Non-convexities, Interest Rates and the Monetary Transmission Process*

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Abstract

This paper studies the effects of interest rate movements on investment. In contrast to other studies, we focus on the implications of non-convex capital adjustment costs at the establishment level for the transmission of interest rate changes, perhaps induced by monetary policy, on investment spending. The dependence of plant-level investment decisions on interest rates is important both for understanding the smoothing effects of interest rate movements and the transmission of monetary policy. We introduce an interest rate process in the form of a stochastic discount factor that directly affects the decisions of the plant. In our analysis, we compare an empirically-based representation of the stochastic discount factor against a model-based representation. We find that that nonconvexities at the plant level have aggregate implications when using empirically consistent stochastic discount factors. As this work proceeds, we will examine how alternative monetary policies, which underly the interest rate process, influence investment behavior.

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*The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City.

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1 Motivation

This paper studies the effects of interest rate movements on plant-level and aggregate investment when adjustment costs are non-convex. The analysis focuses on the dependence of investment decisions on interest rate movements. This dependence is important both for understanding the smoothing effects of interest rate movements and the transmission of monetary policy.

Models of the effects of monetary policy on investment typically assume some form of quadratic adjustment costs. Yet, as is now understood from a number of studies at the establishment level, investment at the plant level is characterized by periods of only minimal changes in the capital stock coupled with intermittent periods of large capital adjustments. These patterns are difficult if not impossible to mimic in the standard quadratic adjustment cost model. Instead, these patterns are captured in models which rely on the presence of non-convex adjustment costs.[1]

It is important to re-evaluate the monetary transmission mechanism given this evidence at the micro level on the significance of non-convexities. For our purposes, we focus on the links from monetary policy to investment through interest rates, leaving aside potential channels through balance sheet effects.

Following the lead of Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008), one might conjecture that the non-convexities are not important for aggregate investment and thus irrelevant in consideration of the monetary transmission mechanism. In those papers, state dependent interest rates are determined in equilibrium. A striking result from this literature is that the absence of aggregate effects of lumpy investment: the aggregate model with non-convexities at the micro-level is essentially indistinguishable from an aggregate model without non-convexities. The key to the result is the response of the interest rate to aggregate shocks and the evolution of the capital stock.[2]

Those results, however, do not quite address the question we raise here. The issue is the interest rate process. Our focus is on the effects of interest rate movements induced by the conduct of monetary policy, rather than those created in the underlying equilibrium of a stochastic business cycle model. It is entirely conceivable that the equilibrium smoothing of

[1] See, for example, the results reported in Caballero and Engel (1999) and Cooper and Haltiwanger (2006).

[2] Cooper and Haltiwanger (2006) find some but not complete smoothing by aggregation just from the idiosyncratic shocks alone with constant interest rates.

plant-level investment through interest rate movements does not arise in a model economy in which interest rates follow, for example, the standard Taylor Rule.

One way to see this difference is by studying aggregate investment in a model with two features. The first, as in Cooper and Haltiwanger (2006), is the presence of non-convexities in the capital adjustment process. The second is a stochastic discount factor which directly effects the decisions of the plant. But, in contrast to Thomas (2002) and the literature that has followed, we focus on representations of the stochastic factor that are empirically based rather than the outcome of a particular stochastic equilibrium model.

The results from this exercise, which is the first part of our paper, indicates that non-convexities at the plant-level have aggregate implications for empirically consistent stochastic discount factors. Put differently, the smoothing results reported by Thomas (2002) and others seem to depend on the stochastic discount factor determined by the equilibrium of that particular model. If instead one looks to the data for a stochastic discount factor, the finding of complete smoothing of plant-level non-convexities no longer holds.

This analysis naturally leads to the second part of our paper: a consideration of monetary policy rules which guide the dependence of interest rates on the aggregate state of the economy as well as the discretion of policy makers. Presumably, these rules underlie the interest rate process we uncover from the data.

Within this monetary policy rules framework, we can also study the effects of monetary policy. Interventions may either be modeled as changes in the policy rule itself or as innovations to the interest rate given a policy function. Our model allows us to them quantify the effects of these forms of monetary policy on plant-level and aggregate investment.

2 Dynamic Capital Demand

Our approach is to begin with a dynamic capital demand problem at the plant-level. For this problem, we follow Cooper and Haltiwanger (2006) for the specification of the adjustment costs. But, in contrast to that analysis, we allow the discount factor to be stochastic. We parameterize this stochastic discount factor from the data. We then study the demand for capital at the plant-level and in the aggregate.

2.1 Plant Level: Dynamic Optimization

Following Cooper and Haltiwanger (2006), the dynamic programming problem is specified as:

$$V(A,\varepsilon,K) = \max\{V^i(A,\varepsilon,K), V^a(A,\varepsilon,K)\}, \qquad \forall(A,\varepsilon,K) \tag{1}$$

where K represents the beginning of period capital stock, A is the aggregate profitability shock, ε is the idiosyncratic profitability shock. The superscripts refer to active investment "a," where the plant undertakes investment to obtain capital stock K' in the next period, and inactivity "i," where no investment occurs. These options, in turn, are defined by:

$$V^i(A,\varepsilon,K) = \Pi(A,\varepsilon,K) + E_{A',\varepsilon'|A,\varepsilon}\left[\tilde{\beta}V(A',\varepsilon',K(1-\delta))\right] \tag{2}$$

and

$$V^a(A,\varepsilon,K) = \max_{K'} \Pi(A,\varepsilon,K) - C(A,\varepsilon,K,K') + E_{A',\varepsilon'|A,\varepsilon}\left[\tilde{\beta}V(A',\varepsilon',K')\right] \tag{3}$$

where $\tilde{\beta}$ is the state dependent discount rate for the establishment. The specification of this discount rate is a key to our analysis of this model.

The model includes three types of adjustment costs which, as reported in Cooper and Haltiwanger (2006), are the leading types of estimated adjustment costs.

$$\begin{aligned} C(A,\varepsilon,K,K') &= (1-\Lambda)\,\Pi(A,\varepsilon,K) - p_b(I>0)(K'+(1-\delta)K) \\ &\quad - p_s(I<0)((1-\delta)K-K') + \frac{\nu}{2}\left(\frac{K'-(1-\delta)K}{K}\right)^2 K \end{aligned} \tag{4}$$

The first is a disruption cost parameterized by Λ. If $\Lambda < 1$, then any level of gross investment implies that a fraction of revenues is lost. The second is the quadratic adjustment cost parameterized by ν. The third is a form of irreversibility in which there is a gap between the buying, p_b, and selling, p_s, prices of capital. These are included in (3) by the use of the indicator function for the buying $(I>0)$ and selling of capital $(I<0)$.

Assume the profit function has the following form

$$\Pi(A,\varepsilon,K) = A\varepsilon K^{\alpha}. \tag{5}$$

This is a reduced-form profit function which can be derived from an optimization problem over flexible factors of production (labor, materials). The parameter α will reflect factor shares as well as the elasticity of demand for the plant's output. Here A is a aggregate profitability shock and ε is the idiosyncratic profitability shock.

