

Optimal Devaluations

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Abstract

We analyze optimal policy in a simple small open economy model with price setting frictions. In particular, we study the optimal response of the nominal exchange rate following a terms of trade shock. We depart from the New Keynesian literature in that we explicitly model internationally traded commodities as intermediate inputs in the production of local final goods and assume that the small open economy takes this price as given. This modification is not only in line with the long standing tradition of small open economy models, but also drastically changes the optimal movements in the exchange rate. We perform our analysis in the tradition of optimal dynamic Ramsey problems, so we characterize optimal allocation and the government policies that implement it. We show that, while we can derive general second best policy principles, the exact way the nominal exchange rate ought to comove with the terms of trade depends on specific details of the model.

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1. INTRODUCTION

The purpose of this paper is to study the optimal movements in the nominal exchange rate, following a change in the price of a commodity a small open economy actively trades in international markets. This question is very important for many economies in the world. Indeed, commodity prices are very volatile over time and, in many cases, exports of commodities are a sizable fraction of foreign trade. In Figure 1, we plot monthly data on prices for a set of commodities for the last 10 years. The prices are expressed in constant dollars, as a fraction of the price at the beginning of the sample, in January 2000. In Table 1 we report the principal commodity exports for a selection of small open economies and their shares in total good exports. We also report the aggregate share of good exports over total exports, and of total exports over gross domestic product.¹

Concern regarding shocks to commodity prices runs very high in the political agenda of these countries. For small open economies (say Chile) a drop in the exportable commodity price (cooper) is seen as recessionary; the same happens following an increase in the price of the importable commodity (oil).² It is precisely to hedge against this uncertainty that in recent years, countries in which the government either owns or taxes the production of a particular commodity, like Norway (oil) and Chile (cooper) passed legislation forcing the Treasury to save in foreign markets in periods when the commodity prices are "high", in order to be able to spend more in times in which the prices are "low".

While it is clear that international commodity prices volatility can give rise to fiscal policies like the ones just described, it is less clear what are the implications, if any, regarding monetary and exchange rate policy. In small open economies (SOE), movements in the nominal exchange rate are important shock absorbers. In a world with fully flexible prices, this should not be important,

¹Total imports of commodities can also be large, but they are not so concentrated in a few goods. That is why we do not report a table similar to Table 1 for imports.

²Chile imported over 90% of the oil consumed during the last 10 years.

but inefficient real effects could follow after shocks in the presence of nominal rigidities as it has been emphasized in the "new open economy macroeconomics" literature. As we will argue, however, the literature has largely ignored the effect of commodity price shocks: this is the main theme of our paper.

The one we address, is a central question for policy design in small open economies. For example, both New Zealand and Chile have explicitly adopted an inflation targeting policy. This means that the Central Bank defines an inflation rate on a particular price index that it will choose to target as its main policy objective. Therefore, the Central Bank abstains from foreign exchange interventions so the nominal exchange rate is fully market determined.

However, Central Bank can typically deviate from that policy, according to the law, under "special circumstances". The Central Bank of Chile did so in April 2008 and announced a program for buying reserves (for an amount close to 40% of the existing stock) after the nominal exchange rate went from over 750 pesos per dollar in March 2003 to below 450 in March 2008. The program was suspended in September with almost 70% completed, after the exchange rate jumped back to around 650 pesos. A new program to buy reserves was announced in January 2011, with a total amount over 40% of the existing stock. The exchange rate last January was around 475 pesos per dollar. The justification used by the board of the Central Bank of Chile was "The international economy presents an unusual state, characterized by high commodity prices, low interest rates, slow recovery of the developed economies and depreciation of the US dollar. In addition, emerging economies are experiencing strong growth and their currencies are appreciating".³

Is this an optimal policy in a small open economy facing commodity prices shocks? The model we analyze in this paper is a step towards providing an answer to that question.

Following the seminal work of Obstfeld and Rogoff (1995, 1996) there has been growing interest

³The statement can be found in <http://www.asipla.cl/2011/01/04/banco-central-interviene-el-mercado-cambiario-con-compra-de-us12-000-millones/>. The translation to English has been made by the authors.

in studying optimal policy in open economies with frictions in the setting of prices or wages. A branch of the literature, like Obstfeld and Rogoff (2000), Engel (2001) and many others⁴ focused on the two-countries case. This literature emphasizes the relationship between the strategic interactions in two-countries models, and optimal exchange rate policy, and in most cases it focused on the flexible versus fixed exchange rate regimes debate. Gali and Monacelli (2005) specifically consider the case of the small open economies and several other papers followed, like Faia and Monacelli (2008) and de Paoli (2009).

The main innovation of our paper relative to the New Keynesian small open economy literature just cited is to explicitly model commodities⁵ as intermediate goods in production. In previous papers, only domestic inputs enter into the production function of domestic final goods. The final goods are produced by local monopolists and are traded internationally. In our model, domestic inputs *and* traded commodities enter the production function of the final goods. Then, as in the previous models, the final goods are produced by local monopolists and can be traded internationally.

As we will clearly demonstrate in the paper, this modification is very important for two reasons. First, in Gali and Monacelli (2005), "adverse" terms of trade shocks—an increase in the price of importables⁶—are, contrary to the concerns mentioned above, expansionary. The reason is that a reduction in the international relative price of local *final* goods implies, via a substitution effect in preferences, an increase in world—and local—demand for the local composite good which in turns increases local production. On the contrary, in our model, when the increase is on the price of the *intermediate* importable—relative to the exportable intermediate—the units of labor required to import one unit of the importable intermediate goes up and it is therefore

⁴An incomplete list also includes Corsetti and Pesenti (2001) and (2005), Devereux and Engel (2003), Benigno and Benigno (2003), Duarte and Obstfeld (2004), Ferrero (2005) and Adao, Correia and Teles (2005).

⁵In our model, commodities are intermediate inputs that are traded internationally in perfectly competitive market, whose prices are exogenous to the the small open economy we consider.

⁶Our model follows the literature in that final goods are produced by monopolists, so the price of exportables is endogenous, even in the small open economy case. Thus, a negative shock to the terms of trade means an increase in the price of importables.

contractionary. Second, in the model without traded commodities, the shock to the terms of trade does not change local costs, so it does not interact in an interesting way with the domestic price frictions.⁷ To the extent that the emphasis is not to discuss the optimal policy response following terms of trade shocks—as it is the case with the previous literature—this may be inessential. Given the emphasis of this paper, this is a key distinction.

On the methodological front, we also depart from the literature - with the exception of Adao, Correia and Teles (2009)⁸—in that we do not allow for lump-sum taxes. We consider distorting fiscal instruments, as in Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991) and Correia, Nicolini and Teles (2008). The main advantage of our approach is that we can fully characterize the optimal allocation and the optimal policy that decentralizes it under mild assumptions regarding preferences. Gali and Monacelli (2007), de Paoli (2009), and others, only solve for linear approximations, very useful to compare alternative rules, but less so to fully characterize the behavior of policy targets—as the nominal exchange rate—as we do.

We study a labor only representative agent economy with final goods produced by monopolistically competitive firms—so firms have power to set prices—and tradable intermediate inputs—so we can analyze the optimal policy response following terms of trade shocks. The intermediate good is produced by perfectly competitive firms that take the international price as given and produce using domestic labor only. The price of the importable intermediate input is also given to the country. It is the introduction of these internationally traded commodities that is novel in our model—relative to the recent literature on small open economies. We also assume that money enters the utility function.⁹

The policy instruments that we consider are consumption and labor income taxes, dividend

⁷This is also a consequence of the Dixit-Stiglitz formulation. As demands faced by monopolist of final goods have constant elasticities, changes in demand affect quantities sold, but not prices, since domestic marginal costs are not affected by changes in the price of foreign final goods.

⁸They also ignore commodities in most of their analysis. This is important for their results, see Hevia and Nicolini (2010a).

⁹This departs from the tradition of the recent literature that usually considers cashless economies. But we will also consider that limiting case.

taxes, and monetary policy. We also allow the government to issue state contingent bonds. With these, the government has a complete set of instruments, as defined in Chari and Kehoe (1999). We abstract from the question of the best intermediate target for monetary policy and also from the question of implementability. We characterize sequences of $\{R_t, M_t, E_t\}_{t=0}^{\infty}$ that are consistent with the optimal allocation, but we abstract from the bigger question of how to implement that allocation. It is well known that while exchange rate rules implement a unique allocation, both interest rate rules and money rules lead, in general, to indeterminacy.

As it is standard in Ramsey analysis, we abstract from time inconsistency and assume full commitment. Thus, whichever role the exchange rate can have in fostering good—or bad!—reputation will be absent in this analysis.

Most of the analysis will be done assuming that some prices must be set in advance. We will also show that if we only consider the limiting cashless economy case—as the New Keynesian literature does—all the results extend to most other forms of price or wage rigidity, including Calvo, the most commonly used.

