

Competitive Poaching in Unsecured Lending*

Lukasz A. Drozd and Ricardo Serrano-Padial

November 24, 2010

ABSTRACT

The paper studies the effects of non-exclusivity of credit card contracts on the provision of insurance through the institution of personal bankruptcy. In our model, lenders can continually observe borrower's time-varying creditworthiness and provide credit to them by undercutting (poaching) the existing lender(s). Contracts are non-exclusive and, to rollover their debt, borrowers may accept multiple credit agreements to economize on the cost of credit. The main result of the paper, which holds for a broad range of parameter values, is that the level of insurance provided under bankruptcy is largely independent from borrowers' preferences and features a bang-bang property: Either too little insurance is provided or, generically, there is overinsurance (potentially severe). Comparing to the exclusivity regime, our results suggest that non-exclusivity regime is unambiguously inferior in terms of welfare. The key novel mechanism of the model is a strategic entry deterrence motive of lenders.

JEL: D1,D8,G2

Keywords: personal bankruptcy, credit cards, credit lines, unsecured credit

*Preliminary and incomplete. Drozd: Finance Department, The Wharton School of the University of Pennsylvania, email: ldrozd@wharton.upenn.edu; Serrano-Padial: Department of Economics, University of Wisconsin - Madison, email: rserrano@ssc.wisc.edu. Drozd acknowledges the financial support provided by the Vanguard Research Fellowship (Rodney L. White Center for Financial Research, Wharton School). We thank George Alessandria, V.V. Chari, Harald Cole, Dirk Krueger, Urban Jermann, Pricila Maziero and Amir Yaron for valuable comments. We are grateful to Matthew Denes for an excellent research assistantship. All remaining errors are ours.

I. Introduction

A statistical US household holds an option to draw as much as \$40k¹ in credit card funds. Perhaps not all, but a significant fraction of these funds can be used to finance day-to-day consumption, and can be defaulted on under Chapter 7 or 13 of the US personal bankruptcy law. By historic standards, this is an unprecedented level of insurance provided by the private credit markets, raising an important question whether the provision this type of insurance is efficient.

By design, the institution of personal bankruptcy in the unsecured credit market is meant to provide state contingency when private contracts cannot be made state contingent. According to the theory of incomplete contracts, bankruptcy protection can result in significant welfare gains, potentially leading to an allocation that exhibits some notion of constrained efficiency or *second best*. The typical conditions to achieve second best are: two-sided commitment contracts that can be signed by lenders and borrowers, and default that is sufficiently costly for borrowers in the absence of any shocks that bankruptcy is intended to insure against. In practice, it has been shown that a simple form of punishment for default can come a long way in insuring particularly severe shocks like persistent job loss, or health and serious family problems (e.g. divorce, unwanted pregnancy)². In terms of policy, this research has provided a powerful argument in support of some form of personal bankruptcy regulations.

In the case of the credit card market, due to the two-sided commitment requirement, the above arguments are somewhat questionable. In fact, the credit card market features the most extreme form of lack of commitment on the borrower side: full non-exclusivity of contracts with increasingly popular options of balance transfers attached to more than 60% of credit card offers. In this context, a characterization of the possible distortions caused by non-exclusivity is very much needed to better understand the economic impact of personal bankruptcy protection laws. This should allow to assess potential solutions, welfare costs, and give policy makers some guidance about what the observable symptoms of the underlying

¹Data for 2007. Source: Page 226 of H.R. 5244, the Credit Cardholder's Bills of Rights: Providing New Protections for Consumers, Hearing Before the Subcommittee on Financial Institutions and Consumers Credit, House Committee on Financial Services, 110 Cong. 109, 226 (April 17, 2008) (testimony of Travis Plunkett, Consumer Federation of America), available at http://www.house.gov/apps/list/hearing/financialsvcs_dem/hr041708.shtml.

²See, for example, Livshits, McGee & Tertilt (2006)

inefficiencies might be.

To study these questions, we develop here an equilibrium theory of balance transfer with multiple- credit line contracts and default. Specifically, in our model borrowers use unsecured credit lines to smooth consumption intertemporally between two periods, and can default on their debts by maxing out on available credit limits. In consistency with the US market, consumers can accept multiple credit lines, and cannot credibly commit to cancel upon previous credit agreements conditionally on receiving new ones. As a result, they take advantage of non-exclusivity if they want to optimize on the interest payments to their existing debt. Lenders observe time-varying credit-worthiness of borrowers and choose whether to extend credit and what kind of contract to offer.

In this environment, we show that non-exclusivity of contracts creates a strategic entrant-incumbent relation between lenders, resulting in a distortionary entry deterrence behavior. This relationship is brought about by the possibility that borrowers may engage in strategic default, i.e. they may default on their debts in the absence of negative shocks. The threat of strategic default limits how much total credit lenders can extend, thereby introducing an incentive for initial lenders to crowd out potential competitors by extending overly generous credit lines. As a result, we show that for most parameter values, the level of insurance implied by bankruptcy in our model is highly distorted as it is essentially independent from borrower's preferences and features a bang-bang property: either no insurance at all is provided or, generically, there is overinsurance (potentially severe).

In particular, our numerical experiments suggest that, under reasonable parameter values, the threat of a balance transfer can make credit limits excessive by as much as 20-30% with respect to the underlying constrained efficient allocation. In the presence of other social insurance programs, we think this result is particularly worrying, as it may unnecessarily hurt intertemporal smoothing and contribute to the fragility of the banking system. For instance, a 20-30% overinsurance raises the cost of borrowing above the optimal level by as much as 25-35%.

In terms of policy prescriptions, our results imply that some form of exclusivity imposed by law (commitment of borrowers) would be unambiguously welfare improving. We find that, while exclusivity is also distortionary in some cases, all distortions implied by the ex-

clusivity regime carry forward to the non-exclusivity regime. In other words, non-exclusivity only adds further distortions to the system. Moreover, we argue that the distortionary effect of exclusivity may be fairly easy to avoid in practice. In addition, in the context of the recently introduced modifications of the bankruptcy law, our model suggests that means testing regulation introduced by the 2005 Act may exacerbate the overinsurance result under non-exclusivity, potentially worsening outcomes in terms of bankruptcy statistics.³ All of these results are in stark contrast with the standard models of consumer bankruptcy which effectively assume exclusivity of contracts for the admissible duration of the borrowing relationship (e.g. Livshits, McGee & Tertilt (2006), Chatterjee, Corbae, Nakajima & Rios-Rull (2007), Athreya (2002)).

Related literature There are three papers that are most closely related to ours: DeMarzo & Bizer (1992), Parlour & Rajan (2001), and Petersen & Rajan (1995). These papers, with the exception of Parlour & Rajan (2001), predominately focus on the corporate finance aspects of the bankruptcy protection.⁴ While exploring effects related to ours, the novelty of our approach relies on (i) the type of contracts considered, and (ii) the possibility of entry deterrence through capacity. Specifically, this literature is based on loan contracts, which in contrast to credit lines, can affect lenders only through moral hazard considerations. While loan contracts do introduce externalities, credit lines allow for business stealing (poaching).

Our paper is also related and complementary to the quantitative modeling of the credit card market (e.g. Rios-Rull & Mateos-Planas (2007), Drozd & Nosal (2007) and Narajabad (2007)). In contrast to this literature, here we allow for multiple credit relations to coexist

³The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (the Act) was signed into law on April 20, 2005, with the explicit intent of discouraging filings under the Bankruptcy Code. The most controversial change was the creation of a means test for eligibility to file under liquidation Chapter 7 (full discharge of debt) versus restructuring under Chapter 13 (partial discharge). The Act requires a comparison of the debtor's income to the median income in the individual's domiciled state. If the debtor's income is above the median, and he is able to pay at least a minimal amount per month to creditors, he is barred from Chapter 7 filing and can only default under Chapter 13. If the debtor fails the test for filing under Chapter 7, there may now also arise a presumption of bad faith. These changes were openly intended to reduce the national default rate under Chapter 7, and limit any potentially fraudulent behavior.

⁴Parlour & Rajan (2001), study the sources of imperfect competition in the presence of a seemingly competitive environment and DeMarzo & Bizer (1992) the effects of sequential competition for loan contracts in presence of moral hazard involved in repayment. In a similar context, Petersen & Rajan (1995) emphasize the benefits of long-term relationship borrowing in the context of small business borrowing.

at the same time.

The link between information and provision of credit considered by us in the numerical section is related to the recent studies exploring such effects in the context of the recent bankruptcy trends. For example, see Livshits, MacGee & Tertilt (2007), Athreya, Tam & Young (2008) and Sanches (2008).

More broadly, our paper is also related to the literature on implicit contracts, which studies, in a different context, the effects of voluntary participation constraints (e.g. Harris & Holmstrom (1982)) on the broadly defined provision of insurance under incomplete contracts.

The structure of the rest of the paper is as follows. In section 2, we present the model and discuss the connection between our key assumptions and the institutional features of the credit card market. In section 3, we prove our main analytic results. In section 4-5, we extend the model and discuss a leading numerical example that illustrates our results.

II. Model

The economy is populated by two types of agents: consumers (households) and lenders. Time is discrete and there are two periods, the current and the future period.

Consumers optimize to smooth consumption across dates and states by saving or borrowing from the lenders. In consistency with the US Bankruptcy Code, they can default on their debts. Their current income is deterministic and is normalized to 1. Future income is random $y \in Y = \{y_1, \dots, y_n\}$, and consumers observe its realization before first period consumption/borrowing choices are made.

Lenders have deep pockets, facing cost of funds normalized to one. Markets are incomplete and unsecured credit is restricted to take the form of non-exclusive credit lines. Lenders have access to a commitment device, which allows them to offer contracts with and without commitment to terms.

A. Timing, information and contractual restrictions

Access to credit is acquired sequentially by consumers in two rounds of Bertrand competition between lenders. Both rounds are assumed to take place in the beginning of the current period, with additional information regarding borrower's future creditworthiness being revealed between the rounds. Specifically, the second round takes place after a public

signal $s \in Y$ about the realization of borrower's future income is revealed to all lenders (and the borrower). The informativeness of the signal is governed by the probability $0 \leq \pi \leq 1$ that the signal is correct.

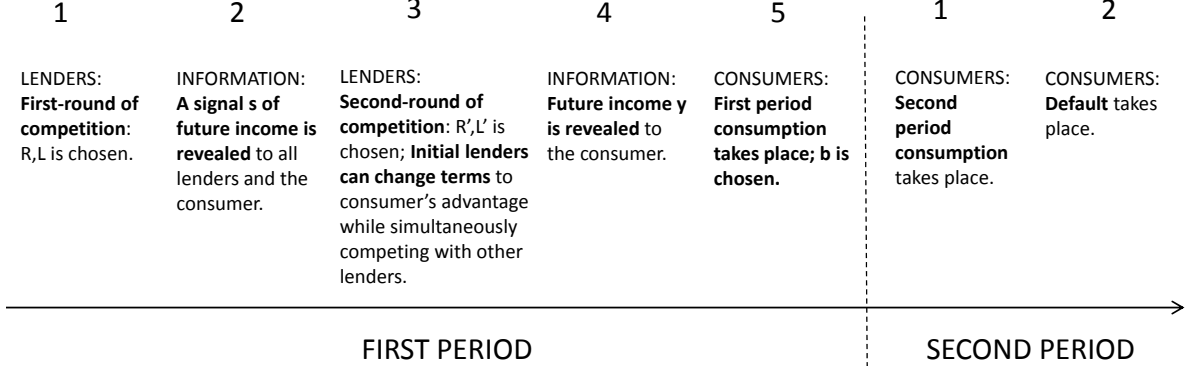


Figure 1: Timing of events.

In the baseline setup, for simplicity, we assume that only one contract per round can be accepted. At the end of our analytical section, we revisit this assumption and show that it is without loss of generality in the case of fully revealing signals and two income states. In addition, we show that, when signals are uninformative, we can still focus on a restricted set of contract combinations exhibiting one contract per round.⁵

Contracts between lenders and consumers take the form of unsecured credit lines $\mathcal{C} = (L, R) \in \mathcal{R}_+^2$, where L denotes a credit limit and $R \geq 1$ includes the principal and the interest rate. As already mentioned, initial lenders, while specifying the initial terms, can, but do not have to commit. Nevertheless, in what follows, by $\mathcal{C}$ we will mean a commitment contract from the first round. This is because, as it will become clear below, the choice of a contract without commitment, or alternatively, sweetening of terms under commitment is equivalent to the initial lenders choosing not to enter ($L = 0$).

B. Consumers

Consumers are born in the current period and live for two periods. They choose the borrowing (saving) level for the current period and decide whether to default on their debts in the future period. The amount of borrowing is constrained by the credit lines made

⁵We also identify, for the case of noisy signals, some key features of equilibrium contracts brought by the possibility of multiple contracts per round.

available to them by lenders in the aforementioned two rounds of competition.

The timing of events is illustrated in Figure 1. At the beginning of the current period, consumers may accept a contract from the initial lenders⁶ This happens before additional information regarding the realization of consumer's future income y is revealed. After the first contract is accepted (can be null), a signal $s \in Y$ about y is observed by all parties, and the second round of competition takes place.

During the second round, the consumers are allowed to accept an additional contract, referred to as the second credit line.⁷ We denote $\mathcal{C}'$ the credit line with the lowest interest rate, which will typically correspond to the second round contract, given the informational advantage enjoyed by second round lenders.

Finally, after learning y , consumers choose current period consumption (c_1); the amount of borrowing/saving (b); future period consumption (c_2); and whether to default or not ($D \in \{0, 1\}$), so as to maximize their lifetime utility⁸

$$u(c), \tag{1}$$

subject to intertemporal aggregation of consumption,

$$c = G(c_1, c_2), \tag{2}$$

current period budget constraint,

$$c_1 = 1 + b, \tag{3}$$

⁶Initial lenders can commit to L, R , if it improves borrower's welfare. See Figure 1.