3 Finding the Stochastic Discount Factor

The optimization problem given in (1) includes a stochastic discount factor, $\tilde{\beta}$. As is customary within an stochastic equilibrium model in which households do not face costs of adjusting their portfolios, rates of return are linked to household preferences through an Euler equation

$$E_t\left[\tilde{\beta}_{t+1}R^j_{t+1}\right]=1 \tag{6}$$

for any asset j. Here

$$\tilde{\beta}_{t+1}\equiv\frac{\beta u'(c_{t+1})}{u'(c_t)} \tag{7}$$

is the standard pricing kernel where c_t is household consumption in period t and $u(\cdot)$ represents utility for a representative household.

We study two specifications of $\tilde{\beta}_{t+1}$. We start with the process for $\tilde{\beta}_{t+1}$ that comes from the data. We then study, following Thomas (2002) and others, a stochastic general equilibrium version of the state dependent interest rates.

For both, we are interested in a state space (capital stock and productivity shocks) representation of this stochastic discount factor. In this way, the state dependent discount factor can be used directly in (1).

3.1 Data Based Approach

We study two data-based approaches to uncovering $\tilde{\beta}_{t+1}$. In the first, we use the Euler equation from (7). The second bypasses the household problem and looks directly at interest rates.

Given data on consumption and some assumptions on preferences, (7) can be used to generate a time series for $\tilde{\beta}_{t+1}$. For this exercise, assume $u(c)=\log(c)$ and set $\beta=0.95$.[3]

[3]Future work will look at the robustness of our results to alternative specifications of preferences and the discount factor.

We compute the empirical relationship between the stochastic discount factor and observables corresponding to key state variables in our model. This relationship is estimated by regressing the stochastic discount factor, $\tilde{\beta}_{t+1}$, on measures of the capital stock, current and future productivity: (K_t, A_t, A_{t+1}). The inclusion of both current period and future period values of the aggregate shock, A_t, are required since $\tilde{\beta}_{t+1}$ measures the realized real return between periods t and $t+1$.

The appendix provides a detailed discussion of our data. For our analysis, we consider A_t to be total factor productivity in period t. With a focus on business cycle dynamics, we model a stationary specification of the shock process, which we parameterize from the data after detrending using the H-P filter. As discussed in the appendix, our results depend on the choice of the parameter of the H-P filter, denoted λ. The choice of this parameter determines directly the serial correlation of the total factor productivity process and consequently the dependence of $\tilde{\beta}_{t+1}$ on the state vector (K_t, A_t, A_{t+1}). The mapping from the values of λ to the serial correlation of the aggregate shock, ρ_A, is given in Table 11 in the Appendix. We therefore report results for three distinct values of ρ_A.

Our results are presented in Table 1. There are four different specifications and three different values of ρ_A for each. For each of these empirical models, we report regression coefficients and goodness of fit measures.

The first two blocks report two different specifications of the dependence of $\tilde{\beta}_{t+1}$ on different parts of the (A_t, A_{t+1}) state space. The model in which $\tilde{\beta}_{t+1}$ depends on (A_t, A_{t+1}) fits better than the model with A_t alone. The results ignore the capital stock since the inclusion of K_t adds very little to the empirical model.

As is made clear in the table, the magnitude of the coefficient depends on the choice of ρ_A. A lower value of this parameter leads to more dependence on A_t and less, in absolute value, dependence on A_{t+1} in the second block of results. This will be important later when we study how investment responds to profitability shocks.

Instead of working through the Euler equation, the last two blocks replace $\tilde{\beta}_{t+1}$ with observed returns on different assets. The argument for doing so is that the household asset pricing model has failed in a number of ways. Thus instead of going through the Euler equation, we can look directly at interest rates that a firm might use to discount profits.

We study two real interest rates, the 30-day T bill and a long term AAA bond. These are *ex ante* rates and thus depend on the current state, A_t. The real rates are constructed

Table 1: Empirical estimates of relationship between state variables and discount factors

Specification	ρ_A	Constant	A_t	A_{t+1}	R^2	$\frac{dE_t[\tilde{\beta}(\cdot)]}{dA_t}$
	0.14	-0.08	0.12		0.02	0.12
		(0.00)	(0.12)			
$\tilde{\beta}(A_t)$	0.45	-0.08	0.01		0.00	0.01
		(0.00)	(0.09)			
	0.84	-0.08	-0.04		0.02	-0.04
		(0.00)	(0.04)			
	0.14	-0.08	0.21	-0.61	0.46	0.12
		(0.00)	(0.09)	(0.09)		
$\tilde{\beta}(A_t, A_{t+1})$	0.45	-0.08	0.24	-0.52	0.49	0.01
		(0.00)	(0.07)	(0.07)		
	0.84	-0.08	0.34	-0.46	0.56	-0.04
		(0.00)	(0.05)	(0.06)		
	0.14	0.02	-0.37		0.05	0.36
		(0.00)	(0.23)			
$r(A_t)$	0.45	0.02	-0.37		0.10	0.36
(30-day T-bill)		(0.00)	(0.16)			
	0.84	0.02	-0.25		0.19	0.24
		(0.00)	(0.07)			
	0.14	0.03	-0.27		0.03	0.25
		(0.00)	(0.21)			
$r(A_t)$	0.45	0.03	-0.30		0.07	0.28
(AAA LT Bond)		(0.00)	(0.15)			
	0.84	0.04	-0.40		0.57	0.37
		(0.00)	(0.05)			

by subtracting realized inflation for the 30-day T bill and long-term inflation expectations from the Survey of Professional Forecasters for the AAA bond from the respective nominal rate. Interestingly, the response of the return to a variation in A_t is generally negative and it is significant for the larger values of λ. Since the interest rate is inversely related to the stochastic discount factor, high realizations of productivity translate into higher discount factors.

3.2 Model Based Approach

An alternative approach is to study the dependence of the stochastic discount factor on the state variables using a model. Table 2 reports results for two models. For each model, we use three different values of ρ_A corresponding to the three different values used in the empirical specification.

The first model, labeled RBC, is the standard real business cycle model building from King, Plosser and Rebelo (1988). Here we see that the return responds positively to both the current profitability shock and the capital stock and negatively with future profitability for all specifications of ρ_A.

The second model, labeled Chat-Coop, comes from Chatterjee and Cooper (1993), which studies a stochastic real business cycle model with monopolistic competition. This environment is closer to the underlying market structure assumed in Cooper and Haltiwanger (2006).

The results for the Chat-Coop specification in Table 2 come from a model with no entry and exit and a markup of 25%.[4]. The results are quite similar to those in the standard RBC model.

3.3 Response of the expected stochastic discount factor to productivity

It is instructive to compare the response of interest rates to the state variables in these models with the results from the data. The key is how the expected stochastic discount factor responds to variations in current productivity. Though the plant-level optimization

[4]The markup is based on a CES specification where the elasticity of substitution is set to 5 for both consumption and capital goods.