The model we analyze can easily accommodate tradable final goods, as in previous literature.¹⁰ It turns out, however, that allowing for tradable final goods complicates the analysis without adding any interesting result. Thus, in the body of the paper we assume that the intermediate goods are the only ones that are tradable. In an appendix, to make the comparison with existing New Keynesian literature more straightforward, we solve the model assuming that final goods are also tradable and show that all results go through.

We start by showing that the general, second best, policy principles of Correia, Nicolini and Teles (2008)¹¹ extend to an open economy: optimal policy ought to fully stabilize the “right” price index and reproduce the flexible prices allocation. In this sense, frictions on the setting of

¹⁰As those are the only goods—besides labor—IN THEIR MODELS they must be tradable.

¹¹For first best examples, see Obstfeld and Rogoff (2002), Woodford (2003), Gali Monacelli (2007) among others.

prices are irrelevant for policy in the open economy case as well.

We then move to the core of the paper and analyze the optimal movements in the nominal exchange rate following foreign price shocks to the domestic economy. In the simplest version of the model, with a single traded commodity, we show that the nominal exchange rate must fully insulate the economy from shocks to the commodity price in order to implement the optimal allocation. In particular, the nominal exchange rate must go up (down) whenever the foreign price of the commodity goes down (up), so as to fully stabilize the price of the commodity in domestic currency. However, in a more general model in which there are two commodities, one importable and one exportable, the (log of the) nominal exchange rate must stabilize a weighted average of the (log of the) prices of both the exportable and the importable commodity in terms of domestic currency. The results are robust to having land in the production function of the commodity or labor in the production function of the intermediate goods.

We also analyze the optimal behavior of the nominal exchange rate following productivity shocks, so as to characterize the optimal cyclical properties of the nominal exchange rate. Following the general second best principle, optimal policy should stabilize the nominal marginal cost of domestic monopolists. It turns out, however, that the application of that principle implies different optimal response of the nominal exchange rate depending on the source of the shock: if there is a negative productivity shock on the production of final goods, the nominal exchange rate must go down, while if it is on the production of the exportable commodity, it must go up. Thus, the optimal cyclical properties of the nominal exchange rate cannot be described in reference to the cycle: the source of the shock generating the cycle matters.

The paper proceeds as follows. Section 2 presents the model and solves for an equilibrium. Section 3 defines the Ramsey problem and establishes necessary conditions for a second best. Section 4 discusses the optimal response of the nominal exchange rate to terms of trade shocks. It also discusses the optimal movements of the nominal exchange rate following productivity shocks and describes the optimal fiscal policy.

2. THE MODEL

The model is composed of a small open economy, which we call home, and the rest of the world. In the benchmark model, only commodities that are used as intermediate inputs in the production of final goods can be internationally traded. (Appendix B considers an extension of the model that also includes trade in final goods.) The domestic economy is inhabited by households, competitive firms that produce one of the tradeable commodities, a continuum of firms that produce differentiated final goods, and a government. Both commodities can be traded with the rest of the world. The government of the home economy sets monetary and fiscal policy and must raise taxes to pay for exogenous expenditures. Monetary policy consists of rules for the money supply—or the nominal interest rate—and the exchange rate. Fiscal policy consists of labor taxes, consumption taxes, export taxes, and dividend taxes.

Time is denoted by $t = 0, 1, \dots, \infty$. We let μ_t denote the state of the economy at period t and $\mu^t = (\mu_0, \mu_1, \dots, \mu_t)$ the history of states up to time t . We denote by $\pi(\mu^t)$ the probability of μ^t conditional on μ_0 . We assume that the state μ_t belongs to a finite set of events for all t .

Households. A representative household has preferences over contingent sequences of a non-traded composite consumption good $C(\mu^t)$, labor $N(\mu^t)$, and real money balances $m(\mu^t)$ represented by the utility function

$$\sum_{t=0}^{\infty} \sum_{\mu^t} \beta^t U(C(\mu^t), N(\mu^t), m(\mu^t)) \pi(\mu^t), \quad (1)$$

where $0 < \beta < 1$ is a subjective discount factor. Real money balances are defined as $m(\mu^t) = M(\mu^t) / [P(\mu^t)(1 + \tau^c(\mu^t))]$ where $M(\mu^t)$ is end-of-period money balances, $P(\mu^t)$ is the home price index the composite good $C(\mu^t)$, and $\tau^c(\mu^t)$ is a consumption tax. Thus, consumers derive utility from the purchasing power of their monetary holdings. The function $U(\cdot)$ is twice differentiable, concave, increasing in consumption, and decreasing in labor.

The composite good $C(\mu^t)$ is defined as

$$C(\mu^t) = \left[\int_0^1 c_i(\mu^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (2)$$

where $c_i(\mu^t)$ denotes consumption of a non-traded final good variety $i \in [0, 1]$, and $\theta > 1$ is the elasticity of substitution across varieties.

Financial markets are complete. We let $B(\mu^t, \mu_{t+1})$ and $B^*(\mu^t, \mu_{t+1})$ denote one-period discount bonds denominated in domestic and foreign currency respectively. These are bonds issued at period t that pay one unit of the corresponding currency at period $t+1$ on the event μ_{t+1} and zero otherwise. The price in terms of domestic currency of the domestic currency bond is denoted by $Q(\mu^{t+1}|\mu^t)$. Likewise, the price in foreign currency of the foreign bond is denoted by $Q^*(\mu^{t+1}|\mu^t)$. For any integer $k > 0$, let $Q(\mu^{t+k}|\mu^t) = Q(\mu^{t+1}|\mu^t) Q(\mu^{t+2}|\mu^{t+1}) \dots Q(\mu^{t+k}|\mu^{t+k-1})$ be the price of one unit of domestic currency at history μ^{t+k} in terms of domestic currency at history μ^t . An analogous definition holds for $Q^*(\mu^{t+k}|\mu^t)$.

An asset traded at time t that pays one unit of home currency at $t+1$ for sure costs $\sum_{\mu_{t+1}} Q(\mu^{t+1}|\mu^t)$. We thus define the gross nominal interest rate as $R(\mu^t) = \left[\sum_{\mu_{t+1}} Q(\mu^{t+1}|\mu^t) \right]^{-1}$.

The household's sequence budget constraint is given by

$$\begin{aligned} & (1 + \tau^c(\mu^t)) \int_0^1 P_i(\mu^t) c_i(\mu^t) di + M(\mu^t) \\ & + \sum_{\mu_{t+1}} [Q(\mu^{t+1}|\mu^t) B(\mu^{t+1}) + S(\mu^t) Q^*(\mu^{t+1}|\mu^t) B^*(\mu^{t+1})] \leq \\ & W(\mu^t) (1 - \tau^n(\mu^t)) N(\mu^t) + M(\mu^{t-1}) + B(\mu^t) + S(\mu^t) B^*(\mu^t), \end{aligned} \quad (3)$$

where $P_i(\mu^t)$ denotes the domestic currency price of variety $i \in [0, 1]$, $S(\mu^t)$ is the nominal exchange rate between domestic and foreign currency, $W(\mu^t)$ is the nominal wage rate, and $\tau^n(\mu^t)$ is a labor income tax. Firms' profits are fully taxed and, therefore, do not enter into the

budget constraint.

Using the budget constraint (3) at periods t and $t + 1$ and rearranging gives the no-arbitrage condition between domestic and foreign bonds

$$Q(\mu^{t+1}|\mu^t) = Q^*(\mu^{t+1}|\mu^t) \frac{S(\mu^t)}{S(\mu^{t+1})}. \quad (4)$$

As long as (4) is satisfied, households are indifferent about holding domestic or foreign bonds.

We thus assume, without loss of generality, that households only hold domestic bonds.

For convenience, we work with the present value budget constraint. Iterating forward on (3) and imposing the no-Ponzi condition $\lim_{t \rightarrow \infty} \sum_{\mu^t} Q(\mu^t|\mu^0) [M(\mu^t) + B(\mu^t)] \geq 0$ gives

$$\sum_{t=0}^{\infty} \sum_{\mu^t} Q(\mu^t|\mu^0) \left[\begin{aligned} & (1 + \tau^c(\mu^t)) \int_0^1 P_i(\mu^t) c_i(\mu^t) di \\ & + M(\mu^t) [R(\mu^t) - 1] / R(\mu^t) - W(\mu^t) (1 - \tau^n(\mu^t)) N(\mu^t) \end{aligned} \right] \leq 0. \quad (5)$$

Here $[R(\mu^t) - 1] / R(\mu^t)$ is the financial cost of holding money and we have assumed that initial financial wealth is zero, namely $M_{-1} + B(\mu^0) + S(\mu^0) B^*(\mu^0) = 0$.