⁷It is assumed that during the second-round consumers can not cancel the first-round credit line. There are two ways of rationalizing this assumption in light of the credit card market. The first is that the borrower enters the model with some debt. Thus, the lack of commitment to close an account only upon receiving a new contract makes it necessary for the borrower to repay debt before receiving a new offer. This is obviously costly. Formally, such extension can be accommodated in our model by setting current period income equal to initial income minus initial debt, and normalizing the resulting net income to one. The second justification is to instead assume that the credit line of the second round lender can not be fully used to finance consumption in the current period. That is, it only gives the option of a balance transfer. This second formulation strengthens our results.

⁸Implicitly, we are assuming that durable consumption can be frictionlessly financed by secured credit (equivalent to rental of durables). As a result, we interpret here consumption c as the sum of consumption of non-durables or the flow of services of durables purchased using perfectly secured credit or rented in the market.

a borrowing constraint implied by the sum of credit limits,

$$b \leq L + L', \quad (4)$$

and the second period budget constraint,

$$c_2 = \begin{cases} \theta_y y - b + D(L + L') & \text{for } b \leq 0, \\ \theta_y y - R'b + D(L + L') & \text{for } 0 < b \leq L', \\ \theta_y y - R(b - L') - R'L' + D(L + L') & \text{for } b \geq L'. \end{cases} \quad (5)$$

REMARK 1. *The future period budget constraint could be written as follows:*

$$b \leq L + \alpha L', \quad (6)$$

where the fraction $\alpha \in (0, 1]$ is intended to capture the idea that consumers could be constrained to use only part of the second credit line to directly finance current period consumption ($\alpha L'$), while the remainder of the line $((1 - \alpha)L')$ may only be used as a balance transfer option to reduce the overall repayment amount. As we stress below (see the companion Remark 2 below), our main results still apply to this case and introducing $\alpha < 1$ increases the likelihood of overinsurance.

The intertemporal aggregator G is assumed to be symmetric ($G(x, y) = G(y, x)$). In addition, u and G are continuously differentiable and strictly concave. Finally, we assume that G is homogeneous of degree one, so that u is the only source of risk aversion. Accordingly, G determines the intertemporal rate of substitution. By using G and u rather than a single utility function of each period's consumption levels, we are able to isolate the role of bankruptcy in the provision of insurance against severe income shocks in the presence of non-exclusive credit lines, since we can look at transfers of aggregated consumption across income paths. Hence, as we show below, we can unambiguously determine whether a particular allocation implies over- or under-insurance compared to the constrained efficient allocation.

As we show in the next lemma, our assumptions are designed to yield a particularly tractable form of the default decision rule. Apart from that, they give rise to the fairly

standard property shared by many models of default: The default set (the range of debt that triggers default) shrinks with the level of income y . This property is necessary to give rise to insurance through bankruptcy, since the lender can exploit the fact that the incentives to repay debts are positively correlated with borrower's future income.

LEMMA 1. *The consumer defaults iff $L + L'$ exceeds*

$$\mathcal{L}_{\max}(y) \leq y - \theta_y y. \quad (7)$$

Proof. Since the utility function is increasing in consumption, we note that the sufficient condition for default on a given income path y is that the budget set expands due to default for all values of b , i.e. for all b

$$\theta_y y - \rho(b, \mathcal{C}, \mathcal{C}') + \mathcal{L}_{\max}(y) > y - \rho(b, \mathcal{C}, \mathcal{C}'),$$

where ρ stands for a repayment function of debt b under contracts $\mathcal{C}, \mathcal{C}'$. Since $\rho(b, \mathcal{C}, \mathcal{C}')$ appears both on the left-hand side and on the right-hand side, the condition does not depend on b , and becomes necessary. $\square$

DEFINITION 1. $\mathcal{L}_{\max}(y)$ is referred to as the **repayment capacity** associated with income path y .

Finally, to formally define the default decision problem faced by the consumer, let $\mathcal{W}(D; y, \mathcal{C}, \mathcal{C}')$ be the *conditional* value function given a fixed default decision D , i.e. the optimal aggregated consumption level conditional on D . The choice of D then satisfies the following

$$\mathcal{V}(y, \mathcal{C}, \mathcal{C}') = \max_{D \in \{0,1\}} [D\mathcal{W}(1; y, \mathcal{C}, \mathcal{C}') + (1 - D)\mathcal{W}(0; y, \mathcal{C}, \mathcal{C}')]. \quad (8)$$

Hence, the value function $\mathcal{V}$ determines consumer's preferences regarding the choice of contracts offered by lenders.

C. Lenders

Lenders compete in a Bertrand fashion in two rounds, both taking place in the first period. The first round takes place in the very beginning, while the second round occurs after the

lenders and the consumer receive a signal about future income—as illustrated in Figure 1, the consumer does not know income yet. Given the public nature of information, we avoid any signalling considerations that would arise under asymmetric information.

The contracts offered by lenders are assumed to constitute a subgame perfect Nash equilibrium. By backward induction, we start defining the equilibrium in the lending market from the second round.

Second round of Bertrand competition In the second round, lenders observe a noisy signal of the borrower’s income realization $s \in S$ and are assumed to know the contract that the borrower accepted in the first round. Given this information, the second-round lenders choose their contract $\mathcal{C}' = (L', R')$ to best respond to the existing contract of initial lender $\mathcal{C}$ and the contracts offered by the lateral competitors. That is, the second-round contract solves

$$\mathcal{C}'(\mathcal{C}, s) = \arg \max_{\mathcal{C}'} E_s \mathcal{V}(y, \mathcal{C}, \mathcal{C}'), \quad (9)$$

subject to the expected zero profit condition given the realized signal s ,

$$E_s \pi'(y, \mathcal{C}, \mathcal{C}'(s)) = 0,$$

where π' is the profit function of second round lenders. On the equilibrium path, second round lenders actually undercut the initial lender due to their informational advantage, i.e. $R \geq R'$, in which case the profit function π' is defined as follows:

$$\pi'(y, \mathcal{C}, \mathcal{C}') = \begin{cases} -DL' & \text{for } b \leq 0, \\ (1-D)(R'-1)b - D(L' - (R'-1)b) & \text{for } 0 < b \leq L', \\ (1-D)(R'-1)L' - D(L' - (R'-1)L') & \text{for } b > L'. \end{cases} \quad (10)$$

The off-equilibrium formulation of the profit function when second-round lenders do not undercut initial lenders can be obtained by reverting the roles of $\mathcal{C}$ and $\mathcal{C}'$. This function states that, in the case of no default ($D = 0$), second round lenders profit on the interest rate $R' - 1$ charged on borrowing up to the specified limit L' , and incur a loss in the case of default ($D = 1$) *equal* to the credit limit granted by them *less* any interest paid on the

rolled-over debt.

First round of Bertrand competition In the first round, lenders choose contract $\mathcal{C}$ to best respond to the expected offering of lateral competitors, as well as the strategy profile of second-round lenders, summarized by $\mathcal{C}'(\mathcal{C}, s)$. Therefore, given the expected value function of the consumer, $E\mathcal{V}(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s))$, the first round contract $\mathcal{C} = (L, R)$ solves

$$\mathcal{C} = \arg \max_{\mathcal{C}} E\mathcal{V}(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s)) \quad (11)$$

subject to the zero profit condition

$$E\pi(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s)) = 0.$$

Since $R \geq R'$ on the equilibrium path, we can write the profit function as follows:

$$\pi(y, \mathcal{C}, \mathcal{C}'(\mathcal{C}, s)) = \begin{cases} -DL & \text{for } b \leq L', \\ (1-D)(R-1)(b-L') - D(L - (R-1)(b-L')) & \text{for } b > L'. \end{cases} \quad (12)$$

As we can see, the initial lenders profit in the case of no default if and only if borrowing b exceeds the second-round lender's credit limit L' . In case of default, the loss is equal to their credit limit *less* any interest paid on the rolled-over debt.

Equilibrium The equilibrium in this economy consists of policy functions

$$D(y, \mathcal{C}, \mathcal{C}'), b(y, \mathcal{C}, \mathcal{C}'), \mathcal{C}, \mathcal{C}'(\mathcal{C}, s),$$

and value functions

$$\mathcal{W}(D, y, \mathcal{C}, \mathcal{C}'), \mathcal{V}(y, \mathcal{C}, \mathcal{C}'), \pi(y, \mathcal{C}, \mathcal{C}'), \pi'(y, \mathcal{C}, \mathcal{C}'),$$

that simultaneously solve the consumer and lender problems described above.

Constrained Efficiency By *constrained efficient allocation (CEA)*, which we use as a benchmark, we mean a solution to a planning problem in which the planner faces the same

resource constraints, information structure and contractual restrictions as the lenders in our model, but controls both credit lines and the default decision D of the agent. Conceptually, we think of CEA as the desired allocation by some benevolent legislators who can design bankruptcy law so that the approval of a bankruptcy filing can be determined ex-post, for instance, after the state is revealed in front of a bankruptcy court. In our setup, this implies that the availability of the default option—or, equivalently, the punishment schedule of θ_y —is conditional on the realized income y . Formally, CEA is the allocation resulting from the following optimization problem:

$$\max_{D(y), \mathcal{C}, \mathcal{C}'(s)} EW(D; y, \mathcal{C}, \mathcal{C}'(s)) \quad (13)$$

subject to the resource feasibility analogous to the zero profit condition of lenders:

$$E[\pi(y, \mathcal{C}, \mathcal{C}'(s))] + E_s[\pi'(y, \mathcal{C}, \mathcal{C}'(s))] = 0, \quad (14)$$

where π, π' have been defined in (12) and (10).

Compared to the problem of initial lenders, note that the constraint efficient allocation not only internalizes all externalities arising under non-exclusivity, but also allows for cross-subsidization of the two credit lines. This last feature will turn out unimportant, since any distortions arising in equilibrium will solely come from the lack of exclusivity.

D. Discussion of Key Assumptions

The key modeling assumptions presented above try to capture several salient features of the credit card market, which we now briefly discuss.

Credit card contracts. Our focus on credit lines exclusively characterized by a credit limit and an interest rate, while not exhausting all the possibilities seen in the market (e.g. annual fees, transaction fees or cash back), is supported by the revenue and cost structure of the credit card issuers in the US. According to data in Table 1, as much as 68% of revenue is accounted for by the pure interest revenue, and interest revenue generates earnings in excess of cost of funds and chargeoffs (including fraud) by as much as 35%, i.e. 17 billion dollars.

This number suggests that interest rate revenue is actually more important source of profits for credit card issuers than the transactional aspects that we ignore in this work (interchange fees). Clearly, given the volume, any miscalculation in this respect may also have disastrous consequences for the lenders.

Table 1: Revenue & cost structure of bank card business in 2003.

Revenue		Cost	
Interest Rate	65.35	Cost of Funds	15.18
Interchange Fees	16.08	Charge-offs	33.18
Penalty Fees	7.71	Operations/Marketing	24.74
Cash-Advance Fees	4.26	Fraud	0.82
Annual Fees	2.41	-	-
Enhancements	0.73	-	-

Data for year 2003 in billions of 2003 dollars. Source: Daly (2004).

Based on this evidence, we conclude that credit limits and interest rates are the defining characteristics of this type of contract.

Commitment. We assume that lenders can credibly commit to terms, if they wish to do so. We think of commitment as being enforced by the reputational considerations of the banks⁹, whose customers expect that terms will remain stable. In fact, according to the consumer survey conducted by the Opinion Research Corporation for the Consumer Federation of America in 2007 (based on 1000 adults), as much as 91% of Americans think it is unfair to raise interest rates or fees at any time for any reason and 76% believe it is very unfair.¹⁰

⁹While lenders did change terms in the past, such a practice has never been widespread (with the exception of the recent crisis). Moreover, as expected when reputation is valued, if terms were changed, borrowers were given an option to opt-out of these changes—sometimes without any consequences. For example, the exact wording of the opt-out option offered by the Citibank during its famous 2008 interest rate hikes was (bottom of the page reads): “Right to Opt Out. To opt out of these changes, you must call or write us by Jan 31 2009. When you do, you must tell us that you are opting out. Call us toll-free at 1-866-565-7030. (Please have your account number available.) (...), **“If you opt out of these changes, you may use your account under the current terms until the end of your current membership year or the expiration date on your card, whichever is later.”** (...)”. According to industry analysts, with the exception of a few credit card issuers, this was a standard practice. A copy of the actual notice is available at the end.

¹⁰See page 19 of the aforementioned document in footnote 1: H.R. 5244, the Credit Cardholder’s Bills of Rights: Providing New Protections for Consumers, Hearing Before the Subcommittee on Financial Institutions and Consumers Credit, House Committee on Financial Services, 110 Cong. 109, 226 (April 17, 2008).

Some form of commitment is now required by law¹¹.

Non-exclusivity. This is the most important feature of our model, which implies the possibility of entry by other lenders. It turns out that one of the key products of every credit card company in the U.S. is a credit card with ‘0% APR on balance transfers’. This product, by design, is targeted to undercut existing lenders by offering an option to transfer current balances to the new card and charging 0% APR on the transferred debt.¹² Free balance transfer option, according to Evans & Schmalensee (2005), is attached to as much as 60% of solicited offers. In 2002, about 17% of outstanding debt has been transferred this way, and the number has been steadily rising since the 90s (also relative to the growth rate of the industry).

Competition under informational asymmetries. We allow for the possibility of entry after the arrival of new information about borrowers’ creditworthiness. In fact, lenders in this industry heavily rely on screening based on a large menu of variables updated almost real time, e.g. all variables pertaining to a borrower’s credit report. In addition, lenders’ screening filters often make use of ancillary data from databases tracking consumer behavior. In this context, we show numerically in *Section E* that our results are robust to the flow of information being noisy. Moreover, increasing the quality of information makes deterring entry more attractive, potentially exacerbating any existing oversinsurance.

