# Unemployment and Endogenous Reallocation over the Business Cycle*

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**Abstract**

This paper presents a tractable stochastic general equilibrium model in which to study different types of unemployment over the business cycle. In our model workers face search frictions on their local labor markets and reallocation frictions across labor markets. The interaction of these two types of frictions is shown to have important implications for the analysis of resource allocation over the business cycle. We focus attention on (i) the impact on the aggregate unemployment rate, and its decomposition into search, rest, reallocation and job separation components; (ii) the expected duration of unemployment; and (iii) how the interaction between search and reallocation frictions affects the cyclicality of job separations and worker reallocations over the business cycle. We use this framework to quantitatively assess the relative contributions of the different sources of unemployment in explaining the unemployment rate and its cyclical properties.

*Keywords*: Unemployment, Business Cycle, Search, Reallocation.

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# 1 Introduction

## 1.1 Motivation and Summary

Understanding what are the main underlying forces that allow unemployed workers and unfilled jobs to coexist has been a topic of longstanding importance in economics. Over the years several equilibrium theories have provided useful insights. For example, the search and matching framework, as described in Pissarides (2001), considers unemployment that arises from informational frictions about job opportunities. Jovanovic (1987), Hamilton (1988), Gouge and King (1997) and more recently Alvarez and Shimer (2011), present a theory in which unemployment arises as workers might decide not to search for jobs on offer, but "rest" until the state of their labor market improves. Shimer (2007) and Ebrahimy and Shimer (2010) present theories of misallocation of workers across labor markets to study mismatch unemployment. A common feature of this literature has been to use a particular source of unemployment to explain qualitatively or quantitatively the cyclical pattern of the observed unemployment rate. Surprisingly, there is considerably less emphasis in studying what are the relative contributions of the different sources of unemployment in explaining the observed unemployment rate and its cyclical properties. Indeed, without such analysis it seems difficult to have a consistent statistical description of the forces that determine the unemployment rate and drive resource reallocation over the business cycle. This paper takes up this issue and puts forward a tractable general equilibrium model of the business cycle in which to jointly study different types of unemployment. We examine in detail the interaction between the different frictions that generate different types of unemployment and explore the efficiency properties of the predicted patterns.

Our model combines the ideas originally set out by Lucas and Prescott (1974) and Pissarides (2001). We consider an island economy in which each island faces an idiosyncratic productivity shock each period. Here an island can be interpreted as the labor market attached to an occupation, sector/industry and/or a geographical location. In each island unemployed workers can decide to (i) apply and search for job opportunities, (ii) not to apply and remain "rest" unemployed or (iii) to reallocate to another island. Employed workers, on the other hand, can decide to separate from their employers and become unemployed. Two forces play an important role in our analysis: search and reallocation frictions. Search frictions operate within each island where the meeting process of workers and firms is governed by a matching function. Reallocation frictions, on the other hand, operate across islands. We consider a reallocation process that takes time and during which the worker remains unemployed. Further, as in Alvarez and Veracierto (1999), a worker's new island is a random draw from the set of islands across the economy and the reallocation decision may or may not turn out to be profitable ex-post (see also Jovanovic, 1987, Shimer, 2007 and Mortensen, 2009).

Using this framework we consider business cycle analysis by introducing aggregate productivity shocks. In particular, we focus attention on block recursive equilibria (see also Menzio and Shi, 2010). That is, equilibria in which the relevant state in each island is characterized only by the aggregate and idiosyncratic productivity shocks.[1] We show existence and uniqueness of such an equilibria. We also show that our equilibrium is efficient when the social planner faces the same search and reallocation frictions as agents in the decentralized economy.[2]

The model allows for a parsimonious analysis of its implications in terms of two functions of aggregate productivity. The reservation island productivity below which workers decide to reallocate, $z^r$, and the reservation island productivity below which workers decide to separate from their jobs, $z^s$. When $z^s > z^r$, rest unemployment occurs in islands with idiosyncratic productivities between $z^r$ and $z^s$. In these islands the state of the labor market is sufficiently depressed for workers not to search but is not bad enough to decide to reallocate. This case occurs, for example, when the explicit cost of reallocation and/or the unemployment benefit are sufficiently high. When $z^s < z^r$, however, rest unemployment does not occur. Unemployment in this case arises only due to search and reallocation frictions and job separation decisions.

We show that search and reallocation frictions interact in a significant way. Namely, search frictions within each island alter the reallocation decision of workers across islands in response to aggregate productivity shocks. When each island is characterized by a competitive labor market an unexpected increase in aggregate productivity tends to reduces worker reallocation across islands. In contrast, when individual labor markets exhibit search frictions, an increase in aggregate pro-

[1] Our model becomes very tractable as we do not have to keep track of the distribution of employed and unemployed workers to determine wages and employment probabilities. Instead, we can first solve for decisions, and then use these decision rules to update the distribution of employed and unemployed workers, block recursiveness.

[2] The efficiency of our equilibrium relies crucially on the arguments put foward by Hosios (1991) and Moen (1997). We chose to follow Moen (1997) competitive search approach. As agents are homogeous within an island the analysis is equivalent to Moen's model with only one sub-market and coordination frictions only operate out of the equilibrium path.

ductivity tends to increase reallocation. Although these results depend on the size of the explicit cost of reallocation and the characteristics of the production function, allowing for search frictions within each island always induces more reallocation of workers than under perfect competition. This prediction is consisted with the empirical evidence documenting the procyclicality of worker flows across occupations (Kambourov and Manovskii, 2009, Moscarini and Thomsson, 2007) and regions (Lkhagvasuren, 2009, and Molloy and Wozniak, 2009) in the US.

The interaction between reallocation and separation decisions, on the other hand, has implications about the cyclical properties of job separations. Given worker reallocation across islands is procyclical, $z^s < z^r$ implies that a positive aggregate productivity shock can induce more separations if the production function exhibits a sufficiently high degree of supermodularity. In this case, workers in islands with idiosyncratic productivity $z^s$ find that the value of becoming unemployed and reallocating can increase more than the worker-firm joint value of continuing production. When $z^s > z^r$, however, job separations are always countercyclical as the benefits of reallocation are no longer present in this island. This interaction relates to the discussion about the "cleansing" and "sullying" effects of recessions (see Mortensen and Pissarides, 1994, Caballero and Hammour, 1994, and Barlevy, 2002). Although in this model we do not consider on-the-job search, procyclicality of workers flows across islands helps resource reallocation in expansions, very much in the same way as job to job transition do in Barlevy (2002).

The island structure of our model enable us to generate aggregate unemployment duration dependence. We show that unemployed workers in islands with a higher idiosyncratic productivity exit unemployment faster. However, changes in aggregate productivity affect the job finding rates across islands differently and hence generate disproportional shifts the aggregate hazard function. In particular, when the production function exhibits a sufficiently low degree of log-supermodularity, the job finding rates in low productivity islands increase proportionally more than in high productivity islands.

Using the solutions for $z^r$ and $z^s$ we provide a decomposition of the evolution of the aggregate unemployment rate over the business cycle. This decomposition distinguishes between search, rest, reallocations and separations motives. We then obtain a measure of workers' mismatch across islands by developing and mismatch index in the spirit of Jackman, Layard and Savouri (1991). We evaluate these decompositions by calibrating our model to match long run features of the US labor market.

The paper is organized as follows. After a brief review of related literature, we set out the model in Section 2. We present the decision problems of workers and firms considering the complete state space. In Section 3, we define and analyse block recursive equilibria. Here we show existence, uniqueness and the efficiency properties of such an equilibrium. Section 4 discusses the implications of the model with respect to the cyclical properties of reallocation and separation flows, the conditions for rest unemployment to arise in equilibrium and duration dependence in unemployment spells. We also analyse a simple extension of our model that considers island-specific human capital. For example, workers might gain occupational, industry or location specific human capital with time spent in an island. Section 5 considers a calibration exercise to further explore the properties of the model. Section 6 concludes. All proof and tedious derivations are relegated to the Appendix.

### 1.2 Related Literature

This paper contributes to the literature that uses the Lucas and Prescott (1974) model to analyse aggregate unemployment. In particular, our paper is closest to Veracierto (2008) which considers a business cycle version of Lucas and Prescott (1974) with random reallocation across islands. A crucial feature of this framework is to assume that the labor market within an island is competitive and reallocation frictions are the only source of market imperfection. Further, as the worker becomes unemployment during the reallocation process, this friction is the only one driving force behind aggregate unemployment. Lucas and Prescott (1974), and others using their framework, refer to the latter as search unemployment. Here, however, we make the distinction between unemployment due to frictions within and across islands. Furthermore, Veracierto (2008) shows that, under reasonable parameter values, a real business cycle model that only considers reallocation frictions generates procyclical unemployment, a counterfactual implication. By introducing search frictions within islands, we show that under reasonable parameter values reallocations across islands are procyclical *and* unemployment is countercyclical.

In a closely related paper, Gouge and King (1997) make a similar point about the inability of the Lucas and Prescott framework to generate countercyclical unemployment (see also Jovanovic, 1987). They consider the Lucas and Prescott model with a two state aggregate and idiosyncratic productivity shock process and introduce rest unemployment within islands. They show that their model can generate procyclical reallocations, while also countercyclical unemployment flows. There are some important difference between our papers. Although Gouge and King only hint about what would happen if each island's labor markets exhibited search frictions, they do not provide a full analysis of its implications as we do in this paper. Further, to preserve tractability, these authors only consider very simple productivity shock processes. We are able to show existence and uniqueness of equilibrium and prove its efficiency by requiring both productivity process to be Markovian and the island productivity shock process to show some persistence in the form of stochastic dominance. Finally, we provide a quantitative evaluation of the model, while Gouge and King only consider the qualititative properties.

Most of the papers that consider rest unemployment also consider the Lucas and Prescott model to allow workers to reallocate across islands and hence consider two types of unemployment simultaneously. We generalise this analysis and consider frictional unemployment within islands. This implies that those unemployed workers "resting" might still not get a job in their own island when conditions improve and become part of those "frictionally" unemployed. Further, we consider job separation decisions as a source of unemployment. As argued by Fujita and Ramey (2009) and Elsby, Michaels and Solon (2009), job separations are strongly countercyclical and account for a significant fraction of the variation of aggregate unemployment over the business cycle. We provide a dynamic decomposition of the unemployment rate into its search, rest, reallocation and separation components. We also obtain a measure of mismatch in our economy.

Finally, Lkhagvasuren (2009) also considers the interaction between reallocation and search frictions in a similar setup. His analysis is focused on explaining the coexistence of large difference in the unemployment rates across US states (the operationalization of 'islands' in his model) and large reallocation flows between them. His model is a steady state model, the specifics of his setup do not allow the type of easily computable equilibrium that we show to exist in this paper, and hence computational concerns do not allow for an investigation of the behavior of local labor market over the business cycle.

## 2 Model

Time is discrete, and goes on forever; it is denoted by $t$. There is a continuum of infinitely lived risk-neutral workers of measure one, located over a continuum of islands, each island indexed by $i$, such that (almost) all islands are home to a continuum of workers of various measure. Workers can be either employed or unemployed in an island. An unemployed worker receives $b$ each period. The wages of employed workers are determined below.

There is also a continuum of risk-neutral firms that live forever. Each firm has one position, and can decide to enter the labor market in an island of choice. The firm needs a worker to produce a good, with a production function $y(p_t, z_{it})$ that is continuous differentiable, strictly increasing in all arguments, where $p_t$ is the aggregate productivity shock (which impacts all islands in the economy) and $z_{it}$ is the island specific productivity component at a given time $t$. We assume that the cross-derivatives of productivities are (weakly) positive. Both types of productivities are drawn from bounded intervals and follow stationary Markov-processes. The initial realizations and any future innovations of $z$'s are iid across islands. All agents discount the future using the same discount factor $\beta$.

A firm can find a worker by posting a vacancy in a particular island, paying a cost $k$. There is no on-the-job search, therefore only unemployed workers can decide to search for vacant jobs. A posted job specifies a wage contract contingent on the sequence of realizations of $p_t$, $z_{it}$ and the duration of the relationship. Let $w_{ift}$ denote the wage paid at firm $f$ in island $i$ at time $t$. We further specify the matching process within each island in the next section.

Once a matched is formed, firms pay workers according to the posted contract, until the match is broken up. The latter can happen with an exogenous (and constant) probability $\delta$, but in addition also occurs if the worker and the firm decide to do so. Once the match is broken, the worker becomes unemployed in the current island and the firm has to decide to reopen the vacancy or not. We assume that if a worker separates from his current employer (voluntarily or not), he stays unemployed until the end of the period. This assumption allows us to have a clear relationship between the unemployment to employment flows (and vice versa) observed in the data and the ones generated by the model.

An important component of workers' choice is the decision to stay in the current labor market or reallocate to a different island. Reallocation, however, involves paying a moving cost $c$. We also assume that informational frictions prevent the worker from going to the island with the best possible conditions. Once the worker decides to move, the conditions in the next island he visits are a random draw from the distribution of labor market conditions that occur in the remaining islands. Further, a worker who decides to reallocate cannot immediately apply for a job and must sit out unemployed in the new island for the rest of the period. This assumption allows us to consider the explicit and implicit costs of reallocation and compare our results with that of Alvarez and Veracierto (1999) and Veracierto (2008).

