

A Theory of Optimal Reserves Allocation and Sudden Stops in Emerging Economies

Sewon Hur

Illeen Kondo

University of Minnesota

Federal Reserve Bank of Minneapolis

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Abstract

A wave of financial crises and sudden stops crippled emerging economies during the period of 1997-2001. Since that time, there has been a remarkable increase of reserve holdings in emerging economies; and there have been virtually no sudden stops in these economies. We argue that, in the presence of debt rollover risk, idle reserves make countries more solvent in more states of the world. This in turn makes sudden stops less likely. We derive optimal reserves-to-debt ratios in a small open economy model with endogenous sudden stop probabilities and interest rate premia. Based on this theory of reserves allocation, we present a dynamic multi-country model with Bayesian learning and a regime switch in the stochastic liquidity shocks. This model can quantitatively account for the rise in reserve holdings and the sudden stop frequencies in emerging economies.

Hur: sewonhur@umn.edu | Kondo: kondo007@umn.edu. All data We thank Cristina Arellano, Tim Kehoe, Fabrizio Perri, Santiago Bazdresch and participants at the Trade Workshop at the University of Minnesota for many valuable comments. We have also benefited from comments by seminar participants at the 2010 Graduate Students Conference at Washington University in St. Louis. The data used in this paper are available at www.econ.umn.edu/~hurxx016. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. All errors remain our own.

1 Introduction

The world witnessed a surge of financial crises and sudden stops in emerging economies during the period of 1997-2001.¹ In contrast, there has been a conspicuous lack of sudden stops since 2002 and emerging economies thus far have not suffered from sudden stops despite the global financial crisis of 2007-2009.² In this paper, we document an increase of reserve holdings relative to external liabilities in emerging economies, and argue that this has made sudden stop occurrences less likely. We build a model of endogenous reserve holdings and sudden stops that can account for both the increase in reserve holdings and the pattern of sudden stop occurrences in emerging economies since 1990.

We consider an environment in which emerging economies borrow from international lenders to finance domestic projects. The governments of these emerging economies hold “reserves” because some foreign lenders who are subject to interim liquidity shocks may refuse to roll over their loans. The domestic project can be partially liquidated to pay back these lenders in the interim, but governments will first pay with their reserve holdings because liquidation can be costly. When reserve levels are inadequate for interim payments, liquidation of the domestic project will inevitably diminish the final returns of the “patient” international lenders. Even so, these lenders choose to optimally roll over as long as the final returns are higher than the world interest rate. However, when the government cannot guarantee such returns, all lenders refuse to roll over their loans in the interim, causing a “sudden stop”. Higher reserves allow governments to absorb larger amounts of interim called debt, and decreases the likelihood of a sudden stop. However, higher reserves imply less capital invested into the domestic project and therefore less final output. Given these tradeoffs, the levels of reserves and debt will then be chosen to maximize expected net output of the domestic project after paying international lenders.

We extend this environment to a dynamic framework with N emerging economies in order to quantitatively account for the increase in reserve holdings and the pattern of sudden stop occurrences. Faced with an unexpected change in the liquidity shock process, the countries gradually learn the true process through Bayesian updating. We argue that sudden stops surged as an outcome of two factors: greater international capital mobility in the globalization era and agents’ initial underestimation of this increased mobility. This is modeled as an unexpected change in the liquidity shock process. This causes an underinvestment in reserve holdings which increases the probability of sudden stops. When governments and investors observe a higher frequency of sudden stops, they use Bayesian updating to learn the new

¹A sudden stop is a sudden slowdown or reversal of capital inflows into emerging economies.

²Some view Greece’s 2010 debt crisis to be a sudden stop. In this paper, we follow standard classifications in not considering Greece as an emerging economy.

process. When agents have fully learned the new regime, governments hold a higher level of reserves and thus sudden stops decrease.

This paper builds on a large body of literature on reserves and sudden stops. For a long time, reserves were seen as an integral part of a country's export promotion strategy: they promote export by slowing appreciation. Dooley, Folkerts-Landau and Garger (2004) recently reiterated this explanation to justify the large foreign reserve holdings of emerging economies, in particular China. As documented by Aizenman and Lee (2007), this export promotion view cannot explain the recent increases in reserves of most countries, including China. In fact, reserves mostly increased long after exports started growing. If reserves serve to promote exports, they should have grown during the export growth.

Heller (1966) and Frenkel and Jovanovic (1981) model reserves as a buffer against exogenous stochastic balance-of-payments (BOP) deficits. In Frenkel and Jovanovic (1981), the government seeks to minimize the one-time adjustment costs that are incurred when reserves dry up. Higher reserves increase the distance-to-adjustment because the exogenous adjustment threshold is hit less often. Reserves however have an opportunity cost represented by the forgone interest earnings. This trade-off determines the optimal reserves held by a government. Therefore, there is no sudden stop per se in this class of models. Reserves serve as an infrequently replenished buffer with costly adjustments. Numerous papers follow this inventory approach to the role of reserves, e.g. Flood and Marion (2001).

More recently, precautionary motives have been explored as a potential key determinant of reserve allocations. Jeanne and Ranciere (2008) and Alfaro and Kanczuk (2007) focus on the consumption smoothing role of foreign reserves against output contractions associated with sudden stops. In Jeanne and Ranciere (2008), a government can purchase a special insurance ("reserves") for which it pays a premium every period until an exogenous event (a "sudden stop") occurs. Using the sovereign default framework of Arellano (2008), Alfaro and Kanczuk (2007) considers the role of reserves in an environment where the government can default on debt. In this setup, reserves serve as a post-default consumption smoothing mechanism since reserves can be used even after a country has defaulted. However, these consumption smoothing models of reserves can neither account for the rise in reserve holdings nor the pattern of sudden stop occurrences.

Our work is closely related to Aizenman and Lee (2007) who use a simple Diamond-Dybvig framework with exogenous interest rate, investment scale, and exogenous sudden stop probability to model reserve hoarding. In Aizenman and Lee (2007), countries face exogenous BOP deficits which must be financed with reserves or by liquidating domestic investments. Reserves hence serve as a cushion against the costly liquidation of productive domestic projects. We also use a Diamond-Dybvig technology specification. Our work

departs from Aizenman and Lee (2007) by considering the joint decision of borrowing and reserves, and by endogenizing the probability of financial crises. We can account for the rise in reserve holdings and the sudden stop occurrences precisely because reserve holdings do affect sudden stop probabilities.

Our contribution to the literature is twofold. First, we develop a theoretical framework capable of analyzing optimal reserves-to-debt ratios in a model with endogenous sudden stop probabilities and endogenous interest rate premia. In this theory, sudden stops arise when all foreign lenders rationally choose not to roll over the entirety of the country's debt. Despite having full commitment, the government's ability to repay its debt is limited by the resources available in the economy. On one hand, reserves protect domestic projects from liquidation and make foreign lenders calmer as the country is solvent in more states of the world. On the other hand, foreign reserves reduce the capital used in the productive sector.

Our second contribution is that we can explain and quantitatively account for the rise in reserve holdings and the sudden stop frequencies in emerging economies. We do so using a dynamic multi-country extension of our model with Bayesian learning and a regime switch in the stochastic liquidity shocks. During the transition when agents are learning about the regime switch, governments underinvest in reserves, leading to an endogenous increase in sudden stop occurrences.

This paper is structured as follows. Section 2 empirically analyzes reserve holdings and sudden stops in emerging economies from 1990-2007. Section 3 provides a simple three-period model of reserves allocation that delivers an optimal reserves-to-debt ratio with endogenous interest rates and sudden stop probabilities. Section 4 presents a multi-country dynamic model with learning and regime change. In Section 5, the model is parameterized and we show it can quantitatively account for the observed stylized facts from section 2. Section 6 concludes.