Table 2: Model-based relationship between state variables and discount factor

Model	ρ_A	A_t	A_{t+1}	K_t	$\frac{dE_t[\tilde{\beta}(A_t,A_{t+1})]}{dA_t}$
RBC	0.14	0.08	-0.39	0.09	0.031
	0.45	0.15	-0.43	0.09	-0.044
	0.84	0.37	-0.59	0.09	-0.121
Chat-Coop	0.14	0.09	-0.41	0.09	0.032
	0.45	0.17	-0.47	0.09	-0.038
	0.84	0.43	-0.64	0.09	-0.110

problem does include a covariance of future values with the stochastic discount factor, looking at the expected discount factor is instructive.

For the data based stochastic discount factors reported in Table 1, we study how $E_t[\tilde{\beta}(A_t, A_{t+1})]$ varies with A_t. The results appear in the last column of that table.

A similar exercise is done with the results from the model based stochastic discount factors. These results are reported in the last column of Table 2.

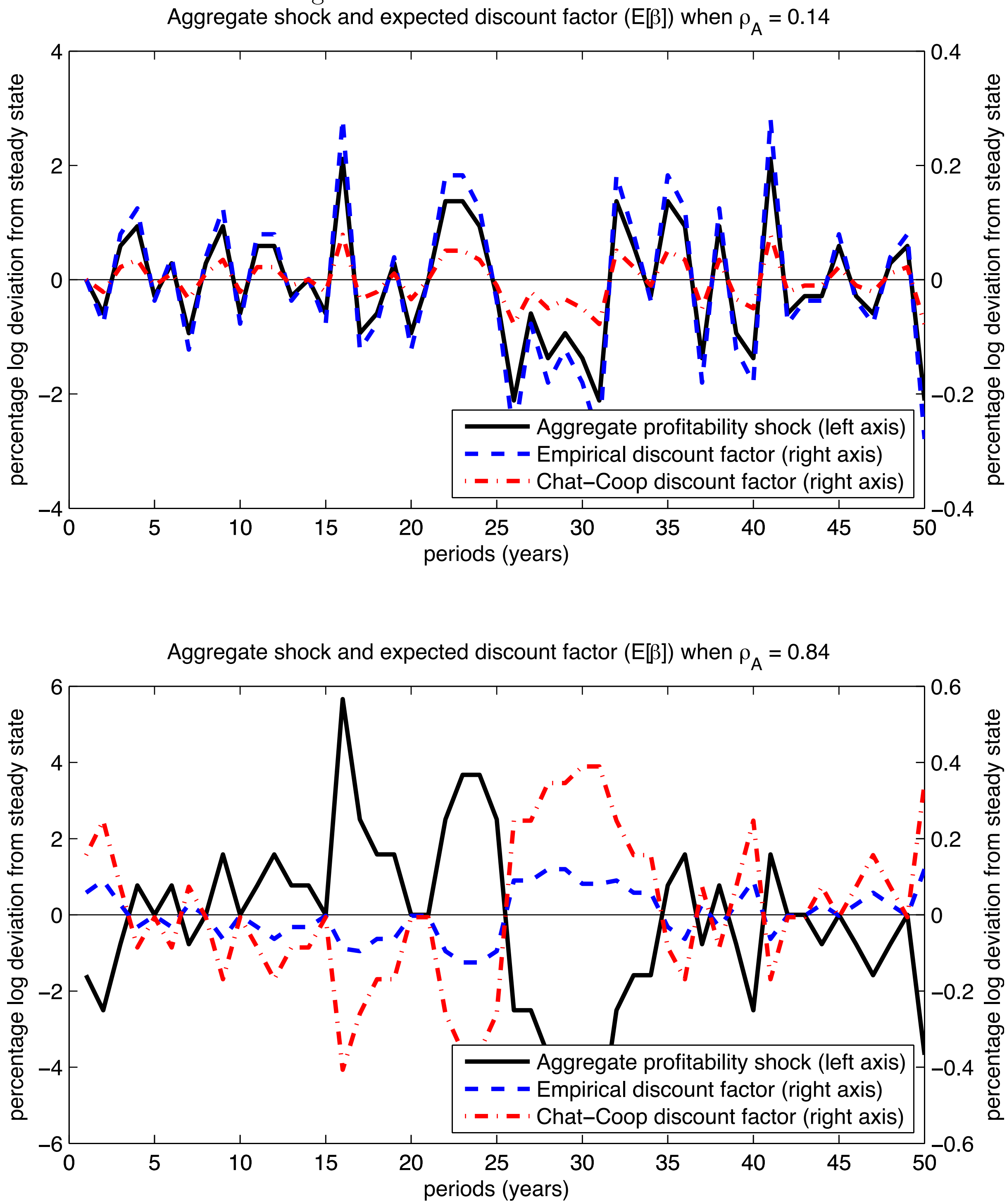
In the first two cases of the $\tilde{\beta}(A_t, A_{t+1})$ estimates, corresponding to $\rho_A = 0.14$ and $\rho_A = 0.45$, the expected stochastic discount factor is increasing in current productivity. The response is slightly negative at $\rho_A = 0.84$ as the negative coefficient on A_{t+1} is given more weight due to the higher serial correlation of the shock.

From the model based results, the expected stochastic discount factor is countercyclical for the $\rho_A = 0.45$ and $\rho_A = 0.84$ cases. This response is more countercyclical than in the empirical based results. At $\rho_A = 0.14$, the stochastic discount factor is also procyclical in the model based estimates.

Figure 1 summarizes these results. At $\rho_A = 0.14$, displayed in the top panel, both stochastic discount factors are procyclical. The data based measure is more volatile than the model based one. For the case of $\rho_A = 0.84$, the stochastic discount factors are both countercyclical but in this case the model-based one is more volatile.

The market returns on both T-bills and long-term bonds imply a strongly procyclical discount factor, regardless of the assumed serial correlation. This effect is stronger than in either the data or model based measures.

Figure 1: Stochastic Discount Factors.



This figure shows the relationship between data and model based stochastic discount factors for two values of ρ_A.

These differences are important for our understanding of how much interest rate movements ultimately smooth the response of plant-level and aggregate investment to profitability shocks. If $\tilde{\beta}$ is positively correlated with the aggregate productivity shock, then the effect of A_t on investment will be magnified. Alternatively, if $\tilde{\beta}$ is negatively correlated with A_t, then aggregate shocks will be smoothed. They also highlight the significance of the serial correlation of the aggregate shocks for the smoothing effects of interest rate movements.

4 Results

Using these processes for the stochastic discount factor, we study the response of investment to shocks. We do so first at the plant level and then in the aggregate.

To obtain these results, we solve (1) for different specifications of the stochastic discount factor reported in Table 1. We then study investment choices at the plant-level and in aggregate. Thus the investment choices depend on empirically relevant representations of the stochastic discount factor.

For these simulations, we follow Cooper and Haltiwanger (2006) and assume $\alpha = 0.58$, $\rho_\varepsilon = 0.885$ $\sigma_\varepsilon = 0.1$ for the idiosyncratic shock process. The adjustment are costs given by $\Lambda = 0.8, \nu = 0.15, qs = 0.98$. In the simulated data set, we follow 1000 plants for 500 periods.

