

Brain versus Brawn: The Realization of Women's Comparative Advantage*

Michelle Rendall[†]
The University of Zurich

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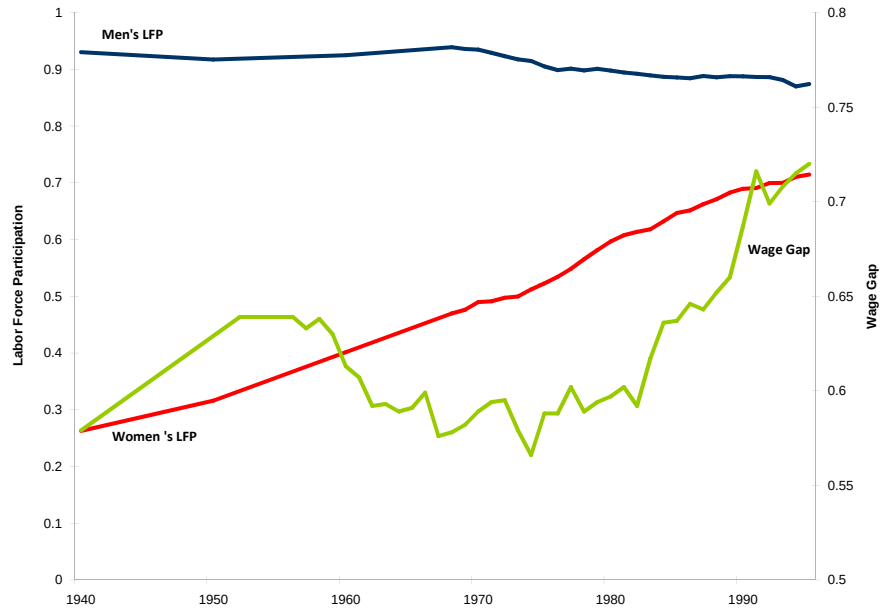
ABSTRACT

This paper estimates how much of the post-World War II evolution in employment and average wages by gender can be explained by a model where changing labor demand requirements are the driving force. I argue that a large fraction of the original female employment and wage gaps in mid-century, and the subsequent shrinking of both gaps, can be explained by labor reallocation from brawn-intensive to brain-intensive jobs favoring women's comparative advantage in brain over brawn. Thus, aggregate gender-specific employment and wage gap trends resulting from this labor reallocation are simulated in a general equilibrium model. This shift in production is able to explain: (1) about 79 percent of the rise in female labor force participation, (2) approximately 37 percent of the stagnation in the average female to male wage ratio from 1960 to 1980, and (3) about 83 percent of the closing wage gap between 1980 and 2005. In contrast, a counterfactual experiment, where agents cannot increase their innate brain abilities through education, fails to match the shape of the wage gap over time, resulting in a stagnant simulated wage gap from the 1960s onward.

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[†]University of Zurich, IEW, Muehlebachstrasse 86 CH-8008 Zurich.
Email: mrendall@iew.uzh.ch. All errors are mine.

Figure 1: *Female Labor Force Participation and Gender Wage Gap*



1 Introduction

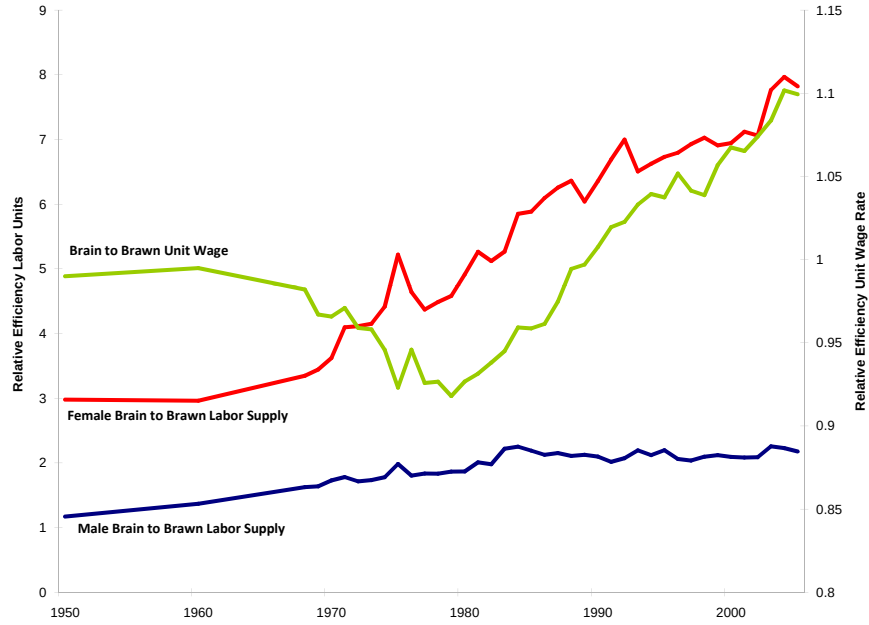
One of the greatest phenomena of the 20th century has been the rise in female labor force participation. Using evidence from United States data, this study develops a general equilibrium model based on the following two facts of labor supply and wages since World War II:

1. Women's labor force participation, aged 25 to 64, rose from 32 percent in the 1950s to 71 percent in 2005 (see Figure 1), while men's labor force participation stayed fairly steady.
2. The gender wage gap, defined as average female to average male wages, changed quickly during the same period. After initially falling from about 64 percent to a low of 59 percent, the gender wage gap began rising again in the mid 1970s reaching around 77 percent by 2005 (see Figure 1).

While it is a popular perception that anti-discrimination laws focused on gender equality were the main reasons behind women's changing labor market participation and earnings, economic studies have found various other reasons played an important role in shaping women's labor market experience, such as changes in women's work experience, education, and occupational mix (see, for example, [Black and Juhn, 2000](#); [Blau, 1998](#); [Mulligan and Rubinstein, 2005](#)). The forces behind the changing female employment and wages should be of particular interest to economists and policy makers alike.

This paper presents evidence from the United States and develops a general equilibrium model where women's improved labor market experience is driven by labor demand changes. I argue that the main factor in improving women's labor market opportunities, and their potential wages, is the shift in labor shares away from brawn-intensive occupations, as suggested by [Galor and Weil \(1996\)](#). The shift in labor shares is modeled by a linear exogenous "skill-biased" technical change, where skilled occupations are those requiring relatively more brain than brawn. This definition

Figure 2: *Efficiency Units and Wages in “Female-Friendly” Occupations*



deviates from the traditional education-based skill classification. For example, while a department store sales worker is usually classified as unskilled, in this study he/she is part of the “skilled” labor force since a sales worker requires almost no physical strength in performing his/her job effectively. More specifically, the model economy consists of two types of occupations, brain-intensive and brawn-intensive. These occupations are aggregated by a CES production function to produce a final market good. Heterogeneous agents differ in their innate intellectual aptitude (brain), physical ability (brawn) and, therefore, in their willingness to work in either occupation or in the labor market at all. Agents maximize consumption over market and home produced goods by allocating time between the labor market and their home. In addition, finitely lived myopic agents can increase their innate brain abilities by choosing to become educated when young.

A selection bias of women into brain-intensive occupations with initially lower wages (discussed in detail later), a rise in the relative returns to these occupations, and a rise in women’s relative labor supply to these occupations since World War II is undisputable (see Figure 2). Therefore, I argue that female labor force participation rose following skill-biased technical change favoring women’s comparative advantage in brain. Following this hypothesis, the wage gap closed for two reasons, (1) a rise in the returns to “female-friendly” occupations and (2) a faster increase in the female to male efficiency-unit labor supply to these occupations. Consequently, the goal of this paper is to estimate the quantitative importance of labor demand changes in explaining the shrinking wage gap and the rise in female labor force participation.

The rise of female labor force participation has been the focus of many recent macroeconomic papers. Some of these studies argue that improvements in home technology, such as the invention and marketization of household appliances (see, for example, [Greenwood, Seshadri, and Yorukoglu, 2002](#), and references therein), or the improvements in baby formulas (see [Albanesi and Olivetti, 2006](#)), enabled women to enter the labor market. While improvements in home technology freed women from time-consuming household chores, theories only focused on home technology improvements do

not and cannot effectively address the evolution of the wage gap over time.

Another set of research argues that certain observed labor market changes, such as the closing wage gap (see [Jones, Manuelli, and McGrattan, 2003](#)) or the increased returns to experience for women (see [Olivetti, 2006](#)), are largely responsible for the rise in female employment. Neither of these studies explain why women suddenly earned higher wages or had higher returns to experience, thus leaving the mechanism behind the closing wage gap unexplained.

To summarize, while previous studies have been successful in explaining part of Fact 1, the rise of the female labor force, they say nothing about the closing gender wage gap beyond taking Fact 2 as given. That is, they only address one aspect of the events shaping women’s labor market experience.

Two recent studies focus on the effects of cultural, social, and intergenerational learning on labor supply (see [Fernández, 2007](#); [Fogli and Veldkamp, 2007](#)). As before, these models are successful in explaining part of the rise in female labor force participation. In addition, [Fogli and Veldkamp \(2007\)](#) extend their theory to explain the evolution of wages through women’s self-selection bias, i.e., the characteristics of working women changed in the 20th century. However, this model is unable to match the complete wage evolution, only matching either the initial stagnation or the later rise.

All previously mentioned studies focus on labor supply side changes while keeping the labor demand constant. Naturally, this leaves one big unexplored fact: the changing labor demand. Two econometric studies analyze the effects of labor inputs in production on the gender wage gap. [Wong \(2006\)](#) finds that skill-biased technical change had a similar impact on men’s and women’s wages and, therefore, cannot explain the closing wage gap. [Black and Spitz-Oener \(2007\)](#) quantify the contribution of changes in specific job tasks on the closing wage gap from 1979 to 1999 for West Germany. The authors find that skill-biased technical change in West Germany, especially through the adoption of computers, can explain about 41 percent of the closing wage gap. While these two studies estimate the effects of relative labor demand changes on the wage gap, both assume an inelastic labor supply. Consequently, they cannot address the non-linear path of average female to male wages stemming from women’s self-selection bias into the labor market.

