

Consumption Risk Sharing, the Real Exchange Rate, and Borders: Why Does the Exchange Rate Make Such a Difference?

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Abstract

This paper explores the nature of consumption risk-sharing within and across countries. A basic prediction of efficient risk sharing is that relative consumption growth rates across countries or regions should be positively related to real exchange rate growth rates across the same areas. We provide a comprehensive investigation of this hypothesis in a multi-country and multi-regional data set. Controlling for consumption comparisons across national borders, we find significant evidence of risk sharing. Incorporating the impact of borders, however, relative consumption growth is negatively related to real exchange rate changes. In line with previous work, we find that the border effect is substantially (but not fully) accounted for by nominal exchange rate variability. We then ask whether standard open economy macro models can explain these features of the data. We argue that they cannot. In order to explain the key role of the nominal exchange rate in deviations from cross country consumption risk sharing, it is necessary to combine multiple sources of shocks, both from supply and demand, ex-ante price setting, and incomplete financial

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markets. The paper develops a model based on these features and investigates its ability to account for the empirical evidence on consumption risk sharing and the role of the nominal exchange rate.

1 Introduction

Many studies of aggregate consumption behavior have documented the failure of naive models of consumption risk-sharing. This is true both within countries (risk-sharing across provinces or states) and across countries. Recognizing that relative consumption prices are time-varying leads to a more elaborate test for consumption risk-sharing, incorporating both within and between country real exchange rates movements. The prediction of this extended model is that relative consumption growth rates (across regions or countries) are highly correlated with movements in real exchange rates. When this is tested, a sharp dichotomy arises between within-country and across-country studies, particularly for across-country groups that have flexible bilateral exchange rates. Within countries, movements in real exchange rates tend to support the hypothesis of some (imperfect) risk-sharing. Across countries however, the real exchange rate plays either no role or a negative role in risk-sharing. This is essentially the well-known ‘Backus Smith’ puzzle (Backus and Smith, 1993). This is particularly true of countries that exhibit substantial fluctuations in nominal exchange rates.¹ Thus, the failure of across-country (as opposed to within-country) risk sharing is proximately due to movements in nominal exchange rates. Countries (or regions) with fixed exchange rates tend to exhibit relative consumption growth rates that are positively correlated with national (or regional) real exchange rates. But in countries with substantial fluctuations in nominal exchange rates the correlation tends to be negative. Can this finding be reconciled with standard models of real exchange rate determination? This paper is an attempt to resolve this question.

We note that this is not quite the same as an attempt to resolve the Backus-Smith problem, or ‘consumption real exchange rate anomaly’ (see Chari, Kehoe, and McGrattan, 2002). Studies that have offered convincing resolutions of this anomaly (see below for a discussion) typically do not make any predictions about the behavior of the nominal exchange rate. Therefore, these studies imply that the relationship

¹See for instance, Hess and Shin (2010) and Hadzi-Vaskov (2008).

between relative consumption and the real exchange rate should be unaffected by the nominal exchange rate regime.

We begin by providing a comprehensive empirical account of the role of the real exchange rate in regional and international risk sharing in a large intra-national and international data-set. The data set contains consumption and bilateral consumption prices (or real exchange rates) at the provincial or state level for a group of countries. We show that for all countries in the sample, there is evidence that the real exchange rate plays a positive role in within country risk-sharing. That is, consumption growth differences across provinces or states are positively correlated with bilateral real exchange rate changes across the same geographical units. But when we include a ‘border dummy’ in the risk sharing regression, indicating that relative consumption growth involves comparisons across countries, the overall relationship between consumption growth and real exchange rate changes falls dramatically, and in most cases is negative. Further investigation reveals that most, (but not all) of this border effect can be attributed to nominal exchange rate volatility.

We then ask, are these results consistent with standard models of international risk sharing and real (and nominal) exchange rate volatility? Most proposed resolutions of the Backus-Smith anomaly have emphasized the joint role of incomplete markets and shocks which generate strong income effects. The intuition is that a country which has a faster growing consumption experiences an appreciating real exchange rate (e.g. Corsetti, Dedola, and Leduc, 2008). But when we extend the anomaly to encompass both regions within a country, and the importance of the nominal exchange rate, these explanations are not complete, since in the standard models, these shocks tend to produce negative consumption real-exchange rate correlations across regions within a country, or across countries, independantly of how the nominal exchange rate is determined. In order to adequately explain the data, it is necessary to allow for a non-trivial role for the nominal exchange rate regime in consumption risk-sharing, since the evidence indicates that the failure of risk-sharing (or equivalently, the large role played by the border) is strongly tied to movements in the nominal exchange rate.

We go on to show that the standard sticky price international macro cannot account for the anomaly. This is because, despite allowing for a non-trivial role for the nominal exchange rate regime, this models predict that if there is a negative correlation between relative consumption growth and the real exchange rate under flexible

exchange rates, there is also a negative correlation under fixed exchange rates, in contrast to the evidence. In fact, to the extent that states (or provinces) are analogous to countries within a fixed exchange rate area, the standard model also predicts a negative correlation between relative consumption growth and real exchange rates between states. The critical requirement in explaining the data is to allow for shocks which cause relative consumption growth to rise and the nominal exchange rate to simultaneously appreciate under flexible exchange rates, but which leave relative inflation rates unchanged (or to increase) under fixed exchange rates.

We amend the standard sticky price open economy model to allow for these features. In particular, we combine the assumptions of a) incomplete financial markets (limited by trade in non-contingent bonds), b) ex-ante staggered price setting in both countries, and c) a combination of persistent productivity shocks and transitory relative demand shocks. We show that this combination leads to a model in which movements in the nominal exchange rate may be dominated by transitory demand shocks, causing relative consumption and the real exchange rate to move in opposite directions, while, controlling for the nominal exchange rate, movements in the real exchange rate attributable to relative inflation rates lead relative consumption and the real exchange rate to move in the same direction. If transitory demand shocks play a significant enough role, then the model predicts that under flexible exchange rates, the correlation between relative consumption growth and the real exchange rate will be negative. But fixing the exchange rate produces a positive correlation. Thus, in principle, we can answer the question of the title - the role of the border in cross country consumption risk sharing is crucially tied the nominal exchange rate regime, and the exchange rate displays characteristics in the model which is akin to that seen in the data.

2 Estimating the border effect

2.1 Key theoretical relationship

In this section, we present a general model of risk-sharing without additional features of production, sticky prices, etc, that are explored in section (3) below. To illustrate the main idea behind the Backus-Smith puzzle, consider a multi-jurisdiction (where a jurisdiction may be a country or region) stochastic model. The utility of a

representative household in jurisdiction $j = 1, \dots, J$ is given by:

$$E_t \sum_{s=0}^{\infty} \beta^s U(C_{j,t+s}, \xi_{j,t+s}), \quad \beta < 1$$

where β is the subjective discount factor, $C_{j,t}$ denotes a composite consumption good in country j . Here $\xi_{j,t}$ represents a jurisdiction specific factor which can affect the marginal utility of consumption, apart from consumption itself. This could represent pure preference shocks, or movements in work-hours when households have non-separable utility. Define $P_{j,t}$ to be the price of a representative consumption basket in jurisdiction j in period t . Also let $S_t^{i,j}$ be the exchange rate that converts prices from country j 's currency to country i 's currency in period t . If jurisdictions are within the same country, then $S_t^{i,j} = 1$. Then the real exchange rate between any two regions i and j in different countries is given by $RE R_t^{i,j} = S_t^{i,j} P_{j,t} / P_{i,t}$, or $RE R_t^{i,j} = P_{j,t} / P_{i,t}$ if i and j are two regions in the same country.

Suppose that there is a complete set of state-contingent securities available to households in all countries. In this case, the key optimality condition is to equate marginal utilities of consumption across countries (or regions), adjusted for differences in price levels, evaluated in a common currency:

$$U_c(C_{i,t}, \xi_{i,t}) RE R_t^{i,j} = U_c(C_{j,t}, \xi_{j,t}). \quad (2.1)$$

This equation must hold in every date and state of the world, between any two countries or regions i and j . It says that in equilibrium, consumption between households i and j must be allocated in a way that marginal utility (converted into the same units using the real exchange rate) is equalized across countries. Say now that $\xi_{j,t} = 1$ for all t .² Then if utility is of a constant relative risk aversion (CRRA) form, with the coefficient of relative risk aversion σ , equation (2.1) becomes

$$\left(\frac{C_{i,t}}{C_{j,t}} \right)^{\sigma} = RE R_t^{i,j},$$

or equivalently in logs

$$\sigma (\ln C_{i,t} - \ln C_{j,t}) = \ln RE R_t^{i,j}.$$

²In the simplest version of the theoretical model below, we will re-introduce preference shocks.

The expression above must also hold in growth rates:

$$\sigma(\Delta \ln C_{i,t} - \Delta \ln C_{j,t}) = \Delta \ln RER_t^{i,j}, \quad (2.2)$$

where $\Delta \ln X_{i,t} = \ln X_{i,t} - \ln X_{i,t-1}$. These expressions establish the close relationship between the real exchange rate and relative consumption between jurisdictions i and j . In particular, it implies that consumption growth should be relatively higher between $t - 1$ and t in jurisdictions whose real exchange rates depreciate during the same period. Therefore, if markets are complete, the correlation, $\rho_{e, c_j/c_i} = \text{corr}(\Delta \ln RER_t^{i,j}, \sigma \Delta \ln \frac{C_{j,t}}{C_{i,t}})$, should be equal to 1, as pointed out by Backus and Smith (1993) and Kollmann (1995). But a version of condition (2.2), defined in terms of conditional expectations, will also hold even under incomplete markets, so long as some financial assets can be traded across countries (see e.g. Obstfeld.). Notice that if relative purchasing power parity (PPP) holds, so that RER is constant, then $\Delta \ln RER_t^{i,j} = 0$. In this case we get a standard risk-sharing result that consumption growth rate should be equal across jurisdictions. This simple implication has been tested extensively in the cross-country context in Asdrubali, Sørensen, and Yosha (1996), Athanasoulis and van Wincoop (2001), Bayoumi and Klein (1997), Hess and Shin (1998), Del Negro (2002), Van Wincoop (1995), Crucini (1999), and others.

2.2 Evidence from US states and Canadian provinces

Equation (2.2) gives us the key testable relationship implied by the model. As is clear from (2.2), the condition can be applied to any two locations of interest: countries, regions, states/ provinces/ prefectures, etc. We use this relationship to study the effect of national and regional risk-sharing between the US, Canada, Germany, Japan and Spain. We begin by focusing on just the US and Canada. This allows us to investigate the impact of the border on risk sharing in a similar manner to studies of deviations of the law of one price across regions within the US and Canada (Engel and Rogers (1996), Gorodnichenko and Tesar (2009)). In the next sub-section we investigate the same question for all five countries in our sample.