4.1 Plant-level Implications

Table 3 reports moments at the plant-level for different interest rate processes.[5] These are the moments used in Cooper and Haltiwanger (2006) for the estimation of adjustment cost parameters.[6] There is an important point to gather from this table: these plant-level moments are essentially independent of the representation of the stochastic discount factor. Hence the parameter estimates from Cooper and Haltiwanger (2006), which assumes a constant discount factor, are robust to analysis allowing a stochastic discount factor.

This does not imply though that the investment decision is independent of the specification of the stochastic discount factor. Table 4 reports results of plant-level regressions on

[5] In these tables, $r(A_t)$ is the 30-day T-bill.

[6] In that analysis, the correlation of profitability and investment was used rather than the correlation of plant-specific profitability and investment. In the data, these correlations are about the same.

current state variables for different measures of the stochastic discount factor and different parameters for ρ_A.

There are three important results here. First, the response of investment to current profitability depends on the specification of the stochastic discount factor. Second, this response depends on ρ_A. Third, the response of investment to aggregate profitability is very different for the data based characterization of the stochastic discount factor compared to the model based version.

The response of investment to A_t is generally positive and significant. One exception is for the fixed β case and the low value of ρ_A. At $\rho_A = 0.14$, the profitability process is closest to iid. This combined with the opportunity cost of investment, $\Lambda < 1$, implies that it is more costly to invest when A_t is high without any apparent future benefit. This result is overturned for larger values of ρ_A and for the other specifications of the stochastic discount factor.

A second exception comes from the comparison of the data and model based versions of the stochastic discount factor. For the former, investment at the plant-level is increasing in A_t while for the latter it is decreasing. Investment is also increasing in the shock when discounting occurs through market determined real return on the 30-day T bill.

Table 3: Plant-level moments from simulated data

Model	ρ_A	mean($\frac{I_i}{K_i}$)	Frac Inactive	Frac -	Spike +	Spike -	mean(abs($\frac{I_i}{K_i}$))	Corr($I_i, I_{i,-1}$)	Corr(I_i, ε_i)
β	0.14	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.19
	0.45	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.19
	0.84	0.090	0.88	0.000	0.12	0.000	0.72	-0.13	0.20
$\tilde{\beta}(A_t, A_{t+1})$ (Empirical)	0.14	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.19
	0.45	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.19
	0.84	0.090	0.88	0.000	0.12	0.000	0.72	-0.13	0.20
$\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop)	0.14	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.19
	0.45	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.19
	0.84	0.090	0.88	0.000	0.12	0.000	0.72	-0.13	0.20
$r(A_t)$	0.14	0.090	0.87	0.000	0.13	0.000	0.72	-0.13	0.18
	0.45	0.090	0.87	0.000	0.13	0.000	0.72	-0.12	0.18
	0.84	0.091	0.87	0.000	0.13	0.000	0.72	-0.12	0.18

Note that the coefficient on the plant-specific shock is positive and almost independent of the stochastic discount factor. The same goes for the plant-specific capital stock. This makes sense since the stochastic discount factor depends on aggregate not plant-specific variables.

Tables 5 and 6 break the investment response into two components. The intensive margin indicating the investment response of the adjusters and the extensive margin regarding the choice to invest or not.

Comparing the intensive margin regressions in Table 5 with the results in Table 4, there are a couple of points to note. For the fixed β case, the response to A is positive for the adjusters. Else there is no change in sign for the response to the shocks. However, for the intensive margin the response to K is almost zero.

Comparing the extensive margin regressions in Table 6 with the results in Table 4, the negative effects of high capital on the adjustment choice is very strong. The adjustment probability is increasing in A_t for all specifications except for the fixed β case with low ρ_A and for all simulations using the Chat-Coop specification of the stochastic discount factor. The opposing sign in the response of the extensive margin to changes in A_t for the two specifications of $\tilde{\beta}\left(A_t, A_{t+1}\right)$ illustrates the important role of the interest rate channel for plant-level investment decisions.

4.2 Aggregate

We aggregate our simulated data to study the response of aggregate investment to profitability shocks for different specifications of the stochastic discount factor. We also ask whether empirically relevant movements in the stochastic discount factor smooth out fluctuations in aggregate investment due to the non-convexities at the plant level.

4.2.1 Basic Moments

Table 7 presents some basic correlations from the aggregated simulated data. The patterns are clearly sensitive to both the specification of the stochastic discount factor and the choice of ρ_A. For the fixed discount factor and a low value of ρ_A, investment is negatively correlated with the aggregate shock, both in terms of the investment rate and in the fraction of adjusters. This, as explained earlier, comes from the opportunity cost of adjusting.

For the data based version of the stochastic discount factor, either from $\tilde{\beta}\left(A_t, A_{t+1}\right)$

Table 4: Plant-level investment regression on simulated data: $I_i(A, \varepsilon_i, K_i)$

Model	ρ_A	A	ε_i	K_i	R^2
	0.14	-12.78	28.47	-0.46	0.36
		(3.22)	(0.26)	(0.00)	
β	0.45	1.36	29.18	-0.46	0.36
		(2.32)	(0.26)	(0.00)	
	0.84	23.59	29.82	-0.43	0.36
		(1.37)	(0.27)	(0.00)	
	0.14	24.58	28.40	-0.46	0.36
		(3.22)	(0.26)	(0.00)	
$\tilde{\beta}(A_t, A_{t+1})$	0.45	7.08	29.10	-0.46	0.37
(Empirical)		(2.32)	(0.26)	(0.00)	
	0.84	13.76	29.74	-0.43	0.36
		(1.37)	(0.27)	(0.00)	
	0.14	-1.97	28.63	-0.46	0.36
		(3.21)	(0.26)	(0.00)	
$\tilde{\beta}(A_t, A_{t+1})$	0.45	-4.70	28.86	-0.45	0.36
(Chat-Coop)		(2.33)	(0.26)	(0.00)	
	0.84	-6.48	30.20	-0.44	0.36
		(1.36)	(0.27)	(0.00)	
	0.14	80.89	27.44	-0.44	0.36
		(3.23)	(0.26)	(0.00)	
$r(A_t)$	0.45	91.40	27.14	-0.43	0.36
		(2.34)	(0.26)	(0.00)	
	0.84	91.15	26.39	-0.41	0.36
		(1.45)	(0.26)	(0.00)	

Table 5: Plant-level investment regression (Adjusters only): $I_i(A, \varepsilon_i, K_i)$

