagrange multiplier μ_t is common for firms of both countries. If firms have the same productivity, same capital and pay the same wage, the first order condition (16) implies that they all choose the same employment. This must be the case in the current period since the stock of capital was chosen in the previous period before the observation of the shock and workers have the same consumption ratio. However, the choice of investment differs as we can verify from equations (18) and (19).*

Therefore, in the current period, both countries experience the same responses of employment and output but different responses of investment. Therefore, starting in the next period firms will have different stocks of capital, which in turn implies different employment levels. However, even if the responses are different starting in the next period, we will see in the next section that the differences are quantitatively small.

4 Quantitative analysis

This section studies the properties of the model quantitatively using a calibrated version of the model. The model is solved numerically using the procedure described in Appendix C.

We think of country 1 as the US and country 2 as the other countries in the group of the seven largest industrialized economies, that is, Canada, Japan, France, Germany, Italy, UK. We refer to this group as G6 countries. The discount factor for workers, δ , and the discount factor for investors, β , are set to target an average interest rate of 2 percent and an average return of 7 percent. In the deterministic steady state the interest rate would be equal to $1/\delta - 1$ and the return on equity would be $1/\beta - 1$. In the stochastic economy the relations between the two parameters and the average returns are more complicated. This requires an iterative procedure where we fix δ and β , solve the model and check whether the average returns meet the targets.

The utility function takes the form $U(c, h) = \ln(c) - \alpha h^{1+1/\eta}/(1 + 1/\eta)$ where η is the Frisch elasticity of labor supply. We set the elasticity to 1. This value is commonly used in macroeconomic studies. The parameter α is set so that working hours are 0.36 on average.

Next we parameterize the production function. The parameter ν is chosen to have a labor income share of 0.7. Without uncertainty, the fraction of output going to workers in the form of wages is equal to $\nu\beta/\delta$.⁶ Given the values of δ and β , we choose ν so that this fraction is equal to 0.7. Of course, in the stochastic economy the average labor share is not exactly 0.7 but the difference is small. Next we impose the return to scale of an individual firm, $\theta + \nu$, to 0.9. Given the value of ν , this pins down the value of $\theta = 0.9 - \nu$.

The stock of capital evolves according to $k' = (1 - \tau)k + \Upsilon(i/k)k$, with the function Υ taking the form

$$\Upsilon\left(\frac{i}{k}\right) = \frac{\phi_1}{1 - \zeta} \left(\frac{i}{k}\right)^{1-\zeta} + \phi_2.$$

This functional form is widely used in the literature. See, for example, Jermann (1998). The parameters ϕ_1 and ϕ_2 are chosen so that in the deterministic steady state the adjustment cost does not play any role, that is, $Q = 1$ and $I = \tau K$. This requires $\phi_1 = \tau^\zeta$ and $\phi_2 = -\zeta\tau/(1-\zeta)$. Thus, we need to choose two parameters, τ and ζ . The first is the depreciation rate which we set to $\tau = 0.02$. The second determines the sensitivity of the adjustment cost which we set to $\zeta = 0.5$.

At this point we are left with the parameters that determine the stochastic properties of the shocks. The ideal approach would be to construct series for the variables z_t and ξ_t from the data for both countries. We could then parameterize the stochastic process according to the properties of these series. While this approach can be easily implemented for the productivity shocks, it is not easily applicable to construct the series of financial shocks. There are two major limitations. The first limitation is that the borrowing limit may not be always binding. Therefore, we cannot use the empirical series of debt to back up the ξ_t series. The second limitation is that there is not comparable data for the financial flows for all the G7 countries. Given these limitations, we use a different approach to parameterize the stochastic process for ξ_t .

Let's start with the productivity series. We construct Solow residuals series for the US and

⁶From the first order condition of labor, equation (4), we derive $wh/F(z, k, h) = \nu(1 - \mu)$, which provides an expression for the labor share. To derive an expression for μ we use condition (5) evaluated in the version of the model without uncertainty. Taking into account that in a deterministic steady state $m' = \beta$ and $R = 1/\delta$, this condition becomes $\beta/\delta = 1 - \mu$. Substituting in the labor share $\nu(1 - \mu)$, we get the expression reported in the main text.

for the aggregate of the remaining G6 countries. Using these series we estimate a two state autoregressive process,

$$\begin{pmatrix} \log(z_{t+1}^{US}) \\ \log(z_{t+1}^{G6}) \end{pmatrix} = \begin{bmatrix} \rho_z & \psi_z \\ \psi_z & \rho_z \end{bmatrix} \begin{pmatrix} \log(z_t^{US}) \\ \log(z_t^{G6}) \end{pmatrix} + \begin{pmatrix} \epsilon_{t+1}^{US} \\ \epsilon_{t+1}^{G6} \end{pmatrix},$$

where the log series are linearly detrended and ϵ_{t+1}^{US} and ϵ_{t+1}^{G6} are mean zero white noises with standard deviation σ_z^{US} and σ_z^{G6} respectively. The estimation returns $\rho_z = 0.98$, $\psi_z = -0.008$, $\sigma_z^{US} = 0.0059$, $\sigma_z^{G6} = 0.0065$. Finally the correlation between residuals is 0.15.