2 Stylized facts about international liquidity in emerging economies

In this section, we document a set of stylized facts regarding foreign reserves, external liabilities, and sudden stops in 24 emerging economies during 1990-2007. We use data on international liquidity from the IMF IFS dataset and the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). The list of emerging economies includes Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Romania,

Russia, Saudi Arabia, South Africa, South Korea, Thailand, and Turkey. This list includes countries appearing in most classifications of emerging countries with the notable exception of Taiwan for which the available data is limited.

2.1 Foreign Reserves

In the IFS dataset, foreign reserves are constructed as Total Reserves minus Gold. This definition includes convertible foreign exchange, SDR holdings, and IMF reserve position. There are two notable facts regarding foreign reserves holdings. The first is that foreign reserves as a percent of GDP in emerging economies are significantly higher than those in developed economies.³ The second is that these ratios have increased in emerging economies while the opposite holds for developed economies. These facts are summarized in Figure 1.

[Figure 1 here]

It is worth noting that this phenomenon of increasing reserves is not limited to just a few countries; in fact, they are increasing in all but one of the 24 emerging economies with Chile being the exception.⁴ This robust observation can be seen in the first 3 columns of Table 1.

[Table 1 here]

2.2 Foreign Reserves and Gross External Liabilities

Here, we document two facts using the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007) for gross external liabilities. External Liabilities are constructed using “Other Investment Liabilities” and the “Debt Securities” item under “Portfolio Investment Liabilities”.

The first fact is that reserves-to-liabilities ratios are much higher in emerging economies than in developed economies. The average of these ratios for emerging economies for 2002-2007 is more than 30 times higher than that for developed economies. The second fact is that reserves-to-liabilities ratios have been increasing in emerging economies while they have been decreasing in developed economies. This observation holds for all but three of the 24 emerging economies with Chile, Hungary, and Colombia being the exceptions. These facts can be seen more succinctly in Figure 2 and with more details in the last 3 columns of Table 1.

[Figure 2 here]

³The developed economies refer to the United States, the United Kingdom, France, and Germany.

⁴Note, however, that Chile had very large reserve holdings as a percent of GDP during 1990-1996.

2.3 Sudden Stops in Emerging Economies

We define a sudden stop to be a sudden slowdown or reversal of capital inflows into emerging economies⁵. Using the sudden stop episodes classified in Durdu, Mendoza, Terrones (2009), there were 15 sudden stop experiences during 1990-2007 across the 24 emerging economies we are studying.⁶ In particular, we divide this timeframe into three periods as shown in Figure 3:

- 1990-1996 is a period of low-frequency sudden stops (with two occurrences),
- 1997-2001 is a period of high-frequency sudden stops (with thirteen occurrences),
- and 2002-2007 is a period of low-frequency sudden stops (with no occurrence).

[Figure 3 here]

3 An illustrative three-period model of optimal reserves allocation

In this section, we introduce a simple three-period model to clearly illustrate our theory of foreign reserves allocation. We describe the environment and the technologies available before stating the optimal debt contract offered by the domestic government to foreign lenders. Reserves holdings emerge as an integral part of the borrowing arrangement.

3.1 Environment

We consider a three-period small open economy model. The country finances long-term domestic projects with short-term foreign debt. The initial period ($t = 0$) is a contracting period during which no production or consumption takes place. In the interim ($t = 1$), foreign lenders can either roll over or call the loan they made at $t = 0$. Production occurs in the final period ($t = 2$), and is used for consumption and for paying lenders.

The small country has a representative agent and a welfare-maximizing government which offers debt contracts to foreign lenders.⁷ There is a continuum of ex-ante identical risk-neutral foreign lenders⁸ indexed by i . Agents consume in period 2 and do not discount.

⁵Calvo et al. (2004) define a sudden stop to be a year-on-year fall in capital flows of at least two standard deviations below the mean.

⁶Sudden Stop episodes: Argentina, Mexico (1994), Indonesia, Malaysia, Philippines, South Korea, Thailand (1997), Brazil, Chile, Colombia, Pakistan, Peru, Russia (1998), Argentina, Turkey (2001).

⁷We can readily write an equivalent decentralized environment with externalities in which domestic banks borrow and lend internationally while the domestic government sets the foreign reserves requirements.

⁸For technical reasons, we assume that the foreigners' capital endowment is finite and large enough.

3.2 Technologies

The technologies available in this model closely mirror Diamond and Dybvig (1983).⁹

In the initial period, using the amount borrowed D and the incoming stock of reserves R_0 , the government chooses reserves R and the long-term investment K subject to the initial resource constraint:

$$R + K \leq R_0 + D. \quad (1)$$

In the interim, reserves R can be used for paying P_1 to lenders who call the debt.⁹ The government may also choose to liquidate L from the long-term investment K . However, liquidation is costly as it yields λL with $\lambda < 1$. This implies that the government will pay interim payments P_1 using reserves, if possible. Any remaining reserves can be stored as R_2 and will be available for use in the final period. The interim constraints are given by:

$$P_1 \leq R + \lambda L, \quad (2)$$

$$R_2 \leq \max \{R - P_1, 0\}. \quad (3)$$

In the final period, the remaining long-term investment $(K - L)$ is used in the productive technology that yields $A(K - L)^\theta$. The remaining reserves and the final output can be used for consumption C and for final payments P_2 to lenders who rolled over in the interim. The final resource constraint is given by:

$$C + P_2 \leq R_2 + A(K - L)^\theta. \quad (4)$$

Assumption 1 (Liquidation costs and decreasing returns to scale)

$$\lambda < 1 \text{ and } \theta < 1.$$

The first condition reflects the idea that it is costly to divest from the long-term investment in the interim. The second condition implies that the productive technology has decreasing returns. This assumption mainly allows us to pin down the long-term investment scale and the borrowing needs in the country.

⁹We assume that the reserves do not earn any return, but this assumption can be relaxed.

3.3 International debt market and limited repayment

We now present the main features of the international debt markets and carefully state the constraints these features induce.

Every period, foreign lenders can lend short-term at the world interest rate r . This effectively represents the outside option of each foreign lender as they assess whether to lend to the domestic country or not. In the initial period, the domestic government offers a contract to the foreign lenders that *promises* a short-term interest rate of $\hat{r}$ on a loan of size D . The return is not guaranteed because there are states of the world where the government cannot pay the full promised amount: $(1 + \hat{r})D$ after one period or $(1 + \hat{r})^2 D$ after two periods. Thus, the government must offer a promised rate $\hat{r}$ that is greater than the world interest rate r . The difference is the interest premium. Since the foreign lenders are risk-neutral, the government will essentially offer a promised interest rate $\hat{r}$ that makes the lender break even on expectation.

In the interim, a fraction φ of foreign lenders receive liquidity shocks, meaning that they must call the loan. By calling a loan, a lender asks the government to pay back the amount borrowed plus the promised interest instead of extending that loan for another period under the same terms. The fraction φ is stochastic and follows a cumulative distribution function F . The remaining fraction $(1 - \varphi)$ of “patient” lenders can choose to call or roll over their loans. Let $\psi_i \in \{0, 1\}$ denote the rollover decision of lender i :

$$\begin{cases} \psi_i = 1 & \text{if lender } i \text{ chooses to call the loan} \\ \psi_i = 0 & \text{if lender } i \text{ chooses to roll over the loan} \end{cases}$$

This implies that the total fraction of lenders who call the loan is: $\psi \triangleq \int \psi_i di$. Also, $\psi \geq \varphi$ since “impatient” lenders all call their loans. Hence, $\psi(1 + \hat{r})D$ represents the total promised payment to lenders calling in the interim, and $(1 - \psi)(1 + \hat{r})^2 D$ is the total promised payment to the remaining lenders.

