Relational Contracts with On-the-Job Search*

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Abstract

This paper characterizes the distribution of jobs in a relational contracting model where underemployed workers compete with unemployed workers in the job market. Such on-the-job search increases turnover and thereby the marginal costs to incentivize effort. This leads firms to replace permanent jobs that pay high wages and command high effort, with temporary jobs that pay low wages and command low effort. This deterioration of job quality reduces the prospects of the unemployed and introduces slack into the workers' incentive constraints. If the number of firms is fixed, firms react by cutting wages, so profits increase while welfare and workers' rents decrease. If the number of firms is endogenous, new firms enter the market, so the loss of high-quality jobs is countered by an increase in the total number of jobs. When unemployed and underemployed workers' are equally skilled in getting job offers, free entry leads to full employment and effort is incentivized by the threat of underemployment rather than unemployment.

1 Introduction

Employment contracts are typically incomplete, lacking explicit incentives for important aspects of a worker's behavior, such as teamwork, initiative and judgment (Williamson (1985)). Firms thus motivate workers through long-term relationships, and the desire to generate incentives and sustain morale becomes a major factor determining wages (Bewley (1999), Fehr and Falk (1999)). The goal of this project is to study the interplay between firm-worker relationships and the market-level interaction between firms. From a firm's standpoint, the longevity of its relationships and the way it motivates its workers depend on job offers by other firms in the market. From the market's perspective, the quality of jobs in the economy and distribution of these jobs depend on firms' choices

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and the underlying contractual problem. By analyzing both intra- and inter-firm interaction we wish to sharpen our understanding of the determinants of turnover and its ramifications on job quality, the role of unemployment in incentivizing workers, and the phenomenon of 'over-qualification'.

We consider an economy with homogeneous firms and workers in which firms motivate their employees through relational contracts and both unemployed and underemployed workers constantly compete for jobs. We characterize the equilibrium job distribution in terms of wages, effort, and turnover. We find that the ability to search on-the-job acts like a non-pecuniary benefit for employees, and makes it profitable for firms to replace permanent, high-quality jobs with temporary, low-quality jobs. With a fixed number of firms, this deterioration of job quality reduces the prospects of unemployed workers and allows firms to cut wages further, and increase profits. With free entry, the potential for increased profit leads new firms to enter at the bottom of the market, so the deterioration in job quality is countered by a reduction of unemployment.

This paper contributes to the literature on relational contracts in large markets, pioneered by Shapiro and Stiglitz (1984) and Macleod and Malcomson (1998), by introducing on-the-job search. Job-to-job transitions are empirically prevalent, accounting for more than 70% of all job transitions (Nagypal (2004)). As discussed above, on-the-job search also significantly changes the theoretical predictions of the model.

In the model, there is measure 1 of identical workers and measure $n < 1$ of identical firms. Each firm has one job. Each period $t \in \{1, 2, \dots\}$ has three stages. First, firm i pays its worker wage w_t. Second, the worker chooses his effort η_t and output is publicly revealed. Third, both firm and worker can terminate the relationship unilaterally; separation also occurs with exogenous probability $1 - \alpha$.

A relational contract between a firm and a worker consists of a sequence of wages and effort requirements that may depend on the worker's tenure. Assuming workers' outside options and strategies are independent of calendar time, Theorem 1 shows that there exists a firm-optimal self-enforcing contract with stationary wages and effort levels. Ideally, the firm would like to back-load wages to extract the worker's rents, but such contracts are not self-enforcing since the firm would deviate by firing expensive old workers and replacing them with cheap new workers.

This stationarity result allows us to analyze the continuum of infinitely repeated games as industry equilibria of a static game where each firm simultaneously chooses a contract, consisting of a wage w and a required effort level η, so as to maximize profits $\pi = \phi(\eta) - w$ subject to its worker's incentive compatibility constraint.[1]

At the end of each period some jobs are vacant because of natural turnover. Workers and jobs are then re-matched in a way that satisfies the following properties:

1. Market Clearing: Every firm finds a worker. At the start of the period some firms have vacancies; all of these vacancies are filled, as are all subsequent vacancies that arise in the matching process.

[1] While we write the model in terms of classical relational contracting, one can interpret the results in terms of a model where agents have reciprocal preferences and exert high effort at a firm that offers high wages and good job opportunities, as captured by the agents' value function (Fehr and Gächter (2000)). In such a model effort is determined by a function analogous to IC, and the qualitative results are unaffected

2. Anonymity: Job offers do not depend on a worker's history with current or previous employers, but may differ for employed and unemployed workers.

3. Individual Rationality: A worker only accepts a job offer if it offers higher utility than his current job.

This formulation nests many interesting matching processes. Under *Shapiro-Stiglitz matching*, only umemployed workers receive job offers; under *fully anonymous matching*, employed and unemployed workers receive the same job offers; under *intern matching* employed workers receive offers first and only left-over entry-level jobs are offered to the unemployed. Formally, the goal of the paper is to characterize how industry equilibria depend on the matching process.

The matching process and the job distribution jointly determine workers' outside options. We use this connection to derive a contract's value to a worker, a worker's incentives to honor the contract, and finally the firm-optimal self-enforcing contract. In equilibrium the continuum of optimal contracts constitutes a fixed point and firms are indifferent between all contracts offered. Theorem 2 shows that industry equilibrium exists and is unique. With Shapiro-Stiglitz matching, all firms offer the same job, and workers never quit. In contrast, with on-the-job search, equilibrium contracts are distributed continuously. The best of these jobs commands the same effort as the unique job in Shapiro-Stiglitz but all other jobs command strictly less effort and have higher turnover. This means that the introduction of on-the-job search crowds out high-quality, permanent jobs with low-quality, temporary jobs.

