Collateral Constraints and Noisy Fluctuations

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Abstract

Collateral constraints on firm-level investment introduce a potentially powerful two-way feedback between the financial market and the real economy. On one hand, real economic activity forms the basis for asset dividends. On the other hand, asset prices affect collateral value, which in turn determines the ability of firms to invest. In this paper I show how this two-way feedback can generate significant expectations-driven fluctuations in asset prices and macroeconomic outcomes when information is dispersed. In particular, I study the implications of this two-way feedback within a micro-founded business-cycle economy in which agents are imperfectly, and heterogeneously, informed about the underlying economic fundamentals. I then show how tighter collateral constraints mitigate the impact of productivity shocks on equilibrium output and asset prices, but amplify the impact of "noise", by which I mean common errors in expectations. Noise can thus be an important source of asset-price volatility and business-cycle fluctuations when collateral constraints are tight.

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1 Introduction

Standard business cycle models generally assume that the economy's financial structure is both indeterminate and irrelevant to real economic outcomes. However, many economists have argued that credit constraints and financial conditions may in fact be key factors in determining business cycle fluctuations. The idea of assigning a more central role to credit-market imperfections in explaining short-run macro fluctuations not only dates back to the work of Fisher and Keynes, but has furthermore gained renewed interest in light of the recent financial crisis. In accordance with this view, this paper demonstrates an important and distinct role financial frictions may serve in generating significant *expectations*-driven business cycles.

An extensive literature on credit-market imperfections has incorporated financial frictions into standard macroeconomic models of the business cycle. In particular, many researchers have explored how collateral constraints interact with aggregate economic activity. In an economy in which firm-level debt must be fully secured by collateral, an endogenous two-way feedback arises between the financial market and the real economy. On one hand, firm output and activity forms the basis for pricing assets. On the other hand, asset prices affect collateral value, which in turn determines the ability of firms to invest. The seminal work of Kiyotaki-Moore (1997) demonstrated how under certain conditions this feedback may amplify the response of the economy to changes in technology or the income distribution. This has launched a wealth of research on collateral constraints; examples include Kiyotaki (1998), Krishnamurthy (2003), Caballero and Krishnamurthy (2001, 2006), Iacoviello (2005). Still others have questioned the quantitative significance of such an amplification mechanism, especially within less stylized environments and under more standard assumptions for preferences and technology (Kocherlakota, 2000; Cordoba and Ripoll, 2004). In any case, most of the existing literature focuses primarily on how collateral constraints affect the response of the economy to aggregate fundamental shocks, e.g. technology, endowments, and preferences, presuming that these are the main drivers of both business cycle and asset price fluctuations.

This comes as no surprise as the current dominant explanation of business cycle fluctuations are changes in fundamentals, the most important being technology shocks. At the same time, many economists find the idea of short-run fluctuations driven entirely by quarterly movements in the technological frontier somewhat implausible. For example, using structural VAR methods researchers have attempted to provide some empirical evidence that only a small fraction of output variation at business-cycle frequencies can be accounted for by technology shocks (Blanchard and Quah, 1989; Shapiro and Watson, 1988; Cochrane, 1994; Galí, 1999). Secondly, the finance literature has found it difficult to explain movements in the stock market on the basis of fundamentals alone; Shiller (1981) and LeRoy and Porter (1981) were the first to document the existence of "excess volatility", or residual variation in asset prices not accounted for by changes in technology or discount factors. Thus in general, while much research has been devoted to understanding the macroeconomic consequences of certain disturbances, the true *sources* of short-run business cycle and asset price fluctuations still remains very much an open question.

This paper addresses these issues by providing a theory of why financial frictions may imply that an important and potentially significant driver of short-run fluctuations in aggregate output and asset prices may simply be "noise". By this I mean noise in available public information, or more generally, correlated errors in agents' expectations of underlying fundamentals. In this paper I show how the feedback between asset prices and the real economy that results from collateral constraints may lead to significant expectations-driven business cycles when one breaks the assumption that information is commonly shared. Importantly, I find that when collateral constraints are tight and information is dispersed, the effects of noise can potentially account for a large portion of both business cycle and asset price fluctuations, while the relative contribution of technological shocks to these short-run fluctuations may be small. Thus, by allowing for heterogeneity in information, I identify a distinct and novel role for financial frictions.