We assume that the government’s ability to honor these promised payments is limited by the resources available in the economy when the payments are due. In other words, lenders who call their loan in the interim can only be repaid using the reserves R and the liquidation λL of invested capital K as in equations (2) and (5). Similarly, lenders who roll over the debt will be paid using long-term output $A(K - L)^\theta$ and remaining reserves R_2 , if any, as in equations (4) and (6). Hence, the liability of the government is limited. The government is however assumed to have a full commitment to honor as much as possible its obligations to the lenders¹⁰. In particular, the government will liquidate as much capital as needed to meet

¹⁰One can generate this restriction by giving the lenders the ability to take over the available assets and

the interim payments, as in equation (7). This implies that the actual payments to lenders calling in the interim P_1 , the actual payments to remaining lenders P_2 , and the government's interim liquidation policy will be given by:

$$P_1 = \min \{ \psi [(1 + \hat{r}) D] , R + \lambda L \} \quad (5)$$

$$P_2 = \min \{ (1 - \psi) [(1 + \hat{r})^2 D] , R_2 + A(K - L)^\theta \} \quad (6)$$

$$L = \min \left\{ \frac{1}{\lambda} \max \{ \psi [(1 + \hat{r}) D] - R , 0 \} , K \right\} . \quad (7)$$

The timeline diagram below provides an overview of the sequence of actions taken in this environment.

[Figure 4 here]

3.4 Foreign lenders' call/rollover and participation decisions

We assume that an individual lender i takes the actions of other investors as given. In particular, they take as given the call/rollover decisions $\{\psi_j(\varphi)\}_{j \neq i}$ as a function of the aggregate liquidity shock φ . This implies that once the aggregate liquidity shock is realized, the fraction of lenders calling the loan $\psi(\varphi) (= \int \psi_j(\varphi) dj)$ can be inferred by each individual lender. We address the issue of potential multiple equilibria by allowing the government to include a call/rollover *recommendation* ψ_i as part of the contract. Of course, this recommendation is not binding, but the foreign lender is willing to follow the recommendation as long as it is individually rational to do so.

Let $V(\psi_i | \varphi)$ denote the "patient" lender's payoff given the call or rollover decision ψ_i :

$$V(\psi_i | \varphi) = \begin{cases} (1 + r) \frac{P_1(\varphi)}{\psi(\varphi)} & \text{if } \psi_i = 1 \\ \frac{P_2(\varphi)}{(1 - \psi(\varphi))} & \text{if } \psi_i = 0 \end{cases}$$

The call/rollover recommendation ψ_i^* is individually rational if it satisfies:

$$\psi_i^* \in \arg \max V(\cdot | \varphi) \quad (8)$$

Moreover, the lender is willing to initially enter the loan contract only if he or she breaks technologies when they ask to be repaid. In that case, the domestic government will effectively repay them to its utmost capacity.

even on expectation:

$$\mathbf{E}_\varphi \left[\varphi \cdot (1+r) \frac{P_1(\varphi)}{\psi(\varphi)} + (1-\varphi) \cdot \max \left\{ (1+r) \frac{P_1(\varphi)}{\psi(\varphi)}, \frac{P_2(\varphi)}{(1-\psi(\varphi))} \right\} \right] \geq (1+r)^2 D \quad (9)$$

3.5 Optimal contract problem and reserves allocation

We can now succinctly state the domestic government's problem.

Given reserve endowments R_0 , an optimal contract $C^* = \{\hat{r}^*, D^*, R^*, K^*, L^*(\varphi), P_1^*(\varphi), P_2^*(\varphi), \psi_i^*(\varphi)\}$ solves

$$\begin{aligned} \max_C \quad & \mathbf{E}_0 \left[U \left(\underbrace{A(K-L)^\theta + R_2}_{\substack{\text{final output} + \\ \text{remaining reserves}}} - \underbrace{P_2(\varphi)}_{\substack{\text{time 2} \\ \text{payments}}} \right) \right] \\ \text{s.t.} \quad & (1), (3), (5), (6), (7), (8), (9) \\ & \text{non-negativity constraints} \end{aligned}$$

3.6 Optimal Contract Characterization

We now characterize the optimal contract. First, we establish some intermediate lemmas that will help us establish the main results of the model.

Assumption 2 (Liquidity shocks occur)

$$\Pr(\varphi > 0) > 0$$

Lemma 1. Reserves, dilution, and sudden stop cutoffs

Let Assumptions 1,2 hold. Then, the optimal contract C^ has cutoff levels $\varphi_L \leq \varphi_D \leq \varphi_{SS}$ such that:*

- (1) *there is interim liquidation if and only if the aggregate shock exceeds φ_L ,*

$$\text{i.e. } L(\varphi) > 0 \iff \varphi \geq \varphi_R$$

(2) *promised final payments are diluted if and only if the aggregate shock exceeds φ_D ,*

$$\text{i.e. } P_2(\varphi) < (1 - \psi(\varphi))(1 + \hat{r})^2 D \iff \varphi \geq \varphi_D$$

(3) *patient lenders will call in the interim if and only if the aggregate shock exceeds φ_{SS} ,*

$$\text{i.e. } \psi(\varphi) \geq \varphi \iff \varphi \geq \varphi_{SS}$$

Proof: The complete proof is not yet available, but our numerical simulations seem to confirm our conjecture. The complete proof will be updated in future versions.

Discussion In Lemma 1, φ_L is the liquidity shock above which reserves are depleted and the government must start liquidating the invested capital to meet the promised payments. Because $\lambda < 1$, the government always uses existing reserves to meet payments before eventually liquidating the invested capital. If the invested capital is large enough, both the interim and final obligations may be fully honored.

φ_D is the liquidity shock above which the government no longer has the resources to pay the full promised amount to the foreign lenders. Patient lenders are willing to roll over their loans as long as the dilution incurred is not too high, as in equation (8). In the meantime, interim payments cannot be diluted. This would otherwise contradict the government's commitment to honor the interim debt obligations.

φ_{SS} is the liquidity shock at which the government cannot deliver sufficient time 2 payments to persuade the patient lenders to roll over the debt. Any liquidity shock higher than φ_{SS} will cause total exit, and we identify this phenomena as a *sudden stop*.

Illustratively, given an allocation of physical resources (R_0, D, K_0) and the promised interest rate $\hat{r}$, the cutoffs are shown in Figures 6-8.

[Figure 5 here]

[Figure 6 here]

[Figure 7 here]

Proposition 1. Existence and uniqueness *Let Assumptions 1,2 hold. Then, the optimal contract problem has a solution C^* and it is unique.*

Proof: Existence follows from Lemma 1 and the concavity of the objective function with respect to D, K , and R . A complete proof for uniqueness is still in progress.

Corollary 1. Endogenous sudden stop probability

The optimal contract C^ induces an ex ante endogenous probability that a sudden stop occurs. That is:*

$$\Pr(\chi = 1) = 1 - F(\varphi_{SS}^*)$$

where $\chi = 1$ if a sudden stop occurs.

Proof: This follows from Lemma 1 and Proposition 1.

4 A multi-country dynamic extension with learning and regime change

The previous section illustrated the main forces determining the optimal allocation of foreign reserves: the delicate interaction between sudden stop probabilities and productive capital use. We now propose a fully dynamic model with N small (emerging) economies that face an unexpected change in the liquidity shock process and gradually learn the true process through Bayesian updating. This framework can quantitatively account for the dynamics of the foreign reserves holdings, external liabilities, and sudden stops.

We argue that sudden stops surged as a result of greater international capital mobility in the globalization era along with agents' initial underestimation of this increased mobility. This increased mobility is modeled as an unexpected change in the liquidity shock process. The extension we propose formalizes a thought experiment in which the unexpected switch occurs in the late nineties. As predicted by our theory, any underestimation of the true process of liquidity shocks will induce an increase in sudden stop occurrences. Countries gradually update their beliefs on the liquidity shock process using Bayes' rule and the endogenous probabilities of sudden stops predicted by our theory.

