Economic Growth with Bubbles

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Abstract

This paper presents a stylized model of economic growth with bubbles. This model views asset price bubbles as a market-generated device to moderate the effects of frictions in financial markets, improving the allocation of investments and raising the capital stock and welfare. (TO BE COMPLETED)

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1 Introduction (preliminary)

Modern economies often experience large movements in asset prices that cannot be explained by changes in economic conditions or fundamentals. It is commonplace to refer to these episodes as asset price bubbles popping up and bursting. Typically, these bubbles are unpredictable and generate substantial macroeconomic effects. Consumption, investment and productivity growth all tend to surge when a bubble pops up, and then collapse or stagnate when the bubble bursts. Here, we address the following questions: What is the origin of these bubbles? Why are bubble episodes unpredictable? How do bubbles affect consumption, investment and productivity growth? In a nutshell, the goal of this paper is to develop a stylized view or model of economic growth with bubbles.

The theory developed here views an asset price as the sum of two components: fundamental and bubble. The fundamental is the net present value of all future dividends, and its size depends on how productive and well managed is the asset. The bubble is a pyramid scheme, and its size depends on the market's expectation of its future size. Throughout most of the paper, we assume that it is possible to strip any asset into these two components. As a result, any asset market can be thought of as the sum of two assets classes: productive assets or "capital" whose price is equal to the fundamental component; and unproductive assets or "bubbles" whose price equals the bubble component. We consider environments with rational, informed and risk neutral investors that follow the simple portfolio rule of holding only those assets that offer the highest expected return. The theoretical challenge is to identify situations in which these investors optimally choose to hold bubbles in their portfolios and then characterize the macroeconomic consequences of their choice.

In a seminal contribution, Samuelson (1958) explained how pyramid schemes could be possible and welfare-enhancing even if all agents are rational and well informed. Tirole (1985) built upon these results to develop a theory of rational asset-price bubbles. The key insight behind this theory can be summarized as follows: to satisfy their need for a store of value, economies might sometimes accumulate capital even if the investment required to maintain this capital exceeds the income that it produces. That is, economies might find themselves using a costly or inefficient store of value. In this situation, a bubble with negligible maintenance costs constitutes a more efficient store of value and can favorably compete with capital. As the bubble displaces capital in the portfolios of investors, it liberates resources that are used to raise consumption and welfare. This explains the origins and effects of bubbles. Since bubbles do not have intrinsic value, their size depends on the market's expectation of their future size. In a world of rational investors, this opens the door for self-fulfilling expectations to play an important role in bubble dynamics and accounts for their unpredictability.

Tirole's model provides an elegant and powerful framework to think about bubbles. However, it fails to fit the facts along two important dimensions. In the first place, the main prediction of the model is that bubbles displace the capital stock, leading to a contraction in output and a simultaneous boom in consumption. This seems at odds with historical evidence, which suggests that bubble episodes tend to be associated with increases in investment and in the capital stock. In the second place, the model predicts that bubbles can only appear in economies in which capital is an inefficient store of value. But Abel et al. (1989) have analyzed a group of developed economies and found that, in all of them, the investment required to keep the capital stock at current levels is less than the income that it produces. A successful model of bubbles must therefore explain how they can lead to expansions in the capital stock and how they can arise in economies in which capital is an efficient store of value. This paper provides such a model.

Our main contribution is to extend the theory of rational bubbles to the case of imperfect financial markets and show that this is enough to reconcile it with the facts. Assume a simple dynamic economy in which some individuals ("entrepreneurs") are able to run productive firms, while others ("unproductives") can only run unproductive firms. Under frictionless financial markets, this heterogeneity in productivity would be inconsequenctial because all of the economy's resources would be channeled to productive firms. But financial markets might be subject to frictions. Agency costs, for example, might prevent unproductives from lending their resources to entrepreneurs: in this case, unproductives might be forced to invest in their own firms and obtain a low rate of return. This could explain why some investors (unproductives) purchase bubbles even if, for the economy as a whole, capital appears to be an efficient store of value. As bubbles displace capital from the portfolios of unproductives, they free resources that could raise consumption. As part of the bubbles might be owned by entrepreneurs, they transfer resources to productive firms and expand productive investment. If the latter increases enough, the extended theory can also explain why bubble episodes raise the capital stock and aggregate output.

The previous argument implicitly assumes that whatever agency costs restrict borrowing and lending, they do not affect the value of the bubble. This seems a reasonable assumption, though. Agency costs increase with the manager's ability to influence the firm's value and decrease with creditors' ability to observe the actions of the manager. Since the value of productive firms depends on their capital and how it is managed, it is likely that managers could influence the firm's value through a variety of channels that are difficult to observe. Since the value of bubbles depends instead on the expectations of a rational market, it is unlikely that managers could have much of an effect on it unless the market decided to use the manager as a sunspot to coordinate expectations. Even in this case, it seems unlikely the manager could exploit this ability to his/her advantage without creditors knowing it.

Our basic results are derived in a very stylized setting. We therefore extend them along two natural dimensions. First, bubbles in our baseline economy always decrease the level of investment, although not necessarily the capital stock. We illustrate that this need not be the case by providing some of our agentes with a simple consumption-savings choice. In such a scenario, the bubble might actually lead to an expansion of investment by increasing the return to savings. Second, the presence of dimishing returns in our baseline economy imply that bubbles affect the steady-state levels of per-capita output and of the capital stock, but not their growth rates. We generalize the production structure of our economy and show that, in the presence of constrant or increasing returns, bubbles have long-term effects on growth.

The paper is organized as follows: Section one presents the basic setup and describes the equilibrium without bubbles. Section two shows that there are additional equilibria with bubbles and formally describes them. Section three discusses the macroeconomic effects of bubbles in a stationary economy. Sections four and five introduce endogenous productivity growth and shows how bubbles and productivity growth interact with each other. Section six provides a discussion of the effects of financial development in the presence of bubbles.

2 The Tirole model

Tirole (1985) showed that bubbles can arise in economies that are dynamically inefficient, i.e. that accumulate too much capital. In Tirole's setup, bubbles crowd out capital, raising the interest rate and welfare. In this section, we derive Tirole's results again using the setup that we will use throughout the paper. This serves a dual purpose. The first one is to provide a quick refresher on the macroeconomic implications of the theory of rational bubbles as it exists now. The conclusion though is dismal, as the theory is poorly equipped to explain even the most basic features of real-world bubble episodes. The second purpose of this section is to provide a natural benchmark against which to compare the findings of the rest of the paper. After all, our goal here is to show why and how removing a single and quite unrealistic assumption of the Tirole model dramatically modifies the predictions of the theory. We are referring, of course, to Tirole's assumption that financial markets are frictionless.

2.1 Basic setup

Consider a country inhabited by overlapping generations of young and old, all with size one. All generations maximize the expected consumption when old: $U_t = E_t \{c_{t+1}\}$, where U_t and c_{t+1} are the welfare and the old-age consumption of generation t.

The output the country is given by a Cobb-Douglas production function of labor and capital: $F(l_t, k_t) = l_t^{1-\alpha} \cdot k_t^{\alpha}$ with $\alpha \in (0,1)$, and l_t and k_t are the country's labor force and capital stock, respectively. All generations have one unit of labor which they supply inelastically when they are young, i.e. $l_t = 1$. The stock of capital in period $t+1$ equals the investment made by generation t during its youth.[1] This means that:

$$k_{t+1} = s_t \cdot k_t^{\alpha} \tag{1}$$

where s_t is the investment rate, i.e. the fraction of output that is devoted to capital formation.

Markets are competitive and factors of production are paid the value of their marginal product:

$$w_t = (1-\alpha) \cdot k_t^{\alpha} \quad \text{and} \quad r_t = \alpha \cdot k_t^{\alpha-1} \tag{2}$$

where w_t and r_t are the wage and the rental rate, respectively.

This stylized setup allows us to derive the key results in a very straightforward and clear-cut fashion. But the reader might wonder whether these results would also hold with more realistic assumptions regarding preferences and technology. The answer is positive, as we show later in the paper. There, we extend the model in two important directions: (i) we consider a more sophisticated production structure in the style of the new growth theory, and study the effects of bubbles on the long-run rate of economic growth; (ii) we also consider alternative preferences that value consumptionin both periods, and we study the effects of bubbles that work through and interest-rate effects on savings.

[1] That is, we assume that (i) producing one unit of capital requires one unit of consumption, and that (ii) capital fully depreciates in production.

2.2 Equilibria

To solve the model, we need to find the investment rate. The old receive capital income but do not save, since they will not be around the following period. The young receive labor income, which is a fixed fraction $1 - \alpha$ of output, and save it to finance old age consumption. What do the young do with their savings? At this point, it is customary to assume that the only option is to invest in capital. This implies that the investment rate is constant, as in the classic Solow (1956) model:

$$s_t = 1 - \alpha \tag{3}$$

We shall instead follow Tirole (1985) and give the young the additional option of purchasing bubbles or pyramid schemes. These are intrinsically useless assets, and the only reason to hold them is to resell them later at a profit. Tirole showed that bubbles could exist in equilibrium if and only if the bubbleless economy is dynamically inefficient. In this setup, this requires that $\alpha < 0.5$.[2] We return to this issue below.

The presence of bubbles has two effects: (i) those that start a bubble receive a pure rent which constitutes an additional source of income; and (ii) everybody has an additional savings option. Let b_t be the aggregate bubble, that is, the total value of the bubbles traded at time t. Some of these bubbles might be old, i.e. created by earlier generations; and some of them might be new, i.e. created by generation t. Bubbles are created randomly and without cost. As a result, new bubbles constitute a pure profit or rent for the generation. We use n_t to denote the value of the new bubbles as a share of the aggregate bubble. Naturally, $n_t \in [0, 1]$.

With this notation at hand, we can write the investment rate as follows:

$$s_t = 1 - \alpha - \frac{(1 - n_t) \cdot b_t}{k_t^\alpha} \tag{4}$$

Equation (4) says that the investment rate equals the income of the young minus their purchases of bubbles, both expressed as a share of output. The income of the young consists of labor income and the value of new bubbles created by them, i.e. $(1 - \alpha) \cdot k_t^\alpha + n_t \cdot b_t$. Since the old do not save, the young must be purchasing the whole aggregate bubble, i.e. b_t. Since $n_t < 1$, the presence of a bubble lowers the investment rate. This is a key feature of Tirole's model, as we shall discuss at

[2] In this model, the steady-state of the economy without bubbles is $k^{SS} = (1 - \alpha)^{\frac{1}{1-\alpha}}$. The golden rule capital stock is $k^{GR} = \alpha^{\frac{1}{1-\alpha}}$. Therefore, the economy is dynamically inefficient if $k^{SS} > k^{GR}$.

length shortly.