Punishment for default. The default (bankruptcy) option is modeled to be consistent with the US Bankruptcy Code (Chapter 7 and Chapter 13) that governs personal default. As is clear from the equations, the consumer is allowed to max out on all available credit cards to obtain an effective transfer *equal* to the total available credit limit *less* the interest

¹¹Credit Card Act of 2009 gives consumers the right to opt out of—or reject—certain significant changes in terms on their accounts. Opting-out means card holders agree to close their accounts and pay off the balance under the old terms. Moreover, interest rate hikes on existing balances are allowed only under limited conditions, such as when a promotional rate ends, there is a variable rate or the cardholder makes a late payment. Interest rates on new transactions can increase only after the first year. Significant changes in terms cannot occur without a 45 days’ advance notice of the change.

¹²Typically, there is an upfront 3-5% fee, and the 0% APR is for a limited time period, ranging between 6 to 21 months. If the borrower keeps the balance for the duration of the offer, the initial fee is still substantially lower than the interest costs associated with prevailing market rates.

rate charged in the process of rolling over the debt from the first to the second period. The pecuniary punishment for defaulting is assumed increasing with income. While we do not model the distinction between chapters, our formulation does incorporate the possibility of forced default under Chapter 13 due to the means testing regulation introduced in 2005—this is interpreted in our model as an increase of the level of income garnishment $1 - \theta_y$ associated to higher income levels.

III. Results

In this section, we present the key results characterizing both the CEA and the equilibrium allocation in our model. To this end, we introduce the following set of simplifying assumptions, which allow us to obtain a closed form characterization, and also help illustrate the results graphically. We later relax, in the numerical section, most of the assumptions introduced here, and show that the results presented here are quite robust. Unless explicitly stated, all proofs are relegated to the Appendix.

ASSUMPTION 1. *G is a symmetric CES aggregator with elasticity of substitution given by $\gamma \geq 1$, i.e.*

$$G(c_1, c_2) = \left(c_1^{\frac{\gamma-1}{\gamma}} + c_2^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}. \quad (15)$$

This aggregator represents the canonical constant returns to scale, concave and symmetric consumption aggregator. The restriction on the elasticity, while not needed for the results presented here, greatly simplifies the proof of existence of a unique CEA (see *Proposition 1* below).

ASSUMPTION 2. *The signal $s \in S$ is fully revealing ($\pi = 1$).*

Assumption 2 focuses attention on the polar case of perfect information revelation. While analytically tractable, this case highlights the role of a key feature of the credit card market: the possibility of entry after the arrival of new information.

ASSUMPTION 3. *There are two income states: $Y = \{y_H, y_L\}$, where $y_L < 1 < y_H$. The probability of y_L occurring is p .*

Two income states allow us to provide a clear intuition of our main results by focusing on the simplest case of insurance provision: under this assumption, we interpret y_L as a

catastrophic income loss (expense shock) that is intended to be insured through the institution of personal bankruptcy by the legislators, and y_H stands for normal times in which there is a need to borrow for life-cycle consumption smoothing purposes.¹³ Accordingly, we also assume that default always occurs in the low state.

ASSUMPTION 4. *Low state is normalized to be always the default state: $\theta_L = 1$. Some repayment capacity is assumed sustainable in the high state: $\theta_H < 1$, implying $\mathcal{L}_{\max}(y_H) > 0$.*

DEFINITION 2. *Given assumptions 3 and 4, let $\mathcal{L}_{\max} \equiv \mathcal{L}_{\max}(y_H)$, and $\theta_H \equiv \theta$.*

Finally, we restrict the range of the repayment capacity in the high state so that the agent saves in the low state. There is a compelling economic argument behind this restriction: if we regard the institution of bankruptcy as insurance against severe income/expense shocks (e.g. persistent job loss or a serious illness), an anticipation of such event should prompt saving in the current period (i.e. in normal times) to mitigate its impact. This effectively implies that default in the low state is ‘non-strategic’, in the sense that the agent does not use part of the proceeds obtained by maxing out and defaulting on her credit lines to increase consumption in normal times. In fact, we deem this restriction as an operational definition of *non-strategic default* in the context of our model.

ASSUMPTION 5. *Income in the low state satisfies $y_L + \mathcal{L}_{\max} \leq 1$.*

This assumption is a sufficient condition that guarantees saving in the state, since the maximum amount of insurance that can be provided in equilibrium, and thus defaulted on, is $\mathcal{L}_{\max}$. Otherwise, if the sum of credit limits of both lender exceeds $\mathcal{L}_{\max}$, the agent defaults in all states, and so lenders can not possibly break even. Consequently, agent’s first period income must exceed second period income if the above condition holds and, given the symmetry of G , the agent will save even when interest rates equal the cost of funds.

A. Constrained Efficient Allocation

We begin by characterizing CEA and showing that it is unique. The results obtained here will later be used as a benchmark for the equilibrium allocation. All our results are illustrated

¹³Most of the results hold if we introduce multiple income states, but this greatly complicates the analysis. We explore this case in the numerical section.

in Figure 2. In our analysis, we focus on the space of intertemporally aggregated consumption (c_H, c_L) in the high and in the low income state, respectively, where $c_H = G(c_{1H}, c_{2H})$ and $c_L = G(c_{1L}, c_{2L})$. This space is particularly convenient, as it focuses on the insurance aspects of our model, i.e. the distribution of consumption across states of the world. To find CEA in this space, we first characterize the preferences (indifference curves), and then the shape of the Resource Feasible Consumption Frontier (RFCF) as defined below.

DEFINITION 3. Resource Feasible Consumption Frontier (RFCF) *is the efficient frontier of all (c_H, c_L) consistent with (14).*

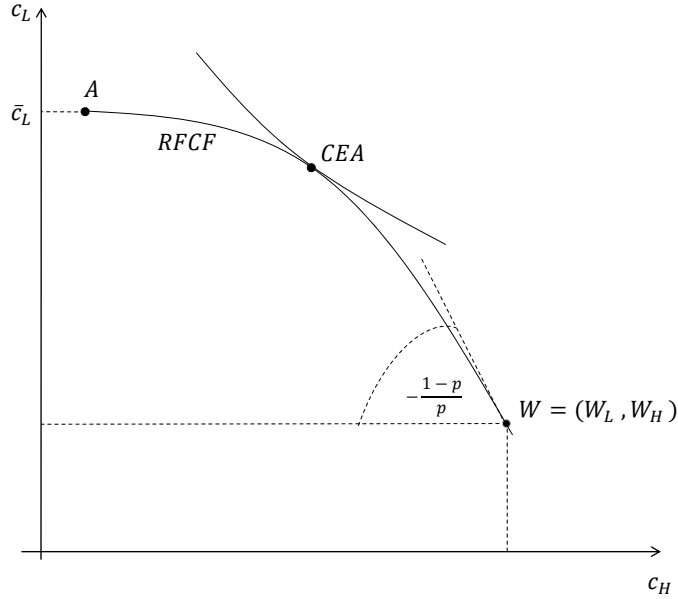


Figure 2: RFCF and CEA.

DEFINITION 4. The indifference curve associated to utility level U is the set of points (c_H, c_L) such that $U = (1 - p)u(c_H) + pu(c_L)$ ($= Eu(c)$).

The properties of the utility function guarantee a map of nicely behaved indifference curves, as established by the next lemma.

LEMMA 2. Under assumptions 1-4, the indifference curves implied by $u(\cdot)$ in the space of c_L, c_H are: (i) strictly increasing towards NE, (ii) strictly convex, and (iii) downward-sloping, with their slope being strictly flatter (steeper) than $-(1 - p)/p$ for all $c_L < (>) c_H$.

The last property stems from the fact that the marginal rate of substitution of c_H w.r.t. c_L is given by $\frac{1-p}{p} \frac{u'(c_H)}{u'(c_L)}$ and u is strictly concave. It is important to point out that indifference curves are flatter than $-(1-p)/p$ if we also impose A5. To see why, notice that resource feasibility requires that, if the consumer defaults $L \leq \mathcal{L}_{\max}$ on the low path, the revenue from charging interest on borrowing on the high path needs to be at least $Lp/(1-p)$. But then, we have that $y_L + L \leq 1$ by A5, and $1 < y_H - Lp/(1-p)$, otherwise no revenues would be raised since the agent does not borrow on the high path. Thus, by G being strictly increasing, it must be that $c_L < c_H$ on the segment of the RFCF satisfying A5.¹⁴

The ratio $-(1-p)/p$ corresponds to the slope of the *Actuarially Fair Transformation Line* (AFTL), i.e. the rate at which consumption would be transferred across states under an actuarially fair insurance policy if markets were complete. This ratio plays a key role in our results. As we show below, while the RFCF is *flatter* than the AFTL, the profit feasible frontier is *steeper* than the AFTL. As a result, the CEA will be an interior point on the RFCF, whereas the equilibrium allocation will always be a corner of the profit feasible frontier, and thus independent from consumer preferences for insurance.

We next proceed with the characterization of the Resource Feasible Consumption Frontier (RFCF), as defined in (3). We first state a technical result that allows us to restrict attention to only a subset of contracts, and then in the following lemma, we characterize the RFCF.

LEMMA 3. *Under assumptions 2-4, the constrained efficient allocation (CEA) features $R' = R$, unless $L = 0$ or $L' = 0$. Moreover, borrowing constraints are never binding in the high state, and in the low state the agent either does not borrow, or a faces marginal interest rate equal to one.*

LEMMA 4. *Under assumptions 1-4, the Resource Feasible Consumption Frontier (RFCF) defined in (3) is a decreasing function $c_L(c_H)$, obeying the following basic properties: (i) it is defined on a non-empty, closed and connected interval $[W_H, \bar{c}_H]$ ($\bar{c}_H > W_H$), (ii) it is continuously differentiable, and (iii) its slope monotonically increases from $-(1-p)/p$ at W_H to 0 at $\bar{c}_H$.*

The above lemma fully characterizes the shape of RFCF, which looks as illustrated in

¹⁴The agent can always set $c_{1H} = c_{1L}$ and have $c_{2H} > c_{2L}$, leading to $c_H > c_L$.

Figure 2. Looking from the bottom, the frontier starts from a no-insurance (no-default) point (W_L, W_H) , where $W_i = (1 + y_i)$ is the ex-post wealth discounted at the cost of funds and credit is provided at the cost of funds. As we move away from (W_L, W_H) , the planner sets higher interest rates and credit limits, which are defaulted on in low state. As a result, consumption in high state falls and consumption in low state increases. Finally, the slope of the frontier reaches a plateau at the top reflecting the fact that for sufficiently high levels of R any further increases of R lower interest revenue. The frontier is concave because higher interest rates imply a progressively higher distortion of the intertemporal margin. This result, while robust numerically, turns out analytically very nuanced. The reason is that increasing the credit limit not only distorts intertemporal smoothing in the high state but also involves taking away resources from a distorted (high) state to a non-distorted (low) state. The latter effect reduces the overall distortion when G exhibits constant returns to scale. To see why note that, by homogeneity of degree one, c_H and c_L can respectively be written as $c_H = E_H \times G(x, 1 - x)$ and $c_L = E_L \times G(1/2, 1/2)$, where E_i stands for the present value of expenditures in state $i = H, L$ discounted at cost of funds and x is the fraction of E_H consumed in period 1. Because $G(x, 1 - x) < G(1/2, 1/2)$ for $R > 1$ (imperfect smoothing), lowering E_H to raise E_L (keeping x fixed) leads to an increase in c_L that outweighs the reduction in c_H . As we show in the proof of the lemma, this effect is not enough to offset the distortion caused by higher interest rates.

The properties of RFCF established above, when combined with the properties of the consumer preferences given in *Lemma 2*, lead to the existence of a unique, interior CEA in the interval $[W_H, \bar{c}_H]$ (see Figure 2). This final conclusion is summarized below.

PROPOSITION 1. *Under assumptions 1-4, the constraint efficient allocation (CEA) exists, is unique, and, as illustrated in Figure 2, lies on the interior of the domain of the RFCF given by Lemma 4 part (i).*

Proof. Follows from *Lemmas 4* and *2* by noting that $W_L < W_H$. □

In the next paragraph, we establish how our constrained efficient allocation relates to equilibrium allocation with full exclusivity.

Relation to Equilibrium under Exclusivity By equilibrium under exclusivity (EA-E), we mean a situation in which the borrowers commit to a particular lender for a specified period of time. In the model, it implies that there is no second round of competition whenever the first round results in a positive credit limit. As we show below, the equilibrium under exclusivity may involve some distortion, but compared to non-exclusivity regime, this distortion is unambiguously smaller¹⁵.

Why exclusivity can be welfare improving? Formally, this follows from the fact that the allocation solves a problem that is very similar to the planner's problem: i.e. it solves (11), but with an exogenously fixed null second-round contract ($L' = 0$) which makes it similar to (13).

The conditions under which EA-E implements CEA are: (a) the agent saves in the low state, (b) credit limits are non-binding in the high state, and (c) the punishment for defaulting in states that the planner chooses to be non-default states is sufficiently high to avoid strategic default, i.e. the non-default states endogenously arising in EA-E coincide with those determined by the planner.

The first and second requirements come from the fact that the planner uses the second credit line to both allow perfect smoothing on the low path ($R'=1$ if $s = y_L$) and relax the borrowing constraint. Since there is no second round in EA-E, the contract terms are the same across paths. Thus, $R > 1$ on the low path, which distorts the intertemporal margin whenever the agent borrows. In addition, the agent may be constrained on the high path by L . However, (b) is satisfied as long as a committed initial lender can sweeten the terms by raising the credit limit. This is because relaxing the borrowing constraint unambiguously raises profits. This is summarized by the proposition below.

PROPOSITION 2. *Exclusivity implements CEA iff the agent saves in the default states, and θ_y is sufficiently large for all $y \in Y$ for which the planner sets $D = 0$, so that the agent does not default when y is realized.*

Proof. Follows from the argument in text above. □

¹⁵This should be fairly obvious. In terms of the underlying contracting problem, non-exclusivity adds an extra voluntary participation constraint.