Undoubtedly trends in demand changes are missing from macroeconomic theory focusing on the rise of the female labor force and the shrinking wage gap. I argue that these trends arise from one underlying economic process: technical change leading to labor reallocation from brawn-intensive to brain-intensive occupations. The mechanism developed in this paper is able to explain: (1) about 79 percent of the rise in female labor force participation, (2) approximately 37 percent of the stagnation in average female to male wages from 1960 to 1980 and (3) about 83 percent of the closing wage gap between 1980 and 2005.

While the empirical results are specific to the United States, the model developed could also be used to study cross-country differences in women’s labor market participation. [Rogerson \(2005\)](#) notes that the change in relative employment of women and the aggregate service share (a brain-intensive sector given data evidence) between 1985 and 2000 are highly correlated at 0.82, concluding that countries which added the most jobs to the service sector also closed the employment gap the most.

As labor demand changes are the key motivation for this study, Section 2 provides further evidence for the changing labor market, focusing on (1) the evolution of physical and intellectual job requirements in the United States over time, (2) women’s self-selection into low-strength jobs due to physical hurdles, and (3) the effects of the changing labor demand for physical and intellectual abilities on female and male wage differentials. The general equilibrium model is outlined in Section 3, and Section 4 provides analytical results of skill-biased technical change on labor demand, labor

supply, and wages. Section 5 discusses the estimation and calibration procedure, and Section 6 presents labor market trends resulting from a linear exogenous skill-biased technical change starting in the 1960s. Lastly, Section 7 discusses extending the model to married households, and Section 8 concludes.

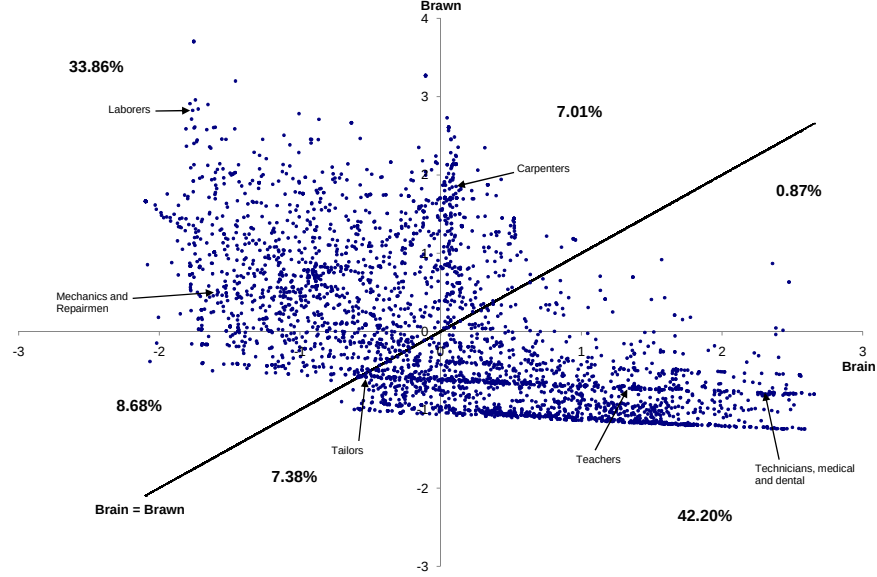
This study’s main contribution is in presenting a theory that simultaneously explains Fact 1, the rise in female employment, and Fact 2, the evolution of the gender wage gap, through a rise in “female-friendly” occupations driven by skill-biased technical change.

2 United States Labor Facts

To explore the relationship between the rise in female labor force participation and changes in labor demand, this study focuses on the relative demand and supply of two types of labor inputs: intellect and physical strength. This study starts from the premise that women have, on average, less brawn than men. One well documented sector where women are barred from certain occupations because of physical strength requirements is the military. For example, a [BBC News Online \(2002\)](#) article notes that starting in 2002 the British military barred women from front-line combat since they failed to pass the required physical test, where, “soldiers under 30 had to carry 20kg of equipment and their rifle while running a mile and a half in 15 minutes, as well as carrying a colleague for 50 yards.” Accepting that women and men have similar levels of brain, men have a comparative advantage in brawn-intensive occupations. However, technological change shifts labor demand toward low-brawn occupations diminishing men’s comparative advantage in the labor market.

Using factor analysis, I obtain brain and brawn estimates by United States census occupation and industry classifications from the 1977 Dictionary of Occupational Title (DOT). The 1977 DOT reports 38 job characteristics for over 12,000 occupations, documenting (1) general educational development, (2) specific vocational training, (3) aptitudes required of a worker, (4) temperaments or adaptability requirements, (5) physical strength requirements, and (6) environmental conditions. For example, general educational development measures the formal and informal educational attainment required to preform a job effectively by rating reasoning, language and mathematical development. Each reported level is primarily based on curricula taught in the United States, where the highest mathematical level is advanced calculus, and the lowest level only requires basic operations, such as adding and subtracting two-digit numbers. Specific vocational preparation is measured in the number of years a typical employee requires to learn the job tasks essential to perform at an average level. Eleven aptitudes required of a worker (e.g., general intelligence, motor coordination, numerical ability) are rated on a five point scale, with the first level being the top ten percent of the population and the fifth level comprising the bottom ten percent of the population. Ten temperaments required of a worker are reported in the 1977 DOT, where the temperament type is reported without any numerical rating. An example of a temperament is the ability to influence people in their opinions or judgments. Physical requirements include a measure of strength required on the job, rated on a five point scale from sedentary to very heavy, and the presence or absence of tasks such as climbing, reaching, or kneeling. Lastly, environmental conditions measure occupational exposure (presence or absence) to environmental conditions, such as extreme heat, cold, and noise. I use factor analysis similarly to [Ingram and Neumann \(2006\)](#) to reduce the dimensionality of DOT job characteristics. Using factor analysis, a linear relationship between normally distributed broad skill categories (e.g., brain, brawn, motor coordination) and the 38 DOT characteristics is estimated from the associated 38 variable correlation matrix. For a detailed explanation of the estimation procedure see Appendix A.

Figure 3: *Brain and Brawn Job Combinations from the 1977 DOT with 1971 CPS Labor Shares*



2.1 Brain and Brawn in the United States

Using maximum likelihood estimation methods, three factors are determined sufficient in capturing the information contained in the 38 DOT characteristics. Given the estimated coefficients (factor loadings) I term these factors: brain, brawn, and motor coordination (see Appendix A Table A.1). These factors are merged with the 1950 and 1960 United States Census data and the 1968 to 2005 Current Population Survey (CPS) data to compute trends over time.¹ Figure 3, which plots all 1977 occupational brain and brawn combinations, clearly depicts the difference in brain and brawn requirements across the economy. Figure 3 also shows aggregate labor shares from the 1971 CPS civilian population. To compute aggregate factor demand changes in the United States over time, 1977 occupation-industry factor estimates are aggregated using United States Census and CPS civilian labor force weights. Figure 4 depicts aggregate factor standard deviations from the mean over time, with a normalized mean of zero in 1950. While motor coordination remains fairly constant over time, the brain supply steadily increases and the brawn supply steadily decreases. This rising trend in the supply of brain versus the falling trend in the supply of brawn is what I term skill-biased technical change. These trends are not specific to the 1977 DOT, since [Ingram and Neumann \(2006\)](#) obtain similar trends over time using the 1991 DOT (see Figure 3 in the referenced paper). Note that using a single DOT survey to determine job requirements implies that the specific job factor requirements did not change over the last five decades. For example, a craftsman utilized the same brawn level in 1950 as in 2005. Ergo, all trends pictured are due to changes in the composition (mix) of occupations within the economy, and the rise in brain and fall in brawn requirements might possibly be greater than shown due to intra occupation skill-biased

¹Census and CPS data is obtained from the IPUMS-USA ([Ruggles, Sobek, Alexander, Fitch, Goeken, Hall, King, and Ronnander, 2004](#)) and the IPUMS-CPS project ([King, Ruggles, Alexander, Leicach, and Sobek, 2004](#)). The IPUMS projects provide a consistent 1950 United States Census classification of occupations and industries over the years, which is used in merging 1977 DOT brain and brawn factors.

