

A Quantitative Analysis of the US Housing and Mortgage Markets and the Mortgage Crisis¹

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Abstract

We construct a quantitative equilibrium model that accounts for the salient features of the US housing market, namely, the homeownership rate, the average foreclosure rate prior to the crisis and the distribution of home-equity ratios across homeowners. Given this steady state, we examine the consequences of a 3 percent unexpected increase in the supply of housing on house prices and foreclosures. We analyze the impact of the tax code, namely the fact that the implicit rental income from housing is not taxed and that the mortgage interest payment is tax deductible, for the steady state of the model as well as for the impact of the unanticipated supply shock. We show that the model is able to account for the observed decline in house prices with a modest increase in the cost of new mortgages following the shock (crisis) and predicts a large increase in foreclosures.

Key Words: Leverage, Foreclosures, Mortgage Crisis

1 Introduction

The goal of this project is two-fold. First, to construct a quantitative equilibrium model of the US housing market that can account for key features of the US housing market, namely, the homeownership rate, the average foreclosure rate prior to the crisis and the home-equity distribution observed in the US data. Second, to see if such a model can account for the steep rise in foreclosures following a modest over-building shock. The analysis is motivated by the recent experience of the housing boom and bust and rising defaults on mortgages in the United States (and elsewhere).

The key elements of our model environment are as follows. We imagine a city with an exogenously given but potentially time-varying stock of housing. The city is populated by a continuum of infinitely lived residents subject to uninsured idiosyncratic shocks to earnings. Residents can buy consumption goods, save in the form of a risk-free savings account with an exogenously given and constant interest rate, and purchase or rent their housing space. If a resident chooses to purchase housing space, he or she can borrow funds from a mortgage market to do so. Residents must pay tax on earnings and interest income. As in the US federal tax code, interest payments on mortgages and property taxes can be deducted from taxable income and the implicit rental income from owner-occupancy is excluded from taxable income.

We model the mortgage market as competitive, with every borrower being charged an interest rate that exactly reflects the borrower's objective probability of default. A key determinant of this default risk is the level of down payment made on the loan - a level that we assume is freely chosen by the borrower. The endogeneity of the down payment will imply that when the risk of default is perceived to be low, the down payment chosen will be low as well and homeowners will be highly leveraged.

We show that this basic model is capable of accounting for the observed facts regarding the US housing market noted above. The tax treatment of housing plays a huge role in bringing the model close to reality. Without the deduction of mortgage interest income, the model would predict way more home equity than we see in the data. And without the exclusion of implicit rental income from taxable income, the model would predict a homeownership rate of zero. The model accounts for the dispersed distribution of home-equity observed in the US data because mortgages are nominal contracts and there is steady increase in the general price level over time, which erodes the real value of debt.

The second goal of this paper is to use the model to study the housing and mortgage crisis in the US. We locate the proximate cause of the mortgage crisis in "overbuilding": a modest increase in the supply of owner-occupied housing that fails to be matched by an increase in demand at the going price. In equilibrium, the price of owner-occupied housing must fall to absorb the increase in supply. Given the transactions costs

of selling and buying homes, the decline in price in the short-run has to exceed the long-run decline in price. We show that a 3 percent excess supply of owner-occupied space along with a modest increase in the cost of new mortgages following the shock (crisis) can account for the observed drop in house prices. This drop in house prices in turn generates a large increase in foreclosures. Our model highlights how leverage and the tax benefits of homeownership conspire to turn a simple adjustment of demand and supply into a crisis. Leverage implies that when the price of housing falls, selling one's home imposes a capital loss. The capital loss of a selling induces homeowners to give up their homes in foreclosure and rent for a while. This further reduces the demand for owner-occupied housing and leads to a further decline in the price of housing.

We also used the model to understand what role an unexpected drop in the inflation rate during the crisis might have played with regard to house prices and foreclosures. We show that a lower inflation rate leads to significantly higher default but only marginally lower house prices. We also investigated what would have happened if foreclosures were prevented altogether. There is still a substantial drop in price but it is lower than in the baseline model. Thus in our model foreclosures are a depressive force on house prices because they lower the demand for owner-occupied housing. Finally, we examine how the over-building shock would have affected the economy if the tax code did not encourage homeownership and leverage. We show that the economy would not have experienced any foreclosures and only a modest decline in house prices.

2 Literature Review

(Not revised) There is a rapidly growing literature on the financial crisis and the mortgage crisis. Much of this literature is empirical. To the best of our knowledge, ours is the first quantitative-theoretic investigation of the interaction between the jump in foreclosures (the mortgage crisis) on the one hand and the crash in house prices (the housing crisis) on the other. We are also the first to analyze the implications of the foreclosure prevention policy on house prices in a fully articulated equilibrium model of the housing and mortgage markets.

We build on a small but growing quantitative-theoretic literature on the housing sector. We have in mind studies such as Gervais (2002), Nakajima (2005), Jeske and Krueger (2005), Guler (2008), Ríos-Rull and Sánchez-Marcos (2008), Corbae and Quintin (2009), Iacoviello and Pavan (2009), and Chambers, Garriga, and Schlagenhauf (2009).