We employ intra-national data on consumption, output and prices in 50 US states during 1969-2006 and in 12 Canadian provinces and territories during 1981-2007.³

³For the US we use retail sales to proxy for private consumption; we construct state-level price

Using this data we compute all possible unique bilateral pairs of differences between log consumption, price and output growth rates. The pairs of states within the US we denote by UU, the pairs of provinces within Canada – by CC and state-province pairs by UC. The summary statistics for our dataset are reported in Table 1. Note that, as to be expected, within country real exchange rates are much less volatile than across country real exchange rates.

Table 1: Sample summary statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Panel 1: US					
$\Delta \ln C_{i,t} - \Delta \ln C_{j,t}$	41650	0.00013	0.05510	-0.44723	0.38587
$\Delta \ln Y_{i,t} - \Delta \ln Y_{j,t}$	41650	0.00001	0.04574	-0.36243	0.42771
$-\Delta \ln RER_t^{i,j}$	41650	-0.00012	0.01166	-0.09220	0.09985
Panel 2: Canada					
$\Delta \ln C_{i,t} - \Delta \ln C_{j,t}$	1674	0.00149	0.01823	-0.06804	0.11139
$\Delta \ln Y_{i,t} - \Delta \ln Y_{j,t}$	1674	0.00200	0.05566	-0.29884	0.43895
$-\Delta \ln RER_t^{i,j}$	1674	0.00058	0.00950	-0.03312	0.03416
Panel 3: US-Canada					
$\Delta \ln C_{i,t} - \Delta \ln C_{j,t}$	13000	0.00253	0.04318	-0.36621	0.28524
$\Delta \ln Y_{i,t} - \Delta \ln Y_{j,t}$	14200	-0.00420	0.05149	-0.72791	0.28154
$-\Delta \ln RER_t^{i,j}$	14200	-0.00091	0.05509	-0.15880	0.16038
Notes: The table reports summary statistics of the presented variables for three samples: all US-US state pairs (Panel 1); all Canada-Canada province pairs (Panel 2); all US-Canada pairs (Panel 3). Obs. refer to the number of observations in each sample; Mean - sample average; Std. Dev. - sample standard deviation; Min-sample minimum; Max-sample maximum.					

To simplify our notation we will use $\Delta c_t^{i,j}$ to denote relative consumption growth between two locations i and j , so that $\Delta c_t^{i,j} = \Delta \ln C_{i,t} - \Delta \ln C_{j,t}$; and $\Delta e_t^{i,j}$ to denote real exchange rate growth between locations i and j , so that $\Delta e_t^{i,j} = \Delta \ln RER_t^{i,j}$. Then based on equation (2.2) we posit the following specification to link relative

indices using consumer price index (CPI) for main metropolitan areas and rural/urban prices; and use Gross State Product to measure output in the 50 states. In all other countries we use final consumption and output from regional accounts and regional consumer price indices. Data details are provided in the Appendix.

consumption growth and real exchange rate growth:

$$\Delta c_t^{i,j} = \beta_0 + \beta_1 \Delta e_t^{i,j} + \beta_2 (\Delta e_t^{i,j} * border_{i,j}) + v_t^{i,j},$$

where $v_t^{i,j}$ is the error term arising due to preference shocks, measurement error, etc. $border_{i,j}$ is the border dummy that takes value of one for all UC location pairs, and a value of zero otherwise. This allows us to focus on the distinction between cross country risk sharing and cross region risk sharing.

This specification restricts the relationship between the real exchange rate and relative consumption to be the same for any two locations in the US, in Canada or any two locations between the US and Canada. However, it is plausible to posit that the same change in the real exchange rate could be associated with different movements in relative consumption depending on the particular locations observed. In the theoretical model below for instance, we allow for preference shocks which may differ between any two locations. But more generally there may be differences in the degree of openness in goods or financial markets between two jurisdictions that are not reflected in changes in the real exchange rate. Distance represents a natural explanatory variable in the studies of the deviations from the law of one price between location pairs. In terms of deviations from risk sharing, distance may seem somewhat less compelling, since a) it may already be incorporated in the movement in real exchange rates, and b) it is a constant, and may not affect the risk sharing relationship when measured in growth rates. Nevertheless, some studies (e.g. Portes and Rey (2005), Okawa and van Wincoop (2010)) have documented the explanatory power of gravity type variables in accounting for financial market integration. To allow for this, we thus amend the basic relationship so as to allow for a distance measure, as in the gravity literature. Our benchmark model specification thus becomes

$$\Delta c_t^{i,j} = \beta_0 + \beta_1 \Delta e_t^{i,j} + \beta_2 (\Delta e_t^{i,j} * border_{i,j}) + \beta_3 \Delta e_t^{i,j} * \ln \tilde{d}_{i,j} + \beta_4 \ln \tilde{d}_{i,j} + v_t^{i,j}, \quad (2.3)$$

where $\ln \tilde{d}_{i,j}$ is the normalized log distance between any two locations i and j , defined as $\ln \tilde{d}_{i,j} = \ln d_{i,j} - \overline{\ln d_{i,j}}$. Here $d_{i,j}$ is the distance between locations i and j , which we proxy using the distance between the capital cities of the US states and Canadian provinces; while $\overline{\ln d_{i,j}} = 7.69$ is the average log distance between all UC pairs. This normalization implies that $\ln \tilde{d}_{i,j}$ is equal to zero at $\ln d_{i,j} = \overline{\ln d_{i,j}}$, and simplifies

interpretation of the β_2 coefficient, which now expresses the average effect of the border for the consumption-RER relationship between any two locations that are $\ln \tilde{d}_{i,j}$ kilometers away. The interaction term between the real exchange rate and distance allows the relationship between $\Delta c_t^{i,j}$ and $\Delta e_t^{i,j}$ to change monotonically with the distance.

Table 2: Estimates of Border Effect: US-Canada

	Pooled (i)	Fixed effects (ii)	Pooled (iii)	Fixed effects (iv)
$\Delta e_t^{i,j}$	0.415*** (0.023)	0.409*** (0.026)	0.307*** (0.023)	0.303*** (0.027)
$\Delta e_t^{i,j} * border_{i,j}$	-0.454*** (0.024)	-0.449*** (0.027)	-0.350*** (0.024)	-0.346*** (0.027)
$\Delta e_t^{i,j} * \ln \tilde{d}_{i,j}$	-0.042*** (0.011)	-0.043*** (0.011)	-0.037*** (0.011)	-0.038*** (0.011)
$\ln \tilde{d}_{i,j}$	-0.001*** (0.000)		-0.001** (0.000)	
$\Delta y_t^{i,j}$			0.155*** (0.006)	0.151*** (0.007)
$\Delta y_t^{i,j} * border_{i,j}$			-0.013 (0.011)	-0.013 (0.014)
$\Delta y_t^{i,j} * \ln \tilde{d}_{i,j}$			-0.010 (0.008)	-0.007 (0.009)
$\Delta e_t^{i,j} + \Delta e_t^{i,j} * border_{i,j}$	-0.039*** (0.007)	-0.039*** (0.006)	-0.043*** (0.007)	-0.043*** (0.006)
N	56324	56324	56324	56324
R^2	0.01	within = 0.0072 between = 0.0275 overall = 0.0074	0.0262	within = 0.0249 between = 0.1876 overall = 0.0261

Notes: The dependent variable is relative consumption growth between locations i and j , $\Delta c_t^{i,j}$. The estimated specification in columns (i)-(ii) is equation (2.3); while in columns (iii)-(iv) it is equation (2.4). Robust standard errors are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.

Our findings from the OLS and fixed effects estimation of equation (2.3) are presented in columns (i) and (ii) of Table 2.⁴ The results in column (i) indicate that the conditional correlation between the growth rates of RER and relative consumption within US and Canada is positive and significant, equal to 0.415 on average. We can also see that the estimated border effect is large and economically significant. In fact, due to this effect, the consumption-RER correlation across countries turns negative, equal to -0.039 on average. Taking (2.2) as our basic theory of risk-sharing, these results suggest that relative prices facilitate risk-sharing within countries, but impede

⁴In the fixed effects regression the fixed effects capture the time-invariant, bilateral-pair specific effects.

risk-sharing across countries. The estimates in column (ii) obtained from the fixed effects regression confirm this finding.

How sensitive are these findings to the assumption of complete access to capital markets? Many studies of risk-sharing, both intra-national and international, have relaxed this assumption and posited the alternative specification in which at least a fraction of consumers do not make consumption plans based on intertemporal optimization, but rather follow rules of thumb, or equivalently, have no ability to borrow and lend at all.⁵ To allow for this, we extend our framework to encompass limited capital market participation. Say that a fraction of households are hand-to-mouth consumers; that is they are restricted to consume only their current income. These households do not have access to capital markets, and therefore cannot participate in international risk-sharing. The testable implication of such a modified model is that relative consumption growth of these hand-to-mouth consumers living in any two locations follows their relative income growth. Let $\Delta y_t^{i,j} = \Delta \ln Y_{i,t} - \Delta \ln Y_{j,t}$ denote the relative consumption growth between locations i and j at time t . Then the relationship in (2.3) must be modified to account for the limited participation as follows:

$$\begin{aligned} \Delta c_t^{i,j} = & \beta_0 + \beta_1 \Delta e_t^{i,j} + \beta_2 (\Delta e_t^{i,j} * border_{i,j}) + \beta_3 \Delta e_t^{i,j} * \ln \tilde{d}_{i,j} + \beta_4 \ln \tilde{d}_{i,j} \\ & + \beta_5 \Delta y_t^{i,j} + \beta_6 (\Delta y_t^{i,j} * border_{i,j}) + \beta_7 \Delta y_t^{i,j} * \ln \tilde{d}_{i,j} + v_t^{i,j} \end{aligned} \quad (2.4)$$

Note that the equation specification in (2.4) allows for the border effect in the consumption-income relationship, and for the interaction term between $\Delta y_t^{i,j}$ and the normalized distance measure to allow the relationship between $\Delta c_t^{i,j}$ and $\Delta y_t^{i,j}$ to change monotonically with the distance. The results from this estimation are presented in columns (iii) and (iv) of Table 2. We find that there is significant positive association between $\Delta c_t^{i,j}$ and $\Delta y_t^{i,j}$ in both pooled and fixed effects specifications. Allowing for limited participation also affects the within-country correlation between $\Delta c_t^{i,j}$ and $\Delta e_t^{i,j}$ as it declines to about 0.307. At the same time, the border effect in the consumption-real exchange rate relationship remains negative and significant. In fact, this effect turns the correlation between $\Delta c_t^{i,j}$ and $\Delta e_t^{i,j}$ for the UC pairs negative and significant, as before. Overall, our estimated border effect in the consumption-real

⁵See for instance, Crucini (1999), Hess and Shin (2000), Hess and Shin (2010), Kollman (2010) and Devereux, Smith and Yetman (2009).

exchange rate risk-sharing remains robust to the inclusion of income.

We also allow the intra-national correlation between $\Delta c_t^{i,j}$ and $\Delta e_t^{i,j}$ to be country-specific by including a product term of the dummy variable for all CC provincial pair with $\Delta e_t^{i,j}$ into the specification (2.4), but find that our estimate of the border effect remains unchanged. These results are presented in Table A1 in the Appendix.