To understand how this job deterioration comes about, consider a single firm's problem. At the optimal contract, the firm equates the marginal benefit of effort and its marginal cost of inducing effort. This marginal cost can be decomposed into direct compensation for the worker's effort and an incentive premium. The marginal premium is higher for a firm with high turnover since a wage raise incentivizes effort today only to the degree that the worker is still employed to collect the raise tomorrow. With Shapiro-Stiglitz matching the retention rate is constant, equal to $1 - \alpha$ for any job, so all firms offer the same job. If we introduce on-the-job search, a firm would deviate and offer a job with lower wage and effort, as shown in Figure 1. This is because on-the-job search lowers the worker's opportunity cost of taking a bad job. Such jobs have lower retention rates and therefore lower effort requirements. In the on-the-job search equilibrium, the retention rate is strictly increasing in job quality, so a better job has lower turnover, a lower cost of incentivizing the worker and thus higher effort. For any quantile of the job distribution, the firm's first order condition is thus satisfied at a different effort level, so effort levels and wages are distributed continuously, as shown in Figure 2.

To study wages, profits and unemployment, we need to take a stance on firm entry. We first consider the conceptually and analytically simpler case with a fixed number of firms $n < 1$. Theorem 3 shows that an increase in on-the-job search (in a sense we make precise) unambiguously increases turnover, decreases welfare and workers' rents but increases firms' profits. The increase in turnover follows mechanically from the increase in on-the-job search, raising the marginal cost of incentive provision and decreasing firms' optimal effort requirements. The deterioration in job quality in turn

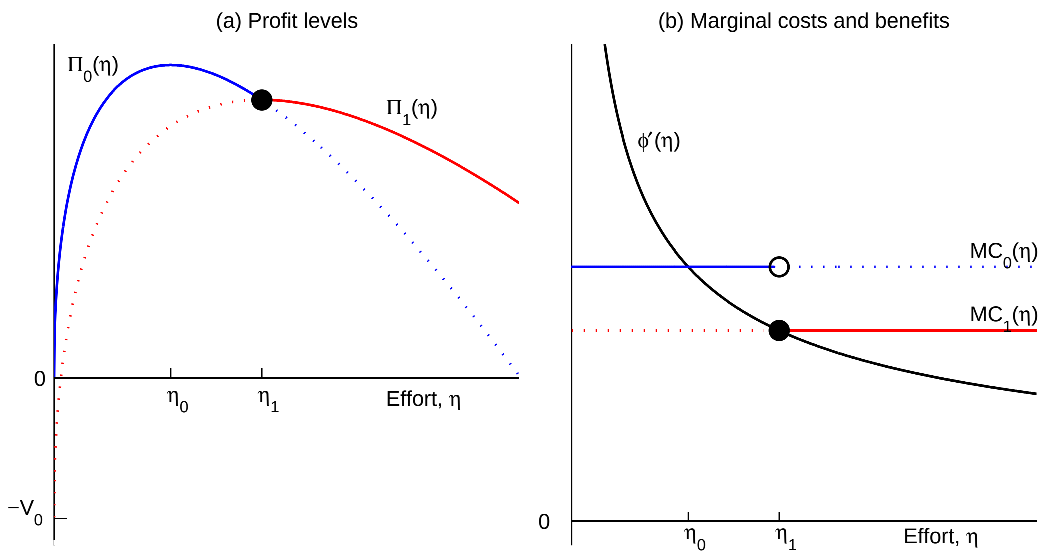

Figure 1: **Non-Existence of Fixed Wage Equilibria.** This illustrates the profit when deviating from the Shapiro-Stiglitz equilibrium with effort η_1. With Shapiro-Stiglitz matching, firms retain their workers with probability α, profits are given by $\Pi_1(\eta)$ and the marginal cost of effort is $\mathrm{MC}_1(\eta)$. With on-the-job search, a firm that offers a lower wage has a lower retention rate but does not have to pay its workers the full opportunity cost of forgone searching V_0. The resulting profit is $\Pi_0(\eta)$ with marginal cost of effort $\mathrm{MC}_0(\eta)$, yielding a profitable deviation. The figure assumes fully anonymous matching, output $\phi(\eta) = \sqrt{2\eta}$, mass of firms $n = 2/5$, breakup rate $1 - \alpha = 1/2$, and discount rate $\delta = 1/2$.