Before defining the equilibrium of this economy, we need an additional piece of notation. Let z_t be the state of the economy, with $z_t \in \{F, B\}$. We say that the economy is in the *fundamental* state if there are no bubbles:

$$b_t = n_t = 0 \quad \text{if } z_t = F \tag{5}$$

We say instead that the economy is in the *bubbly* state if there are bubbles. Since individuals are risk-neutral, all bubbles (and therefore the aggregate bubble, too) must promise the same expected return as investing in capital:

$$E_t \left\{ \frac{b_{t+1} \cdot (1 - n_{t+1})}{b_t} \right\} = \alpha \cdot k_t^{\alpha-1} \quad \text{if } z_t = B \tag{6}$$

The LHS of Equation (6) shows the return to the bubble, which consists on its price appreciation over the holding period. The purchase price of the bubble is b_t, and the selling price is $b_{t+1} \cdot (1 - n_{t+1})$. The RHS of Equation (6) shows the return to capital, which consist on the rental since each unit of capital costs one unit of consumption and it fully depreciates in one period.

In the fundamental state the probability that any existing useless asset becomes a bubble in the future must be zero. The reason is simple: a rational individual would be willing to pay a positive price for an asset if there is some probability that this asset commands a positive price tomorrow.[3] When the economy transitions from the fundamental to the bubbly state, new bubbles appear. After this, new bubbles might appear as well for as long as the economy remains in the bubbly state. These new bubbles did not exist before. We can think of them as being created (randomly and without cost!) by the young of the corresponding generation.

We are ready to define a competitive equilibrium of this economy. For a given initial capital stock $k_0 > 0$, a competitive equilibrium is a feasible sequence $\{k_t, s_t, b_t, n_t, z_t\}_{t=0}^{\infty}$ satisfying Equations (1), (4), (5) and (6). A sequence $\{k_t, s_t, b_t, n_t, z_t\}_{t=0}^{\infty}$ is feasible if, for all t, the following inequalities hold: (i) $k_t \geq 0$, (ii) $b_t \geq 0$, (iii) $n_t \in [0, 1]$ and $n_t = 1$ if $z_{t-1} = F$ and $z_t = B$. As Tirole showed, this economy generically has many competitive equilibria.

[3] See Diba and Grossman (1987).

2.3 Rational Bubbles

There is a useful trick that allows us to characterize the set of rational or equilibrium bubbles. Define x_t as the aggregate bubble as a share of the wage, i.e. $x_t \equiv \frac{b_t}{(1-\alpha) \cdot k_t^{\alpha}}$. Then, combine Equations (1), (4), (5) and (6) to find that:

$$x_t = n_t = 0 \quad \text{if } z_t = F \tag{7}$$

$$E_t \{x_{t+1} \cdot (1 - n_{t+1})\} = \frac{\alpha}{1-\alpha} \cdot \frac{x_t}{[1 - x_t \cdot (1 - n_t)]} \quad \text{if } z_t = B \tag{8}$$

Any feasible sequence for $\{x_t, n_t, z_t\}_{t=0}^{\infty}$ that satisfies Equations (7) and (8) is part of one equilibrium.[4] To streamline the presentation, we assume throughout that: (i) the only randomness in this economy relates to the dates in which bubble episodes start and end, and (ii) throughout a bubble episode, the probability of the bubble bursting and the rate at which new bubbles are created are constant, i.e. $\Pr_t(z_{t+1} = F \,| z_t = B) = p$ and that when $z_t = B$, then $n_t = n$ if $z_{t-1} = B$ and $n_t = 1$ if $z_{t-1} = F$.[5] These additional assumptions imply that we can rewrite Equation (8) as follows:

$$x_{t+1} = \frac{\alpha}{1-\alpha} \cdot \frac{x_t}{(1-p) \cdot (1-n) \cdot [1 - x_t \cdot (1-n)]} \quad \text{if } z_t = B \tag{9}$$

Let us consider next the type of bubbles that can arise under these assumptions.

Rationality imposes two restrictions to the type of bubbles that can exist. First, the bubble must grow fast enough or otherwise the young will not be willing to purchase it. Second, the bubble cannot grow too fast or otherwise the young will not be able to purchase it. We can build some intuition about how these restrictions shape the set of equilibrium bubbles with the help of Figure 1. The solid line shows, for any initial bubble x_t, the value of x_{t+1} that leaves the young indifferent between buying the bubble or investing in capital.

The left panel of Figure 1 shows the case in which our economy is dynamically efficient, i.e. $\alpha > 0.5$.[6] It is straightforward to show that, in this case, bubbles cannot arise. The proof is by contradiction. Assume a bubble episodes starts with $x_t < \frac{1}{1-n}$. The young are willing to buy the bubble only if it keeps growing as a share of output at a rate dictated by Equation (9). But this

[4] A sequence $\{x_t, n_t, z_t\}_{t=0}^{\infty}$ is feasible if (i) $0 \leq x_t \cdot (1 - n_t) \leq 1 - \alpha$, and (ii) $n_t \in [0, 1]$ and $n_t = 1$ if $z_{t-1} = F$ and $z_t = B$.

[5] Of course, these assumptions rule out many interesting equilibria. We chose here to preserve the simplest possible structure to convey our main results.

[6] The steady-state of the economy without bubbles is $k^{SS} = (1-\alpha)^{\frac{1}{1-\alpha}}$. The golden rule capital stock is $k^{GR} = \alpha^{\frac{1}{1-\alpha}}$. The economy is dynamically efficient if $k^{SS} < k^{GR}$.

means that the bubble will eventually outgrow the savings of the young in finite time. Since this is not possible, it follows that the only equilibrium is $z_t = F$ for all t. That is, rational bubbles cannot arise if the economy is dynamically efficient.

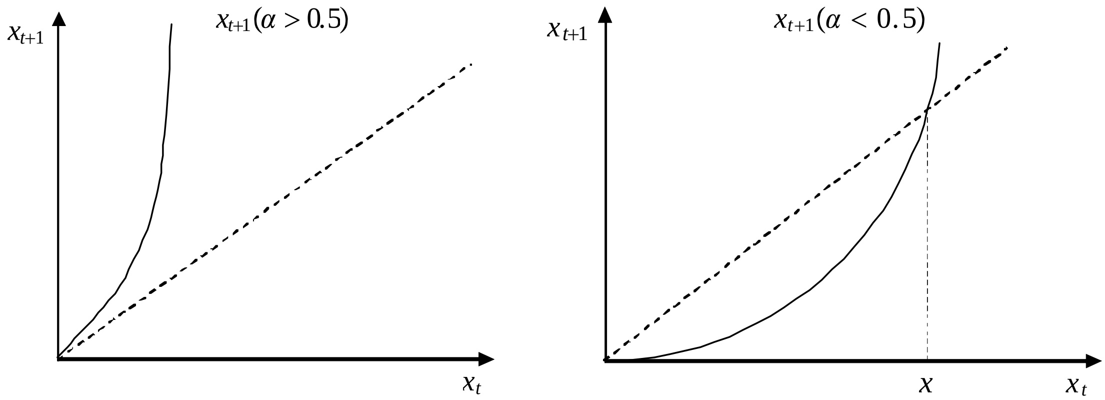

Figure 1

The right panel of Figure 1 shows instead the case in which the economy is dynamically inefficient, i.e. $\alpha < 0.5$. Tirole showed that in this case there is a stationary bubble and a continuum of non-stationary bubbles that asymptotically disappear.[7] The stationary bubble is given by:

$$x = \frac{1}{1-n} \cdot \left[1 - \frac{\alpha}{(1-\alpha)\cdot(1-p)\cdot(1-n)}\right] \tag{10}$$

Equation (10) implies that the stationary bubble is decreasing in both p and n. Increases in p imply a higher probability that the bubble bursts, thereby decreasing its expected return. Increases in n also decrease the expected return of the bubble: they do so by increasing the speed at which an existing bubble is "diluted" through bubble creation. Bubbles with $n = p = 0$ exist, as we have said, when $\alpha < 0.5$. Bubbles with higher levels of n and/or p require lower levels of α to exist. In the limit, bubbles with $n \to 1$ or $p \to 1$ exist only if $\alpha \to 0$.

In addition to the stationary bubble of Equation (10), there is a continuum of non-stationary bubbles. Let date 0, be the date in which a bubble episode starts. Each of the non-stationary bubbles is associated with a different initial value in the interval $x_0 \in (0, x)$. In equilibrium, all of these non-stationary bubbles grow at a rate below that of the economy and therefore vanish over time.

[7] Tirole considered only the case in which $p = n = 0$. This means that bubble episodes in his setup last forever.

2.4 Macroeconomic effects of bubbles

We are ready to describe the macroeconomic implications of bubbles. To do this, combine Equations (1), (4), and use the definition of x_t to find that:

$$k_{t+1} = [1 - (1 - n_t) \cdot x_t] \cdot (1 - \alpha) \cdot k_t^{\alpha} \tag{11}$$

Equation (11) describes the evolution of the capital stock, for a given bubble. As we have already mentioned, we shall consider the particular case of the stationary bubble:

$$x_t = \begin{cases} 0 & \text{if } z_t = F \\ \frac{1}{1-n_t} \cdot \left[1 - \frac{\alpha}{(1-\alpha)\cdot(1-p)\cdot(1-n_t)}\right] & \text{if } z_t = B \end{cases} \quad \text{and} \quad n_t = \begin{cases} 1 & \text{if } z_t = B \text{ and } z_{t-1} = F \\ n & \text{if } z_t = z_{t-1} = B \end{cases} \tag{12}$$

Equations (11) and (12) provide a full description of the dynamics of the economy. This model therefore rigorously encapsulates the notion of bubble episodes as shocks to the economy.