In this section, we set up the extended environment. We present numerical results in the next section.

4.1 Environment

We consider N identical small economies indexed by $j = 1, \dots, N$. Time is infinite, discrete and indexed by $t = 0, 1, \dots, \infty$. Each country is populated by an infinitely-lived representative agent and a welfare-maximizing domestic government. There is a continuum of infinitely lived risk-neutral foreign lenders $i \in [0, 1]$. Agents discount future consumption by the discount factor β .

Each time period t is divided into three stages, $s = 0, 1, 2$. Each period t effectively encaps-

ulates the three stages of the previous section's basic model:

- $s = 0$ is the initial contracting stage
- $s = 1$ is the interim stage during which liquidity shocks are realized, and lenders make call/rollover decisions
- $s = 2$ is the final production and consumption stage.

Within each period t , the technologies available at a stage s are identical to those in the previous section with the addition of an inter-period reserves savings technology. As in the previous section, at $s = 2$, the government can use final output $A(K - L)^\theta$ and remaining reserves R_2 for consumption and final payments P_2 . In addition, the government may choose to save part of the remaining reserves for next period. The choice of reserves to carry over R'_0 is constrained by:

$$R'_0(\varphi) \leq R_2(\varphi) - \max \left\{ P_2(\varphi) - A(K - L(\varphi))^\theta, 0 \right\} \quad \forall \varphi \quad (10)$$

Shocks and Information Structure The aggregate liquidity shock in country j at time t is denoted by $\varphi_t^j \in [0, 1]$. The N aggregate shocks $\{\varphi_t^j : j = 1 \dots N\}_{t=0}^\infty$ are independent and identically distributed across countries and time¹¹. These aggregate liquidity shocks follow a common stochastic process with cumulative distribution function F_{σ_t} with higher values of σ_t indicating higher likelihood of large aggregate shocks¹². As in the basic model, a fraction φ_t^j of foreign lenders lending to country j receive liquidity shocks and must call the debt in the interim.

We assume $\sigma_t \in \{\sigma_L, \sigma_H\}$ with $\sigma_L < \sigma_H$. This regime parameter σ_t is unobserved and unknown to the agents, though agents share a common belief ρ_t at time t ¹³:

$$\rho_t \triangleq \Pr(\sigma_t = \sigma_L)$$

At the end of each period t , agents observe the sudden stop occurrences in the N countries. Using these sudden stop occurrences and the endogenous sudden stop probabilities,¹⁴ agents

¹¹This setup can easily be extended to internationally and/or serially correlated aggregate shocks.

¹²That is: $\sigma_L \leq \sigma_H \Rightarrow F_{\sigma_L}(\varphi) \geq F_{\sigma_H}(\varphi) \quad \forall \varphi$

¹³Given a belief ρ_t , the (subjective) probability distribution function f_t of the aggregate liquidity shock is:

$$f_t(\varphi) = \rho_t \cdot dF_{\sigma_L}(\varphi) + (1 - \rho_t) \cdot dF_{\sigma_H}(\varphi)$$

¹⁴See Corollary 1.

update their beliefs according to Bayes' rule.¹⁵

4.2 Optimal dynamic stochastic contracts

An important difference with the basic model is the endogeneity of reserve endowments. In the basic model, the reserve endowment was exogenous; in the dynamic model, governments will face a consumption/savings decision and will choose the reserve endowments of the following period. As in the previous section, let $C = \{\hat{r}, D, R, K, L(\varphi), P_1(\varphi), P_2(\varphi), \psi_i(\varphi), \mathbf{R}'_0(\varphi)\}$. For each belief ρ about the prevailing liquidity shock regime, we can characterize the path of the optimal dynamic stochastic contracts $\{\mathbb{C}_t^{*j} | \rho\}_{j,t}$ by solving the following functional equation:

$$\begin{aligned}
 W(R_0; \rho) &= \max_{C, R'_0} \mathbb{E} \left[U \left(\underbrace{A(K-L)^\theta + R_2(\varphi)}_{\text{output} + \text{remaining reserves}} - \underbrace{P_2(\varphi)}_{\text{final payments}} - \underbrace{\mathbf{R}'_0(\varphi)}_{\text{reserves for next period}} \right) + \beta W(\mathbf{R}'_0(\varphi); \rho) \right] \\
 &\text{s.t.} \\
 &\quad (1), (3), (5), (6), (7), (8), (9), \text{ (10)} \\
 &\quad \text{non-negativity constraints}
 \end{aligned}$$

Recall that at the end of each period t , the common belief of the agents is updated to ρ_{t+1} using the sudden stop occurrences and sudden stop probabilities in the N countries according to Bayes' Rule¹⁶. Hence, given a sequence of sudden stop occurrences $\{\chi_t\}_t$, an initial belief ρ_0 , and initial reserve endowments $\{R_{-}^j\}_{j=1}^N$, the realized sequence of optimal

¹⁵If we denote $\chi_t \triangleq \{\chi_t^j\}_{j=1}^N \in \{0, 1\}^N$ as the vector of sudden stops where $\chi_t^j = 1$ if a sudden stop occurs in country j at time t , then agents can use the sudden stops vector χ_t , and the endogenous sudden stop probabilities $\Pr(\chi_t^j = 1 | \sigma)$ to update their beliefs according to Bayes' rule

$$\rho_{t+1} = \frac{\rho_t \Pr(\chi_t | \sigma_L)}{\rho_t \Pr(\chi_t | \sigma_L) + (1 - \rho_t) \Pr(\chi_t | \sigma_H)}$$

¹⁶Denoting $\chi_t \in \{0, 1\}^N$ as the vector of sudden stops, Bayesian updating is done following:

$$\rho_{t+1} = \frac{\rho_t \Pr(\chi_t | \sigma_L)}{\rho_t \Pr(\chi_t | \sigma_L) + (1 - \rho_t) \Pr(\chi_t | \sigma_H)}$$

contracts $\{\mathbb{C}_t^{*j}\}_{j,t}$ is well-defined and can be fully characterized using the functional equation solutions and Bayes' rule.

5 Quantitative analysis

In this section, we discuss the quantitative results of a carefully parametrized model. Our simulations show that our extended model can account for the stylized facts we documented.

In particular, we simulate the following thought experiment. We assume that the period of 1990-1996 was an era of relatively low volatility in international capital movements, i.e. a σ_L regime. By 1997, globalization and widespread financial liberalization allowed less restrictive capital movements but governments and investors underestimated the increase in capital mobility, i.e. there is an unexpected change to a σ_H regime. Based on our theory, this will cause an underinvestment in reserve holdings which increases the probability of sudden stops. Governments and investors, seeing the rise in sudden stops, update their common belief about the prevailing regime. By 2002, agents have fully learned the new regime; as a result, reserves-to-debt is higher and sudden stops decrease.

5.1 Parametrization and functional forms

A period in the model is assumed to be a quarter. We choose $N = 24$ as we have 24 emerging economies in our dataset.

The domestic agents are assumed to have a constant relative risk aversion utility function¹⁷:

$$U(c) = \frac{1}{1-\gamma} (c + c_{\min})^{1-\gamma}$$

with

$$\begin{cases} \Pr(\chi_t | \sigma_L) \triangleq \prod_{j=1}^N \left\{ \left[1 - F_{\sigma_L}(\varphi_{ss,t}^j) \right] \cdot \chi_t^j + F_{\sigma_L}(\varphi_{ss,t}^j) \cdot (1 - \chi_t^j) \right\} \\ \Pr(\chi_t | \sigma_H) \triangleq \prod_{j=1}^N \left\{ \left[1 - F_{\sigma_H}(\overline{\varphi_{ss,t}^j}) \right] \cdot \chi_t^j + F_{\sigma_H}(\overline{\varphi_{ss,t}^j}) \cdot (1 - \chi_t^j) \right\} \end{cases}$$

where $\varphi_{ss,t}^j$ refers to the sudden stop cutoff induced by the solution to $[\mathcal{RE}]$ at $(R, \rho_t) = (R_{2,t-1}^j, \rho_t = 1)$ and $\overline{\varphi_{ss,t}^j}$ refers to the sudden stop cutoff induced by the solution to $[\mathcal{RE}]$ at $(R, \rho_t) = (R_{2,t-1}^j, \rho_t = 0)$. The agents take these endogenous cutoffs as given.