We should stress that the above proposition is reminiscent of a fairly standard fact in this literature: as mentioned in the introduction of the paper, bankruptcy can, under certain conditions, implement CEA when two-sided commitment of contracts are allowed, and default is punished sufficiently to discourage strategic non-repayment in normal times (high income states). An analog of this result holds in the standard Eaton & Gersovitz (1981) framework.

Moreover, in our view, high punishment is not a serious limitation to implement CEA through exclusivity. Clearly, if a severe problem of insufficient punishment arises, binding credit constraints will be observed, and legislators will likely be able to correct the problem by tightening the bankruptcy law. In fact, the means testing regulation introduced in 2005 is an attempt to do so. Under exclusivity, such modification can be effective in tightening bankruptcy law.

Finally, before we turn to the characterization of the equilibrium allocation under non-exclusivity, we develop here a sufficient condition, independent from preferences, guaranteeing that CEA lies in the range consistent with *Assumption 5*, i.e. the agent saves at CEA.

LEMMA 5. *A sufficient condition for the agent to save in CEA is:*

$$y_H \leq \left(\sqrt{\frac{2p}{1-p}} + 1 \right)^2, \quad y_L \leq 1 - \frac{1-p}{p} \frac{(\sqrt{y_H} - 1)^2}{2}.$$

The above condition essentially requires that the shock that is intended to be insured through bankruptcy (low income state in our model) is severe enough so that, in anticipation of it, the agent decides to save. This condition is only tight for $\gamma = 1$ and becomes quite loose for higher values of γ . The next example illustrates this and shows that the range of y_L and y_H satisfying *Assumption 5* can be quite big.

EXAMPLE 1. *Let $p = 3\%$ and $\gamma = 2$. Then, the first inequality in Lemma 5 implies $y_H \leq 1.56$. Thus, consider $y_H = 1.35$. Then, the second inequality implies $y_L \leq 0.58$. The actual bounds are $y_H \leq 1.83$; and $y_L \leq 0.79$ for $y_H = 1.35$.*

B. Equilibrium Under Non-Exclusivity

In this section, we characterize the equilibrium allocation under fully revealing signals and non-exclusive contracts (EA-NE). In the numerical section, we extend our results to noisy

signals. Again, we focus attention on the space (c_H, c_L) , and characterize the profit feasible consumption frontier (PFCF). PFCF is defined as the locus of intertemporally aggregated consumption allocations implied by the zero profit contracts of the initial lenders, given the best response of the second-round lenders. The formal definition of PFCF is stated below.

DEFINITION 5. *PFCF is defined as the set of all (c_H, c_L) consistent with (12), given that $C'(\mathcal{C}, s)$ solves (9).*

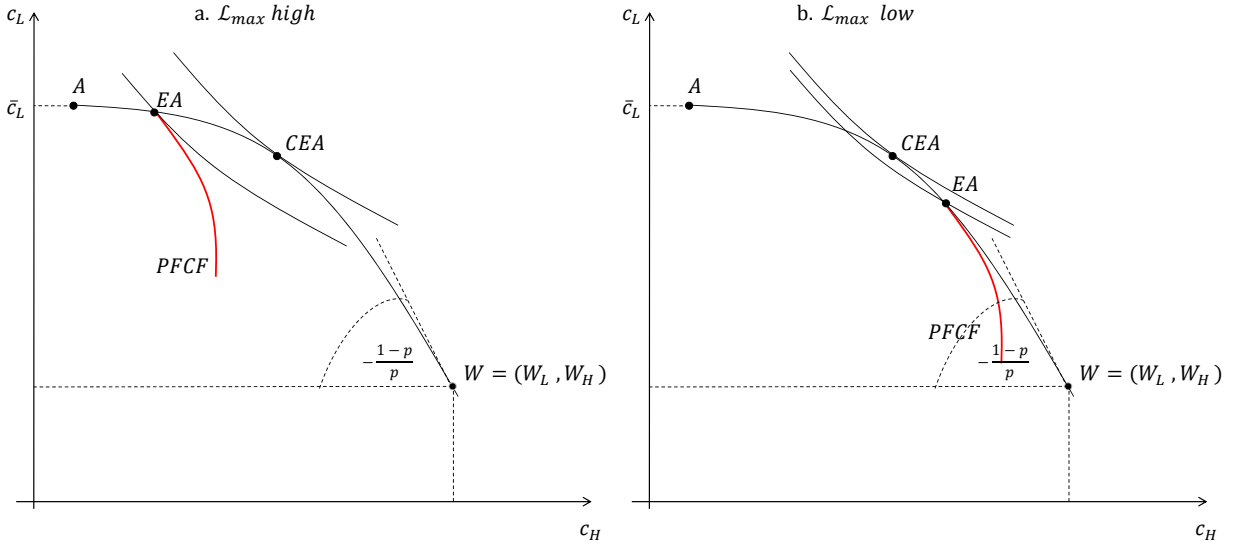


Figure 3: Equilibrium allocation under non-exclusivity.

As the next lemma shows, except for a very narrow range of parameter values, the PFCF is *globally steeper* than both the indifference curves of the consumer and the RFCF characterized in the previous section. This is the main result of our paper because it immediately implies that the equilibrium point must lie at the top of PFCF—as it is illustrated in Figures 3a and 3b, unless there is a complete shutdown of insurance, in which case the equilibrium allocation coincides with the no-insurance point (W_H, W_L) . Since the exact position of top of the PFCF is determined exclusively by profit feasibility, and is thus independent from borrower's preferences for insurance, it implies that the non-exclusivity regime can potentially be highly distortionary.

In terms of the exact outcome observed in equilibrium, three cases may arise depending on the value of the total repayment capacity $\mathcal{L}_{max}$ of the borrower:

1. For moderate levels of $\mathcal{L}_{max}$, it may be feasible for the initial lender to break even at $L = \mathcal{L}_{max}$, and because at this point there is no entry in the second round ($L' = 0$), this point is on the RFCF as long as the agent is not credit constrained. If it lies to the left of CEA, as illustrated in panel *a* of Figure 3, there is overinsurance in equilibrium. On the other hand, there is underinsurance whenever it lies to the right of CEA (panel *b* of Figure 3). In this case, the allocations under exclusivity and under non-exclusivity coincide, but not when there is overinsurance, since the allocation under exclusivity coincides with CEA by *Proposition 2*.
2. The second case involves $L = \mathcal{L}_{max}$ that is either very low so that the agent is credit constrained, or so high that yields negative profits for the lender, which leads to some entry in the second round ($L < \mathcal{L}_{max}$). In both cases the equilibrium allocation still corresponds to the top of the PFCF but it now lies strictly below the RFCF. This is because the planner can either relax the credit constraint or lower the marginal interest rate (see *Lemma 3* above).
3. Finally, the third case is a complete shutdown of insurance, which arises if the candidate equilibria of the type mentioned above feature such severe overinsurance or lay well below the RFCF that the borrower may prefer the no-insurance point W .

Since the repayment capacity depends only on the punishment for defaulting in the high income state, which case actually arises is effectively determined by θ , with only the switch between insurance and no-insurance being dependent on the consumer preferences for insurance (indifference curves). The formal results leading to this conclusion rely on the next lemma.

LEMMA 6. *Under A1-5 there exist $\underline{\mathcal{L}}$ and $\overline{\mathcal{L}}$ satisfying $\underline{\mathcal{L}} > \left(1 - \frac{1}{2} \frac{p}{1-p}\right) \overline{\mathcal{L}}$ such that, for all $\mathcal{L}_{max} \notin (\underline{\mathcal{L}}, \overline{\mathcal{L}})$, the PFCF has the following properties:*

- (i) *it shares at most one point with the RFCF, corresponding to the credit limit of the initial lender fully exhausting the repayment capacity of the consumer on high income path, i.e. $L = \mathcal{L}_{max}$ and $L' = 0$,*
- (ii) *it is continuously differentiable everywhere but the endowment point, and*
- (iii) *its slope is globally lower than the actuarially fair ratio $-(1-p)/p$.*

Given the above lemma, together with the fact that the RFCF and indifference curves are flatter than the AFTL (*Lemmas 2 and 4*), we next establish our main result: for most values of the repayment capacity, the equilibrium allocation is a corner solution, i.e. the initial lender sets L as high as it is profit feasible, typically at $\mathcal{L}_{max}(y_H)$, or there is a complete insurance shutdown.

PROPOSITION 3. *Under the restrictions of Lemma 6, the equilibrium under non-exclusivity (EA-NE) exists, is unique and features either insurance shutdown or entry deterrence by the initial lenders, in the sense that L is equal to the highest level that is profit feasible.*

As we can see, the above proposition covers most of the parameter range, with the exception of a tiny interval of $\mathcal{L}_{max}$. To give a sense how significant this restriction is, consider the following reasonable parametrization of the model: probability of default state $p = 3\%$, intertemporal elasticity $\gamma = 2$, high income $y_H = 1.35$. In such case, the excluded length of the interval of $\mathcal{L}_{max}$ in which our results may not apply is strictly smaller than $0.015\bar{\mathcal{L}} = 0.002$, where $\bar{\mathcal{L}} = 0.14$. We conclude that this restriction is very slack. We next discuss the intuition behind all these results, including why our result does not hold on this interval.

REMARK 2. *Propositions 2 and 3 still apply to the case in which the second credit line represents a balance transfer offer and cannot be fully used for intertemporal smoothing. In this case, as pointed in Remark 1 the credit constraint can be written as $b \leq L + \alpha L'$ with $\alpha \in (0, 1]$. The introduction of $\alpha < 1$ only affects allocations with positive entry (i.e. $L' > 0$), including the insurance shutdown point. To see why, notice that allocations exhibiting full deterrence are unaffected by α given that $L' = 0$. However, allocations with $L' > 0$ become less attractive for the agent as we lower α , since she progressively becomes credit constrained. In addition, some allocations with $L > 0$ (and $L' > 0$) may no longer be feasible, given that lowering α makes it harder for first period lenders to break even.¹⁶ As a consequence, introducing $\alpha < 1$ makes full deterrence, and thus overinsurance, more likely to arise in equilibrium.*

¹⁶ The reason is that the borrowing level b may be low if the agent is credit constrained and, since she can still transfer balances up to L' to the second round line, there may be little or no balance left in the first round line.

C. Discussion of the Results

The central result of our paper is that the RFCF and the PFCF are separated by the actuarially fair ratio. This basic property disconnects the insurance level from preferences in our model, and implies that the sole determinant is the repayment capacity $\mathcal{L}_{max}$ or, alternatively, the default punishment in non-default states. Here we provide the intuition behind this key result.

As already mentioned, the RFCF turns out always flatter than the AFTL because every marginal unit of consumption transferred to the low state essentially requires a higher interest rate to satisfy resource feasibility. This implies that the planner not only takes away an actuarially fair amount of resources from the agent in the high state, but also distorts intertemporal consumption, leading to a trade-off of resources between states that is less favorable than the one under complete markets.

In stark contrast, initial lenders can crowd out second round entrants by raising credit limits, which allows them to transfer resources to the low state while *lowering* the interest rate, thereby *reducing* the distortion already in place. Hence, the transfer of resources from the high to the low state via default is done at a better than fair rate.

To understand this implication intuitively, recall that the possibility of a strategic default of the consumer on the high income path places a limit on how much overall credit can be extended. Given this, initial lenders see in our model that granting a higher credit limit L generally raises the amount borrowed from them due crowding out of future lenders. This is because second round lenders will need to be more cautious in extending credit to avoid strategic default. If this effect is sufficiently strong, they may be able to lower the interest rate.

Formally, the additional revenue generated by $\Delta L > 0$ is given by

$$(R - 1)\left(1 + \frac{\partial b(y_H)}{\partial L}\right)\Delta L, \quad (16)$$

and it covers the additional losses generated by default if

$$(1 - p)(R - 1)\left(1 + \frac{\partial b(y_H)}{\partial L}\right) > p. \quad (17)$$

The condition for this to happen is the fact that the initial interest rate R must be consistent with zero profit condition, i.e.

$$(1 - p)(R - 1)(b(y_H) - (\mathcal{L}_{max} - L)) = pL.$$

From this equation we note that the key determinant of the profit feasible range of interest rate is the utilization rate of the credit line. For the initial lender, it is given by

$$\mathcal{U} = \frac{b(y_H) - (\mathcal{L}_{max} - L)}{L}.$$

It is easy to see that, for (17) to hold, utilization can not be too high, but for sufficiently low utilization rate the conditions must hold. In particular, we note that under full utilization we must have $R - 1 = \frac{p}{1-p}$, and so the condition (17) is surely violated given that $\frac{\partial b(y_H)}{\partial L} \leq 0$. On the other hand, when utilization rate is close to zero, interest rate must be very high for lenders to break even, assuring the above condition. Our proof essentially establishes exactly the range of utilization rates (parameters that determine it) for which the result holds and for which it does not. Quite surprisingly, the interval on which the result breaks down turns out to be really small. In particular, going back to the example given above, the utilization levels associated with the excluded interval range between 98% and 100% of utilization rate.

Given the wide range of parameters under which our results hold, we expect our results to extend to a much broader class of economic environments or variation of our model. This is because the only link to the specifics of our model is how a change of the credit limit affects the condition (17) through the derivative $\frac{db}{dL}$, where b is derived from standard preferences for intertemporal smoothing.

D. Extension: Multiple contracts within each round

Our first extension concerns relaxing the restriction of a single contract per round by allowing multiple contracts under sequential competition. The exact setup is summarized in the next assumption.

ASSUMPTION 6. *Within each round, lenders (publicly) post their contracts, and consumers sequentially choose which contracts to accept. Consumers can sign multiple contracts*

within each round. After a contract is signed, this information is revealed to all lenders, who can adjust the terms of their posted (unsigned) contracts. The process continues until consumers exit the market.