Figure 2 shows the macroeconomic effects of a bubble. Assume initially that the economy is in the fundamental state so that the appropriate law of motion is the one labeled k_{t+1}^F. This law of motion for the fundamental state is that of the standard Solow model. Since the initial capital stock is below the Solow steady state, i.e. $k_t < k^F \equiv (1-\alpha)^{\frac{1}{1-\alpha}}$, the economy is growing at a positive rate. When a bubble pops up, it crowds out capital in the portfolio of the young and reduces the investment rate. The law of motion for the bubble state is now the one labeled k_{t+1}^B. The picture has been drawn so that, at the time of the bubble, the capital stock is above the bubbly steady state, i.e. $k_t > k^B \equiv \left(\frac{\alpha}{(1-p)\cdot(1-n)}\right)^{\frac{1}{1-\alpha}}$. As a result, growth turns negative. Throughout the bubbly episode the capital stock and output decline. Eventually, the bubble bursts and the economy starts growing again.

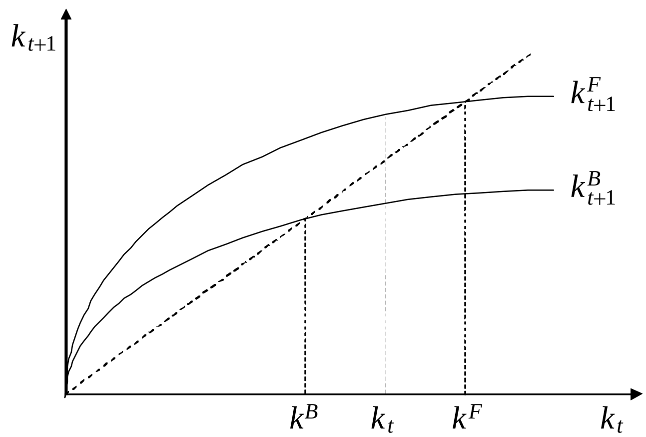

Figure 2

Despite their negative effect on capital accumulation and output, bubbles in this model lead to an increase in consumption and welfare. In this regard, there are two main effects of bubbles. The first such effect is that, when a bubble appears, the economy receives a positive wealth shock or transfer from the future. This is the central feature of a pyramid scheme where the initiator claims that, by making him/her a payment now, the other party earns the right to receive a payment from a third person later. By successfully creating and selling a bubble, young individuals have assigned themselves and sold the "rights" to the savings of a generation living in the very far future or, to be more exact, living at infinity. This appropriation of rights is a pure windfall or wealth gain for the lucky young. This is why bubbles increase the consumption of the generation that creates it.

The second effect of the appearance of a bubble is to eliminate investment in capital that was dynamically inefficient. In the absence of the bubble, we have seen that the steady-state capital stock of our economy equals $(1-\alpha)^{\frac{1}{1-\alpha}}$. This implies that, at each point in time, the young invest $(1-\alpha)^{\frac{1}{1-\alpha}}$ in capital accumulation whereas the old receive $\alpha \cdot (1-\alpha)^{\frac{\alpha}{1-\alpha}}$ in terms of capital income: whenever $\alpha < 0.5$, the latter amount is lower than the former and the resources devoted by the economy to the accumulation of physical capital exceed the resources that it recovers from such investment. Clearly, welfare could be improved in this economy if the young reduced their investment in capital and transferred the available resources directly to the old. And this is exactly what is made possibly by the appearance of bubbles. In the bubbly state, each generation reduces its investment in capital and receives in return a transfer from the next generation. Since the interest rate increases with the appearance of the bubble, the economy achieves a higher level of expected consumption and welfare.

2.5 Discussion

Tirole's (1985) model provided an elegant and powerful framework to think about bubbles. In this framework, however, the sole role of bubbles is to eliminate inefficient investment. This implies that its main macroeconomic implications are at odds with the facts along two key dimensions:

1. In the first place, and as we have already mentioned, the main prediction of the model is that bubbles lead to simultaneous contractions in output and booms in consumption. When a bubble appears in the basic model the capital stock falls, the interest rate increases, but the consumption level of all generations expands. This is contrary to historical evidence, which suggests that bubble episodes tend to be associated with increases in consumption but also in investment and the capital stock. A successful model of bubbles must therefore explain how they can lead to such increases despite displacing capital in the portfolios of investors.

2. In the second place, the model predicts that bubbles can only appear in dynamically inefficient economies. But Abel et al. (1989) have analyzed a group of developed economies and found no instance of dynamic inefficiency: in all of these economies, the resources devoted to capital accumulation were found to be consistently below the income produced by such investment. A successful model of bubbles must therefore explain how they can arise even in situations in which capital is an efficient store of value.

We now wish to argue that these important discrepancies between the model and the facts rest on one important assumption: financial markets are frictionless. Once this assumption is removed, we will show that bubbles can perform a double role, eliminating inefficient investments but also intermediating between unproductives and entrepreneurs. In this case, it is possible for bubbles to (i) appear even in economies in which all investments are dynamically efficient, and to (ii) lead to expansions in output and the capital stock.

3 A model with financial frictions

We have thus far assumed that the stock of capital at $t+1$ equals the investment made by generation t during its youth. This trivially follows if all young individuals have access to a technology for producing one unit of capital with one unit of consumption. But it may also follow if only a fraction of the young have access to such a technology, as long as there are financial markets that enable these individuals to undertake all of the economy's investment. We can interpret the baseline model of the previous section along these lines. Here, we wish to depart from this baseline by introducing financial frictions that limit trade between individuals of the same generation. As we shall see, these have important implications for the existence and macroeconomic implications of bubbles.

3.1 Modified setup

We modify our baseline economy by explicitly introducing heterogeneity in the productivity of investment among the young. It is assumed that a measure $\varepsilon < 0.5$ of the young, "entrepreneurs", can produce one unit of capital with one unit of the consumption good. The remaining young, "unproductives", have instead access to an inferior technology that produces $\delta < 1$ units of capital with one unit of the consumption good. In the presence of frictionless financial markets, it can be shown that this heterogeneity is inconsequential: in equilibrium, unproductives lend their resources to entrepreneurs and these invest on everyone's behalf. Our baseline economy, as we have already mentioned, can be interpreted in this way. We now explore the other extreme, in which there are no financial markets: in this case, no borrowing or lending is possible between unproductives and entrepreneurs and it is no longer true that the stock of capital at $t+1$ equals the investment made by generation t during its youth. Instead, the stock of capital at $t+1$ can now be expressed as:

$$k_{t+1} = s_t \cdot A_t \cdot k_t^{\alpha}, \tag{13}$$

where s_t is the investment rate, i.e. the fraction of output that is devoted to capital formation and A_t is the average productivity of such investment. In other words, in the presence of financial frictions, the evolution of the capital stock depends not only on the level but also on the composition of investment.

3.2 Equilibria

To solve this modified model, we need to find both the investment rate and the average productivity of investment. As in the baseline model, the young save a fraction $1-\alpha$ of output for consumption during old age. Also as in the baseline model, they must decide what to do with these savings: they may invest them in capital formation or use them to purchase a bubble. In the modified economy, though, there are different types of individuals that may make different portfolio choices and this affects the productivity of investment.

If the only option of the young is to invest in capital, the investment rate is simply $1-\alpha$ while the average productivity of investment is simply the population average,

$$A_t = \overline{A} = (1 - \varepsilon) \cdot \delta + \varepsilon.$$

We shall consider instead the possibility that the young can also purchase bubbles. To do so, there is one additional assumption that needs to be specified. One of the effects of bubbles is that their creation generates rents. Whereas in the baseline model the distribution of these rents is inconsequential, this is no longer true in our modified economy: in the absence of financial markets, the distribution of wealth – and hence of these rents – between entrepreneurs and unproductives is crucial to characterize the equilibrium. To simplify the exposition, we assume throughout that the rents from bubble creation are completely appropriated by entrepreneurs. Consequently, we henceforth assume that the income of entrepreneurs in any period t is given by $(w_t + n_t \cdot b_t)$.[8]

Whenever the young devote resources to purchasing the bubble, these resources are diverted away from physical investment: as in the baseline economy, the investment rate in capital accumulation is therefore still defined as in Equation (4). Due to the heterogeneity in investment productivity, though, not all of the young in our modified economy find bubbles equally attractive. The return to investing in a bubble equals its price appreciation and it is naturally the same for all individuals. The return of investing in capital, however, now differs across types of individuals. This implies that the expected return that must be promised by the bubble depends on the identity of its marginal buyer. Formally,

$$E_t \left\{ \frac{b_{t+1}}{b_t} \cdot (1 - n_{t+1}) \right\} \begin{cases} = \delta \cdot \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \; b_t < (1-\varepsilon) \cdot w_t \\ = \left[\delta \cdot \alpha \cdot k_{t+1}^{\alpha-1}, \alpha \cdot k_{t+1}^{\alpha-1} \right] & \text{if } \; b_t = (1-\varepsilon) \cdot w_t \\ = \alpha \cdot k_{t+1}^{\alpha-1} & \text{if } \; b_t > (1-\varepsilon) \cdot w_t \end{cases} \quad \text{if } z_t = B. \tag{14}$$

The LHS of Equation (14) represents the expected return to the bubble. The RHS of Equation (14) represents the returns to capital accumulation for different types of individuals. The first row of the RHS says that, if the return to the bubble equals the return to capital of unproductives, then no entrepreneurs purchase the bubble: hence, the bubble cannot exceed the total income of unproductives. The last row of the RHS says that, if the return to the bubble equals the return to capital of entrepreneurs, then unproductives must be spending all of their income in the bubble:

[8] It is clearly important for our results that some of the rents from bubble creation are appropriated by entrepreneurs. Nothing substantial would change, however, if a fraction of these rents were also appropriated by unproductives (see Martin and Ventura (2010)).

hence, the bubble can be no smaller than the total income of this group. Finally, the middle row of the RHS says that, when the return of the bubble is higher than the return to capital of unproductives but lower than the corresponding return of entrepreneurs, the bubble must exactly equal the income of unproductives.