Preview of the Model. The starting point of this paper is a micro-founded one-sector real business cycle model in which firm debt must be fully secured by collateral.[1] I abstract from capital to simplify the analysis, but assume firms must borrow in order to pay their workers.[2] As in most models of collateral constraints, in this economy there exists a durable asset which plays a dual role: it serves both as collateral to secure firm debt, and its value depends on real economic activity.[3] The endogeneity of the collateral value is effectively what creates the two-way feedback between the financial market and the real economy: the tighter the constraint, the less any individually-constrained firm may produce; at the same time, the less the entire economy produces, the tighter the constraints. In this respect this model is very similar to the mechanism present in Kiyotaki-Moore (1997) as well as the rest of the literature on collateral constraints.

One key difference between this model and Kiyotaki-Moore (1997), however, is that Kiyotaki-Moore allows firms to be highly levered, in that they have borrowed heavily against the value of their collateral. For this reason they obtain amplification of productivity shocks, yet many have convincingly argued that this effect must be quantitatively small (Kocherlakota, 2000; Cordoba and Ripoll, 2004). In this paper I abstract from the effects of leverage. I do so in order to isolate the contribution of the essential ingredients considered in this model. This model thus incorporates the salient feature that endogenous credit constraints create a feedback between the financial market and the real economy, but it is designed so that the Kiyotaki-Moore leveraged amplifier of productivity shocks is absent. I could argue that this doesn't seem to be such a terrible assumption in its own right, as demonstrating the quantitative significance of the Kiyotaki-Moore amplifier is clearly a challenge. Nevertheless the main reason for making this modelling choice is that it allows me to focus solely on the contribution of the information structure. By construction,

[1] The underlying assumption that motivates this type of credit constraints is that lenders cannot force borrowers to repay their debts unless the debts are secured.

[2] I can show in an extension of the model that the inclusion of capital does not affect the paper's main qualitative results.

[3] Most collateral-constraint models allow the asset to be an input to production. Here I instead assume workers obtain utility from consuming its dividends. In either case, the main reason for this assumption is to make the asset price sensitive to aggregate economic activity.

then, my benchmark economy is one in which collateral constraints have no effect on the business cycle properties of equilibrium output when information is complete.

As anticipated, a key ingredient in this paper is the *heterogeneity* of information. I show how this type of informational constraint interacts with the aforementioned financial friction. Unlike all previous models of collateral constraints, I allow for production, employment, and asset demand decisions to take place under dispersed information about the underlying fundamentals. I then show how the introduction of dispersed information creates a distinct and important role for financial frictions with respect to the equilibrium behavior of the economy.

Finally, I assume that the durable asset is traded among firms and workers across all islands. Therefore the price of this asset is observed by all agents in the economy and in equilibrium partially aggregates the dispersed information.

Preview of the Results. First, as a benchmark I demonstrate that under complete information, by construction tighter collateral constraints have no effect on the business cycle properties of the equilibrium. The focus of this paper, then, is to explore the implications of collateral constraints when information among agents is heterogeneous. The main results that obtain under dispersed information are the following:

(i) *Noise as a Source of the Business Cycle.* The combination of endogenous collateral constraints and dispersed information can lead to significant noise-driven fluctuations in aggregate output and asset prices. Specifically, I find that tighter collateral constraints mitigate the impact of productivity shocks on equilibrium output and asset prices, but amplify the impact of noise, and hence lead to a greater relative contribution of noise to business-cycle fluctuations. Noise can thus be a significant source of asset-price and output volatility when collateral constraints are tight.

(ii) *Positive Co-Movement.* Noise results in positive co-movement in employment, output, consumption, and asset prices. These fluctuations are independent of the underlying productivity shock; they are merely the product of common errors in agents' expectations.

(iii) *Labor Wedge.* Noise contributes to movements in the measured "labor wedge". In particular, the measured labor wedge is positively correlated with fluctuations in output when the main driver of the business cycle is noise.