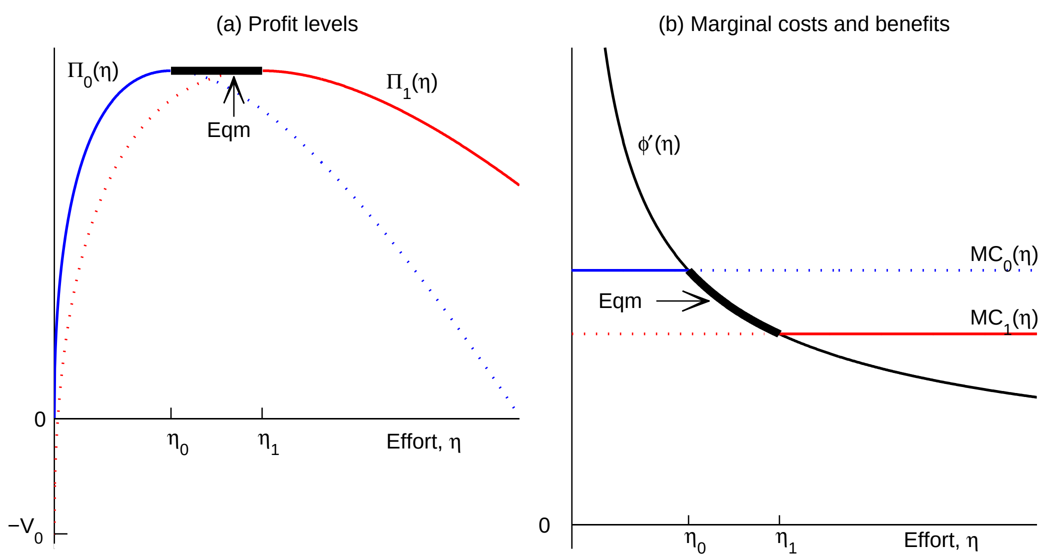

Figure 2: **Equilibrium Contracts.** In this figure, $\Pi_1(\eta)$ and $\Pi_0(\eta)$ are profit functions for firms with the lowest and highest turnover rates. The distribution of contracts is determined so that each firm equates the marginal cost and benefit of effort, and achieves the same level of profits. When compared to Figure 1, the distribution of jobs has deteriorated, lowering V_0 and raising $\Pi_1(\eta)$.

reduces the prospect of unemployed workers and increases the relative value of a job with given wage and effort. The worker's incentive constraint is thus slack and all firms can cut wages. Thus, workers suffer twice from an increase in on-the-job search: First through the replacement of high-wage, high-quality jobs by low-wage, low-quality jobs, and second through the ensuing wage cuts. Profits on the other hand increase, because they are held constant in the first step and increase in the second.

With free entry, the slack in workers' incentive constraints is met by additional firms entering at the bottom of the market, rather than incumbent firms increasing profits by cutting wages. The loss of high quality jobs is thus compensated by a reduction in unemployment and the welfare effect is ambiguous. Unlike in the classical treatment by Shapiro and Stiglitz (1984), entry can lead to full employment. Effort in good jobs is then incentivized by underemployment rather than unemployment.

Empirical Evidence. The model is based on the premise that higher wages raise the value of a job and encourage effort. This is consistent with evidence that high-wage plants have fewer disciplinary actions (Raff and Summers (1987), Cappelli and Chauvin (1991)), that wages are positively correlated with a worker's reported commitment and effort (Levine (1993)), and that firms give bonuses and refuse to cut pay in order to motivate workers (Bewley (1999)). These correlations could be the result of reciprocal preferences as found in the experiments of Fehr and Falk (1999). Alternatively, they could result from efficiency wages, consistent with the findings that unemployment increases effort (Bewley (1999, Table 8.3)), reductions in employment protection increase effort (Jacob (2010)), and that effort declines at the end of a relationship (Hansen (2009)).

Our results are consistent with the large unexplained cross-industry and cross-employer wage differentials (Krueger and Summers (1988), Mortensen (2003)), and job ladders where a large fraction of wage growth occurs at job transitions, and jumps are more frequent and larger at the start of a worker's career (Topel and Ward (1992)). The model is particularly applicable to professional occupations, where effort is more subjective (Milgrom and Roberts (1992, p. 258)) and the rate of job-to-job transitions is particularly high (Topel and Ward (1992), Nagypal (2005)). It helps explain why such jobs have higher levels of residual wage inequality (Lemieux (2006)) and give rise to job ladders. Indeed, Bulow and Summers (1986, p. 378) write that, "job ladders appear in parts of the economy where individual performance is difficult to disentangle". For example, in his study of academic economists, Oyer (2006) finds that a poor initial placement leads to a lower later placement and lower productivity, as predicted by the model.

Related Literature. This paper builds on the relational contracting models of Shapiro and Stiglitz (1984) and Macleod and Malcomson (1998). These models assume that firms only recruit from the pool of unemployed. They show how high wages can motivate workers, and argue that unemployment is a necessary component of any equilibrium.[2] Intuitively, agents only work if shirking is punished by a spell of unemployment. These models have given rise to the view that "If all firms were identical, one

[2]Macleod and Malcomson (1998) exhibits two types of equilibria. In the first type, some workers are unemployed; in the second type, some firms are unemployed.

would not expect to see different firms paying different wages even if efficiency wages were important," Krueger and Summers (1988, p. 261). This paper shows that when the employed compete for jobs, firms offer different jobs. As a result, equilibrium unemployment falls, with incentives in a good job maintained by fear of being fired and having to start over in a bad job.

On-the-job search also gives rise to wage dispersion in the seminal model of Burdett and Mortensen (1998).[3] In their paper, higher wages attract more workers, so a firm facing a degenerate wage distribution can increase its profit by increasing its wage and obtaining a larger workforce. In our model, higher wages incentivize more effort from the firm's single worker, so a firm facing the degenerate wage distribution of Shapiro-Stiglitz can increase its profit by decreasing its wage and partially compensating its workers with the non-pecuniary benefit of on-the-job search. Thus, in Burdett and Mortensen (1998), on-the-job search increases competition, raising wages and lowering profits; to the contrary, in our paper, on-the-job search increases turnover and reduces morale, lowering wages and raising profits. In practice, both market- and intra-firm forces are important: Bewley's survey (1999, Tables 7.1, 11.2) reveals that employers' wage setting is influenced by turnover (58% of firms), the ability to hire (57%) and morale (32%), with most firms refusing to cut pay during a recession because it would hurt morale and demotivate workers (69%).

Overview. Section 2 introduces the bare-bones model. The subsequent sections analyze the infinite game between the continua of firms and workers in three steps. In Section 3 we focus on the interaction between a single firm and the sequence of workers it is matched with over time, taking other firms' strategies as given. We show that we can restrict attention to stationary contracts. In Section 4, we elaborate on the matching process given stationary contracts. In Section 5 we solve for industry equilibrium as the unique fixed point of the continuum of firm-optimal stationary contracts.