Equation (14) establishes a relationship between the size of the bubble and the productivity of investment in capital accumulation. As the bubble grows in size, it absorbs first the resources of the unproductives, thereby increasing the share of investment that is carried out by entrepreneurs. This leads to an increase in the average productivity of total investment, which can now be expressed as:

$$A_t = \min\left\{\frac{(\varepsilon + (1-\varepsilon)\cdot\delta)\cdot w_t + (n_t - \delta)\cdot b_t}{w_t - (1-n_t)\cdot b_t}, 1\right\}. \tag{15}$$

In the fundamental state, $b_t = n_t = 0$ and the average productivity of investment is simply equal to the population average. In bubbly states, the bubble increases the productivity of investment through two channels. On the one hand, it displaces a disproportionately high share of unproductive investment. On the other hand, it makes it possible for entrepreneurs to expand productive investment by providing them with the rents from bubble creation. Once the bubble completely absorbs the income of the unproductives, only entrepreneurs invest in physical capital and the average productivity of investment equals one.

The previous analysis indicates that, in the presence of financial frictions, bubbles have two conflicting effects on capital accumulation and output. On the one hand, they have the usual contractionary effect of reducing inefficient investments thereby reducing s_t, i.e. the share of output that is invested in capital accumulation. On the other hand, they now have the additional expansionary effect of intermediating resources between unproductives and entrepreneurs, which increases the average productivity of investment A_t. If this last effect dominates, bubbles lead to an increase in $s_t \cdot A_t$, thereby expanding the capital stock. A necessary and sufficient condition for this is that

$$n > \delta. \tag{16}$$

Equation (16) has a natural economic interpretation. Every unit of the consumption good that is spent by unproductives in purchasing the bubble has a direct and negative impact on the capital stock, which decreases by δ. A fraction n of this unit, however, is transferred to entrepreneurs and it is therefore used to increase the capital stock. Whenever $n > \delta$, this last effect dominates and bubbles expand the capital stock.[9]

We are now ready to define the competitive equilibria of this modified economy. For a given initial capital stock $k_0 > 0$, a competitive equilibrium is a feasible sequence $\{k_t, s_t, A_t, b_t, n_t, z_t\}_{t=0}^{\infty}$ satisfying Equations (4), (5), (13), (14) and (15). A sequence $\{k_t, s_t, A_t, b_t, n_t, z_t\}_{t=0}^{\infty}$ is feasible if, for all t, the following inequalities hold: (i) $k_t \geq 0$, (ii) $b_t \geq 0$, (iii) $n_t \in [0, 1]$ and $n_t = 1$ if $b_{t-1} = 0$ and $b_t > 0$. Exactly as our baseline economy, this modified version of our model generically has many equilibria.

3.3 Rational bubbles

By combining equations Equations (4), (13) and (14) we find that, in the modified economy,

$$x_t = n_t = 0 \quad \text{if } z_t = F \tag{17}$$

$$E_t\{x_{t+1} \cdot (1 - n_{t+1})\} \begin{cases} = \frac{\delta \cdot \alpha \cdot x_t}{[(\varepsilon + (1-\varepsilon)\cdot\delta) + (n_t - \delta)\cdot x_t]\cdot(1-\alpha)} & \text{if } \; x_t < (1-\varepsilon) \\ = \left[\frac{\delta \cdot \alpha \cdot x_t}{[(\varepsilon + (1-\varepsilon)\cdot\delta) + (n_t - \delta)\cdot x_t]\cdot(1-\alpha)}, \frac{\alpha \cdot x_t}{[1 - x_t \cdot (1-n_t)]\cdot(1-\alpha)}\right] & \text{if } \; x_t = (1-\varepsilon) \\ = \frac{\alpha \cdot x_t}{[1 - x_t \cdot (1-n_t)]\cdot(1-\alpha)} & \text{if } \; x_t > (1-\varepsilon) \end{cases} \quad \text{if } z_t = B. \tag{18}$$

Any feasible sequence for $\{x_t, n_t, z_t\}_{t=0}^{\infty}$ that satisfies Equations (17) and (18) is part of one equilibrium. We will rely on these equations in order to characterize the possible existence and macroeconomic effects of equilibrium bubbles.

In our baseline economy, bubbles were shown to be possible only in the presence of dynamic inefficiency, i.e. when the resources devoted to capital accumulation are higher than the income produced by such investment. In that economy, with only one investment technology, the concept of dynamic inefficiency was simple and depended only on the total amount of investment as captured by α. Also simple were the characteristics and effects of bubbles. In our modified economy, the existence of heterogenous technologies along with financial frictions imply that the concept of dynamic inefficiency is less clear-cut: it now depends not only on the total amount of investment as captured by α, but also on the average productivity of investment and thus on δ. These parameters are the defining features of the economy with financial frictions, which provides a substantially enriched view of bubbles and their effects. This section outlines the main aspects of such a view,

[9] Clearly, necessary conditions for this are $\delta < 1$ and $n > 0$. If $\delta = 1$, all individuals are equally productive and bubbles cannot increase average productivity. If $n = 0$, there are no rents from bubble creation that can be used by entrepreneurs to expand productive investment and bubbles cannot increase the capital stock.

providing a separate analysis of contractionary and expansionary bubbles. To simplify the exposition, we focus throughout on the stationary bubbles of the type considered in Section 2, for which n and p remain constant throughout bubbly episodes.

3.3.1 Contractionary bubbles

We begin by characterizing contractionary bubbles due to their similarity to those analyzed in Section 2. In equilibrium, these bubbles must satisfy Equation (17) along with the requirement that $n < \delta$. They must grow fast enough for unproductive individuals to purchase them, thereby satisfying,

$$x_{t+1} \geq \frac{\delta \cdot \alpha}{(1-n) \cdot (1-p) \cdot (1-\alpha)} \cdot \frac{x_t}{[(\varepsilon + (1-\varepsilon) \cdot \delta) + (n-\delta) \cdot x_t]}. \tag{19}$$

Equation (19) describes the combinations of x, p and n that make bubbles attractive for young unproductives. At the same time, these bubbles cannot grow too fast or otherwise the young will not be able to purchase them. These conditions jointly determine the combinations (α, δ) for which contractionary bubbles can exist, which are given by

$$\alpha \leq \max_{n,p} \left[\frac{(\varepsilon + (1-\varepsilon) \cdot \delta) \cdot (1-n) \cdot (1-p)}{\delta + (\varepsilon + (1-\varepsilon) \cdot \delta) \cdot (1-n) \cdot (1-p)} \right] \tag{20}$$

$$\leq \alpha_C(\delta) = \frac{\varepsilon + (1-\varepsilon) \cdot \delta}{\varepsilon + (2-\varepsilon) \cdot \delta}. \tag{21}$$

The mapping $\alpha_C(\delta)$ denotes – for any given value of δ – the value of α below which unproductive investment is dynamically inefficient.[10] Equation (21) therefore implies that contractionary bubbles can only arise in economies that display some dynamically inefficient investment. It is depicted graphically in Figure 3, in which the shaded area illustrates combinations (α, δ) satisfying $\alpha < \alpha_C(\delta)$: the shaded area below the mapping contains all combinations of (α, δ).

[10] This also implies that, for $\alpha > \alpha_C(\delta)$, all investments in the economy are dynamically efficient.

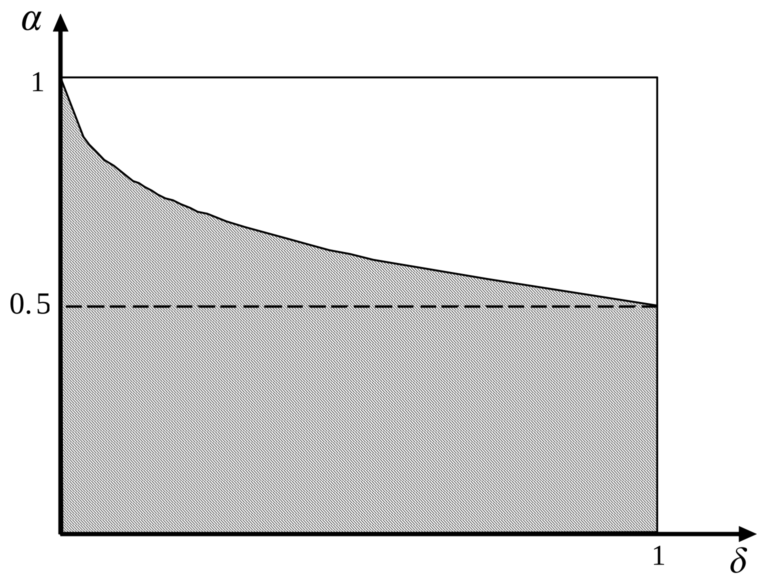

Figure 3

Figure 3 has a very natural interpretation. When $\delta \to 1$, we are in the baseline economy and bubbles can only exist if $\alpha < 0.5$. As δ decreases, so does the return to the investment of unproductives and this increases the potential appeal of bubbles: consequently, there is an expansion in the range of values of α that make bubbles possible. Finally, when $\delta \to 0$ unproductives have no way to save for old age and bubbles may therefore exist regardless of α.

The set of stationary bubbles satisfying Equation (21) contains a great deal of diversity in terms of bubble size, which can be lower than, equal to, or greater than the savings of unproductive individuals. We can build some intuition about these stationary bubbles with the help of Figure 4. As usual, the solid line shows – for any initial bubble x_t – the value of x_{t+1} that leaves the young indifferent between buying the bubble or investing in capital. The discontinuity captures the fact that, in order for entrepreneurs to start purchasing the bubble, there must be a jump in its expected return.