Figure 4: *Standard Deviations of Labor Input Supply Over Time*

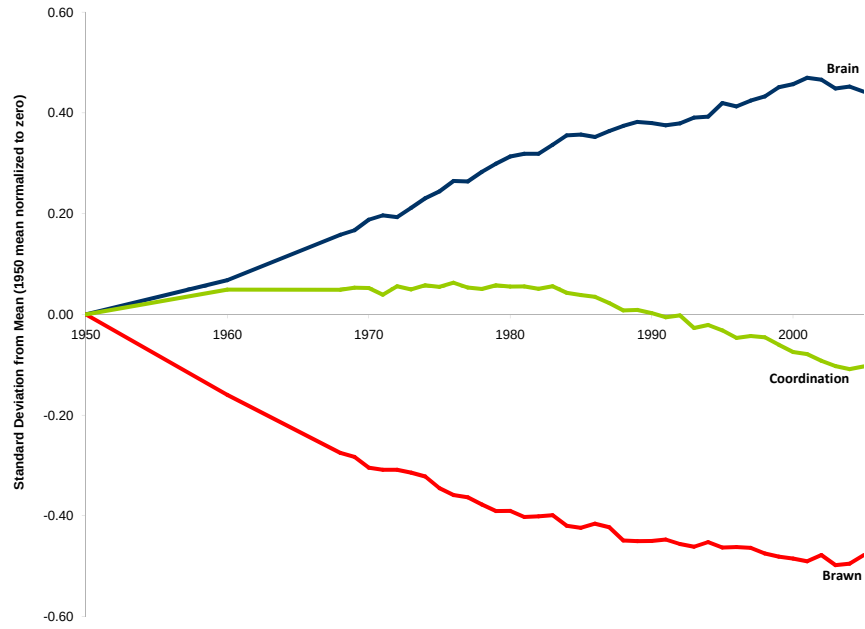
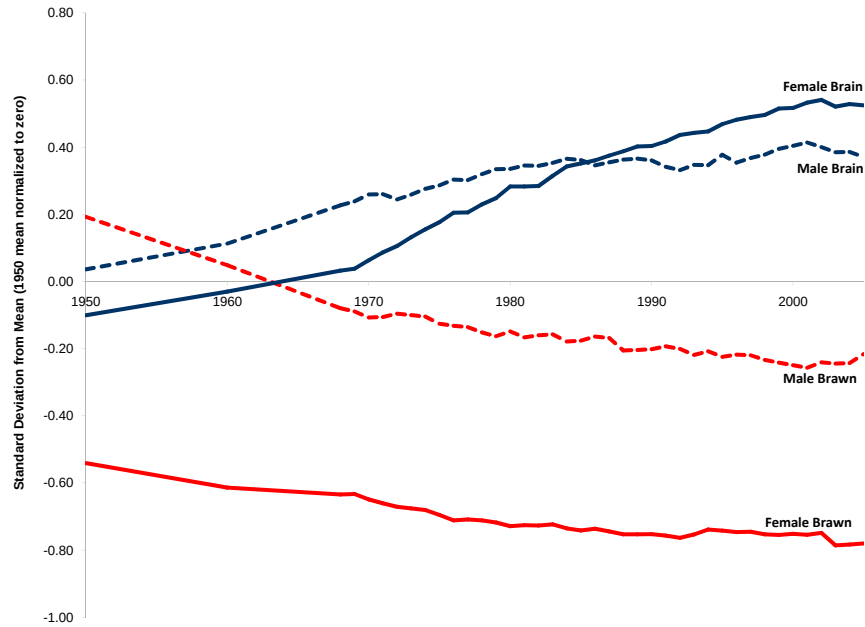


Figure 5: *Standard Deviation of Labor Input Supply by Gender*



technical changes. Figure 5 depicts brain and brawn standard deviations by gender over time, with the selection of women into low-brawn occupations clearly evident. Given women's lower innate brawn levels, this bias toward low brawn occupations can be either due to employee self-selection or employer discrimination. Additionally, the total brain supply has risen continuously since the 1950s, with women's brain supply surpassing men's by the 1980s. This trend could possibly be linked with

increased educational investment.

2.2 Wage Decomposition

The pictured brain and brawn trends suggest a strong relationship between the rise of female employment and skill-biased technical change. The combined effect of changes in relative factor prices and factor supplies by gender on the wage gap are computed from the following wage decomposition,

$$(\bar{w}_{m,T} - \bar{w}_{f,T}) - (\bar{w}_{m,0} - \bar{w}_{f,0}) = \sum_j \bar{p}_j \{ (\bar{F}_{j,m,T} - \bar{F}_{j,m,0}) - (\bar{F}_{j,f,T} - \bar{F}_{j,f,0}) \} + \dots \\ \sum_j (\bar{F}_{j,m} - \bar{F}_{j,f}) (p_{j,T} - p_{j,0}) \quad \text{for } j=\{brain, brawn\}, \quad (1)$$

where subscript 0 denotes the base year; $\bar{w}_{g,T}$ is the average natural logarithmic wage of gender g at time T ; p_j is factor j 's return; and $\bar{F}_{j,g}$ is the average supply of factor j by gender g . Variables without time subscripts are averages of the two years, 0 and T . Unlike Black and Spitz-Oener (2007), factor returns are not allowed to vary across gender, since I argue men's and women's wages only differ because of their relative brain and brawn supplies.² Average factor demands by gender can be computed from the brain and brawn estimates using United States Census and CPS weights over time. Using standard explanatory variables (e.g., age, education) and an individual's brain, brawn, and motor coordination factor supplies, a log-linear wage regression is estimated to obtain factor returns. The resulting coefficients on brain and brawn are taken as a proxy of factor returns (see Appendix B, Table B.1 for coefficient estimates). The percentage contribution to movements in the wage gap through changes in relative factor supplies between men and women is captured in the first term of equation (1). The second term measures the percentage contribution to movements in the wage gap through changes in factor returns. These "quantity" and "price" percentages, combined, measure the total percentage contribution to changes in the wage gap resulting from skill-biased technical change between period 0 and period T. Table 1 provides a breakdown of these contributions for two time periods: 1950 to 1980 and 1980 to 2005.

Table 1: *Wage Gap Change Decomposition*

Percent Contribution	1950-1980 ^a	1980-2005
Relative Brain Supply	-0.91	13.58
Relative Brain Prices	1.05	2.39
Relative Brawn Supply	11.07	-0.19
Relative Brawn Prices	-47.99	13.42
Total	-36.79	29.20

Notes: Regression source data 1950 Census and 1980, 2005 CPS.

^aWage gap widened during this period

Changes in brain and brawn over time can explain about one-third of the changes in female to male average wages. As the gender wage gap widened from 1950 to 1980 the total contribution was

²Allowing factor returns to differ by gender results in slightly higher contributions of relative price and supply changes on the evolution of the gender wage gap.

negative, with 37 percent of the widening wage gap mainly explained by rising returns to brawn. During this time period a fall in male brawn supply actually prevented the gap from widening further. From 1980 to 2005, the second period under consideration, the wage gap closed considerably. Relative female to male brain supply growth and falling returns to brawn had approximately equal impacts on the convergence of female to male wages.

Given the above facts, I argue that beginning in the 1950s women entered the labor market and their average wages improved due to the rise of brain-intensive occupations, which complemented women's comparative advantage. The remainder of this paper is devoted to the development of a model consistent with:

1. The rise of a brain-intensive sector;
2. The rise in women's labor force participation;
3. Rising average female wages primarily driven by brain supply and brawn price changes; and
4. An initial wage gap stagnation.

3 General Equilibrium Model

The simulated economy consists of a unit measure of agents,³ and two types of occupations, one brain-intensive and the other brawn-intensive. The two occupations' outputs are aggregated to a final market good, which is consumed by households. Agents can choose to work in the labor market or the home, and substitute consumption between market and home produced goods.

3.1 Household Maximization

Given evidence on the intensive and extensive margin of labor supply,⁴ it is assumed that agents can either work full-time in the labor market or not at all, $\ell_k = \{0, 1\}$ for agent k . Moreover, it is assumed that market and home produced goods are perfect substitutes

$$U(c, c_h) = \ln(c + c_h). \quad (2)$$

Agent k maximizes this utility function subject to a standard budget constraint, the home production technology, and a time constraint,

$$c_k \leq \ell_k \omega_k \quad (3)$$

$$c_{h,k} = A_h (1 - \ell_k) \quad (4)$$

$$\ell_k = \{0, 1\}. \quad (5)$$

Agent k can earn the wage $\omega_k = \psi(b_k, r_k)$, a function of his/her innate brain and brawn abilities in the labor market. To determine this functional form it is necessary to first describe the firm's problem. Lastly, given the discrete labor choice, agents work in the labor market if and only if

$$\omega_k > A_h. \quad (6)$$

³While the rise in labor force participation was considerably greater for married women, adding married couples does not provide any further dynamics to the model.

⁴Single employed women worked nearly 40 hours per week in 1950 and slightly less than 40 hours per week in 2005, while married women worked about 38 hours per week both in 1950 and 2005.

3.2 Production Process

There are two types of occupations, a brain-intensive production process, b , and a brawn intensive production process, r . Each production process only uses one of the inputs, brain $B_b \equiv B$ or brawn $R_r \equiv R$, where B and R are the aggregated individual labor supplies of brain and brawn. These brain and brawn units are combined in a CES production function to produce the final market good,

$$Y = \left(\lambda_b (A_b B)^\phi + \lambda_r (A_r R)^\phi \right)^{1/\phi}, \quad (7)$$

where A_j is occupation j 's factor productivity; $\epsilon_\phi = \frac{1}{1-\phi}$ is the elasticity of substitution between the two occupations; and λ_j is occupation j 's production share, with $\lambda_b + \lambda_r = 1$. A change in $\frac{A_b}{A_r}$ over time represents the exogenous skill-biased technical progress.

The relative wage follows from the cost minimization of the final good,

$$w = \frac{w_b}{w_r} = \frac{\lambda_b}{\lambda_r} \left(\frac{A_b}{A_r} \right)^\phi \left(\frac{B}{R} \right)^{\phi-1}, \quad (8)$$

with w_b and w_r representing the wages for brain and brawn occupations, respectively. The relative wage is a function of relative factor productivity as well as relative quantities supplied. Using equation (8), and the aggregate production function (7), an occupation's demand of efficiency-units per one unit of aggregate good is,

$$l_j = \frac{L_j}{Y_t} = (A_j)^{\epsilon_\phi-1} \left(\frac{\lambda_j}{w_j} \right)^{\epsilon_\phi} \left[\lambda_b^{\epsilon_\phi} \left(\frac{A_b}{w_b} \right)^{\epsilon_\phi-1} + \lambda_r^{\epsilon_\phi} \left(\frac{A_r}{w_r} \right)^{\epsilon_\phi-1} \right]^{-1/\phi}, \quad (9)$$

where L_j equals either B or R , and the term in brackets is the unit cost of the aggregate production.

3.3 Wages and the Distribution of Brain and Brawn

We can now explicitly state an agent's wage, ω_k , which is determined by his/her innate brain and brawn ability. From the firm's problem it follows that $\omega_k = \max\{w_b b_k, w_r r_k\}$. Moreover, brain and brawn are jointly distributed $(b_k, r_k) \sim \mathcal{A}_g(b, r)$ with differing distributions by gender. Since the premise of this study is the lack of women's brawn, the two gender distributions, $\mathcal{A}_m(b, r)$ and $\mathcal{A}_f(b, r)$, only differ in their distribution of brawn, $\mathcal{R}_g$. Consequently, the distribution of brain, $\mathcal{B}$, and the correlation of brain and brawn, ρ , are identical for men and women.