2 Model

Basics. The economy consists of measure 1 of identical workers and measure $n < 1$ of identical firms. Each firm has one job. Time t is discrete and infinite, $t \in \{1, 2, \ldots\}$. The timing of each period t is summarized in Figure 3. Both firms and workers discount the future at rate $\delta \in (0, 1)$.

Job Market Matching. At the start of each period some firms have vacancies that are filled through a matching process that satisfies the three properties satisfied in the introduction: market clearing, anonymity, and individual rationality. We elaborate on the matching process in Section 4.

Jobs. In each period, first the firm pays the worker wage $w \in [0, \infty)$. Second, the worker chooses observable but non-contractible effort $\eta \in [0, \infty)$ and produces output $\phi(\eta)$ for a surplus of $\phi(\eta) - \eta$. Third, at the end of the period, either party can terminate the relationship; separation also occurs

[3] Wage dispersion is also seen in Albrecht and Vroman (1992), Peters (2008), and Galenianos and Kircher (2009). As in Burdett and Mortensen (1998), these papers are market-based with high wages helping a firm hire more or better workers.

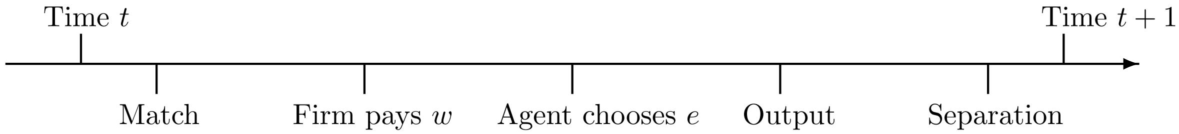

Figure 3: **Timeline.**

with exogenous probability $1-\alpha$. The firm's profit and worker's rent are given by

$$\pi := \phi(\eta) - w \qquad \text{and} \qquad u := w - \eta.$$

For convenience, we assume that the production function $\phi(\cdot)$ is continuously differentiable and strictly concave with $\phi(0) = 0, \phi'(0) = \infty, \lim_{\eta\to\infty} \phi'(\eta) = 0$.

3 Single Firm's Problem

In this section we focus on a single firm and capture the interaction with other firms through abstract outside options. We first argue that we can restrict attention to stationary contracts and then analyze the first-order condition for the firm-optimal contract.

During the matching stage, at the beginning of period t, let W be the continuation value of the best outside job offer to the firm's worker. $W \in [0, \infty)$ is distributed with cdf F^e; the distribution may have an atom at 0, representing the possibility that the worker gets no offer this period. Draws from F^e are i.i.d. across periods and we assume that F^e is stationary and anonymous, i.e. it does not depend on t or the worker's history. If the firm loses its worker, either at the end of period $t-1$ or in the matching stage of period t, it fills its vacancy with a new worker.

The rest of the period t transpires as above: first the firm pays its worker w, then the worker exerts effort η, and finally worker, firm or nature can terminate the relationship. In case of termination, the worker starts the next period with continuation value $V^{\varnothing}$, which like F^e is stationary and anonymous, while the firm gets matched to its next worker.

3.1 Stationary Contracts

We focus on *contract-specific* strategies in which the actions of firm and worker do not depend on the identity of the worker, calendar time, or history outside the current relationship. Thus, wage w_τ in period τ of a contract only depends on $(w_1, \eta_1, \cdots, w_{\tau-1}, \eta_{\tau-1})$, effort η_τ also depends on w_τ, and separation decisions additionally depend on η_τ.

We define a *self-enforcing contract* as a subgame-perfect equilibrium in pure contract-specific strategies. A self-enforcing contract remains self-enforcing if deviations off the equilibrium path are punished in the harshest possible way. For our simple game, this means that a worker who receives an off-equilibrium wage w_τ shirks and quits, and the firm fires a worker after a deviation from equilibrium effort η_τ.[4]

[4] We restrict attention to self-enforcing contracts without voluntary terminations on the equilibrium path. Such

Let Π_τ and W_τ be post-matching continuation values in period τ of contract.[5] We call a contract *stationary* if $w_1 = w_2 = \cdots = w$ and $\eta_1 = \eta_2 = \cdots = \eta$, and *firm-optimal* if it is self-enforcing and maximizes Π_1 among all self-enforcing contracts.

Theorem 1 (Stationary Contracts) *For stationary conditions $V^\varnothing, F^e$, there exists a stationary firm-optimal contract.*

Proof Sketch. If $\langle w_\tau, \eta_\tau \rangle_\tau$ is a firm-optimal contract, then it must be that $\Pi_s = \Pi_1$ for all $s \in \mathbb{N}$. If $\Pi_s > \Pi_1$, then the contract $\langle \widetilde{w}_\tau, \widetilde{\eta}_\tau \rangle$ 'that starts with period s', i.e. $\widetilde{w}_\tau = w_{\tau+s-1}, \widetilde{\eta}_\tau = \eta_{\tau+s-1}$ is self-enforcing with $\widetilde{\Pi}_1 = \Pi_s > \Pi_1$. If $\Pi_s < \Pi_1$, then the firm can increase its continuation payoff in period s by firing the current worker and restarting the relationship with a new worker.

As continuation profits are constant, the static contract with wage w_{τ^*} and effort η_{τ^*} where τ^* maximizes workers rents $w_\tau - \eta_\tau$, is self-enforcing and has profit Π_1. $\square$

We see below that workers' command rents beyond their outside option in firm-optimal contracts. The proof of Theorem 1 shows that the firm cannot expropriate such rents by backloading wages, because backloading creates an incentive for the firm to fire expensive, old workers and replace them with cheap, new workers.

From now on, we restrict attention to stationary firm-optimal contracts that maximize firm profit subject to the worker's incentive, and participation constraint.