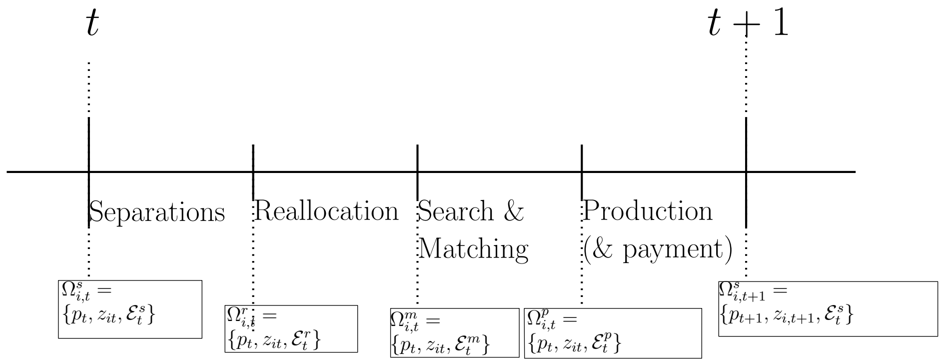


Figure 1: Timing

Given the above considerations, Figure 1 summarizes the timing of the events within a period conditional on the state in island $i$ at time $t$. A period is subdivided into four stages: separation, reallocation, matching and production. Let $u^x_{it}$ and $e^x_{it}$ denote the measure of unemployed and employed workers at the *beginning* of stage $x$ in island $i$ and period $t$. Also let $\mathcal{E}^x_t$ denote the distribution of unemployed and employed workers over the different islands at the beginning of stage $x$ in period $t$. The state of island $i$ at the beginning of the separation stage is described by the vector $\Omega^s_{it} = \{p_t, z_{it}, \mathcal{E}^s_t\}$. After within-island separations of worker-firm matches, the state vector is updated to $\Omega^r_{it} = \{p_t, z_{it}, \mathcal{E}^r_t\}$, describing the state of island $i$ at the beginning of the reallocation stage. During the reallocation stage, unemployed workers decide whether to continue on their own islands or to move to another island. After reallocations take place the state vector is once again updated such that at the beginning of the matching stage the state of the island is given by $\Omega^m_{it} = \{p_t, z_{it}, \mathcal{E}^m_t\}$. During the matching stage firms decide to enter the island $i$ and post $v_{it}$ vacancies. After matching the resulting state vector is given by $\Omega^p_{it} = \{p_t, z_{it}, \mathcal{E}^p_t\}$, which describes the state of the island at the beginning of the production stage. Here unemployed workers receive $b$, employed workers receive their wages and any unfilled vacancies are destroyed. The number of unemployed and employed workers at the beginning of the production stage then gives the number of unemployed and employed workers at the beginning of the next period. Hence, $u^p_{it} = u^s_{it+1}$ and $e^p_{it} = e^s_{it+1}$.

## 2.1 Posting and Matching

In each island firms post contracts to which they are committed. Unemployed workers and advertising firms then match with frictions as in Moen (1997). In particular, in each island there is a continuum of (potentially inactive) sub-markets, one for each expected lifetime value $\tilde{W}$ that could potentially be offered by a vacant firm. After firms have posted a contract in the sub-market of their choice, workers $u$ can choose which sub-market to visit. Once in their preferred sub-market, workers and firms meet according to a constant returns to scale matching function $m(u, v)$, where $u$ is the measure of workers searching in the sub-market, and $v$ the measure of firms which have posted a contract in this sub-market.[3] From this matching function one can easily derived the workers' job finding rate

$$\lambda(\theta) = m(1, v/u), \text{ with } \theta = v/u,$$

and the vacancy filling rate

$$q(\theta) = m(u/v, 1)$$

in the sub-market. The matching function and the job finding and vacancy filling rates are assumed to have the following properties: (i) they are twice-differentiable functions, (ii) nonnegative on the relevant domain, (iii) $m(0, 0) = 0$, (iv) $q(\theta)$ is strictly decreasing, and (v) $\lambda(\theta)$ is strictly increasing and concave.

We impose two restrictions on beliefs off-the-equilibrium path. Workers believe that, if they go to a sub-market that is inactive on the equilibrium path, firms will show up in such measure to have zero profit in expectation. Firms believe that, if they post in an inactive sub-market, a measure of workers will show up, to make *the measure of deviating firms* indifferent between entering or not. We assume, for convenience, that the zero-profit condition also holds for deviations of a single agent: loosely, the number of vacancies or unemployed, and therefore the tightness will be believed to adjust to make the zero-profit equation hold.

Although our model is set up in the language of a competitive search model a la Moen (1997), we will see that in equilibrium only one sub-market opens in each island and the preceeding matching process is equivalent to consider the Pissarides (2001) setup with random search and Nash bargaining under the Hosios (1991) condition on each island to guarantee efficiency. We chose to follow Moen (1997) for two reasons. The first is that it guarantees efficiency in the posting and matching stage and helps us to guarantee efficiency in our model. The second is that it is explicit about workers' strategies to visit different sub-markets. This is useful as it allows us to jointly consider search and rest unemployment in a simple way. The decision to rest arises when a worker decides not to visit

[3] Alternatively, we can consider the matching function associated with coordination frictions when workers use symmetric mixed application strategies, as in Burdett, Shi and Wright (2001).

any sub-market.

### 2.2 Worker's problem

Conditional on the state of the island at the beginning of the production stage as summarized by $\Omega_{it}^{p}$, consider the value function of an unemployed worker

$$W^U(\Omega_{it}^p) = b + \beta\mathbb{E}[W^R(\Omega_{it+1}^r)]. \tag{1}$$

The value of unemployment consists of the flow benefit of unemployment $b$ this period, plus the discounted expected value of being unemployed at the beginning of next period's reallocation stage,

$$W^R(\Omega_{it+1}^r) = \max_{\rho(\Omega_{it+1}^r)}\{\rho(\Omega_{it+1}^r)R(\Omega_{jt+1}^r) + (1-\rho(\Omega_{it+1}^r))\mathbb{E}[S(\Omega_{it+1}^m) + W^U(\Omega_{it+1}^m)]\},$$

where $\rho(\Omega_{it+1}^r)$ takes the value of one when the worker decides to reallocate and zero otherwise.

Equation (1) includes continuation values for each possible realization in the matching and reallocation stages. In particular, $R(.)$ denotes the expected benefit of reallocation. Given that workers who reallocate have to sit out one period of unemployment in the new island, this benefit is given by

$$R(\Omega_{jt+1}^r) = -c + \mathbb{E}_{\Omega_{jt+1}^p}[W^U(\Omega_{jt+1}^p)].$$

The expected value of staying and searching on the island is given by $\mathbb{E}[S(\Omega_{it+1}^m) + W^U(\Omega_{it+1}^m)]$. In this case, $W^U(\Omega_{it+1}^m) = \mathbb{E}[W^U(\Omega_{it+1}^p)]$ describes the expected value of not finding a job on the same island, while $S(\Omega_{it+1}^m)$ summarizes the expected value *added* of finding a new job on the island. The reallocation decision is captured by the choice between $R(\Omega_{jt+1}^r)$ and the expected payoff of search on the current island.[4]

To derive $S(.)$ recall that $\lambda(\theta(\Omega_{it}^m, W_f))$ denotes the probability with which the worker meets a firm $f$ in the sub-market associated with the promised value $W_f$ and tightness $\theta(\Omega_{it}^m)$. Further, let $\alpha(W_f)$ denote the probability of visiting such a sub-market. From the set $\mathcal{W}$ of promised values which are offered in equilibrium by firms in this island, the worker only visits with positive probability those sub-markets for which the associated $W_f$ satisfies

$$W_f \in \arg\max_{\mathcal{W}} \lambda(\theta(\Omega_{it+1}^m, W_f))(W_f - W^U(\Omega_{it+1}^m)) \equiv S(\Omega_{it+1}^m).$$

[4]Notice that, after paying the reallocation cost $c$, the worker randomly draws a new island with state vector $\Omega_{jt+1}^p$ and, from the next period onwards, any subsequent decisions in the chosen island are the same as the ones described above.

Hence equation (1) incorporates the worker's optimal application decisions to those active sub-markets in island $j$ after he has reallocated, or in island $i$ if he did not move. Further, when the set $\mathcal{W}$ is empty, the expected value added of finding a job in the island is zero, $S(.) = 0$ for all sub-markets in the island, and the worker chooses $\alpha(W_f) = 0$ for all $W_f$. In this case the worker does not visit any sub-market and becomes rest unemployed.

Now consider the value function at the beginning of production stage of an employed worker in a contract that currently has a value $\tilde{W}_f(\Omega^p_{it})$. Similar arguments as before imply that

$$\tilde{W}_f(\Omega^p_{it}) = w_{ift} + \beta \mathbb{E}\Big[\max_{d(\Omega^s_{it+1})} \{(1 - d(\Omega^s_{it+1}))\tilde{W}_f(\Omega^s_{it+1}) + d(\Omega^s_{it+1}) W^U(\Omega^s_{it+1})\}\Big], \tag{2}$$

where $d(\Omega^s_{it+1})$ take the value of $\delta$ when $\tilde{W}_f(\Omega^s_{it+1}) \geq W^U(\Omega^s_{it+1})$ and the worker decides not to quit into unemployment and the value of one otherwise. In equation (2), the wage payment $w_{ift}$ at firm $f$ is contingent on state $\Omega^p_{it}$, while the second term describes the worker's option to quit in the separation stage the next period. Note that $W^U(\Omega^s_{it+1}) = \mathbb{E}[W^U(\Omega^p_{it+1})]$ as a worker who separates must stay unemployed in his current island for the rest of the period and $\tilde{W}_f(\Omega^s_{it+1}) = \mathbb{E}[\tilde{W}_f(\Omega^p_{it+1})]$ as the match will be preserved after the separation stage.

### 2.3 Firm's problem

Given state vector $\Omega^p_{it}$, consider a firm $f$ in island $i$, currently employing a worker who has been promised a value $\tilde{W}_f(\Omega^p_{it}) \geq W^U(\Omega^p_{it})$. The expected lifetime discounted profit of this firm can be described recursively as

$$\begin{aligned} J(\Omega^p_{it}; \tilde{W}_f(\Omega^p_{it})) \quad = \quad & \max \Big\{ y(p_t, z_{it}) - w_{ift} + \beta \mathbb{E}\Big[\max_{\sigma(\Omega^s_{it+1})} \Big\{(1 - \sigma(\Omega^s_{it+1}))J(\Omega^s_{it+1}; \tilde{W}_f(\Omega^s_{it+1})) \\ & + \sigma(\Omega^s_{it+1})\tilde{V}(\Omega^s_{jt+1})\Big\}\Big]\Big\}, \end{aligned} \tag{3}$$

where $\sigma(\Omega^s_{it+1})$ takes the value of $\delta$ when $J(\Omega^s_{it+1}; \tilde{W}_f(\Omega^s_{it+1})) \geq \tilde{V}(\Omega^s_{jt+1})$ and the value of one otherwise and $\tilde{V}(\Omega^s_{jt+1}) = \max\{V(\Omega^s_{jt+1}), 0\}$. Here the first maximization is over the wage payment $w_{ift}$ and the promised lifetime utility to the worker $\tilde{W}_f(\Omega^p_{it+1})$. The second maximization refers to the firm's layoff decision.

Equation (3) is subject to the restriction that the wage paid today and tomorrow's promised values have to add up to today's promised value $\tilde{W}_f(\Omega^p_{it})$, according to equation (2). Moreover, the workers' option to quit into unemployment, and the firm's option to lay off the worker imply the following participation constraint

$$(J(\Omega^s_{it+1}; \tilde{W}_f(\Omega^s_{it+1})) - \tilde{V}(\Omega^s_{jt+1})) \cdot \left(\tilde{W}_f(\Omega^s_{it+1}) - W^U(\Omega^s_{it+1})\right) \geq 0, \tag{4}$$

with complementary slackness. Thus firms can only get nonnegative profits in the future if the worker is promised to be at least as well off as in unemployment; likewise, this condition rules out promised values that lead to strictly negative profits, because the firm will opt to render the job vacant. When a firm opts for the latter, or in case of an exogenous break-up, the firm could decide to take its vacancy to a different market $j$ instead. Thus, $V(\Omega^s_{jt})$ refers to the value of an unfilled vacancy in market $j$ at time $t$ with island-specific state vector $\Omega^s_{jt}$. Of course, the firm can always withdraw the vacancy from the economy, and obtain zero profits.

Note that the solution to (3) gives the wage payments during the match (for each realization of $\Omega^p_{it}$ for all $t$). In turn these wages pin down the expected lifetime profits at any moment during the relation, and importantly also at the start of the relationship, where the promised value to the worker is $\tilde{W}_f$.