In terms of modeling the mortgage market, we follow Jeske and Krueger (2005) and Guler (2008) in assuming that each loan is competitively priced to reflect the objective probability of default on the loan (individualized or risk-based pricing) and in assuming that a borrower controls the objective probability of default by her

choice of down payment.

In terms of modeling the housing market, we follow Gervais (2002) and Chambers, Garriga, and Schlagenhaut (2009) in conceiving of the housing market as a market for homogeneous housing *space*, as opposed to houses. This conceptualization is attractive in that there is *one* housing market price – the price per square foot of housing space – to be determined each period.

The role of aggregate shocks in a model of mortgage lending and housing has been analyzed in Ríos-Rull and Sánchez-Marcos (2008). Their analysis is explicitly stochastic, and they employ a forecasting equation (for future house prices) of the type developed in Collusi (2006) to get around the high dimensionality of the state vector. In contrast, our analysis is not explicitly stochastic – we analyze the impact of an unanticipated shock to housing supply – but we work out the perfect foresight equilibrium path following the shock.

3 Environment

We will study the housing equilibrium in a representative city. Time is discrete and indexed by $t = 0, 1, 2, \dots$. The city has a fluctuating stock of housing space $H(t) > 0$.

3.1 People

There is a fixed continuum of individuals. Individuals derive utility from the consumption of a homogeneous consumption good and the service flow from housing space. Let c_t denote consumption of the homogeneous good in period t , let h_t denote the consumption of housing space in period t . Then an individual values the consumption stream $c = \{c_0, c_1, c_2, \dots\}$ and $h = \{h_0, h_1, h_2, \dots\}$ according to:

$$U(c, h, e) = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t), 0 < \beta < 1, \nu \geq 0. \quad (1)$$

We assume that

$$u(c_t, h_t, 0) = [c_t^{1-\theta} h_t^\theta]^{1-\gamma} / (1-\gamma). \quad (2)$$

We assume that people must either own their housing space or rent it.

Each resident independently draws an earnings level w from a finite-state Markov process with non-negative positive support $W \subset R_+$. The probability that $w_{t+1} = w'$ given $w_t = w$ is $F(w', w)$.

3.2 Market Arrangement

The homogeneous consumption/endowment good is the *numeraire* good. Period t prices are expressed in period t consumption goods. There are four markets in this economy.

1. There is a market for owner-occupied housing in which the price per unit of housing space in period t is $p(t)$. An individual who buys k' units of housing pays a purchase price of $p(t) \cdot k'$. We assume that owner-occupied housing comes in discrete sizes given by the finite set K . We assume that the depreciation rate on housing is random: houses depreciate by δ_j with probability ξ_j , $j = H, L$, $\delta_H > \delta_L$. That is, houses either depreciate at a high rate or a low rate.
2. Second, there is a market for rental housing in which the rent per unit of housing space in period t is $z(t)$. An individual who rents h units of housing space in period t pays $z(t) \cdot h$ as rent.
3. Third, there is a market for risk-free deposits. The nominal interest rate on deposits is $i(t)$, where $(1 + i(t)) = (1 + r)(1 + \pi(t))$. The real interest rate r and the inflation rate $\pi(t)$ is taken as exogenous.
4. Finally, there is a market for mortgages where individuals borrow in nominal terms to fully or partially fund the purchase of a house. For tractability we assume that all mortgages are perpetuities: Each mortgagee pays some fixed nominal agreed-to amount X each period unless the mortgagee defaults or the mortgage is paid off. In the case of default, the financial intermediary gets ownership of k and the defaulter pays a utility cost κ that is drawn from a continuous distribution $\Phi(\kappa)$. In the case where the mortgage is paid off, the lender receives the present value of the promised nominal payment X discounted at the nominal risk-free rate. Because of the possibility of default, the (unit) price q of a mortgage depends on the amount of housing pledged as collateral k' , the payment amount X promised in perpetuity whose real value in the current period is x (since the price of consumption goods today is normalized to 1), the individual's post-purchase savings a' , his current earnings w , and the time period t . An individual who takes out a mortgage in period t $q(w, a', x, k', t) \cdot x$ in the current period.

3.3 Government Sector

There is also a government sector that levies taxes on income. For simplicity, we assume that government consumption of goods does not have to provide any benefits to households. The amount of taxes G to be paid by an individual in nominal terms is modeled after the US tax code:

$$\int_0^{\max\{0, W + \omega i(t)A - \max[X + \rho P(t)k, S]\}} T(Y) \cdot Y dY \quad (3)$$

where S is the standard deduction, $\rho P(t)k$ is property tax, $1 - \omega$ is the fraction of assets on which interest earnings are tax deferred, $T(\cdot)$ is the marginal tax rate and weakly increasing in taxable income (we assume that tax brackets move up with inflation). In real terms, an individual's tax liability is given by:

$$g(w, a, x, k) = \int_0^{\max\{0, w + \omega ia / (1 + \pi) - \max[x + \rho p(t)k, s]\}} \tau(y) \cdot y dy \quad (4)$$

where $\tau(\cdot)$ is the marginal tax rate when income is measured in current period consumption good.¹

3.4 Financial Intermediaries

Financial intermediaries take in deposits, sell mortgages, and own the housing space rented by people. All intermediaries can borrow or lend funds in a world credit market at a given risk-free interest rate $\bar{r} > 0$. We will assume that there is one representative risk-neutral intermediary that takes all prices as given.