Households choose $C(\mu^t)$, $c_i(\mu^t)$, $N(\mu^t)$, and $M(\mu^t)$ to maximize (1) subject to (2) and (5). Optimal choices are summarized by the following conditions. The demand for variety $c_i(\mu^t)$ conditional on $C(\mu^t)$ is given by

$$c_i(\mu^t) = C(\mu^t) \left(\frac{P_i(\mu^t)}{P(\mu^t)} \right)^{-\theta} \quad \text{for } i \in [0, 1] \quad (6)$$

where $P(\mu^t)$ is a price index defined as

$$P(\mu^t) = \left[\int_0^1 P_i(\mu^t)^{1-\theta} di \right]^{1/(1-\theta)}. \quad (7)$$

The optimal choice of the aggregates $C(\mu^t)$, $N(\mu^t)$, and $m(\mu^t)$ satisfy the first order conditions

$$\frac{\beta^t \pi(\mu^t) U_C(\mu^t)}{P(\mu^t)(1 + \tau^c(\mu^t))} = \frac{Q(\mu^t | \mu^0) U_C(\mu^0)}{P(\mu^0)(1 + \tau^c(\mu^0))}, \quad (8)$$

$$-\frac{U_N(\mu^t)}{U_C(\mu^t)} = \frac{W(\mu^t)(1 - \tau^n(\mu^t))}{P(\mu^t)(1 + \tau^c(\mu^t))}, \quad (9)$$

$$\frac{U_m(\mu^t)}{U_C(\mu^t)} = \frac{R(\mu^t) - 1}{R(\mu^t)}, \quad (10)$$

where, here and throughout the paper, $U_C(\mu^t)$ denotes the partial derivative of $U(\cdot)$ with respect to C evaluated at μ^t . An analogous definition is used for all partial derivatives.

Government. The government consumes an exogenous sequence $G(\mu^t)$ of a composite good identical to that of the households

$$G(\mu^t) = \left[\int_0^1 g_i(\mu^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \quad (11)$$

Given prices $P_i(\mu^t)$, the government minimizes the cost of acquiring the composite $G(\mu^t)$. Cost minimization then implies

$$g_i(\mu^t) = G(\mu^t) \left(\frac{P_i(\mu^t)}{P(\mu^t)} \right)^{-\theta} \quad \text{for } i \in [0, 1]. \quad (12)$$

To finance these expenditures, the government sets fiscal and monetary policy. Monetary policy consists of rules for the stock of money $M(\mu^t)$, the nominal interest rate $R(\mu^t)$, and the exchange rate $S(\mu^t)$. As is well known, the government can only choose one of these instruments; the others are determined endogenously in equilibrium. Which particular instrument is used by the government is irrelevant for our results. Fiscal policy consists of labor income taxes $\tau^n(\mu^t)$, consumption taxes $\tau^c(\mu^t)$, and dividend taxes $\tau^d(\mu^t)$. The government is able to issue bonds denominated in domestic and foreign currency, denoted by $B^g(\mu^t)$ and $B^{g*}(\mu^t)$ respectively. We

also assume that dividend taxes are set at 100 percent, or $\tau^d(\mu^t) = 1$ for all μ^t .¹² A government policy is denoted by $\omega \equiv \{M, \tau^n, \tau^c, \tau^d, B^g, B^{g*}\}$, where ω is a shorthand notation for the complete contingent plan $\omega(\mu^t)$ for all μ^t .

Commodities Sector. There are two tradeable commodities, denoted by x and z , that are used as intermediate inputs in the production of domestic final goods. The domestic economy, however, is able to produce only the x commodity; the z commodity must be imported. We denote by $P^x(\mu^t)$ and $P^z(\mu^t)$ the domestic currency prices of commodities x and z respectively.

The technology used to produce the x commodity is given by

$$X(\mu^t) = A^x(\mu^t) n(\mu^t) \quad (13)$$

where $n(\mu^t)$ is labor and $A^x(\mu^t)$ is a productivity shock. Competitive pricing then requires

$$P^x(\mu^t) A^x(\mu^t) = W(\mu^t). \quad (14)$$

Because the two commodities can be freely traded, the law of one price holds, namely

$$\begin{aligned} P^x(\mu^t) &= S(\mu^t) P^{x*}(\mu^t) \\ P^z(\mu^t) &= S(\mu^t) P^{z*}(\mu^t), \end{aligned} \quad (15)$$

where $P^{x*}(\mu^t)$ and $P^{z*}(\mu^t)$ are the foreign currency prices of the x and z commodities.

Domestic Final Goods Sector. Domestic final goods are produced using the commodi-

¹²If dividend taxes cannot be set at 100 percent, optimal monetary policy will use the inflation tax to partially tax monopolistic profits (Schmitt-Grohe and Uribe 2004). We believe that taxing profits is not the natural objective of monetary policy and thus assume that pure rents can be fully taxed.

ties x and z . Variety $i \in [0, 1]$ is produced using the Cobb-Douglas technology

$$y_i(\mu^t) = \frac{A^y(\mu^t) x_i(\mu^t)^\eta z_i(\mu^t)^{1-\eta}}{\eta^\eta (1-\eta)^{1-\eta}}$$

where $y_i(\mu^t)$ denotes output of variety i , $A^y(\mu^t)$ is a productivity shock common across varieties, and $0 < \eta < 1$.¹³

This production function and the law of one price (15) imply that the nominal marginal cost function can be written as $MC(\mu^t) = S(\mu^t) MC^*(\mu^t)$, where $MC^*(\mu^t)$ denotes the marginal cost measured in foreign currency and is defined as

$$MC^*(\mu^t) = \frac{P^{x*}(\mu^t)^\eta P^{z*}(\mu^t)^{1-\eta}}{A^y(\mu^t)}.$$

Cost minimization also implies that final goods firms choose the ratio of inputs

$$\frac{z_i(\mu^t)}{x_i(\mu^t)} = \left(\frac{1-\eta}{\eta} \right) \frac{P^{x*}(\mu^t)}{P^{z*}(\mu^t)} \text{ for } i \in [0, 1]. \quad (16)$$

Introducing (16) into the production function then gives

$$y_i(\mu^t) = x_i(\mu^t) A^y(\mu^t) [P^{x*}(\mu^t) / P^{z*}(\mu^t)]^{1-\eta} / \eta. \quad (17)$$

Varieties are produced by monopolists, some of whom are constrained to set prices in advance. Let $0 < \alpha < 1$ and assume that firms with index $i \in [0, \alpha]$, called ‘sticky firms’, set prices at period t before observing the state μ_t . On the other hand, firms in the interval $(\alpha, 1]$, called ‘flexible firms’, are allowed to set prices at t conditional on μ_t . Although the model exhibits complete technological and demand symmetry, the existence of price setting frictions introduces heterogeneity across firms.

¹³Our results generalize to any constant returns to scale production function.

Monopolist i faces the demand $c_i(\mu^t) + g_i(\mu^t)$ which, using (6) and (12), is given by

$$y_i^d(\mu^t) = \left(\frac{P_i(\mu^t)}{P(\mu^t)} \right)^{-\theta} [C(\mu^t) + G(\mu^t)]. \quad (18)$$

Flexible firms maximize dividends period by period by setting prices according to¹⁴

$$P_i(\mu^t) = \frac{\theta}{\theta - 1} S(\mu^t) MC^*(\mu^t) \quad \text{for } i \in (\alpha, 1]. \quad (19)$$

Sticky firms set prices at $t \geq 1$ conditional on information available up to period $t-1$. After μ_t is realized, sticky firms satisfy demand at the previously chosen price. Their problem is equivalent to choosing prices at $t-1$ to maximize the value of the next period's dividends

$$\max_{P_i(\mu^{t-1})} \sum_{\mu^t | \mu^{t-1}} Q(\mu^t | \mu^{t-1}) [P_i(\mu^{t-1}) - S(\mu^t) MC^*(\mu^t)] y_i^d(\mu^t).$$

The pricing rule is then

$$P_i(\mu^{t-1}) = \frac{\theta}{\theta - 1} \sum_{\mu^t | \mu^{t-1}} \nu(\mu^t) S(\mu^t) MC^*(\mu^t) \quad \text{for } i \in [0, \alpha], \quad (20)$$

where

$$\nu(\mu^t) = \frac{Q(\mu^t | \mu^{t-1}) P(\mu^t)^\theta [C(\mu^t) + G(\mu^t)]}{\sum_{\mu^t | \mu^{t-1}} Q(\mu^t | \mu^{t-1}) P(\mu^t)^\theta [C(\mu^t) + G(\mu^t)]}.$$

That is, the optimal price is set as a mark-up over a weighted average of the marginal cost across states. We assume that all sticky firms inherit a common price P_{-1}^h in period 0.