Now consider a firm posting a vacancy. Given cost $k$, a firm can choose an island where to locate its vacancy, knowing both the aggregate and the island-specific productivities and the number of unemployed workers at the beginning of the matching stage on each island. Further, for each island and with current state vector $\Omega^m_{it}$, the firm has to decide which $\tilde{W}_f$ to post given $q(\theta(\Omega^m_{it}, \tilde{W}_f))$, the associated job filling probability. Note that this probability summarizes the pricing behavior of other firms and the visiting strategies of workers. Along the same line as above, the expected value of a vacancy in island $i$ solves the Bellman equation

$$V(\Omega^m_{it}) = -k + \max_{\tilde{W}_f} \left\{ q(\theta(\Omega^m_{it}, \tilde{W}_f)) J(\Omega^m_{it}, \tilde{W}_f) + (1 - q(\theta(\Omega^m_{it}, \tilde{W}_f))) \mathbb{E}_{\Omega^p_{jt}}[V(\Omega^p_{jt})] \right\}.$$

We assume that in each island there is free entry of firms posting vacancies, which implies that $V(\Omega^x_{it}) = 0, \ \forall \ \Omega^x_{it}, i, t$ at any stage $x$. The free entry condition then simplifies the vacancy creation condition to

$$k = \max_{\tilde{W}_f} q(\theta(\Omega^m_{it}, \tilde{W}_f)) J(\Omega^m_{it}, \tilde{W}_f).$$

## 2.4 Worker flows

Until now, we have taken as given the state vectors $\Omega^s_{it}, \Omega^r_{it}, \Omega^m_{it}, \Omega^p_{it}$ and their evolution to discuss agents' optimal decisions. As mentioned earlier $p_t, z_{it}$ follow exogenous processes. However, the evolution of the number of unemployed and employed workers is a result of optimal vacancy posting, visiting strategies, separation and reallocation decisions. We now turn to analyze how these measures evolve. Changes over time in the unemployment and employment rates in an island $i$ are described by the sum of four types of flows. The within-market flows of unemployment to employment and vice versa. The between-market flows of unemployed and the direct flow of employed workers who separate from their current employment to look for jobs as unemployed workers in other islands (after paying cost $c$).

Consider an island $i$ at the beginning of period $t$ with state vector $\Omega^s_{it}$. Assume that on such an island all firms during the matching stage offer the same $\tilde{W}^*_f(\Omega^m_{it})$. As shown below, this will be indeed the case in equilibrium. Given $u^s_{it}$ and $e^s_{it}$, the number of unemployed workers in this island at the beginning of the reallocation state is given by

$$u^r_{it} = \Big(\delta + (1-\delta)\mathbb{I}[\tilde{W}^*_f(\Omega^s_{it}) < W^U(\Omega^s_{it})]\Big)e^s_{it} + u^s_{it}.$$

where $\mathbb{I}$ denotes a standard indicator function which takes the value of one when the inequality in square brackets is true and zero otherwise. The first term takes into account that a measure $\delta e^s_{it}$ of employed workers gets displaced, while the rest of employed workers quit to unemployment if is optimal to do so. The number of unemployed $u^r_{it}$ is given by summing this flow to the number of unemployed at the beginning of the period. The number of employed at the beginning of the reallocation stage, on the other hand, is simply given by $e^r_{it} = e^s_{it} - (u^r_{it} - u^s_{it})$.

Now consider the number of unemployed and employed workers at the beginning of the matching stage. To derive these numbers we have to consider the flows between islands. It is important to remember that only those unemployed workers at the beginning of the period in each island, $u^s_{it}$, are allowed to reallocate. The flow from any island $i$ to another island $j$ is then given by

$$outflow(i,j) = u^s_{it}\mathbb{I}[R(\Omega^r_{jt}) > \mathbb{E}[S(\Omega^m_{it}) + W^U(\Omega^m_{it})]]dF_j.$$

This expression captures the transitions of the unemployed from island $i$ to island $j$, where $dF_j$ is the probability of drawing island $j$ after deciding to reallocate. Since islands' identities are on the unit interval and are drawn randomly using a uniform distribution, $dF_j = 1$. The inflow into island $i$ is given by

$$inflow(i) = \int_0^1 outflow(j,i)dj.$$

Hence the number of unemployed workers at the beginning of the matching stage is

$$u^m_{it} = u^r_{it} + inflow(i) - outflow(i,j),$$

from which only $u^s_{it} - outflow(i,j)$ are allowed to search for jobs, since workers that reallocated to this island at time $t$ have to wait until the following period to search for jobs. Note that the number of employed workers at the beginning of the matching period is the same as the number of employed workers at the beginning of the reallocation period; that is, $e^m_{it} = e^r_{it}$.

Finally, at the beginning of the production stage the number of unemployed workers is given by

$$u^p_{it} = u^m_{it} - \lambda(\theta(\Omega^m_{it}, \tilde{W}^*_f))[u^s_{it} - outflow(i,j)].$$

The case in which no worker decided to visit the sub-market is capture by the possibility that $\theta(\Omega^m_{it}, \tilde{W}^*_f) = \lambda(\theta(\Omega^m_{it}, \tilde{W}^*_f)) = 0$.[5] Hence when there is rest unemployment in the island we have that $u^p_{it} = u^m_{it}$. The number of employed workers is given by

$$e^p_{it} = e^m_{it} + \lambda(\theta(\Omega^m_{it}, \tilde{W}^*_f))[u^s_{it} - outflow(i,j)]$$

and $e^p_{it} = e^m_{it}$ in the case of rest unemployment.

# 3 Equilibrium

We look for an equilibrium in which the value functions and decisions of workers and firms only depend on the productivity in the aggregate and on the island. Moreover, we are also looking for equilibria where the values offered to all employed workers at a given moment on a given island are equal. Under these considerations the following describe the candidate equilibrium value functions

$$\begin{aligned} W^U(p,z) = b + \beta \mathbb{E}_{p',z'} \Bigg[ \max_{\rho(p',z')} \Bigg\{ \rho(p',z') \left[ -c + \int W^U(p', z'_i) dF(i) \right] + \quad (5) \\ (1-\rho(p',z')) \left[ \max_{W^{E\prime}} \left\{ \lambda(\theta(p',z',W^{E\prime}))W^{E\prime} + (1-\lambda(\theta(p,z,W^{E\prime})))W^U(p',z') \right\} \right] \Bigg\} \Bigg] \end{aligned}$$

$$W^E(p,z) = w(p,z) + \beta \mathbb{E}_{p',z'} \max_{d(p',z')} \left\{ (1-d(p',z'))W^E(p',z') + d(p',z')W^U(p',z') \right\} \quad (6)$$

$$J(p,z,\tilde{W}^E) = \max_{\{w, \tilde{W}^{E\prime}(p',z')\}} \left\{ y(p,z) - w + \beta \mathbb{E}_{p',z'} \max_{\sigma(p',z')} \{(1-\sigma(p',z'))J(p',z',\tilde{W}^{E\prime}(p',z'))\} \right\} \quad (7)$$

$$V(p,z,\tilde{W}) = -k + q(\theta(p,z,\tilde{W}))J(p,z,\tilde{W}) = 0, \quad (8)$$

where $\tilde{W}^E$, $w$ and $\tilde{W}^{E\prime}$ must satisfy (6) and the maximization in (7) is subject to the participation constraint (4).

[5] As shown later, rest unemployment occurs for sufficiently low values of $z$. In this case, new firms will not enter these islands and hence setting $\theta(\Omega^m_{it}, \widetilde{W}^*_f) = \lambda(\theta(\Omega^m_{it}, \widetilde{W}^*_f)) = 0$.

The main simplification we achieve by focusing attention in this type of equilibria is that we do not need to keep track of the measures of unemployed and employed workers on each island or their flows between islands to derive agents' decision rules. In turn, this implies that equilibrium outcomes can now be derived in two steps. In the first step, decision rules are solved independently of the heterogeneity distribution that exists across agents and islands using the above four value functions. Once those decision rules are determined, we fully describe the dynamics of these distributions using the workers' flow equations.

This recursive property is common in many search models and in particular in those based on Pissarides (2001). In these models the free entry condition determines the labor market tightness (the key variable of the model) without taking into account the number of unemployed or employed workers in the labor market. These measures are derived using the flow equations that describe workers' transition between employment and unemployment once labor market tightness is obtained.[6]

More recently, Menzio and Shi (2010) show that it is likewise possible to apply this property to a business cycle model within a directed search framework where workers search on the job. In their model there is a unique labor market in which workers with different contracts endogenously sort into different sub-markets. They show that free entry of firms implies that labor market tightness, contract values and separation decisions can be solved independently of the distribution of match quality and contract values across all employed workers. Once again the workers' flow equations are then used to solve for the equilibrium behavior of the entire economy. Menzio and Shi termed this kind of equilibria 'block recursive'. We follow this convention.

Our model differs from that of Menzio and Shi because workers on the same island do not engage in on-the-job search and only differ in their employment status, and thus in the matching stage, there is no need to have them separate into different sub-markets (on the same island). As a result we do not require directed search to get tractability in our model. More importantly, our model differs from both Pissarides and Menzio and Shi because workers are able to reallocate to a *randomly drawn* different island.

**Definition 1.** *A Block Recursive Equilibrium (BRE) in our island economy is a set of value functions* $W^U(p,z)$, $W^E(p,z)$, $J(p,z,W^E)$, *workers' policy functions* $d(p,z)$, $\rho(p,z)$, $\alpha(p,z)$ *(resp. sepa-*

[6] Shimer (2005) and Mortensen and Nagypal (2007) provide recent examples of how this property is preserved when analyzing the canonical search and matching model in a business cycle context.

*ration, reallocation and visiting strategies), firms' policy functions $\tilde{W}_f(p,z)$, $\sigma(p,z,W^E)$, $w(p,z,W^E)$, $\tilde{W}^{E\prime}(p,z,W^E)$ (resp. contract posted, layoff decision, wages paid, and continuation values promised), tightness function $\theta(\tilde{W},p,z)$, matching probabilities $\lambda(\theta),q(\theta)$, laws of motion of $y_{it},p_t$, $F_z(.),F_p(.)$, and a law of motion on the distribution of unemployed and employed workers over islands $\tilde{u}(.)$ : $\mathcal{F}^{[0,1]} \to \mathcal{F}^{[0,1]}$ and $\tilde{e}(.) : \mathcal{F}^{[0,1]} \to \mathcal{F}^{[0,1]}$, such that*

1. *$\theta(p,z,\tilde{W})$ results from free entry condition $V(p,z,\tilde{W})=0$, if $\theta(p,z,\tilde{W})>0$ and $V(p,z,\tilde{W}) \leq 0$ if $\theta(p,z,\tilde{W})=0$, defined in (8), and given value function $J(p,z,\tilde{W})$.*

2. *Matching probabilities $\lambda(.)$ and $q(.)$ are only functions of labor market tightness $\theta(.)$, according to the definitions in section 2.1.*

3. *Given firms' policy functions, laws of motion $F_z,F_p$, and implied matching probabilities from functions $\lambda(.)$, the value function $W^E$ and $W^U$ satisfy (6) and (5), while $d(.),\rho(.),\alpha(.)$ are the associated policy functions.*

4. *Given workers optimal separation, reallocation and application strategies, implied by $W^E(.)$ and $W^U(.)$, and the laws of motions on $p_t,z_{it}$, firms' maximization problem is solved by $J(.)$, with associated policy functions $\{\sigma(.),w(.),\tilde{W}^{E\prime}(.)\}$.*

5. *$\tilde{W}^{E\prime}(p,z)=W^E(p,z)$.*

6. *$\tilde{u}$ and $\tilde{e}$ map initial distributions of unemployed and employed workers (respectively) over islands into next period's distribution of unemployed and employed workers over islands, according to policy functions and exogenous separation, and then according to equations in section 2.4.*

### 3.1 Characterization

We start the characterization of equilibria by showing that in each matching stage, firms offer a unique $\tilde{W}_f$ with associate tightness $\tilde{\theta}(p,z)$. To do so, consider an island $i$ that is characterized by state vector $(p,z)$. For any promised value $W^E$, the joint value of the match is defined as $W^E + J(p,z,W^E) \equiv \tilde{M}(p,z,W^E)$. Lemma 1 now shows that under risk neutrality the value of a match is constant in $W^E$ and $J$ decreases one-to-one with $W^E$.

**Lemma 1.** *The joint value $\tilde{M}(p,z,W^E)$ is constant in $W^E \geq W^U(p,z)$ and hence we can uniquely define $M(p,z) \overset{def}{=} \tilde{M}(p,z,W^E)$, $\forall\ M(p,z) \geq W^E \geq W^U(p,z)$ on this domain. Further, $J_W(p,z,W^E) =$*

*$-1,\ \forall\ M(p,z) > W^E > W^U(p,z)$, and endogenous match breakup decisions are efficient from the perspective of the match.*

The proof of Lemma 1 crucially relies on the firms' ability to offer workers intertemporal wage transfers such that the value of the match is not affected by the (initial) promised value. Lemma 2 now shows that firms offer a unique $\tilde{W}_f$ in the matching stage and there is a unique $\theta$ associated with it.