3.4 Decentralized Equilibrium

An equilibrium, given wages $\{w_b, w_r\}$, exists and is defined by:

1. The demand for market goods, c_k , the production of household goods, $c_{h,k}$, and the supply of labor, $\ell_{g,k}$, that maximizes household utility;
2. The demand for labor inputs, B and R , that minimizes the final good's cost function; and
3. Factor returns, $\{w_b, w_r\}$ that clear,
 - (a) The labor market, $B_{hh} = B$ and $R_{hh} = R$; and
 - (b) The goods market, $C_{hh} = Y$,
 where B_{hh} , R_{hh} , and C_{hh} are aggregate household supply and demand levels obtained by integrating labor demand and market consumption of individuals over the brain and brawn distribution of all working agents.

4 Analytical Dynamics

Data presented in Section 2 clearly depicts that labor moved away from brawn and toward brain. Any technical change, defined as a change in A_b and A_r , mimicking the movement from brawn-intensive to brain-intensive occupations must increase the relative demand for the brain-intensive efficiency units of labor. I analyze the changes in labor demand, supply, and wages resulting from a “one time” change in relative factor productivity, $\frac{A_b}{A_r}$. The dynamics of a steady change in relative technology parameters can be simply deduced by allowing this one time change to occur repeatedly, where $A_{j,t} = A_{j,t-1} (1 + \gamma_j)$ with γ_j defined as sector technology growth rates for $j = \{b, r\}$.

4.1 Relative Labor Demand

The relative labor demand follows from the unit labor demands in equation (9),

$$\frac{B}{R} = \left(\frac{A_b}{A_r} \right)^{\epsilon_\phi - 1} \left(\frac{\lambda_b w_r}{\lambda_r w_b} \right)^{\epsilon_\phi}. \quad (10)$$

Taking the derivative of this relative demand with respect to $\frac{A_b}{A_r}$, *ceteris paribus*, results in the Proposition 1.

Proposition 1

A rise in relative factor productivity of brain-intensive occupations increases relative labor demand efficiency units if $\epsilon_\phi > 1$, implying the two occupations are substitutes in the aggregate production process, since

$$\frac{\partial \frac{B}{R}}{\partial \frac{A_b}{A_r}} = (\epsilon_\phi - 1) \left(\frac{A_b}{A_r} \right)^{\epsilon_\phi - 2} \left(\frac{\lambda_b w_r}{\lambda_r w_b} \right)^{\epsilon_\phi}. \quad (11)$$

Representing Proposition 1 graphically (see Figure 6), technological change shifts labor demand to the right. Thus, the relative quantity of brain-intensive to brawn-intensive labor efficiency units at any given wage ratio increases, and, as a consequence, the equilibrium wage $\frac{w_b}{w_r}$ rises as long as an outward shift in labor supply does not offset the increase in labor demand. The relative wage equation (8) shows that a rise in $\frac{B}{R}$ will offset relative demand increases since $(\phi - 1) < 0$.

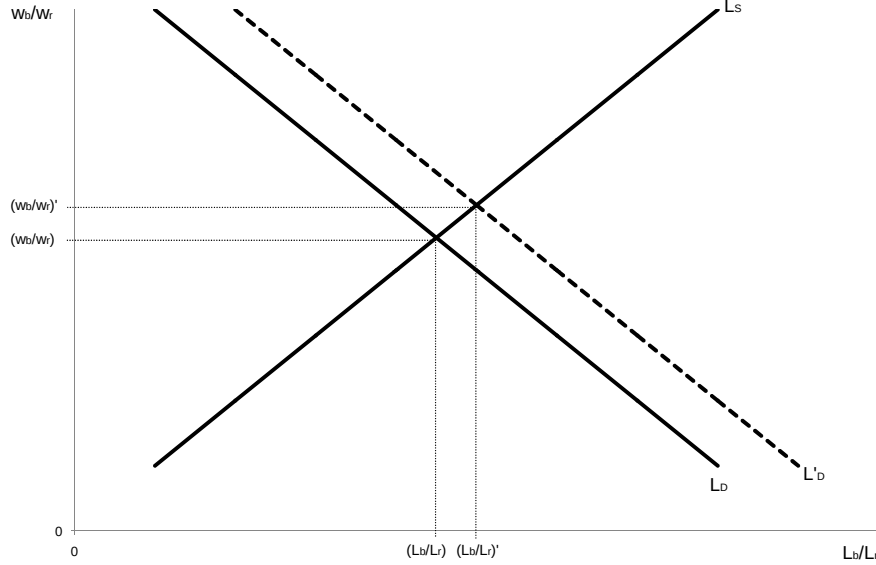
4.2 Labor Supply Decision

At the equilibrium wage rate, a change in relative factor productivity has no effect on the labor supply threshold $\omega_k > A_h$. Therefore, the relative labor supply does not shift and relative wages rise. However, a rise in the relative wage will change the type of person who enters the labor market, since the effect on ω_k will depend on an agent’s innate brain and brawn levels. By normalizing $w_r = 1$, an agent with relatively low brain but high brawn will see no change in his/her labor threshold, while an agent with relatively high brain will experience a rise in ω_k and, therefore, might change his/her labor supply decision. An agent works in a brain-intensive occupation if and only if

$$\frac{b_k}{r_k} > \frac{w_r}{w_b}. \quad (12)$$

To illustrate the effects of a rise in relative wages on the labor supply decision by gender, the following section elaborates on the dynamic effects by assuming two independent uniform distributions for brain and brawn. Brain and brawn are independently uniformly distributed with $B_g \sim [\underline{B}, \bar{B}]$

Figure 6: *Impact of Technological Change on Labor Demand and Relative Wage*



and $R_g \sim [\underline{R}_g, \bar{R}_g]$ for gender $g = \{m, f\}$, where $\bar{R}_g = \bar{R} + x_g$, $\underline{R}_g = \underline{R} + x_g$, and the only difference between men and women is the mean brawn level, $x_m > x_f \geq 0$.

The gender-specific labor force participation, LFP_g , is defined as,

$$LFP_g = \int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{A_h}{w_b}} a_g(b, r) db dr + \int_{\frac{A_h}{w_r}} \int_{\frac{A_h}{w_r}} a_g(b, r) db dr, \quad (13)$$

where $a_g(b, r)$ is the joint probability density function. The first term represents all agents that work in brain occupations, given home productivity and wages. The second term represents all remaining working agents, i.e., agents that work in either occupation. To not trivialize the results, it is assumed that $\underline{B} < \frac{A_h}{w_b}$ and $\underline{R}_g < \frac{A_h}{w_r}$, that is, some agents will not work in brain-intensive and/or brawn-intensive occupations. Given these special distributional assumptions, $LFP_m > LFP_f$.

Proposition 2

Women are less likely than men to work in the labor market, since

$$\frac{\partial LFP_g}{\partial x} = \left(\frac{A_h}{w_b} - \underline{B} \right) \frac{1}{(\bar{B} - \underline{B})(\bar{R} - \underline{R})} > 0. \quad (14)$$

Proposition 3

As the returns to brain increase, ceteris paribus, the employment gap will shrink, since

$$\frac{\partial LFP_g}{\partial w_b} = \frac{\frac{A_h}{p_b^2} \left(\frac{A_h}{w_r} - \underline{R}_g \right)}{(\bar{R} - \underline{R})(\bar{B} - \underline{B})} > 0 \quad (15)$$

and

$$\frac{\partial^2 LFP_g}{\partial w_b \partial x} = -\frac{A_h}{w_b^2} \frac{1}{(\bar{B} - \underline{B})(\bar{R} - \underline{R})} < 0. \quad (16)$$

To summarize, increased demand for low-brawn occupations, coupled with their rising returns, leads to a shrinking gender employment gap given women's comparative advantage in brain.

4.3 Wage Gap Evolution

The wage gap is defined as average female to average male wages in terms of average factor supplies to each occupation,

$$\frac{\bar{w}_f}{\bar{w}_m} = \frac{\pi_f w \bar{B}_f + (1 - \pi_f) \bar{R}_f}{\pi_m w \bar{B}_m + (1 - \pi_m) \bar{R}_m}, \quad (17)$$

where $\bar{B}_g$ is the average brain level conditional on the working population of gender g in brain occupations, $E\left(b_{g,k} \mid \frac{b_{g,k}}{r_{g,k}} > \frac{w_r}{w_b} \wedge \omega_{g,k} > A_h\right)$. Similarly, $\bar{R}_g$ is the average brawn conditional on the working population of gender g in brawn occupations, π_g is the fraction of working agents of gender g working in brain-intensive occupations, and $w = \frac{w_b}{w_r}$ is the relative wage.

There are two opposing effects shaping the evolution of the wage gap, a “price effect” and a “supply effect.”

Proposition 4

A rise in the relative wage results in a closing wage gap if

$$\frac{\pi_f}{1 - \pi_f} \frac{\bar{B}_f}{\bar{R}_f} > \frac{\pi_m}{1 - \pi_m} \frac{\bar{B}_m}{\bar{R}_m}. \quad (18)$$

Thus, Proposition 4 holds if a greater fraction of women work in brain-intensive occupations and their average relative brain to brawn efficiency-unit labor supply is relatively higher than men's, which I call the price effect. However, this ignores any self-selection bias.

A rise in w_b raises wages for agents with relatively high brain to brawn ability levels. Moreover, a rise in w_b , *ceteris paribus*, also enables agents with a comparative advantage in brain, but lower brain ability compared to the working population, to enter the labor market. Consequently, the average brain supply, $\bar{B}_g$, in the labor market may fall with a rise in relative wages. The fall in average brain supply, however, will be greater for women than men. This second supply effect can be illustrated by returning to the simplified example of the uniform distributions.