Foreign Sector Feasibility. Net exports measured in foreign currency are given by the

¹⁴Due to full taxation of dividends, pricing and production decisions are indeterminate. We think of firms maximizing after-tax dividends for $\tau_j^d(\mu^t) < 1$, and then consider the limit as $\lim_{j \rightarrow \infty} \tau_j^d(\mu^t) = 1$ for all μ^t .

value of net exports of commodity x minus the value of imports of commodity z

$$NX^*(\mu^t) = P^{x*}(\mu^t) \left[X(\mu^t) - \int_0^1 x_i(\mu^t) di \right] - P^{z*}(\mu^t) \int_0^1 z_i(\mu^t) di. \quad (21)$$

Thus, the net foreign assets of the country, denoted by $B^*(\mu^t)$, evolve according to

$$B^*(\mu^t) + NX^*(\mu^t) = \sum_{\mu^{t+1}|\mu^t} B^*(\mu^{t+1}) Q^*(\mu^{t+1}|\mu^t).$$

Solving this equation from period 0 forward, gives the economy foreign sector feasibility constraint measured in foreign currency at time 0

$$\sum_{t=0}^{\infty} \sum_{\mu^t} Q^*(\mu^t|\mu^0) NX^*(\mu^t) + B_0^* = 0. \quad (22)$$

3. THE RAMSEY PROBLEM

An equilibrium with sticky prices is defined in the usual fashion: an allocation and price system such that households maximize utility, firms maximize profits, interest rates satisfy $R(\mu^t) \geq 1$, all no-arbitrage conditions are satisfied, and all market clearing conditions hold; namely, constraint (22), $B^g(\mu^t) + B(\mu^t) = 0$, $B^{g*}(\mu^t) + B^*(\mu^t) = 0$, $N(\mu^t) = n(\mu^t)$, and $y^d(\mu^t) = y(\mu^t)$. By Walras's law, we ignore the government budget constraint as it is implied by the household's budget constraint and the feasibility conditions.¹⁵

Following the tradition of the Ramsey literature, we assume that the government is able to commit to a particular policy ω chosen at period zero. Let $a(\omega)$ denote the set of equilibrium allocations given policy ω . The image of the mapping $a(\omega)$, denoted by $\Omega(\alpha)$, is called the set of implementable allocations. That is, $\Omega(\alpha)$ is the set of allocations that can be implemented as an equilibrium for some government policy ω . In general, this set depends on the degree of price

¹⁵Here we are assuming, without loss of generality, that the government holds all net foreign assets.

stickiness α . The optimal allocation is the implementable allocation $a^R \in \Omega(\alpha)$ that maximizes the household's utility (1). The optimal policy ω^R is the government policy that implements the optimal allocation a^R as an equilibrium with sticky prices.

To characterize the optimal policy, the Ramsey taxation literature finds necessary and sufficient conditions that an allocation has to satisfy to be implementable as an equilibrium (Lucas and Stokey, 1983; Chari and Kehoe, 1999). In our model, however, these sufficient conditions cannot be characterized in terms of the allocation alone. The constraints imposed by the price setting restrictions make the equilibrium set $\Omega(\alpha)$ a difficult object to analyze. We thus follow a different approach and define a *relaxed* set $\tilde{\Omega}$ that contains the set of equilibrium allocations $\Omega(\alpha)$ for every degree of price stickiness α . The set $\tilde{\Omega}$ is defined in terms of necessary conditions that any equilibrium allocation must satisfy.

We first state these necessary conditions and next define the relaxed set $\tilde{\Omega}$.

Proposition 1. *Given prices $P_i(\mu^t)$ for $i \in [0, 1]$, any equilibrium allocation satisfies*

$$U_m(\mu^t) \geq 0 \quad (23)$$

$$\sum_{t=0}^{\infty} \sum_{\mu^t} \beta^t \pi(\mu^t) [U_C(\mu^t) C(\mu^t) + U_N(\mu^t) N(\mu^t) + U_m(\mu^t) m(\mu^t)] = 0, \text{ and} \quad (24)$$

$$NX^*(\mu^t) = P^{x*}(\mu^t) A^x(\mu^t) N(\mu^t) - MC^*(\mu^t) D(\mu^t) [C(\mu^t) + G(\mu^t)], \quad (25)$$

where

$$D(\mu^t) = \int_0^1 \left(\frac{P_i(\mu^t)}{P(\mu^t)} \right)^{-\theta} di \quad (26)$$

is an index of price dispersion across domestic final good firms. This index satisfies $D(\mu^t) \geq 1$, with equality if and only if $P_i(\mu^t) = P(\mu^t)$ for all $i \in [0, 1]$.¹⁶

Proof: See Appendix A.

¹⁶Formally, the equality must hold almost everywhere, which means for all $i \in [0, 1]$ except for those in a set of zero Lebesgue measure.

Condition (24) summarizes the household's optimization problem, and condition (25) rewrites the net exports equation using market clearing conditions and conditional input demands. In this equation, $MC^*(\mu^t)D(\mu^t)$ represents the unit cost in foreign currency of producing the home composite good. This cost is composed of two terms. The first is the marginal cost in foreign currency, $MC^*(\mu^t)$, which depends on technological factors and exogenous shocks. The second term, $D(\mu^t) \geq 1$, captures an additional cost of producing the home composite good that arises because firms that are otherwise identical are (potentially) producing different quantities due to the price setting restriction.

Definition: Given B_0^* , the relaxed set $\tilde{\Omega}$ is the set of aggregate allocations $\{C(\mu^t), N(\mu^t), m(\mu^t), NX^*(\mu^t)\}$ such that there are prices $P(\mu^t)$ and $P_i(\mu^t)$ for $i \in [0, 1]$ for which conditions (22), (23), (24), (25), and (26) hold, and the prices $P_i(\mu^t)$ and $P(\mu^t)$ satisfy

$$P(\mu^t) = \left[\int_0^1 P_i(\mu^t)^{1-\theta} di \right]^{1/(1-\theta)}. \quad (27)$$

The relaxed set $\tilde{\Omega}$ imposes less restrictions on the allocation than the equilibrium set $\Omega(\alpha)$. In effect, the relaxed set allows for firm specific prices $P_i(\mu^t)$, so it disregards the constraints imposed by the price setting restrictions (19) and (20) and by the no-arbitrage condition (4). Thus, the set of equilibrium allocations with sticky prices is contained in the relaxed set for any degree of price stickiness, $\Omega(\alpha) \subseteq \tilde{\Omega}$. Suppose now that $\tilde{a}$ is the allocation in the relaxed set $\tilde{\Omega}$ that maximizes the household's utility. It then follows that any equilibrium allocation delivers utility no greater than that attained under the allocation $\tilde{a}$. We will show that there is a government policy that implements $\tilde{a}$ as an equilibrium allocation with sticky prices for any degree of price stickiness. Therefore, $\tilde{a}$ is the optimal allocation with sticky prices.

The Relaxed Ramsey Problem. In this section we first characterize the relaxed Ramsey allocation $\tilde{a}$, and then show that it can be implemented as an equilibrium with sticky prices.

The relaxed Ramsey problem is to choose an aggregate allocation, domestic currency prices $P_i(\mu^t)$ and $P(\mu^t)$ for all $i \in [0, 1]$ and μ^t in the relaxed set $\tilde{\Omega}$ that maximizes the household's utility (1). Consider first the optimal price dispersion $D(\mu^t)$ satisfying (27).

Suppose that the relaxed Ramsey problem induces some heterogeneity in prices $P_i(\mu^t)$ for some μ^t , so that $D(\mu^t) > 1$. We arrive at a contradiction by noting that, by decreasing $D(\mu^t)$, the planner can increase the foreign currency value of its net exports (25). In particular, by Proposition 1, $D(\mu^t)$ is minimized subject to (27) when $P_i(\mu^t) = P(\mu^t)$ for all i , so that $D(\mu^t) = 1$. Thus, the relaxed Ramsey problem imposes equal prices across domestic final good firms at all dates and states.

Using $D(\mu^t) = 1$, we characterize the relaxed Ramsey allocation using Lagrangian methods. To that end, we define the following pseudo-utility function,

$$V(C, N, m) = U(C, N, m) + \lambda [U_C(C, N, m)C + U_N(C, N, m)N + U_m(C, N, m)m] \quad (28)$$

where λ is the Lagrange multiplier on constraint (24). We think of the planner as valuing consumption plans according to the pseudo-utility function. When $\lambda > 0$, this pseudo-utility function internalizes the impact that distorting taxes have on equilibrium utility.