The estimation shows that there is very low cross-country comovement between the Solow residuals of the two countries and they have very similar standard deviations. Therefore, the process for the productivity variables can be well approximated by symmetric and independent first order autoregressive processes with autocorrelation parameter $\rho = 0.98$ and standard deviation of residuals $\sigma = 0.0062$.

Let's turn now to the financial shocks where we take a simple approach. We assume that the variable ξ_t follows symmetric and independent autoregressive processes in both countries, that is,

$$\begin{aligned} \log(\xi_{t+1}^{US}) &= (1 - \rho_\xi)\bar{\xi} + \rho_\xi \log(\xi_t^{US}) + \varepsilon_{t+1}, \\ \log(\xi_{t+1}^{G6}) &= (1 - \rho_\xi)\bar{\xi} + \rho_\xi \log(\xi_t^{G6}) + \varepsilon_{t+1}. \end{aligned}$$

Here ε_{t+1} are mean zero shocks with standard deviation σ_ξ .

We choose $\bar{\xi}$, ρ_ξ and σ_ξ so that the level, persistence and standard deviation of debt generated by the model are approximately equal to the statistics for the US debt. In particular, the average value of the enforcement parameter $\bar{\xi}$ affects the average leverage. Higher is the value of $\bar{\xi}$ and higher is the average leverage. We set this parameter to 0.5, implying an average ratio of total debt over physical capital, $(b_{t+1}/R_t + y_t)/k_{t+1}$, slightly lower than 0.5.⁷ The remaining two parameters ρ_ξ and σ_ξ are chosen to replicate the autocorrelation and standard deviation for the stock of credit market liabilities in the nonfinancial business sector. Data is from the Flows of Funds Accounts. This is obtained by setting $\rho_\xi = 0.95$ and $\sigma_\xi = 0.012$. The full list of parameter values are reported in Table 1.

⁷The average ratio is smaller than 0.5 since the enforcement constraint is only occasionally binding.

Table 1: List of parameters

Discount factor for households/workers, δ	0.994
Discount factor for entrepreneurs, β	0.984
Utility parameter, α	0.300
Production technology, θ	0.200
Production technology, ν	0.700
Depreciation rate, τ	0.020
Capital adjustment cost, ζ	0.050
Productivity persistence, ρ_z	0.980
Productivity volatility, σ_z	0.006
Credit average, $\bar{\xi}$	0.500
Credit persistence, ρ_ξ	0.950
Credit volatility, σ_ξ	0.012