The first result highlights the main contribution of this paper. This is the first paper, to my knowledge, to provide a theory of how financial frictions may imply that movements in both the business cycle and asset prices may be driven primarily by noise, as opposed to fundamentals. In this sense, collateral constraints play a distinct role in comparison to the earlier literature. This paper thus highlights the importance of considering heterogeneous information when firms are subject to collateral constraints.

The intuition for this result begins with the fact that asset prices, and hence the value of collateral, are based on average expectations of aggregate economic activity. When one relaxes the assumption of common knowledge and allows for heterogeneity of information, agents rely more on common sources of information regarding aggregate productivity, and less on private sources of information. This is because common sources of information are relatively better predictors of aggregate economic activity. As a result, asset prices become relatively more sensitive to this noise and less sensitive to fundamentals.

Consider then a small negative expectational error about technology, common to all agents. If agents believe common information will affect output, the small error depresses asset prices relatively more than what is dictated by its informational content about fundamentals alone. The decline in asset prices, however, lowers collateral values, which in turn leads to a fall in output. Thus, due to collateral constraints, the behavior of asset prices affects the behavior of real output. Moreover, the expected decline in output depresses asset prices even further. The two-way feedback just described amplifies the small expectational error, and in this way expectations about aggregate production become in many ways self-fullfilling.

Now consider a small positive innovation in aggregate productivity. Heterogeneity of information implies that this shock is not common knowledge. Hence even if agents on all islands are sufficiently well-informed of this innovation, because of lack of common knowledge asset prices are not extremely responsive. However, if asset prices move relatively little in response to the productivity innovation, constrained firms cannot react to the increase in productivity. But this inertia in expected output reinforces the dampened response of asset prices. Therefore, the two-way feedback generated by the collateral constraints mitigates the impact of the productivity shock.

In equilibrium, this behavior makes both asset prices and output more sensitive to noise and less sensitive to fundamentals. A robust prediction then of this model is that the tighter the collateral constraints, i.e. the more asset prices matter for firm production, the greater the relative contribution of noise to business cycle and asset price volatility. Thus, noise could very well be an important and dominant source of business cycle fluctuations.

The second and third main results then describe the implications of noise in this model in terms of observables.

The second result is notable as it shows how the noise-driven movements highlighted in this paper are a possible interpretation of what many believe as a major source of business cycle fluctuations–namely, "demand shocks". Using structural VAR methods, many researchers claim to identify a certain type of shock, orthogonal to technology, which causes positive comovement in aggregate macroeconomic variables including employment, output and consumption. They moreover find that these shocks may account for the majority of short-run fluctuations. While New-Keynesians have designated these as "demand shocks", their structural interpretation remains a contentious issue. This model offers a possible explanation for these fluctuations. The noise-driven movements documented here resemble demand-driven fluctuations in the following sense. They have many of the same features often associated with such shocks: they contribute to positive co-movement in employment, output and consumption, yet at the same time are orthogonal to underlying movements in productivity.

The third result relates the implications of noise in this model to the large body of research documenting the observed variation in the "labor wedge" over the business cycle (Hall, 1997; Rotemberg and Woodford, 1999; Chari, Kehoe, and McGrattan, 2007; Shimer, 2009).[4] Chari, Kehoe, and McGrattan (2007), in particular, show that a large class of models are equivalent to a frictionless representative-agent model of the business cycle in which various types of time-varying wedges (labor, investment, efficiency) distort the equilibrium decisions of the agents. They furthermore document the empirical contribution of each wedge over certain business cycle episodes as well as over the entire postwar period and find that the efficiency and labor wedges together account for the bulk of fluctuations while the investment wedge plays essentially no role. While their methodology is not meant to identify the source of the business cycle, it does suggest that the most promising models, in terms of relating to the data, are ones in which the underlying sources and frictions manifest themselves as labor wedges. Thus, this model may be useful for understanding certain business cycle periods, such as the recent crisis, in which the labor wedge plays an important role empirically.