We also attach multipliers $\varphi(\mu^t)$ to the non-negativity interest rate constraint (23), and a multiplier ξ to the feasibility constraint (22) after using $D(\mu^t) = 1$. The first order necessary conditions for $C(\mu^t)$, $N(\mu^t)$, and $m(\mu^t)$ are then given by

$$\beta^t \pi(\mu^t) V_C(\mu^t) = \xi Q^*(\mu^t | \mu^0) MC^*(\mu^t) \quad (29)$$

$$-\beta^t \pi(\mu^t) V_N(\mu^t) = \xi Q^*(\mu^t | \mu^0) P^{x*}(\mu^t) A^x(\mu^t) \quad (30)$$

$$V_m(\mu^t) + \varphi(\mu^t) U_{mm}(\mu^t) = 0. \quad (31)$$

These conditions, constraints (22), (24), and (25); and the complementary slackness condition $\varphi(\mu^t) U_m(\mu^t) = 0$ determines $\{C(\mu^t), N(\mu^t), m(\mu^t)\}$ and the multipliers λ , ζ , and $\varphi(\mu^t)$.¹⁷

We now state the main proposition of this section

Proposition 2. *There is a government policy ω^R that implements the relaxed Ramsey allocation $\tilde{a}$ as an equilibrium with sticky prices. Therefore, $\tilde{a} = a^R$ is the optimal allocation of the sticky prices economy. Furthermore, the policy ω^R that implements the optimal allocation is independent of the degree of price stickiness α .*

Proof: See Appendix A

The intuition for the last result is as follows. The relaxed set $\tilde{\Omega}$ does not restrict the behavior of final goods firms. Because any pair of final goods $c_i^h(\mu^t)$ and $c_{i'}^h(\mu^t)$ for $i \neq i'$ are imperfect substitutes in the consumption aggregator $C^h(\mu^t)$, it is optimal, if possible, to induce a symmetric allocation so that $c_i(\mu^t) = C(\mu^t)$ for all $i \in [0, 1]$ and μ^t . But a symmetric allocation requires $P_i(\mu^t) = P(\mu^t)$ for all i . Therefore, if a positive fraction of firms is constrained to set prices in advance, optimality then requires the prices of all domestic final goods to depend on information available up to period $t - 1$. We thus have

Corollary 1. *Final good prices must be perfectly forecastable one period in advance.*

4. OPTIMAL EXCHANGE RATE FLUCTUATIONS.

In this section we consider how different specifications of the basic model have different implications for optimal exchange rates. We will show that the optimal volatility and the covariance with the terms of the exchange rate depends on particular details of the model, such as which commodities enter the production function of the intermediate goods or on the source of the shock to the terms of trade (an increase in the price of the imported commodity or a decrease in

¹⁷Extending the methods of Lucas and Stokey (1983), we can also show that $\lambda > 0$ as long as the government's initial holding of domestic and foreign bonds is smaller than the present value of government expenditures.

the price of the exported commodity).

However, a general principle immediately follows after Proposition 2.

Corollary 2. *Fixed exchange rate regimes are not optimal.*

Note, indeed, that as optimality implies that goods prices must be perfectly forecastable, the following must hold

$$S(\mu^t) MC^*(\mu^t) - E_{t-1} S(\mu^t) MC^*(\mu^t) = 0$$

so

$$S(\mu^t) = \frac{1}{MC^*(\mu^t)} E_{t-1} S(\mu^t) MC^*(\mu^t).$$

As $MC^*(\mu^t)$ depends on the shock, the nominal exchange rate must depend on the shock also.¹⁸

More formally, the pricing equation (19) becomes

$$P(\mu^{t-1}) = \frac{\theta}{\theta - 1} S(\mu^t) \frac{P^{x*}(\mu^t)^\eta P^{z*}(\mu^t)^{1-\eta}}{A^y(\mu^t)}. \quad (32)$$

Then, for $P(\mu^{t-1})$ to be independent of μ_t , the exchange rate $S(\mu^t)$ must absorb all shocks to the marginal cost. Namely, unforeseen changes in commodity prices $P^{x*}(\mu^t)$ and $P^{z*}(\mu^t)$, and productivity $A^y(\mu^t)$.

We now discuss how this principle translates into movements in the nominal exchange rate.

Case 1. Only one commodity

Consider first a model with just the commodity x by taking the limit as $\eta \rightarrow 1$ in the benchmark model. Although a terms of trade shock is not clearly defined, this example is still useful to understand the mechanics of the model.

¹⁸This results is in - partial - contrast with Adao, Correia and Teles. The reason is, precisely, the introduction of commodities. For details, see Hevia and Nicolini (2011).

Evaluating (32) at $\eta = 1$ implies

$$P(\mu^{t-1}) = \frac{\theta}{\theta - 1} S(\mu^t) P^{x*}(\mu^t) / A^y(\mu^t)$$

Suppose that productivity is constant in both sectors, say at $A^y(\mu^t) = A^x(\mu^t) = 1$. In this special case, the previous equation shows that the nominal exchange rate must stabilize $S(\mu^t) P^{x*}(\mu^t)$. On the other hand, the nominal exchange rate must also absorb productivity shocks in the final good sector, $A^y(\mu^t)$, but not in the commodity sector, $A^x(\mu^t)$.

Case 2. Benchmark model with two commodities

This case makes clear that the response of the nominal exchange rate to a negative terms of trade shock depends on the origin of the shock. Consider the case in which $A^y(\mu^t) = A^x(\mu^t) = 1$. Equation (32) then implies

$$\begin{aligned} \log P(\mu^t) - \mathbb{E}_{t-1} \log P(\mu^t) &= 0 \\ \log S(\mu^t) - \mathbb{E}_{t-1} \log S(\mu^t) &= -\eta [\log P^{x*}(\mu^t) - \mathbb{E}_{t-1} \log P^{x*}(\mu^t)] \\ &\quad - (1 - \eta) [\log P^{z*}(\mu^t) - \mathbb{E}_{t-1} \log P^{z*}(\mu^t)]. \end{aligned}$$

where $\mathbb{E}_{t-1}(\cdot)$ denotes expectation conditional on μ^{t-1} . Thus, innovations to the logarithm of final good prices must be zero, and innovations to the logarithm of nominal exchange rates must be a weighted average of innovations to the logarithm of commodity prices.

Note that following a drop in the price of the exportable commodity $P^{x*}(\mu^t)$, the nominal exchange rate must optimally go up. On the other hand, if there is an increase in the price of the importable commodity $P^{z*}(\mu^t)$, the nominal exchange rate must optimally go down. In both cases, however, the terms of trade go down.

Case 3. Model with land as an input in the production of commodity x

The benchmark model ignores the fact that many commodities are produced with land. Here we argue that our results hold as long as the government is able to fully tax the rents on land.

The technology to produce the domestic commodity is now $X(\mu^t) = A^x(\mu^t) n(\mu^t)^\rho l(\mu^t)^{1-\rho}$, where $l(\mu^t)$ denotes land and $0 < \rho < 1$. Letting $V(\mu^t)$ denote the rental rate of land, the firm's optimality conditions are given by

$$\begin{aligned}\rho P^x(\mu^t) A^x(\mu^t) n(\mu^t)^{\rho-1} l(\mu^t)^{1-\rho} &= W(\mu^t) \\ (1-\rho) P^x(\mu^t) A^x(\mu^t) n(\mu^t)^\rho l(\mu^t)^{-\rho} &= V(\mu^t).\end{aligned}$$

We assume that households own the land, but that all the rents of land are fully taxed—for otherwise the government will use monetary policy to partially tax these rents. Under this assumption, the household's problem does not change. The problem of final good firms does not change either. Finally, normalizing the total amount of land to one and using the equilibrium condition $l(\mu^t) = 1$, net exports are given by

$$NX(\mu^t) = P^{x*}(\mu^t) A^x(\mu^t) N(\mu^t)^\rho - MC^*(\mu^t) D(\mu^t) [C(\mu^t) + G(\mu^t)].$$

We thus can prove results analogous to Propositions 1 and 2, and the implications for final good prices and the optimal exchange rate and are identical to those in the benchmark economy without land.

Case 4. Model with labor in the production of final goods

Consider now the case where labor is also an input in the final goods sector. Let $n_i^y(\mu^t)$ and $n_i^x(\mu^t)$ denote the amount of labor allocated to the final good and exportable commodity sectors respectively, and suppose that the technology to produce final goods is now

$$y_i(\mu^t) = \frac{A^y(\mu^t) x_i(\mu^t)^{\eta_1} n_i^y(\mu^t)^{\eta_2} z_i(\mu^t)^{1-\eta_1-\eta_2}}{\eta_1^{\eta_1} \eta_2^{\eta_2} (1-\eta_1-\eta_2)^{1-\eta_1-\eta_2}}.$$

Cost minimization then implies $MC(\mu^t) = P^x(\mu^t)^{\eta_1} W(\mu^t)^{\eta_2} P^z(\mu^t)^{1-\eta_1-\eta_2} / A^y(\mu^t)$, Using (14) then gives

$$MC(\mu^t) = P^x(\mu^t)^\eta P^z(\mu^t)^{1-\eta} \frac{A^x(\mu^t)^{\eta_2}}{A^y(\mu^t)} \quad (33)$$

where we define $\eta = \eta_1 + \eta_2$. Thus, if $A^x(\mu^t) = A^y(\mu^t) = 1$, the model collapses to the benchmark economy without productivity shocks and the optimal response of the nominal exchange rate to terms of trade shocks is identical to that of Case 2 above.