4.1 Results

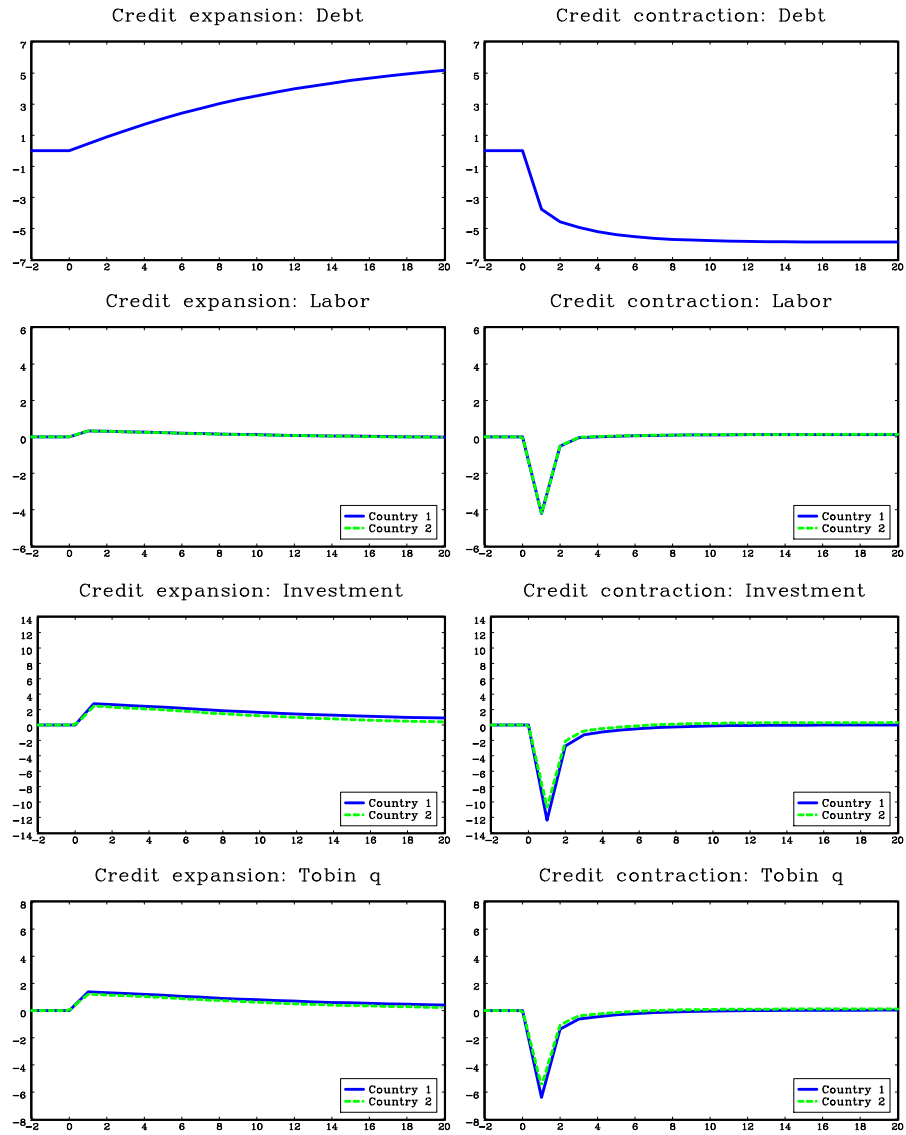
Appendix C describes the computational procedure. To implement this procedure we discretize the space for the exogenous and endogenous states. All the exogenous states z_t , z_t^* , ξ_t^* and ξ_t are approximated with a three-state Markov chain (see Tauchen (1986)). The spaces for the endogenous states, $\tilde{b}_t$ and s_t are each discretized on a grid containing eleven points. Values outside the grids are determined through linear interpolation.

We present the results outlining five main properties. In particular, we emphasize (i) the asymmetric response to shocks; (ii) the countercyclicity of labor productivity in response to credit shocks; (iii) the international spillover of credit shocks; (iv) the severity of crisis after long periods of credit and macroeconomic booms; (v) the importance of credit shocks for the volatility of labor and asset prices.

Asymmetry: Figure 9 plots the impulse responses to a credit shock. The responses are generated starting from the limiting equilibrium in which the economy converges after a long series of draws $\xi_t = \xi_2$. Starting from this equilibrium we consider two cases. In the first case, starting at time 1, the economy experiences a sequence of $\xi_t = \xi_3$ leading to a credit expansion. In the second case the economy experiences a sequence of draws $\xi_t = \xi_1$ leading to a credit contraction. There is no uncertainty in productivity z_t which is constant and equal to its mean.

In response to the credit expansion (left panels of Figure 9) we see a gradual increase in the stock of debt and a persistent expansion in labor, investment and Tobin's q . The magnitude of the macroeconomic expansion, however, is not large at impact. The macroeconomic expansion induced by the credit boom arises through the following mechanism. As firms take

Figure 9: Impulse responses to credit expansions and contractions.

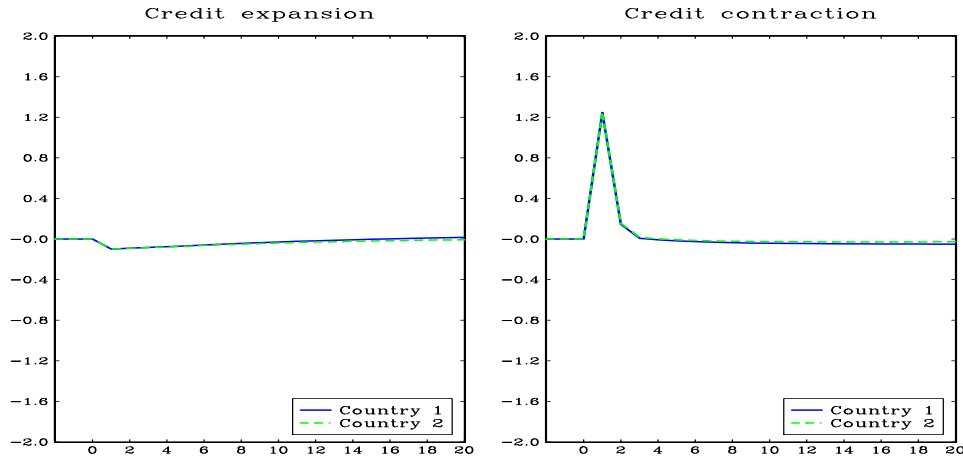


on more debt, they pay more dividends, increasing the discount factor m' . Thanks to the lower discounting, firms invest more. At the same time, the higher borrowing from firms must be matched by higher lending (savings) from workers. This implies that workers will experience consumption growth. At impact this increases the labor supply through the impact that lower consumption today has on the disutility of labor.