2.2.1 What drives the border effect?

We now investigate the source of the negative border effect. That is, what explains the negative relationship between consumption and the real exchange rate across borders? For this purpose we decompose the real exchange rate into its components as follows. Recall the definition of the real exchange rate: $RE R_t^{i,j} = P_{j,t} S_t^{i,j} / P_{i,t}$. Taking logs and first-differencing we get

$$\Delta \ln RE R_t^{i,j} = \Delta \ln(P_{j,t}/P_{i,t}) + \Delta \ln S_t^{i,j},$$

where the first term on the right-hand-side captures movements in the the real exchange rate due to changes in the relative prices, while the second term is due to the movements in the nominal exchange rate. Table 3 reports the summary statistics for the three terms above.

Table 3: Decomposing RER movements for UC location pairs

Variable	Obs	Mean	Std. Dev.	Min	Max
	(i)	(ii)	(iii)	(iv)	(v)
$\Delta \ln RE R_t^{i,j}$	13000	-0.00091	0.05509	-0.15880	0.16038
$\Delta \ln(P_{j,t}/P_{i,t})$	13000	-0.00068	0.01771	-0.09098	0.09093
$\Delta \ln S_t^{i,j}$	13000	-0.00023	0.05120	-0.06895	0.11342

Notes: The table reports summary statistics of the presented variables for a sample of all US-Canada pairs. Obs. refer to the number of observations in each sample; Mean - sample average; Std. Dev. - sample standard deviation; Min-sample minimum; Max-sample maximum.

As can be seen from column (iii) the real exchange rate across borders is very volatile and the majority of this volatility comes from nominal exchange rate movements. At the same time, relative prices across countries exhibit more volatility than their intra-national counterparts: compare the volatility of 1.77% for UC location pairs with 1.17% for UU location pairs and 0.95% for CC location pairs. Next we

amend our specification in equations (2.3) and (2.4) to include the growth rate in nominal exchange rates. This allows us to assess the relative contribution of real exchange rate components – relative prices and nominal exchange rate – to the consumption–real exchange relationship. The results are presented in Table 4.

Table 4: Estimates of the Border Effect: RER Decomposition

	Pooled	Fixed effects	Pooled	Fixed effects
	(i)	(ii)	(iii)	(iv)
$\Delta e_t^{i,j}$	0.429*** (0.023)	0.424*** (0.026)	0.320*** (0.023)	0.318*** (0.027)
$\Delta e_t^{i,j} * border_{i,j}$	0.033 (0.031)	0.043 (0.035)	0.114*** (0.030)	0.125*** (0.034)
$\Delta e_t^{i,j} * \ln \tilde{d}_{i,j}$	-0.056*** (0.011)	-0.057*** (0.012)	-0.050*** (0.011)	-0.051*** (0.011)
$\Delta y_t^{i,j}$			0.156*** (0.007)	0.152*** (0.007)
$\Delta y_t^{i,j} * border_{i,j}$			-0.024*** (0.010)	-0.024* (0.013)
$\Delta y_t^{i,j} * \ln \tilde{d}_{i,j}$			-0.005 (0.007)	-0.002 (0.009)
$\Delta \ln S_t^{i,j}$	-0.551*** (0.021)	-0.555*** (0.024)	-0.525*** (0.021)	-0.532*** (0.022)
N	56324	56324	56324	56324
R^2	0.020	0.015	0.033	0.033

Notes: The dependent variable is relative consumption growth between locations i and j , $\Delta c_t^{i,j}$. The estimated specification in columns (i)-(ii) is equation (2.3); while in columns (iii)-(iv) it is equation (2.4). Both are modified to include the growth rate of the nominal exchange rate. Robust standard errors are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.

Columns (i) and (ii) provide estimates of specification (2.3), while columns (iii) and (iv) allow for market segmentation and thus summarize the estimates of specification (2.4), both amended to include the nominal exchange rate growth rate between regions i and j . The key result that stands out from Table 4 is that border effect turns positive when we control for nominal exchange rate movements. The coefficient on the nominal exchange rate growth, in turn, is negative and significant. This suggests that the negative border effect estimated in Table 2 is primarily due to the nominal exchange rate movements. This supports the findings of Hess and Shin (2010) and

Hadzi-Vaskov (2008) who look at cross country regressions in which different bilateral pairs of countries have differences in their exchange rate volatility. Thus, without at this stage suggesting causation, the finding seems to indicate that country pairs with higher nominal exchange rate will deviate more from the benchmark model of full risk sharing. As we see in the next section, there are a number of alternative theoretical interpretations of this finding.

To sum up so far, our findings seem to suggest that the relative price movements facilitate consumption risk-sharing across Canadian provinces, and across the U.S. states; while they obstruct consumption risk-sharing across the Canada-US border. Moreover, most of this border effect can be attributed to nominal exchange rate variability.

2.3 Additional evidence

We next expand our sample of countries to include Japanese prefectures, Spanish states and German bundeslaender. Our data for Germany covers 16 bundeslaender over 1995-2007 period; for Japan our dataset covers 47 prefectures over 1990-2005 period; while for Spain we have data for 18 autonomous communities over 1995-2004 period. For each country our dataset includes information on consumption, price indices and output in each regional unit.

Table 5 presents our estimates of the border effect for the five countries comprising our full sample. Columns (i) and (ii) report our estimates of equations (2.3), while columns (iii) and (iv) report the estimates for equation (2.4).

Consistent with our findings for the US and Canada, the intra-regional consumption-real exchange rate correlation for our sample of countries is positive and significant, equal to about 0.35 in a benchmark specification, and to about 0.28 when we control for the effect of output on consumption. This correlation declines dramatically when we account for the border. In particular, the cross-country consumption-real exchange rate correlation is -0.02 when estimated from a pooled regression, and increases to -0.01 in the fixed effects specification (robust to the inclusion of output). Table 6 confirms that the majority of this drop in the correlation is due to nominal exchange rate movements – the coefficient on the variable $(\Delta \ln S_t^{i,j})$ is negative and significant. It is also worthwhile to note that the coefficient on the $\Delta c_t^{i,j} * border_{i,j}$ variable is negative and significant, implying that the movements of cross-border relative prices

Table 5: Estimates of Border Effect, all countries

	Pooled	Fixed effects	Pooled	Fixed effects
	(i)	(ii)	(iii)	(iv)
$\Delta e_t^{i,j}$	0.349*** (0.019)	0.373*** (0.023)	0.275*** (0.019)	0.298*** (0.022)
$\Delta e_t^{i,j} * border_{i,j}$	-0.371*** (0.020)	-0.385*** (0.024)	-0.291*** (0.020)	-0.305*** (0.023)
$\Delta e_t^{i,j} * \ln \tilde{d}_{i,j}$	0.021*** (0.003)	0.026*** (0.004)	0.019*** (0.003)	0.022*** (0.004)
$\Delta y_t^{i,j}$			0.169*** (0.006)	0.161*** (0.008)
$\Delta y_t^{i,j} * border_{i,j}$			-0.036*** (0.008)	-0.050*** (0.012)
$\Delta y_t^{i,j} * \ln \tilde{d}_{i,j}$			0.040*** (0.004)	0.039*** (0.005)
$\Delta e_t^{i,j} + \Delta e_t^{i,j} * border_{i,j}$	-0.022*** (0.004)	-0.012*** (0.004)	-0.016*** (0.004)	-0.008** (0.004)
N	156509	156509	156509	156509
R2	0.003	0.003	0.050	0.047

Notes: The dependent variable is relative consumption growth between locations i and j , $\Delta c_t^{i,j}$. The estimated specification in columns (i)-(ii) is equation (2.3); while in columns (iii)-(iv) it is equation (2.4). Robust standard errors are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.

reduce international risk-sharing relative to the intra-regional relative prices.

Overall, our empirical analysis provides us with several insights into the consumption-RER relationship: (i) Within countries this correlation is positive and significant, implying some amount of intra-regional risk-sharing. This risk-sharing, however, is far from perfect; (ii) Across countries, the consumption-real exchange rate correlation is significantly smaller than within countries. It is in fact negative between US and Canada, and in the full sample of countries; (iii) The majority of the decline in international risk-sharing relative to the intra-national risk-sharing is due to nominal exchange rate co-moving negatively with relative consumption; (iv) International relative price movements (controlling for nominal exchange rate changes) hinder international risk-sharing. We next develop a theoretical framework to shed some light on these empirical findings.

Table 6: Estimates of the Border Effect: RER Decomposition, all countries

	Fixed effects (i)	Fixed effects (ii)
$\Delta e_t^{i,j}$	0.384*** (0.023)	0.308*** (0.022)
$\Delta e_t^{i,j} * border_{i,j}$	-0.127*** (0.026)	-0.061*** (0.025)
$\Delta e_t^{i,j} * \ln \tilde{d}_{i,j}$	0.038*** (0.004)	0.034*** (0.004)
$\Delta y_t^{i,j}$		0.159*** (0.008)
$\Delta y_t^{i,j} * border_{i,j}$		-0.049*** (0.012)
$\Delta y_t^{i,j} * \ln \tilde{d}_{i,j}$		0.037*** (0.005)
$\Delta \ln S_t^{i,j}$	-0.293*** (0.011)	-0.278*** (0.010)
N	156509	156509
R2	0.001	0.022

Notes: The dependent variable is relative consumption growth between locations i and j , $\Delta c_t^{i,j}$. The estimated specification in columns (i)-(ii) is equation (2.4). Both are modified to include the growth rate of the nominal exchange rate. Robust standard errors are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.

3 Consumption Risk-Sharing with Sticky Prices

From the previous section, it is apparent that the presence of the nominal exchange rate plays a key role in empirical tests of the risk sharing relationship between bilateral consumption differences and real exchange rate changes. What can account for this? One obvious suggestion is that goods prices are sticky, and so substantial real exchange rate adjustment, in the short run, can be achieved only by nominal exchange rate movement. But while this may *facilitate* an explanation, it is not clearly a complete resolution of the problem. Many sticky price models (e.g. Clarida, Gali, and Gertler, 2002, Devereux and Engel, 2003) exhibit volatile real and nominal exchange rates, but still have the property that the cross country risk sharing condition between consumption and real exchange rates holds exactly. On the other hand, many models

in the literature which offer potential resolutions to the Backus Smith puzzle have no role at all for the nominal exchange rate, and so cannot offer a robust explanation of the findings of the previous section in the sense that they imply that eliminating exchange rate volatility by fixing the exchange rate between two countries would have no implications for the empirical tests of risk-sharing.

In this section, we attempt to narrow this gap by combining features of the previous literature on the Backus Smith puzzle with a fairly standard two country ‘New Keynesian’ model with gradual price adjustment. We then ask whether this model exhibits the property that the sign of the consumption growth real exchange rate correlation depends upon the exchange rate regime. That is, we ask whether the apparent failure of efficient consumption risk sharing can be attributed to the presence of nominal exchange rate volatility?