3.2 Analyzing the First-Order Condition

We now analyze the first-order condition of the firm-optimal contract. To simplify notation we write a stationary contract $\langle u, \eta \rangle$ as function of rents $u = w - \eta$ rather than the wage w. We also write the outside options in terms of a current per-period rent u rather than a continuation value V, write F^e as a distribution of u, and let $V(u)$ be the continuation value of an employed worker with contract $\langle u, \eta \rangle$ at the beginning of the period, before matching occurs.

The firm-optimal contract $\langle u, \eta \rangle$ maximizes firm profits

$$\pi = \phi(\eta) - \eta - u$$

subject to incentivizing effort and participation

$$-\eta + \alpha\delta V(u) \geq \alpha\delta V^\varnothing \qquad \text{(IC)}$$

$$u + \alpha\delta V(u) \geq \alpha\delta V^\varnothing \qquad \text{(IR)}$$

terminations unravel cooperation and the only such self-enforcing contract has $w = 0$, $\eta = 0$, and thus $\pi = 0$ for one period before terminating.

[5] Explicitly, (for what it's worth)

$$\begin{aligned} W_\tau &= w_\tau - \eta_\tau + \delta\left(\alpha \int \max\{W_{\tau+1}; W\} dF^e(W) + (1-\alpha)V^\varnothing\right) \\ \Pi_\tau &= f(\eta_\tau) - w_\tau + \delta\left(\alpha F^e\left(W_{\tau+1}\right)\Pi_{\tau+1} + (1 - \alpha F^e\left(W_{\tau+1}\right))\Pi_1\right). \end{aligned}$$

As long as wages are positive IR is implied by IC. At the maximum, IC binds and we define $u_*(\eta)$, the rents to incentivize effort η, by $\eta = \alpha\delta(V(u_*(\eta)) - V^{\varnothing})$. The firm's problem is then to choose η to maximize $\phi(\eta) - \eta - u_*(\eta)$.

To analyze the first-order-condition we expand the value function $V(u)$ into current rents and continuation value:

$$V(u) = \int \max\{u + \delta\alpha V(u); x + \delta\alpha V(x)\} dF^e(x) + \delta(1-\alpha)V^{\varnothing}.$$

The marginal value with respect to current rents is given by[6]

$$V'(u) = (1 + \delta\alpha V'(u))F^e(u) = \frac{F^e(u)}{1 - \delta\alpha F^e(u)},$$

where $\alpha F^e(u)$ is firm's retention rate and $\delta\alpha F^e(u)$ the effective discount rate in the relationship. Holding tight the incentive constraint, this implies that the marginal cost of effort to the firm

$$\frac{d}{d\eta}(\eta + u_*(\eta)) = 1 + \frac{1}{\delta\alpha V'(u_*(\eta))} = \frac{1}{\delta\alpha F^e(u_*(\eta))}$$

exceeds one and increases in the turnover rate $1-\alpha F^e(u_*(\eta))$, so the firm always chooses η inefficiently low. Intuitively, tomorrow's wages incentivize today's effort, so the incentive premium equals one period's interest on the cost of effort and the effective interest rate increases in turnover.

The necessary first-order condition for the optimality of contract $\langle u, \eta \rangle$ is that $u = u_*(\eta)$ is defined as above and

$$\phi'(\eta) = \frac{1}{\delta\alpha F^e(u_*(\eta))}.$$

This equation shows that the solution to the firm's problem need not be unique because both marginal benefits and marginal costs decrease in effort. Indeed, in Section 5 we show that with on-the-job search firms must be indifferent between a continuum of contracts in equilibrium.

4 Job Market Matching

Absent voluntary terminations, there are αn filled jobs, $(1-\alpha)n$ vacancies, and $1 - \alpha n$ unemployed workers at the beginning of each period. By the assumptions of individual rationality and anonymity, the job market transitions can be fully described by the distribution of job offers to employed workers $F^e(u)$ and unemployed workers $F^{\varnothing}(u)$.

We derive the offer distributions $F^e, F^{\varnothing}$, from an exogenous fundamental of our model, the matching technology. The matching technology is modeled as an increasing matching function $\psi : [0,1] \to [0,1]$ with $\psi(z) \leq z$, that measures the comparative advantages of employed and unemployed workers on the labor market via $F^{\varnothing}(u) = \psi(F^e(u))$. If $1 - z$ percent of employed workers get an

[6] If the distribution F^e has an atom at u, the value function V has a kink at u but left-, and right-sided derivative are given by the left-, and right-sided limit of this equation.

offer better than u, then so do $1-\psi(z)$ percent of unemployed workers. The matching function $\psi(\cdot)$ together with the distribution of jobs $F(u)$ and market clearing for jobs better than u

$$\underbrace{(1-\alpha n)(1-\psi(F^e(u)))}_{\text{unemployed}}+\underbrace{\alpha n F(u)(1-F^e(u))}_{\text{employed below } u}=\underbrace{(1-\alpha)n(1-F(u))}_{\text{vacancies above } u} \tag{4.1}$$

uniquely determines the distributions of job offers to the employed $F^e(u)$ and the unemployed $F^{\varnothing}(u)=\psi(F^e(u))$.[7]

Equation (4.1) defines the retention rate $F^e(u)$ at job u solely based on the quantile $q=F(u)$ of the job, so we disentangle the effect of matching technology ψ and job distribution $F^e(u)$ by writing

$$F^e(u)=\beta(F(u)),$$

where retention as a function of the quantile $\beta=\beta(q)$ is determined solely by the matching function $\psi(\cdot)$. Our assumptions that unemployed workers get better offers than employed workers, $\psi(z)\leq z$, and that there is unemployment, $n<1$, imply that the retention rate is strictly positive even at the worst job, $\beta(0)>0$.