The proposition below shows that, in the case of perfectly revealing signals analyzed above, the restriction to one contract per round was without loss of generality. In the case of non-revealing signals, this is also true, but with an additional restriction on the space of feasible contracts.

PROPOSITION 4. *If A6 is satisfied and signals are either fully revealing ($\pi = 1$) or non-revealing ($\pi = 0$), it is without loss of generality to assume that, after a consumer accepts the first contract, no further contracts are accepted in equilibrium. In addition, for all $\pi \in [0, 1]$, the space of contracts must be restricted so that, if n contracts are accepted, either $\sum_{i=1}^n L_i(s) = \mathcal{L}_{\max}(y_h)$ for at least one $s \in S$, or in case $\sum_{i=1}^n L_i(s) < \mathcal{L}_{\max}(y_H)$ for all $s \in S$, there is full utilization, i.e. $\sum_{i=1}^n L_i(s) = b(y_H; s)$ for all $s \in S$.*

REMARK 3. *The above argument of a single contract per round does not readily generalize to the case of $0 < \pi < 1$. The following example illustrates the problem that arises in this case. Suppose that two lines are offered in the first round, with $R_1 < R_2$, and that the second round lender enters only if $s = y_H$ offering $L' = \mathcal{L}_{\max} - L_1 - L_2$. Furthermore, suppose both first round lines are utilized on the high income path when $s = y_L$, and only the one characterized by lowest interest rate R_1 is utilized if $s = y_H$. In this case, both credit lines can theoretically break even because y_H is still possible when $S = L$. As a result, the argument showing that the deviation contract considered in the proof of Proposition 4 is better can not be applied.¹⁷*

Finally, an analogous statement applies to the planning problem that defines CEA.

PROPOSITION 5. *If Assumptions 3 and 4 are satisfied and $\pi \in \{0, 1\}$, it is without loss of generality that the planner uses only one contract per round.*

Proof. Omitted. □

¹⁷ A counterexample showing that two first round contracts yield strictly higher Ec_H for the same insurance level is available from the authors.

E. Numerical Analysis

We illustrate our analytic results by considering a concrete numerical example with revealing, partially revealing and non-revealing signals. To accommodate the case of partially revealing signals, in contrast to previous sections, we analyze the allocations in the space of expected utility for each income realization. This space is more convenient with partially revealing signals, as depending on the realization of the signal and entry, aggregated consumption may vary in each state with the signal realization. In the space of expected utilities level curves are linear, with a slope equal to the actuarially fair ratio $-(1-p)/p$. Formally, the space of allocations is represented by tuples (u_L, u_H) such that $u_i = E_s[u(G(c_{1i}(s), c_{2i}(s)))]$, $i = y_H, y_L$, where $s \in Y$ and E_s denotes expectation conditional on signal realization s .

In terms of parameters, we follow here the leading example from the analytic section. Specifically, we assume two income states, $y_H = 1.35, y_L = 0.75$, no punishment for default in low state $\theta_L = 1$, some punishment in the high state $\theta_H = 0.8$, probability of low income p set equal to 2%, and intertemporal elasticity $\gamma = 2$. The utility function u is assumed standard: CRRA with elasticity of substitution $\sigma = 2$.

Our goal is to compare outcomes under exclusivity and non-exclusivity, and also show the effects of increasing signal precision on entry—which likely occurred in the 90s due to the widespread implementation and enhancement of an automatic credit scoring system.

The numerical solution for the case of perfectly revealing signals is given in Table 2 and, in the space defined above, the outcomes are additionally illustrated in *Figure 4a*. The figure shows: (i) the set of resource feasible allocations (blue dots), (ii) the constraint efficient allocation (CEA) with an indifference curve passing through it, (iii) the profit feasible set under non-exclusivity (red stars), and (iv) the equilibrium allocation under non-exclusivity (EA) with an indifference curve passing through it.

Case 1: Perfectly revealing signals ($\pi = 1$) The profit feasible set illustrates in a stark way how entry deterrence motives severely skews allocations under non-exclusivity to lie at the very top of the resource feasible set, i.e. to allocations exhibiting little entry by second round lenders. Essentially, the profit feasible consists of points at the top, and the no-insurance point at the bottom-right. Since this case obeys all the assumptions imposed

by us in the analytical section, not surprisingly, the selected equilibrium point in this case is $\mathcal{L}_{max}$, and features significant overinsurance. Specifically, we can calculate using Table 2 that first round credit limits under non-exclusivity are in excess by about 25% of what is optimal, while exclusivity delivers CEA.

Case 2: Partially revealing signals ($\pi < 1$) We next study how the model is affected by reducing signal precision. We consider two cases, labeled, signals with precision $\pi = 0.5$ and non-revealing signals ($\pi = 0$). The former is shown in *Figure 4b* while the latter corresponds to *Figure 4c*.

As we can see, whereas the allocation when the signal is noisy ($\pi = 0.5$) still exhibits full entry deterrence, the level of overinsurance is drastically mitigated in the case of non-revealing signals. Moreover, if we allow multiple contracts per round by adding the restriction stated in *Proposition 4*, the equilibrium allocation exhibits extreme underinsurance, as it is illustrated in *Figure 4d*. That is, the equilibrium outcome changes to an inefficient one, since multiple contracts remove all points that do not imply full utilization. In this case, the level of insurance is given by the optimal borrowing level at the actuarially fair rate in the high state. Consequently, it is also independent from consumer's preferences for insurance.

As the figures show, the impact lowering signal precision is that it becomes more and more possible for the second-round lender to also provide insurance. Notice that a set of points gradually appears at the bottom right corner as precision goes down. At these points, entry of the insurer is either limited or non-existent altogether. In the former case, initial lenders make profits on the high income path only when the signal is low and second-round lenders do not enter. Due to the noise, however, second-round lenders also provide insurance with some (small) probability. Such allocations are appealing for consumers, only when the probability of getting insurance from second round lenders is sufficiently high or, equivalently, π is sufficiently low. In the extreme case of $\pi = 0$, second round lenders take over and become the sole providers of insurance.

Another effect of noisy information, which turns unimportant in our numerical solution, is that with noisy signal it is no longer feasible for the planner to separate credit provision for intertemporal smoothing and insurance. Consequently, as precision of information worsens,

points at the bottom of the resource feasible set are gradually wiped out. This should be intuitive: With noisy signals, all lenders (or the planner), regardless of whether they want it or not, are exposed to default probability. Consequently, very low levels of insurance result in lower consumption in the high state because they require low credit limits, which hurt intertemporal smoothing on the high income path.

Independently, the figures also illustrate how increasing the informational advantage enjoyed by second round lenders can lead to a transition from underinsurance to overinsurance. Interpreted loosely, this effect is suggestive of how improvements in screening technology that occurred during the 90s might have possibly contributed towards the higher charge-off rates observed in the unsecured credit market. In our example, when we raise π from 0 to 0.5 the charge-off rate increases from 2% to about 4.9%—as it roughly did between late 80s and the end of 90s. Thus, although the example is very stylized, it reveals the model’s potential to fit existing bankruptcy trends.

IV. Conclusions

We have provided a detailed study of the effects of non-exclusivity on the provision of insurance through the institution of personal bankruptcy. In light of our results we believe further work is needed to identify the optimal regulatory framework for the credit card market. This is because, in the context of our model, introducing some sort of borrowers’ commitment may be welfare enhancing. This could be done, for instance, by having exclusivity imposed by law (borrowers commit to lenders for a specified period of time), and allowing for the possibility of charging a fee if a borrower decides to switch to another lender before the original contract’s expiration date.

It is important to stress that our results are fairly limited to the credit card market, and do not necessarily extend to other defaultable credit relations. The main effects we identify in this paper crucially depend on the contracts being restricted to non-exclusive credit lines, featuring

no practical possibility of charging substantial fixed fees¹⁸ or prepayment penalties.¹⁹ It is also crucial that borrowers cannot commit to terminate an existing account upon receiving a contract from another lender. The presence of pre-existing debt and the lack of borrower's commitment make our modeling assumptions better fit the credit card market than other credit markets.

¹⁸One way of resolving the contracting problem we consider is to charge a large fixed fee upon contract initiation. However, such practices do not take place in the credit card market. Annual fees contribute to as little as 3 percent of revenue of credit card companies (see source in text). Our conjecture is that the reason is an adverse selection problem that such a contract would implicate. In the model, instead of modeling such problems explicitly, we assume it is not possible to impose fixed fees.

¹⁹Unlike in some other markets, in the credit card market, lenders do not seem to have the ability of detecting a hidden prepayment, in which the borrower finances all current consumption using a new credit card, while repaying quickly the old one. Because of this problem, implementation of such fees is highly problematic.

Appendix

Proof of Lemma 2: (i)-(ii) are trivially implied by the properties of u and G . (iii) follows from the arguments given right after the statement of the lemma. $\square$

Proof of Lemma 3: The first few properties are trivial. First, the interest rate distortion can only arise in the high state (non-default state). This is because whenever the planner dictates the agent to default in the low state, it is optimal for the planner to use the second credit line ($L = 0$, $L' > 0$), and while setting $R' > 1$ on the high income path, set $R' = 0$ on the low income path (default path). Only this way the planner can eliminate a wasteful intertemporal distortion in the state in which the transfer is meant to be paid. It is also trivial to establish that the planner will always set L' in high state so that the borrowing constraint is non-binding.

To establish the last property ($R = R'$), consider by contradiction that the agent has two contracts (with positive limit) and the interest rate on them does differ. Two cases can arise: (i) there is no borrowing on one of the credit lines or (ii) there is borrowing using both credit lines. Since (i) is trivial, consider (ii) and denote the marginal interest rate by $R_m \equiv \max\{R, R'\}$.

To show that the hypothesis leads to a contradiction, consider instead one deviation contract that sets the same credit limit ($L + L'$), but makes the interest rate charged on the entire range equal to some $\bar{R}$. Let $\bar{R}$ be the lowest possible rate that is resource feasible.

First, note that $\bar{R}$ is obviously strictly lower than R_m and strictly higher than $\min\{R, R'\}$. Clearly, by increasing the interest rate of the lower interest rate credit line the revenue associated with this line goes up, allowing the marginal interest rate to actually fall. Since borrowing b in high state only goes up for a lower marginal R , higher revenue is assured for R sufficiently close to R_m .

Second, note that the deviation contract must imply actually higher consumption level in high state (and same in low state). To this end, observe that the present value of income $1 + y_1$ less the present value of consumption $c_1 + c_2$ on the high income path must equal to $p(L + L'(s = y_L))$ (expected amount defaulted on in the low state). Next, define an auxiliary level of income $y' = y - p(L + L'(s = y_L))$, and by setting $R = 1$, evaluate consumption at the optimal choices of b in the high state both under the deviation contract b^{**} and the original set of 2 contracts b^{***} from the formula: $c = G(1 + b, y' - b)$. Clearly, by definition of y' , consumption at these points is exactly equal to the one from the original problem, but the deviation contract, by properties of G , must lie closer to the well-defined and interior in this case maximizing auxiliary value b defined by $b^* = \arg \max_b G(1 + b, y' - b)$: i.e. $b^{***} < b^{**} < b^*$ (*). The inequality (*) is trivially implied by the consumer's first order condition $G_1(1 + b, y' - b) = RG_2(1 + b, y' - b)$, resource feasibility of the original contracts implying $y' > 0$, non-binding borrowing constraint in the high state, and the strict concavity of the symmetric aggregator G . By strict concavity of G , $G(b^{***}) < G(b^{**})$, which shows that the deviation contract unambiguously raises consumption in both states. $\square$

LEMMA 7. Fix $L' \in [0, (y_H - 1)/2]$ and $R \geq 1$, and let $b(R)$ satisfy

$$R = \frac{G_2(1 + b(R), y_H - R(b(R) - L') - L')}{G_1(1 + b(R), y_H - R(b(R) - L') - L')}. \quad (A1)$$

Then, under A1, the function $TIR(R) := (R - 1)(b(R) - L')$ is (i) zero and strictly increasing at $R = 1$, and (ii) strictly quasiconcave and continuously differentiable in R .

Proof. Given A1, we can express (A1) as follows:

$$R^\gamma = \frac{y_H - TIR(R) - b(R)}{1 + b(R)},$$

which, combined with $TIR(R) = (R-1)(b(R) - L')$, leads to

$$TIR(R) = (R-1) \frac{y - L' - R^\gamma(L' + 1)}{R^\gamma + R}.$$

It is easy to see that TIR is continuously differentiable, and equals zero at $R = 1$ and at $R = \left(\frac{y_H - L'}{1 + L'}\right)^{1/\gamma}$. In addition, after some tedious calculation it can be shown that

$$TIR'(R) = \frac{(y_H - L')f(R) - (L' + 1)g(R)}{(R^\gamma + R)^2},$$

where $f(R) = R^\gamma(\gamma - R(\gamma - 1)) + R$, and $g(R) = R^{\gamma+1}(R^\gamma + R\gamma - \gamma + 1)$. Notice that $TIR(R)$ is increasing at $R = 1$ for some non-zero credit limits: i.e. $TIR'(1) = y_H - 1 - 2L' > 0$ for any $0 < L' < (y_H - 1)/2$. Moreover, for $\gamma \geq 1$, we know $f(R)$ is concave and initially increasing, while $g(R)$ is strictly convex and increasing. Thus, given $(y_H - L')f(1) > (L' + 1)g(1)$, $TIR'(R)$ crosses the horizontal axis *at most* once from above. Given the fact that $TIR(R)$ is equal to zero at $R = 1$, while approaching 0 from above (in this case) at R arbitrarily high, $TIR'(R)$ crosses the horizontal axis *exactly* once. Hence, $TIR(R)$ is strictly quasi-concave on the relevant domain. $\square$

Proof of Lemma 4: By Lemma 3, the borrowing constraint never binds in the high state, and without loss of generality we can assume that the planner only uses the second round contract, with R' , L' dependent on the realization of the signal (to relax borrowing constraint in high state when needed), and the interest rate set equal to $R' = 1$ in the low state (implying no interest distortion in the low state).