As we show, the answer is nuanced, and requires a balance of a number of competing mechanisms. Ultimately, the question can be resolved down to the following requirement. We need to isolate a mechanism whereby, under a floating exchange rate system, the preponderance of shocks produces a negative correlation between relative consumption growth and the real exchange rate, with this negative correlation itself being accounted for by movements in the nominal exchange rate (or, equivalently, the correlation between consumption growth and the real exchange rate, conditional on the nominal exchange rate, is positive). At the same time it should be the case that under the same composition of shocks, a policy that fixes the nominal exchange rate changes the sign of the consumption growth and real exchange rate correlation to a positive number. As we show below, it is not the case that simply adding together the assumption of price stickiness with a menu of shocks that product a negative correlation can satisfy this joint desiderata.

To show this, we start with a ‘bare-bones model’ which has both; a) nominal prices stickiness, and b) a shock which can produce a negative correlation between consumption growth and the real exchange rate. In this simple model, we show that the introduction an exchange rate peg, which eliminates nominal exchange rate flexibility cannot resolve the puzzle in the sense that it cannot reverse the sign of the consumption growth real exchange rate correlation. We then introduce an additional mechanism that does help to achieve this sign reversal. Following this, we develop a more elaborate model, more closely related to the literature, identifying the key requirements that are necessary to reconcile the model with the empirical findings.

3.1 A bare-bones model

To see that sticky prices alone cannot explain the importance of the nominal exchange rate, we take the simplest possible New Keynesian open economy model; basically an extension of the Clarida, Gali, and Gertler (2002) framework. Say that there are two countries, home and foreign. Let the utility of a representative infinitely lived home household evaluated from date 0 be defined as:

$$U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t \xi_t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right), \quad \beta < 1. \quad (3.5)$$

where C_t is the composite home consumption bundle, and N_t is home labour supply. The variable ξ_t represents a preference, or ‘demand’ shock, changing the intertemporal relative valuation of period by period utilities.⁶ Composite consumption is defined as $C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}$, $v \geq 1$, indicating the possibility for home bias in preferences.⁷ C_{Ht} is the consumption of the home country composite good by the home household, and C_{Ft} is consumption of the foreign composite good. C_{Ht} and C_{Ft} are defined over the range of home and foreign differentiated goods with elasticity of substitution θ between goods. The aggregate (CPI) price index for the home country is $P = P_H^{\frac{v}{2}} P_F^{1-\frac{v}{2}}$. Demand for individual differentiated home and foreign goods and home and foreign composite goods may be obtained from these functions in the usual way. Each firm faces a demand elasticity of θ .

In this example, we make the assumption of a complete set of state contingent international assets markets which implies that state-contingent nominal marginal utility is equated across countries, so that:

$$\xi_t C_t^{-\sigma} = \xi_t^* C_t^{*- \sigma} \frac{S_t P_t^*}{P_t} = \xi_t^* C_t^{*- \sigma} T_t^{v-1}, \quad (3.6)$$

where S_t is the nominal exchange rate (home price of foreign currency), $P_t^* = P_F^{*v/2} P_H^{*1-v/2}$ is the foreign CPI, and $T = \frac{S P_F^*}{P_H}$ is the home country terms of trade. Thus, the real exchange rate becomes T_t^{v-1} . Implicit in this condition is the assumption that the law of one price holds, so that $P_F = S P_F^*$ and equivalently for home goods.

⁶See for instance, Stockman and Tesar (1994) for an early application of preference shocks in multi-country DSGE models.

⁷We define $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$.

Note that in this simple example model, the only way in which there can arise a negative correlation between relative consumption and the real exchange rate is due to the presence of country-specific preference shocks ξ_t and ξ_t^* . In the extended model below, we depart from this specification, so that even in the absence of preference shocks, there may arise a negative correlation between realized relative consumption differences and the real exchange rate.

We assume that households also have access to a market in domestic nominal government bonds, each of which pays an interest rate of R_t in all states of the world. Thus we can define an Euler equation for nominal bond pricing given by:

$$\frac{\xi_t C_t^{-\sigma}}{P_t} = R_{t+1} E_t \beta \frac{\xi_{t+1} C_{t+1}^{-\sigma}}{P_{t+1}}. \quad (3.7)$$

Foreign households preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1-v/2)$ on the foreign (home) composite good in preferences.

Assume that production technologies for all goods are linear in labour, so that for a home good i , we have production $Y(i)$ as:

$$Y_{Ht}(i) = A_t N_t(i),$$

where A_t is a common economy-wide productivity shock that applies to all home firms. Each home firm re-sets its price according to a Calvo pricing policy, where the probability of re-adjusting its price is $1 - \kappa$ in each period. The optimal price for a typical firm i in the home country is:

$$\tilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0} m_{t+j} \kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{Ht+j}(i)}{E_t \sum_{j=0} m_{t+j} \kappa^j Y_{Ht+j}(i)}. \quad (3.8)$$

where m_t is a stochastic discount factor defined in the Appendix, and W_t is the nominal wage.⁸

In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1 - \kappa) \tilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (3.9)$$

⁸We assume that there is an optimal subsidy in place that eliminates the distortionary effect of the price markup.

The behaviour of foreign firms and the foreign good price index may be described analogously.

Assume that the home country monetary authority follows a Taylor rule, which targets the inflation rate of the home good $\Pi_{Ht} = \frac{P_{Ht}}{P_{Ht-1}}$. The foreign country however follows a rule which puts some weight on the nominal exchange rate. Specifically, assume that the foreign nominal interest rate R_{t+1}^* is set so that:

$$R_{t+1}^* = \beta^{-1} (\Pi_{Ft}^*)^\gamma (\Pi_{St})^{-\delta}, \quad (3.10)$$

where $\gamma > 1$, $\delta \geq 0$, $\Pi_{St} = \frac{S_t}{S_{t-1}}$, and S_t is the nominal exchange rate (home price of foreign currency). Thus, the higher is δ , the more weight the foreign monetary authority places on exchange rate changes, and the closer the foreign monetary rule approximates an exchange rate peg.

Market clearing conditions are given by:

$$Y_{Ht} = \frac{v}{2} \frac{P_t}{P_{Ht}} C_t + \left(1 - \frac{v}{2}\right) \frac{S_t P_t^*}{P_{Ht}} C_t^*. \quad (3.11)$$

Here $Y_{Ht} = V_t^{-1} \int_0^1 Y_{Ht}(i) di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta} di$. It follows that home country employment (employment for the representative individual home household) is given by $N_t = \int_0^1 N(i) di = A_t^{-1} Y_{Ht} V_t$.

In the same manner, we may write the aggregate market clearing condition for the foreign good as

$$Y_{Ft} = \frac{v}{2} \frac{P_t^*}{P_{Ft}^*} C_t^* + \left(1 - \frac{v}{2}\right) \frac{P_t}{S_t P_{Ft}^*} C_t^*. \quad (3.12)$$

The full solution of the model is defined in the Appendix. Here we present the log linear approximation around an efficient, zero-inflation steady state. As usual in the open economy new Keynesian models, we may define an inflation equation as a forward looking relationship in home and foreign output rates. Let $x = \ln(X_t/\bar{X})$ be the log deviation of any variable from its steady state (except for inflation and nominal interest rates, which are in levels).

We can use (3.6), (A8) and (3.12) to solve for home and foreign consumption and

the terms of trade as a function of aggregate home output. Then substituting into the linear approximation of (A5) gives the inflation equation for the home country:

$$\pi_{Ht} = k((\phi + \omega(1 + D))y_{Ht} + y_{Ft}\omega(D - 1) + \vartheta(\varepsilon_t - \varepsilon_t^*) - (1 + \phi)a_t) + \beta E_t \pi_{Ht+1} \quad (3.13)$$

where, $k = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$, $\omega = \frac{\sigma}{2D}$, $\vartheta = \frac{(2-v)(v(\sigma-1)+1)}{2D}$, and $D = \sigma v(2 - v) + (1 - v)^2$.

Likewise, using (A3) we may define the home country dynamic IS equation as:

$$\begin{aligned} & E_t(y_{Ht+1} - y_{Ht})(D + 1) + E_t(y_{Ft+1}^* - y_{Ft}^*)(D - 1) \\ & - \frac{(D+v-1)}{\sigma} E_t(\varepsilon_{t+1} - \varepsilon_t) - \frac{(D-v+1)}{\sigma} E_t(\varepsilon_{t+1}^* - \varepsilon_t^*) \\ & = \omega^{-1} E_t(r_t - \pi_{Ht+1}) \end{aligned} \quad (3.14)$$

where r_t is the home country nominal interest rate, and the interest rate rule implies

that: $r_t = \gamma \pi_{Ht}$. An equivalent set of conditions hold for the foreign country, except we define the foreign interest rate rule as $r_t^* = \gamma \pi_{Ft} - \delta(\tau_t - \tau_{t-1}) + \delta(\pi_{Ft} - \pi_{Ht})$. This captures the degree to which the foreign monetary authority targets the exchange rate.⁹ Here τ_t is the terms of trade in terms of log deviations. It may be shown that in this simple model, the terms of trade follows the process:

$$\tau_t = -\frac{(v-1)}{D}(\varepsilon_t - \varepsilon_t^*) + \frac{\sigma(y_{Ht} - y_{Ft})}{D} \quad (3.15)$$

In addition, the linear approximation to (3.6) gives relative consumption as:

$$c_t - c_t^* = \frac{1}{\sigma}(\varepsilon_t - \varepsilon_t^*) + \frac{(v-1)}{\sigma}\tau_t \quad (3.16)$$

Equations (3.13) and (3.14), and the equivalent for the foreign country, along with the interest rate rules, and the terms of trade equation (3.15) may be solved for the path of inflation and output levels consequent on the shocks to preferences and technology.

⁹We could also have assumed that the exchange rate policy is symmetric, so that each monetary authority places some weight on the exchange rate in setting its interest rate. The results would be the same in this case.

First take the case with fully flexible exchange rates, so that $\delta = 0$. We may then take the difference between (3.13) and its foreign counterpart, and (3.14) and its foreign counterpart, to express the two country model in terms of differentials in inflation and output levels, i.e. $\Delta\pi_t = \pi_{Ht} - \pi_{Ft}^*$ and $\Delta y_t = y_{Ht} - y_{Ft}$ as follows:

$$\Delta\pi_t = k\left(\phi + \frac{\sigma}{D}\right)\Delta y_t + 2\vartheta\Delta\varepsilon_t - (1 + \phi)\Delta a_t + \beta E_t\Delta\pi_{t+1} \quad (3.17)$$

$$(\gamma\Delta\pi_t - E_t\Delta\pi_{t+1}) = \frac{\sigma}{D}E_t(\Delta y_{t+1} - \Delta y_t) - \frac{(v-1)}{D}E_t(\Delta\varepsilon_{t+1} - \Delta\varepsilon_t) \quad (3.18)$$

where, $\Delta a_t = a_t - a_t^*$ and $\Delta\varepsilon_t = \varepsilon_t - \varepsilon_t^*$.