¹⁷This formulation does not yield a constant relative risk aversion coefficient in the strict sense. The relative risk aversion is constant in terms of the total consumption level $c + c_{\min}$. This minimum consumption level $c_{\min}$ is needed to induce a bounded utility when $c = 0$. Otherwise, since sudden stops are probable, the problem would not be well defined. Moreover, one can think of $c_{\min}$ as the output from the other part of the economy that is not financed through foreign lending.

Here, we only report results for the risk neutral case ($\gamma = 0$) as our results are not very sensitive to the relative risk aversion coefficient and the effective consumption lower bound. In the risk neutral case, this lower bound does not affect the optimal contract.

The liquidity shock process is an important element of the model. We assume the aggregate liquidity shock distributions $(F_{\sigma_L}, F_{\sigma_H})$ belong to the class of Generalized Bounded Pareto distributions on $[0, 1]$:

$$F_{\sigma}(\varphi) \triangleq 1 - (1 - \varphi)^{\frac{1}{\sigma}}$$

An increase in σ shifts the cumulative distribution function F_{σ} to the left. The switch from σ_L to σ_H therefore reflects the increase in capital mobility. We chose this kind of power law distribution to reflect the idea that high aggregate liquidity shocks are very unlikely.

The parameters β , r , σ_L , and σ_H are then set to match some facts regarding international liquidity.

In particular, we set β to match average interest rates of 2% in emerging economies over 1990-2007, r to match the risk-free rate of 1%, while σ_L and σ_H are set to match average reserves-to-debt ratios in the emerging economies for the periods of 1990-1996 and 2002-2007 respectively. We follow Ennis and Keister (2003) to set the divestment cost $1 - \lambda$ to be 30%. We follow Atkeson and Kehoe (2005) to set θ to 0.85 and we assign an arbitrary value of A to 1.5.¹⁸ The parameters are summarized in Table 2.

[Table 2 here]

5.2 Quantitative results

We consider $N = 24$ identical economies starting with different initial foreign reserves¹⁹. As the N economies experience different aggregate liquidity paths, their reserves holdings and sudden stops paths also evolve differently. The results shown are the average across a large number of simulated paths for the N countries.

As can be seen in Figure 10, our model is able to replicate the pattern of low frequency sudden stops during 1990-1996, high frequency sudden stops in the transition (1997-2001), and low frequency after the transition (2002-2007). During the transition, governments are underinvesting in reserves, thereby increasing the probability of sudden stops. Once the governments have learned of the regime change to higher liquidity shocks, they choose to hold a higher level of reserves, thus returning sudden stop probabilities to lower levels.

¹⁸The quantitative results are not sensitive to changes in A . We plan to provide a detailed sensitivity analysis in future versions of this paper.

¹⁹The initial levels of foreign reserves were generated by simulating, for a few periods, the model with initial zero foreign reserves.

[Figure 8 here]

In our theory, misaligned beliefs beget “abnormal” sudden stop occurrences. Our extended model is able to replicate the surge in sudden stops and the subsequent stabilization because our theory of optimal reserves allocation endogenizes the sudden stops probabilities. As sudden stops occurrences increase, reserves dry up more often but governments keep ramping up their foreign holdings as a result of updated beliefs. In this model, model reserves do not serve as a *post*-sudden stop insurance. Instead, in contrast to most consumption smoothing theories of reserves allocation, reserves play an active role of preventing sudden stop occurrences and they do not help increase consumption after sudden stops.

Table 3 summarizes our key results. One drawback of the results is that the speed at which agents learn the true process is quite fast: this leads to governments increasing reserves faster and sudden stops ceasing sooner than 2002 as seen in the data. Also, since $\sigma_L < \sigma_H$, the post-crisis era is characterized by slightly more sudden stops than the pre-crisis era. Of course, both periods feature much less sudden stops than the crisis/adjustment era. The extended model also predicts a rise in country-specific interest rate premium during the surge of sudden stops. However, in the absence of risk aversion in the agents’ preferences, the premia we generate are very modest and we do not report these results here.

[Table 3 here]

6 Conclusion

In this paper, we have studied empirically and theoretically the joint dynamics of external liabilities, foreign reserves, and sudden stops in emerging economies. Using international liquidity data for 24 emerging economies, we document that reserves holdings as a percent of GDP and reserves-to-external liability ratios have dramatically increased in emerging countries from 1990 to 2007. We also present the time series of sudden stop occurrences: there were virtually no sudden stops in these emerging economies except during 1997-2001.

We then develop a small open economy model where reserves endogenously affect the probability of sudden stops. In our model, foreign lenders choose to roll over their loans as long as their returns are not undermined by the divestments made to repay lenders calling in the interim. Sudden stops occur when all foreign lenders choose to call the loans. On one hand, reserves protect domestic projects from liquidation and make foreign lenders calmer as the country is solvent in more states of the world. On the other hand, foreign reserves incur opportunity costs by reducing the capital used in the productive sector. Consequently, the model yields an endogenous probability of sudden stop and optimal reserves-to-debt ratios.

Furthermore, we explore the empirical validity of this channel. In particular, the model implies that any underestimation of the true process of liquidity shocks will induce a higher likelihood of sudden stops. We propose a dynamic multi-country model with Bayesian learning and a regime switch in the stochastic liquidity shocks. With the gradual learning of the true regime, we obtain a transition path during which sudden stops surge. The optimal reserves-to-debt ratios are higher at the end of the transition as seen in the data. The calibrated model generates levels that are similar to the three stylized facts we documented.

This paper therefore provides a useful theory of optimal reserves allocation and sudden stops. Our model however is highly stylized. It does not include many relevant features that can affect our results such as a domestic sector or multiple debt maturities. For instance, Cole and Kehoe (2000) suggest that the composition of a country's debt portfolio matters for sudden stop occurrences. We leave these questions for future research.