In the remainder of the proof, we proceed by establishing the following properties: (1) the frontier defined as $c_L(c_H)$ is continuous function and it is defined on a non-empty, compact and connected set $c_H \in [\underline{c}_H, W_H]$, (2) $c_L(c_H)$ is decreasing, continuously differentiable and strictly concave; (3) the lowest point is a no-insurance point $W = (W_L, W_H)$, where $W_i = (1 + y_i)G(1, 1)/2$, (4) in the left-hand side neighborhood of the no-insurance point, the slope of the frontier is $-(1 - p)/p$ and zero at $\bar{c}_H$.

For what follows, we define the interest rate revenue for the high state: $TIR = (R-1) \max\{b(R, y_H), 0\}$. By resource feasibility, this revenue is paid out to consumer in the low state, i.e. $L'(s = y_L) = \frac{1-p}{p}TIR$. Given the interest rate in the low state is 1, consumption in the low state is thus given by: $c_L = (1 + y_L + \frac{1-p}{p}TIR)/2$, and is linear in TIR .

To establish (1), we note that in the case the agent borrows, borrowing is given by

$$\frac{y_H - R^\gamma}{R^\gamma + R},$$

implying that interest revenue is given by

$$TIR = (R-1) \max \left\{ \frac{y_H - R^\gamma}{R^\gamma + R}, 0 \right\}. \quad (A2)$$

For $R = 1$, $TIR = 0$. Moreover, by Lemma 7, TIR is first increasing w.r.t. R and then decreasing, equals zero for all $R \geq y_H^{1/\gamma}$, thus reaching a unique maximum at some $\bar{R} > 0$. Denote the corresponding value by $\bar{TIR}$. Clearly, the range of $R \in [1, \bar{R}]$ defines the frontier. After plugging in the underlying policy to the aggregator $G(1 + b(R), y_H - Rb(R))$, we obtain the corresponding interval

for c_H . This interval clearly is non-empty, connected and compact. It follows from the continuity of the functions determining c_H .

To show (2-4), we note that the equilibrium can equivalently be represented as follows (note that we use *Lemma 3*). Given G that is homogeneous of degree 1, we can define total expenditures in the high state as $E = 1 + y_H - TIR$, and note that the equilibrium is fully characterized by the following conditions: (i) the first order condition of the consumers, implying

$$R = \frac{G_1(x, 1-x)}{G_2(x, 1-x)} = \left(\frac{1-x}{x} \right)^{\frac{1}{\gamma}},$$

and

$$x = \frac{1}{R^\gamma + 1}, \quad (\text{A3})$$

where x stands for a fraction of total expenditures E consumed in each period (note: $c_1 = Ex, c_2 = E(1-x)$); (ii) the definition of total consumption in the high state given by

$$c_H = E \times G(x, 1-x);$$

(iii) total consumption in the low state $c_L = (1 + y_L + \frac{1-p}{p}TIR)G(1,1)/2$, the definition of total expenditures, (iv) $E = 1 + y_H - TIR$, and the definition of TIR , (v) $TIR = (R-1)(1-xE)$.

To prove property (2), notice first that since TIR is continuously differentiable and strictly increasing in $[1, \bar{R}]$, its inverse $R(TIR)$ is thus well-defined, increasing and continuously differentiable w.r.t. TIR . In addition, x is strictly decreasing and continuously differentiable in TIR , which implies that $c_H = E \times G(x, 1-x)$ is strictly decreasing and continuously differentiable w.r.t. TIR , and thus has a well-defined inverse function $TIR(c_H)$. Given the linearity of c_L w.r.t. TIR , we conclude that the frontier, defined by $c_L(c_H) = c_L(TIR(c_H))$, is a decreasing and continuously differentiable function (w.r.t. c_H).

To show strict concavity of $c_L(c_H)$, given the linearity of c_L w.r.t. TIR , we need to prove strict concavity of c_H w.r.t. TIR . It turns out that it suffices to prove that borrowing in the high state is a decreasing and *concave* function of TIR . This is true as long as $\gamma \geq \underline{\gamma}$ with $\underline{\gamma} < 1$. To see why, notice that

$$\frac{d^2 c_H}{dTIR^2} = \frac{dG(x, 1-x)}{dx} (Ex'' - 2x') + E \frac{d^2 G(x, 1-x)}{dx^2} (x'^2),$$

where $x' = dx/dTIR$ and $x''^2 x/dTIR^2$. The last term is negative from strict concavity of G w.r.t. x , while the first term is negative if $Ex'' - 2x' < 0$. Furthermore, notice that $b(TIR) = Ex - 1$ where, abusing notation, $b(TIR)$ denotes borrowing in the high state as a function of TIR . Concluding, we have: $b''(TIR) = Ex'' - 2x'$, implying $Ex'' - 2x' < 0$ whenever $b'' < 0$, which means b is concave w.r.t. TIR . Hence, given b is concave w.r.t. TIR , c_H must be concave w.r.t. TIR .

Concavity of $b(TIR)$: let $h(TIR) = TIR + b(TIR)$. Notice that we can express the first order condition as

$$\left(\frac{h(TIR)}{b(TIR)} \right)^\gamma = \frac{1-x}{x} = \frac{y_H - h(TIR)}{1 + b(TIR)}. \quad (\text{A4})$$

If we rearrange this expression and omit b and h arguments to ease notation, we get

$$\frac{1+b}{b^\gamma} = \frac{y_H - h}{h^\gamma}.$$

Differentiating both sides with respect to TIR , we get

$$\frac{\gamma + (\gamma - 1)b}{b^{\gamma+1}} b' = \frac{\gamma y_H - (\gamma - 1)h}{h^{\gamma+1}} h'.$$

If we plug $h' = 1 + b'$ and $(h/b)^\gamma$ from (A4) into this expression and rearrange, we get that, for $\gamma \neq 1$,

$$\frac{1 + b'}{b'} = \left(\frac{y_H - h}{1 + b} \right)^{\frac{1}{\gamma}} \left(\frac{y_H - h}{\frac{\gamma}{\gamma-1} y_H - h} \right) \left(\frac{\frac{\gamma}{\gamma-1} + b}{1 + b} \right), \quad (\text{A5})$$

while, for $\gamma = 1$ we get

$$\frac{1 + b'}{b'} = \left(\frac{y_H - h}{1 + b} \right)^2 \left(\frac{1}{y_H} \right). \quad (\text{A6})$$

We need to consider two cases: $\gamma \geq 1$ and $\gamma < 1$. When $\gamma \geq 1$, the right hand side of both expressions is always positive. Therefore, we must have that $1 + b' = h' < 0$ (i.e. h is decreasing in TIR), given that $b' < 0$ since b is decreasing in R and TIR is increasing in R in the relevant range. When b and h are both decreasing in TIR , it is easy to see that the right hand side of (A5) is increasing in TIR . The first fraction increases with TIR since $y_H - h$ goes up as h goes down while $1 + b$ decreases as b increases. The second fraction is decreasing in h since $\gamma/(\gamma - 1) > 1$, and thus increasing in TIR . Finally, the third fraction is also increasing in TIR given that it is decreasing in b for $\gamma > 1$. Accordingly, $(1 + b')/b'$ must go up with TIR , which can only happen if b' goes down with TIR , i.e. b is concave in TIR . The same reasoning applies to the case of $\gamma = 1$.

Now consider the case of $\gamma < 1$. We can rewrite (A5) as follows:

$$\frac{1 + b'}{b'} = \left(\frac{y_H - h}{1 + b} \right)^{\frac{1}{\gamma}+1} \left(\frac{\frac{\gamma}{1-\gamma} - b}{\frac{\gamma}{1-\gamma} y_H + h} \right). \quad (\text{A7})$$

As long as $b \leq \frac{\gamma}{1-\gamma}$ the right hand side is positive and thus, h is decreasing in TIR . But that implies that both fractions in the RHS are positive and increasing in TIR , leading to b' decreasing in TIR . To find $\underline{\gamma}$, notice that the maximum value of b is $(y_H - 1)/2$. Thus, the condition $b \leq \frac{\gamma}{1-\gamma}$ will be satisfied as long as $(y_H - 1)/2 \leq \frac{\gamma}{1-\gamma}$, i.e. as long as $\gamma \geq (y_H - 1)/(y_H + 1) = \underline{\gamma}$. To show the non-tightness of the condition, notice that for $\gamma = 2$, the tight restrictions are $y_H \leq$

To prove property (3) notice that since, c_H is decreasing in TIR its highest value happens at $TIR = 0$, i.e. when $R = 1$, leading to perfect smoothing in both states. That is, consumption in each period is equal to $(1 + y_i)/2$ and aggregated consumption is given by $W_i = G((1 + y_i)/2, (1 + y_i)/2) = (1 + y_i)G(1, 1)/2$, given that G is homogeneous of degree one.

Finally, we show (4) by noticing that the slope of $c_L(c_H)$ can be obtained by computing $(dc_L/dTIR)/(dc_H/dTIR)$. These derivatives are given by

$$\frac{dc_L}{dTIR} = \frac{1 - p}{p} \frac{G(1, 1)}{2},$$

and

$$\frac{dc_H}{dTIR} = -G(x, 1 - x) + E \frac{dG(x, 1 - x)}{dx} x'.$$

At the insurance point we have that $x = 0.5$ and $\frac{dG(x, 1-x)}{dx} = 0$ by the symmetry of G . Therefore, $dc_H/dTIR = G(1, 1)/2$ at the insurance point and the slope of $c_L(c_H)$ at that point is $-(1 - p)/p$.

At the other end of the interval, i.e. at the point associated with $\overline{TIR}$, c_L reaches a maximum since further increases in the interest rate beyond $\overline{R}$ lead to both lower c_H and lower TIR . Therefore, the slope of $c_L(c_H)$ at such point must be zero. This concludes the proof of the lemma. $\square$

Proof of Lemma 5. The condition in the lemma is obtained by calculating the highest feasible credit limit when $\gamma = 1$. This credit limit is equal to $\frac{1-p}{p}\overline{TIR}$, where $\overline{TIR}$ is the maximum total interest revenue. Given (A2), it is easy to check that, if $\gamma = 1$ we must have

$$\overline{TIR} = \frac{1-p}{p} \frac{(\sqrt{y_H} - 1)^2}{2}.$$

The first part of the condition guarantees that the highest feasible credit limit is less than one (i.e. the first period income) so that Y_L is non-negative. The second part implies that, given y_H , the sum of y_L plus the highest feasible credit limit is less than or equal to one, so the agent wants to save. Since total interest revenue is decreasing in γ , so are feasible credit limits, so this condition suffices for the agent to save for all $\gamma \geq 1$. $\square$

Proof of Lemma 6: We proceed by proving first properties (i) and (iii), and then part (ii). For the moment assume differentiability throughout.

Property (i) follows from *Lemma 3*. First note that, if we fix $L = \mathcal{L}_{max}$ and the consumer is not credit constrained, the initial lender chooses R by solving the same constrained maximization problem as the planner when $L' = 0$. Thus, the point corresponding to zero profit contract at $L = \mathcal{L}_{max}$ must lie on RFCF. When $L < \mathcal{L}_{max}$, second round lenders extend credit up to $L' = \mathcal{L}_{max} - L$ at cost of funds while $R > 1$. But then, for fixed c_L , the borrower faces a higher marginal rate in equilibrium, compared to the rate offered by the planner. Given this, as long as L is not binding, the argument used in the proof of *Lemma 3* applies, and the equilibrium contracts must deliver lower aggregated consumption on the repayment path (c_H) than the corresponding point at the RFCF. If the consumer is credit constrained in equilibrium, since the consumer is never credit constrained in the RFCF, aggregated consumption on the repayment path must be lower in the PFCF, compared to the point on the RFCF associated to c_L .

To prove **Property (ii)**, we need first to characterize candidate equilibrium contracts given L . It is easy to check that the only contract combinations that lead to points on the PFCF are $C = (L, R_0(L, \mathcal{L}_{max}))$ and $C' = (\mathcal{L}_{max} - L, 1)$, where $R_0(L, \mathcal{L}_{max})$ is the lowest interest rate satisfying $E[\pi(y, C, C')] = 0$, i.e. the initial lender's zero profit interest rate. Clearly, Bertrand competition implies that no positive profits can be made, i.e. $R = R_0(L, \mathcal{L}_{max})$ and $R' = 1$; moreover, the second round lender will extend as much credit as possible without inducing default on the repayment path, i.e. $L' = \mathcal{L}_{max} - L$.

Having identified candidate contracts leading to points on the PFCF, we proceed in three steps. In the first step (*Lemma 8*), we show that the slope of the PFCF is steeper than the AFTL whenever it is feasible for the first period lender to increase the credit limit without increasing the interest rate. While this is always true under full utilization, we show in the second step (*Lemma 9*) that, when the credit limit is not binding, this can only happen if the zero profit interest rate is above a threshold (R_{min}), which is independent of L . In the final step, we show that if $\mathcal{L}_{max}$ is below a certain threshold ($\underline{\mathcal{L}}$) there is always full utilization for all feasible L , and that, if $\mathcal{L}_{max}$ is above another threshold ($\overline{\mathcal{L}}$), any feasible contract must involve $R \geq R_{min}$. Thus, whenever $\mathcal{L}_{max}$ lies outside $(\underline{\mathcal{L}}, \overline{\mathcal{L}})$ the PFCF must be steeper than the AFTL.

LEMMA 8. Let $C = (L, R_0(L, \mathcal{L}_{max}))$ and $C' = (L', 1)$, where $L' = \mathcal{L}_{max} - L$. Then, if $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} \leq 0$ the slope of the PFCF is strictly lower than the actuarially fair ratio $-(1-p)/p$.