4 Decision Problems

4.1 People

The state variables for individuals are $w, a, x, k, \delta_h, \kappa, t$ and whether the individual is excluded from the mortgage market or not. Consider first the decision problem of an individual who does not own housing and who is not excluded from the mortgage market due to prior default. For this person $k = 0$ and $x = 0$ and the person may choose to rent or she may choose to buy. Since the person is not excluded from the mortgage market she can borrow to purchase a house. In this case, if the individual chooses to purchase, she solves:

$$M_1(w, a, x = 0, k = 0; t) = \max_{c \geq 0, k' \in K, x' \geq 0, a' \geq 0} \{u(c, k') + \beta E_{w', \delta', \kappa' | w} V(w', a', x', k', \delta', \kappa'; t + 1)\}$$

$$c = w - g(w, a, x = 0, k') + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - p(t)[1 + \chi_B]k' + q(w, a', x', k'; t) \cdot x'$$

where χ_B is the percentage transactions cost of purchasing a house, $\tilde{r}$ is the rate of return on assets on which taxes are deferred.

If the individual is excluded from the mortgage market due to a prior default but chooses to purchase a house, she solves:

$$M_1^D(w, a, x = 0, k = 0; t) = \max_{c \geq 0, k' \in K, a' \geq 0} \{u(c, k') + \beta E_{w', \delta', \kappa' | w} V(w', a', x' = 0, k', \delta', \kappa'; t + 1)\}$$

$$c = w - g(w, a, x = 0, k') + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - p(t)[1 + \chi_B]k'.$$

¹In the computation we approximate the return on assets that are subject to income tax as $\omega a(r + \pi)$.

We assume that if an excluded individual purchases a house, he is no longer excluded from a mortgage market (the default flag is gone).²

If the individual is not excluded from the mortgage market and chooses to rent, she solves

$$M_0(w, a, x = 0, k = 0; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} V(w', a', x' = 0, k' = 0; t + 1)\}$$

$$c = w - g(w, a, x = 0, k' = 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h$$

If the individual is excluded from the mortgage market and chooses to rent, she solves

$$M_0^D(w, a, x = 0, k = 0; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w'|w} \lambda V^D(\cdot; t + 1) + (1 - \lambda)V(\cdot; t + 1)\}$$

$$c = w - g(w, a, x = 0, k' = 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - z(t)h$$

where λ is the probability the individual remains excluded from the mortgage market.

Then

$$V(w, a, x = 0, k = 0; t) = \max \{M_1(w, a, x = 0, k = 0; t), M_0(w, a, x = 0, k = 0; t)\}$$

And

$$V^D(w, a, x = 0, k = 0; t) = \max \{M_1^D(w, a, x = 0, k = 0; t), M_0^D(w, a, x = 0, k = 0; t)\}$$

Consider next the decision problem of an individual who owns a house and has an outstanding mortgage. The household may choose to keep the current house, sell it, or default on the mortgage. If he chooses to keep the house, he solves:

$$K_0(w, a, x, k, \delta; t) = \max_{c \geq 0, a' \geq 0} \{u(c, k) + \beta E_{w', \delta', \kappa' | w} V(w', a', x/(1 + \pi(t)), k, \delta', \kappa'; t + 1)\}$$

$$c = w - g(w, a, x, k) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - x - \delta h$$

If he chooses to sell, he solves:

$$K_1(w, a, x, k, \delta; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) + \beta E_{w', |w} V(w', a', x' = 0, k' = 0; t + 1)\}$$

$$c = w - g(w, a, x, 0) + a(1 + \omega r + (1 - \omega)\tilde{r}) - a' - x + p(t)[1 - \chi_S]k - \bar{q}x - z(t)h - \delta h$$

Here χ_S is the percentage cost of selling a house. Selling the house requires the individual to buy back the perpetuity x at the nominal risk-free interest rate.³

²This assumption is really without any loss of generality because given the substantial transactions costs of purchasing and selling a home, individuals purchase homes for a long duration of time. By the time they need to make another purchase, their exclusion flag would typically be gone. Thus the individuals will behave as if they do not have a default flag.

³Ideally we should require the individual to buy back the perpetuity at $\bar{q} \cdot x$ where $\bar{q}$ is the price the individual paid when he issued the perpetuity. But implementing this would require expanding the state vector.

If the household chooses to default on the mortgage, he solves:

$$K_D(w, a, x, k, \kappa; t) = \max_{c \geq 0, h \geq 0, a' \geq 0} \{u(c, h) - \kappa + \beta E_{w'|w}[(1 - \lambda)V(\cdot; t + 1) + \lambda V^D(\cdot; t + 1)]\}$$

$$c = w - g(w, a, 0, 0) + a(1 + \omega r + (1 - \omega)\bar{r}) - a' - z(t)h$$

Foreclosure results in the individual losing the house as well as the mortgage and being excluded from the mortgage market for some random length of time.