7 Appendix 1: Figures and Tables

Figure 1. Foreign Reserves

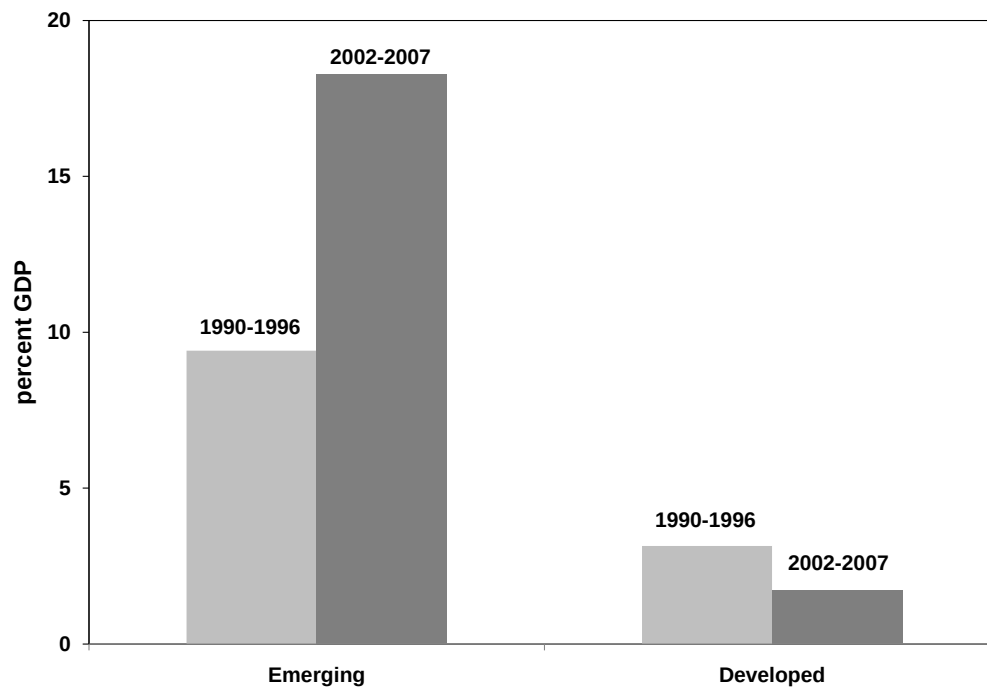


Figure 2. Foreign Reserves / Gross External Liabilities

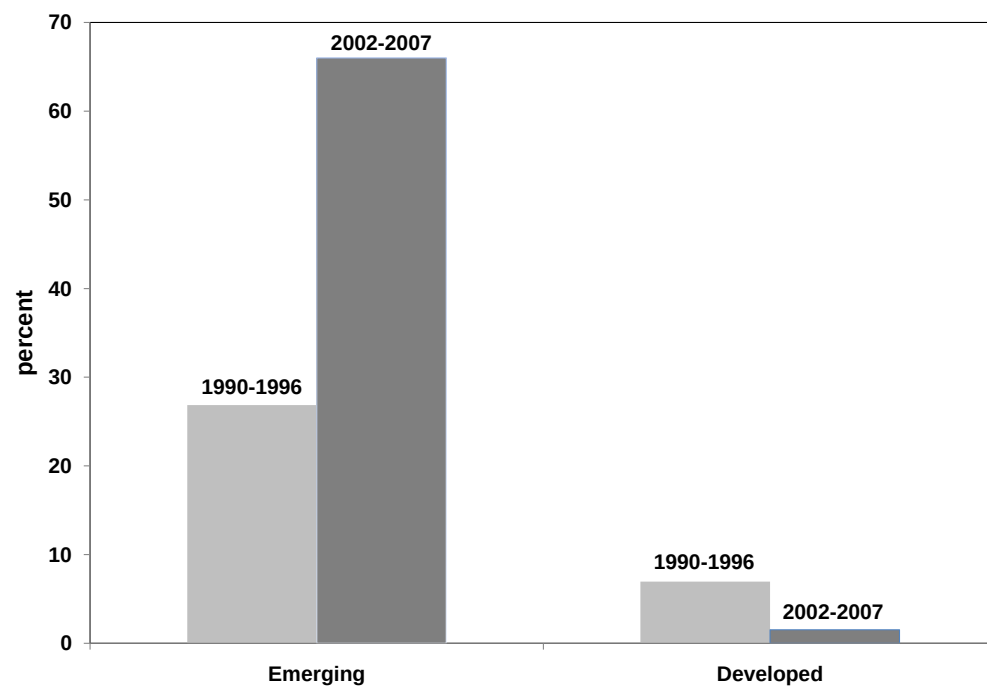


Figure 3. Sudden Stops in Emerging Economies

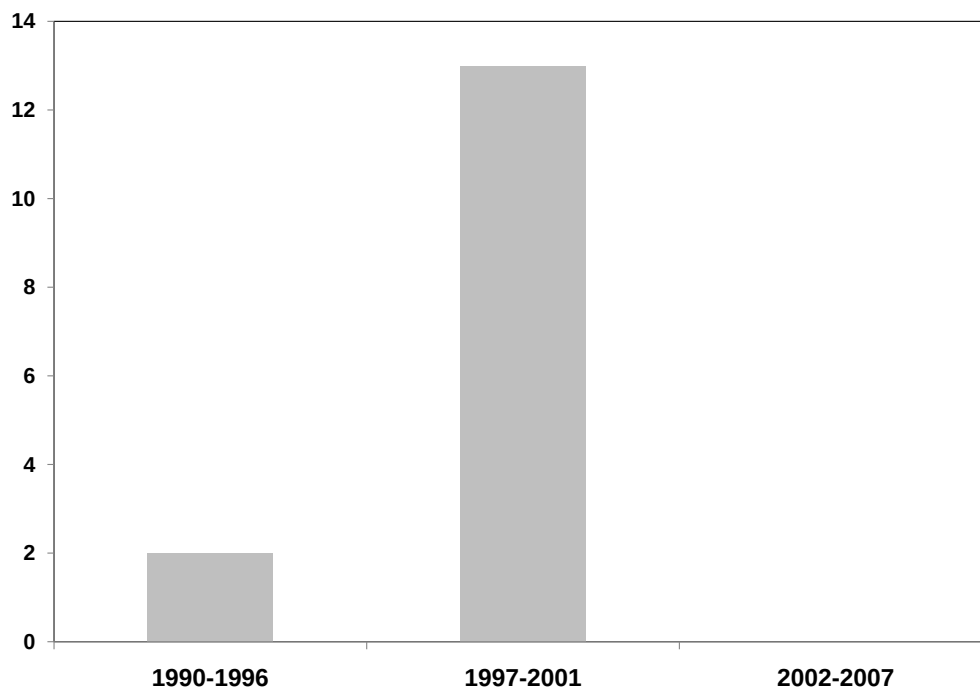


Figure 4. Timeline

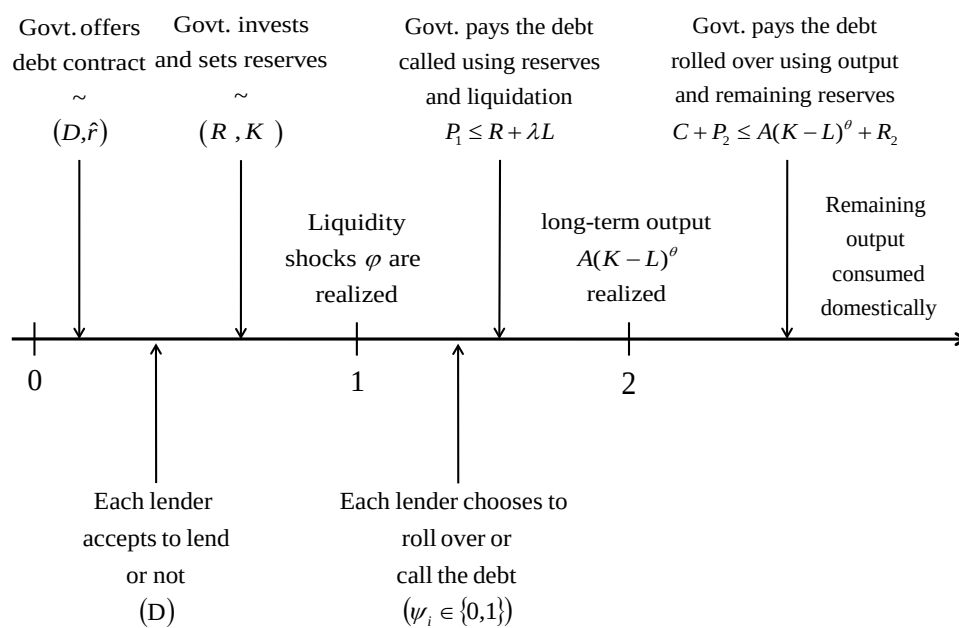


Figure 5. Fraction of K_0 liquidated

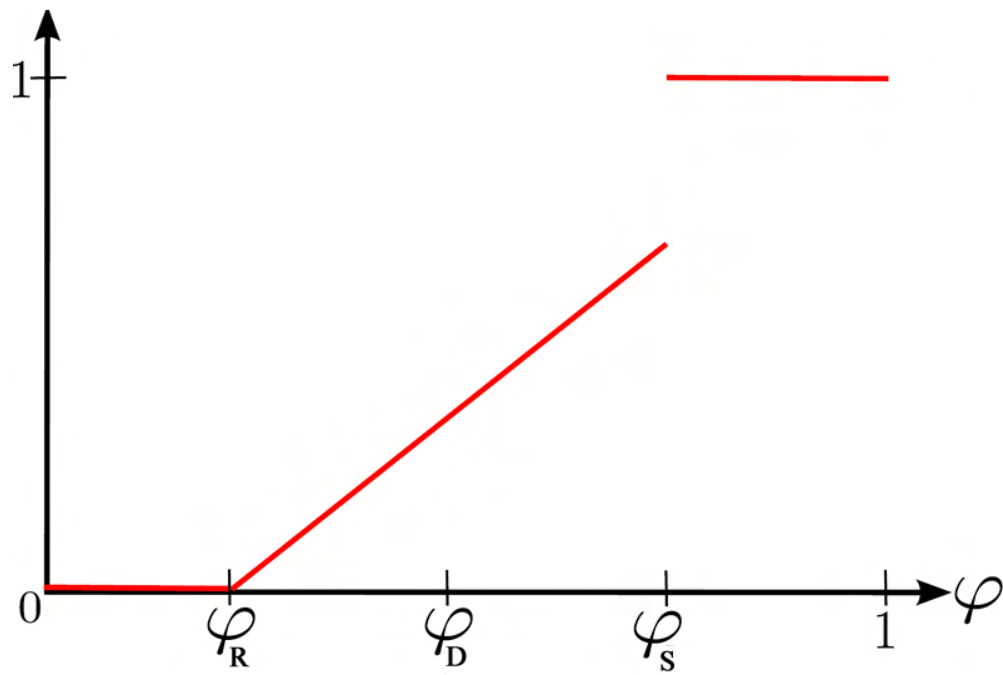


Figure 6. Fraction of lenders who call the debt

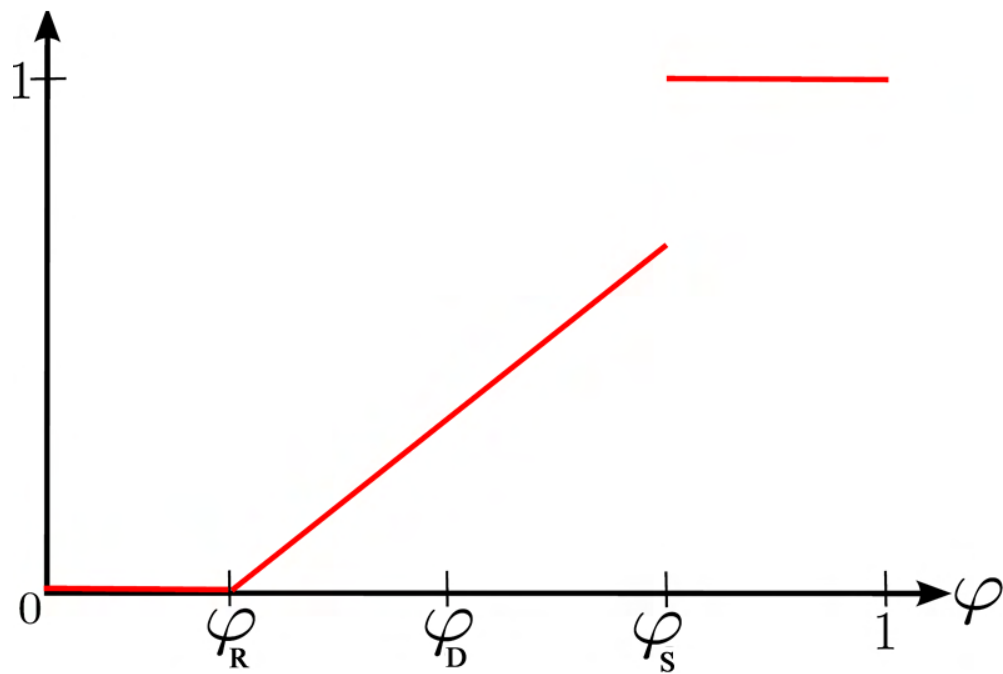


Figure 7. Payoffs for roll vs. call when $\varphi_i = 0$

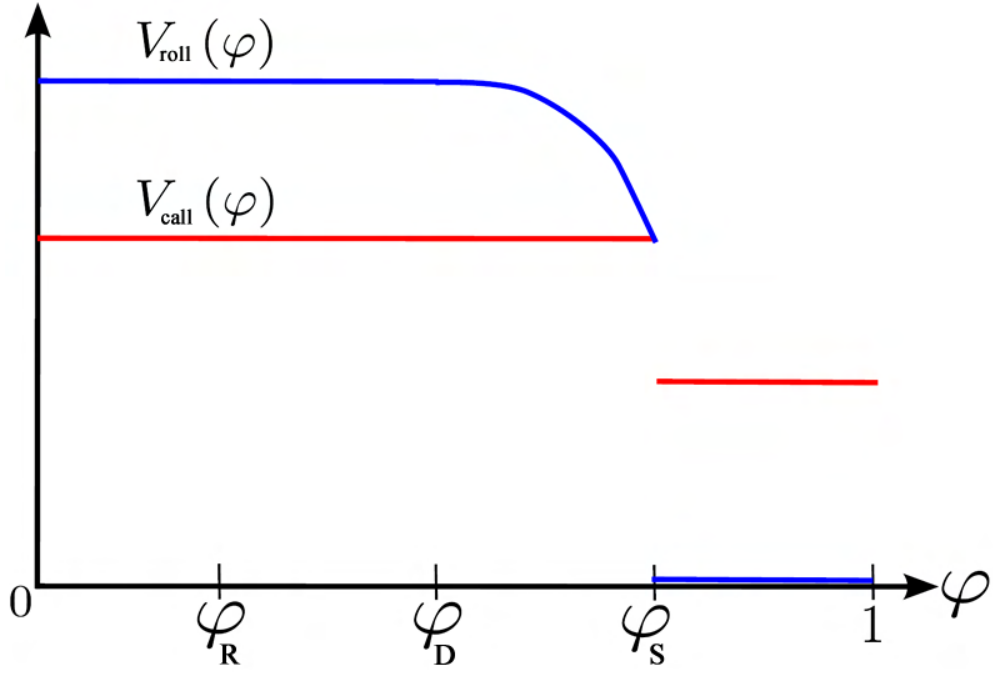


Figure 8. Reserves-to-Debt and Sudden Stops

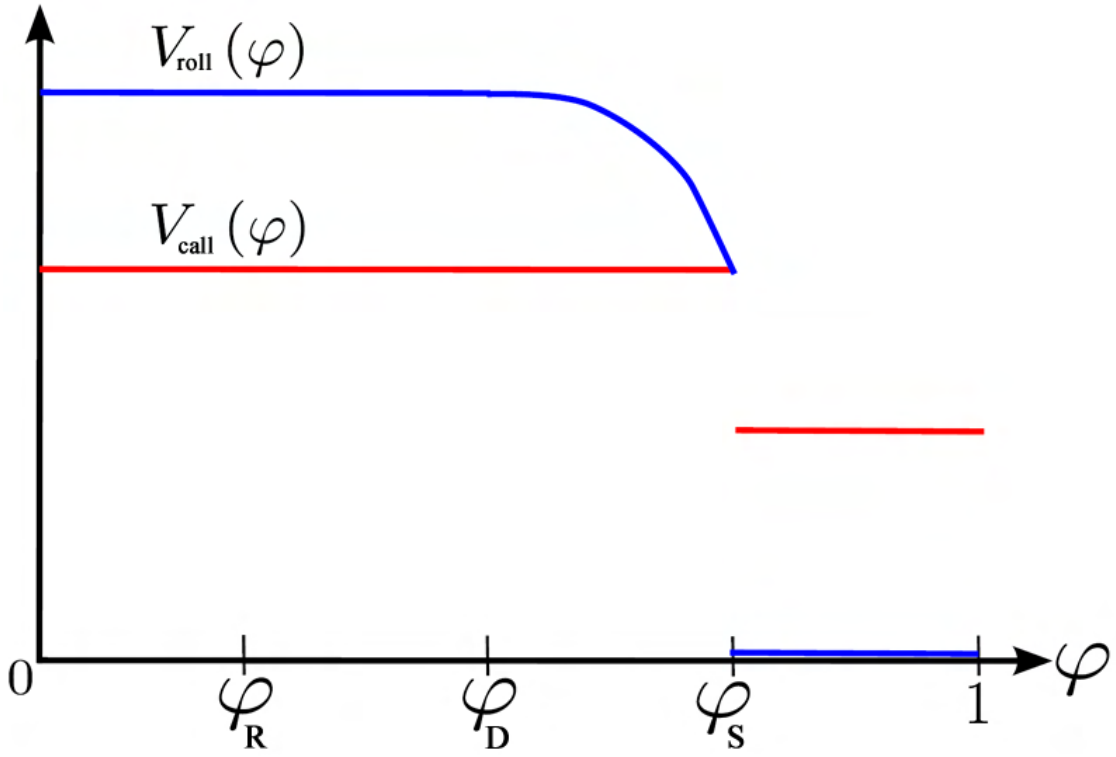


Table 1.

	Foreign Reserves/GDP			Foreign Reserves/External Liabilities		
	1990-1996	2002-2007	% Change	1990-1996	2002-2007	% Change
<i>Emerging</i>						
Argentina	4.9%	12.5%	154%	14.3%	21.4%	50%
Brazil	4.8%	8.3%	73%	19.5%	35.3%	81%
Chile	20.1%	15.7%	-22%	50.9%	39.0%	-23%
China	8.1%	32.9%	306%	54.3%	271.3%	400%
Colombia	9.7%	10.5%	8%	35.6%	35.1%	-1%
Czech Republic	18.3%	25.2%	38%	57.2%	77.5%	36%
Egypt	18.8%	19.3%	3%	35.2%	65.5%	86%
Hungary	15.8%	16.7%	5%	26.6%	26.2%	-2%
India	3.4%	18.2%	429%	11.8%	101.0%	754%
Indonesia	6.7%	12.4%	86%	11.7%	25.2%	116%
Korea	5.5%	24.6%	345%	28.0%	97.4%	248%
Malaysia	28.7%	47.0%	64%	77.0%	125.6%	63%
Mexico	4.6%	8.1%	78%	12.2%	40.9%	235%
Morocco	10.4%	28.7%	175%	15.9%	99.0%	522%
Pakistan	1.8%	10.2%	472%	4.3%	28.2%	552%
Peru	11.1%	18.2%	64%	16.7%	49.1%	195%
Philippines	8.1%	17.2%	113%	13.3%	28.4%	113%
Poland	7.0%	14.0%	101%	15.9%	35.8%	125%
Romania	4.0%	18.1%	352%	22.7%	53.9%	138%
Russia	3.2%	22.9%	615%	7.3%	68.5%	836%
Saudi Arabia	7.3%	9.4%	28%	56.2%	87.3%	55%
South Africa	1.0%	7.0%	591%	4.7%	34.9%	646%
Thailand	19.8%	30.6%	54%	41.8%	112.1%	168%
Turkey	3.9%	10.6%	171%	13.0%	25.5%	96%
<i>Developed</i>						
France	2.1%	1.7%	-19%	3.8%	1.3%	-66%