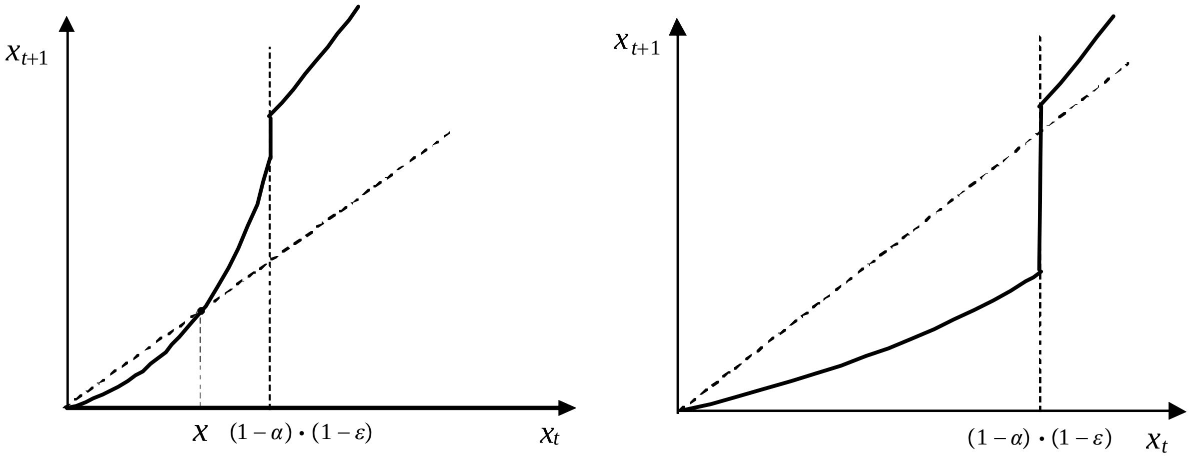

Figure 4

The left panel of Figure 4 displays the case of an interior stationary bubble that absorbs the savings of unproductives only partially, which is formally given by

$$x = \frac{1}{\delta - n} \cdot \left[(\varepsilon + (1-\varepsilon)\cdot\delta) - \frac{\alpha \cdot \delta}{(1-\alpha)\cdot(1-p)\cdot(1-n)} \right]. \tag{22}$$

The right panel, in turn, depicts a stationary bubble of size $(1-\varepsilon)$ that fully absorbs the savings of unproductives. In both cases, there also exists a continuum of non-stationary bubbles that disappear asymptotically.[11] A third possibility, not depicted in the figure, corresponds to a stationary bubble of size greater than $(1-\varepsilon)$ that absorbs all the savings of unproductives and some of the savings of entrepreneurs.[12] This concludes our characterization of contractionary bubbles.

3.3.2 Expansionary bubbles

In our modified economy, bubbles may be expansionary when $n > \delta$. In this case, their act predominantly as intermediaries of resources between unproductives and entrepreneurs, partially overcoming the adverse effects of financial frictions. Besides the requirement that $n > \delta$, expansionary bubbles must satisfy Equation (17). Like their contractionary sisters, they must grow fast enough to be attractive to young unproductives, but not too fast or otherwise the young will not be able to purchase them. These conditions jointly determine the set of combinations (α, δ) that

[11] Each of these non-stationary bubbles is associated with a different initial value in the interval $x_0 \in (0, x)$, where x denotes the stationary bubble.

[12] To be precise, this last group includes also some bubbles with $n > \delta$. These bubbles, as we argue in the next section, are typically expansionary. They can nonetheless be contractionary if entrepreneurs invest heavily in them. We do not develop this possibility formally because doing so does not yield new insights regarding the macroeconomic effects of bubbles.

admit the existence of expansionary bubbles, which is given by

$$\alpha \leq \max_{n,p} \left[\frac{(\varepsilon + (1-\varepsilon) \cdot n) \cdot (1-n) \cdot (1-p)}{\delta + (\varepsilon + (1-\varepsilon) \cdot n) \cdot (1-n) \cdot (1-p)} \right], \text{ subject to } n > d. \tag{23}$$

Solving for the optimization problem, Equation (23) becomes,

$$\alpha \leq \alpha_E(\delta) = \begin{cases} \frac{1}{1+4\cdot(1-\varepsilon)\cdot\delta} & \text{if } \delta \leq \frac{0.5-\varepsilon}{1-\varepsilon} \\ \frac{(\varepsilon+(1-\varepsilon)\cdot\delta)\cdot(1-\delta)}{\delta+(\varepsilon+(1-\varepsilon)\cdot\delta)\cdot(1-\delta)} & \text{if } \delta > \frac{0.5-\varepsilon}{1-\varepsilon} \end{cases}, \tag{24}$$

which provides a necessary and sufficient condition for the feasibility of expansionary bubbles.[13] This condition is depicted in shaded area below:

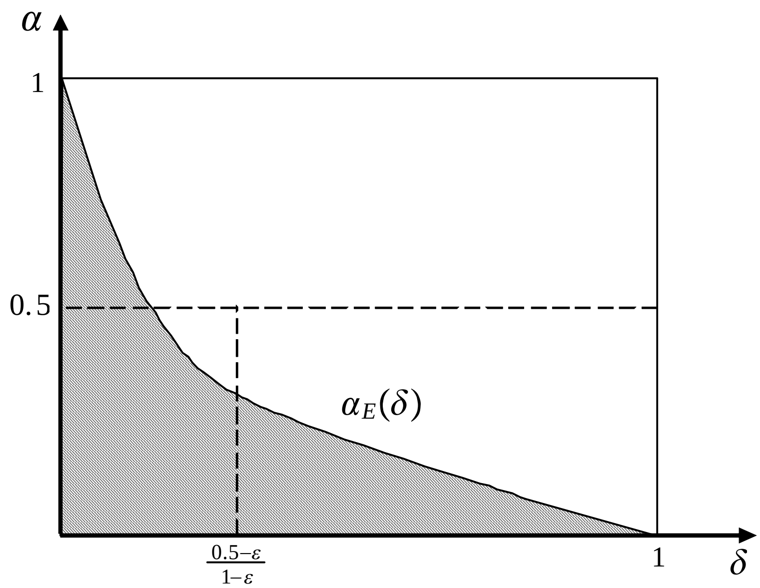

Figure 5

When $\delta \to 1$, we are in the baseline economy and expansionary bubbles are not possible: clearly, if there is no heterogoneity in productivity, bubbles can only be contractionary. As δ decreases, there is room for expansionary bubbles with $n > \delta$ to arise. When δ is high, however, so are the values of n required by these bubbles: this implies that their return must necessarily be low and,

[13]It might not seem immediately clear that Equation (24) is sufficient for the existence of expansionary bubbles. The reason is that even bubbles for which $n > \delta$ may have contractionary effects: this may happen once their size exceeds the income of the unproductive and they start to crowd out entrepreneurial investment. For a given combination (α, δ), however, any such bubble can be made to have expansionary effects by decreasing its appeal (for example, by increasing p) until entrepreneurs stop purchasing it.

consequently, they can only arise for low values of α. Decreases in δ therefore lead to an increase in $\alpha_E(\delta)$ until, when $\delta \to 0$, expansionary bubbles become possible regardless of α.

Depending on their size, expansionary bubbles might completely eliminate unproductive investment or they may do so only partially. In the former case, the formal expression for a stationary bubble is given by

$$x = \max\left\{(1-\varepsilon), \frac{1}{1-n} \cdot \left[1 - \frac{\alpha}{(1-\alpha)\cdot(1-p)\cdot(1-n)}\right]\right\}, \tag{25}$$

which is illustrated in Figure 6 below. The left panel of the Figure represents the first term of Equation (25), characterizing a stationary bubble that exactly eliminates unproductive investment. The right panel, in turn, depicts the second term of Equation (25) and it characterizes a stationary bubble that eliminates some entrepreneurial investment as well. In this case, the stationary bubble is formally the same as in the baseline model of Section 2. In both cases, there also exists a continuum of non-stationary bubbles that disappear asymptotically.[14]

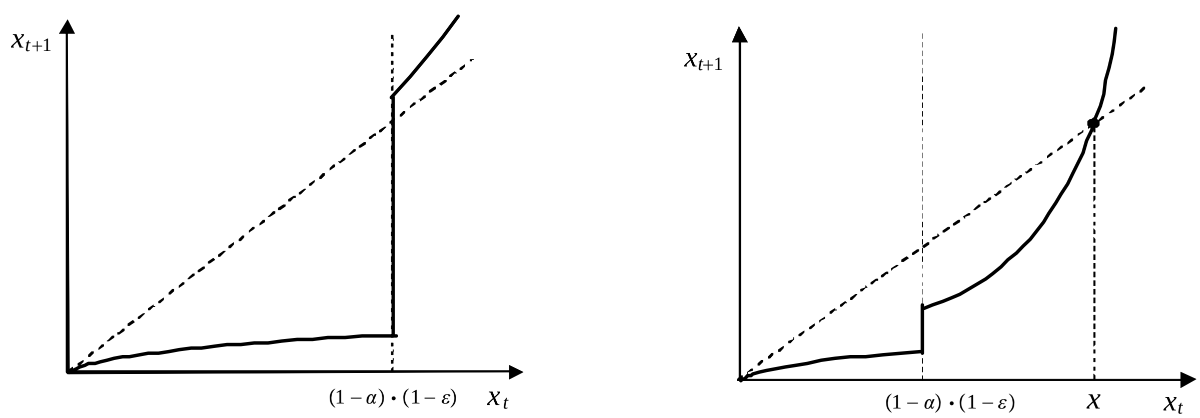

Figure 6

There is also a second type of expansionary bubble, an interior one that absorbs the resources of unproductives only partially. Equation (18) yields the following expression for a stationary bubble when $n > \delta$,

$$x = \frac{1}{n-\delta} \cdot \left[\frac{\alpha\cdot\delta}{(1-\alpha)\cdot(1-p)\cdot(1-n)} - (\varepsilon + (1-\varepsilon)\cdot\delta)\right], \tag{26}$$

which is illustrated in Figure 7 below. Whenever it exists, this stationary bubble is always accom-

[14]Each of these non-stationary bubbles is associated with a different initial value in the interval $x_0 \in (0, x)$, where x denotes the stationary bubble.

panied by a second one of size greater than or equal to $(1-\varepsilon)$: in the figure, x_2 is used to illustrate this second stationary bubble.

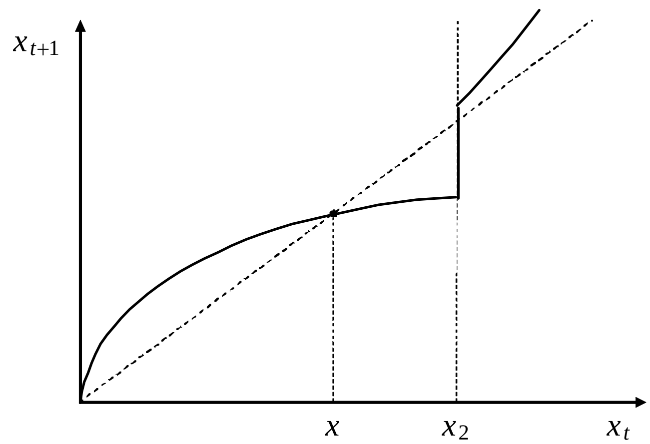

Figure 7

It is interesting to note that, unlike the contractionary bubbles of the baseline economy, the stationary bubble of Equation (26) is locally stable. Whenever this bubble exists, there is also a continuum of non-stationary bubbles that converge to it. Letting 0 denote the date in which a bubble episode starts, each of these non-stationary bubbles is associated with a different initial value in the interval $x_0 \in (0, x_2)$.