Then for $k > 0$ and $x \geq 0$

$$V(w, a, x, k, \delta, \kappa; t) = \max \{K_0(w, a, x, k, \delta; t), K_1(w, a, x, k, \delta; t), K_D(w, a, x, k, \kappa; t)\}$$

4.2 Financial Intermediaries

The representative financial intermediary accepts deposits, buys mortgages, and buys and rents housing. We will assume that the intermediary chooses how much to engage in these activities on the basis of the expected return in each of these activities. Denote the intermediary's expected net rate return (profits) on a deposit of size a by $\nu(a)$, the net expected return on a mortgage with characteristics w, x', a', k' and t by $\nu(w, x', a', k', t)$ and the net expected return on the purchase of h units of housing in period t by $\nu(h, t)$. Correspondingly, let $m(a)$, $m(w, x', a', k', t)$ and $m(h, t)$ denote the measure (more precisely, the density or pdf) of such contracts acquired by the financial intermediary. Then, the financial intermediary's decision problem is:

$$\nu(t) = \max_{\{m(a), m(w, x', a', k', t), m(h, t)\}} \left\{ \int \nu(a)m(da) + \int \nu(w, x', a', k', t)m(dw, dx', da', dk', t) + \int \nu(h, t)m(dh, t) \right\}$$

For there to be a (bounded) solution to this problem the net expected returns on each type of asset must be non-positive. For deposits this requires $\nu(a) = a - [a(1 + r)]/(1 + \bar{r}) \leq 0$ for all a . This requirement reduces to

$$(1 + r) \geq (1 + \bar{r}). \quad (5)$$

For housing, this requires that $\nu(h, t) = z(t) \cdot h - \rho p(t)h + [p(t + 1) \cdot (1 - \delta)h]/(1 + \bar{r}) - p(t)h \leq 0$ for all h (financial intermediaries do not pay any cost for selling houses). This requirement reduces to

$$p(t) \geq z(t) + [p(t + 1)(1 - \delta)/(1 + \bar{r})] \quad (6)$$

For mortgages, the expression for net return is more involved. When the intermediary acquires a mortgage it gives up $q(w, a', x', k', t) \cdot x'$ in goods. Next period, if the individual defaults, the intermediary receives $p(t+1)[1 - \chi_{LD}]k'$ where χ_D is the cost of foreclosure to the intermediary; if the individual sells the property, the intermediary receives $x \cdot \bar{q}$, where $\bar{q}$ is the present discounted value of one nominal unit promised in perpetuity⁴; and if she neither defaults nor sells, the intermediary receives x plus the value of the continuing mortgage, which is then given by $q(w', a(w', a', x, k', \delta', \kappa', t+1), x, k', t+1) \cdot x$. The requirement that the expected net return from a mortgage $\nu(w', a', x', k', t)$ be non-positive becomes

$$\begin{aligned} q(w, a', x', k', t) x' &\geq (1+r)^{-1} \times \\ &E_{w', \delta', \kappa' | w} \{ d(w', a', x', k', \delta, \kappa; t+1) p(t+1) [1 - \chi_D] k' + \\ &s((w', a', x', k', \delta, \kappa; t+1)) \bar{q}(t+1) x' + \\ &(1 - d(\cdot; t+1))(1 - s(\cdot; t+1)) q(w', a(w', a', x', k', \delta', t+1), x', k', t+1) x' \} \end{aligned} \quad (7)$$

5 Equilibrium

An equilibrium consists of a sequence of rental pricing function $\{R^*(t)\}$, a sequence of housing price functions $\{P^*(t)\}$, a deposit interest rate r^* , a sequence of mortgage price functions $\{q^*(k, x, a, w, t)\}$, a sequence of distributions $\mu^*(k, x, a, w, t)$ of people over the state space, and a sequence of decision rules for excluded and non-excluded individuals such that:

1. The decision rules are optimal given r^* , $z^*(t)$, $p^*(t)$, $q^*(t)$.
2. The net returns (5)-(7) are zero.
3. Demand for housing equals supply, that is, $\int h^*(w, a, x, k, \delta, \kappa; t) \mu^*(dw, da, dx, dk, d\delta, d\kappa; t) = H(t)$
4. The sequence of distributions $\mu^*(w, a, x, k, \delta, \kappa; t)$ is implied by the decision rules.

6 Parameter Selection and Calibration

Turning first to the Markov process for earnings, we assume that log earnings follow an AR1 process:

$$\ln(w_t) = \psi \ln(w_{t-1}) + \epsilon_t \quad (8)$$

⁴ $\bar{q}(t) = 1/(1+i(t+1)) + 1/(1+i(t+1))(1+i(t+2)) + \dots$

Several studies have estimated this process for the US using PSID earnings data. Estimates of ρ and the standard deviation of ϵ (σ_ϵ) vary across studies. We set $\psi = 0.97$ and $\sigma_\epsilon = 0.129$. This AR1 process is then approximated by a 17-state Markov chain.