Finally, in delivering these results this paper makes two methodological contributions to the recent growing literature on dispersed information in macro. There are a number of papers, most notably Morris and Shin (2002), which highlight the potential implications of heterogeneous information in a more abstract class of games that feature strategic complementarity. In relation to this body of work, one of the contributions of this paper is that it provides a novel microfoundation for these games. By this I mean the following: I show how the equilibrium allocation of the fully micro-founded business-cycle economy featured here can be represented as the Bayesian-Nash equilibrium of a game similar to those considered in the more abstract game-theoretic settings. This representation is useful, as the more abstract insights of this earlier work aids one in understanding how the two-way feedback between the financial market and the real economy results in noisy fluctuations when combined with dispersed information. In this respect, this paper is complementary to Angeletos and La'O (2009), in that it considers a micro-founded real business cycle model in which agents have dispersed information about underlying fundamentals. However, the micro-foundations and hence the mechanisms considered in these two papers are substantially different. Angeletos-La'O focuses on the effects of specialization in a multisector economy, and how trade across sectors creates interlinkages among firms. This paper instead considers a standard one-sector economy and highlights the two-way feedback that naturally arises between the financial and real sides of the economy when firms are constrained by collateral.

Secondly, the environment here features an asset whose price both aggregates and conveys information across islands. While most models with dispersed information abstract from this complication, this model calls for financial markets, and thus I demonstrate how one can allow for observable asset prices in a dispersed information environment yet preserve tractability. This is important, as one possible concern with heterogeneous information models is that their effects

[4] The labor wedge is defined as the wedge between the marginal rate of substitution and the marginal product of labor that would obtain in a frictionless representative-agent model of the business cycle.

may disappear when financial market prices aggregate information. I show here that this concern is misguided; the effects of noise are robust to the inclusion of observable asset prices.

Layout. This paper is organized as follows. The remainder of the introduction discusses the related literature. Section 2 introduces the model and describes the economic environment. In Section 3, I define and characterize the equilibrium of the economy. In Section 4 I consider the complete information benchmark. Section 5 explores the mechanism underlying the main results of this model in a simplified environment. In Section 6, I describe the method I use to obtain an approximate solution to the general equilibrium of the full model. I then present the results of this solution in Section 7 and highlight the main contributions of this paper. Section 8 concludes. Finally, the appendix contains all proofs which are not presented in the text.

Related Literature. This paper is related to the literature on financial frictions in the business cycle. Within this literature, there are two general strands. One strand builds on the work by Townsend (1979) and Gale and Hellwig (1985) and focuses on the costly state verification problem between creditors and debtors. Papers such as Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), Cooley, Marimon, and Quadrini (2004), Gertler, Gilchrist, and Natalucci (2007), and Christiano, Motto, and Rostagno (2008) build this problem into macroeconomic models, emphasizing the interaction between interest rate speads and aggregate quantities. The key mechanism here involves the link between "external financing premium" (the difference between the cost of funds raised externally and the opportunity cost of funds internal to the firm) and the net worth of borrowers.

This paper, however, is more closely related to another strand of the financial frictions literature: one that emphasizes collateral constraints in models of the business cycle. Examples in this literature include Kiyotaki-Moore (1997), Kiyotaki (1998), Kocherlakota (2000), Krishnamurthy (2003), Caballero and Krishnamurthy (2001, 2006), Cordoba and Ripoll (2004), Iacoviello (2005), Liu, Wang, and Zha (2009). In this class of models, a firm's debt level is constrained by the value of its collateral. This paper incorporates a similar collateral constraint friction into a business cycle model, but focuses on its interaction with heterogeneous information.

There is a wealth of empirical evidence that point towards the importance of financial frictions in aggregate fluctuations. First, there is considerable evidence of balance-sheet effects on investment, see Fazzari, Hubbard and Petersen (1988), Gertler, Hubbard, and Kashyap (1991), Gilchrist and Himmelberg (1995), Hubbard, Kashyap and Whited (1995), as well as on inventories, see Kashyap, Lamont and Stein (1994). Secondly, some authors have found that smaller firms are more sensitive to the business cycle; see Gertler and Gilchrist (1994), and Sharpe (1994).[5] Finally, Rajan and Zingales (1998) provide empirical evidence for a causal effect of financial development on growth.