**Lemma 2.** *Assume free entry of firms, $J_W(p,z,W^E) = -1$ for each $p, z$, and a matching function that exhibits constant returns to scale, with a vacancy filling function $q(\theta)$ that is nonnegative and strictly decreasing, while the job finding function $\lambda(\theta)$ is nonnegative, strictly increasing and concave. If the elasticity of the vacancy filling rate is weakly negative in $\theta$, there exists a unique $\theta^*(p,z)$ and $W^*(p,z)$ that solve (**??**), subject to (**??**).*

The requirement that the elasticity of the job filling rate with respect to $\theta$ is non-positive is automatically satisfied when $q(\theta)$ is log concave, as is the case with the urn ball matching function.[7] Alternatively, one can use the Cobb-Douglas matching function as it implies a constant $\varepsilon_{q,\theta}(\theta)$. Both matching functions imply that the job finding and vacancy filling rates have the properties described in Lemma 2 and hence guarantee a unique pair $\tilde{W}_f, \theta$. To simplify the analysis that follows, we assume a Cobb-Douglas matching function. Using $\eta$ to denote the (constant) elasticity of the job finding rate with respect to $\theta$, we find the well-known division of the surplus according to the Hosios' rule

$$\eta(W^E - W^U(p,z)) - (1-\eta)J(p,z,W^E) = 0. \tag{9}$$

Finally, since in every period there is only one $\tilde{W}_f$ offered in the matching stage, a worker's visiting strategy, $\alpha$, is to visit the sub-market associated with $\tilde{W}_f$ with probability one when $S(p,z) > 0$ and not to visit any sub-market when $S(p,z) = 0$.

The last step in our characterization is to derive the reallocation and separation policy functions, $d(p,z)$, $\sigma(p,z)$ and $\rho(p,z)$. Lemmas 3 and 4, below, show that for every $p$, there exists a (potentially trivial) reservation productivity $z^s(p)$ below which any match, if it exists, is broken up such that $d(p,z) = \sigma(p,z) = 1$ for all $z < z^s(p)$ and $d(p,z) = \sigma(p,z) = \delta$ otherwise. Further, for every $p$, there exists a reservation productivity $z^r(p)$ such that $\rho(p,z) = 1$ (a worker reallocates) for all $z < z^r(p)$ and $\rho(p,z) = 0$ (a worker does not reallocate) otherwise.

[7] The urn-ball matching function is the one that arises endogenously within a directed search model a la Burdett, Shi and Wright (2001). In this case $q$ exhibits a negative elasticity, $\frac{-\frac{1}{\theta}}{e^{1/\theta}-1} - \frac{\frac{1}{\theta^2}e^{1/\theta}}{(e^{1/\theta}-1)^2} < 0$.

### 3.2 Existence

To show existence of equilibrium it is useful to consider the operator $T$ mapping a value function $\tilde{M}(p,z,n)$ for $n=0,1$ into the same function space such that $\tilde{M}(p,z,0)=M(p,z)$, $\tilde{M}(p,z,1)=W^U(p,z)$ and

$$T(\tilde{M}(p,z,0)) = y(p,z) + \beta\mathbb{E}_{p',z'}\Big[\max_{d^T}\{(1-d^T)M(p',z') + d^T W^U(p',z')\}\Big]$$

$$T(\tilde{M}(p,z,1)) = b+\beta\mathbb{E}_{p',z'}\Big[\max_{\rho^T}\{(\rho^T\left(\int W^U(p',\tilde{z})dF(\tilde{z})-c\right)+(1-\rho^T)(S^T(p',z')+W^U(p',z'))\}\Big]$$

where by virtue of the free entry condition

$$S^T(p',z') \overset{def}{=} \max_{\theta(p',z')}\Big\{\lambda(\theta(p',z'))\Big(M(p',z')-W^U(p',z')\Big)-\theta(p',z')k\Big\}.$$

A fixed point $\tilde{M}(p,z,n)$, $n=0,1$ describes the problem faced by unemployed workers and firm-worker matches in the decentralized economy. In the proof of Proposition 1 we show that all equilibrium functions and the evolution of the economy can be derived completely from the fixed point of the mapping $T$. For that purpose, we assume that the probability distribution of tomorrow's $z$ conditional on today's $z$ first-order stochastically dominates the corresponding distribution conditional on a $z'$ that is lower today.

**Assumption 1.** *$F_z(z_{it+1}|p_t,z_{it}) < F_z(z_{it+1}|p_t,z'_{it})$, for all $i, z_{it+1}, p_t$ if $z_{it} > z'_{it}$.*

Thus, a higher island-specific productivity today leads, on average, to a higher productivity tomorrow and hence the ranking of island-specific $z$ productivity is -in this sense- persistent. The next result derives the essential properties of $T$.

**Lemma 3.** *$T$ is (i) a well-defined operator mapping functions from the closed space of bounded continuous functions $\tilde{M}$ into itself, (ii) a contraction and (iii) maps functions $M(p,z)$ and $W^U(p,z)$ that are increasing in $z$ into itself.*

A direct implication of the above Lemma is that the optimal reallocation policy is a reservation-$z$ policy as described above, as both $S(p,z)$ and $W^U(p,z)$ are increasing in $z$, but $R(p,z_j)$ is constant. The next result implies that the optimal quit policy is also a reservation-$z$ policy as described above.

**Lemma 4.** *If $\delta+\lambda(\theta(p,z))<1$, $M(p,z)-W^U(p,z)$ in the fixed point of $T$ is increasing in $z$.*

Lemma 4 and equations (8) and (9) together imply that in each island labor market tightness $\theta(p,z)$ and the job finding rate $\lambda(\theta(p,z))$ are also increasing functions of $z$ if $\delta + \lambda(\theta(p,z)) < 1$.[8]

Note that the above policy functions describe the decision rules in our candidate equilibrium. Since $\tilde{W}(p,z) = M(p,z) - J(p,z,\tilde{W})$ and $J(p,z,\tilde{W}) = (1-\eta)(M(p,z) - W^U(p,z)) = k/q(\theta(p,z,\tilde{W}))$, $\tilde{W}(p,z)$, $J(p,z,\tilde{W})$ and $\theta(p,z,\tilde{W})$ can be constructed from $M(p,z)$ and $W^U(p,z)$. This is done in the proof of Proposition 1, where the existence and uniqueness of equilibrium are 'inherited' from the existence and uniqueness of the fixed point of the mapping $T$.

**Proposition 1.** *A Block Recursive Equilibrium exists and is unique.*

### 3.3 Planner's Problem and Efficiency

The social planner, currently in the production stage, in this economy solves the problem of maximizing total discounted output. Namely,

$$\max_{\{d_{it}(\Omega_t^s),\rho_{it}(\Omega_t^r),v_{it}(\Omega_t^m),\alpha_i(\Omega_t^m)\}} \mathbb{E}\left[\sum_t \beta^t \int_I [u_{it}b + e_{it}y(p_t, z_{it}) - (c\rho_{it}u_{it} + kv_{it})]\, di\right]$$

subject to the laws of motion and initial conditions

$$\begin{aligned} u_{it+1} &= (1-\rho_{it})u_{it} + (e_{it} - e_{it+1}) + \int_I \rho_{jt}u_{jt}dj \\ e_{it+1} &= (1-d_{it})e_{it} + (1-\rho_{it})u_{it}\lambda\left(\frac{v_{it}}{(1-\rho_{it})u_{it}}\right) \\ &\mathcal{E}_0 \text{ given}, \ v_{i0} = 0, \text{ for all } i, \end{aligned}$$

where $I$ denotes the set of islands in the economy and $\theta_{it} = v_{it}/(1-\rho_{it})u_{it}$.

**Proposition 2.** *The equilibrium identified in Proposition 1 is efficient.*

The crucial insight behind Proposition 2 is that the social planner's value functions are linear in the number of unemployed and employed on each island. The remaining dependence on $p$ and $z_i$ is equivalent to the one derived from the fixed point of $T$.[9] Given the value functions of unemployed workers and worker-firm matches, the outcome at the matching stage is efficient and the Hosios' condition is thus satisfied. Proposition 2 also implies that workers' reallocation decisions are efficient.

[8] In the quantitative part of the paper we show that this parametric restriction is easily satisfied in the data.

[9] Menzio and Shi, (2010), use this property to establish block recursivity in their stochastic on-the-job search model.

This is intuitive as the value of an unemployed worker who always remains on the island equals the shadow value of this worker in the social planner's problem, and reallocation decisions are made by comparing the expectation over the value of unemployment at other islands with the value of unemployment on the current island.

# 4 Implications

In this section we explore three main implications of our model. First we show that the interaction of within island and across island frictions implies that worker reallocation flows become more procyclical. We then show the conditions under which job separation flows are countercyclical. We consider the conditions under which rest unemployment occurs in our model. We also present a brief extension of our theory that considers island specific human capital. This can be interpreted as occupational, industry and/or location specific human capital. Finally, we show that the model is able to generate duration dependence in unemployment spells and analyze how it evolves along the business cycle. To gain some intuition about the forces at work, we consider a one-time unexpected permanent change in productivity $p$.[10]

In what follows it will be useful to note that in our model wages are described by a standard Pissarides style wage equation[11]

$$w(p,z) = (1-\eta)y(p,z) + \eta b + \beta(1-\eta)\theta(p,z)k.$$

Using the free-entry condition and the Cobb-Douglas specification for the matching function we have that $\theta$ then solves

$$\theta(p,z)^{\eta-1}\frac{\eta(y(p,z)-b)-\beta(1-\eta)\theta(p,z)k}{1-\beta(1-\delta)} - k \equiv E(\theta;p,z) = 0,$$

where differentiation implies that $\theta$ is increasing in both $p$ and $z$,

$$\theta_x(p,z) = \frac{\theta(p,z)y_x(p,z)}{w(p,z)-b}, \tag{10}$$

and the subscript $x$ denotes differentiation with respect to $x = p, z$.

[10] Since the equilibrium value and policy functions only depend on $p$ and $z$, analyzing the change in the expected value of unemployment and joint value of the match after a one-time productivity shock is equivalent to compare those values at the steady states associated with each productivity level. This follows as the value and policy functions jump immediately to their steady state level, while the distribution of unemployed and employed over islands takes time to adjust.

[11] A formal derivation of this equation can be found in Appendix B.

### 4.1 Cyclicality of Worker Reallocation Flows

We first turn to analyze whether a higher aggregate productivity leads to more or less reallocation, given the same initial distribution of employed and unemployed workers over islands.[12] Note that at islands where the island-specific productivity equals $z^r$ (the reservation productivity for the reallocation decision) it holds that the value of reallocation equals the value of staying and searching in the local labor market,

$$\int_{\underline{z}}^{\bar{z}} W^U(p,z)dF(z) - c = W^U(p,z^r) + \lambda(\theta(p,z^r))(W^E(p,z^r) - W^U(p,z^r)). \tag{11}$$

In a stationary environment, described by $p, z$, the value of unemployment at islands with $z < z^r$ is given by $W^U(p,z) = W^U(p,z^r)$.[13] On the other hand, the value of unemployment at islands with $z \geq z^r$ is given by

$$W^U(p,z) = \frac{b + \beta\lambda(\theta(p,z))(W^E(p,z) - W^U(p,z))}{1-\beta}. \tag{12}$$

Equation (11) can then be expressed as

$$\frac{(1-\eta)k}{\eta}\left(\beta\int_{\underline{z}}^{\bar{z}} \max\{\theta(p,z),\theta(p,z^r)\}dF(z)\right) - c(1-\beta) = \frac{(1-\eta)k}{\eta}\theta(p,z^r), \tag{13}$$

where the LHS describes the net benefit of reallocating to a different island and the RHS the benefit of staying in the same island.[14] Hence the response to a positive (and permanent) productivity shock is more reallocation (a higher reservation productivity) if $dz^r/dp > 0$.

Proposition 3, below, derives the conditions under which procyclical reallocation arises, taking into account on the one hand that the value of switching to become unemployed on an island with a higher $z$ than the current island is increasing in $p$; and on the other that this gain realizes only

[12] Recall that upon deciding to reallocate, a worker pays a cost $c$, is not allowed to work in the market while reallocating, and arrives at a randomly drawn new location only at the end of the period, after production has already taken place.

[13] This follows since over this range of $z$'s, $\int_{\underline{z}}^{\bar{z}} W^U(p,z)dF(z) - c \geq W^U(p,z) + S(p,z)$ and unemployed workers prefer to reallocate the period after arrival. The stationary version of (5) then implies $W^U(p,z) = W^U(p,z^r)$ for all $z < z^r$.