In contrast, the responses to a credit contraction (right panels of Figure 9) display a very different pattern. The stock of debt declines much more drastically. Also, the responses of labor, investment and Tobin's q are much larger at impact but not persistent. Therefore, there is a very strong asymmetry in the responses to credit shocks. We would like to point out that, as stated in Proposition 3.1, the credit shock has the same impact on the production of the two countries in the first period but different impacts on investment. However, Figure 9 shows that the differences are quite small.

Countercyclicality in labor productivity: Figure 10 plots the impulse responses of labor productivity generated using the approach described above. As in the previous figure we see an asymmetry between credit expansions and credit contractions. More importantly, a credit expansion generates a decline in labor productivity while a credit contraction generates an increase. This is important for capturing one of the facts outlined in the first section of the paper, that is, the countercyclicality of labor productivity.

Figure 10: Impulse responses to credit expansions and contractions.



Macroeconomic comovement: Figure 11 plots the impulse responses of labor, investment and Tobin's q to a credit contraction in country 1, differentiating the cases of financial autarky and capital mobility. Since the shock is only to country 1, in the autarky regime country 2 is not affected by the shock. In the regime with capital mobility, instead, the shock impacts country 2 in the same way it impacts country 1 (international spillover). It can be noticed, however, that the effect on country 1 is smaller when the economies are financially integrated. Therefore, from a macroeconomic perspective, capital markets integration allows for an international sharing of credit shocks. This helps us understanding the high international comovement observed in the recent crisis.

Credit booms and severity of recessions: Figure 12 plots the impulse responses to a credit expansion that later reverts back to the initial level. Starting from an equilibrium to which the economy converges after a long series of $\xi_t = \xi_2$, we assume that at time 1 the economy experience a draw of $\xi_t = \xi_3$ (credit expansion). This higher value of ξ is drawn for several period after which it reverts back to $\xi_t = \xi_2$. We consider several durations for the credit expansions, before the reversal: 5 quarters, 20 quarters and 60 quarters.

The key finding is that the severity of the credit contraction increases with the duration of the credit expansion. After a protracted credit boom, the economy accumulates large leverages. Then, when the shock reversal arrives, the required de-leveraging is much more severe generating a stronger macroeconomic contraction. In this way the model can explain why recessions that arise after long periods of financial expansions tend to be more damaging.

Volatility of labor and asset prices: Table 2 reports the standard deviations of various variables. Three versions of the economy are considered: the economy with productivity shocks only; the economy with credit shocks only; and the economy with both shocks. The statistics are computed after detrending the simulated series with a band-pass filter that preserves cycles of 1.5-8 years (Baxter and King (1999)).

Two properties are especially noticeable. First, the model with credit shocks can generate much higher volatility of labor, bringing the model closer to the data. Second, credit shocks also generate a high volatility of asset prices. In particular, in the version of the model with only credit shocks, the value of the firm becomes more than four times as volatile as output. In contrast, with only productivity shocks, the value of the firm is less volatile than output. Therefore, credit shocks are important for understanding the high volatility of labor and asset

Figure 11: Impulse responses to credit contractions in closed and open economies.