Optimal movements on the nominal exchange rate following productivity shocks

While the main focus of this paper is the effect of terms of trade shocks, the model has also implications regarding the optimal movements on the exchange rate following productivity shocks. As before, the general second best principle derived in Proposition 2 does not translate into simple properties moments like the volatility of the exchange rate or its comovement with output: the source of the shock is important.

To see this, we focus in the economy with two commodities and labor in the production of final goods and the exportable commodity—that is, case 4 above. In this case, equation (33) and the pricing equation (19) imply

$$P(\mu^{t-1}) = \frac{\theta}{\theta - 1} S(\mu^t) P^{x*}(\mu^t)^\eta P^{z*}(\mu^t)^{1-\eta} \frac{A^x(\mu^t)^{\eta_2}}{A^y(\mu^t)},$$

or, ignoring changes in commodity prices $P^{x*}(\mu^t)$ and $P^{z*}(\mu^t)$,

$$\log S(\mu^t) - \mathbb{E}_{t-1} \log S(\mu^t) = -\eta_2 [\log A^x(\mu^t) - \mathbb{E}_{t-1} \log A^x(\mu^t)] + [\log A^y(\mu^t) - \mathbb{E}_{t-1} \log A^y(\mu^t)].$$

Therefore, the optimal exchange rate policy must also absorb fluctuation in $A^y(\mu^t)$ and $A^x(\mu^t)$. If there is a negative productivity shock in the final goods sector, so that $A^y(\mu^t)$ drops, the optimal exchange rate must go down, while if it is in the production of the exportable commodity,

so that $A^x(\mu^t)$ drops, it must go up. This implies that the optimal cyclical properties of the nominal exchange rate cannot be described in reference to the cycle alone. That is, the source of the shock matters.

OPTIMAL FISCAL POLICY

In this section we briefly discuss the decentralization of fiscal policy and argue that, in general, to implement the optimal allocation the planner needs state contingent labor and consumption taxes.

In the proof to Proposition 2 we show that, given the Ramsey allocation, optimal taxes satisfy

$$\frac{1 + \tau^c(\mu^{t+1})}{1 + \tau^c(\mu^t)} = \frac{V_C(\mu^t)}{V_C(\mu^{t+1})} \frac{U_C(\mu^{t+1})}{U_C(\mu^t)} \quad (34)$$

$$\frac{1 - \tau^n(\mu^t)}{1 + \tau^c(\mu^t)} = \frac{\theta}{\theta - 1} \frac{U_N(\mu^t)}{U_C(\mu^t)} \frac{V_C(\mu^t)}{V_N(\mu^t)}. \quad (35)$$

where the terms on the right side of these equations are evaluated at the Ramsey allocation a^R .

These conditions pin down the optimal taxes. The first condition summarizes the no-arbitrage condition between domestic and foreign bonds. Therefore, (34) requires that consumption taxes must move to make the optimal allocation consistent with the absence of arbitrage. In particular, given a consumption tax at time zero, say $\tau_0^c = 0$, (34) determines recursively the sequence of consumption taxes that decentralize the allocation. Next, given the proposed sequence of consumption taxes, (35) pins down the required labor tax $\tau^n(\mu^t)$. Labor taxes arise for two reasons. First, to eliminate the distortions induced by the monopolistic producers. Because $\theta/(\theta - 1) > 1$, eliminating monopolistic distortion requires a constant subsidy to labor; this is a standard result in the literature. And second, because the household's marginal rate of substitution of labor for consumption, $U_N(\mu^t)/U_C(\mu^t)$, could be different from that of the planner, $V_N(\mu^t)/V_C(\mu^t)$. In general, these discrepancies vary across time and states and, therefore, require labor taxes that

vary across time and states as well.¹⁹

CONCLUSIONS

To be done.

¹⁹One can prove, however, that if preferences are of the form $U(C, N, m) = C^{1-\sigma} / (1 - \sigma) - N^\psi / \psi + u(m)$, optimal consumption taxes are zero, and optimal labor taxes are constant through time and across states. Therefore, the planner does not introduce any distortion in the no-arbitrage condition between domestic and foreign bonds and implements a constant labor tax.

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APPENDIX A: PROOFS.

Proof of Proposition 1: To simplify notation, here we omit the argument μ^t and write $x(\mu^t) = x_t$ for any variable x . Condition (23) follows from imposing $R_t \geq 1$ on (10). Condition (24) follows from introducing (8), (9), and (10) into (5) evaluated at equality.

To verify (25) write net exports as

$$\begin{aligned} P_t^{x*}[X_t - \int_0^1 x_{it} di] - P_t^{z*} \int_0^1 z_{it} di &= P_t^{x*} A_t^x N_t - P_t^{x*} \int_0^1 x_{it} di / \eta \\ &= P_t^{x*} A_t^x N_t - MC_t^* \int_0^1 y_{it} di \\ &= P_t^{x*} A_t^x N_t - MC_t^* D_t [C_t + G_t]. \end{aligned}$$

The first equality uses (13), the equilibrium condition $n_t = N_t$, and (16). The second equality uses (17) and the definition of marginal cost in foreign currency. The last equality uses (18) and the definition $D_t = \int_0^1 (P_{it}/P_t)^{-\theta} di$.

It remains to prove that $D_t \geq 1$ with equality if and only if $P_{it} = P_t$ almost everywhere. Let $w_{it} = P_{it}^{1-\theta}$ and note that $P_{it}^{-\theta} = w_{it}^{\theta/(\theta-1)}$ is a strictly convex function of w_{it} . Therefore, Jensen's inequality implies

$$\int_0^1 P_{it}^{-\theta} di = \int_0^1 w_{it}^{\theta/(\theta-1)} di \geq \left(\int_0^1 w_{it} di \right)^{\frac{\theta}{\theta-1}} = P_t^\theta$$

with equality if and only if $P_{it} = P_t$ almost everywhere. $\square$

Proof of Proposition 2: We find a government policy and a price system that implement $\tilde{a}$ as an equilibrium allocation. Throughout the proof, all expressions are evaluated at the allocation $\tilde{a}$. The interest rate $R(\mu^t)$ consistent with $\tilde{a}$ follows from (10)

$$R(\mu^t) = \frac{U_c(\mu^t)}{U_c(\mu^t) - U_m(\mu^t)}. \quad (\text{A1})$$

The optimal allocation requires that all final good firms set the same price. This happens only if $P_i(\mu^t) = P(\mu^{t-1})$ for all i . Thus

$$P(\mu^{t-1}) = \frac{\theta}{\theta-1} S(\mu^t) MC^*(\mu^t). \quad (\text{A2})$$

Next, use (8) at periods t and $t + 1$ to get

$$\frac{\beta \pi (\mu^{t+1}|\mu^t) U_C (\mu^{t+1})}{P (\mu^{t+1})} \frac{P (\mu^t)}{U_C (\mu^t)} \frac{1 + \tau^c (\mu^t)}{1 + \tau^c (\mu^{t+1})} = Q (\mu^{t+1}|\mu^t) \quad (\text{A4})$$

Introducing this expression into (4), and using (A2) and (29) results in

$$\frac{1 + \tau^c (\mu^{t+1})}{1 + \tau^c (\mu^t)} = \frac{V_C (\mu^t)}{V_C (\mu^{t+1})} \frac{U_C (\mu^{t+1})}{U_C (\mu^t)}. \quad (\text{A5})$$

The right side depends only on the allocation. Thus, given any arbitrary consumption tax at $t = 0$, say $\tau_0^c = 0$, this equation determines recursively $\tau^c (\mu^t)$.

Conditions (9), (14), (15), (29), (30), and (A2) imply

$$\frac{1 - \tau^n (\mu^t)}{1 + \tau^c (\mu^t)} = \frac{U_N (\mu^t)}{U_C (\mu^t)} \frac{V_C (\mu^t)}{V_N (\mu^t)} \frac{\theta}{\theta - 1},$$

which, given $\tau^c (\mu^t)$, pins down the optimal labor tax $\tau^n (\mu^t)$.