[14] This equation is obtain by noting that (11) can be expressed as

$$\beta\int_{\underline{z}}^{\bar{z}}\Big(\max\{\lambda(\theta(p,z))(W^E(p,z) - W^U(p,z)), \lambda(\theta(p,z^r))(W^E(p,z^r) - W^U(p,z^r))\}\Big)dF(z)$$
$$= \lambda(\theta(p,z^r))(W^E(p,z^r) - W^U(p,z^r)) + c(1-\beta).$$

Using $\eta\lambda(\theta)(W^E(p,z) - W^U(p,z)) = (1-\eta)\lambda(\theta)J(p,z) = (1-\eta)\theta(p,z)k$, we have (13).

one period after arriving to the new island (as workers cannot search during the same period they arrived to the island), and that the cost of missing out on one period of higher productivity also goes up with $p$.[15]

Proposition 3 also compares the cyclicality of reallocation in our setting (where there is search frictions on islands) with a setting where markets on the islands are competitive. To make precise the comparison, consider the same environment as above, with the exception that workers can match instantly with firms.[16] This implies that every worker will earn his marginal product $y(p, z)$. Importantly, we keep the reallocation frictions the same: workers who reallocate have to forgo production for a period, and arrive at a random island at the end of the period. In the simple case of permanent productivity $(p, z)$, the value of being in island $z$, conditional on $y(p, z) > b$ is $W^c(p, z) = y(p, z)/(1 - \beta)$, where to simplify we have not consider job destruction shocks.[17]

Block recursiveness, given the free entry condition, is preserved, so again, decisions are only functions of $(p, z)$. Unemployed workers optimally choose to reallocate, and the optimal policy is a reservation quality, $z_c^r$, characterized by the following equation

$$(b - c)(1 - \beta) + \beta \int \max\{y(p, z), y(p, z_c^r)\} dF(z) = y(p, z_c^r). \tag{14}$$

The LHS describes the net benefit of switching islands, while the RHS the value of of staying employed earning $y$ in the (reservation) island.

**Proposition 3.** *Given an increase in aggregate productivity:*

1. *Search frictions on the island make reallocation more procyclical relative to the competitive benchmark case with the same $F(z)$ and the same initial reservation productivity $z^r = z_c^r$.*

2. *With search frictions, if the production function is modular or supermodular (i.e. $y_{pz} \geq 0$), there exists a $c \geq 0$ under which reallocation is procyclical. With competitive markets on islands, if the production function is modular, reallocation is countercyclical, for any $\beta < 1$ and $c \geq 0$.*

[15] The absence of the qualification $\delta + \lambda(\theta) < 1$ is because in Lemma 2 we put very little restrictions on the stochastic process for $z$. Here, with a one-time unexpected increase, we do not need this restriction.

[16] As before, we assume free entry (without vacancy costs), and constant returns to scale production.

[17] Note that if island productivity was stochastic, "rest" unemployment can occur on these competitive islands (as in Jovanovic 1987, Hamilton 1988, Alvarez and Shimer 2010), where workers prefer to stay on the island, but enjoy $b$ at home rather than working.

Note that the first part of the Proposition does not say anything about the sign of $dz^r/dp$ or $dz_c^r/dp$ and hence if reallocation is procyclical our countercyclical in either the frictional or the competitive case. It does imply, however, that $dz^r/dp > dz_c^r/dp$ at $z^r = z_c^r$ and, hence, that search frictions within islands make reallocation more attractive to worker. The crucial difference between the two cases arises since the benefits of reallocation increase proportionally more when labor markets are frictional than when they are competitive. In particular, with competitive markets a higher aggregate productivity increases the expected gain of reallocation only through an increase in wages relative to the reservation island, $\mathbb{E}[y_p(p,z)/y_z(p,z^r) \mid z \geq z^r]$. With search frictions a higher aggregate productivity increases both wages and the probability of finding employment, leading to $\mathbb{E}[(\theta(p,z)/\theta(p,z^r))(y_p(p,z)/(w(p,z)-b))((w(p,z^r)-b)/y_z(p,z^r)) \mid z \geq z^r]$. The term $(\theta(p,z)/\theta(p,z^r))$ shows the increase in the job finding rate relative to the reservation island, while the other terms describe the proportional increase in wages relative to the reservation island. Since workers are paid less than their marginal product, this proportional increase is higher in the frictional case, generating an extra benefit for reallocation.

The second part of the Proposition presents restrictions on the production technology that guarantee countercyclical reallocation with competitive labor markets, but is able to generate procyclical reallocation in the frictional case. It is useful to note, however, that in both cases $dz^r/dp$ is more likely to be positive when the production function exhibits a higher degree of supermodularity. For example, when $p$ and $z$ are complements in total output (i.e. $y = pz$), the benefits of reallocation are $z/z^r > 1$ times higher than in the case in which $p$ and $z$ are perfect substitutes (i.e. $y = p + z$).

## 4.2 Countercyclicality of Job Separations Flows

Procyclicality of reallocation flows does not imply procyclicality of separation flows, as we will argue below. However, it can add a force that pushes in a procyclical direction, because the increased attractiveness of reallocation can feed back into separation decisions. How strong this force is depends on whether there is "rest unemployment", i.e. there are islands where workers prefer to stay while without a job for a time without relocating, and thus $z^s(p) > z^r(p)$. In this case, job separations are countercyclical, because the value of being unemployed does not depend directly on the value of relocation, but on the island-specific value of unemployment, which rises less with p than the value of the match $M(p,z)$. This implies that for a higher $p$, the value of a match $M(p,z)$ will equal the value of unemployment $W^U(p,z)$ at a lower island $z$. In our case of a one-time aggregate productivity shock, with permanent island components $z$ of productivity, all islands with rest unemployment have the same value of unemployment: $\underline{W}^u = b/(1-\beta)$. Then it's easy to derive the slope of $z^s(p)$ from $M(p, z^s(p)) = W^u(p, z^s(p) = \underline{W}^u$

$$M(p,z) = y(p,z) + \beta[(1-\delta)M(p,z) + \delta W^U(p,z)] \Rightarrow (1-\beta)\underline{W}^u = y(p, z^s(p)) \tag{15}$$

$$\Rightarrow \frac{dz^s(p)}{dp} = -\frac{y_p(p, z^s(p))}{y_z(p, z^s(p))} \tag{16}$$

When $z^r(p) > z^s(p)$ and any worker that becomes unemployed in islands with $z \in [z^s(p), z^r(p)]$ prefers to reallocate, countercyclical job separations cannot be guaranteed. As long as the island is on or below the relocation cutoff, the value of unemployment now is the value of relocating (after sitting out one period of unemployment before being able to relocate), $R(p) \stackrel{def}{=} W^u(p, z^r(p))$. We can show that the slope of $z^s(p)$ an affine combination of (16) and the slope of $z^r(s)$

**Lemma 5.** *With permanent island-specific productivity, and $z^s(p) < z^r(p)$ for $p$, it holds that*

$$-\frac{y_p(p, z^s(p))}{y_p(p, z^r(p))} + \frac{\beta\lambda(\theta(p, z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p, z^r(p))}\left(1 + \frac{y_z(p, z^r(p))}{y_p(p, z^r(p))}\frac{dz^r(p)}{dp}\right) = \frac{y_z(p, z^s(p))}{y_p(p, z^r(p))}\frac{dz^s(p)}{dp}. \tag{17}$$

*Proof.* Note that $R(p) = \frac{b+\beta\theta(p,z^r(p))k(1-\eta)/\eta}{1-\beta}$. The derivative of this function with respect to $p$ equals

$$\frac{\beta k(1-\eta)}{(1-\beta)\eta}\frac{\theta}{w(p, z^r(p)) - b}\left(y_p(p, z^r(p)) + y_z(p, z^r(p))\frac{dz^r(p)}{dp}\right). \tag{18}$$

Since $w(p, z^r(p)) - b = (W^e(p, z^r(p)) - W^u(p, z^r(p)))(1-\beta(1-\delta)+\beta\lambda(\theta(p, z^r(p))))$ and $\frac{\theta\beta k(1-\eta)}{(1-\beta)\eta} = \beta\lambda(\theta(p, z^r(p)))(W^e(p, z^r(p)) - W^u(p, z^r(p))$, we find that (18) reduces to

$$\frac{\beta\lambda(\theta(p, z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p, z^r(p))}\left(y_p(p, z^r(p)) + y_z(p, z^r(p))\frac{dz^r(p)}{dp}\right). \tag{19}$$

From the cutoff condition for separation, we find $(1-\beta)R(p) = y(p, z^s(p))$. Taking the derivative with respect to $p$ implies the left side equals (19) and the right side equals $y_p(p, z^s(p)) + y_z(p, z^s(p))\frac{dz^s(p)}{dp}$. Rearranging yields (17). □

The first term $\frac{y_p(p,z^s(p))}{y_p(p,z^r(p))}$ is less than one when the production function is (super)modular, and $z^r(p) > z^s(p)$, and $\frac{\beta\lambda(\theta(p,z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z^r(p))}$ the second term is positive, and its magnitude depends on $dz^r(p)/dp$ (normalized by $\frac{y_z(p,z^r(p))}{y_p(p,z^r(p))}$). This means that some extent the procyclicality of reallocation can feed back into the separation decisions.

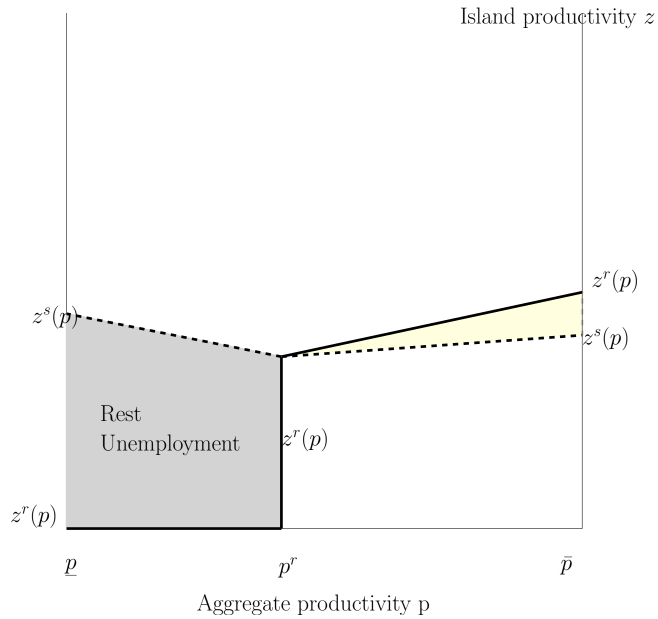


Figure 2: Reservation values $z^s(p), z^r(p)$ for the no-shock case

Alternatively, we can derive the slope explicitly as

$$\frac{dz^s}{dp} = -\frac{\frac{y_p(p,z^s)}{1-\delta} - \beta\frac{1-\eta}{\eta}\frac{\theta(p,z^r)y_p(p,z^r)}{w(p,z^r)-b}k}{\frac{y_z(p,z^s)}{1-\delta} - \beta\frac{1-\eta}{\eta}\frac{\theta(p,z^r)y_z(p,z^r)}{w(p,z^r)-b}k}.$$

It is not difficult to verify that $dz^s/dp$ becomes negative when the production technology has a sufficiently low degree of supermodularity, which naturally also from the previous section relates to lower slope of the $z^r(p)$ curve.

In figure 2 we draw the reservation island productivities below which workers separate from existing matches ($z^s$), and below which workers starts searching for jobs on other islands ($z^r$), as a function of the aggregate productivity. In addition to the distribution of workers over islands, these two functions determine the transition flows as the aggregate productivity varies. In the case of the figure, reallocation is procyclical, it shuts down completely below aggregate productivity, $p^r$; at this productivity it starts (increasing discretely), and then continues to increase as the aggregate productivity increases even more. At all aggregate productivities there is search and reallocation unemployment, where the search unemployment would be countercyclical in the absence of increased inflows due to endogenous job separations. Below $p^r$ rest unemployment occurs, the more the worse the aggregate economy is doing.

### 4.3 Rest Unemployment and its causes

Rest unemployment occurs to the left of the intersection of the $z^r(s)$ and the $z^s(p)$, provided the $z^r(p)$ is not too downward-sloping. However, above we discussed conditions that generate the empirically more relevant case of procyclical reallocation. Rest unemployment responds directly to the cost of reallocation, the benefits received when unemployed, and the persistence of the island-specific productivity.

First, note that in the world of a one-time aggregate productivity shock with permanent island productivity components, the $z^s(s)$ curve (in the absence of reallocation) is simply given implicitly by $b = y(p, z^s(p))$. Given $p$ we can make any $\hat{z}$ low enough (because there always is the time cost of relocating) the reservation cutoff for reallocation by choosing $c$ large enough, given a $b$. Keeping $c$ constant, a rise in $b$ will also lead to a rise in rest unemployment. One can derive that (keeping everything else constant), $dz^s(p)/db = y_z^{-1}$, while $dz^r(p)/db = y_z^{-1}(1 - \beta F(z^r) - \int_{z^r(p)}^{\bar{z}} \frac{\theta(p,\tilde{z})(w(p,z^r)-b)}{\theta(p,z^r)(w(p,\tilde{z})-b)} dF(\tilde{z})$ from $d\theta/db = -d\theta/dy$. This means that an increase in b will lift the separation cutoff more than the relocation cutoff, and therefore increase the tendency towards rest unemployment.

Finally, the incidence of rest unemployment also depends on the extent of the volatility of the island-specific productivity (with regression towards the mean). If it is high, it becomes more attractive to wait out the bad shock, because the island will return to better times faster, in expectation.

To see this explicitly, consider the previous environment, with one change: with probability $p$ the island-specific productivity is redrawn, for once and for all, for all islands, at the beginning of next period. Intuitively, the possibility that a bad island will turn out to experience a good draw will make unemployed workers less likely to move, and employed workers less likely to quit. We want to show here that the first effect is dominant, and therefore, increased uncertainty will lead to more rest unemployment on the margin.