Setting aside the parameters of the income tax schedule, our model economy has 14 other parameters. These include 3 preference parameters (β, θ, γ) , 6 parameters related to housing transactions $(\chi_S, \chi_B, \{\delta_j, xi_j\}, j = H, L)$, 2 related to costs of foreclosures (λ, χ_D) , and 3 related to financial markets $(\omega, \tilde{r}, r)$.

The parameter values we pick are motivated by the relevant micro studies. Here we summarize the selections in the following table.

Table 1

Parameter	Value	Description
λ	0.5	probability of re-entry after default
$\tilde{r}$	0.04	risk-free real interest rate
ψ	0.97	autocorrelation of earnings
σ	0.129	sd of innovation to earnings shock
δ	0.015	depreciation of rentals each year
δ_H	0.15	high depreciation rate for homeowners
θ	0.20	risk-aversion coefficient over utility
γ	2.0	risk-aversion coefficient
π	0.025	steady-state inflation
χ_B	0.01	cost of buying
χ_S	0.06	cost of selling
χ_F	0.15	foreclosure cost
β	0.9556	beta
ρ	0.0138	property tax rate
$\tilde{r}$	0.0281	real after-tax annual return on long-term investment
ω	0.60	portion of asset return in long-term tax-deferred investment

We need to specify the tax schedule $\tau(\cdot)$ and the standard deduction s . The tax schedule is chosen to match the tax table for 1998. In our model, people are viewed as individuals (this seems consistent with the earnings data). But we will take individuals to be married. Hence, the tax table we use is the tax table for married filing separately. According to the Census Bureau, male median income of year-round full-time workers age 25 and older in 1998 was \$37,906 and that of females was \$27,956. We use the average of these two numbers, which is \$32,931, as the median income of an individual filing for taxes. Normalizing the tax brackets for 1998 by this estimate of median income, we obtain the following tax schedule $\tau(\cdot)$:

Table 2

Tax Brackets	Tax Rate
0 - 0.64	0.15
0.64 - 1.55	0.28
1.55 - 2.37	0.31
2.37-4.23	0.36
4.23 -	0.396.

And normalizing the standard deduction for a married person filing separately by median income gives $s = 0.1116$.⁵

Our calibration recognizes that only a portion of the nominal returns on financial assets is taxed at the relevant income tax rate. The remaining portion gets taxed at a lower rate because some of the return on assets is in the form of capital gains (which is typically taxed at a lower rate) and both the capital gains as well as dividends and interests on assets which are in retirement accounts are not taxed each year. In our calibration we assume that 40 percent of the earnings on financial are taxed at the relevant income tax rate and the remaining 60 percent is taxed at a flat rate of 0.23. The calculations that lead to these numbers is described in the Appendix.

The remaining 3 parameters (β, δ_L, ξ_H) are determined by computing the steady state of the model economy for a given housing stock and setting these parameters so as to match the ratio of mean financial asset to mean income, the homeownership rate, and the steady-state default rate. In computing the ratio of financial assets to income, we ignored the top 3 percent of the wealthiest households since it is well-known that this class of models cannot match the upper tail of the wealth distribution. We chose to target averages over the period 1993-2003 so as not to distort our parameter choices by booms and busts in the residential real estate markets.

We solve for the steady state by normalizing price of housing services, namely, rents. First we normalize the price of rentals. Given the Cobb-Douglas form in the utility function, a renter will always spend a fixed proportion of his current expenditures on rents, and a fixed portion on the other good. Given the accrued flow of rental income to the risk-neutral landlord, subtracting the cost of depreciation of houses and taxation of houses, this gives a discounted value of a unit of rental to the landlord. Although we have not modeled a construction sector explicitly, it is assumed that in steady state the return of a unit should be the same

⁵Our tax schedule overstates the taxes paid by low-income people because we ignore the earned income tax credit. However, what is important for our study is the tax benefit of owner-occupied housing and this benefit is *not* affected by the earned income tax credit. This is because the credit is calculated on a person's adjusted gross income and, therefore, does not depend on whether the household rents or owns.

whether it is sold to a landlord who has an intention to rent it out or it is bought by a home-owner. This implies that if a construction sector were to exist in the model, it would be indifferent toward providing an extra unit of housing to the home-owner market or to the rental market (but landlords do not pay cost of real-estate agents, while home-owners pay). Thus, in steady state, the housing stock is effectively endogenous. When we compare alternative steady states, we follow the same procedure. Thus all steady state comparisons should be viewed as comparisons of long-run equilibrium in the housing market. If the cost of constructing houses does not change from one equilibrium to the next, this would be exactly the long run equilibrium. Finally, note that the price-rent ratio is independent of the normalized rental price. If the normalized rental price goes up by $x\%$, in the new steady state all households (whether they are renters or home-owners) would consume $x\%$ less housing units and nothing else would change.