For illustration purposes, assume that both preference shocks and technology shocks follow an AR(1) process with persistence μ . Then there is a simple analytical solution to the system (3.17)-(3.18). With this, we may then use (3.15) and (3.16) to obtain the solution for relative consumption and the terms of trade as:

$$c_t - c_t^* = \frac{[(1 - \beta\mu)(1 - \mu) + k(\gamma - \mu)(1 + \phi v)]}{\Delta_1}\Delta\varepsilon_t + \frac{k(v-1)(\gamma - \mu)(1 + \phi)}{\Delta_1}\Delta a_t \quad (3.19)$$

$$\tau_t = \frac{-k(\sigma + \phi(v-1))(\gamma - \mu)}{\Delta_1}\Delta\varepsilon_t + \frac{k\sigma(\gamma - \mu)(\sigma + \phi(v-1))}{\Delta_1}\Delta a_t \quad (3.20)$$

where $\Delta_1 = \sigma(1 - \beta\mu)(1 - \mu) + k(\gamma - \mu)(\sigma + \phi D)$.

Shocks to preferences of the home country increase home relative consumption, but cause a terms of trade appreciation. Shocks to relative home technology also increase home relative consumption, but cause a terms of trade deterioration. As a result, the correlation between relative consumption and the real exchange rate (which is $v - 1$ times the terms of trade) may be negative or positive, depending upon the dominance of preference shocks relative to technology shocks. From (3.19) and (3.20) however, it is clear that the correlation will be affected by the degree of price stickiness. As the Calvo price rigidity parameter κ rises, k falls, and both relative consumption and the terms of trade are less and less affected by shocks to technology. As a result, the consumption real exchange rate correlation is more likely to be negative, the greater is the degree of price stickiness. This is our first key observation; the observations of deviations from the condition for risk sharing are likely to be affected by the degree

of price stickiness, and as a consequence, the stance of monetary policy.

How does a fixed exchange rate affect the correlation between relative consumption and the real exchange rate? With a fixed exchange rate, relative prices can adjust only through domestic and foreign inflation rates. The log change in the exchange rate, defined as π_{st} , must be zero. This implies that:

$$\pi_{st} = \tau_t - \tau_{t-1} + \pi_{Ht} - \pi_{Ht}^* = 0. \quad (3.21)$$

Thus, the lagged terms of trade acts as a state variable in the model. Substituting (3.21) and (3.15) into (3.13), we obtain a single equation determining the terms of trade under a fixed exchange rate:

$$\tau_t - \tau_{t-1} = \frac{k(\phi D + \sigma)}{\sigma} \tau_t + \frac{k(\phi(v - 1) + \sigma)}{\sigma} \Delta \varepsilon_t - k(1 + \phi) \Delta a_t + \beta E_t(\tau_{t+1} - \tau_t) \quad (3.22)$$

From (3.22), the terms of trade will adjust gradually to preference or technology shocks, since adjustment must take place through domestic inflation differentials rather than movements in exchange rates. Then, relative consumption can be solved using (3.16). Intuitively, a fixed exchange rate slows down the response of the terms of trade and relative consumption to both shocks.

What does this simple model imply regarding the consumption-real exchange rate correlation? We may first illustrate the impact of preference and technology shocks on relative consumption and the terms of trade under the two alternative exchange rate regimes in Figure 1. For this figure we use the parameter values in Table 7, which represent a very standard calibration. The discount factor is set at 0.99. The labor supply elasticity ϕ is set at unity. The value for κ is set to .75, giving an average degree of price stickiness of four quarters. The elasticity of intertemporal substitution is set at 0.5 so that $\sigma = 2$. The degree of home bias in preferences is set at $v = 1.5$, so that imports are 25 percent of GDP in steady state. The persistence of both shocks is set at 0.9. Under a fixed exchange rate, δ is set at a high value so that the nominal exchange rate is constant (the actual value for δ is irrelevant once is it set high enough).

Figures 1(a) and 1(b) show respectively the impact of a negative relative home preference shock on relative consumption and the real exchange rate under inflation

Table 7: Baseline model calibration

β	0.99	σ	2	ρ	.01
ϕ	1	v	1.5	δ	8
κ	0.75	μ	0.9		

targeting and a fixed exchange rate, while figures 1(c) and 1(d) show the equivalent response following a negative technology shock. Figures 1(a) and 1(c) show, as indicated by equations (3.19) and (3.20), that preference shocks and technology shocks have opposite effects on the real exchange rate, but affect consumption in the same direction. A negative correlation between relative consumption and the real exchange rate obtains when preference shocks tend to dominate. Figures 1(b) and 1(c) show that, under a fixed exchange rate, the impact on the real exchange rate is substantially dampened. Following a preference shock the real exchange rate depreciates initially, followed by a further depreciation. A technology shock causes an initial appreciation followed by further appreciation. The impact on relative consumption is magnified in response to a preference shock, since real the exchange rate cannot adjust to cushion the impact of the shock. On the other hand, the impact of the technology shock on relative consumption is dampened in a fixed exchange rate regime, because the reduction in the response of the real exchange rate means that there is a smaller response in relative output levels through expenditure switching from foreign towards home goods.

How does the exchange rate peg affect the implied consumption real exchange rate correlation? The sign of the correlation is determined predominantly by the relative volatility of preference shocks to technology shocks. We choose these relative volatilities so that, given other parameters in Table 7, the baseline consumption real exchange rate correlation is equal to -0.1 under an inflation targeting rule, and the volatility of relative consumption is set at 0.02. Table 8 gives the model's implications for the volatility of consumption, the real exchange rate, and the consumption-real exchange rate correlation, under an inflation targeting rule and under a fixed exchange rate policy.

Under the inflation targeting rules, the volatility of the real exchange rate is just over 3 percent. A fixed exchange rate reduces the volatility of the real exchange rate, but also increases the volatility of relative consumption. But despite a lower volatility of the real exchange rate, the consumption-real exchange rate correlation actually

Table 8: Policies in the bare-bones model with baseline calibration

Policy	stdev($c - c^*$)	stdev(RER)	corr($c - c^*, RER$)	corr($\varepsilon - \varepsilon^*, RER$)
$\delta = 0$	.020	.032	-0.12	-0.67
$\delta = 8$	.024	.020	-0.20	-0.53

falls (gets more negative). Therefore this model, even with sticky prices, does not capture the empirical property documented in the previous section - fixing the nominal exchange rate actually reduces the consumption real exchange rate correlation here, while in the data we see the opposite. Hence, the negative consumption-real exchange rate correlation is not driven by the nominal exchange rate in this model, even though prices are sticky, and the exchange rate regime itself does have real effects.

To see why this model cannot reproduce the empirical importance of the nominal exchange rate regime we found in the data, take the following decomposition of the consumption real exchange rate correlation, obtained using equation (3.16):

$$\text{corr}(c_t - c_t^*, RER_t) = \frac{\left(\sqrt{\text{var}(RER_t)} + \sqrt{\text{var}(\varepsilon_t - \varepsilon_t^*)} \text{corr}(\varepsilon_t - \varepsilon_t^*, RER_t) \right)}{\sigma \sqrt{\text{var}(c_t - c_t^*)}} \quad (3.23)$$

Recall that the real exchange rate in this simple model is just $(v - 1)\tau_t$. The correlation is a function of the standard deviation of the real exchange rate, the standard deviation of relative consumption and relative preference shocks, and the correlation of the real exchange rate and relative preference shocks. For this correlation to be negative in the first place, it must obviously be that $\text{corr}(\varepsilon_t - \varepsilon_t^*, RER_t) < 0$, so that demand shocks lead to a real appreciation. The value for $\text{corr}(\varepsilon_t - \varepsilon_t^*, RER_t)$ in this example is given in the rightmost column of Table 8. Fixing the exchange rate reduces the absolute value of $\text{corr}(\varepsilon_t - \varepsilon_t^*, RER_t)$, and in principle does the same for $\text{corr}(c_t - c_t^*, RER_t)$. But the fixed exchange rate also reduces $\sqrt{\text{var}(RER_t)}$, as shown in column 3 in Table 8. This tends to make $\text{corr}(c_t - c_t^*, RER_t)$ more negative. In this example, the second factor dominates the first factor, and the value of $\text{corr}(c_t - c_t^*, RER_t)$ falls when we move to a fixed exchange rate. Hence, the bare-bones sticky price model produces a consumption real exchange rate correlation that moves in the wrong direction when we move from inflation targeting to fixed exchange rates.

How can we amend the model to make it consistent with the empirical findings? From the decomposition above, the critical requirement is that $\text{corr}(\varepsilon_t - \varepsilon_t^*, RER_t)$ fall by more relative to the fall in $\sqrt{\text{var}(RER_t)}$ when we move from inflation targeting to fixed exchange rates. One way to facilitate this is by changing the nature of the price adjustment mechanism. Now, instead of the basic Calvo price adjustment model, we follow Woodford (2003) in assuming that newly price setting firms at time t must set prices *before* they observe time t shocks to preferences or productivity. In the approximated model, this leads to a change in the inflation equation for the home country from (3.13) to:

$$\pi_{Ht} = E_{t-1}k((\phi + \omega(1 + D))y_{Ht} + y_{Ft}\omega(D - 1) + \vartheta(\varepsilon_t - \varepsilon_t^*) - (1 + \phi)a_t) + \beta E_{t-1}\pi_{Ht+1} \quad (3.24)$$

Likewise, (3.14) is replaced by:

$$\Delta\pi_t = E_{t-1}k((\phi + \frac{\sigma}{D}))\Delta y_t + 2\vartheta\Delta\varepsilon_t - (1 + \phi)\Delta a_t + \beta E_{t-1}\Delta\pi_{t+1} \quad (3.25)$$

This alters the dynamics of the model, because nominal prices cannot immediately adjust to shocks, even for firms that are re-setting their price. Figure 2 shows how the impulse response to preference and technology shocks are affected, both under inflation targeting and under fixed exchange rates. The responses under inflation targeting differ only slightly from these in the baseline model - there is one period of adjustment after which new prices are updated. Under a fixed exchange rate, however, the real exchange rate cannot adjust at all in response to a shock. This significantly reduces the impact of a preference shock on the real exchange rate (panel (b) of Figure 2), thus reducing the (absolute) value of $\text{corr}(\varepsilon_t - \varepsilon_t^*, RER_t)$.

Table 9 illustrates the effect of this alternative price setting assumption (all other parameter and shock settings are as in Table 7).

Table 9: Policies in the bare-bones model with ex-ante pricing

Policy	stdev($c - c^*$)	stdev(RER)	corr($c - c^*, RER$)	corr($\varepsilon - \varepsilon^*, RER$)
$\delta = 0$	.020	.030	-0.12	-0.65
$\delta = 8$	.025	.018	0.03	-0.31

As before, moving from inflation targeting to a fixed exchange rate increases the

volatility of relative consumption and reduces the volatility of the real exchange rate. But the key difference from Table 8 is that the fixed exchange rate policy leads to a large fall in $\text{corr}(\varepsilon - \varepsilon^*, RER)$. Using the decomposition from (3.23), this leads to a reversal in the sign of $\text{corr}(c - c^*, RER)$. Hence, $\text{corr}(c - c^*, RER)$ becomes positive, and in principle, the bare-bones model amended to allow for ex-ante price setting can be made consistent with evidence on the importance of the nominal exchange rate for the consumption-real exchange rate correlation.