Germany	3.9%	1.8%	-54%	8.7%	1.5%	-83%
United Kingdom	3.6%	1.8%	-49%	2.4%	0.6%	-73%
United States	1.0%	0.5%	-46%	3.2%	0.8%	-74%

Table 2. Parameter Values

Name	Symbol	Value
Discount factor	β	0.98
World interest rate	r	0.01
Low liquidity shock parameter	σ_L	0.1
High liquidity shock parameter	σ_H	0.32
Divestment parameter	λ	0.7
Production exponent	θ	0.85
Production parameter	A	1.5

Table 3. Summary of Numerical Results

	1990-1996	1997-2001	2002-2007
<i>Data</i>			
Reserves-to Debt Ratio	27%	36%	65%
Sudden Stops	2	13	0
<i>Model</i>			
Reserves-to Debt Ratio	27%	51%	65%
Sudden Stops	0.03	5.84	0.94

8 Appendix 2: Computational methods

This appendix describes the numerical methods used to compute the optimal contract. In fact, the government's problem cannot be neatly solved analytically because of the break-even constraint and the discontinuities in the rollover policy functions (see Figures 5-7).

Let $C = \{\hat{r}, D, R, K, L(\varphi), P_1(\varphi), P_2(\varphi), \psi_i(\varphi), \mathbf{R}'_0(\varphi)\}$. For each belief ρ about the prevailing liquidity shock regime, the optimal contract $\{\mathbb{C}^* | \rho\}$ solves the following functional equation:

$$\begin{aligned}
 W(R_0; \rho) &= \max_{C, R'_0} \mathbb{E} \left[U \left(\underbrace{A(K-L)^\theta + R_2(\varphi)}_{\text{output} + \text{remaining reserves}} - \underbrace{P_2(\varphi)}_{\text{final payments}} - \underbrace{\mathbf{R}'_0(\varphi)}_{\text{reserves for next period}} \right) + \beta W(\mathbf{R}'_0(\varphi); \rho) \right] \\
 \text{s.t.} \quad & (1), (3), (5), (6), (7), (8), (9), \mathbf{(10)} \\
 & \text{non-negativity constraints}
 \end{aligned}$$

Below, we show the key steps of the algorithm used to find the optimal contracts.