Table 3

Parameter	Value	Description	Data	Model
β	0.9556	discount factor	avg asset to avg inc ratio = 1.5	1.496
δ_L	0.0075	low depreciation rate for homeowners	homeownership rate=0.664	0.664
ξ_H	0.055	probability of high depreciation rate	steady-state default rate = 0.0135	0.0135
			avg housing wealth to avg income = 1.33	1.265
			avg home equity = 0.62	0.565
			price-rental ratio = ?	14.6

Table 3 displays the steady-state model statistics and the parameter values that generate them. The steady-state statistics come reasonably close to their target values.

First, one may ask, why do people in our model have an incentive to purchase homes? We have assumed that there is no direct benefit to homeownership and in our calibration there is no “rental externality”: The average depreciation of rental properties is 0.0150 and that of owner-occupied units is 0.0153. Furthermore, in computing the solution to our model, we assume that households that rent can choose their h freely, while those that purchase their homes must choose from a finite set of house sizes. Thus, homeownership affords no direct utility benefit, involves somewhat faster depreciation and less flexibility (in terms of house sizes) than renting, and involves the payment of significant transactions costs in its acquisition. Nevertheless, the majority of people choose to purchase their homes. This is because of a key tax benefit of homeownership: the implicit rental income from ownership is not counted as part of income and therefore not taxed. This exemption means that people – especially those in the higher tax brackets – have a strong incentive to purchase their homes.⁶ The deductibility of mortgage interest payments encourages these households to

⁶Let’s say the household is deciding between saving in risk-free asset or saving in a home. When the household saves in risk-free asset it pays taxes over the nominal interest return. If the household saves by buying a house, the return to that saving

borrow to finance the purchase of their homes as opposed to paying for the purchase from accumulated assets. Both of these effects kick in more strongly for richer households because their tax rate is higher and the mortgage interest payment deductions are more likely to exceed standard deductions as they buy bigger homes. Given this, home-owners are concentrated among richer households. In our model, the average income of homeowners is 1.97 times the average income of renters. Our model is also consistent with the observation that owner-occupants consume more housing space, on average, than renters. In our equilibrium the per capita housing space of renters is 54? percent of per capita housing space of owner occupants. There are two reasons for this: First, high earners choose to buy houses which makes the housing space of owner- occupants larger than that of renters. In addition, the tax benefit of owner-occupancy makes owner-occupants consume more housing than renters.

To see how tax incentives encourage households to buy bigger homes, Table 4 shows how the steady state is altered if these tax benefits were to be eliminated. If the mortgage deduction is eliminated, the homeownership declines from 0.664 to 0.583. Average equity rises to more than 95 percent since there is no benefit to taking on leverage. In addition, average housing consumption declines by 7.3 percent. If the implicit rental income from owner-occupancy is taxed in exactly the same fashion as returns from financial assets, the incentive to own homes goes away entirely. Furthermore, average housing consumption declines by 14.5 percent relative to the baseline model.

Table 4

Statistic	Baseline	No Mort Ded	No Mort Ded and Taxes on Implicit Rental Inc
Homeownership rate	0.664	0.583	0
Avg Home Equity	0.565	0.960	0
Avg Housing Cons	0.893	0.903	0.876

Steady state inflation also influences house market outcomes as well. Table xx also compares the baseline steady state with the steady state with inflation rate of 4 percent. There are two effects. First, a higher inflation rate increases the nominal interest rate and, therefore, the tax benefits of the mortgage deduction. The benefits of the mortgage deduction increase with inflation because nominal interest payments are tax-deducted, not real. This encourages households to take on more debt and buy bigger houses. On the other hand, higher inflation erodes the value of debt faster and thus causes households to accumulate home-equity at a faster rate. In the experiment, the second effect dominates and average home-equity goes up along with house-size and the home-ownership rate.

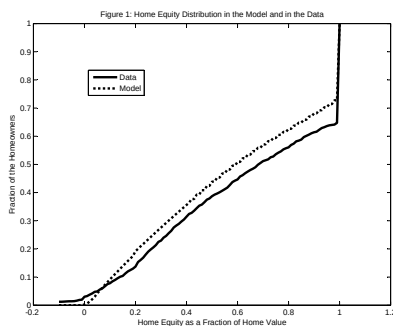
Given the tax incentives of home-ownership why do households do not hold a much bigger portion of their comes as (implicit) rental income and appreciation in the value of the house, both of which are not taxed. So there is additional tax benefit of home-ownership.

wealth in housing? In the model, we constrain households to own only one house, but given that the mentioned incentives apply only to the first homes of households, this is without loss of generality. In the steady state economy, there is no appreciation of house-prices, so the benefit to having home equity occurs because the implicit return in rental income from owning the house is not taxed, while if that equity was transferred to other assets, the returns would have been taxed. But still the size of the house people wish to own is bounded by the utility they get from the consumption of the house. Although the implicit rental income from home-ownership is not taxed, it comes with a big constraint which is that return has to be spent on housing consumption. So although home-owners consume bigger houses than renters (as it is implicitly subsidized), how much they invest in housing is bounded by how much of their income they would like to allocate to housing consumption. The weight of housing in the Cobb-Douglas utility function is 20%, which results in households to invest around a third of their wealth in home-equity which is very close to the data.