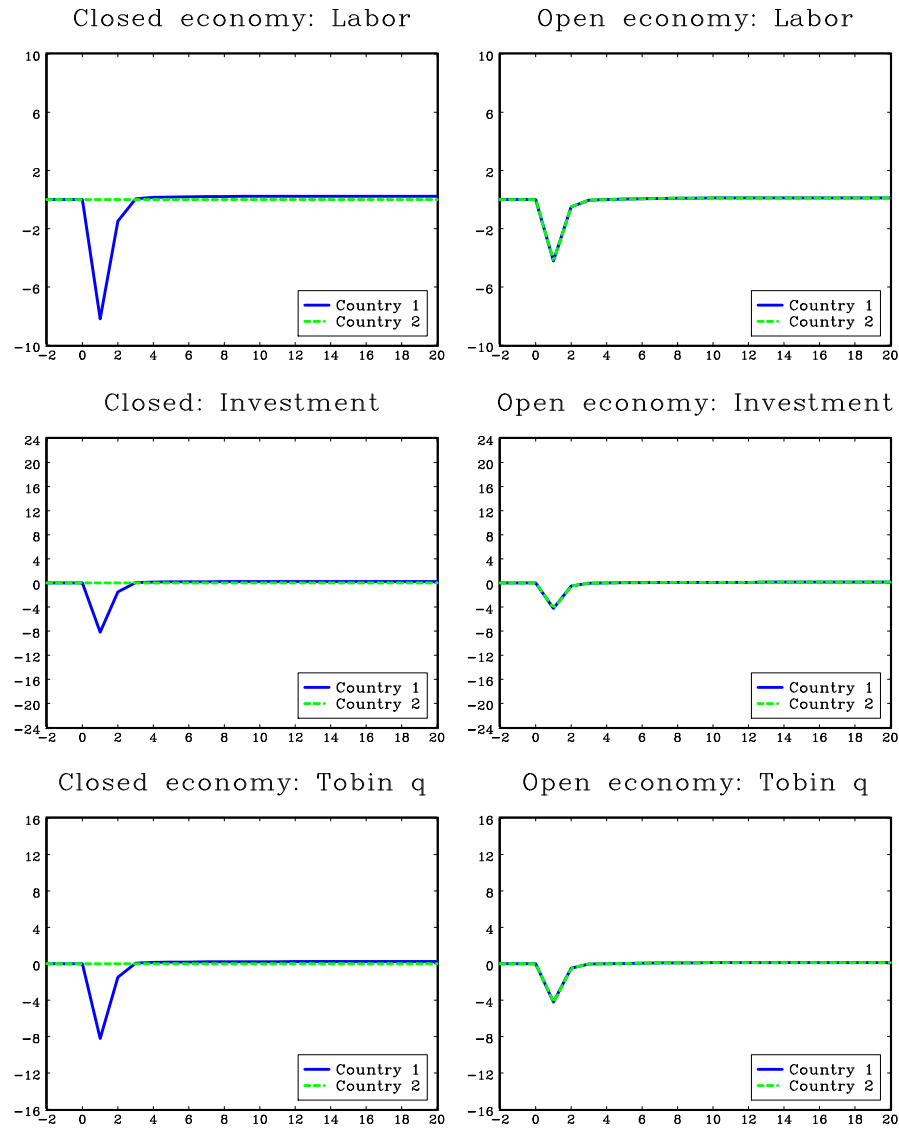


Figure 12: Duration of credit expansions and severity of contractions.