The sector specific labor force participation is simply,

$$\pi_g = \frac{\int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{A_h}{w_b}}^{\frac{A_h}{w_r}} a_g(b, r) db dr + \int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{w_r}{w_b} r_{g,k}}^{\frac{w_r}{w_b} r_{g,k}} a_g(b, r) db dr}{LPF_g}. \quad (19)$$

The mean brain and brawn levels of gender g equal,

$$\bar{B}_g = \frac{\int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{A_h}{w_b}}^{\frac{A_h}{w_r}} b_{g,k} a_g(b, r) db dr + \int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{w_r}{w_b} r_{g,k}}^{\frac{w_r}{w_b} r_{g,k}} b_{g,k} a_g(b, r) db dr}{\int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{A_h}{w_b}}^{\frac{A_h}{w_r}} a_g(b, r) db dr + \int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{w_r}{w_b} r_{g,k}}^{\frac{w_r}{w_b} r_{g,k}} a_g(b, r) db dr} \quad (20)$$

and

$$\bar{R}_g = \frac{\int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{w_r}{w_b} r_{g,k}}^{\frac{w_r}{w_b} r_{g,k}} r_{g,k} a_g(b, r) db dr}{\int_{\frac{A_h}{w_r}}^{\frac{A_h}{w_b}} \int_{\frac{w_r}{w_b} r_{g,k}}^{\frac{w_r}{w_b} r_{g,k}} a_g(b, r) db dr}, \quad (21)$$

respectively. Using these identities, the gender wage gap can be written as,

$$\frac{\bar{w}_f}{\bar{w}_m} = \frac{w \frac{B_f}{LFP_f} + \frac{R_f}{LFP_f}}{w \frac{B_m}{LFP_m} + \frac{R_m}{LFP_m}}, \quad (22)$$

where B_g and R_g equal the numerator of the conditional expectations, which are the total brain and brawn supplies by gender g .

Given the distributions of brawn, that is, men's higher average brawn levels ($x_m > x_f$), the total brawn supply of men is greater than that of women ($R_m > R_f$) as long as some agents prefer to work in the brawn-intensive sector ($w_r \bar{R}_g > w_b \underline{B}$). Similarly, the total brain supply is greater for women than men as long as some agents prefer to work in the brawn-intensive sector than stay at home ($w_r \bar{R}_g > A_h$). More importantly, a rise in the returns to brain-intensive occupations will have a different effect on the average brain supplied by each gender, $\frac{B_g}{LFP_g}$.

Proposition 5

A rise in the relative wage results in a stagnant/widening wage gap when

$$\frac{\partial B_f / LFP_f}{\partial w_b} < \frac{\partial B_m / LFP_m}{\partial w_b}. \quad (23)$$

More specifically,

$$\frac{\partial B_g / LFP_g}{\partial w_b} = \frac{1}{LFP_g^2} \left(\frac{1}{3} \frac{w_r^2}{w_b^3} \left(\bar{R}_g^3 - \left(\frac{A_h}{w_b} \right)^3 \right) LFP_g + \frac{A_h}{w_b^2} \left(\frac{A_h}{w_r} - \bar{R}_g \right) \left(\frac{A_h}{w_b} LFP_g - B_g \right) \right), \quad (24)$$

where all terms are positive except for the last term, $\frac{A_h}{w_b} LFP_g - B_g$, which can be positive or negative. Since $LFP_f < LFP_m$ and $B_f > B_m$ from above, this last term, which potentially slows the growth in the conditional mean brain supply, is smaller or negative for women compared to men. However, as women's and men's total and sectoral-specific labor force participation rates converge over time, this term will take the same value for men and women.

In summary, the price effect will close the wage gap, while the supply effect will widen the wage gap. The supply effect will dominate when women's labor force participation is considerably lower than men's, but will slowly disappear as these labor force participation rates converge. The natural evolution of these effects will initially cause a fall, or stagnation, of average female to male wages, which will close as the price effect begins to dominate.

These analytical results suggest that a model differentiating between brain-intensive and brawn-intensive jobs should replicate the initial United States employment and wage differences across gender. Moreover, it should reproduce the subsequent evolution of the female labor force participation rate and the gender wage gap, including some initial stagnation in average female wages as observed during the 1960s and 1970s.

4.4 Simulation Model Modifications

Two model modifications are introduced to match relevant United States data targets in the calibration. First, brain-intensive and brawn-intensive occupations utilize both input factors in linear combinations. Therefore, agents' efficiency wages are

$$\omega_k = \max\{w_b(\alpha_b b_k + (1 - \alpha_b)r_k), w_r(\alpha_r b_k + (1 - \alpha_r)r_k)\}, \quad (25)$$

where $\alpha_b > \alpha_r$. For the simplified example from Section 4.2, equation (25) implies $\alpha_b = 1$ and $\alpha_r = 0$. Linearity in brain and brawn inputs allows the aggregation of individual labor efficiency units by occupation,

$$L_j = \alpha_j B_j + (1 - \alpha_j) R_j, \quad \text{for } j = \{b, r\}. \quad (26)$$

An agent chooses to work in a brawn-intensive occupation if and only if

$$\frac{b_k}{r_k} \frac{w_b \alpha_b - w_r \alpha_r}{w_r (1 - \alpha_r) - w_b (1 - \alpha_b)} > 1. \quad (27)$$

The numerator is the difference in potential earnings of his/her brain ability between brain and brawn occupations, and the denominator is the difference in potential earnings between his/her brawn ability in brawn to brain occupations. If this ratio is greater than one, i.e., the additional returns to brain in brain-intensive occupations are greater than the additional returns to brawn in brawn-intensive occupations, the agent chooses to work in a brain occupation.

The second modification extends the model with an education choice allowing agents to increase their innate brain level. This modification enables the model to match the observed trend in brain supply in the United States more precisely (see Figure 4). Finitely lived myopic agents can choose to become educated when young at a cost of b^η , where $\eta < 0$. Education increases an agents brain endowment to B_e , such that all educated agents have the same brain level. However, education is cheaper for agents with initially higher levels of brain. Given the myopic nature of agents, agent k who lives N periods chooses to become educated when,

$$\begin{aligned} & \frac{1 - \beta^N}{1 - \beta} \max \{ \ln (\max \{ w_b (\alpha_b B_e + (1 - \alpha_b) r_k), w_r (\alpha_r B_e + (1 - \alpha_r) r_k) \}), \ln (A_h) \} - b_k^\eta > \dots \\ & \frac{1 - \beta^N}{1 - \beta} \max \{ \ln (\max \{ w_b (\alpha_b b_k + (1 - \alpha_b) r_k), w_r (\alpha_r b_k + (1 - \alpha_r) r_k) \}), \ln (A_h) \}, \end{aligned} \quad (28)$$

where β is the discount factor. The first line of equation (28) represents the lifetime utility of being educated, and the second line defines the lifetime utility of being uneducated. Since agents with high brawn, who prefer to work in brawn-intensive occupations, have less to gain from education, equation (28) is less likely to hold. In the context of this study, where men have on average higher brawn levels than women, fewer men will obtain education. As a consequence, average female brain supply, $\bar{B}_f$, surpasses average male brain supply, $\bar{B}_m$, once the returns to brain are sufficiently high to compensate for the cost of education. This is consistent with the United States brain supply trends (see Figure 5), where women's average brain supply exceeds men's average brain supply by the end of the 1980s. Therefore, in addition to the price effect, the "education effect" also contributes to the closing gender wage gap once the supply effect subsides.

5 Calibration

Simulating the model over time requires the calibration of individuals' brain and brawn distributions, and several household and production parameters. Given the pronounced hump-shape in the wage gap between 1940 and 1960, possibly due to the effects of World War II, the model is matched to various 1960 data targets.

5.1 Production Parameter Estimation

To determine the production parameters, A_b and A_r , their growth rates, γ_b and γ_r , and the substitution parameter, ϕ , the regression of [Katz and Murphy \(1992, pg. 69\)](#) is reestimated, where skilled labor is defined as brain-intensive labor and unskilled labor is defined as brawn-intensive labor. Occupations are sorted by their relative brain to brawn inputs in such a way that occupations with $b > r$ are brain-intensive and occupations with $b < r$ are brawn-intensive (see Figure 3). Full-time workers⁵ are grouped according to their age group (eight five-year intervals from 25 to 64 years old), gender, education (less than high school, high school, some college, college), race (white, black, other), marital status (married, single), sector (industry, services), and the type of occupation (brain-intensive, brawn-intensive). I follow [Hansen \(1993\)](#) in estimating labor efficiency units at time t as

$$E_t = \sum_k \delta_k L_{t,k}, \quad (29)$$

where $L_{t,k}$ is the total labor supply of group k and δ_k is the group's weight. Weights are determined by

$$\delta_k = \frac{\bar{\omega}_k}{\bar{\omega}}, \quad (30)$$

the average hourly wage of group k over the average hourly wage of the whole population (across individuals over the entire time period). The resulting relative unit wage of brain over brawn and relative efficiency unit labor supply is shown in Figure 2. This study assumes a log-linear skill-biased technical change over time,

$$\ln \left(\frac{A_b}{A_r} \right)_t = \zeta_0 + \zeta_1 t + \eta_t, \quad (31)$$

as in [Krusell, Ohanian, Ríos-Rull, and Violante \(1997\)](#). Taking the natural logarithm of the relative wage equation (8), and inserting equation (31), leads to the following regression estimation,

$$\ln \left(\frac{w_b}{w_r} \right)_t = a_0 + a_1 t + a_2 \ln \left(\frac{E_b}{E_r} \right)_t, \quad (32)$$

where $a_0 = \ln \left(\frac{\lambda_b}{\lambda_r} \right) + \phi \zeta_0$, $a_1 = \phi \zeta_1$, $a_2 = \phi - 1$, and $\frac{E_b}{E_r}$ is the relative efficiency unit of brain-intensive to brawn-intensive occupations. Table B.2 in Appendix B provides the regression estimates. By normalizing ζ_0 to zero, the parameter values of the annual skill-biased technical change growth rate, $\gamma_b - \gamma_r$, and the substitution parameter, ϕ , are obtained (see Table 2).