We say that a matching process satisfies *on-the-job search* if the matching function $\psi(\cdot)$ has full range. This implies that F^e is strictly increasing on the support of F and captures the idea that there is no interval of jobs $[u,u']$ that is offered exclusively to unemployed workers. Also we say that there is *more on-the-job search* under $\widetilde{\psi}$ than under ψ, if $\widetilde{\psi}(z)\geq\psi(z)$ for all $z\in[0,1]$. This translates to lower retention rates in the sense that $\widetilde{\beta}(q)\leq\beta(q)$ for all $q\in[0,1]$.

We illustrate the formalism with the help of three examples:

Shapiro-Stiglitz matching: Employed workers never receive any job offers, so $\psi_{\text{SS}}\equiv 0$ on $[0,1]$. The retention rate is then given by $\beta_{\text{SS}}(\cdot)\equiv 1$ on $[0,1]$. This process does not satisfy on-the-job search.

Fully anonymous matching: Employed and unemployed workers receive job offers from the same distribution, so $\psi_{\text{FA}}(z)=z$ for all $z\in[0,1]$. This process satisfies on-the-job search.

Intern matching: All jobs are first offered to employed workers, so $\psi_{\text{IM}}\equiv 1$ on $[0,1]$. The retention rate $\beta_{\text{IM}}(\cdot)$ is then zero for jobs $q\in[0,1-\alpha]$. This process is not covered by our assumption $\psi(z)\leq z$ but we return to it when we relax this assumption.

By imposing $\psi(q)\leq q$ we assume that job offers to unemployed workers first-order-stochastic-dominate job offers to employed workers. This restriction, which rules out intern matching, is imposed for ease of exposition in Sections 5 and 6 and will then be removed.

[7] If no such $F^e(u)\in[0,1]$ exists we define $F^e(u)$ as the supremum that satisfies (4.1) with '$\geq$'.

5 Equilibrium

In this section we integrate the the analysis of a single firm's problem from Section 3 with the market analysis in Section 4. We say that a mass n of stationary contracts $\langle u, \eta \rangle$ is an *industry equilibrium* if every contract $\langle u, \eta \rangle$ is firm-optimal when the distribution of outside offers F^e is derived from the matching technology ψ and the marginal distribution over rents F, and the value of unemployment $V^{\varnothing}$ is given by

$$V^{\varnothing} = \int (u + \delta\alpha V(u))\, dF^{\varnothing}(u) + \delta \left(1 - \frac{n - \alpha n}{1 - \alpha n}\alpha\right) V^{\varnothing}. \tag{5.1}$$

If more than one contract is offered in equilibrium, then all such contracts must yield the same profit. Hence, there exists $\pi \geq 0$ such that for all equilibrium contracts $\langle u, \eta \rangle$

$$u = u_\pi(\eta) := \phi(\eta) - \eta - \pi. \tag{5.2}$$

Firm-optimality implies that every equilibrium contract $\langle u, \eta \rangle$ maximizes firm profit $\phi(\eta) - \eta - u_*(\eta)$ where worker's rents $u = u_*(\eta)$ are given by the worker's IC constraint

$$\eta = \delta\alpha(V(u) - V^{\varnothing}). \tag{5.3}$$

The (right-sided) first-order condition for this optimization problem is given by[8]

$$\delta\alpha\beta(F(u))\phi'(\eta) = 1. \tag{5.4}$$

Equation (5.4) can be interpreted as a marginal IC constraint in the following sense: If (5.4) holds on a connected set of contracts $\left\{\langle u_\pi(\eta), \eta \rangle : \eta \in [\underline{\eta}, \overline{\eta}]\right\}$ and one of the contracts $\langle u_\pi(\eta), \eta \rangle$ is firm-optimal, then all of them are.

Equations (5.2), (5.3), and (5.4) pin down an unique equilibrium. First, the indifference condition, equation (5.2), introduces a free parameter π and reduces the problem to finding a one-dimensional distribution over workers' per-period utilities $F(u)$. Second, the marginal incentive constraint, equation (5.4), together with $u = u_\pi(\eta)$ determines $F(u)$. Third, the incentive constraint, equation (5.3), for any contract offered $\langle u_\pi(\eta), \eta \rangle$ determines the free parameter π. To understand this third step, note that an increase in profits π comes at the expense of workers' rents, so value functions $V(u)$ are decreasing in π for all employed workers. Unemployed workers' value $V^{\varnothing}$ is also decreasing in π, but less so because they experience the decrease in rents with a delay. Thus $V(u) - V^{\varnothing}$ is linearly decreasing in π, and there is a unique value of π for which the IC constraints are met with equality. To summarize:

Theorem 2 (Equilibrium Characterization) *(a) Industry equilibrium exists and is unique.*

(b) The support of effort levels $[\underline{\eta}, \overline{\eta}]$ is given by $\underline{\eta} = (\phi')^{-1}(1/\delta\alpha\beta(0))$ and $\overline{\eta} = (\phi')^{-1}(1/\delta\alpha)$; on

[8] The left-sided first-order condition $\lim_{\varepsilon\to 0} \delta\alpha\beta(F(u - \varepsilon))\phi'(\eta) = 1$ further constrains optimal contracts when F has an atom at u.

the support the distribution of effort levels is given by equation (5.4). If there is on-the-job search the distribution is continuous.

Proof Sketch. Part (a) follows form the preceding discussion.