Now sum (A4) over all $\mu^{t+1}|\mu^t$, use $\sum_{\mu^{t+1}|\mu^t} Q (\mu^{t+1}|\mu^t) = 1/R (\mu^t)$ and equations (A1), (A2), and (A5) to get

$$P (\mu^t) = P (\mu^{t-1}) \frac{U_c (\mu^t)}{U_c (\mu^t) - U_m (\mu^t)} \sum_{\mu^{t+1}|\mu^t} \frac{MC^* (\mu^{t+1})}{MC^* (\mu^t)} Q^* (\mu^{t+1}|\mu^t)$$

Given P_{-1} , this equation determines recursively the price $P (\mu^t)$. Given the sequence of prices $P (\mu^t)$, (A2) determines the nominal exchange rate $S (\mu^t)$ that justifies the proposed price sequence. Also, given $P (\mu^t)$ and the sequence of consumption taxes, equation (A4) determines the bond price $Q (\mu^{t+1}|\mu^t)$. Next, the proposed exchange rate and (15) pin down the domestic currency prices $P^x (\mu^t)$, and $P^z (\mu^t)$. The nominal wage $W (\mu^t)$ follows from (14). The Ramsey allocation $\tilde{m} (\mu^t)$, final good prices, and consumption taxes determines the money supply $M (\mu^t) = \tilde{m} (\mu^t) [P (\mu^t) (1 + \tau^c (\mu^t))]$.

The household's present value budget constraint at time t is

$$\sum_{j=0}^{\infty} \sum_{\mu^{t+j}|\mu^t} Q (\mu^{t+j}|\mu^t) \left[\begin{array}{c} M (\mu^{t+j}) [R (\mu^{t+j}) - 1] / R (\mu^{t+j}) + P (\mu^{t+j}) C (\mu^{t+j}) \\ - W (\mu^{t+j}) N (\mu^{t+j}) (1 - \tau^n (\mu^{t+j})) \end{array} \right] = M (\mu^{t-1}) + B (\mu^t).$$

Thus, given the proposed prices, taxes, and money supply the previous equation pins down the domestic bond holdings $B (\mu^t)$.

Likewise, iterating on the net foreign assets accumulation equation from any μ^t forward gives

$$\sum_{j=0}^{\infty} \sum_{\mu^{t+j}|\mu^t} Q^*(\mu^{t+j}|\mu^j) NX^*(\mu^{t+j}) + B^*(\mu^t) = 0,$$

which determines the total net foreign asset position $B^*(\mu^t)$.

Because all equilibrium conditions are satisfied at the proposed prices and policies, the relaxed Ramsey allocation $\tilde{a}$ can be implemented as an equilibrium with sticky prices. $\square$

APPENDIX B: A MODEL WITH TRADED FINAL GOODS

Here we extend the benchmark model to include trade in final goods. The home economy imports a foreign final good and exports the continuum of domestic final good varieties. To keep the model as simple as possible, we assume that there is a constant elasticity foreign demand for the goods produced at home.

We endow the planner with an export tax to obtain results analogous to propositions 1 and 2.²⁰ Without that additional instrument, the planner faces a trade-off between stabilizing domestic prices and an extracting rents from foreign consumers by using monetary policy to replicate the export tax. As we argued in the case of dividend taxes, we let the government to set an export tax because we believe these taxes are the natural instrument to discuss optimal tariff issues.

The household's problem changes as follows: the consumption index $C(\mu^t)$ that enters the utility function (1) is now a function $H(C^h(\mu^t), C^f(\mu^t))$ of a composite home good $C^h(\mu^t)$ and a foreign final good $C^f(\mu^t)$. The function $H(\cdot)$ is increasing in each argument, homogeneous of degree one, and has an associated marginal cost function $h(P^h(\mu^t), P^f(\mu^t))$ increasing in each argument and homogeneous of degree one.²¹ Here, $P^h(\mu^t)$ and $P^f(\mu^t)$ denote the home currency price of the home composite and foreign final goods respectively. The composite home good is given by $C^h(\mu^t) = [\int_0^1 c_i^h(\mu^t)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ and the price index, by $P^h(\mu^t) = [\int_0^1 P_i^h(\mu^t)^{1-\theta} di]^{1/(1-\theta)}$. The household's budget constraint (3) now includes expenditures on the foreign final good which are also taxed at the rate $\tau^c(\mu^t)$.

The demand for home final goods now satisfy $c_i^h(\mu^t) = C^h(\mu^t) (P_i^h(\mu^t)/P^h(\mu^t))^{-\theta}$ for all

²⁰The government could use an import tariff instead. Indeed, what is needed is an instrument that is able to drive a wedge between the relative price of domestic to foreign final goods at home and abroad.

²¹This specification includes all aggregators used in the literature, for example, Cobb-Douglas or constant elasticity of substitution aggregators.

$i \in [0, 1]$. In addition, the demand for the aggregates $C^h(\mu^t)$ and $C^f(\mu^t)$ is given by

$$\begin{aligned} C^h(\mu^t) &= C(\mu^t) \phi^h(P(\mu^t)/P^h(\mu^t)) \\ C^f(\mu^t) &= C(\mu^t) \phi^f(P(\mu^t)/P^f(\mu^t)), \end{aligned}$$

where the functions $\phi^h(\cdot)$ and $\phi^f(\cdot)$ are increasing, and $P(\mu^t)$ is the price index of the composite good $C(\mu^t)$, defined as $P(\mu^t) = h(P^h(\mu^t), P^f(\mu^t))$. Given these conditional demands, the optimality conditions for $C(\mu^t)$, $N(\mu^t)$, and $m(\mu^t)$ are still given by (8), (9), and (10).

We assume that the government also consumes domestic and foreign final goods according to $G(\mu^t) = H(G^h(\mu^t), G^f(\mu^t))$, where $G(\mu^t)$ is an exogenous stochastic process and $G^h(\mu^t) = [\int_0^1 g_i^h(\mu^t)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$. Cost minimization then implies $g_i^h(\mu^t) = G^h(\mu^t) (P_i^h(\mu^t)/P^h(\mu^t))^{-\theta}$ for all $i \in [0, 1]$.²²

There is foreign demand for the composite home good given by

$$C^{h*}(\mu^t) = K^* (P^{h*}(\mu^t)/P^{f*}(\mu^t))^{-\gamma}, \quad (36)$$

where $P^{h*}(\mu^t)$ and $P^{f*}(\mu^t)$ are the foreign currency prices of the domestic composite and foreign final goods, $\gamma > 1$, and $K^* > 0$. When $\gamma < \infty$, the home country has monopoly power over the final goods it sells to the rest of the world.

The foreign demand of domestic final good $i \in [0, 1]$ is given by

$$c_i^{h*}(\mu^t) = C^{h*}(\mu^t) (P_i^{h*}(\mu^t)/P^{h*}(\mu^t))^{-\theta}, \quad (B1)$$

where $P_i^{h*}(\mu^t)$ is the foreign currency price of domestic variety i and $P^{h*}(\mu^t) = [\int_0^1 P_i^{h*}(\mu^t)^{1-\theta} di]^{\frac{1}{1-\theta}}$.

The government imposes a tax $\tau^h(\mu^t)$ on final goods exported to the rest of the world. Therefore, law of one price on domestic and foreign final goods requires

$$\begin{aligned} P_i^h(\mu^t) &= S(\mu^t) P_i^{h*}(\mu^t) (1 - \tau^h(\mu^t)) \\ P_i^f(\mu^t) &= S(\mu^t) P_i^{f*}(\mu^t). \end{aligned} \quad (B2)$$

Domestic final good producer i faces the demand

$$y_i^h(\mu^t) = \left(\frac{P_i^h(\mu^t)}{P^h(\mu^t)} \right)^{-\theta} [C^h(\mu^t) + G^h(\mu^t) + C^{h*}(\mu^t)],$$

²²Our results do not change if the government only consumes domestic final goods.

where we used the conditional demands of households, the government, and foreign consumers; and the laws of one price (B2). Therefore, flexible firms set prices according to

$$P_i^h(\mu^t) = \frac{\theta}{\theta - 1} S(\mu^t) MC^*(\mu^t) \quad \text{for } i \in (\alpha, 1],$$

and sticky firms set prices according to

$$P_i^h(\mu^{t-1}) = \frac{\theta}{\theta - 1} \sum_{\mu^t | \mu^{t-1}} \nu(\mu^t) S(\mu^t) MC^*(\mu^t) \quad \text{for } i \in [0, \alpha],$$

with weights given by

$$\nu(\mu^t) = \frac{Q(\mu^t | \mu^{t-1}) P^h(\mu^t)^\theta [C^h(\mu^t) + G^h(\mu^t) + C^{h*}(\mu^t)]}{\sum_{\mu^t | \mu^{t-1}} Q(\mu^t | \mu^{t-1}) P^h(\mu^t)^\theta [C^h(\mu^t) + G^h(\mu^t) + C^{h*}(\mu^t)]}.$$

In this model we can still prove a version of Proposition 1 describing necessary conditions of an equilibrium. In this version of the proposition, we let $D(\mu^t) = \int_0^1 (P_i^h(\mu^t) / P^h(\mu^t))^{-\theta} di$ and net exports are given by

$$\begin{aligned} NX^*(\mu^t) = & \kappa(\mu^t) \left[P^{h*}(\mu^t)^{1-\gamma} - MC^*(\mu^t) D(\mu^t) P^{h*}(\mu^t)^{-\gamma} \right] + P^{x*}(\mu^t) A^x(\mu^t) N(\mu^t) \\ & - P^{f*}(\mu^t) [C^f(\mu^t) + G^f(\mu^t)] - [C^h(\mu^t) + G^h(\mu^t)] MC^*(\mu^t) D(\mu^t), \end{aligned} \quad (\text{B3})$$

where $\kappa(\mu^t)$ depends on K^* and $P^{f*}(\mu^t)$. We next define a relaxed set $\tilde{\Omega}$ as we did in the benchmark economy. In this case, however, in addition to $C(\mu^t)$, $N(\mu^t)$, and $m(\mu^t)$, the planner also chooses the allocations $C^h(\mu^t)$, $C^f(\mu^t)$, $G^h(\mu^t)$, and $G^f(\mu^t)$, and the foreign currency price of the domestic final good $P^{h*}(\mu^t)$.