3.3.3 Discussion: bubbles and dynamic efficiency

We now combine the analysis of the previous two sections and provide a full characterization of bubbles in our economy. Figure 8 jointly displays constraints $\alpha_C(\delta)$ and $\alpha_E(\delta)$, which respectively represented by the dotted andstraight lines:

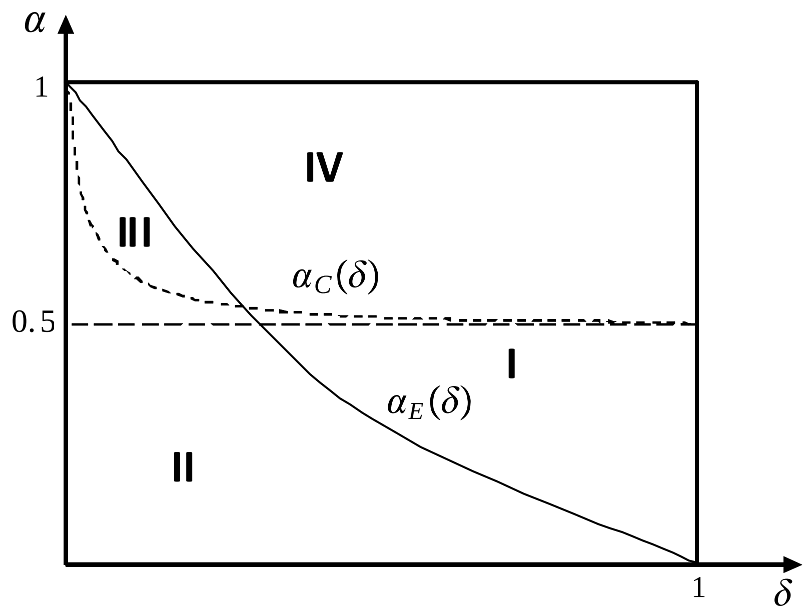

Figure 8

These constraints partition the set of all possible combinations (α, δ) into four regions. It follows from the previous sections that Regions I-III allow for the existence of bubbles, whereas Region IV does not. Among the former, different regions admit different types of bubbles. Regions I and II, in which $\alpha < \alpha_C(\delta)$ and unproductive investment is dynamically inefficient, are compatible with the existence of contractionary bubbles. Region II also admits expansionary bubbles, since $\alpha < \alpha_E(\delta)$ within its bounds. But the most surprising insights emerge from Region III, which is nonempty given our assumption that $\varepsilon < 0.5$. This region is consistent with the existence of bubbles even though $\alpha > \alpha_C(\delta)$ and all of the economy's investments are dynamically efficient. Moreover, any stationary bubble within this region must necessarily be expansionary. The rationale for this is intuitive. Bubbles are only be appealing to individuals with dynamically inefficient investment opportunities. When $\alpha > \alpha_C(\delta)$, there are no such individuals in the fundamental state. In order to exist, bubbles must therefore render some investments dynamically inefficient. But only expansionary bubbles can do this: by increasing the capital stock and lowering the interest rate, these bubbles decrease the appeal of investments in physical capital and thus generate their own demand.

4 The macroeconomic effects of bubbles revisited

We are now ready to analyze the macroeconomic effects of bubbles in our modified economy. By combining Equations (13), (4) and using the definition of x_t we can write:

$$k_{t+1} = [(\varepsilon + (1-\varepsilon)\cdot\delta) + (n_t - \delta)\cdot x_t]\cdot(1-\alpha)\cdot k_t^{\alpha} \tag{27}$$

Equation (27) describes the evolution of the capital stock as a function of an equilibrium bubble. Naturally, the impact of the bubble on the capital stock depends on whether it is expansionary or not. We first consider the less familiar case of expansionary bubbles. We then analyze the consequences of contractionary bubbles, which are closer to the baseline model of Section 2.

4.1 Expansionary Bubbles

Consider the case of an expansionary bubble in Region III. For simplicity, we focus on the following stationary bubble:

$$x_t = \begin{cases} 0 & \text{if } z_t = F \\ x = \frac{1}{n-\delta}\cdot\left[\frac{\alpha\cdot\delta}{(1-\alpha)\cdot(1-p)\cdot(1-n)} - (\varepsilon + (1-\varepsilon)\cdot\delta)\right] & \text{if } z_t = B \end{cases} \quad \text{and} \tag{28}$$

$$n_t = \begin{cases} 1 \quad \text{if } z_t = B \text{ and } z_{t-1} = F \\ n \qquad \text{if } z_t = z_{t-1} = B \end{cases} \tag{29}$$

Equations (27) and (28) provide a full description of the dynamics of this economy. The situation they depict is one of an interior stationary bubble that partially absorbs the savings of the unproductives.

Figure 6 shows the macroeconomic effects of such a bubble. Assume initially that the economy is in the fundamental state so that the appropriate law of motion is the one labeled k_{t+1}^F. This law of motion for the fundamental state is that of the standard Solow model. Since the initial capital stock is above the Solow steady state, i.e. $k_t > k^F \equiv \left[A\cdot(1-\alpha)\right]^{\frac{1}{1-\alpha}}$, the economy is growing at a negative rate. When a bubble pops up, it crowds out capital in the portfolio of the unproductive young and reduces the investment rate. However, the bubble also redistributes resources towards entrepreneurs, increasing the productivity of investment and – given that $n > \delta$ – expanding the capital stock. The law of motion for the bubble state is now the one labeled k_{t+1}^B. The picture has been drawn so that, at the time of the bubble, the capital stock is below the bubbly steady state, i.e. $k_t < k^B \equiv \left[\frac{\alpha \cdot \delta}{(1-p)\cdot(1-n)}\right]^{\frac{1}{1-\alpha}}$. As a result, growth turns positive. Throughout the bubbly episode the capital stock and output increase. Eventually, the bubble bursts and the economy starts contracting again.

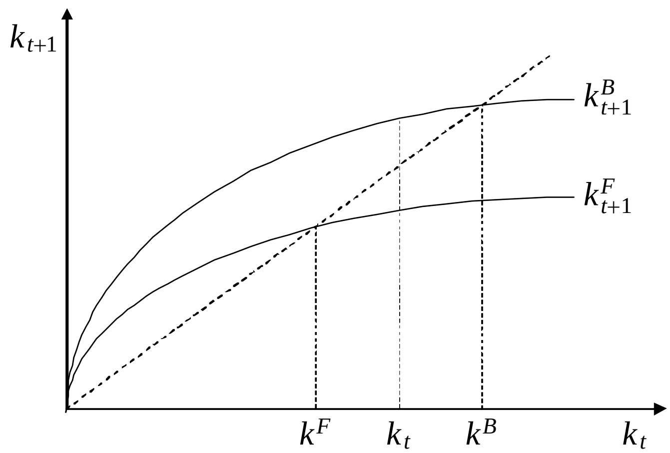

Figure 9

In Region III, bubbles thus increase the productivity of investment, the capital stock and output. As in the baseline case, there are two main effects of bubbles. The first such effect is that the appearance of a bubble implies a positive wealth shock or transfer from the future. Under our assumptions, this transfer is a pure windfall or wealth gain for the young entrepreneurs. Since it is the young unproductives who purchase the bubble, this gain could be as a large as their savings and this might be a substantial fraction of the economy. Since young entrepreneurs do not care about young age consumption, they save all this gain and invest it in capital so as to increase their old age consumption. This is why, when a bubble appears, there is a spike in the productivity of investment and in the total consumption of the generation that creates the bubble.

The second effect of the appearance of a bubble is an increase in the efficiency at which the economy operates. The economy not only gains rights to the future, but by trading them across generations also obtains the further benefit of displacing relatively inefficient investments. Entrepreneurs obtain a direct gain from the bubble, since they appropriate some of the "rights" to the future whenever $n > 0$. As for unproductives, the bubble has conflicting effects on their welfare. On the one hand, it increases their wages by expanding the capital stock. On the other, this very expansion of the capital stock decreases the interest rate and hence the return to their savings. Since $\alpha > 0.5$ in this region, the former effect can be shown to dominate and hence bubbles increase the expected consumption of all individuals. Otherwise, expansionary bubbles generate welfare gains for entrepreneurs but they might lead to welfare losses for unproductives.[15]

The bursting of the bubble is akin to losing the rights to the future and therefore creates opposite wealth and efficiency gains. Suddenly, old unproductives find themselves unable to sell the bubble and experience a windfall or wealth loss. Since this was their only source of old age income, their consumption drops to zero. After the bursting, young unproductives no longer have the option of buying the bubble and go back to their investments; while young entrepreneurs lose their rents and must reduce their efficient investment. When the bubble bursts, then, efficiency declines and so do the capital stock and consumption.

4.2 Contractionary bubbles (rewrite this and appendix)

The case contractionary bubbles follows closely the analysis of the baseline model of Section 2. These bubbles have a negative effect on capital accumulation and output, and their macroeconomic effects are essentially the ones outlined in Section 2.4.

There are, however, two important differences between these contractionary bubbles and those of the baseline economy. The first difference is that, in the presence of financial frictions, bubbles displace a disproportionately high share of low productivity investment: hence, they increase the productivity of investment even though their overall effect on the capital stock and on output is contractionary. This positive effect of bubbles on productivity is absent in the baseline model of Section 2, in which the productivity of investment is constant.

A second important difference has to do with the welfare effects of contractionary bubbles. In our modified economy, these bubbles can lead to a decrease in the expected consumption of all individuals.[16] To see this, consider the case of an interior contractionary bubble in Regions I or II, which partially absorbs the savings of unproductives. This bubble contracts the capital stock and thus equilibrium wages while it increases the returns to savings: if $\alpha > 0.5$, the former effect dominates and the bubble in principle leads to a fall in the expected consumption of all individuals. This fall in expected consumption might be offset, in the case of entrepreneurs, through the rents from bubble creation captured by n. For low values of n, however, an interior contractionary bubble will lead to a generalized fall in welfare. This somewhat surprising result arises even though $\alpha <$

[15] Aggregate consumption, however, must necessarily increase as a consequence of this bubble (see Section 7.1 in the Appendix).