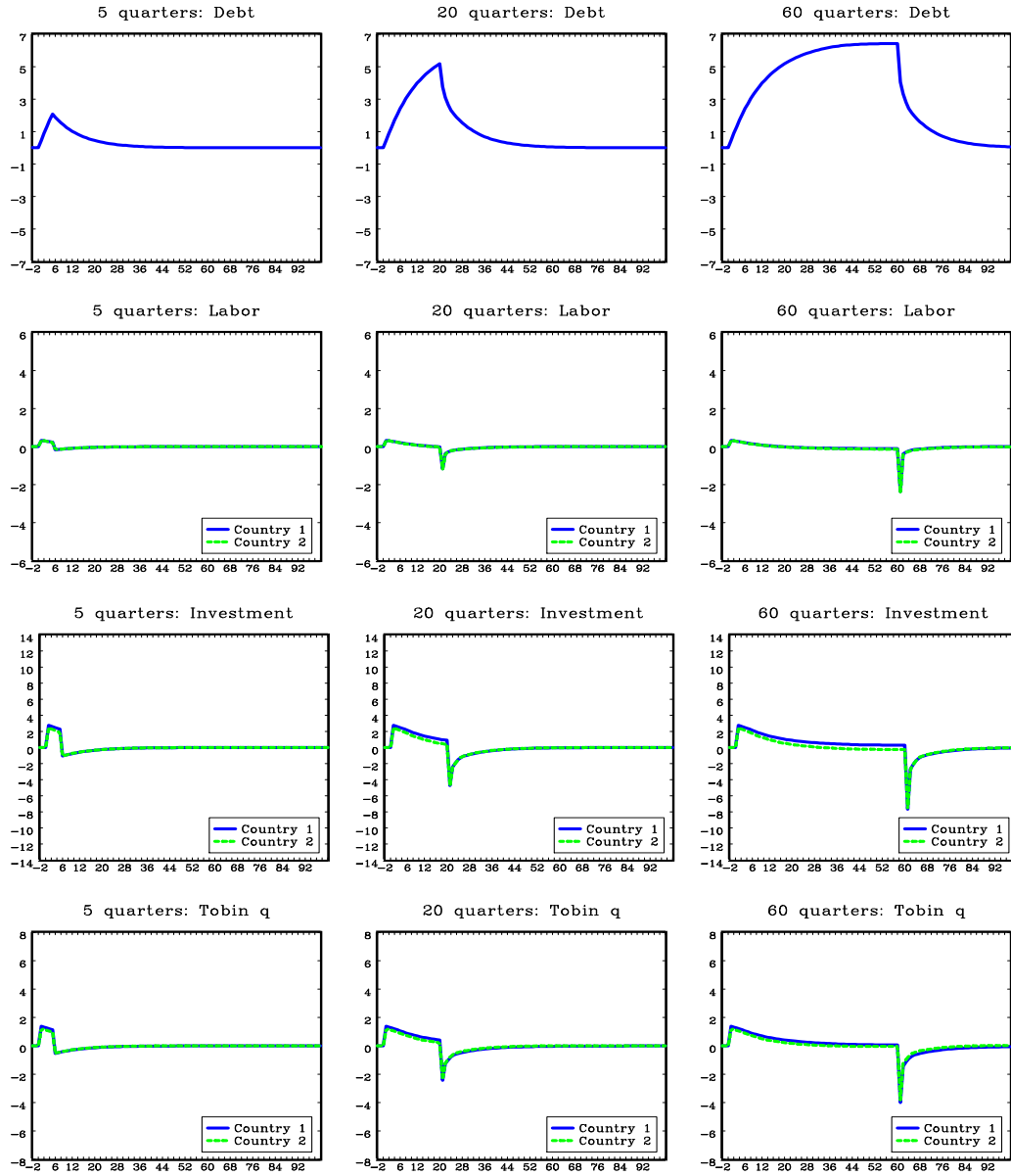


Table 2: Standard deviations of key variables from detrended simulated series.

	Productivity shocks only	Credit shocks only	Both shocks
Output	0.57	0.37	0.60
Consumption	0.31	0.19	0.50
Labor	0.14	0.53	0.43
Investment	0.60	1.70	1.62
Firms' value	0.38	1.74	1.52
Tobin's q	0.30	0.84	0.81
% Nonbinding	99.93	94.87	95.73

prices observed in the data.

5 Conclusion

The recent decade and in particular the 2008-2009 crisis has been characterized by an historically high degree of international synchronization. Furthermore, the internationalization of the recent crisis has taken place in an environment where the association between labor productivity and labor input is at historically low levels for the US and, on average, for other industrialized countries. Motivated by these observations we have considered a theoretical environment in which shocks to credit is one of the driving forces of the business cycle. These shocks affect the real sector of the economy through a credit channel: booms enhance the borrowing capacity of firms and in the general equilibrium they lead to higher employment, production but lower productivity of labor. The opposite arises after a credit contraction. Interestingly, credit shocks generate business cycle fluctuations that are asymmetric, *i.e.*, contractions are sharper than expansions. Furthermore, they generate high volatility in asset prices.

Within this framework we have shown that, when countries are financially integrated, credit shocks that are specific to one country affect the employment and production of other countries, with significant macroeconomic spillovers. At the same time, these shocks generate a negative correlation between labor productivity and working hours. On the contrary, country-specific productivity shocks do not generate large cross-country co-movement in real macroeconomic variables unless the shocks are internationally correlated. However, if productivity shocks are

correlated across countries and they are the major source of business cycle fluctuations, it is difficult to reconcile the fact that the correlation of labor productivity with hours is low and it has further declined in recent years.

Of course, other shocks besides the ones considered in this paper could generate large cross-country comovement and weak correlation between productivity and labor. However, it is not obvious how the most common shocks that have been studied in the literature (for example labor-wedge shocks like those considered by Smets and Wouters, 2007 or by Chari, Kehoe and McGrattan 2007) can generate international co-movement unless the shocks are internationally correlated. We conclude by noting that our research points the importance of credit markets disturbances but it does not attempt to explain the deep causes of these disturbances. In order to assess the appropriate policy responses to these disturbances future research should attempt to better understand how and why these shocks arise.

Appendix

A First order conditions

Consider the optimization problem (1) and let λ and μ be the Lagrange multipliers associate with the two constraints. Taking derivatives we get:

$$\begin{aligned} d : \quad & 1 - \lambda = 0 \\ h : \quad & \lambda[F_h(z, h) - w] - \mu F_h(z, h) = 0 \\ b' : \quad & (1 + \phi\mu)Em'V_{b'}(\mathbf{s}'; b') + \frac{\lambda}{R} = 0 \end{aligned}$$

The envelope condition is:

$$V_b(\mathbf{s}; b) = -\lambda$$

The above conditions can be re-arranged as in (4) and (5).