3.2 A more general model with incomplete markets and non-traded goods

While the previous model set out the ingredients necessary to account of the importance of the nominal exchange rate for the consumption-real exchange rate correlation, it did not allow for a substantial quantitative evaluation of the impact of the exchange rate regime under sticky prices. For instance, the real exchange rate in that model is driven only by differences in the composition of the consumption basket across countries, and in addition, we assumed that international financial markets were complete. In this section, we develop a more elaborate model which is closer in structure to the current literature. We amend the model to allow for incomplete financial markets. In addition, the model is driven by different types of technology shocks, which facilitates a negative consumption-real exchange rate correlation even in the absence of preference shocks. The structure of the model is similar to that of Benigno and Thoenissen (2008).

We briefly sketch out the structure of the extended model, leaving the detailed description to the Appendix. Again, there are two countries, home and foreign, with the population of each country is normalized at unity. We define home country preferences in the following way:

$$U_t = E_0 \sum_{\tau=0}^{\infty} \theta_{\tau} \xi_t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi_N \frac{N_{Nt}^{1+\phi}}{1+\phi} - \chi_H \frac{N_{Ht}^{1+\phi}}{1+\phi} \right). \quad (3.26)$$

We now assume that the composite consumption good is defined as:

$$C_t = \left(\varrho^{\frac{1}{\varphi}} C_{Tt}^{1-\frac{1}{\varphi}} + (1-\varrho)^{\frac{1}{\varphi}} C_{Nt}^{1-\frac{1}{\varphi}} \right)^{\frac{\varphi}{1-\varphi}},$$

where C_{Tt} and C_{Nt} represent respectively, the composite consumption of traded and non-traded goods. The elasticity of substitution between traded and non-traded goods is φ . Traded consumption in turn is decomposed into consumption of home goods, and foreign goods, as follows:

$$C_{Tt} = \left(\left(\frac{v}{2} \right)^{\frac{1}{\lambda}} C_{Ht}^{1-\frac{1}{\lambda}} + \left(1 - \frac{v}{2} \right)^{\frac{1}{\lambda}} C_{Ft}^{1-\frac{1}{\lambda}} \right)^{\frac{\lambda}{1-\lambda}},$$

where λ is the elasticity of substitution between home and foreign traded good. Again, in each case, we assume that consumption sub-aggregates are differentiated across the consumption of individual goods, with elasticity of substitution $\theta > 1$ across goods. We follow Gertler, Gilchrist, and Natalucci (2007) in assuming that the household supplies specialized separately to the non-traded and traded goods sector, so that N_{Nt} represents labor supply to non-traded firms and N_{Ht} denotes labor supply to traded goods firms. Finally, because the incomplete markets environment with constant time discount factor exhibits non-stationarity, we assume an endogenous time discount factor in the following way:

$$\Theta_{\tau+1} = \Theta_{\tau} \beta C_{A\tau}^{-\eta} / \bar{C}_A^{-\eta}, \quad \Theta_0 = 1, \quad (3.27)$$

where $0 < \eta < \rho$, $0 < \beta < 1$, C_A is aggregate home consumption and $\bar{C}_A$ is a constant.¹⁰

These consumption aggregates imply the following price index definitions:

$$P_t = \left(\varrho P_{Tt}^{1-\varphi} + (1 - \varrho) P_{Nt}^{1-\varphi} \right)^{\frac{1}{1-\varphi}},$$

$$P_{Tt} = \left(\frac{v}{2} P_{Ht}^{1-\lambda} + \left(1 - \frac{v}{2} \right) P_{Ft}^{1-\lambda} \right)^{\frac{1}{1-\lambda}},$$

where P_{Tt} and P_{Nt} represent traded and non-traded price levels, and P_{Ht} and P_{Ft} are retail prices of home exportables and foreign importables. We define the real exchange rate as before:

$$RER_t = \frac{S_t P_t^*}{P_t}.$$

We assume that international financial markets are incomplete in the sense that

¹⁰Following Schmitt Grohe and Uribe (2003), θ_{τ} is assumed to be taken as exogenous by individual decision makers. The impact of individual consumption on the discount factor is therefore not internalised.

financial trade takes place via non-contingent one period nominal bonds, denominated in home currency. The home budget constraint is given by:

$$P_t C_t + B_{t+1} = W_{Nt} N_{Nt} + W_{Ht} N_{Ht} + \Pi_t + R_t B_t. \quad (3.28)$$

where B_{t+1} indicates bond holdings of the home household W_{Nt} (W_{Ht}) represents the wage in the non-traded (traded) sector, and Π_t represents profits from the home firm plus net transfers from the government.

Household's choose consumption of individual goods, labor supply in each sector, and bond holdings in the usual way. Preferences, budget constraints, and choices of foreign households are determined in an analogous fashion. The critical difference from the previous section is that we no longer employ the full risk sharing condition given by (3.6). Instead, given an integrated world bond market the state by state risksharing condition (3.6) is replaced with the conditional risk sharing condition given by:

$$E_t \frac{\xi_{t+1} C_{t+1}^{-\sigma}}{\xi_t C_t^{-\sigma}} \frac{P_t}{P_{t+1}} = E_t \frac{\xi_{t+1}^* C_{t+1}^{*- \sigma}}{\xi_t^* C_t^{*- \sigma}} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \quad (3.29)$$

As in the recent literature (see e.g. Corsetti, Dedola, and Leduc (2008), Kollmann (2009), Benigno and Thoenissen (2008)), this condition implies that up to a first order approximation, expected consumption growth differentials across countries are positively related to expected changes in the real exchange rate. In our case, this relationship is conditional on the expected differences in the growth of preference shocks. In the analysis below, we will report results for the model with and without the presence of preference shocks.

Firms in each sector produce with linear technologies. A typical firm in the non-traded (traded) sector has production function $Y_{Nt}(i) = A_{Nt} N_{Nt}(i)$, ($Y_{Ht}(i) = A_{Ht} N_{Ht}(i)$). Thus, there are two technology shocks - shocks to the non-traded sector A_{Nt} , and to the traded sector A_{Ht} . These shocks play substantially different roles in determining the consumption-real exchange rate correlation.

Firms in each sector set prices following a Calvo price adjustment specification, and we assume again that prices must be set before information on shocks for the period is available. We allow for differences in the Calvo probabilities of price adjustment across traded and non-traded goods sector firms.

Monetary policy is set in the same way as before. The home country monetary

authority follows an inflation targeting rule, except that it targets the *consumer* price inflation so that the nominal interest rate in the home economy is $r_t = \rho + \gamma\pi_t$, where $\pi_t = p_t - p_{t-1}$. The foreign monetary rule is given by $r_t^* = \rho + \gamma\pi_t^* + \delta(S_t - S_{t-1})$.

Finally, goods market clearing conditions are given as:

$$\begin{aligned} Y_{Ht} &= C_{Ht} + C_{Ht}^*, \\ Y_{Ft}^* &= C_{Ft} + C_{Ft}^*, \\ Y_{Nt} &= C_{Nt}, \\ Y_{Nt}^* &= C_{Nt}^*. \end{aligned} \tag{3.30}$$

3.3 The consumption-real exchange rate correlation in the extended model

The full linear approximation of the model is more involved than the simple framework of the last subsection. There are two forward looking inflation equations in each country - for the traded goods and non-traded goods sector separately. In addition, the real exchange rate is now determined jointly by movements in the terms of trade and the internal relative price ratio of traded to non-traded goods. The linear approximation to (3.29) is given by:

$$E_t(\Delta c_{t+1} - \Delta c_{t+1}^*) = \frac{1}{\sigma} E_t(\Delta \varepsilon_{t+1} - \Delta \varepsilon_{t+1}^*) + \frac{1}{\sigma} E_t \Delta RER_{t+1} \tag{3.31}$$

where $\Delta c_{t+1} = c_{t+1} - c_t$, etc. The real exchange rate is defined by:

$$RER_t = \rho(\tau_{Nt}^* - \tau_{Nt}) + (1 - \rho)(v - 1)\tau_t + \rho\tau_t \tag{3.32}$$

Here $\tau_{Nt} = p_{Nt} - p_{Ht}$, $\tau_{Nt}^* = p_{Nt}^* - p_{Ft}^*$, and τ_t is the terms of trade as defined before. Since prices are fixed in advance, the impact of shocks on the real exchange rate can come about only through movements in the nominal exchange rate, and this changes only the terms of trade τ_t in the above decomposition.

Unlike the previous model, this extended model allows for a negative correlation between relative consumption and the real exchange rate, even in the absence of preference shocks. As shown by Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008), even if relative consumption and real exchange rates are positively correlated in conditional expectations, they may be ex-post negatively correlated due

to the presence of unanticipated shocks which move relative consumption and the real exchange rate in different directions. A clear example in the present model is a shock to the productivity of the traded good sector. In an incomplete markets setting, this would be expected to raise home consumption relative to foreign consumption, but due to an increased relative demand for non-traded goods, it can also lead to a real exchange rate appreciation.

There are three types of shocks in the extended model; shocks to preferences as before, and shocks to productivity in each of the sectors. We let each shock be AR(1) with persistence μ_ε , μ_N , and μ_T for shocks to preferences, non-traded and traded goods productivity, respectively, with standard deviations of the innovations given by σ_ε , σ_N , and σ_T respectively.

We must extend the calibration of Table 7 for this extended model. Table 10 describes the extended calibration.

Table 10: Extended model calibration

β	0.99	σ	2	ρ	0.7	κ_T	0.67	μ_N	0.3	σ_N	.7
ϕ	1	φ	.5	v	1.25	δ	0, 8	μ_T	0.85	σ_T	1.9
η	.01	λ	2	κ_N	0.8	μ_ε	0.22	σ_ε	.65		

We set $\eta = 0.01$, following Devereux and Sutherland (2008), in order to induce stationarity. The elasticity of substitution between non-traded and traded goods is usually estimated to be substantially below unity, so we set $\varphi = 0.5$, midway between the estimates of Stockman and Tesar (1995) and Mendoza (1995). The elasticity of substitution between home and foreign goods is set at 2, a benchmark estimate. We assume that the non-traded goods sector is 70 percent of GDP, a common estimate for the US economy. The degree of home bias in the traded goods sector is set at $v = 1.25$, half of the estimate of the previous simple model, since the traded goods sector is much smaller in this model.