Step 0: State space We choose appropriate grids for $\{R_0, K, R\}$ as well as a grid for the shocks φ . In particular, the grids are chosen to have more points at lower values as the curvature of the objective function is higher. Note that using Lemma 1, the state-contingent policy functions $\{L(\varphi), P_1(\varphi), P_2(\varphi), \psi_i(\varphi)\}$ can be formulated analytically given $\{\hat{r}, R_0, K, R\}$. The discrete grid for the shocks is only used for numerical integration. Finally, we also have a grid for beliefs ρ since we need to find an optimal contract for each belief.

Step 1: Feasible set We first find all the incentive feasible contracts. We just need to find all the feasible $\{\hat{r}, R_0, K, R\}$ points since the shock-contingent policies can be expressed analytically. Given $\{R_0, K, R\}$, $\hat{r} \geq r$ is found under the property that the participation constraint holds with equality. $\hat{r}$ is therefore the value that solves the corresponding equation. The expected value of the investor is computed using the trapezoidal method with unit spacing. When that solution is less than r , the corresponding feasible $\hat{r}$ is set to r .

Step 2: Policy function iteration Once the incentive feasible contracts are found, the Bellman equations are solved via policy function iteration. We again use the trapezoidal method for the integration.

We repeat these steps for all the points on the grid of beliefs. Using Bayes' rule with the computed optimal contracts, we then simulate a path of shocks and the induced paths for reserves and sudden stops. During our simulations, the policy functions corresponding to off-grid belief values were approximated using the closest belief grid point.

Our program has been parallelized to significantly reduce computation time. We have also performed additional checks to ensure the accuracy of the solution we find: finer grid, differently spaced grid points, different convergence bounds. Our source code is available upon request.

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