Proof. We need to consider 2 cases: partial and full utilization of the credit limit L . We can calculate the slope of the PFCF using $\frac{dc_L}{dL} / \frac{dc_H}{dL}$. Notice that, since the consumer saves on the default path, we have that $c_1(y_L) = c_2(y_L) = (1 + y_L + L)/2$. Accordingly, the slope of the PFCF for given L is given by

$$\frac{\frac{dc_L}{dL}}{\frac{dc_H}{dL}} = \frac{\frac{G(1,1)}{2}}{\frac{dc_H}{dL}}.$$

Thus, we need to show that

$$\frac{dc_H}{dL} > -\frac{p}{1-p} \frac{G(1,1)}{2}. \quad (\text{A8})$$

In order to so, notice that, since G is homogeneous of degree one, we can write consumption on the repayment path as

$$c_H = (c_1 + c_2)G(x, 1 - x),$$

where $x = c_1/(c_1 + c_2)$, with $c_1 = 1 + b$ and $c_2 = y_H - (R - 1)(b - (\mathcal{L}_{max} - L)) - b$. Notice that, as long as $R > 1$ we have that $c_1 < c_2$ (imperfect smoothing) and thus $x < 1/2$. Moreover, by the zero profit condition we have that

$$(R - 1)(b - (\mathcal{L}_{max} - L)) = \frac{p}{1-p}L,$$

and therefore

$$c_1 + c_2 = 1 + y_H - \frac{p}{1-p}L.$$

Given this, we have that

$$\frac{dc_H}{dL} = -\frac{p}{1-p}G(x, 1 - x) + (c_1 + c_2)\frac{dG(x, 1 - x)}{dx} \frac{dx}{dL}.$$

The first thing to notice is that $G(x, 1 - x) < G(1, 1)/2$ and $\frac{dG(x, 1 - x)}{dx} > 0$ for all $x < 1/2$ given *Assumption 1*. In addition, we have that $\frac{dx}{dL} \geq 0$ as long as $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} \leq 0$. To see why, notice that, since G_i are homogeneous of degree zero, we can express the borrower's FOC as

$$G_1(x, 1 - x) = RG_2(x, 1 - x). \quad (\text{A9})$$

Clearly, if R goes down x must go up by concavity of $G(x, 1 - x)$. Given all this, it is clear that (A8) is satisfied when $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} \leq 0$.

In the case of full utilization, i.e. $b = \mathcal{L}_{max}$, $R_0 = 1/(1-p)$ for all L and therefore $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} = 0$. In this context, if the borrowing constraint is strictly binding we have that c_1 does not change while c_2 decreases after an increase in L , causing x to go up. Thus, the above reasoning follows through since $dx/dL > 0$ in this case too. $\square$

Now we need to show that under the conditions stated in the main lemma, either there is full utilization or $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} < 0$. The following lemma establishes that this is true whenever the zero profit interest rate is above a well-defined threshold that only depends on p . Thus, when the insurer raises L she must lower her R to satisfy the zero profit condition, leading to $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} < 0$.

LEMMA 9. *Fix any $L < \mathcal{L}_{max}$ and R . Let $\mathcal{C}' = (\mathcal{L}_{max} - L, 1)$. Then, if there is partial utilization,*

then $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} \leq 0$ for

$$R \geq R_{min} := \frac{1 + a^* - p}{1 + a^* - (2 + a^*)p},$$

where a^* satisfies $R_{min} = G_1(1, a^*)/G_2(1, a^*)$. Moreover, R_{min} is unique, independent of L , and strictly lower than $\frac{2-p}{2-3p}$.

Proof. By implicitly differentiating the zero profit condition of the first period lender with respect to L , we get that

$$\frac{\partial R_0}{\partial L}(b - (\mathcal{L}_{max} - L)) + (R_0 - 1) \left(\frac{\partial b}{\partial L} + 1 \right) = \frac{p}{1-p}.$$

Accordingly, $\frac{\partial R_0}{\partial L} \leq 0$ whenever

$$(R_0 - 1) \left(1 + \frac{\partial b}{\partial L} \right) \geq \frac{p}{1-p}. \quad (\text{A10})$$

To get $\partial b/\partial L$ notice that, since R is fixed, the ratio $a = c_2/c_1$ does not change with L . This is because the borrower's FOC is given by $R = G_1(1, a)/G_2(1, a)$ since G_i is homogeneous of degree zero. Thus, we must have that

$$0 = \frac{\partial a}{\partial L} = \frac{-c_1 (R \partial b/\partial L + (R - 1)) - c_2 \partial b/\partial L}{c_1^2},$$

which leads to

$$\frac{\partial b}{\partial L} = -\frac{R - 1}{a + R}. \quad (\text{A11})$$

Substituting for $\partial b/\partial L$ in (A10) and rearranging we get the condition stated in the lemma. In order to show that this interest rate threshold is unique notice that a satisfies $R_0 = G_1(1, a)/G_2(1, a)$. The RHS of this equality is strictly increasing in a while $\frac{1+a-p}{1+a-(2+a)p}$ is strictly decreasing in a . Thus, the $a = a^*$ at which the FOC is satisfied given this interest rate is unique and so it is R_{min} . Since $a > 1$ for $R > 1$, R_{min} is clearly lower than $(2-p)/(2-3p)$. $\square$

The last part of the proof of Property (ii) is to show that there is (i) there is an interval $[\underline{\mathcal{L}}, \bar{\mathcal{L}}]$ such that for all $\mathcal{L}_{max}$ outside of it, we get that $\frac{\partial R_0(L, \mathcal{L}_{max})}{\partial L} \leq 0$. Specifically, we show that (i) there exists a lower bound on $\mathcal{L}_{max}$ below which we get full utilization; and (ii) an upper bound beyond which non-negative profits can only be sustained if $R > R_{min}$.

To show that there exists a lower bound on $\mathcal{L}_{max}$ below which we get full utilization for all $L \leq \mathcal{L}_{max}$, notice that borrowing goes up if we increase L while keeping R fixed. Thus, if there is full utilization at $L = \mathcal{L}_{max}$ there is full utilization at all L lower than $\mathcal{L}_{max}$. In addition, the zero profit interest rate associated with full utilization is $1/(1-p)$. Profits are negative for interest rates lower than $1/(1-p)$. Given this, it is clear that we get full utilization whenever $\mathcal{L}_{max}$ is less than $\underline{\mathcal{L}} = \max\{0, b^*\}$, where b^* is given by

$$(1-p)G_1(1 + b^*, y_H - b^*/(1-p)) = G_2(1 + b^*, y_H - b^*/(1-p)). \quad (\text{A12})$$

In order to prove the existence of an upper bound on $\mathcal{L}_{max}$ above which the first period lender must charge an interest rate above R_{min} to break even, we make the following argument. Assume that first period lender profit are strictly decreasing in $\mathcal{L}_{max}$ for any L and R as long as there is partial utilization. Then, we can find an $\mathcal{L}_{max} = \bar{\mathcal{L}}$ such that, by Lemma 9, profits are zero at

$R = R_{min}$ for all $L \in (0, \mathcal{L}_{max}]$. Not only that, but also profits at $R = R_{min}$ will be negative for all $L \in (0, \mathcal{L}_{max}]$ if $\mathcal{L}_{max} > \bar{\mathcal{L}}$. But then, the only way the first period lender can break even when $\mathcal{L}_{max} > \bar{\mathcal{L}}$ is by charging an interest rate higher than R_{min} . This is because, for any L associated to a point in the PFCF, profits are increasing in R . To see why notice that the total interest revenue, given by

$$TIR = (R - 1)(b - (\mathcal{L}_{max} - L)),$$

is equal to zero and increasing at $R = 1$, and strictly quasiconcave in R by *Lemma 7*. Thus, since $L = (1 - p)TIR/p$ at the PFCF, for any $L > 0$ such that the associated TIR is decreasing in R , we can find a strictly lower interest rate associated to the same TIR , at which TIR is increasing. Therefore, by Bertrand competition, the contract associated to the point in the frontier must be the one with the lower interest rate and thus profits must be increasing in R .

Therefore, all that we need to show is that profits are strictly decreasing in $\mathcal{L}_{max}$. The partial derivative of profits w.r.t $\mathcal{L}_{max}$ is given by

$$\frac{\partial \pi(y_H, \mathcal{C}, \mathcal{C}')}{\partial \mathcal{L}_{max}} = (R - 1) \left(\frac{\partial b}{\partial \mathcal{L}_{max}} - 1 \right).$$

Notice that, since a does not change as long as R remains fixed, $\partial b / \partial \mathcal{L}_{max}$ satisfies $\partial a / \partial \mathcal{L}_{max} = 0$, which leads to

$$\frac{\partial b}{\partial \mathcal{L}_{max}} = \frac{R - 1}{a + R}.$$

Therefore, we have that

$$\frac{\partial \pi(y_H, \mathcal{C}, \mathcal{C}')}{\partial \mathcal{L}_{max}} = -(R - 1) \frac{a + 1}{a + R} < 0.$$

The upper bound $\bar{\mathcal{L}}$ is given by the $\mathcal{L}_{max}$ associated with zero profits when the first period lender offers a credit limit equal to $\mathcal{L}_{max}$ and charges R_{min} . Therefore, the upper bound $\bar{\mathcal{L}}$ satisfies

$$\begin{aligned} 0 &= (1 - p)(R_{min} - 1)b(y_H, (\bar{\mathcal{L}}, R_{min}), (0, 1)) - p\bar{\mathcal{L}} \\ &= (1 - p)(R_{min} - 1) \frac{y_H - a^*}{a^* + R_{min}} - p\bar{\mathcal{L}}, \end{aligned}$$

where a^* satisfies $R_{min} = G_1(1, a^*)/G_2(1, a^*)$. From equations (A10) and (A11) evaluated at R_{min} and a^* we get the following expression for $\bar{\mathcal{L}}$:

$$\bar{\mathcal{L}} = \max \left\{ 0, \frac{y_H - a^*}{1 + a^*} \right\}.$$

To finish the proof of Property (ii), we need to show that $\underline{\mathcal{L}} > \left(1 - \frac{1}{2} \frac{p}{1-p}\right) \bar{\mathcal{L}}$. This inequality comes from the fact that the borrowing level when $L = \mathcal{L}_{max}$ satisfying the FOC is decreasing in R for all L . This is because L and $\mathcal{L}_{max}$ drop out of the FOC whenever $L = \mathcal{L}_{max}$. Given this and

the above derivation of $\underline{\mathcal{L}}$ and $\bar{\mathcal{L}}$ we have that

$$\begin{aligned}\bar{\mathcal{L}} &= \frac{1-p}{p}(R_{min}-1)b(y_H, (\bar{\mathcal{L}}, R_{min}), (0, 1)) \\ &< \frac{1-p}{p}(R_{min}-1)b(y_H, (\underline{\mathcal{L}}, 1/(1-p)), (0, 1)) \\ &= \frac{1-p}{p}(R_{min}-1)\underline{\mathcal{L}} = \frac{(1-p)(1+a^*)}{1+a^*-(2+a^*)p}\underline{\mathcal{L}} < \frac{2-2p}{2-3p}\underline{\mathcal{L}},\end{aligned}$$

where the last part comes from the fact that $a^* > 1$ and that the expression multiplying $\underline{\mathcal{L}}$ is decreasing in a^* .

It is important to note that whenever $\mathcal{L}_{max} \geq \bar{\mathcal{L}}$, there is partial utilization for all $L \in (0, \bar{\mathcal{L}}]$. This is because when $\mathcal{L}_{max} = \bar{\mathcal{L}}$ and $R = R_{min}$ the borrowing level exactly equals $\bar{\mathcal{L}}$ when $L = 0$.²⁰ Given this, partial utilization follows from the fact that borrowing goes down with L and that $\partial b / \partial \mathcal{L}_{max} < 1$.

Now let's return to **Property (iii)**, i.e. the differentiability of the PFCF. The frontier is made of the combination (c_H, c_L) associated to contracts $(L, R_0(L, \mathcal{L}_{max}))$ plus the endowment point $(\frac{y_L+1}{2}, \frac{y_H+1}{2})$. We now show that the part of the PFCF not including the endowment point is continuously differentiable. Given that outside $(\underline{\mathcal{L}}, \bar{\mathcal{L}})$, we either get full utilization or partial utilization for all L , both c_L and c_H are continuously differentiable in L . Differentiability of c_L is obvious. Under full utilization, we have that $R = 1/(1-p)$ for all L and thus $c_H = G(1 + \mathcal{L}_{max}, y_H - Lp/(1-p) - \mathcal{L}_{max})$, which is continuously differentiable in L with $\partial c_H / \partial L < 0$ by A1.

Under partial utilization, zero profit implies that interest revenue equals $Lp/(1-p)$. In addition, since the interest revenue is continuously differentiable, strictly quasiconcave and strictly increasing in $[1, \bar{R}]$ with $\bar{R}$ being its global maximizer, the zero profit interest rate $R_0(L, \mathcal{L}_{max})$ and thus the associated borrowing level are continuously differentiable in L . Therefore, since $c_H = G(1 + b, y_H - Lp/(1-p) - b)$ with its arguments being continuously differentiable, we have that c_H is continuously differentiable in L with $\partial c_H / \partial L < 0$ in the relevant range of L . Given the above results, this range is given by the compact interval $[\underline{L}(\mathcal{L}_{max}), \bar{L}(\mathcal{L}_{max})]$, where $\underline{L}(\mathcal{L}_{max})$ is the lowest L yielding zero profits and $\bar{L}(\mathcal{L}_{max}) = R_0^{-1}(\bar{R}; \mathcal{L}_{max})$. $\square$

Proof of Proposition 4: In the case of non-revealing signal, our model is essentially equivalent to an environment with just the first round of competition, and so make things simpler, assume there is only one round and potentially two contracts possible (R_1, L_1) , and signed later, (R_2, L_2) . We will show that wlog we can assume $L_2 = 0$, with an additional restriction that $L_1 = \mathcal{L}_{max}(y_H)$ or $L_1 = b(y_H)$. So, by the way of contradiction, assume this is not the case: $L_2 > 0$, and $L_1 < \mathcal{L}_{max}(y_H)$. To the end of the proof, we will consider $L_1 < L_{max}$ with $L_2 = 0$ to show $L_1 = b(y_H)$. If default does not happen for $L_1 + L_2$, Bertrand competition will require that $R_1 = R_2 = 1$. Thus, we can wlog assume only one contract is offered instead of two. Suppose default does happen in low state. Three cases are possible: (i) $R_1 = R_2$, (ii) $R_1 > R_2$, (iii) $R_1 < R_2$, which is analogous to (ii).