Model	ρ_A	A	ε_i	K_i	R^2
β	0.14	4.54	25.54	0.02	0.90
		(2.57)	(0.45)	(0.01)	
	0.45	6.70	26.47	0.03	0.93
		(1.59)	(0.39)	(0.01)	
	0.84	26.11	29.58	0.02	0.98
		(0.56)	(0.23)	(0.01)	
$\tilde{\beta}\left(A_t, A_{t+1}\right)$ (Empirical)	0.14	17.11	24.60	0.04	0.90
		(2.59)	(0.45)	(0.01)	
	0.45	11.57	27.03	0.01	0.93
		(1.58)	(0.39)	(0.01)	
	0.84	18.42	29.61	0.03	0.98
		(0.50)	(0.21)	(0.00)	
$\tilde{\beta}\left(A_t, A_{t+1}\right)$ (Chat-Coop)	0.14	5.28	25.25	0.03	0.90
		(2.51)	(0.44)	(0.01)	
	0.45	2.88	26.63	0.03	0.93
		(1.58)	(0.39)	(0.01)	
	0.84	2.80	29.67	0.03	0.99
		(0.45)	(0.19)	(0.00)	
$r\left(A_t\right)$	0.14	32.77	24.14	0.05	0.89
		(2.91)	(0.44)	(0.01)	
	0.45	57.48	26.46	0.02	0.90
		(2.40)	(0.42)	(0.01)	
	0.84	83.49	27.72	0.03	0.93
		(1.71)	(0.37)	(0.01)	

Table 6: Linear probability regression using plant-level simulated data (extensive margin)

Model	ρ_A	A	ε_i	K_i	R^2
β	0.14	-0.67	1.25	-0.84	0.37
		(0.15)	(0.01)	(0.01)	
	0.45	0.02	1.27	-0.84	0.37
		(0.10)	(0.01)	(0.01)	
	0.84	1.00	1.26	-0.81	0.37
		(0.06)	(0.01)	(0.01)	
$\tilde{\beta}(A_t, A_{t+1})$ (Empirical)	0.14	1.17	1.25	-0.84	0.37
		(0.15)	(0.01)	(0.01)	
	0.45	0.29	1.26	-0.84	0.37
		(0.10)	(0.01)	(0.01)	
	0.84	0.58	1.24	-0.79	0.37
		(0.06)	(0.01)	(0.01)	
$\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop)	0.14	-0.11	1.26	-0.85	0.37
		(0.15)	(0.01)	(0.01)	
	0.45	-0.29	1.26	-0.83	0.37
		(0.10)	(0.01)	(0.01)	
	0.84	-0.33	1.27	-0.81	0.37
		(0.06)	(0.01)	(0.01)	
$r(A_t)$	0.14	3.86	1.20	-0.82	0.37
		(0.15)	(0.01)	(0.01)	
	0.45	4.26	1.17	-0.79	0.38
		(0.10)	(0.01)	(0.01)	
	0.84	3.97	1.11	-0.76	0.37
		(0.06)	(0.01)	(0.01)	

or directly from observed interest rates, investment is positively correlated with aggregate profitability. As noted earlier in the discussion of Table 1, increases in profitability increase $\tilde{\beta}$ when $\rho_A = 0.14$ and $\rho_A = 0.45$ so that the investment response to changes in A_t is magnified by the changes in the stochastic discount factor. For $\rho_A = 0.84$, an increase in A_t does lead to a fall in $\tilde{\beta}(A_t, A_{t+1})$ but as was clear in Figure 1 this effect is small relative to the variability of the model based stochastic discount factor.

The results are quite different in the case of the model-based stochastic discount factor. Here investment is negatively correlated with the profitability shock. This comes from the interaction of the opportunity cost of adjustment and the fact that for $\rho_A = 0.45$ and $\rho_A = 0.84$, $\tilde{\beta}(A_t, A_{t+1})$ falls as A_t increases.

Note that these differences in the determination of the stochastic discount factor carry over to other moments. For the empirically based measured of $\tilde{\beta}(A_t, A_{t+1})$, the fraction of adjusters is positively correlated with the aggregate profitability shock. This correlation is negative for the model based versions. While there is a difference in the sign on the extensive margin, the mean investment rate for those who adjust is positively correlated with A in both specifications of $\tilde{\beta}(A_t, A_{t+1})$.

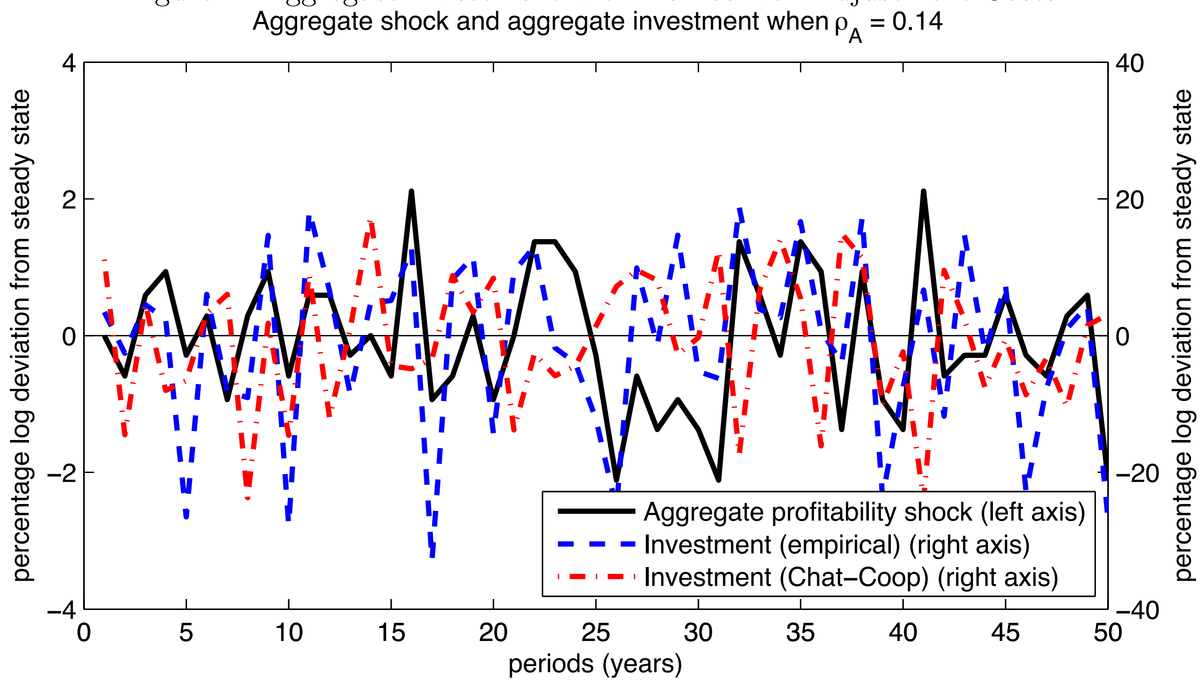
Table 8 reports regressions of the future aggregate capital stock on the current state for different values of ρ_A for different values of the stochastic discount factor. Here we see that the coefficient on current profitability is positive for the empirically based stochastic discount factor but negative for the model based stochastic discount factor. Table 9 is the same exercise on actual data. The coefficient on A_t is positive throughout, consistent with the data based version of the stochastic discount factor.