The bounds in part (b) follow from the first-order condition at the worst job $\phi'(\underline{\eta}) = 1/\delta\alpha\beta(0)$ and the best job $\phi'(\overline{\eta}) = 1/\delta\alpha\beta(1)$. As no worker leaves the best job during the matching phase, the highest effort level $\overline{\eta} = (\phi')^{-1}(1/\delta\alpha)$ does not depend on the matching technology and is always the same as in Shapiro-Stiglitz. The lowest effort level $\underline{\eta} = (\phi')^{-1}(1/\delta\alpha\beta(0))$ on the other hand depends on the matching technology, but is strictly positive, given that $\beta(0) > 0$.

The distribution $F(u)$ must be continuous to satisfy (5.4) for all equilibrium contracts. Marginal productivity $\phi'(\eta)$ is decreasing continuously, so the retention rate $\beta(F(u_\pi(\eta)))$ must increase continuously as well. Since β is strictly increasing under on-the-job search, $F(\cdot)$ must be continuous. $\square$

The incentive constraint and the first-order condition, equations (5.3) and (5.4) have important implications for the comparative statics of on-the-job search.

Theorem 3 (Comparative Statics of OJS) *When on-the-job search increases:*

(a) Turnover increases for all jobs.

(b) Output and surplus decrease for all jobs.

(c) Rents and continuation values decrease for all employed and unemployed workers.

(d) Profits increases for all firms.

Proof Sketch. (a) Consider an increase in on-the-job search from matching technology ψ to $\widetilde{\psi}$, i.e. with $\widetilde{\psi}(z) \geq \psi(z)$ for all z. By definition, the retention rate decreases, i.e. $\widetilde{\beta}(q) \leq \beta(q)$ for all q, and turnover increases.

(b) By the marginal IC constraint (5.4) the retention rate has to be equal at every effort level, $\beta(F(u_\pi(\eta))) = \widetilde{\beta}(\widetilde{F}(u_{\widetilde{\pi}}(\eta)))$, so $\widetilde{F}(u_{\widetilde{\pi}}(\eta)) \geq F(u_\pi(\eta))$ for all η and the effort under $\widetilde{\psi}$ is lower (in distribution) than under ψ. As effort is always below first-best, a reduction in effort reduces surplus.

(c) and (d) We investigate how the shift of effort levels affects the incentive constraint (5.3) at the highest job $\langle u_\pi(\overline{\eta}), \overline{\eta}\rangle$. In the original equilibrium with matching technology ψ the incentive constraint is binding at the best job $\langle u_\pi(\overline{\eta}), \overline{\eta}\rangle$. When we change the matching function from ψ to $\widetilde{\psi}$ but keep profits π, and job distribution F fixed, the value of unemployment $V^{\varnothing}$ decreases as the unemployed receive worse job offers under $\widetilde{\psi}$. When we shift the job distribution from F down to $\widetilde{F}$, but keep profits π and matching function $\widetilde{\psi}$ fixed, the value of unemployment $V^{\varnothing}$ decreases further.

This introduces slack into the top IC constraint $\overline{\eta} = \alpha\delta(V(u_\pi(\overline{\eta})) - V^{\varnothing})$ because $V(u_\pi(\overline{\eta}))$ decreases by less than $V^{\varnothing}$. To understand this, note that value at the top is linked to $V^{\varnothing}$ through the Bellman equation

$$V(u_\pi(\overline{\eta})) = u_\pi(\overline{\eta}) + \delta\left(\alpha V(u_\pi(\overline{\eta})) + (1-\alpha)\, V^{\varnothing}\right)$$

so the partial derivative of $V(u_\pi(\overline{\eta}))$ with respect to $V^\varnothing$ is less than one. Indeed, the incentive constraint is now equally slack at all jobs $\langle u_\pi(\eta), \eta\rangle$. Thus the firms can cut wages and increase profits from π to some level $\widetilde{\pi}$ until IC binds. □

Theorem 3 and its proof follow the discussion in the introduction. An increase in on-the-job search increases turnover, and thereby the marginal cost of effort. When some firms accordingly substitute good stable jobs with bad temp jobs, the prospects of unemployed workers get worse. This introduces slack into workers' incentive constraints and allows firms in a second step to cut wages. Hence, workers lose twice: First when firms substitute some stable jobs with temp jobs, and then again when all firms cut wages. Firms win because one firm's lower wage reduces the value of unemployment and exerts a positive externality on other firms who can cut wages and increase profits accordingly. On-the-job search allows firms to reap this positive externality in equilibrium.

6 Entry

So far we have treated the number of firms in the economy $n < 1$ as exogenous, enabling a clean analysis of on-the-job search. We now endogenize entry to integrate another fundamental inter-firm aspect of the labor market with intra-firm level analysis. Theorem 5 shows that entry leads to a more nuanced, and maybe more plausible evaluation of on-the-job search: Rather than incumbent firms cutting wages, we now find that additional firms enter the industry as a reaction to the profitable opportunities created by on-the-job search and the deterioration in job quality.

While the matching technology $\psi(\cdot)$ is fundamental and does not depend on n, the retention rate $\beta = \beta(q, n)$ does depend on n via equation (4.1)

$$(1-\alpha n)(1-\psi(\beta(q,n))) + \alpha n q(1-\beta(q,n)) = (1-\alpha)n(1-q).$$

We adapt the definition of industry equilibrium as follows: A distribution of stationary contracts $\langle u, \eta\rangle$ is an *industry equilibrium* if every contract $\langle u, \eta\rangle$ is firm-optimal and generates profits equal to $\pi = 0$, when the distribution of outside offers $F^e(u)$ is derived from the matching technology $\psi(z)$ and the marginal distribution over rents $F(u)$, and the value of unemployment $V^\varnothing$ is given by equation (5.1).