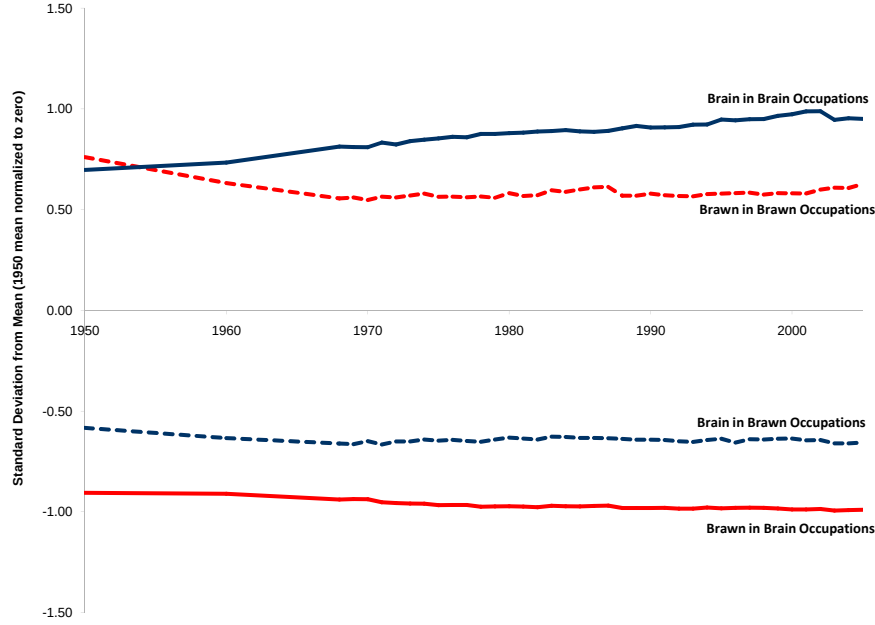
Table 2: *Baseline Parameters*

Production Parameters		
Substitution Parameter	ϕ	0.6032
Difference in Annual Relative TFP Growth Rate	$\gamma_b - \gamma_r$	0.0147

Additionally, the relative factor productivity, $\frac{A_b}{A_r}$, is normalized to one in 1960 and λ_b is set to match the 1960 labor share of brain-intensive occupations in the economy, which is about 51 percent.

⁵Full-time workers are defined as working at least 39 weeks and 35 hours per week (prior to 1976 only hours worked prior to the survey week are recorded).

Figure 7: *Factor Standard Deviations by Brain-Intensive and Brawn-Intensive Occupations*



Lastly, the productivity parameters within occupations, α_b and α_r , are matched to brain and brawn standard deviations in 1960 for each occupation (see Figure 7), together with the remainder of the parameters determining the distribution of brain and brawn of all individuals (see Section 5.2). The fairly steady brain and brawn standard deviations over time suggest that the grouping of occupations is fairly robust over time and appropriate for the simulation exercise.⁶

5.2 Agents' Ability

Brain and brawn are assumed to be joint normally distributed with correlation ρ . This assumption requires six parameter estimates: the mean of brain, μ_b ; the standard deviation of brain, σ_b ; the two means of brawn, $\mu_{r,m}$ and $\mu_{r,f}$; the standard deviation of brawn, σ_r ; and the correlation, ρ . Nine data targets are selected to match nine parameters - the six parameters above, plus α_b and α_r from Section 5.1, and home productivity, A_h . The specific 1960 United States data targets are:

1. Female labor force participation;
2. Standard deviation of male brain supply;
3. Standard deviation of female brain supply;
4. Standard deviation of male brawn supply;
5. Standard deviation of female brawn supply;
6. Standard deviation of the brain-intensive occupation's brain supply;

⁶Other statistics (e.g., standard deviation, minimum, maximum) of this specific occupational classification are also fairly steady over time.

7. Standard deviation of the brain-intensive occupation's brawn supply;
8. Standard deviation of the brawn-intensive occupation's brain supply; and
9. Standard deviation of the brawn-intensive occupation's brawn supply.

The standard deviations of brain and brawn by occupation provide a good representation of the economy. The standard deviations of male and female brain and brawn measure the main differences between gender in this study, i.e., women's lower brawn supplies, and men's and women's similar brain endowments. Lastly, η and B_e are matched to the difference in the standard deviations of female to male brain in 2005 and the difference in female to male brain-intensive labor shares in 2005. Both these measures, combined with the 1960 data targets, provide valuable information on the differences between men's and women's brain supply over time.

Parameters are obtained from performing simulated annealing. To check the robustness of the estimates, the calibration is repeated numerous times with different initial parameter values chosen randomly from a grid of plausible values. The labor market trends discussed below are robust to all calibrations.

5.3 1960 Model Moments and Calibrated Parameters

Before analyzing the resulting employment and wage trends, Table 3 provides the parameter estimates and specific data targets of the calibration. The model closely matches the brain and brawn standard deviations for both occupations and women. While men's brain and brawn levels are not matched, this calibration still captures the differences between men and women. That is, women supply considerably less brawn, but similar brain. The model is unable to match the initial female labor force participation, underestimating it by nine percentage points. However, the model is able to generate a large difference in average female to male wages, where women earn about 66 percent of men's wages (four percentage points higher than in the data). Note that the wage gap is not a data target in the calibration.

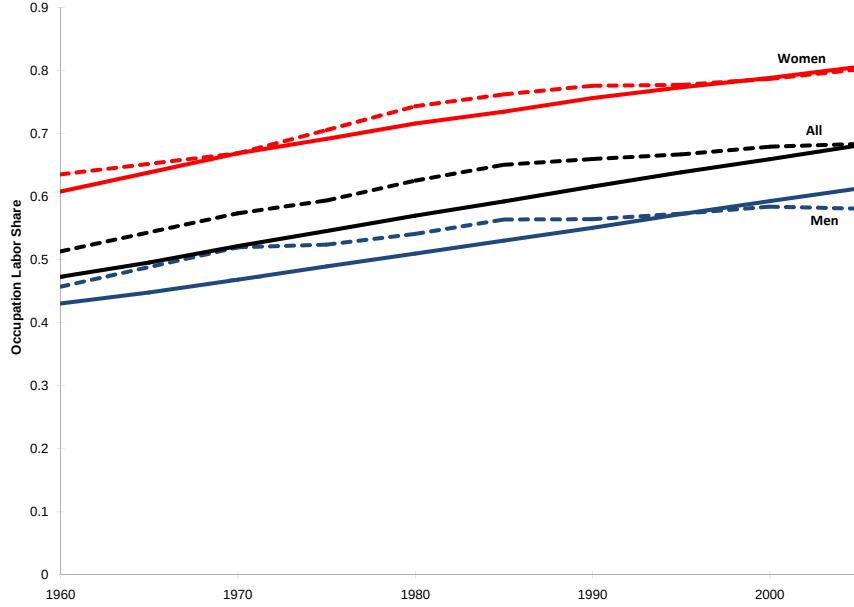
Table 3: *Moments and Parameter Estimates*

Moment	1960s Data	Model	Parameter Estimates
Brain Occupation Labor Share	0.51	0.48	$\lambda_b = 0.47$
Women Labor Force Participation	0.4	0.31	$A_h = 1.54$
Female Brain Standard Deviation	-0.03	-0.04	$\mu_b = 2.70$
Male Brain Standard Deviation	0.11	0.02	$\mu_{r,m} = 2.29$
Female Brawn Standard Deviation	-0.61	-0.61	$\mu_{r,f} = 0.76$
Male Brawn Standard Deviation	0.05	0.19	$\sigma_b = 2.03$
Brawn-intensive Occupation's Brain Standard Deviation	-0.63	-0.63	$\sigma_r = 1.03$
Brain-intensive Occupation's Brain Standard Deviation	0.73	0.71	$\rho = -0.98$
Brawn-intensive Occupation's Brawn Standard Deviation	0.63	0.63	$\alpha_b = 0.47$
Brain-intensive Occupation's Brawn Standard Deviation	-0.91	-0.91	$\alpha_r = 0.24$

6 Main Results

The results presented in this section show that the mechanism highlighted in this study does well in matching rising female employment rates in the United States. Moreover, the estimated growth

Figure 8: *Simulated Brain-intensive Occupation Shares*



rate difference between brain and brawn-intensive occupations, $\gamma_b - \gamma_r = 0.0147$, does extremely well in matching the rise in brain-intensive labor shares, not only for the economy as a whole, but also for men and women (see Figure 8, where dashed lines are the simulated labor share trends). In addition, the base model with education matches both the shape and magnitude of the wage gap from 1960 to 2005. In contrast, the counterfactual model without education is unable to match the wage gap evolution beyond the period of stagnant average female to male wages.

6.1 Simulated Employment and Wage Gap Trends

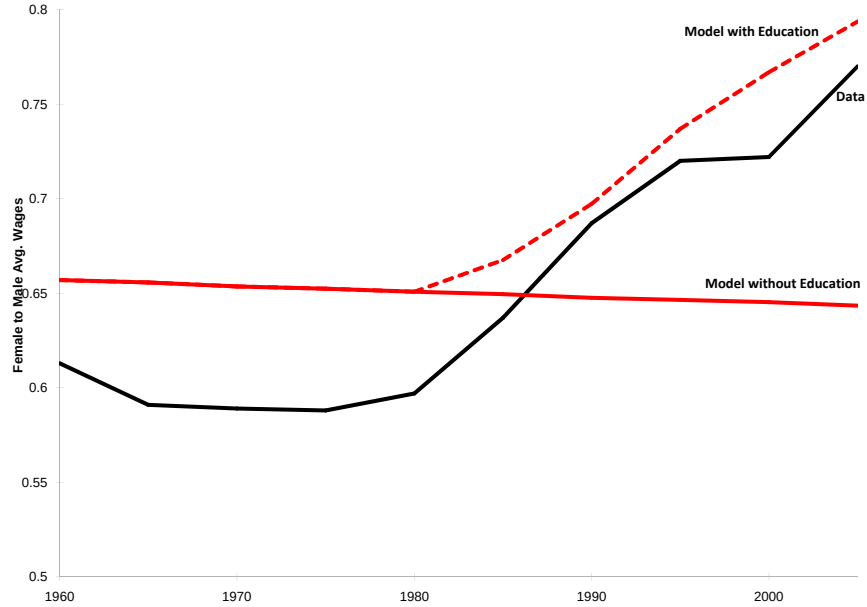
This model generates a linear rise in female labor force participation. Table 4 provides 1960 and 2005 labor force participation rates for women from the base model and the counterfactual model without education.