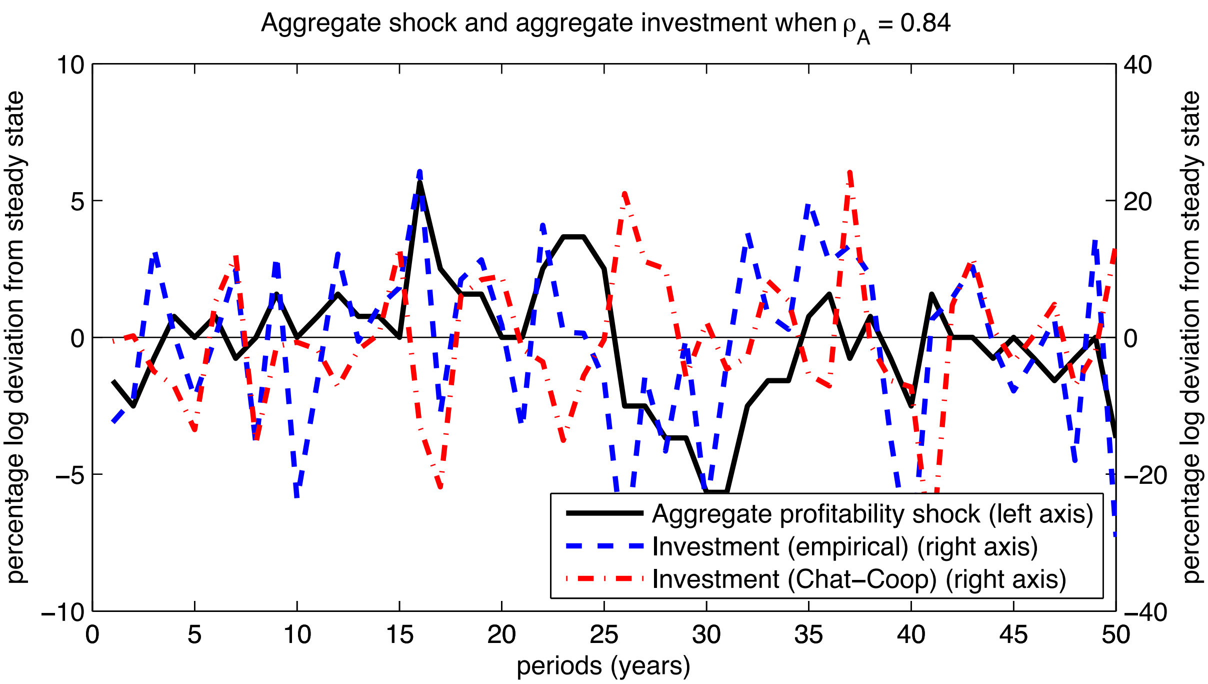
Figure 2 highlights the importance of the stochastic discount factor for aggregate investment. This is for the baseline non-convex adjustment cost case. As in Table 7, the two investment series respond very differently to variations in aggregate profitability. In many cases, the model and empirically based versions of the stochastic discount factor leads to opposite movements of aggregate investment in response to profitability shocks. This is, again, consistent with the differences reported at the plant-level in Table 4.

4.2.2 Role of Adjustment Costs

Returning to the point emphasized in Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008), we can use this model to study the smoothing effects of the data

Figure 2: Aggregate Investment with Non-convex Adjustment Costs.





This figure shows aggregate investment and the aggregate shock for the baseline model of non-convex adjustment costs. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$.

Table 7: Correlation of aggregates with aggregate profitability (A) in simulated data

Model	ρ_A	I	K	$\frac{I}{K}$	Fraction of adjusters	mean($\frac{I_i}{K_i}$)
β	0.14	-0.51	0.03	-0.50	-0.55	0.61
	0.45	-0.05	0.15	-0.06	-0.09	0.54
	0.84	0.55	0.73	0.43	0.46	0.09
$\tilde{\beta}(A_t, A_{t+1})$ (Empirical)	0.14	0.61	0.15	0.58	0.62	0.01
	0.45	0.31	0.26	0.27	0.29	0.41
	0.84	0.47	0.69	0.36	0.36	0.47
$\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop)	0.14	-0.19	0.10	-0.20	-0.23	0.49
	0.45	-0.31	0.04	-0.30	-0.36	0.65
	0.84	-0.27	-0.22	-0.24	-0.33	0.83
$r(A_t)$	0.14	0.79	0.10	0.76	0.78	-0.74
	0.45	0.73	0.36	0.69	0.70	-0.80
	0.84	0.62	0.71	0.52	0.53	-0.78

based and model based stochastic discount factors. To do so, we compare the aggregate properties of investment in our baseline model with non-convex adjustment costs with the aggregate behavior of investment in a model with no adjustment costs. From the results of this literature, one would predict that the non-convexity does not matter in the aggregate.

We base this comparison in part on a the evolution of the aggregate capital stock. If the non-convexity at the plant-level has no aggregate implications, then the state dependent evolution of the capital stock should be the same as in the model without any adjustment costs. This is consistent with the finding in Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008).

For this exercise, we use the model based stochastic discount factor from the model of Chatterjee and Cooper (1993). That model has no adjustment costs. Motivated by the findings in Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008), this is the appropriate discount factor to use under the null hypothesis that the non-convex adjustment costs have no aggregate implications.

Our results for the case of no adjustment costs are summarized in Table 10. This is

Table 8: Aggregate capital regression on simulated data: Non-Convex Adjustment Costs

Model	ρ_A	A_t	K_t	R^2
β	0.14	-0.33	0.71	0.64
		(0.02)	(0.03)	
	0.45	-0.00	0.77	0.60
		(0.02)	(0.03)	
	0.84	0.63	0.50	0.96
		(0.02)	(0.01)	
$\tilde{\beta}(A_t, A_{t+1})$ (Empirical)	0.14	0.60	0.58	0.72
		(0.03)	(0.02)	
	0.45	0.18	0.72	0.68
		(0.02)	(0.03)	
	0.84	0.35	0.59	0.90
		(0.02)	(0.02)	
$\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop)	0.14	-0.10	0.78	0.60
		(0.02)	(0.03)	
	0.45	-0.12	0.77	0.63
		(0.02)	(0.03)	
	0.84	-0.09	0.74	0.66
		(0.01)	(0.03)	
$r(A_t)$	0.14	2.22	0.37	0.92
		(0.03)	(0.01)	
	0.45	2.59	0.40	0.95
		(0.04)	(0.01)	
	0.84	2.49	0.48	0.98
		(0.04)	(0.01)	

Table 9: Aggregate Capital Regression: Actual Data

ρ_A	Constant	A_t	K_t	R^2
0.14	-0.00	0.23	0.23	0.16
	(0.00)	(0.08)	(0.12)	
0.45	-0.00	0.19	0.74	0.54
	(0.00)	(0.09)	(0.09)	
0.84	-0.00	0.13	0.98	0.90
	(0.00)	(0.06)	(0.05)	

comparable to Table 8 which is based on simulated data from a model with non-convex costs at the plant-level.