And why is there default in the steady state economy? One reason is that we have random depreciation of homes that hit home-owners with some exogenous probability. We interpret this random depreciation as random changes in local house prices. But the existence of the random shock is not sufficient to generate defaults. The additional assumption is that after such a depreciation, if there is a default on debt, the further foreclosure cost to the bank is small enough (in the paper we assume this to be zero). So in the beginning of the contract, the bank expects that if the depreciation shock happens early on in the contract (when the equity of the home-owner in the house is still low) the household will dump the house to the bank. In return the bank charges the household with a higher interest payment spread through the years. Given that the bank is risk-neutral and the depreciation shocks are independent across homeowners, it is optimal for the bank to take on the risk of a bad depreciation shock. However, if there is a further cost of foreclosure incurred, the optimal arrangement will deviate from this, with the bank asking for higher premiums to bear the additional cost. The higher cost of loans will motivate households to borrow less and, therefore, default with lower frequency.

Figure 1 displays the home-equity distribution in the model for households that do own a mortgage. In the model about 27 percent of homeowners do not have a mortgage as compared to about 35 percent in the data. The model under predicts the mass at very low home-equity ratio but over predicts it higher home equity ratios. But over all, the fit between the two distributions is fairly good. What factors determine the home-equity distribution in the model? The home-equity distribution at the time of the buying of a house is U-shaped. A large portion of households buy houses with very little equity and these are mostly high-income households who wish to utilize the tax deduction of mortgage interest payments since the interest payments are likely to exceed the standard deduction. The middle is relatively empty, with the second big mass coming at the other end with households choosing very high equity rates. These are middle to low

income households who wish to buy smaller houses, and for whom – had they taken out a big mortgage – the interest payments would have fallen below the standard deduction. When the mortgage payment is below or very close to standard deduction, it is more beneficial for the household to invest more of its existing savings into the house, and get the tax-free implicit rental returns from that savings. The average income of people taking out (new) mortgages with less than 50 percent home-equity is 1.62 times the average income of households taking out new mortgages with more than 50 percent home-equity and the average house size of the former group is about 1.37 times the average house size of the latter group. Aside from these two important effects, the decision of how much to borrow to finance the purchase of the house also depends on the household’s financial wealth: Some households get a large loan because they wish to become home-owners but do not have the financial wealth for a large down-payment. The second important factor affecting home-equity distribution is inflation. Because the mortgage contract is nominal, inflation reduces the real value of debt and, therefore, increases the real value of home-equity. The fairly uniform nature of the home-equity distribution results from this steady erosion of the real value of mortgage debt.



7 Over-Supply and the Foreclosure Crisis

7.1 Baseline

In this section we study how the economy responds to an unanticipated increase in the stock of housing. The interpretation is that during the boom, when house prices were inflated, more houses were built than was consistent with demand. In our analysis, we assume that the supply of owner-occupied housing space increases by 3 percent. Furthermore, for this part of our analysis we make some further assumptions regarding

the house market.

First, we assume that the owner-occupied segment of the housing market is segregated from the rental market. All previously owner-occupied housing space and the additional 3 percent increase in this space must remain owner-occupied during the transition to the new steady state. The segregation also applies to the new steady state. This assumption is meant to capture the idea that it is difficult to rent out housing meant for owner-occupancy. In steady state, we had equalized the return to a landlord whether he chooses to rent out the house or sell the house. This assumption is reasonable because we would expect all investments to be done to equalize those two values. But when there is an unanticipated shock there is no reason to expect that the returns from renting and selling should be equal. Thus, after the shock, there are two separate housing markets and the price in each market is determined by demand and supply for housing space in the respective market. Following the shock, the supply of housing in each of the two markets stays constant.⁷

Second, we assume that for the first 4 years following the shock, households that choose to foreclose on their homes get to live in their house rent-free with some fixed probability. This assumption is meant to capture the fact that the foreclosure process takes longer when there is a large number of households declaring foreclosure (as will, in fact, happen following the housing supply shock). In normal times a foreclosure takes about 6 months to complete. During the crisis, foreclosures have been taking an additional 9 months on average. This extra time for foreclosure is taken into account by setting the probability with which a person declaring foreclosure gets to stay in the house rent-free for another additional year to 0.75.

Finally, we assume that following shock, there is an additional cost that must be paid to get a mortgage. This cost remains constant for the first 4 periods following the shock and then declines geometrically at the rate of 80 percent. In our calibration, we choose this cost to pin down an initial decline in the price of owner-occupied housing of 19 percent. Calibrated in this way, the additional cost is roughly equal to a 1 percentage point increase in the cost of funds for a mortgage beyond the risk-free rate. This additional cost is meant to capture the disruption in the flow of funds to the mortgage market following the mortgage crisis. All else remaining the same, if this cost is eliminated, housing prices will not fall as much as 19 percent.