B First order conditions for the model with capital

Differentiating the firm's problem (15) with respect to $h_t, \tilde{b}_{t+1}, \tilde{i}_t, \tilde{k}_{t+1}$, we get:

$$F_h(z_t, \tilde{k}_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \quad (20)$$

$$\frac{1 - \mu_t}{R_t} + g_t^{-\sigma} E\tilde{m}_{t+1}\tilde{V}_b(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) = 0 \quad (21)$$

$$Q_t \Upsilon' \left(\frac{\tilde{i}_t}{\tilde{k}_t} \right) = 1 \quad (22)$$

$$Q_t = \xi_t \mu_t + g_t^{-\sigma} E\tilde{m}_{t+1}\tilde{V}_k(\tilde{\mathbf{s}}_{t+1}; \tilde{k}_{t+1}, \tilde{b}_{t+1}) \quad (23)$$

where μ_t is the lagrange multiplier associated with the enforcement constraint and Q_t (Tobin's q) is the lagrange multiplier associated with the law of motion of capital. The multiplier associated with the budget constraint is 1. For the foreign country we have the same conditions but with country specific variables denoted with the start superscript.

The envelope conditions are:

$$\tilde{V}_b(\tilde{s}_t; \tilde{k}_t, \tilde{b}_t) = -1 \quad (24)$$

$$\tilde{V}_k = (1 - \mu_t)F_k(z_t, \tilde{k}_t, h_t) + \left[1 - \tau + \Upsilon\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right) - \Upsilon'\left(\frac{\tilde{i}_t}{\tilde{k}_t}\right)\frac{\tilde{i}_t}{\tilde{k}_t} \right] Q_t \quad (25)$$

Substituting the envelope conditions and imposing the equilibrium conditions $k_t = K_t$ and $\tilde{k}_t = s_t$, we obtain (16)-(19).

C Dynamic system and solution approach

We will use the bar sign to denote aggregate worldwide variables normalized by the worldwide stock of capital. For example, the $\bar{d}_t$ is the normalized worldwide dividend, defined as:

$$\bar{d}_t = \frac{d_t + d_t^*}{K_t + K_t^*} \equiv \tilde{d}_t + \tilde{d}_t^*.$$

The full list of equilibrium conditions are:

$$1 = \delta g_t^{-1} R_t E_t \left(\frac{\bar{c}_{t+1}}{\bar{c}_t} \right)^{-1} \quad (26)$$

$$\tilde{c}_t^* = \chi \tilde{c}_t \quad (27)$$

$$\tilde{w}_t h_t + \tilde{w}_t^* h_t^* + \bar{b}_t = \bar{c}_t + \frac{g_t \bar{b}_{t+1}}{R_t} \quad (28)$$

$$\bar{b}_t + \bar{d}_t + \bar{i}_t = F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) - \tilde{w}_t h_t - \tilde{w}_t^* h_t^* + \frac{\bar{g}_t \bar{b}_{t+1}}{R_t} \quad (29)$$

$$g_t(\xi_t s_{t+1} + \xi_t^* s_{t+1}^*) \geq \frac{g_t \bar{b}_{t+1}}{R_t} + F(z_t, s_t, h_t) + F(z_t^*, s_t^*, h_t^*) \quad (30)$$

$$(1 - \mu_t) \bar{d}_t^{-\sigma} = \beta g_t^{-\sigma} R_t E_t \bar{d}_{t+1}^{-\sigma} \quad (31)$$

$$\alpha h_t^\gamma = \frac{\tilde{w}_t}{\tilde{c}_t} \quad (32)$$

$$\alpha (h_t^*)^\gamma = \frac{\tilde{w}_t^*}{\tilde{c}_t^*} \quad (33)$$

$$g_t s_{t+1} = (1 - \tau) s_t + \Upsilon \left(\frac{\tilde{i}_t}{s_t} \right) s_t \quad (34)$$

$$g_t s_{t+1}^* = (1 - \tau) s_t^* + \Upsilon \left(\frac{\tilde{i}_t^*}{s_t^*} \right) s_t^* \quad (35)$$

$$F_h(z_t, s_t, h_t) = \frac{\tilde{w}_t}{1 - \mu_t} \quad (36)$$

$$F_h(z_t^*, s_t^*, h_t^*) = \frac{\tilde{w}_t^*}{1 - \mu_t} \quad (37)$$

$$Q_t \Upsilon' \left(\frac{\tilde{i}_t}{s_t} \right) = 1 \quad (38)$$

$$Q_t^* \Upsilon' \left(\frac{\tilde{i}_t^*}{s_t^*} \right) = 1 \quad (39)$$

$$Q_t = \xi_t \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\} \quad (40)$$

$$Q_t^* = \xi_t^* \mu_t + \beta g_t^{-\sigma} E \left(\frac{\bar{d}_{t+1}}{\bar{d}_t} \right)^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\} \quad (41)$$

Equations (26)-(41) form a dynamic system composed of 16 equations. Given the states $z_t, \xi_t, z_t^*, \xi_t^*, \bar{b}_t, s_t$, the unknown variables are $h_t, h_t^*, c_t, c_t^*, w_t, w_t^*, i_t, i_t^*, Q_t, Q_t^*, g_t, \mu_t, R_t, \bar{d}_t, \bar{b}_{t+1}, s_{t+1}$. Therefore, we have a dynamic system of 16 equations in 16 unknowns.