The degree of price rigidity is likely to be substantially higher in the non-traded goods sector than in the traded goods sector. Nakamura and Steinsson (2008) measure the median duration of fixed prices in the US service sector to be 5 quarters, and 3 quarters for the non-service sector. We use these measures to set $\kappa_N = 0.8$ and $\kappa_T = 0.67$. In measuring the persistence and volatility of productivity shocks to the traded and non-traded sector, we follow Benigno and Thoenissen (2008). They

measure traded sector productivity shocks to have persistence 0.85 and standard deviation of 1.9 percent, while non-traded shocks are much less persistent and less volatile, with $\mu_N = 0.3$ and $\sigma_T = 0.7$. Finally, in measuring preference shocks, we follow Smets and Wouters (2007) estimates of Euler equation shocks for the US economy. They estimate these shocks are about half as volatile as productivity shocks, and have very low persistence with an AR(1) coefficient of 0.22.

3.4 Results with the extended model

Figure 3 shows the impulse responses to the three types of shocks in the extended model. Preference shocks act in similar ways to the previous model, except that their impact on the real exchange rate now is substantially lower, since prices in the traded goods sector are significantly more flexible than before, and there is less of an affect on the terms of trade due to the fact that home bias is lower in this model. The exchange rate regime in this extended model has only a limited impact on the response to preference shocks, for the same reasons.

A negative shock to the traded goods productivity causes a fall in home relative consumption, and a real exchange rate depreciation. Given the persistence of shocks, and open international bond markets, these shocks have very persistent affects in the model. The fixed exchange rate leads to a zero impact effect on the real exchange rate. Thus, we would anticipate that a fixed exchange rate would tend to reduce the degree of negative correlation between relative consumption and the real exchange rate to the extent that the correlation is driven by traded goods productivity shocks.

Finally, a negative shock to non-traded sector productivity reduces home relative consumption and causes a real exchange rate appreciation. This has a substantially more transitory effect than the shock to traded goods productivity, since the degree of persistence in the shock is much less.

Table 11 reports the simulation results for the extended model. As before, we report the results for the standard deviation of relative consumption, the real exchange rate, and the correlation between relative consumption and the real exchange rate. The volatility of relative consumption and the real exchange rate are substantially less in this model. Nevertheless, as before, in both the case without preference shocks and with preference shocks, the impact of an exchange rate peg is to increase the correlation between relative consumption and the real exchange rate. Hence, again

in this more elaborate model, we find that the model can quantitatively account to a substantial degree for the finding that the Backus-Smith ‘puzzle’ is a puzzle of floating versus fixed exchange rate jurisdictions. Particularly in the case without preference shocks, we find that the empirical findings of departures from consumption risk sharing associated with movements in nominal exchange rates are explained by the endogenous change in the consumption real exchange rate correlation that occurs as a result of a fixed exchange rate regime.

Table 11: Policies in the extended model

No preference shocks			
Policy	$\text{stdev}(c - c^*)$	$\text{stdev}(RER)$	$\text{corr}(c - c^*, RER)$
$\delta = 0$	.0016	.0017	-0.08
$\delta = 8$	.0020	.0014	0.29
With preference shocks			
$\delta = 0$	0.0090	.0017	-0.08
$\delta = 8$	0.0092	.0014	0.06

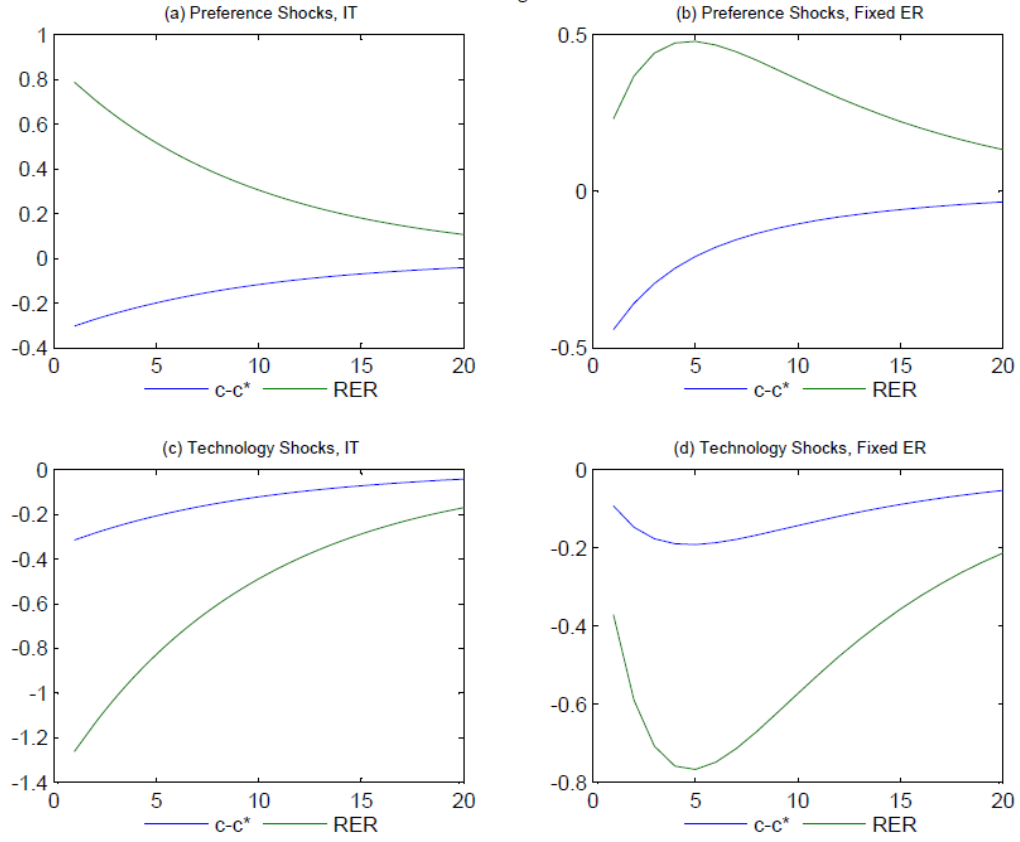
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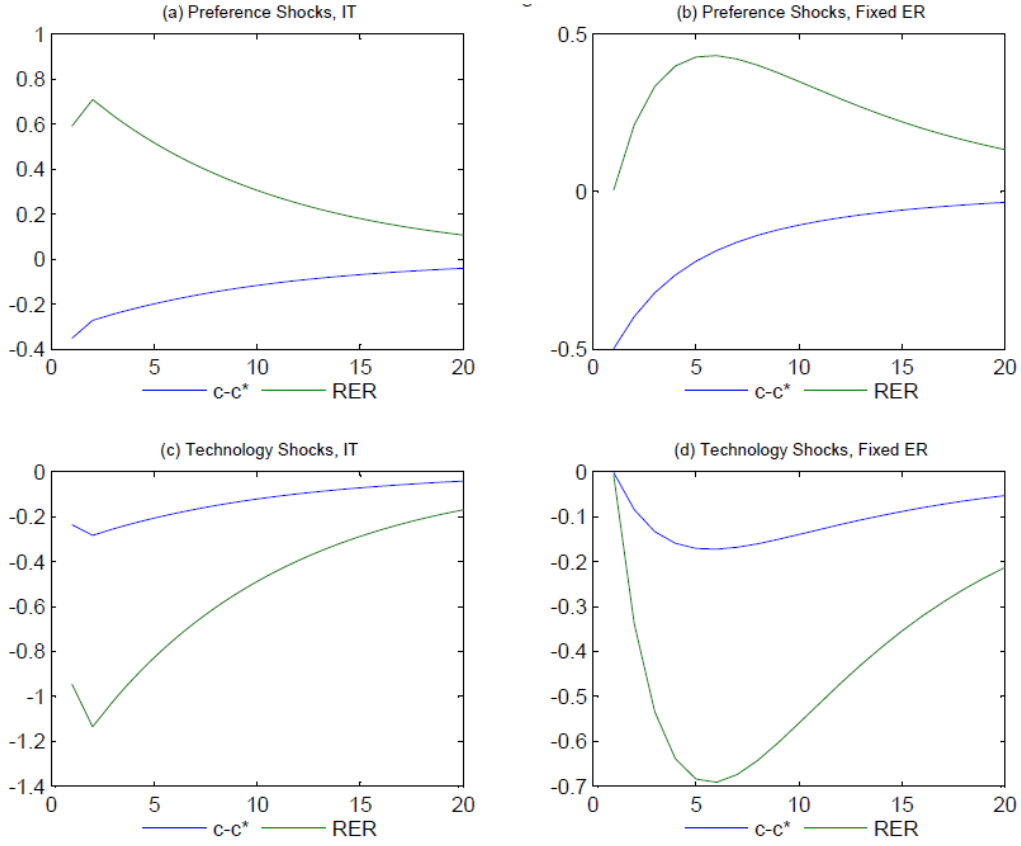
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Figure 1: Impulse responses in the bare-bones model



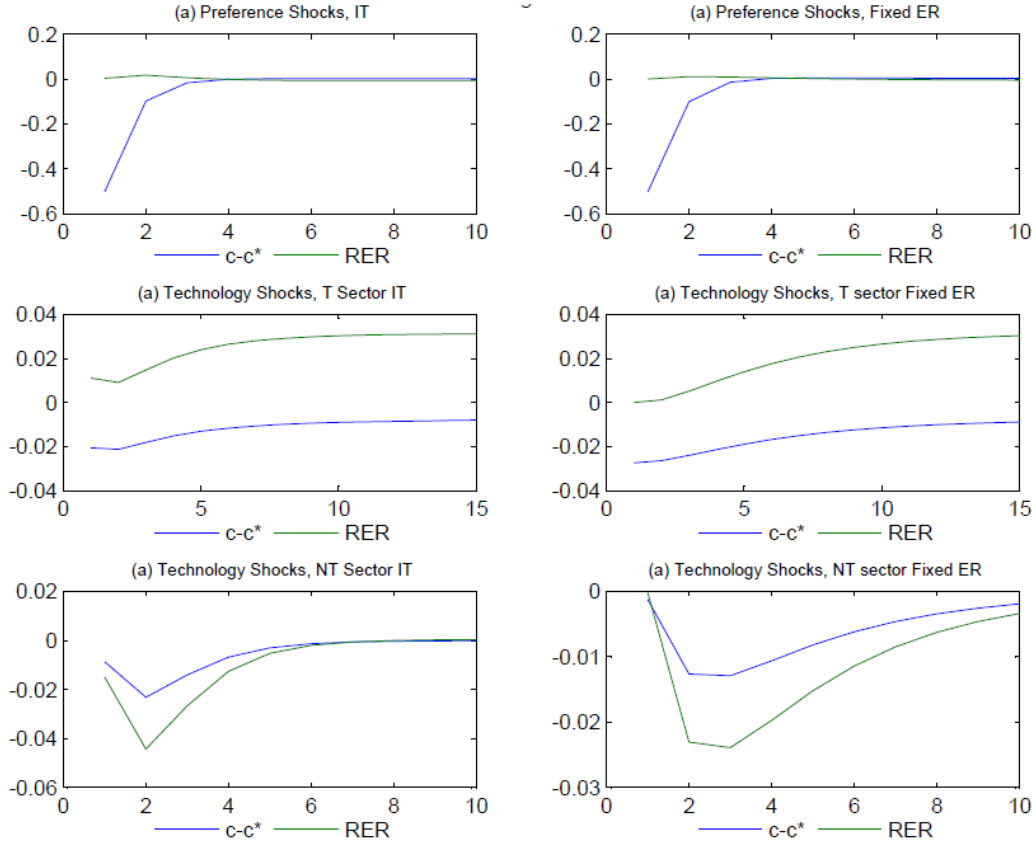
Notes: The figures above present the impulse responses of home country's relative consumption and RER to preference and technology shocks in a bare-bones model under benchmark calibration. Panels on the left ((a) and (c)) refer to the inflation targeting regime, while panels on the right ((b) and (d)) are the responses under a fixed exchange rate regime.