We now turn to understanding the dynamics of house prices and foreclosures following the shock. In steady state, the increase in the supply of owner-occupied housing has benign effects. The 3 percent increase in housing supply leads to roughly 3 percent decline in the price of owner-occupied housing and the excess supply is absorbed through increase in the average housing space occupied by owners and an increase in the fraction of people who become homeowners. However in the period of the shock, the effects are much more

⁷As in steady state, we assume that whichever entity owns the housing space must incur the depreciation expense, which keeps the stock of housing constant through time.

dramatic. Because it is costly for owner-occupants to increase the demand for housing space (due to the transactions costs of selling and buying new homes), the demand for housing space does not rise as much with a decline in the price of housing. Consequently, housing prices have to drop a lot to motivate owner-occupants and those who prefer to rent to absorb the excess supply of housing. But there is a natural lower bound to what can happen to prices in the first period of the shock. Since the owner of the property can always leave the house empty for one period, the implicit return from holding on to the house for one period must always be nonnegative. In equilibrium, the implicit return is strictly positive so no housing space is left empty.

Given the sharp decline in houses prices in the period of the shock, there is an incentive for leveraged households to declare default. One reason is that the homeowner has a chance to live free of rent for one year. The changes in prices of houses and rents add further incentives for default. Households have three choices: keep their house and continue to pay their mortgage, sell or default. It is easiest to describe the trade-off between selling and default and between keeping and default separately. For a household that prefers selling over keeping, we need to compare selling versus defaulting. For this household selling would dominate default if the equity in the house is high. The reasoning is that they will get more from the sale (after transaction costs) than what they have to pay back to the bank. If renting for free for some time is not too attractive, they will sell instead of defaulting. This condition is satisfied most easily if equity is high. For a household that prefers keeping over selling, we need to compare keeping versus defaulting. For this household there are several ways to describe their incentive to default. If equity is low, and the household is making a large mortgage payment, the household might think that, as houses have gotten cheaper, it can get an equivalent house for a cheaper price and lower its mortgage payments. Another inducement to default is that rents go down during the period of the shock. When rents are down, renting, instead of holding on to the current house and continuing with high mortgage payments gets to be more attractive.

There is also a feedback between default and house prices. When houses prices drop unexpectedly incentives for default go up. As default goes up, there may be feedback into the housing market. It depends on whether, an additional foreclosure increases or decreases the demand for owner-occupied housing space. If the alternative to defaulting is selling, then all else remaining the same, preventing foreclosures would decrease the demand for owner-occupied housing space because sellers have to rent for one period. If the alternative to defaulting is keeping the house, then preventing foreclosure will increase the demand for owner-occupied housing space. It turns out the alternative to foreclosure in most cases is keeping the house. Thus, preventing foreclosures results in an increase in the demand for owner-occupied housing space in the period of the shock. The drop in the price of owner-occupied housing space is now only 16 percent as opposed to 19 percent in the baseline model.

7.2 Inflation and the Mortgage Crisis

In this subsection, we study of the effects a lower inflation path on house prices and foreclosures. We assume that in the period of the shock, the anticipated inflation rate going forward falls to 1 percent for 4 years and then recovers back to the steady state value of 2.5 percent.

In order to do this we need to be clear about the nature of the mortgage contract. We assume that the contracts written prior to the shock stipulate that upon sale of the house, the present discounted value of the outstanding payment stream is evaluated at the current market interest rate. In steady state this is equivalent to the present discounted value being calculated at the time the mortgage is written. However, when the shock hits, the nominal interest rate at which the payment stream is evaluated is now (unexpectedly) lower – because anticipated inflation is lower.

The lower inflation path increases the default rate from 17 percent to 24 percent. This is intuitive: With a lower inflation rate, the real value of mortgage debt does not erode as rapidly as in the baseline model. Thus, the value of keeping the house is lower. And, the value from selling the house decreases as well because the present discounted value of the outstanding loan to be repaid upon sale is now higher. For both reasons, more households find default a better option.

Interestingly, the higher default rate does not have much of an impact on the price of houses. House prices fall by about 20 percent now as opposed to 19 percent in the baseline model. It appears that the supply of home-buyers is fairly elastic at this price: as the price drops slightly the excess supply of housing stemming from higher default is soaked up by new buyers and existing buyers buying bigger homes.

7.3 Mortgage Deduction and the Mortgage Crisis

In this section, we study how the crisis would have fared if there was no mortgage incentive to take on leverage. As noted in the discussion of the steady state of the baseline model, eliminating the mortgage deduction lowered the incentive to own homes and greatly lowered the incentive to take on mortgage debt. Thus average home equity is much higher and the average size of owner-occupied housing is also lower. Also, given the very high home equity there is no default in steady state.

A 3 percent increase the supply of owner-occupied housing lowers the price by about 10 percent. The drop in price does not create any incentive to default, so there is no increase in foreclosures in the period of the shock. The drop in price in the period of the shock exceeds the steady state drop for the same reasons as in the baseline model: The costs of selling and buying homes make the demand for owner-occupied housing

space insensitive to a change in the price in the short run.

8 Conclusion

To be added

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