²⁰Since $\partial b / \partial L = -(R_{min}-1)/(a^* + R_{min})$ is constant for all L , the borrowing level when $L = 0$ is given by

$$b(y_H, (\bar{\mathcal{L}}, R_{min}), (0, 1)) - \bar{\mathcal{L}} \frac{\partial b}{\partial L} = \frac{y_H - a^*}{a^* + R_{min}} + \frac{y_H - a^*}{1 + a^*} \frac{R_{min} - 1}{a^* + R_{min}} = \frac{y_H - a^*}{1 + a^*} = \bar{\mathcal{L}}.$$

Case (i) is analogous to non-default case: no decisions will be affected and, if both lenders make zero profits, so does one cumulative single lender. Case (ii) must imply that $b(y_H) > L_2$. Since default happens, the initial lender could not make zero profits otherwise. This means that the marginal interest rate the borrower faces on high income path is necessarily R_1 . It is trivial to show that it is possible to find one credit line $\mathcal{C}_{dev} = (R_2 < R < R_1, L = L_1 + L_2)$, such that the borrower would be strictly better off, and (R, L) will give zero profit to the single lender. To see this, just note that the total revenue raised by the zero profit lender on the high income path on the contract (R, L) must be pL , and so it is the same as the total revenue with two contracts. Now, this means that the total amount of wealth used for consumption on high income path is the same in two cases, and since the marginal interest rate is lower, the borrower must be better off. Moreover, since the interest rate R is lower than marginal one in case with two contracts, the consumer must borrow more — allowing (R, L) to unambiguously break even at $R < R_2$. To show that the single contract in this particular case must feature $L_1 = b(y_H)$, note that $L_1 > b(y_H)$ leaves space for entry that gives exactly zero profits at the same interest rate as long as $L_2 = b(y_H) - L_1$ — thus, for one contract assumption to be wlog, we must restrict space of contracts to force $L_1 = b(y_H)$. The last argument extends to the case of $\pi > 0$: if the aggregate credit limit is strictly below $\mathcal{L}_{\max}(y_H)$ for all s and a contract is not fully utilized in some s , then there is opportunity for profitable entry by a same round lender that will offer a smaller credit limit (so that it is fully utilized) and a lower interest rate than the one associated to the under-utilized contract.

In the case of perfectly revealing signals, given the above discussion, it is sufficient to consider possibility of multiple contracts only in the first round, and assume that in the second-round there is a unique contract that satisfies: $L' = \mathcal{L}_{\max} - L$, if $s = y_H$ (and $y = y_H$ due to revealing signal), and $L' = 0$ otherwise. Going back to the first round, by contradiction, we assume two contracts are signed in the first round R_1, L_2 and a later contract, R_2, L_2 . As above, this contract can be improved by offering $\mathcal{C}_{dev}$ listed above. The rest is analogous to the arguments given above, and will be omitted. $\square$

References

- Athreya, Kartik. 2002. "Welfare Implications of the Bankruptcy Reform Act of 1999." *Journal of Monetary Economics* 49:1567–1595.
- Athreya, Kartik B., Xuan S. Tam & Eric R. Young. 2008. "A Quantitative Theory of Information and Unsecured Credit." *Richmond Fed Working Paper* 08-06.
- Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima & Jose-Victor Rios-Rull. 2007. "A Quantitative Theory of Unsecured Credit with Risk of Default." *forthcoming, Econometrica*.
- Daly, James J. 2004. "Smooth Sailing." *Credit Card Management* 17(2):30–33.
- DeMarzo, Peter M. & David Bizer. 1992. "Sequential Banking." *Journal of Political Economy* 100:41–61.
- Drozd, Lukasz A. & Jaromir B. Nosal. 2007. "Competing for Customers: A Search Model of the Market for Unsecured Credit." *unpublished manuscript*.
- Eaton, Jonathan & Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *The Review of Economic Studies* 110:289–309.
- Evans, David S. & Richard Schmalensee. 2005. *Paying with Plastic: The Digital Revolution in Buying and Borrowing*. The MIT Press.
- Harris, Milton & Bengt Holmstrom. 1982. "A Theory of Wage Dynamics." *Review of Economic Studies* 49:315–333.
- Livshits, Igor, James MacGee & Michele Tertilt. 2007. "Costly Contracts and Consumer Credit." *manuscript*.
- Livshits, Igor, James McGee & Michelle Tertilt. 2006. "Consumer Bankruptcy: A Fresh Start." *American Economic Review* 97(1):402–418.
- Narajabad, Borghan N. 2007. "Information Technology and the Rise of Household Bankruptcy." *Rice University, manuscript* 08-06.
- Parlour, Christine & Uday Rajan. 2001. "Competition in Loan Contracts." *American Economic Review* 91(5):1311–1328.
- Petersen, Mitchell A. & Raghuram G. Rajan. 1995. "The Effect of Credit Market Competition on Lending Relationships." *The Quarterly Journal of Economics* 110:407–443.
- Rios-Rull, J.V. & X. Mateos-Planas. 2007. "Credit Lines." *University of Minnesota, manuscript*.
- Sanches, Juan. 2008. "The Role of Information in Consumer Debt and Bankruptcy." *University of Rochester-manuscript* 08-06.

Table 2: EA and CEA in benchmark case: numerical example with revealing signal.

Variable	$\pi = 1$		$\pi = 0.5$	
	y_L	y_H	y_L	y_H
<i>A. Equilibrium under Non-exclusivity</i>				
L_{max}	0.00	0.27	0.00	0.27
R	1.05	1.05	1.05	1.05
L	0.27	0.27	0.27	0.27
R'	-	-	-	-
L'	0.0	0.0	0.0	0.0
Default decision	1	0	1	0
Borrowing (b)	0.0	0.112	0.0	0.112
<i>B. Equilibrium under Exclusivity</i>				
L_{max}	0.00	0.27	0.00	0.27
R	1.034	1.034	1.034	1.034
L	0.217	0.217	0.217	0.217
L'	0.0	0.0	0.0	0.0
Default decision	1	0	1	0
Borrowing (b)	-.0163	0.1312	-.0163	0.1312
<i>C. Constrained Efficient Allocation</i>				
L_{max}	0.00	0.27	0.0000	0.27
R	1.034	1.034	1.034	1.034
L	0.217	0.217	0.217	0.217
L'	0.0	0.0	0.0	0.0
Default decision	1	0	1	0
Borrowing (b)	-.0163	0.131	-.0163	0.131

The value of the parameters is given in text.

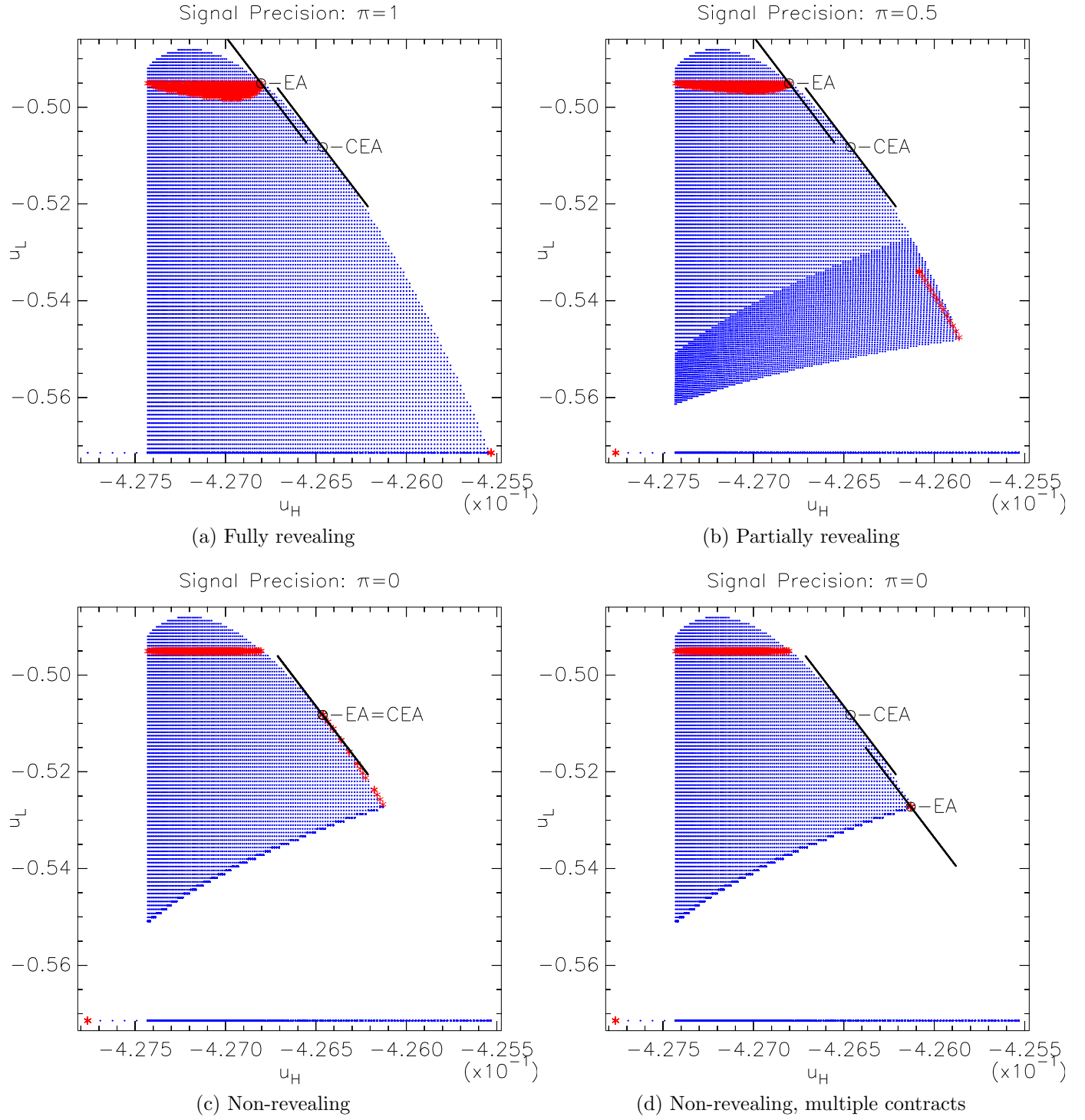


Figure 4: Feasible sets and equilibria for different signal precisions.

Citibank (South Dakota), N.A.
c/o Customer Service Center
P.O. Box 6148
Sioux Falls, SD 57117-6148

Regarding your Citibank account ending in

Notice of Change in Terms and Right to Opt Out

The Changes. We are changing your Card Agreement. The changes will be effective for all billing periods beginning on or after December 3, 2008. The changes will be effective whether or not you receive a billing statement.

I. We are changing how we calculate your variable APR for purchases:

We are increasing your variable APR for purchases. Your purchase APR will equal the LIBOR Rate plus 11.99%, with a minimum APR of 14.99%. As of September 17, 2008, this purchase APR is 14.99%. This APR equals a daily periodic rate of 0.0411%.

II. We are changing how we calculate your variable APR for cash advances:

We are increasing your variable APR for cash advances. Your cash advance APR will equal the LIBOR Rate plus 19.99%, with a minimum APR of 21.99%. As of September 17, 2008, this cash advance APR is 22.73%. This APR equals a daily periodic rate of 0.0623%.

III. We are changing the following sections regarding APRs:

Default APR. All your APRs (including promotional APRs) on all balances may automatically increase to the default APR if you default under any Card Agreement you have with us because you

- do not make the minimum payment when due,
- go over the credit line, or
- make a payment to us that is not honored.

The default APR equals the greater of (1) the LIBOR Rate plus up to 26.99% or (2) up to 29.99%. We set your default APR by reviewing the seriousness of your default with us and your credit history. The default APR takes effect as of the first day of the billing period in which you default. We may lower the APR for new purchases and/or cash advances if you meet the terms of all Card Agreements that you have with us for six billing periods in a row. However, the default APR will continue to apply to your existing balances until they are paid in full, unless we tell you otherwise.

IV. We are changing how we calculate the fee for foreign purchases. This fee will be called the Transaction Fee for Foreign Purchases. The description in your Card Agreement of this fee will be as follows:

Transaction Fee for Foreign Purchases. We add a fee of 3% of the U.S. dollar amount of each purchase made outside the U.S., whether made in U.S. dollars or in a foreign currency. This fee is a **FINANCE CHARGE**.

Right to Opt Out. To opt out of these changes, you must call or write us by January 31, 2009. When you do, you must tell us that you are opting out. Call us toll-free at **1-866-565-7030**. (Please have your account number available.) Write us at Customer Service Center, P.O. Box 6218, Sioux Falls, South Dakota, 57117-6218. Include your name, address, and account number on your letter.

If you opt out of these changes, you may use your account under the current terms until the end of your current membership year or the expiration date on your card, whichever is later. We will close your account at that time. You must then repay the balance under the current terms.

We hope you choose to take full advantage of your Citi Card revolving line of credit and all the benefits and services we offer you. Please save this notice for future reference.

Figure 5: Example of an opt-out option offered voluntarily by a major credit card lender before the Credit CARD Act of 2009 went into effect.