[16] Section 7.1 in the Appendix provides a formal analysis of the welfare effects of bubbles in this type of economies.

$\alpha_C(\delta)$ and the bubble therefore displaces unproductive investment that is dynamically inefficient. The problem is that the contraction of unproductive investment, although beneficial for the first generation of unproductives that purchases the bubble, eventually leads to a decrease in wages that contracts productive entrepreneurial investment as well. Ultimately, this effect ends up being detrimental for the steady-state welfare of unproductives.

4.3 Discussion

This section has discussed the macroeconomic effects of bubbles in an economy characterized by the presence of financial frictions. In this environment, in which there are no financial markets to channel resources from unproductive individuals to productive entrepreneurs, we have shown that bubbles can partially carry out this much needed intermediation. Moreover, this simple model suffices to overturn the two major sources of criticism for the model of Tirole:

1. Bubbles need not lead to contractions in the capital stock or in output. Although they do displace physical investment from the portfolio of unproductives, it is clear by now that this need not reduce the aggregate capital stock. To begin with, the appearance of the bubble creates wealth and part of it can be used for investment. In the specific model presented here, this wealth falls fully into the hands of young entrepreneurs and is therefore invested productively. In addition, continuous bubble creation redistributes wealth from the old to the young and the latter might invest some of it. In the model, this redistribution goes fully from old unproductives to young entrepreneurs who, once again, invest all of it productively.

2. An economy need not be dynamically inefficient in the sense of Abel et al. (1989) in order for bubbles to be possible. Indeed, we have shown a much stronger result. Bubbles may arise even if all of the economy's investments – when individually considered – are dynamically efficient in the aforementioned sense. In this case, bubbles are expansionary and they generate their own demand by lowering the interest rate and rendering some investments inefficient.

The critiques to the Tirole model stem from the assumption that financial markets are frictionless and all investors face the same rate of return. Under this assumption, the bubble must displace capital from the portfolios of all investors and therefore reduce the aggregate capital stock. Under this assumption, all investments are equally efficient and we can rule out the existence of inefficient investments by looking only at economy-wide averages. But financial frictions create heterogeneity in rates of return. Unless one is willing to assume that financial markets are frictionless, it makes sense to think that rational asset price bubbles can raise the capital stock and cannot be ruled out with available empirical methods.

5 Bubbles and long run growth

The model of Section (3) has been useful to illustrate how bubbles can be expansionary and how bubbly periods can be associated with high levels of economic activity. In that model, however, the macroeconomic effects of bubbles are necessarily transitory: in the fundamental state, all of these effects are gradually washed away until there are no traces left of past bubbles. But this need not be the case. We now modify our model to show how the macroeconomic effects of bubbles can be long-lasting, leaving traces on the long-run levels and growth rates of output and of the capital stock. To do so, we generalize the productive structure of our economy to allow for the possibility of constant or increasing returns to capital.

5.1 Generalized production structure

We maintain the model of Section 3 in all regards but one. We generalize the production structure by assuming that the production of the final good consists of assembling a continuum of intermediate inputs, indexed by $m \in [0, m_t]$. This variable, which can be interpreted as the level of technology in period t, will be obtained endogenously as part of the equilibrium. The production function of the final good is given by the following symmetric CES function:

$$y_t = \widehat{\psi} \cdot \left(\int_0^{m_t} q_{tm}^{\frac{1}{\mu}} \cdot dm \right)^{\mu}, \tag{30}$$

where $\hat{\pi} > 1$ is a productivity parameter, q_{tm} denotes units of the variety m of intermediate inputs, and $\mu/(1-\mu)$ is the elasticity of substitution. Throughout, we assume that final good producers are competitive, and we normalize the price of the final good to one.

Production of intermediate inputs requires labor and capital. In particular, each type of intermediate input $m \in [0, m_t]$ is produced according to the following production function,

$$q_{tm} = \begin{cases} (l_{tm,v})^{1-\alpha} \cdot (k_{tm,v})^{a} & \text{if } q_{tm} > 0 \\ 0 & \text{if } q_{tm} = 0 \end{cases}, \tag{31}$$

where $l_{tm,v}$ and $k_{tm,v}$ respectively denote the use of labor and capital to cover the variable costs of producing variety m. Besides this use of factors, the production of any given variety requires the payment of a fixed cost f_{tm} given by

$$f_{tm} = \begin{cases} 1 = (l_{tm,f})^{1-\alpha} \cdot (k_{tm,f})^{a} & \text{if } q_{tm} > 0 \\ 0 & \text{if } q_{tm} = 0 \end{cases}, \tag{32}$$

where $l_{tm,f}$ and $k_{tm,f}$ respectively denote the use of labor and capital to cover the fixed costs of producing variety m. To simplify the model, we assume that input varieties become obsolete in one generation and, as a result, all generations must incur these fixed costs. It is natural therefore to assume that the production of intermediate inputs takes place under monopolistic competition and free entry.

This production structure is a special case of that considered by Ventura (2005).[17] He shows that, under the assumptions made, it is possible to rewrite Equation (30) as

$$y_t = \psi \cdot (k_t)^{\alpha \cdot \mu}, \tag{33}$$

where ψ is a positive constant and μ is a measure of the market size effects.[18] Maintaining the assumption of competitive factor markets, factor prices can now be expressed as follows

$$w_t = (1-\alpha) \cdot \psi \cdot (k_t)^{\alpha \cdot \mu}, \tag{34}$$

$$r_t = \alpha \cdot \psi \cdot (k_t)^{\alpha \cdot \mu - 1}. \tag{35}$$

Equation (33) captures the fact that, with this productive structure, there are two opposing effects of increasing the stock of physical capital. On the one hand, such increases have the standard effect of decreasing the marginal product of capital due to diminishing returns. On the other hand, increases in the stock of capital now expand the varieties of inputs produced in equilibrium, which has an indirect and positive effect on the marginal product of capital. If diminishing returns are strong and market size effects are weak ($\alpha \cdot \mu < 1$) increases in physical capital reduce the marginal product of capital: in this case, the analysis of Sections 3 and 4 apply. If instead diminishing returns are weak and market size effects are strong ($\alpha \cdot \mu \geq 1$) increases in physical capital raise

[17] The equations that follow are formally derived in Section 7.2 in the Appendix.

[18] Formally,

$$\psi = \widehat{\psi} \cdot (\mu)^{\mu} \cdot (1-\mu)^{1-\mu}.$$

the marginal product of capital. This last case is the one we now wish to analyze.

5.2 Equilibrium Bubbles

Regardless of the specific value of $\alpha \cdot \mu$, the law of motion of capital of the economy is given by

$$k_{t+1} = s_t \cdot A_t \cdot \psi \cdot (k_t)^{\alpha \cdot \mu}, \tag{36}$$

where s_t and A_t are as defined in Equations (4) and (15), respectively. In order to be appealing to the young, the equilibrium return of the bubble must now satisfy

$$E_t \left\{ \frac{b_{t+1}}{b_t} \cdot (1 - n_{t+1}) \right\} \begin{cases} = \delta \cdot \alpha \cdot \psi \cdot (k_t)^{\alpha \cdot \mu - 1} & \text{if } b_t < (1-\varepsilon) \cdot w_t \\ = \left[\delta \cdot \alpha \cdot \psi \cdot (k_t)^{\alpha \cdot \mu - 1}, \alpha \cdot \psi \cdot (k_t)^{\alpha \cdot \mu - 1} \right] & \text{if } b_t = (1-\varepsilon) \cdot w_t \\ = \alpha \cdot \psi \cdot (k_t)^{\alpha \cdot \mu - 1} & \text{if } b_t > (1-\varepsilon) \cdot w_t \end{cases} \text{if } z_t = B, \tag{37}$$

which is a generalization of Equation (14). Since output is also a function of $\alpha \cdot \mu$, though, the equilibrium law of motion for x_t is still correctly defined by Equations (17) and (18), while the definition of Regions I-IV is exactly as before. Formally, then, the analysis of equilibrium bubbles in this economy is exactly as the one of Section 3.3 and we will not repeat it here. The only difference with our previous results is that, under constant or increasing returns to capital, the bubble affects not only the long-run levels of output and capital but also the long-run rates of growth.

We are now ready to define the competitive equilibria of this economy. For a given initial capital stock $k_0 > 0$, a competitive equilibrium is a feasible sequence $\{k_t, s_t, A_t, b_t, n_t, z_t\}_{t=0}^{\infty}$ satisfying Equations (4), (5), (15), (36) and (37). A sequence $\{k_t, s_t, A_t, b_t, n_t, z_t\}_{t=0}^{\infty}$ is feasible if, for all t, the following inequalities hold: (i) $k_t \geq 0$, (ii) $b_t \geq 0$, (iii) $n_t \in [0,1]$ and $n_t = 1$ if $b_{t-1} = 0$ and $b_t > 0$.

5.2.1 Constant returns to capital ($\alpha \cdot \mu = 1$)

We focus first on the case in which the diminishing returns effect and the market size effect exactly offset one another, so that the law of motion of capital becomes

$$k_{t+1} = s_t \cdot A_t \cdot \psi \cdot k_t. \tag{38}$$

In such an economy, the effect of bubbles on the macroeconomy operates not through the interest rate but rather through the growth rates of the capital stock and of output. We illustrate this once more by focusing on the least familiar case of expansionary bubbles in Region III. In the fundamental state, the growth rate of output equals $\psi \cdot \overline{A} \cdot (1-\alpha)$ and, since $\alpha > \alpha_C(\delta)$, both types of investments are dynamically efficient. Since there are no individuals with dynamically inefficient investments in the fundamental state, any bubble must generate such investments in order for it to exist. Under constant returns, only an expansionary bubble can do this by increasing the average productivity of investment and hence the growth rate of the economy.

Figure 7 illustrates the macroeconomic effects of an expansionary bubble in such an economy when $\alpha \cdot \mu = 1$: in particular, it depicts the effects of a stationary bubble like the one characterized in Equation (28). Assume initially that the economy is in the fundamental state so that the appropriate law of motion is the one labeled k_{t+1}^F. This law of motion for the fundamental state corresponds to a constant growth rate of $\psi \cdot \overline{A} \cdot (1-\alpha)$. When a bubble pops up, it crowds out capital in the portfolio of the unproductive young and reduces the investment rate. However, the bubble also redistributes resources towards entrepreneurs, increasing the productivity of investment and – given that $n > \delta$ – expanding the growth rate of the economy. The law of motion for the bubble state is now the one labeled k_{t+1}^B, which corresponds to a constant growth rate of $[\psi \cdot \alpha \cdot \delta / ((1-p) \cdot (1-n))]$. As a result, the growth rate of the capital stock and of output increase throughout the bubbly episode. Eventually, the bubble bursts and the economy starts growing at a lower rate once more. The bubble thus increases the growth rate of the economy and it has a permanent effect on the level of output.