This paper furthermore contributes to the growing literature on informational frictions in macro.[6] Within this context, this paper follows in methodology lines similar to Lucas (1972),

[5] However, recent work by Chari, Christiano, and Kehoe (2009), find evidence towards the contrary.

[6] Phelps (1970), Lucas (1972, 1975), Barro (1976), King (1982), Townsend (1983). More recent papers include

Barro (1976), Townsend (1983) and others, by introducing a geographic segmentation of information among islands to formalize the dispersion of information. This paper thus fits in with the recent revival of models of dispersed information, which has been duly influenced by the work of Morris and Shin (2002), Sims (2003), and Woodford (2003).

In considering expectations-driven business cycles, this paper is related to the literature on news-driven business cycles; see, for example, Beaudry Portier (2004, 2006), Christiano et al.(2007), Jaimovich and Rebelo (2008), and Lorenzoni (2009). These papers also generate certain types of expectations-driven fluctuations. However, the mechanisms considered in these papers differs significantly from the one considered here. The news-shock literature focuses on how expectations of future productivity may drive current production within representative-agent environments. Thus, their results do not rest on the heterogeneity of information. Here, I show that the heterogeneity of information is key in bringing about a self-fulfilling feature of expectations: agents need not make large forecast errors when information is dispersed, they merely coordinate on the same information.

Finally, this paper is also very related to an important contribution by Chen and Song (2010). Chen and Song (2010) study an economy in which a fraction of firms face financial constraints. They then show how news of future productivity relaxes these financial constraints, allowing capital to flow to constrained projects. The reallocation of capital generates an increase in current aggregate TFP as well as positive co-movement in macro aggregates. Because there is no uncertainty in their model, news-driven fluctuations are in fact correlated with future productivity, despite the fact that they are independent of current productivity. In this sense, unlike this paper, the positive-comovement generated in Chen and Song (2010) is remains correlated with fundamentals. If they were to add uncertainty under symmetric information to their model, errors in expectations of future productivity would drive current production. However, in this case, as in the news shocks literature, expectations would have no self-fulfilling role.

2 The Model

There is a (unit-measure) continuum of households, or "families", each consisting of a continuum of worker-members. There is a continuum of "islands", which define the boundaries of local labor markets as well as the "geography" of information: information is symmetric within an island, but asymmetric across islands. Each island is inhabited by a competitive representative firm which pro-

Adam (2007), Amador and Weill (2007, 2008), Amato and Shin (2006), Angeletos and Pavan (2004, 2007, 2009), Bacchetta and Wincoop (2005), Collard and Dellas (2005), Hellwig (2002, 2005), Hellwig and Veldkamp (2008), Hellwig and Venkateswara (2008), Klenow and Willis (2007), Lorenzoni (2008, 2009), Luo (2008), Mackowiak and Wiederholt (2008, 2009), Mankiw and Reis (2002, 2006), Mertens (2009), Morris and Shin (2002, 2006), Moscarini (2004), Nimark (2008), Reis (2006, 2008), Rondina (2008), Sims (2003, 2006), Van Nieuwerburgh and Veldkamp (2006), Veldkamp (2006), Veldkamp and Woolfers (2007), and Woodford (2003a, 2008), Angeletos, Lorenzoni, Pavan (2009), Angeletos La'O (2009a, 2009b) duces a consumption good; goods produced among islands are perfect substitutes.[7] Households are indexed by $k \in K = [0,1]$, its members by $j \in J = [0,1]$; and islands, as well as the representative firm on each island, are indexed by $i \in I = [0,1]$.

There are two stages. In stage 1, each household sends a worker to each of the islands. Once workers reach these islands, the representative firm on each island is endowed with a durable good, which I henceforth refer to as land, and learns of its own productivity for producing the commodity. This information is also revealed to all workers within the island. Thus, at this point workers and firms on each island have complete information regarding local productivity and land, but incomplete information regarding the productivities and land endowments of other islands.