The argument here follows Alvarez and Shimer, Hamilton and Jovanovic, however, it involves one additional complication: being in a match on particular island confers an advantage upon the employed worker versus the unemployed worker on this island. This makes the employed worker more conservative in his breakup decisions when there is a chance that the island productivity improves. We show that the unemployed worker becomes even more conservative, and thus that the range of rest unemployment is larger.

### 4.4 Island Specific Human Capital

Kambourov and Manovskii (2009) argue that there are substantial returns to occupational tenure, in the order of 20% for a tenure of ten years. We can study this as a productivity increase that is island-specific, while keeping the productivity of the worker in the other islands constant.

The importance of island-specific human capital is that it creates heterogeneity among workers, where for example, the younger group is more responsible for the reallocation flows, while the older, experienced, group, is more likely to become rest unemployed. This smooths out the rest, search and reallocation behavior over the business cycle.

### 4.5 Unemployment Duration Dependence

The island structure of our model enable us to generate another important feature of the labor market: aggregate unemployment duration dependence.[18] Unemployed workers in a labor market with a higher island specific productivity $z$ will have a higher job finding rate and hence leave unemployment faster than those unemployed workers in islands with lower $z$. This follow directly from $\lambda'(\theta) > 0$ and (10), which shows that $\theta(p, z)$ is an increasing function of $z$, given $p$.

Furthermore, for a given $z$, a higher aggregate productivity leads to an increase in the job finding rate and a corresponding reduction the unemployment duration. However, changes in aggregate productivity affect the job finding rate across islands differently. For example, differentiation of (10) yields

$$\theta_{pz}(p, z) = \frac{\theta y_p y_z (y(p, z) - w(p, z))}{(w(p, z) - b)^3} + \frac{\theta y_{pz}}{w(p, z) - b}.$$

Given the production function is modular or supermodular, $\lambda'(\theta) > 0$ implies that higher aggregate productivity leads to a *higher* increase in the job finding rates in islands with larger idiosyncratic productivities (i.e. $\theta_{pz} > 0$). In terms of proportional changes, on the other hand, labor market tightness may increase proportionally more in low productivity islands than in high productivity islands if the degree of log-supermodularity of the production function is sufficiently small. To see this, consider $\varepsilon_{\theta p}$, the elasticity of $\theta$ with respect to $p$. Using (10) it easy to verify that

$$\varepsilon_{\theta p} = \frac{p y_p}{(1 - \eta)(y(p, z) - b + \beta\theta(p, z)k)}.$$

Differentiation with respect to $z$ then yields

$$\varepsilon_{\theta pz} = -\frac{p(1 - \eta)}{(w(p, z) - b)^2} \left[ y_p y_z \left( 1 + \frac{\beta k \theta(p, z)}{w(p, z) - b} \right) - y_{pz}(y(p, z) - b + \beta k \theta(p, z)) \right],$$

[18]See Shimer (2007) for a similar way of generating duration dependence.

which is strictly negative when the production function $y(p,z)$ is log-submodular $y_p y_z > y_{pz} y$, log-modular $y_p y_z = y_{pz} y$ or log-supermodular, $y_p y_z > y_{pz} y$, but in the latter case with the qualification that $0 < y_{pz} y - y_p y_z < b y_{pz} - (y_{pz}(w-b) - y_p y_z)(\beta k \theta)/(w-b)$.[19] In these cases, we obtain that labor market tightness in low productivity islands is more sensitive to changes in aggregate productivity than in high productivity islands.

# 5 A Calibrated Example

In this section we analyze the properties of the model quantitatively. The block recursive structure allow us to solve the equilibrium computationally from a fixed point of a mapping by simply iterating on the value function with two state variables ($p$ and $z$) only. (This stands in contrast, e.g. to Lkhagvasuren 2010, who is only able to solve a model with relocation flows in the absence of aggregate shocks - citing computational difficulties).

In particular, we parameterize the model to match salient long-run features of the US labor market. We assume that $p$ and $z$ satisfy the following autoregressive processes

$$x_{t+1} = \rho_x x_t + \epsilon_x,$$

where $\rho_x$ and $\epsilon_x$ describe the persistence parameter and variance of the process, respectively, for $x = p, z$. We set the values of $\rho_p$ and $\epsilon_p$ to $xx$ and $xx$; while $\rho_z$ and $\epsilon_z$ are $xx$ and $xx$. To approximate the resulting process for $y$, we impose the same autoregressive process described above.

We set the time period to a week, we set the average working life to 40 years, with a constant probability of death. We set the discount rate such that the implied yearly interest rate is 4%. We calibrate a model with experience accumulation, where the returns to occupational tenure are taken from Kambourov and Manovskii (2009). The returns to 5-year tenure are set at 12%, and those of 10-year occupational tenure at 18%.

We estimate the value of the parameters by simulated minimum distance[20]. We use the following set of moments: (i) a monthly average breakup rate of 0.8%, (ii) a monthly average job finding rate of 45%, and the average unemployment rate, (iii) the variance and persistence of aggregate labor productivity, (iv) the empirical elasticity of the matching function, (v) the average reallocation rate

[19] For example, the supermodular functions $y(p,z) = p + z$ and the modular function $y(p,z) = pz$ are log-modular and log-submodular, respectively.

[20] The parameters and results presented here are the result of a preliminary parametrization

| parameters | value |
|---|---|
| $k$ | 2.0 |
| $c$ | 2.5 |
| $\sigma_z$ | 0.985 |
| $\rho_z$ | 0.06 |
| $\sigma_p$ | 0.990 |
| $\rho_p$ | 0.008 |
| $\eta$ | 0.6 |
| $b$ | 0.70 |
| $\delta$ | 0.0085 |

(15%-20% yearly), (vi) conditional on one occupational switch, the average total switches in a four year period. The last moment was documented in Kambourov and Manovskii (2009), and we use it here (as it was used in their paper) to pin down the volatility of $\sigma_z$. We set $b$ to 50% of average productivity.

### 5.0.1 Some Initial Results

We report the results of a (preliminary) parametrization. The fit of the targeted moments is reasonable to good, with an important exception of the volatility of aggregate productivity, which is too volatile. Hence we will not discuss the amplification properties of the model here. Reallocation is procyclical (a correlation with labor productivity of 0.13), separation is countercyclical (a correlation of -0.10). At the same time a tight relationship between u and v is preserved (-0.81), thus hinting that the model with island heterogeneity with endogenous separations is consistent with a strong Beveridge curve – a common issue with models that incorporate endogenous inflows in the pool of unemployed, is the loss of the prominent Beveridge curve.

## 5.1 Decomposition of the Unemployment Rate

We now turn to analyze the aggregate unemployment rate in our economy. We first consider a dynamic decomposition of the unemployment rate based on the flow equations described in section 2.4 and the reservation productivities, $z^r$ and $z^s$. We then consider a decomposition based on the cross-sectional distribution of unemployed workers in the economy, to disentagle the degree of mismatch across islands. Finally we use the above parametrisation to quantitatively assess these decompositions.

### 5.1.1 Dynamic Decomposition

Consider the case in which $z^s > z^r$. For all islands with idiosyncratic productivities $z > z^s$, the sources of unemployment are (i) search frictions, workers not being lucky enough to get a job, (ii) reallocation frictions, workers transiting from one island to another and (iii) exogeneous job separations. The flow equations in section 2.4 then imply that, given the unemployment and employment rates at the start of the period, $u_t$ and $e_t$, respectively, the next period unemployment rate in any island with $z > z^s$ is given by

$$u_{t+1}(z) = (1 - \lambda(\theta(p, z)))u_t(z) + \int_{\underline{z}}^{z^r} u_t(z')dF(z') + \delta e_t(z). \tag{20}$$

For islands that exhibit productivities $z \in [z^r, z^s]$ we have that the sources of unemployment are (i) rest unemployment, (ii) reallocation frictions and (iii) endogenous separations. Hence, the next period unemployment rate in any of these islands is given by

$$u_{t+1}(z) = u_t(z) + \int_{\underline{z}}^{z^r} u_t(z')dF(z') + e_t(z). \tag{21}$$

Note that in this case, all employed workers decide to separate. For islands with $z < z^r$ the only source of unemployment is due to reallocation frictions since workers that reallocate arrive randomly to this islands every period and those that started unemployed reallocated some where else. The next period unemployment rate in any of these islands is given by

$$u_{t+1}(z) = \int_{\underline{z}}^{z^r} u_t(z')dF(z'). \tag{22}$$

Integrating across islands then gives the dynamics of the unemployment rate in the economy.

When $z^r > z^s$ we have that $u_{t+1}(z)$ is given by equation (20) for those islands with $z > z^r$. For islands with $z \in [z^s, z^r]$ we have that the unemployment pool is made up of those employed workers that are exogenously loose their jobs and those workers that arrive due to reallocation. Namely,

$$u_{t+1}(z) = \int_{\underline{z}}^{z^r} u_t(z')dF(z') + \delta e_t(z). \tag{23}$$

For islands with $z < z^s$ we have that $u_{t+1}(z)$ is given by equation (22). As before, integrating across islands gives the dynamics of the aggregate unemployment rate for this case.

We use these decompositions to show how the proportions of workers that are unemployed due to search frictions, reallocation frictions, job destruction or are rest unemployed change over the business cycle.

### 5.1.2 Mismatch

We now turn to analyse the degree of mismatch in our economy. Here reallocation frictions prevent workers from going to the island with the best conditions. If the planner could eliminate these frictions, then the allocation that maximises output would be to move all workers to that island. Free entry would guarantee that enough firms post vacancies in this island. Aggregate unemployment would then be determined by the degree of search frictions present in such an island. Hence, our model implies that any measure of mismatch should compare this unemployment rate with the unemployment rates across islands obtained when reallocation frictions are present. This implication follows from the (simplifying) assumption that within islands all workers have the same productivity. In this section we depart from this implication and study a mismatch index based on the aggregate unemployment rate that would prevail if all islands had the same productivity $\widetilde{z}$, the average productivity of the ergodic distribution of $z$. This allows us capture a better measure of inequality across islands unemployment rates. Let $u(\widetilde{z})$ denote the aggregate unemployment rate associated with $\widetilde{z}$. Following Jackman, Layard and Savouri (1991) our measure of mismatch is given by the following variance formula:

$$M_t(p, \widetilde{z}) = \int_{\underline{z}}^{\overline{z}} [u_t(z) - u(\widetilde{z})]^2 dF(z). \tag{24}$$

## 6 Conclusions

In this paper we have presented a tractable general equilibrium framework to study the evolution of aggregate unemployment over the business cycle by considering different sources of unemployment. We focused on workers' decisions to search, rest, reallocate and separate as causes of unemployment. The model provides a tractable analysis of the interaction between search and reallocation frictions. We show that when search frictions are present in local labor markets, worker reallocation is more procyclical than if labor markets are competitive. This is consistent with the observed procyclicality of workers across occupations and regions. Further, we present a decomposition of the unemployment rate into its constituent parts and provide a measure of mismatch for our economy. We then calibrate our model and provide quantitative evaluation of its implications.

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# A Proofs

**Proof of Lemma 1** Consider a firm that promised $W \geq W^u(p, z)$ to the worker such that the expected payoff to the firm is given by $J(p, z, W)$ solving (3). Now consider an alternative offer $\hat{W} \neq W$ which is also acceptable to the unemployed worker, provides the same contingent continuation values $\tilde{W}^{E'}(p', z'|p, z, W)$ to the worker as $W$ and implies $J(p, z, \hat{W})$ solves equation (3). Risk neutrality then implies that

$$J(p, z, W) \geq J(p, z, \hat{W}) + (\hat{W} - W),$$

if the firm provides the worker with $\hat{W}$ using the optimal policy associated with providing $W$. Note that the last term in the RHS of the inequality makes up for the difference in value by offering the worker a payment (reduction) today. Similarly, if the firm provides the worker with $W$ using the optimal policy associated with $\hat{W}$, we have that

$$J(p, z, \hat{W}) \geq J(p, z, W) - (\hat{W} - W).$$

Hence it must be that $J(p, z, \hat{W}) = J(p, z, W) + W - \hat{W}$ for *all* $M(p, z) \geq W, \hat{W} \geq W^u$. Differentiability of $J$ with slope -1 follows immediately. Moreover, $M(p, z, W) = W + J(p, z, \hat{W}) + \hat{W} - W = M(p, z, \hat{W}) \equiv M(p, z)$. Finally, if $W'(p', z') < W^u(p', z')$ is offered tomorrow while $M(p', z') > W^u(p', z')$, it is a profitable deviation to offer $W^u(p', z')$, since $M(p', z') - W^u(p', z') = J(p', z', W^u(p', z')) > 0$ is feasible. This completes the proof of Lemma 1.

**Proof of Lemma 2** Since we confine ourselves to one island, with known continuation values $J(w, p, z)$ and $W^u(p, z)$ in the production stage, we drop the dependence on $p, z$ for ease of notation. Free entry implies $k = q(\theta)J(W) \Rightarrow \frac{dW}{d\theta} < 0$. Notice that it follows that the maximand of workers in (**??**), subject to (**??**) is continuous in $W$, and provided $M > W^u$, has a zero at $W = M$ and at $W = W^u$, and a strictly positive value for intermediate $W$: hence the problem has an interior maximum on $[W^u, M]$. What remains to be shown is that the first order conditions are sufficient for the maximum, and the set of maximizers is singular.