As in Section 5, the distribution of equilibrium contracts $\langle u, \eta\rangle$ is determined by zero profits

$$u = u_0(\eta) := \phi(\eta) - \eta \tag{6.1}$$

the marginal IC constraint at all contracts

$$F(u_0(\eta)) = (\beta(\cdot, n))^{-1}(1/\delta\alpha\phi'(\eta)) \tag{6.2}$$

and the IC constraint (5.3) at the top

$$\overline{\eta} = \alpha\delta\left(V(\overline{u}) - V^\varnothing\right) \tag{6.3}$$

where $\overline{\eta} = (\phi')^{-1}(1/\delta\alpha)$ and $\overline{u} = u_0(\overline{\eta})$.

With free entry, the Bellman equation at the best job $\langle \overline{u}, \overline{\eta} \rangle$

$$V(\overline{u}) = \overline{u} + \delta \left(\alpha V(\overline{u}) + (1-\alpha) V^{\varnothing} \right)$$

connects the value of the top job $V(\overline{u})$ and the value of unemployment $V^{\varnothing}$ in a linear way. Simple rearranging implies that IC at the top (6.3) is satisfied if the value of unemployment equals

$$V^{\varnothing} = \frac{\overline{u} + \overline{\eta}\,(1 - 1/\alpha\delta)}{1-\delta}, \tag{6.3b}$$

and IC at the top (6.3) is violated (slack) if $V^{\varnothing}$ is greater (smaller) than this value.

Theorem 4 (Equilibrium Characterization) *Industry equilibrium with entry exists and is unique. With anonymous matching, there is full employment: $n = 1$. With less on-the-job search, some unemployment remains: $n < 1$.*

Proof Sketch. To construct a set of contracts satisfying equations (6.1), (6.2), and (6.3b), we proceed in a similar manner to Section 5. Let the number of firms $n = n_{\mathrm{SS}}$ be as in the Shapiro-Stiglitz equilibrium, and let $F(\cdot, n_{\mathrm{SS}})$ be the distribution of contracts defined by (6.2) given retention rates $\beta(\cdot, n_{\mathrm{SS}})$. For matching process and job distribution as in Shapiro-Stiglitz, $V^{\varnothing}$ is at its equilibrium value (6.3b). Changing the matching process to $\beta(\cdot, n_{SS})$ and the job distribution to $F(\cdot, n_{\mathrm{SS}})$ both decrease $V^{\varnothing}$, so firms enter.

The reason that $V^{\varnothing}$ increases in n rests on two arguments. First, entry adds job offers at the bottom of the job distribution because the lowest job is decreasing in n; this helps unemployed workers because accepting these offers is voluntary. Second, transition rates between unemployment and existing jobs are fixed by the marginal IC constraint (6.2), independent of n. $\square$

When evaluating the consequences of on-the-job search in Theorem 5 we compared the distribution of turnover, effort, wages and profits based on first-order-stochastic-dominance. With a variable number of firms n this comparison is not feasible and we focus instead on the absolute number of jobs above a quality threshold.

Theorem 5 (Comparative Statics of OJS) *An increase in on-the-job search:*

(a) Strictly increases the number of jobs n, but

(b) Strictly decreases the number of jobs $n(1 - F(u_0(\eta), n))$ above any job $\langle u, \eta \rangle$ in the original support.

Proof Sketch. Part (a) follows from the proof of Theorem 4. For (b), remember that the retention rate at job $\langle u_0(\eta), \eta \rangle$ is determined by (6.2). Fixing n, an increase in on-the-job search $\widetilde{\beta} \leq \beta$ increases $F(u_0(\eta))$ and decreases the number of jobs above η. The subsequent increase in n discussed in the proof of Theorem 4 decreases competition for jobs because previously unemployed workers are now employed and thus do not compete as well on the matching market. This effectively decreases demand for jobs above η. To keep the retention rate at job $\langle u_0(\eta), \eta\rangle$ equal to $1/\delta\alpha\phi'(\eta)$, the supply of jobs above η, i.e. $n(1 - F(u_0(\eta), n))$ must decrease. $\square$

Both with a fixed number of firms and with entry, on-the-job search causes firms to replace stable, high-quality jobs with temporary, low-quality jobs. The key difference is in the knock-on effect of this deterioration. With a fixed number of firms, firms cut wages, reinforcing the detrimental effects on worker welfare and leading to the stark results in Theorem 3. With free entry, on-the-job search incentivizes entry and the loss of high-quality jobs is balanced by a reduction in unemployment.

We conjecture that the equilibrium effect of on-the-job search on the average worker $n\int(\phi(\eta) - \eta)dF(u_0(\eta))$ is negative. The rationale behind this conjecture is that the reduction in unemployment and the loss of good jobs exactly cancel from the perspective of an unemployed worker, whose value in equilibrium is given by (6.3b), independently of the matching process. When compared to the average worker, an unemployed worker is more concerned with the lower end of the job distribution than its top end. Thus he benefits more from the additional jobs at the bottom of the distribution and suffers less from the quality deterioration at the top of the distribution. This suggests that, also with entry, the utility of the average worker decreases in on-the-job search.

7 Conclusion

Going beyond this paper we wish to investigate the optimal hiring policies and investigate the phenomenon of 'over-qualification' (Bewley (1999)). In order to study these issues we envisage a model with two-sided heterogeneity in which firms and workers engage in temporary relationships and sustain incentives through relational contracts. In contrast to the search-theoretic models pioneered by Shimer and Smith (2000), a low-ranked firm may not want to hire and train a high-ranked worker only for him to be poached by a higher ranked firm soon after. By making firms reluctant to bid for such 'over-qualified' workers, we conjecture that frequent matching will undermine job quality, stifle competition and depress wages for all workers when compared to the predictions of traditional matching models (e.g. Shapley and Scarf (1974)).

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