Figure 2: Impulse responses in the bare-bones model with ex-ante pricing



Notes: The figures above present the impulse responses of home country's relative consumption and RER to preference and technology shocks in a bare-bones model with ex-ante pricing. Panels on the left ((a) and (c)) refer to the inflation targeting regime, while panels on the right ((b) and (d)) are the responses under a fixed exchange rate regime.

Figure 3: Impulse responses in the extended model



Notes: The figures above present the impulse responses of home country's relative consumption and RER to negative preference and sectoral technology shocks in an extended model. Panels on the left refer to the inflation targeting regime, while panels on the right are the responses under a fixed exchange rate regime.

A Appendix

Preferences of the home country are defined as in (3.5) of the text, where the composite consumption term is defined as: $C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}$, $v \geq 1$. In turn, C_H and C_F are defined over the range of home and foreign differentiated goods with elasticity of substitution θ between goods, so that:

$$C_H = \left[\int_0^1 C_H(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \quad C_F = \left[\int_0^1 C_F(i)^{1-\frac{1}{\theta}} di \right]^{\frac{1}{1-\frac{1}{\theta}}}, \quad \theta > 1.$$

The price indices for home and foreign consumption composites are then defined as:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{\frac{v}{2}} P_F^{1-\frac{v}{2}}$.

The household's implicit labour supply is determined by the condition:

$$C_t^{-\sigma} W_t = P_t N_t^\phi \tag{A1}$$

The risk-sharing condition is given by

$$\xi_t C_t^{-\sigma} = \xi_t^* C_t^{*- \sigma} T_t^{v-1}, \tag{A2}$$

where $T = \frac{SP_E^*}{P_H}$.

The Euler equation for nominal bond pricing is given by:

$$\frac{U_C(C_t, \xi_t)}{P_t} = E_t \beta \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}} R_{t+1}. \tag{A3}$$

Foreign household preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1-v/2)$ on the foreign (home) composite good in preferences.

Firm i in the home economy uses the production function:

$$Y_t(i) = A_t N_t(i), \tag{A4}$$

The home firm has period t profits defined by $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_tH_t(i)(1 - s_t)$, where s_t is a wage subsidy offered to each home firm by the home government, financed with lump-sum taxation. When it can adjust its price, the home firm sets the new price, denoted $\tilde{P}_{Ht}(i)$, so as to maximize the present value of profits evaluated using the stochastic discount factor $m_{t+j} = \frac{P_t}{U_C(C_t, \varepsilon_t)} \frac{U_C(C_{t+j}, \xi_{t+j})}{P_{t+j}}$. This leads to the optimal price setting condition as follows:

$$\tilde{P}_{Ht}(i) = \frac{\theta}{\theta - 1}(1 - s_t) \frac{E_t \sum_{j=0} m_{t+j} \kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{Ht+j}(i)}{E_t \sum_{j=0} m_{t+j} \kappa^j Y_{Ht+j}(i)}. \quad (\text{A5})$$

All home firms that can adjust their price choose the same price. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1 - \kappa)\tilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (\text{A6})$$

The optimal subsidy will be set so as to offset the monopoly markup exactly, so that $s = 1/\theta$ will obtain in each country for all periods.

The home country monetary policy follows a modified Taylor rule, given by:

$$R_t^r = \beta^{-1} (\Pi_{Ht})^\gamma \quad (\text{A7})$$

where $\gamma > 1$. The foreign monetary authority follows the rule (A7) of the text.

Each home country firm i faces demand for its good from home consumers, foreign consumers and its home government. We can define equilibrium in the market for good i as

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta} \left[\frac{v}{2} \frac{P_t}{P_{Ht}} C_t + \left(1 - \frac{v}{2} \right) \frac{S_t P_t^*}{P_{Ht}} C_t^* \right], \quad (\text{A8})$$

Let $x = \ln(X_t/\bar{X})$ be the log deviation of any variable from its steady state (except for inflation and nominal interest rates, which are in levels). Taking a linear approximation of (A1), (A5) and (A6) around the zero inflation steady state, we derive the standard forward looking inflation equation, given by:

$$\pi_{Ht} = k(\phi y_{Ht} + \sigma c_t + (1 - \frac{v}{2})\tau_t - (1 + \phi)a_t) + \beta E_t \pi_{Ht+1} \quad (\text{A9})$$

where $k = \frac{(1-\beta\kappa)(1-\kappa)}{\kappa}$.

Using (A2), (A8) aggregated across all firms i , and the equivalent for the foreign country, we can solve for c_t and τ_t in terms of y_{Ht} , y_{Ft}^* , ε_t , and ε_t^* . This gives:

$$c_t = \frac{v(2-v)}{2D}(\varepsilon_t - \varepsilon_t^*) + \frac{n_t(D+v-1)}{2D} + \frac{n_t^*(D-(v-1))}{2D} \quad (\text{A10})$$

$$c_t^* = -\frac{v(2-v)}{2D}(\varepsilon_t - \varepsilon_t^*) + \frac{n_t(D+v-1)}{2D} + \frac{n_t^*(D-(v-1))}{2D} \quad (\text{A11})$$

$$\frac{\tau_t}{2} = -\frac{(v-1)}{D}(\varepsilon_t - \varepsilon_t^*) + \frac{\sigma(n_t - n_t^*)}{D} \quad (\text{A12})$$

Here, we define $D = \sigma v(2-v) + (1-v)^2$. Then, using (A10)-(A12), re-writing (A9), we have

$$\pi_{Ht} = k[(\phi + \omega(1+D))y_{Ht} + y_{Ft}\omega(D-1)] + \vartheta(\varepsilon_t - \varepsilon_t^*) - (1+\phi)a_t] + \beta E_t \pi_{Ht+1} \quad (\text{A13})$$

where $\omega = \sigma/2D$, and $\vartheta = \frac{(2-v)(v(\sigma-1)+1)}{2D}$. Thus, domestic inflation is driven by both home and foreign output, preference shocks, and home productivity shocks.

In a similar manner, we can derive the ‘dynamic IS equation’, using (A10)-(A12) and (A3). This gives

$$E_t(y_{Ht+1} - y_{Ht})(D+1) + E_t(y_{Ft+1}^* - y_{Ft}^*)(D-1) \quad (\text{A14})$$

$$\begin{aligned} & -E_t(\varepsilon_{t+1} - \varepsilon_t)\frac{(D+v-1)}{\sigma} - E_t(\varepsilon_{t+1}^* - \varepsilon_t^*)\frac{(D-v+1)}{\sigma} \\ & = \omega^{-1}E_t(r_t - \pi_{Ht+1}) \end{aligned} \quad (\text{A15})$$

An identical set of derivations gives the equations for the foreign economy as:

$$\pi_{Ft}^* = k[(\phi + \omega(1+D))y_{Ft} + y_{Ht}\omega(D-1)] - \vartheta(\varepsilon_t - \varepsilon_t^*) - (1+\phi)a_t^*] + \beta E_t \pi_{Ft+1}^* \quad (\text{A16})$$

$$E_t(y_{Ft+1} - y_{Ft})(D+1) + E_t(y_{Ht+1} - y_{Ht})(D-1) \quad (\text{A17})$$

$$\begin{aligned} & -E_t(\varepsilon_{t+1}^* - \varepsilon_t^*)\frac{(D+v-1)}{\sigma} - E_t(\varepsilon_{t+1} - \varepsilon_t)\frac{(D-v+1)}{\sigma} \\ & = \omega^{-1}E_t(r_t^* - \pi_{Ft+1}^*) \end{aligned} \quad (\text{A18})$$

This system may be solved for the variables y_{Ht} , y_{Ft} , π_{Ht} , and π_{Ft}^* . Following this, we may solve for (3.19)-(3.20) in the text.

Table 12: Estimates of Border Effect: US-Canada, allowing for within country heterogeneity

	Pooled (i)	Fixed effects (ii)	Pooled (iii)	Fixed effects (iv)
$\Delta e_t^{i,j}$	0.4171*** (0.0233)	0.4128*** (0.0271)	0.3075*** (0.0233)	0.3051*** (0.0272)
$\Delta e_t^{i,j} * border_{i,j}$	-0.4558*** (0.0243)	-0.4521*** (0.0278)	-0.3502*** (0.0242)	-0.3481*** (0.0279)
$\Delta e_t^{i,j} * \ln \tilde{d}_{i,j}$	-0.0534*** (0.0111)	-0.0550*** (0.0116)	-0.0472*** (0.0109)	-0.0489*** (0.0111)
$\Delta y_t^{i,j}$			0.1598*** (0.0067)	0.1568*** (0.0078)
$\Delta y_t^{i,j} * border_{i,j}$			-0.0185* (0.0109)	-0.0193 (0.0146)
$\Delta y_t^{i,j} * \ln \tilde{d}_{i,j}$			-0.0061 (0.0077)	-0.0036 (0.0095)
<i>CC dummy</i>	0.0018*** (0.0005)		0.0019*** (0.0005)	
$\Delta e_t^{i,j} * CC$	-0.1796*** (0.0503)	-0.2103*** (0.0493)	-0.1430*** (0.0495)	-0.1656*** (0.0506)
$\Delta y_t^{i,j} * CC$			-0.0921*** (0.0117)	-0.0965*** (0.0187)
$\Delta e_t^{i,j} + \Delta e_t^{i,j} * border_{i,j}$	-0.0387*** (0.0067)	-0.0393*** (0.0065)	-0.0427*** (0.0066)	-0.0430*** (0.0064)
$\Delta e_t^{i,j} + \Delta e_t^{i,j} * border_{i,j} + \Delta e_t^{i,j} * CC$	-0.2183*** (0.0507)	-0.2496*** (0.0497)	-0.1858*** (0.0500)	-0.2086*** (0.0510)
<i>N</i>	56324	56324	56324	56324
<i>R</i> ²	0.01	0.01	0.03	0.03

Notes: The dependent variable is relative consumption growth between locations i and j , $\Delta c_t^{i,j}$. The estimated specification in columns (i)-(ii) is equation (2.3); while in columns (iii)-(iv) it is equation (2.4). We allow for within country heterogeneity in C-RER correlation by including a dummy variable for CC pairs and its interaction with $\Delta e_t^{i,j}$. Robust standard errors are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.