Solving the worker's problem of posting an optimal value subject to tightness implied by the free entry condition yields the following first order conditions (with multiplier $\mu$):

$$\begin{aligned}\lambda'(\theta)[W - W^U] - \mu q'(\theta)J(W) &= 0\\ \lambda(\theta) - \mu q(\theta)J'(W) &= 0\\ k - q(\theta)J(W) &= 0\end{aligned}$$

Using the constant returns to scale property of the matching function, one has $q(\theta) = \lambda(\theta)/\theta$. This implies, combining the three equations above, to solve out $\mu$ and $J(W)$,

$$0 = \lambda'(\theta)[W(\theta) - W^U] + \frac{\theta q'(\theta)}{q(\theta)}k \equiv G(\theta),$$

where we have written $W$ as a function of $\theta$, as implied by the free entry condition. Then, one can derive $G'(\theta)$ as

$$G'(\theta) = \lambda''(\theta)[W(\theta) - W^U] + \lambda'(\theta)W'(\theta) + \frac{d\varepsilon_{q,\theta}(\theta)}{d\theta},$$

where $\varepsilon_{q,\theta}(\theta)$ denotes the elasticity of the vacancy filling rate with respect to $\theta$ and

$$\frac{d\varepsilon_{q,\theta}(\theta)}{d\theta} = \frac{q'(\theta)k}{q(\theta)} + \frac{\theta[q''(\theta)q(\theta) - q'(\theta)^2]k}{q(\theta)^2}.$$

Since the first two terms in the RHS are strictly negative, $G'$ is strictly negative when $\varepsilon_{q,\theta}(\theta) \leq 0$. The latter then guarantees there is a unique $\tilde{W}_f$ and corresponding $\theta$ that maximizes the worker's problem. This completes the proof of Lemma 2.

**Proof of Lemma 3** First we show that the operator $T$ maps continuous functions into continuous functions. Note that $\theta \in [0, 1]$, for all $p, z$ and $W^U(p, z)$, $M(p, z)$ and $\lambda(\theta)$ are continuous functions. The Theorem of the Maximum then implies that $S(p, z)$ is also a continuous function. That $T$ maps continuous functions into continuous functions then follows as the $\max\{M(p', z'), W^U(p', z')\}$ is also a continuous function. Moreover, since the domain of $p, z$ is bounded, the resulting continuous functions are also bounded.

To show that $T$ defines a contraction, consider two functions $\tilde{M}, \tilde{M}'$, such that $\|\tilde{M} - \tilde{M}'\|_{\sup} < \varepsilon$. Then it follows that $\|W^U(p, z) - W^{U\prime}(p, z)\|_{\sup} < \varepsilon$ and $\|M(p, z) - M'(p, z)\|_{\sup} < \varepsilon$, where $W^U, M$

are part of $\tilde{M}$ as defined in the text. Since $\|\max\{a,b\} - \max\{a',b'\}\| < \max\{\|a-a'\|, \|b-b'\|\}$, as long as the terms over which to maximize do not change by more than $\varepsilon$ in absolute value, the maximized value does not change by more $\varepsilon$. The only maximization for which it is nontrivial to establish this is $\max\{\int W^U(p,z)dF(z) - c, S(p,z) + W^U(p,z)\}$. The first part can be established readily: $\|\int \Big(W^U(p,z) - W^{U\prime}(p,z)\Big) dF(z)\| < \varepsilon$. We now show that this property holds for $\|S(p,z) + W^U(p,z) - S'(p,z) - W^{U\prime}(p,z)\|$.

Consider first the case that $M - W > M' - W'$. Then, we must have $\varepsilon > W' - W \geq M' - M > -\varepsilon$. Construct $M'' = W' + (M - W) > M'$ and $W'' = M' - (M - W) < W'$. Call $S(M - W)$ the maximized surplus $\max_\theta\{\lambda(\theta)(M - W) - \theta k\}$ and $\theta$ the maximizer; likewise $S(M' - W')$ and $\theta'$. Then

$$\begin{aligned} -\varepsilon < S(M' - W'') + W'' - S(M - W) - W &\leq S(M' - W') + W' - S(M - W) - W \\ &\leq S(M'' - W') + W' - S(M - W) - W < \varepsilon \end{aligned}$$

where $S(M' - W'') = S(M - W) = S(M'' - W')$ by construction. Note that the outer inequalities follow because $M - M' > -\varepsilon, W' - W < \varepsilon$.

Likewise, consider the case where $M' - W' > M - W \geq 0$. Then

$$\begin{aligned} \varepsilon > S(M' - W'') + W'' - S(M - W) - W &> S(M' - W') + W' - S(M - W) - W \\ &> S(M'' - W') + W' - S(M - W) - W > -\varepsilon \end{aligned}$$

Hence $\|S(p,z) + W^U(p,z) - S'(p,z) - W^{U\prime}(p,z)\| < \varepsilon$. It then follows that $\|T(\tilde{M}(p,z,1)) - T(\tilde{M}'(p,z,1)\| < \beta\varepsilon$ for all $p, z$, and $\|\tilde{M} - \tilde{M}'\| < \varepsilon$. Hence, the operator is a contraction.

It is now trivial to show that if $M$ and $W^U$ are increasing in $z$, $T$ maps them into increasing functions. This follows since the $\max\{M(p',z'), W^U(p',z')\}$ is also an increasing function. Assumption 1 is needed so higher $z$ today implies (on average) higher $z$ tomorrow. Since the value of reallocation is constant in $z$, the reservation policy for reallocation follows immediately. This completes the proof of Lemma 3.

**Proof of Lemma 4** $T$ maps the subspace of functions $\tilde{M}$ into itself with $M(p,z)$ increasing weakly faster in $z$ than $W^U(p,z)$. To show this let $M(p,z) - W^U(p,z)$ be weakly increasing in $z$ and $z^s$ denote the reservation productivity such that for and $z < z^s$ a firm-worker match decide to terminate the match. Using $\max_\theta\{\lambda(\theta)(M - W^U) - \theta k\} = \lambda(\theta^*)(M - W^U) - \lambda'(\theta^*)(M - W^U)\theta^* =$

$\lambda(\theta^*)(1-\eta)(M - W^U)$, we construct the following difference

$$T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1) = y(p,z) - b + \beta\mathbb{E}_{p',z'}\Big[(1-\delta)\max\{M(p',z') - W^U(p',z'), 0\} \\ - \max\Big\{\int W^U(p',\tilde{z})dF(\tilde{z}) - c - W^U(p',z'), \lambda(\theta^*)(1-\eta)\big(M(p',z') - W^U(p',z')\big)\Big\}\Big].$$

The first part of the proof shows the conditions under which $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is weakly increasing in $z$. Consider the range of $z \in [\underline{z}, z^r)$, where $z^r < z^s$. In this case, the term under the expectation sign in the above equation reduces to $-\int W^U(p',\tilde{z})dF(\tilde{z}) + c + W^U(p',z')$. It is then immediate that when $W^U$ increases in $z'$, this term also increases in $z$. Since $y(p,z)$ is increasing in $z$, then $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is also increasing in $z$. Now suppose $z \in [z^r, z^s)$. In this case, the term under the expectation sign becomes zero (as $M(p',z') - W^U(p',z') = 0$) and $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is weakly increasing in $z$ in this range. Next suppose that $z \in [z^s, z^r)$. In this case, the term under the expectation sign reduces to $(1-\delta)(M(p',z') - W^U(p',z'))$. Since $M(p,z) - W^U(p,z)$ is weakly increasing in $z$, $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is weakly increasing in $z$ in this case. Finally consider the range of $z \geq z^r > z^s$ or $z \geq z^s > z^r$, such that there employed workers do not quit nor reallocate. In this case the term under the expectation sign equals $(1-\delta)[M(p',z') - W^U(p',z')] - \lambda(\theta^*)(1-\eta)[M(p',z') - W^U(p',z')]$. When $M - W^U$ is increasing in $z$, a sufficient condition for the latter term to also increase in $z$ is that $1 - \delta - \lambda(\theta^*) > 0$. In this case $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is increasing in $z$ when such a condition holds.

The set of functions with increasing differences between the first and second coordinate is closed in the space of bounded and continuous functions. In particular, consider the set of functions $F \stackrel{def}{=} \{f \in \mathcal{C} | f : X \times Y \to \mathbb{R}^2, |f(x,y,1) - f(x,y,2)| \text{ increasing in } y\}$, where $f(.,.,1), f(\cdot,\cdot,2)$ denote the first and second coordinate, respectively, and $\mathcal{C}$ the metric space of bounded and continuous functions endowed with the sup-norm.

The next step in the proof is to show that fixed point of $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is also weakly increasing in $z$. To show we first establish the following result.

**Lemma A.1:** *$F$ is a closed set in $\mathcal{C}$*

*Proof.* Consider an $f' \notin F$ that is the limit of a sequence $\{f_n\}, f_n \in F, \forall n \in \mathbb{N}$. Then there exists an $y_1 < y$ such that $f'(x,y_1,1) - f'(x,y_1,2) > f'(x,y,1) - f'(x,y,2)$, while $f_n(x,y_1,1) - f_n(x,y_1,2) \leq f_n(x,y,1) - f_n(x,y,2)$, for every $n$. Define a sequence $\{s_n\}$ with $s_n = f_n(x,y_1,1) - f_n(x,y_1,2) - f_n(x,y,1) - f_n(x,y,2)$. Then $s_n \geq 0$, $\forall n \in \mathbb{N}$. A standard result in real analysis guarantees that for any limit $s$ of this sequence, $s_n \to s$, it holds that $s \geq 0$. Hence $f'(x, y_1, 1) - f'(x, y_1, 2) \leq f'(x, y, 1) - f'(x, y, 2)$, contradicting the premise. $\square$

Thus, the fixed point exhibits this property as well and the optimal quit policy is a reservation-$z$ policy given $1 - \delta - \lambda(\theta^*) > 0$. Furthermore, since $\lambda(\theta)$ is concave and positively valued, $\lambda'(\theta)(M - W^U) = k$ implies that job finding rate is also (weakly) increasing in $z$. This completes the proof of Lemma 4.

**Proof of Proposition 1** The proof is basically an exercise to construct candidate equilibrium functions from the fixed point value and policy functions of $T$, and then verify these satisfy all equilibrium conditions. From the fixed point functions $M(p, z)$ and $W^U(p, z)$ with policy functions $\gamma_\theta^T(p, z)$ and $\gamma_W^T(p, z)$ define the function $J(p, z, W) = \max\{M(p, z) - W, 0\}$, and $\theta(p, z, W)$ and $V(p, z, W)$ from $0 = V(p, z, W) = -k + q(\theta(p, z, W))J(p, z, W)$. Also define $W^E(p, z) = M(p, z) - k/q(\gamma_\theta^T(p, z)) = \gamma_W^T(p, z)$ if $M(p, z) > W^U(p, z)$, and $W^E(p, z) = M(p, z)$ if $M(p, z) \leq W^U(p, z)$, using $W^U(p, z)$ from the fixed point. Finally, define $\delta(p, z) = \delta^T(p, z)$, $\sigma(p, z) = \delta^T(p, z)$, $\rho(p, z) = \rho^T(p, z)$, $\tilde{W}^{E\prime}(p', z') = \gamma_W^T(p', z')$, $\tilde{W}^f = \gamma_W^T(p, z)$ and $w(p, z)$ derived from (6) given all other functions.

Now (6) is satisfied by construction. Given the construction of $J(p, z, W)$, $\theta(p, z, W)$ indeed satisfies the free entry condition. $J(p, z, W)$ is satisfied if we ignore the maximization problem. However, $w(p, z, W^E)$, $\tilde{W}^{E\prime}(p', z'|p, z, W^E)$ satisfying (6) all yield the same $J(p, z, W^E)$ as long as $M(p, z) \geq W^E > W^U(p, z)$, $M(p', z') \geq \tilde{W}^{E\prime}(p', z'|p, z, W^E) \geq W^U(p, z)$, which is indeed the case. Hence, $J(p, z, W)$ is also satisfies (7), provided the separation decisions coincide, which is the case as the matches are broken up if and only if it is efficient to do so according to $M(p, z)$ and $W^U(p, z)$.

Given the constructed $W^U(p, z)$, the constructed $\rho(p, z)$ also solves the maximization decision in the decentralized setting. Finally, we have to verify $W^U(p, z)$. It's easy to see that this occurs if $S(p, z) = S^T(p, z)$. Consider the unemployed worker's application maximization problem that gives $S(p, z)$,

$$\max_{\{\tilde{\theta}(p,z), \tilde{W}(p,z)\}} \lambda(\tilde{\theta}(p, z))(\tilde{W}(p, z) - \tilde{W}^U(p, z)),$$

subject to

$$J(p, z, \tilde{W}(p, z))q(\tilde{\theta}(p, z)) - k = 0.$$

From Lemma 1, we know that $\tilde{W}(p, z) = M(p, z) - J(p, z, W(p, z))$. Substitute in the latter equation to get rid of $J$, and we see that the maximization problem for $S^T(p, z)$ is equivalent to the problem for the worker in the competitive equilibrium. Finally, $\tilde{W}^f(p,z)$ is consistent with profit maximization and thus here with the free entry condition, since any $W \in [W^U(p,z), M(p,z)]$ by construction of $\theta(p,z,W)$ is made consistent with free entry.