The computational procedure is based on the approximation of four functions:

$$\Gamma_1(\mathbf{s}_{t+1}) = \bar{c}_{t+1}^{-1}$$

$$\Gamma_2(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma}$$

$$\Gamma_3(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}, s_{t+1}, h_{t+1}) - \frac{\tilde{i}_{t+1}}{s_{t+1}} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}}{s_{t+1}} \right) \right] Q_{t+1} \right\}$$

$$\Gamma_4(\mathbf{s}_{t+1}) = \bar{d}_{t+1}^{-\sigma} \left\{ (1 - \mu_{t+1}) F_k(z_{t+1}^*, s_{t+1}^*, h_{t+1}^*) - \frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} + \left[1 - \tau + \Upsilon \left(\frac{\tilde{i}_{t+1}^*}{s_{t+1}^*} \right) \right] Q_{t+1}^* \right\}$$

The procedure starts with a guess for the values of the approximated functions $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$ and $\Gamma_4(\mathbf{s}_{t+1})$. We first form a two dimensional grid for the endogenous states $\bar{b}$ and s . Then for each realization of the exogenous shocks— $z_t, \xi_t, z_t^*, \xi_t^*$ —we guess the values taken by the above functions over the grid points. Values outside the grid are obtained through bi-linear interpolation. Once we know these functions, we can solve for the 18 unknowns of the system (26)-(41) at each grid point and for each realization of the shocks. In finding the solutions we check whether the enforcement constraint is binding ($\mu_t > 0$) or not binding ($\mu_t = 0$). We then use the solutions found at each grid point to update the guess for the three functions. We keep iterating until the guesses for $\Gamma_1(\mathbf{s}_{t+1})$, $\Gamma_2(\mathbf{s}_{t+1})$, $\Gamma_3(\mathbf{s}_{t+1})$ and $\Gamma_4(\mathbf{s}_{t+1})$ at each grid point are equal to the values obtained by solving the dynamic system.

References

- Backus, D. K., Kehoe, P. J. and F. E. Kydland (1992). International Real Business Cycles. *Journal of Political Economy*, 100(4), 745-75.
- Baxter, M. and R. G. King (1999). Measuring Business Cycles: Approximate Band-Pass Filters for Macroeconomic Time Series. *Review of Economics and Statistics*, 81(4), 575-93.
- Chari, V. Kehoe P. and E. McGrattan (2007) Business Cycle Accounting, *Econometrica*, 75(3), 781-836
- Dedola L. and G. Lombardo (2010). Financial frictions, Financial integration and the international propagation of shocks. Unpublished manuscript, European Central Bank.
- Devereux, M. B. and J. Yetman (2010). Leverage Constraints and the International Transmission of Shocks. Unpublished manuscript, University of British Columbia and Bank of International Settlements.
- Enders, Z., Kollmann, R. and G. J. Muller (2010). Global Banks and International Business Cycle. Unpublished manuscript ECARES, Universite Libre de Bruxelles.
- Gali, J. and L. Gambetti (2009) On the Sources of the Great Moderation. *American Economic Journal: Macroeconomics*, 1(1), 26-57.
- Heathcote, J. and F. Perri (2004), Financial Globalization and Real Regionalization. *Journal of Economic Theory*, 119(1), 207-43.
- Helbling, T., Huidrom, R., Kose, M. A. and C. Otrok (2010). Do Credit Shocks Matter? A Global Perspective. Unpublished manuscript, International Monetary Fund and University of Virginia.
- Imbs, J. (2010). The First Global Recession in Decades. Unpublished manuscript, HEC Lausanne.
- Jermann, U. (1998). Asset Pricing in Production Economies. *Journal of Monetary Economics*, 41(2), 257-75.
- Jermann, U. and V. Quadrini (2009). Macroeconomic Effects of Financial Shocks, NBER Working Paper No. 15338.

- Kiyotaki, N. and J. H. Moore (1997). Credit Cycles. *Journal of Political Economy*, 105(2), 211-48.
- Kiyotaki, N. and J. H. Moore (2008). Liquidity, Business Cycle, and Monetary Policy. Unpublished manuscript, Princeton University and Edinburgh University.
- Mendoza, E. (2010). Sudden Stops, Financial Crises and Leverage. *American Economic Review*, forthcoming.
- Morgan, P., Rime, B. and P. Strahan, (2004), Integration and State Business Cycles. *Quarterly Journal of Economics*, 119(3), 1555-85.
- Smets, F. R. and R. Wouters (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, 97(3), 586-606.
- Tauchen, G. (1986). Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. *Economic Letters* 20(2), 177-81.