Table 4: *Change in Women's Labor Force Participation*

	United States (%)	Base Model (%)	Counterfactual Model (%)
1960	40.12	31.37	31.37
2005	71.39	56.10	54.85
Percent Explained		79.12	75.11

Both the base model and the model without education generate a large linear rise in female labor force participation, explaining about 75 to 79 percent of the total rise observed in the data. The rise in labor force participation is almost identical across the two models, suggesting that the rise in the returns to brain, rather than the modeled educational choice, is the primary driving force behind women's labor force participation. Men's labor force participation is 100 percent in the

Figure 9: *Simulated Wage Gap*



model economy, in comparison to United States male labor force participation rates of 92 percent and 87 percent in 1960 and 2005, respectively.

The wage gap evolution, however, differs considerably between the two models. In the counterfactual model the supply effect dominates throughout the entire period, resulting in a virtually flat wage gap (see Figure 9). A large fraction of the stagnant wage gap in the counterfactual model is driven by the fact that women's average brain supply does not surpass men's average brain supply. Figure 10 shows female and male brain and brawn supply standard deviations over time. Although women's brain supply eventually exceeds that of men in the data, the model without education is unable to generate this effect. Therefore, I calibrate the base model with education to match the difference between men's and women's 2005 brain standard deviations. That is, given women's comparative advantage in brain, women are more likely to increase their educational investment once the returns to brain rise. As a consequence, women eventually surpass men in average brain supplies.

From the 1960s to the 1980s the model with education perfectly parallels the counterfactual model (see Figure 9), with both models producing virtually stagnant gender wage ratios. The models generate a 0.6 percentage point decrease in average female to male wages during these three decades, compared to a 1.6 percentage point fall in the data. Therefore, about 38 percent of the fall in the United States female to male wage ratio is explained by the models. However, starting around 1980, the base model is able to simulate most of the closing gender wage gap observed in the United States. The base model generates a rise of 14 percentage points in average female to male wages from 1980 to 2005, compared to a 17 percentage point increase in the data, thus replicating about 83 percent of the closing wage gap during this time period. The model with education generates a closing wage gap through its ability to match the faster relative rise in female brain supply, which ultimately exceeds men's average brain supply by about 0.15 standard deviations in 2005.

Figure 10: *Simulated Standard Deviation of Labor Input Supply by Gender*

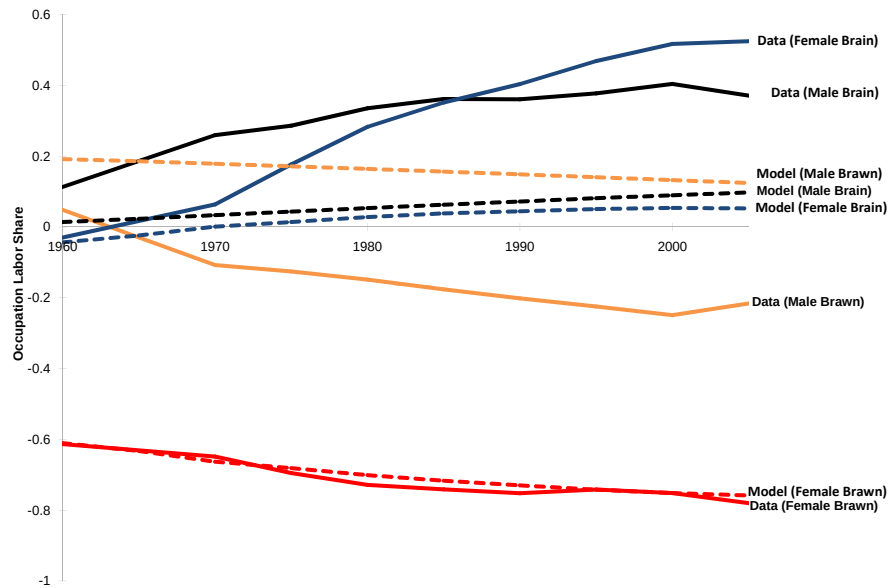
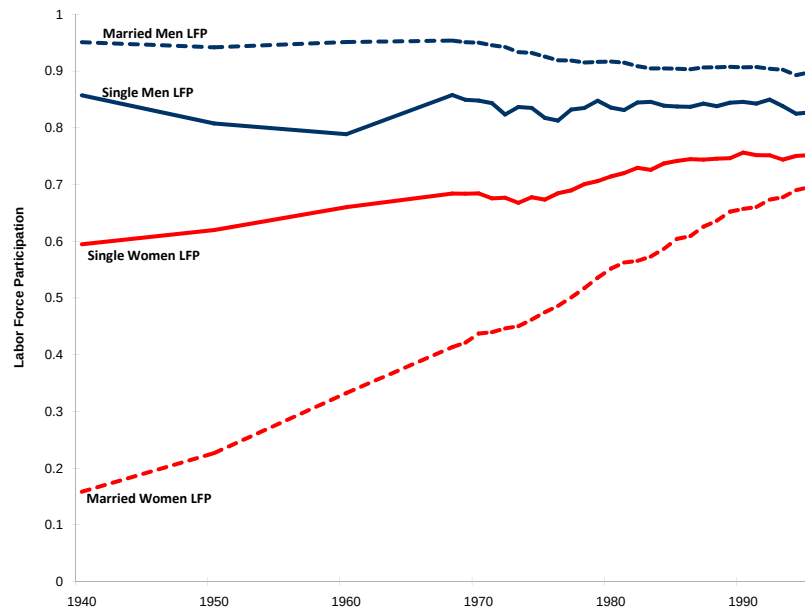


Figure 11: *Labor Force Participation by Gender and Marital Status*



7 Extension: Married Households

I have, thus far, ignored addressing differences in married versus single women's labor force participation (see Figure 11). To model differences in labor force participation between single and married households, the assumption of perfect substitution between market and home production

must be relaxed. If households maximize a CES utility function, where market and home goods are gross substitutes, the labor threshold, $\omega_g > T_\ell(A_h)$, will differ across married and single households. The single household's labor supply decision is identical to Section 3 assuming a discrete labor choice. Therefore, $\omega_g > A_h$ still determines an agent's decision to work or stay at home. A married household, however, now has the following utility function

$$U(c, c_h) = \ln \left((c^\nu + c_h^\nu)^{1/\nu} \right), \quad (33)$$

where the substitution between market and home goods equals $\epsilon_\nu = \frac{1}{1-\nu}$. In a static environment, married household k maximizes this utility function subject to the budget constraint, household production function, and time constraints,

$$\max_{\{c_k, c_{h,k}, \ell_{f,k}, \ell_{m,k}\}} U(c_k, c_{h,k}) \quad (34)$$

s.t.

$$c_k \leq \ell_{f,k} \omega_{f,k} + \ell_{m,k} \omega_{m,k}, \quad (35)$$

$$c_{h,k} = A_h (1 - \ell_{f,k} + 1 - \ell_{m,k}) \quad (36)$$

$$\ell_{f,k} = \{0, 1\} \text{ and } \ell_{m,k} = \{0, 1\}. \quad (37)$$

With perfect substitution in home production, households specialize with the higher wage earner, $\omega_{1,k} \geq \omega_{2,k}$, entering the labor market first. In households with equal wage rates the primary worker is assumed to be male, $1 = m$. The primary wage earner of household k works in the market if and only if

$$\omega_{1,k} > A_h (2^\nu - 1)^{1/\nu}.^7 \quad (38)$$

The secondary wage earner enters the market if and only if the above condition is satisfied in addition to

$$\omega_{2,k} > (\omega_{1,k}^\nu + A_h^\nu)^{1/\nu} - \omega_{1,k}. \quad (39)$$

That is, the secondary agent's labor supply decision is also dependent on his/her spouse's wage. The higher a spouse's wage the less likely the secondary worker is to enter the labor market due to imperfect substitution between market and home consumption. Formally, the derivate of the right hand side with respect to $\omega_{1,k}$ is

$$\omega_{1,k}^{\nu-1} (\omega_{1,k}^\nu + A_h^\nu)^{1/\nu-1} - 1, \quad (40)$$

which is positive as long as $A_h > 0$. This dependence on spousal wages incentivizes married women to stay at home unless their wages are very attractive. However, the general mechanism behind the closing wage and employment gaps will not change. Due to the computational burden of calibrating the married household model, I leave this extension for future research. Moreover, there is little evidence about the appropriate matching function of brain and brawn abilities between spouses, except for some evidence of assortive matching in educational attainment.