Comparing these tables, it is clear that the results are very different for all of the specifications of the stochastic discount factor. This is perhaps not as surprising for the fixed β treatment since smoothing by aggregation is not feasible.

Even for the model based specification of the stochastic discount factor, the results with the non-convex adjustment costs are far from those without any adjustment costs. The response of the future capital stock to a variation in current profitability is negative for all values of ρ_A in the case of non-convex adjustment costs and the model based stochastic discount factor. These responses are all positive in the case of no-adjustment costs.

Moreover, the results are quite different for the empirically based stochastic discount factor. There is no evidence here that interest rate movements smooth out the non-convexities at the plant-level so that aggregate capital movements are essentially identical to those in a model without adjustment costs.

The differences in aggregate investment under the different combinations of adjustment costs and stochastic discount factors are further illustrated in Figures 3 and 4. Not surprisingly, aggregate investment is considerably more volatile in the absence of adjustment costs. The role of adjustment costs is to dampen the response of investment to the aggregate shock. These differences are apparent for both values of ρ_A in Figure 3.

Even for the model based version of the stochastic discount factor in Figure 4, these differences remain. There is no basis for the conclusion that these two models are observationally

Table 10: Aggregate capital regression on simulated data: no adjustment costs

Model	ρ_A	A_t	K_t	R^2
β	0.14	0.77	0.24	0.68
		(0.03)	(0.03)	
	0.45	0.35	0.84	0.73
		(0.02)	(0.03)	
	0.84	0.10	1.81	0.95
		(0.01)	(0.03)	
$\tilde{\beta}(A_t, A_{t+1})$ (Empirical)	0.14	0.08	3.11	0.90
		(0.01)	(0.05)	
	0.45	0.19	1.38	0.83
		(0.02)	(0.03)	
	0.84	0.18	1.20	0.91
		(0.02)	(0.03)	
$\tilde{\beta}(A_t, A_{t+1})$ (Chat-Coop)	0.14	0.40	1.03	0.68
		(0.03)	(0.04)	
	0.45	0.60	0.37	0.66
		(0.03)	(0.03)	
	0.84	0.73	0.03	0.54
		(0.03)	(0.02)	
$r(A_t)$	0.14	0.01	7.85	0.98
		(0.01)	(0.05)	
	0.45	0.01	8.42	0.99
		(0.00)	(0.04)	
	0.84	0.01	6.81	1.00
		(0.00)	(0.03)	

equivalent at the aggregate level.

These results are important in two ways. First, it is clear that the findings of Thomas (2002), Khan and Thomas (2003) and Khan and Thomas (2008) do not hold in our environment. This is partly due to the use of data based stochastic discount factor and partly due to differences in the specification of adjustment costs.[7]

Second, to the extent non-convexities in plant-level investment matter for aggregate investment behavior, the conduct of monetary policy must take these factors into account. We turn to that topic next.

5 Monetary Policy

[to be completed]

6 Conclusion

This paper studies the implications of interest rate movements on investment. In particular, we highlight how changes in interest rates influences the effects of profitability shocks on investment. We see that the smoothing effects of interest rates depends on the determination of that process. If, as we have emphasized here, the state dependent discount factor is determined from the data, then there is little smoothing of investment due to interest rate movements.

As this work proceeds, we will turn to an analysis of monetary policy which presumably underlies the interest rate process uncovered in the data. We can use our model to see how alternative monetary policies can influence investment behavior.

[7] Relative to our model, Thomas (2002) has a lump-sum non-convex adjustment cost rather than the opportunity cost of investment. Further, that model has no quadratic adjustment costs and no irreversibility. There are no iid shocks to profitability but instead there are plant-specific stochastic adjustment costs. The numerical analysis assumes $\rho_A = 0.9225$. The plant's stochastic discount factor comes from the solution of the stochastic general equilibrium model. Understanding which of these model components is the source of the differences in results remains an open question.

7 Appendix: TFP process

The measure of total factor productivity used in this analysis is constructed as a Solow residual following Stock and Watson (1999). The calculation includes nonfarm real GDP (source: BEA), nonfarm payroll employment (source: BLS), real nonresidential private fixed capital stock (source: BEA and authors' calculations), and a labor share of 0.65. The data sample is annual frequency from 1948 to 2008.

Our analysis is focused on business-cycle dynamics, so here we abstract from long-term growth in TFP by detrending the data. Various approaches have been used in the literature for detrending, so we employ three different detrending specifications to examine the sensitivity of the results. The first approach focuses specifically on business cycle frequencies (between 3 and 8 years). For this case, we detrend using the HP filter with the λ parameter set to 7, which closely approximates a band-pass filter on annual data. The second approach is to remove a linear trend from the data, which we approximate by setting the HP filter parameter for λ to 100,000. The third approach uses an intermediate value of λ that is commonly used to filter annual data, $\lambda = 100$.

The parameters of the TFP process are estimated based on a log-normal AR(1) specification.

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}, \ \ \epsilon_A \sim N(0, \sigma^2_{\epsilon_A}) \tag{8}$$

Estimates of the shock process parameters are displayed in Table 11 for the three different detrending specifications. The estimate of serial correlation in TFP, ρ_A, is very sensitive to the detrending specification. If focusing on business cycle frequencies, $\lambda = 7$, there is little serial correlation in detrended TFP. On the other hand, detrended TFP has a much higher serial correlation when approximating the removal of a simple time trend ($\lambda = 100,000$). We consider the process parameter estimates from all three detrending specifications in our analysis to examine the role of the detrending assumption in modeling the relationship between interest rates and investment decisions.