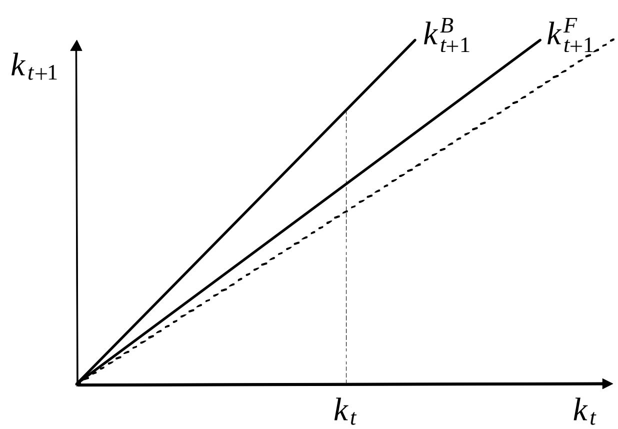

Figure 7

The previous narrative of macroeconomic effects applies equally well to other expansionary bubbles. Contractionary bubbles, on the other hand, have the opposite macroeconomic effects. In these cases, bubbly periods are associated to decreases in the growth rates of the capital stock and of output and they have permanent adverse effects on the equilibrium levels of both variables.

5.2.2 Increasing returns to capital ($\alpha \cdot \mu > 1$)

In economies in which the market size effect is strong enough to overcome the presence of diminishing returns, the law of motion of capital is given by Equation (36) with $\alpha \cdot \mu > 1$. In this case, bubbly episodes can have permanent effects not just on the levels of capital and output but also on their growth rates. We illustrate this through two examples below.

Figure 8 illustrates the manner in which an expansionary bubble can have permanent effects on the growth rate when $\alpha \cdot \mu > 1$. For simplicity, we consider once again the effects of an expansionary bubble like the one characterized in Equation (28). Assume initially that the economy is in the fundamental state so that the appropriate law of motion is the one labeled k^F_{t+1}. Since the initial capital stock is below the steady state, i.e. $k_t < k^F \equiv \left[\psi \cdot \overline{A} \cdot (1-\alpha)\right]^{\frac{1}{1-\alpha \cdot \mu}}$, the economy is growing at a negative rate: in a sense, the economy is caught in a "poverty trap", which it could exit by accumulating capital in excess of k^F. When an expansionary bubble pops up, it crowds out capital in the portfolio of the unproductive young while it redistributes resources towards entrepreneurs. The law of motion for the bubble state is now the one labeled k^B_{t+1}. The picture has been drawn so that, at the time of the bubble, the capital stock is above the bubbly steady state, i.e. $k_t > k^B \equiv \left[\frac{\psi \cdot \alpha \cdot \delta}{(1-p)\cdot(1-n)}\right]^{\frac{1}{1-\alpha \cdot \mu}}$. As a result, growth turns positive. Throughout the bubbly episode the capital stock and output increase. Eventually, the bubble bursts but the economy may keep on growing if the capital stock at the time of bursting exceeds k^F. In this case, the appearance of a bubble has a permanent positive effect on the economy's growth rate.

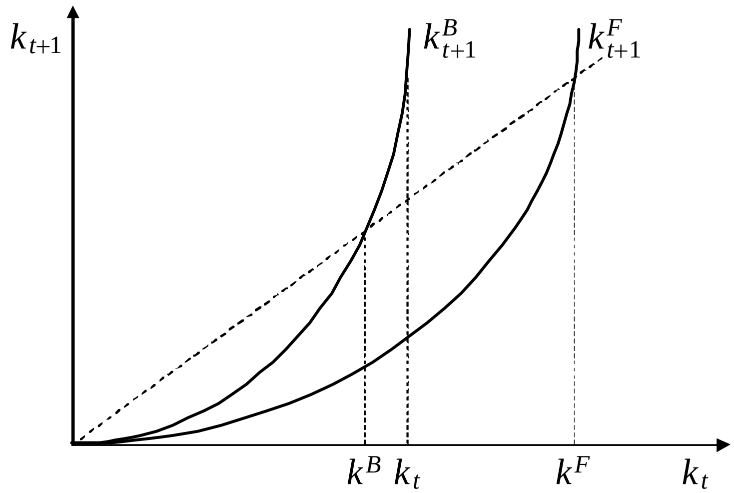

Figure 8

Naturally, it is also possible for bubbles to have a permanently negative effect on the growth rate of an economy. Figure 9 illustrates such an example for the simple case of a contractionary bubble that fully absorbs the resources of unproductives, i.e. a stationary bubble given by $x = (1-\varepsilon)$. The economy is initially in the fundamental state so that the appropriate law of motion is the one labeled k^F_{t+1}. Since the initial capital stock is above the steady state, i.e. $k_t > k^F \equiv \left[\psi \cdot \overline{A} \cdot (1-\alpha)\right]^{\frac{1}{1-\alpha\cdot\mu}}$, the economy is growing at a positive rate. When a contractionary bubble pops up, the law of motion becomes the one labeled k^B_{t+1}. The picture has been drawn so that, at the time of the bubble, the capital stock is below the bubbly steady state, i.e. $k_t > k^B \equiv [\psi \cdot \varepsilon \cdot (1-\alpha)]^{\frac{1}{1-\alpha\cdot\mu}}$. As a result, growth turns negative. Throughout the bubbly episode the capital stock and output decrease. Eventually, the bubble bursts but the economy may keep on contracting if the capital stock at the time of bursting lies below k^F. In this case, the appearance of a bubble has a permanent negative effect on the economy's growth rate, and will cause it to contract indefinitely.

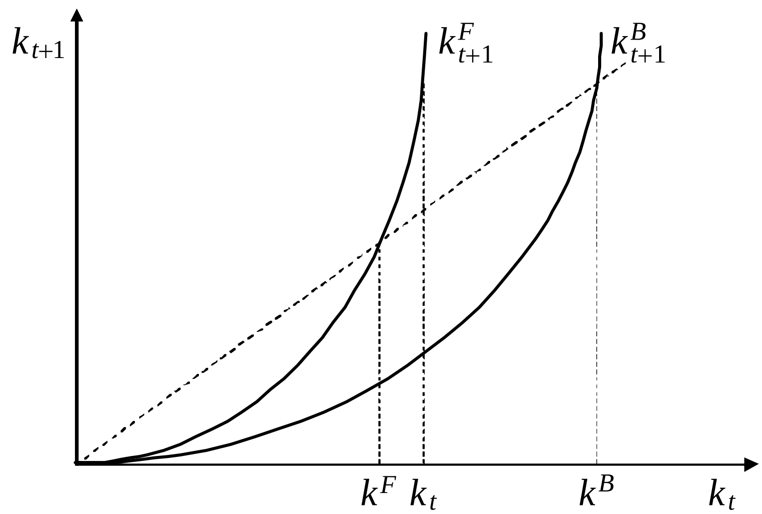

Figure 9

6 Further issues and research agenda

To be completed

References

TO BE COMPLETED

7 Appendix

7.1 Welfare effect of bubbles

TO BE COMPLETED

7.2 Generalized production structure

Profit maximization by producers of the final good yields the following demand function for intermediate good m,

$$p_{tm} = \left(\frac{q_{tm}}{y_t} \right)^{\frac{1-\mu}{\mu}} \cdot (\hat{\pi})^{\frac{1}{\mu}}. \tag{39}$$

Profit maximization by producers of intermediate good m can in turn be stated as

$$\max_{l_{tm,v}, l_{tm,f}, k_{tm,v}, k_{tm,f}} \left[q_{tm} \cdot p_{tm} - (l_{tm,v} + l_{tm,f}) \cdot w_t - (k_{tm,v} + k_{tm,f}) \cdot r_t \right]$$

subject to Equations (31), (32) and (39). Replacing the first and the last of these constraints into the objective function, this optimization problem yields the following first-order conditions:

$$\begin{aligned} \frac{p_{tm} \cdot q_{tm} \cdot (1-\alpha)}{\mu} &= l_{tm,v} \cdot w_t, \\ \frac{p_{tm} \cdot q_{tm} \cdot \alpha}{\mu} &= k_{tm,v} \cdot r_t, \\ \lambda \cdot (1-\alpha) &= l_{tm,f} \cdot w_t, \\ \lambda \cdot \alpha &= k_{tm,f} \cdot r_t, \end{aligned}$$

where λ denotes the multiplier on the constraint imposed by Equation (32). This constraint along with the first-order conditions imply,

$$\lambda = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \cdot \left(\frac{r_t}{\alpha} \right)^{\alpha}, \tag{40}$$

$$p_{tm} = \mu \cdot \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \cdot \left(\frac{r_t}{\alpha} \right)^{\alpha}, \tag{41}$$

which, along with the zero profit condition of intermediate goods producers yield

$$q_{tm} = \frac{1}{\mu - 1}. \tag{42}$$

Equation (42) says that, conditional on it being produced, the amount produced of any given intermediate good is constant. We can use it to derive the number of varieties being produced in equilibrium. To do so, we replace Equations (40)-(42) in the factor demands implied by the first-order conditions in order to obtain the following expressions for equilibrium in the markets for labor and capital:

$$\begin{aligned} m_t \cdot \frac{\mu}{\mu-1} \cdot \left(\frac{w_t}{1-\alpha}\right)^{-\alpha} \cdot \left(\frac{r_t}{\alpha}\right)^{\alpha} &= 1, \\ m_t \cdot \frac{\mu}{\mu-1} \cdot \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \cdot \left(\frac{r_t}{\alpha}\right)^{\alpha-1} &= k_t. \end{aligned}$$

These conditions can be combined to derive the equilibrium number of varieties,

$$m_t = \frac{\mu-1}{\mu} \cdot k_t^{\alpha},$$

which, when replaced in the production function of Equation (30), delivers Equation (33).