8 Conclusion

The purpose of this study is to assess the importance of labor demand changes on women's labor force participation and wages. For proper policy development, it is necessary to establish the

⁷Note this is similar to the single agent's labor supply threshold, except for a scaling factor of $(2^\nu - 1)^{1/\nu}$.

extent to which the female labor market experience has been shaped by discrimination or other factors. This study focuses on the changes in occupational brain and brawn input requirements, and their effect on women’s labor force participation and average wages. A considerable rise in brain and fall in brawn requirements is estimated from the 1977 DOT. Preliminary time trends and wage regression estimates suggest these labor demand changes have had a sizable impact on women’s wages and employment. Using a Mincer-type wage regression to estimate brain and brawn factor returns, I find the fall in relative brawn prices and the rise in female to male brain supplies to explain about 30 percent of both the initial stagnation and later rise of the post-World War II United States wage gap. The simulation of the general equilibrium model provides further insight into the dynamics of these labor demand changes, and their quantitative impact on women’s labor force participation and the closing wage gap. Calibrating the model to the 1960s United States economy shows that skill-biased technical change is able to replicate about 79 percent of the rise in female labor force participation. While the model without education is unable to generate a closing wage gap, the base model with an educational choice is able to generate a similar trend as observed in the data. This model explains about 37 percent of the initial fall and 83 percent of the later rise in the female to male wage ratio.

Clearly, the simple model presented in this paper, abstracting from many other potential factors influencing men’s and women’s labor market experiences, is unable to explain the complete evolution of the labor market over the last five decades. While this model is successful in explaining a significant portion of the changes in women’s labor market experience, it fails to match certain aspects of men’s labor market experience.

Some questions remain for future research. This study does not differentiate between married and single households. While theory suggests the general trends will still hold for a model differentiating between married and single households, I would like to quantify the explanatory power of a model accounting for marriage. Secondly, the model has made some simplifying assumptions, such as modeling skill-biased technical change as an exogenous process. The next research step is endogenizing this process by developing a model where the entrance of women into the labor force possibly spurs the skill-biased technological change observed in the data. Moreover, the educational choice in this study is very simplistic. A more realistic and richer educational investment choice over an agent’s lifetime should be of interest. Lastly, the model calibrated to the 1960s United States economy is unable to match men’s declining brawn supply, suggesting the above model should be modified to better match this trend.

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Appendix A: Factor Estimation

I estimate brain and brawn requirements for United States census occupation and industry classifications from the 1977 Dictionary of Occupational Title (DOT).⁸ This DOT survey set is particularly useful since, (1) it is readily available in an electronic format, (2) it has been merged with the 1971 Current Population Survey (CPS) allowing for civilian employment population weighted results, and (3) it lies mid-way through the period under study (the late 1970s). To estimate brain and brawn levels over time and gender I use factor analysis as in [Ingram and Neumann \(2006\)](#). Factor analysis is a technique to reduce a large number of variables, called characteristics, within a dataset to a few unobserved random variables, called factors. The 1977 DOT reports 38 job characteristics for over 12,000 occupations (see Section 2 for detail on these characteristics). These characteristics capture the heterogeneity across jobs and workers. While they measure different specific job requirements, they can be grouped into broader categories of skills in terms of their common underlying dimensions. This grouping reduces the dimensionality of heterogeneity allowing factor requirements to be matched in a simple general equilibrium model.

Factor analysis uses the correlation matrix of a set of dependent variables to uncover the functional form of some undefined independent variables. In the general specification the characteristics, C_i , are modeled as linear combinations of the independent variables or factors, f_i , plus an error term ϵ_i ,

$$C_i = \mu + \Lambda F_i + \epsilon_i \quad \text{for } i=1, \dots, N, \quad (\text{A.1})$$

where N equals the number of occupations; C_i is the vector of characteristics (38×1); μ is the vector of characteristic means (38×1); Λ is a vector of coefficients ($38 \times n_f$) called factor loadings; $F_i = (f_1, f_2, \dots, f_{n_f})'$ is a vector of the factors ($n_f \times 1$); and $\epsilon_i \sim N(0, \Sigma)$ is the uncorrelated error vector, with Σ being the diagonal variance covariance matrix.

To preform factor analysis certain variables of the DOT need to be rescaled, for example, the variable documenting a job's location is coded $I=indoors$, $O=outdoors$, and $B=both\ indoors\ and\ outdoors$. I follow [Vijverberg and Hartog \(2005\)](#) in rescaling all variables. Additionally, to obtain population representative estimates, the occupations in the DOT must be weighted. As the DOT itself does not record the number of workers for a given job, the 1971 CPS merge is used. In the 1977 DOT, the *Committee on Occupational Classification and Analysis of the National Academy of Sciences funded by the Department of Labor and the Equal Employment Opportunity Commission* merged the 12,431 1977 DOT jobs to 7,289 unique occupation-industry pairs from the 1970 United States Census providing 1971 CPS weights of the civilian labor force. The reduction from 12,431 to 7,289 is the result of more detailed occupational classifications in the DOT. For example, while there is only one "waiter/waitress" category in the census classification, the DOT contains multiple categories, such as "waiter/waitress formal", "waiter/waitress, head", "waiter/waitress, take out."

Since only information on the characteristics is available, this information is used to estimate both, Λ and F_i from

$$E\left(\hat{C} - \mu\right)\left(\hat{C} - \mu\right)' = \Lambda E\left(\hat{F}\hat{F}'\right)\Lambda' + \Sigma, \quad (\text{A.2})$$

that is, the covariance in the 38 characteristics can be explained by a reduced number of factors, where $\hat{C} = [C_1 C_2 \dots C_N]$ and $\hat{F} = [F_1 F_2 \dots F_N]$. It is clear that Λ , $E\left(\hat{F}\hat{F}'\right)$, and Σ are not separately identifiable from this expression. Therefore, factor analysis generally assumes factors to follow a standardized normal distribution, which allows for the identification of Σ . To separately

⁸Data, including documentation, is available from the Inter-university Consortium for Political and Social Research (ICPSR).

identify Λ and $E(\hat{F}\hat{F}')$ additional restrictions must be imposed. In standard factor analysis the covariance between factors is set to zero,

$$E(\hat{F}\hat{F}') = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & \cdots & & 1 \end{bmatrix}, \quad (\text{A.3})$$

allowing both Λ and Σ , which is diagonal by assumption, to be identified separately. In this specification each characteristic is a function of all factors. In practice, the first factor estimate will explain the maximum possible covariance between the characteristics. The second factor is estimated to explain the maximum covariance remaining, and so on. A maximum of 38 factors could be estimated, in which case 38 factors are necessary to explain the covariance between all characteristics. In this study three factors explain most of the characteristics' covariance structure (over 93 percent of the total covariance).⁹ After performing initial factor analysis as described above, the first factor is positively related to intellectual characteristics and negatively correlated with both motor coordination and physical characteristics, making it difficult to interpret the factor consistently. Therefore, I reestimate the factors assuming they are correlated, similarly to [Ingram and Neumann \(2006\)](#). However, for identification purposes, job characteristics that explain one factor are restricted and cannot explain another factor. For example, mathematical development only explains a job's intellectual requirements directly, while it is only informative on the job's physical requirements through the correlation of the aggregate brain and brawn factor. Table A.1 provides the classification of characteristics across factors as well as the factor loading coefficients, which are used to determine factor estimates for each occupation-industry combination present in the 1971 CPS. Given the grouping of characteristics and the estimates of factor loadings, I call the three factors brain, motor coordination, and brawn. Brain, brawn, and motor coordination trends over time (see Figure 4) are robust to either the standard identification restriction of uncorrelated factors or my reestimated identification of correlated factors.

⁹[Ingram and Neumann](#) use the 1991 DOT with over 53 characteristics, primarily expanded by detailing physical and environmental characteristics, to estimate a total of four factors: (1) intelligence, (2) clerical skill, (3) gross motor skill, and (4) ability to deal with physically and hazardous work.

Table A.1: *Factor Loading Estimates (Λ)*

Job Characteristic	Coefficient (Λ_i)
Brawn Factor	
Repetitive Work	0.30406
Climbing/Balancing	0.77651
Stooping/Kneeling/Crouching/Crawling	0.81000
Strength Requirement	0.88075
Environmental Exposure ^a	0.77673
Indoor or Outdoor Work	0.68110
Brain Factor	
Reasoning Development	0.96668
Mathematical Development	0.89217
Language Development	0.95275
Specific Vocational Preparation	0.77567
General Intelligence	0.94685
Verbal Aptitude	0.94068
Numerical Aptitude	0.83968
Clerical Aptitude	0.70447
Talking and Hearing	0.57950
Performs Variety of Duties	0.24961
Directing/Controlling	0.61560
Interpreting Feelings/Ideas/Facts	0.18598
Influencing People	0.37265
Making Evaluations Based on Judgment	0.60055
Making Judgments/Decisions	0.43480
Dealing with People	0.49332
Motor Coordination Factor	
Seeing	0.77650
Spatial Aptitude	0.43418
Form Perception	0.71349
Motor Coordination	0.84869
Finger Dexterity	0.88302
Manual Dexterity	0.66313
Eye-Hand-Foot Coordination	0.07607
Color Discrimination	0.37763
Attaining Precise Tolerances	0.72865
Reaching/Handling/Fingering/Feeling	0.50627
Making Decisions based on Measurable Criteria	0.30894

Notes: Estimated using maximum-likelihood factor analysis.

^aEnvironmental conditions, such as the presence of heat, cold, and humidity, were combined to one variable prior to the estimation.

Appendix B: Regression Estimates

Table B.1: *Factor Price Estimates* (p_b, p_r)

	1950-1980	1980-2005
Brain	0.1138981* (0.0002042)	0.153931* (0.0000898)
Brawn	0.0446319* (0.0001997)	0.13126* (0.0000963)
Brain $\times$ T	0.0629789* (0.0002157)	0.0555917* (0.0001103)
Brawn $\times$ T	0.0793846* (0.0002184)	-0.0600552* (0.0001233)
R-squared	0.3170	0.2576

Notes: * Statistically significant at the 1 percent confidence level. Standard errors are in parenthesis. The regression also includes controls for age, age squared, years of education, marital status, race, region, motor coordination factor, and a T-year dummy.

Table B.2: *Production Regression Estimates*

Constant	0.0416971 (0.0393867)
Time Trend	0.0088655* (0.000657)
Brain to Brawn Labor	-0.3967528* (0.0593553)
R-squared	0.739

Notes: * Statistically significant at the 1 percent confidence level. Robust standard errors are in parenthesis.