As in the benchmark model, the relaxed Ramsey problem implies $D(\mu^t) = 1$. This requires that all domestic final good producers set the same nominal price. In addition, the optimal price $P^{h*}(\mu^t)$ maximizes the foreign currency value of net exports. According to (B3), this requires

$$P^{h*}(\mu^t) = \frac{\gamma}{\gamma - 1} MC^*(\mu^t).$$

That is, the optimal price is set as a constant mark-up over the marginal cost of producing the home final good measured in foreign currency. The rest of the relaxed Ramsey problem is computed along the lines described in benchmark economy.

We note that, in implementing the relaxed Ramsey allocation, the planner needs to use the export tax. In effect, the optimal price $P^{h*}(\mu^t)$, the optimal pricing rules of flexible firms, and the law of one price (B2) imply

$$\tau^h(\mu^t) = 1 - \frac{\theta}{\theta - 1} \frac{\gamma - 1}{\gamma}.$$

To understand this equation, suppose that $\theta/(\theta - 1) \cong 1$. If the monopoly power of the country vanishes, so that $\gamma \rightarrow \infty$, the optimal export tax converges to zero. On the other hand, if the monopoly power of the country increases without bound, so that $\gamma \rightarrow 1$, the optimal export tax converges to one. The term $\theta/(\theta - 1)$ appears in the optimal tariff equation because domestic monopolistic producers already impose a mark-up over marginal costs. In fact, for any θ there is a $\bar{\gamma}$ such that for all $\gamma > \bar{\gamma}$, the export tax is negative. In this case the planner subsidizes exports because the mark-up charged by domestic producers is too high relative to the elasticity of foreign demand.

Using these results we can prove a version of Proposition 2 and its corollary: the relaxed Ramsey allocation can be implemented as an equilibrium and domestic final good prices are perfectly forecastable one period in advance. Moreover, because the pricing equation of domestic final good firms is identical to that in the benchmark economy, the decentralization of the optimal nominal exchange rate coincides in both models.

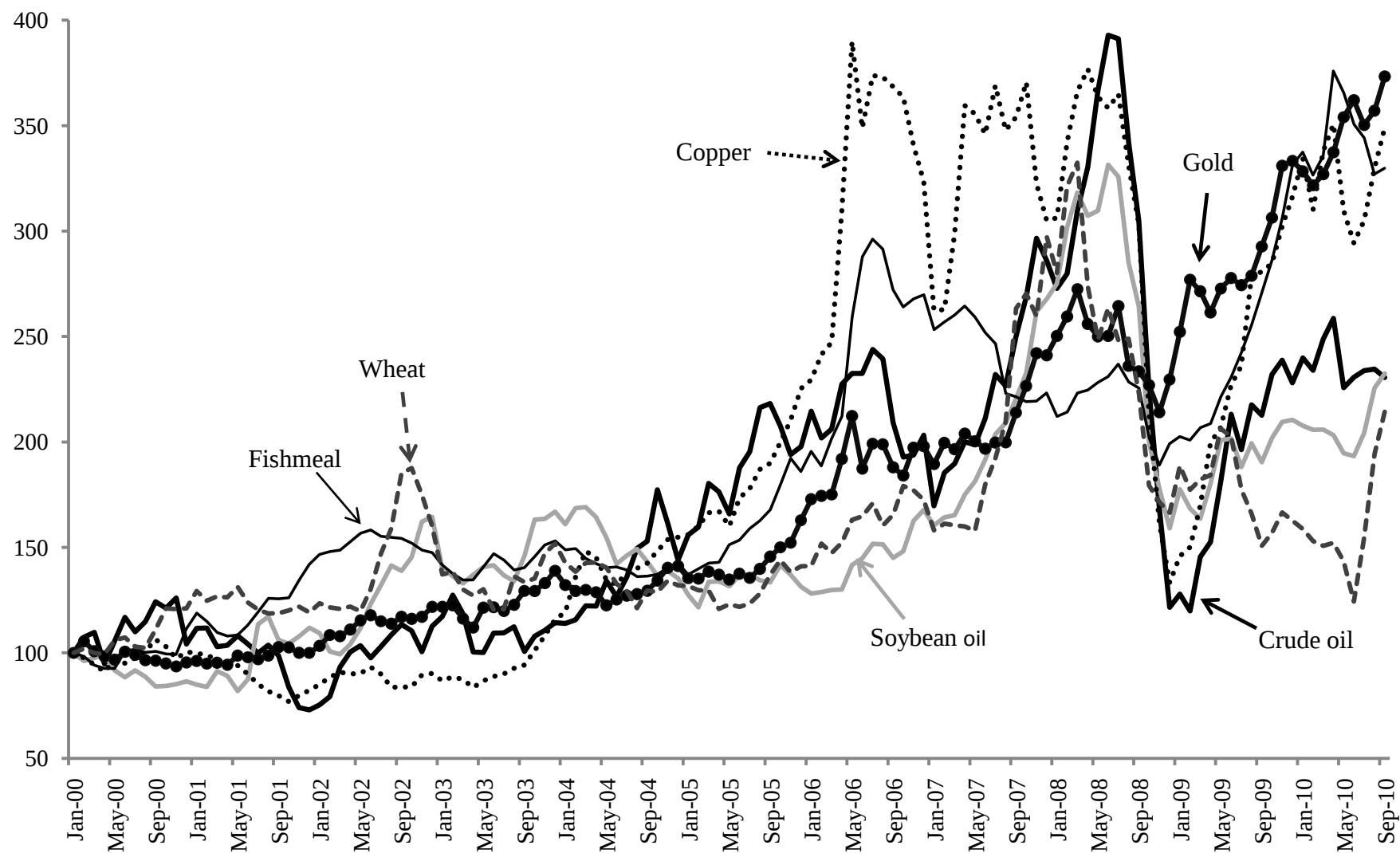
To finish this section, we note that if the government does not have access to export taxes or tariffs, the planner faces a trade-off between stabilizing domestic final good prices and extracting monopolistic rents from foreign consumers. This is the problem studied by De Paoli (2009).

TABLE 1. Principal commodity exports in selected countries

Panel A	Principal commodity exports (monthly averages since Jan 2000)			Share in good exports (%)			
	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>C1</i>	<i>C2</i>	<i>C3</i>	<i>Total</i>
Argentina	Soybean and products	Petroleum and products	Wheat	23	9	4	36
Australia	Coal	Iron ore	Gold	14	9	5	28
Brazil	Soybean and products	Petroleum and products	Iron oxides	9	8	7	24
Chile	Copper	Marine products		45	7	-	52
Iceland	Marine products	Aluminium		53	25	-	78
New Zealand	Diary produce	Meat and edible offal	Wood and products	19	13	7	39
Norway	Petroleum and products	Marine products		57	5	-	62
Peru	Copper	Gold	Marine products	20	19	8	47
Panel B	Aggregate shares (%)						
	<u><i>Goods/Total Exports</i></u>			<u><i>Total Exports/GDP</i></u>			
Argentina	87			22			
Australia	78			20			
Brazil	87			13			
Chile	83			39			
Iceland	65			37			
New Zealand	74			30			
Norway	76			44			
Peru	87			22			

Sources: National statistics agencies. Columns labeled *C1-C3* report the most important commodities and their shares in total exports of goods. Column labeled *Total* reports the share of the three principal commodities on total good exports. Commodity exports data are monthly and the last observation varies by country: Argentina, Jan2000 - Jun2010; Australia, Jan2000 - Oct2010; Brazil, Jan2000 - Oct2010; Chile, Jan2000 - Nov2010; Iceland, Jan2000 - Oct2010; New Zealand, Jan2000 - Oct2010; Norway, Jan2000 - Oct2010; and Peru, Jan2000 - Sep2010.

FIGURE1. Selected commodity prices



Source: World Bank's Global Economic Monitor. Prices are expressed at constant US dollars and normalized to 100 in January 2000.