Table 11: Parameter estimates for Solow-residual technology process

λ	ρ_A	σ_{ϵ_A}
7	0.14	0.012
100	0.45	0.015
100000	0.84	0.018

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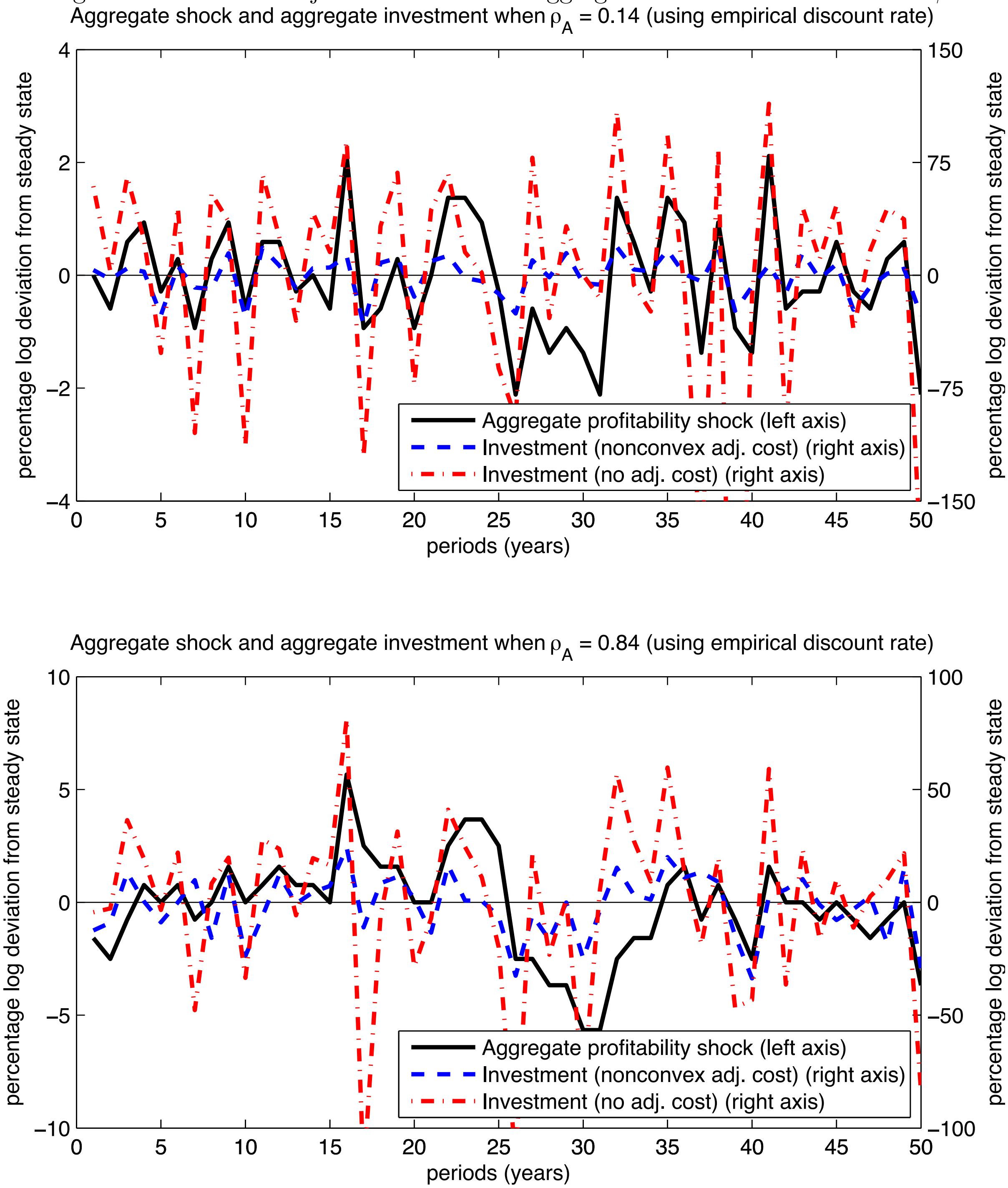
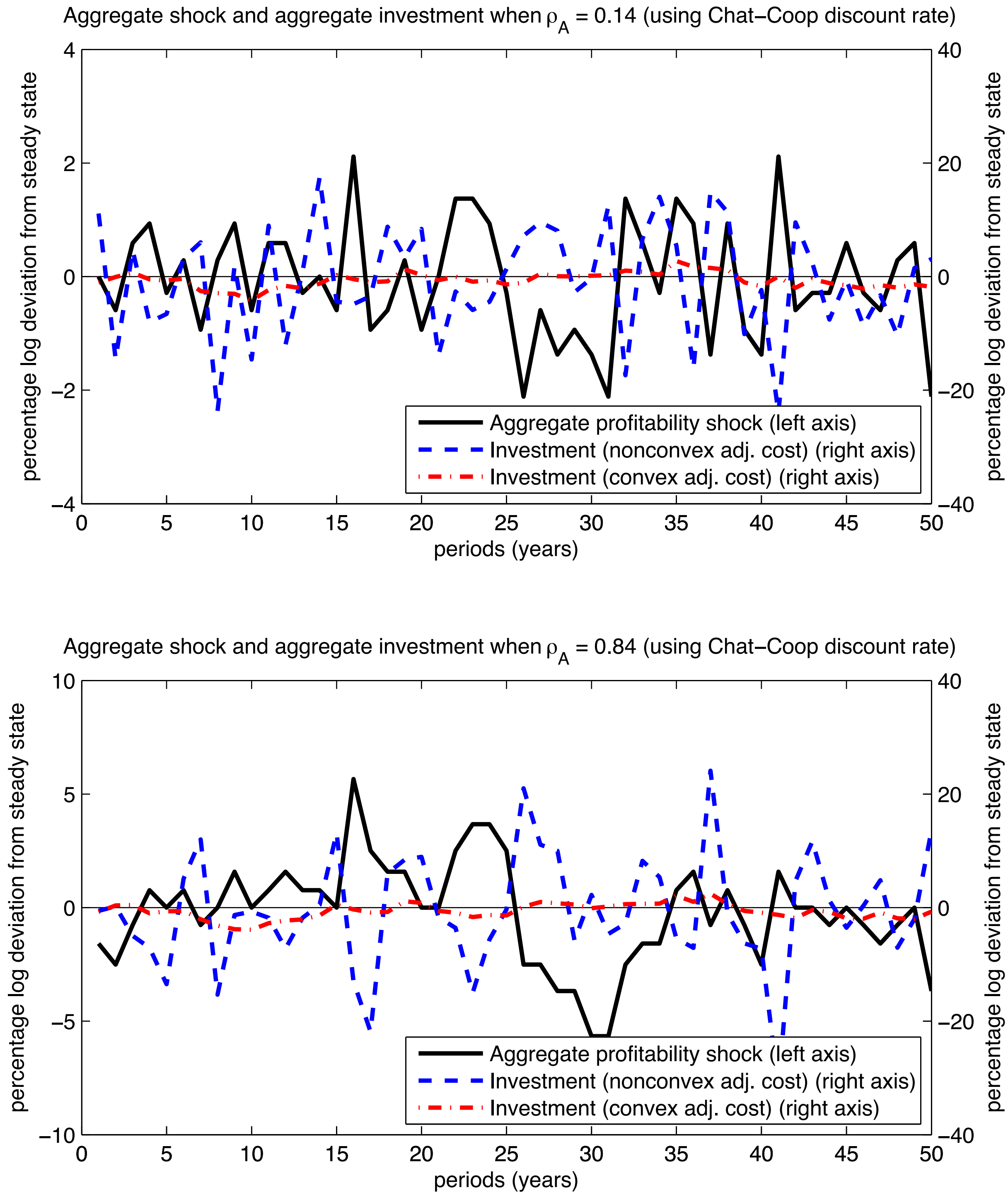
Figure 3: Effects of Adjustment Costs on Aggregate Investment: Data Based $\tilde{\beta}$



This figure shows aggregate investment and the aggregate shock for the baseline model of non-convex adjustment costs and for the case of no adjustment costs. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$. In both cases, the stochastic discount factor is based on the data.

Figure 4: Effects of Adjustment Costs on Aggregate Investment: Model Based $\tilde{\beta}$



This figure shows aggregate investment and the aggregate shock for the baseline model of non-convex adjustment costs and for the case of no adjustment costs. The top panel is for the case of $\rho_A = 0.14$ and the bottom panel is for the case of $\rho_A = 0.84$. In both cases, the stochastic discount factor is based on the model.