Hence, the constructed value functions and decision rules satisfy all conditions of the equilibrium, and the implied evolution of the distribution of employed and unemployed workers will also be the same.

Uniqueness follows from the same procedure in the opposite direction, by contradiction. Suppose the equilibrium is not unique. Then a second set of functions exists that satisfy the equilibrium conditions. Construct $\hat{M}$ from these. Since in any equilibrium the breakup decisions have to be efficient, the reallocation decision and application is captured in $T$, $\hat{M}$ and $\hat{W}^U$ must be fixed point of $T$, contradicting the uniqueness of the fixed point established by Banach's Fixed Point Theorem. This completes the proof of Proposition 1.

**Proof of Proposition 2** Consider the mapping $T^{SP}$, with 'aggregate' states at the moments of decision making abbreviated to $(p', \{z'_i, e'_i, u'_i\}_I) \stackrel{def}{=} \mathcal{S}'$. The values are 'measured' at the beginning of the period, and tomorrow is denoted by a prime.

$$T^{SP}W^{SP}(p, \{z_i, e_i, u_i\}_I) = \max_{\{d_i(\mathcal{S}'), \rho_i(\mathcal{S}'), v_i(\mathcal{S}')\}} \int_I (u_i b + e_i y(p, z_i)) di$$
$$+ \beta \mathbb{E}_{\mathcal{S}'} \left[ -\left( c \int_I \rho_i(S') u_i di + k \int_I v_i(S') di \right) + W^{SP}(p', \{z'_i, e'_i, u'_i\}_I) \right]$$

subject to

$$u'_i = (1-\rho_i)u_i + (e_i - e'_i) + \int_I \rho_j u_j dj$$
$$e'_i = (1-d_i)e_i + (1-\rho_i)u_i \lambda \left( \frac{v_i}{(1-\rho_i)u_i} \right)$$
$$\mathcal{S}_0 \text{ given}, \; v_{i0} = 0, \; \forall i.$$

Note that the decisions of the social planner here are: (i) reallocate people on an island ($\rho_i$), (ii) break up matches ($d_i$), (iii) set the number of vacancies for the unemployed ($v_i$). With $v_i = \theta_i(1-\rho_i)u_i$, we can change the last decision variable to the tightness, by substitution.

The next step is to show that as $W^{SP}$ is linear in $u_i$ and $e_i$, then $T^{SP}$ maps this function into a function that is likewise linear in these variables. Linearity of $W^{SP}$ implies that it can be written as

$$W^{SP}(\mathcal{S}) = \int_I \left( W^U(p, z_i) u_i + M(p, z_i) e_i \right) di.$$

Moreover, under linearity the value of reallocation for $u$ workers leaving their island is $\int_I W^U(p, z_j)udj - uc$,, and hence we can write

$$T^{SP}W^{SP}(p, \{z_i, e_i, u_i\}_I) = \max_{\substack{d_i(\mathcal{S}'), \rho_i(\mathcal{S}') \\ v_i(\mathcal{S}')}} \int_I \Bigg( u_i b + \beta \mathbb{E}_{p', z_i'} \Bigg[ \left( \int_I W^U(p', z_j')dj - c \right) \rho_i(\mathcal{S}')u_i$$

$$+ (1-\rho_i(\mathcal{S}'))u_i \left[ \lambda\left(\theta_i(\mathcal{S}')\right) M(p', z_i') - \theta_i(\mathcal{S}')k + (1-\lambda\left(\theta_i(\mathcal{S}')\right))W^U(p', z_i') \right] \Bigg]$$

$$+ e_i(p, z_i)y(p, z_i) + \beta \mathbb{E}_{p', z_i'} \left[ e_i(p', z_i') \left[ (1-d_i(\mathcal{S}'))M(p', z_i') + d_i(\mathcal{S}')W^U(p', z_i') \right] \right] \Bigg) di$$

Further we can completely isolate the terms with $u_i$ and $e_i$ and within these terms we can isolate $u_i$ and $e_i$ and take the maximization over the remaining terms such that

$$T^{SP}W^{SP}(p, \{z_i, e_i, u_i\}_I) = \int_I \left[ W^U_{max}(p, z_i)u_i + M_{max}(p, z_i)e_i \right] di$$

where

$$W^U_{max}(p, z_i) = \max_{\substack{\rho_i(\mathcal{S}') \\ v_i(\mathcal{S}')}} \Bigg\{ b + \beta \mathbb{E}_{p', z_i'} \Bigg[ \left( \int_I W^U(p', z_j')dj - c \right) \rho_i(\mathcal{S}')$$

$$+ (1-\rho_i(\mathcal{S}')) \left[ \lambda(\theta_i(\mathcal{S}') \left[ M(p', z_i') - W^M(p', z_i') \right] - \theta_i(\mathcal{S}')k + W^U(p', z_i') \right]$$

$$M_{max}(p, z_i) = \max_{d_i(\mathcal{S}')} \Bigg\{ y(p, z_i)$$

$$+ \beta \mathbb{E}_{p', z_i'} \Bigg[ \left( d_i(\mathcal{S}')W^U(p', z_i') + (1-d_i(\mathcal{S}'))M(p', z_i') \right) \Bigg] \Bigg\}$$

The maximized value depends only on $p$ and $z_i$, and hence $T^{SP}$ maps a value function that is linear in $u_i$ and $e_i$ into a value function with the same properties. Moreover, using the definitions of $W^U_{max}$ and $M_{max}$ it follows that from the fixed point of the mapping $T^{SP}$ we can derive a $W^{U*}_{max}$ and $M^*_{max}$ that constitutes a fixed point to $T$, and vice versa. Hence, the allocations of the fixed point of $T$ are allocations of the fixed point of $T^{SP}$, and hence the equilibrium allocation is the efficient allocation. This completes the proof of Proposition 2.

**Proof of Proposition 3** The reservation island productivity for the competitive and search case, satisfies, respectively,

$$b + \beta \int_{\underline{z}}^{\bar{z}} \frac{\max\{y(p, z), y(p, z_c^r)\}}{1-\beta} dF(z) - \frac{y(p, z_c^r)}{1-\beta} - c_c = 0$$

$$\frac{(1-\eta)k}{\eta} \left( \beta \int_{\underline{z}}^{\bar{z}} \frac{\max\{\theta(p, z), \theta(p, z^r)\}}{1-\beta} dF(z) - \frac{\theta(p, z^r)}{1-\beta} \right) - c_s = 0$$

Using (10), the response of the reservation island productivity, for the competitive, and the frictional case, is then given by

$$\frac{dz_c^r}{dp} = \frac{\beta F(z_c^r)\frac{y_p(p,z_c^r)}{y_z(p,z_c^r)} + \beta\int_{z_c^r}^{\bar{z}} \frac{y_p(p,z)}{y_z(p,z_c^r)} dF(z) - \frac{y_p(p,z_c^r)}{y_z(p,z_c^r)}}{1-\beta F(z_c^r)} \tag{25}$$

$$\frac{dz^r}{dp} = \frac{\beta F(z^r)\frac{y_p(p,z^r)}{y_z(p,z^r)} + \beta\int_{z^r}^{\bar{z}} \frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}\frac{y_p(p,z)}{y_z(p,z^r)} dF(z) - \frac{y_p(p,z^r)}{y_z(p,z^r)}}{1-\beta F(z^r)} \tag{26}$$

Choosing $c_c, c_s$ appropriately such that $z_c^r = z^r$, the above expressions imply that $\frac{dz^r}{dp} > \frac{dz_c^r}{dp}$ if $\frac{\theta(p,z)}{w(p,z)-b} > \frac{\theta(p,z^r)}{w(p,z^r)-b},\ \ \forall\ z > z^r$. Hence we now need to show that $\frac{\theta(p,z)}{w(p,z)-b}$ is increasing in $z$.

$$\frac{d\left(\frac{\theta(p,z)}{w(p,z)-b}\right)}{dz} = \frac{\theta y_z(p,z)}{(w-b)^2} - \theta\left(\frac{(1-\eta)+(1-\eta)\beta\frac{\theta}{w-b}k}{(w-b)^2}\right) y_z(p,z),$$

which has the same sign as $\eta - (1-\eta)\beta k\frac{\theta}{w-b}$ and the same sign as

$$\begin{aligned}&\eta(1-\eta)(y(p,z)-b) + \eta(1-\eta)\beta\theta k - (1-\eta)\beta\theta k\\ &\qquad = (1-\eta)(\eta(y(p,z)-b) - (1-\eta)\beta\theta k).\end{aligned}$$

But $\eta(y(p,z)-b) - (1-\eta)\beta\theta k = y(p,z) - w > 0$ and thus we have established Part *1* of the Proposition.

For Part *2*, note that modularity implies that $y_p(p,z) = y_p(p,\tilde{z}),\ \forall z > \tilde{z}$; while supermodularity implies $y_p(p,z) \geq y_p(p,\tilde{z}),\ \forall z > \tilde{z}$. Hence modularity implies

$$\frac{dz_c^r}{dp} = \frac{1}{1-\beta F(z_c^r)}\frac{y_p(p,z_c^r)}{y_z(p,z_c^r)}\left(\beta F(z_c^r) + \beta\int_{z_c^r}^{\bar{z}}\frac{y_p(p,z)}{y_p(p,z_c^r)}dF(z) - 1\right) < 0,\ \forall\ \beta < 1.$$

In the case with frictions,

$$\frac{dz^r}{dp} = \frac{1}{1-\beta F(z^r)}\frac{y_p(p,z^r)}{y_z(p,z^r)}\left(\beta F(z^r) + \beta\int_{z^r}^{\bar{z}}\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}\frac{y_p(p,z)}{y_p(p,z^r)}dF(z) - 1\right).$$

If we can show that the integral becomes large enough, for $c$ large enough, to dominate the other terms, we have established the claim. First note that $\frac{y_p(p,z)}{y_p(p,z^r)}$ is weakly larger than 1, for $z > z^r$ by the (super)modularity of the production function. Next consider the term $\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}$. Note that

$$\lim_{z\downarrow y^{-1}(b;p)}\frac{\theta(p,z)}{w(p,z)-b} = \frac{\lambda(\theta(p,z))}{1-\beta+\beta\lambda(\theta(p,z))} = 0,$$

because $\theta(p,z)\downarrow 0$, as $y(p,z^r)\downarrow b$. Hence, fixing a $z$ such that $y(p,z) > b$, $\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)} \to \infty$, as $y(p,z^r)\downarrow b$. Since this holds for any $z$ over which is integrated, the integral term becomes unboundedly large, making $dz^r/dp$ strictly positive if reservation $z^r$ is low enough. Since the integral rises continuously but slower in $z^r$ than the also continuous term $\frac{\theta(p,z^r)}{1-\beta}$, it can be readily be established that $z^r$ depends continuously on $c$, and strictly negatively so as long as $y(p, z^r) > b$ and $F(z)$ has full support. Moreover, for some $\bar{c}$ large enough, $y(p, \underline{z}^r) = b$. Hence, as $c \uparrow \underline{z}^r$, $\frac{dz^r}{dp} > 0$.

This completes the proof of Proposition 3.

# B Omitted Derivations

**Derivation of the 'Pissarides wage equation'** Given that an employed worker value in steady state is

$$W^E(p,z) = w(p,z) + \beta(1-\delta)W^E(p,z) + \beta\delta W^U(p,z),$$

then

$$W^E(p,z) - W^U(p,z) = w(p,z) - b - \beta\lambda(\theta(p,z))(W^E(p,z) - W^U(p,z)) + \beta(1-\delta)(W^E(p,z) - W^U(p,z)),$$

or

$$W^E(p,z) - W^U(p,z) = \frac{w(p,z) - b}{1 - \beta(1-\delta) + \beta\lambda(\theta(p,z))}.$$

From the combination of the free entry condition and the Hosios condition, we have

$$\eta \frac{w(p,z) - b}{1 - \beta(1-\delta) + \beta\lambda(\theta(p,z))} = (1-\eta)k/q(\theta(p,z)). \tag{27}$$

Moreover, from the value of the firm, we have

$$\frac{k}{q(\theta(p,z))} = \frac{y(p,z) - w(p,z)}{1 - \beta(1-\delta)}$$

Solving the latter equation for $w(z)$ gives

$$w(p,z) = y(p,z) - \frac{k}{q(\theta(p,z))}(1 - \beta(1-\delta)).$$

Substituting this in (27), we find

$$\eta(y(p,z) - b) - \frac{k}{q(\theta(p,z))}(1 - \beta(1-\delta)) - \beta\theta(p,z)(1-\eta)k = 0. \tag{28}$$

If we replace the middle term with $y(p,z) - w(p,z)$, we get the desired wage equation.