Firms and workers then trade claims on land in a centralized (economy-wide) market; in particular, firms sell land claims, workers buy these claims, and the land price adjusts so as to clear the market. After the economy-wide land market has cleared, local (within-island) labor markets open: workers decide how much labor to supply, firms decide how much labor to demand, and local wages adjust so as to clear the local labor market. Finally, after employment and production choices are sunk, workers return home and the economy transits to stage 2.

In stage 2, all information that was previously dispersed becomes publicly known, and the commodity market opens. Quantities are now pre-determined by the exogenous productivities and the endogenous employment choices made during stage 1, but the price of the consumption good adjusts to clear the market. Finally, consumption of goods and housing takes place.

Households. The utility of household k is given by

$$\mathcal{U}_k = C_k^{1-\gamma} H_k^{\gamma} - N_k$$

where C_k is the household's consumption of the identical good produced by firms, and H_k and N_k are composites of the housing purchased and the labor supplied by the household's various workers in stage 1. The household has Cobb-Douglas utility in consumption and housing, where $\gamma \in (0,1)$ is the income share of land. The composites H_k and N_k are given by the following

$$H_k = \left[\int_J h_{jk}^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\nu}{\nu-1}}, \qquad N_k = \int_J n_{jk} dj$$

where n_{jk} is the amount of labor supplied by family member j during stage 1 and h_{jk} is that family member's purchase of land claims.[8] Here $\nu > 0$ is the elasticity of substitution across housing purchased by different members of the household.

Households own equal shares of all firms in the economy. Accordingly, the budget constraint of household k is given by the following

$$PC_k + Q \int_J h_{jk} dj \leq \int_I \pi_i di + \int_J w_j n_{jk} dj$$

[7] One may think of this as a continuum of identical competitive firms within an island. To supress notation I will refer to this continuum as a single competitive representative firm.

[8] Here utility is quasilinear in labor, but that assumption can easily be relaxed.

where P is the price of the consumption good, Q is the price of housing, π_i is the profit of firm i, and w_j is wage of family member j.

Finally the objective of the household is simply to maximize expected utility subject to the budget and informational constraints faced by its members. Here, one should think of the members of each family as solving a team problem: they share the same objective (family utility) but have different information sets when making their labor-supply and asset-demand choices. Formally, during stage 1 the household sends off its workers to different islands with bidding instructions on how to supply labor and demand assets as a function of (i) the information that will be available to them on their island, (ii) the wage in their local labor market, and (iii) the price of the asset. In stage 2, the workers return home and the household collects all labor income and firm profits and consumes.

Firms. The output of the representative firm on island i is given by

$$y_i = A_i n_i^\theta \tag{1}$$

where A_i is the productivity on island i, n_i is the firm's employment, and θ parameterizes the degree of diminishing returns to production. Each firm is endowed with L_i units land, which they sell on the centralized land market at price Q. Thus, the firm's realized profits are given by

$$\pi_i = P y_i - w_i n_i + Q L_i$$

The firm's objective is to maximize expected shareholder value.

I make the following critical assumption about firms: the firm cannot precommit to not run away with the revenues from its sales in stage 2. The goods, however, must be produced by workers in stage 1, before the product markets open. This poses a problem as workers, in light of the firm's commitment problem, refuse work in stage 1 unless paid up front.

To get around this problem, firms may use their land holdings as collateral to pay workers. This implies that the firm may hire as much employment as it likes, as long as the cost of doing so does not exceed the value of its land. The firm's problem then is given by the following. The competitive representative firm on island i, taking prices as given, maximizes expected profits weighted by the representative household's marginal value of wealth, λ,

$$\max \mathbb{E}_i \left[\lambda \; \pi_i\right]$$

subject to its production function (1) and subject to its collateral constraint given by

$$w_i n_i \leq \chi Q L_i \tag{2}$$

where $\chi > 0$ is the fraction of land which is collateralizable. The collateral constraint (2) dictates that the firm's cost of employment in stage 1 cannot exceed the value of its land, where land is valued mark-to-market at the price Q.

Labor, Land, and Product Markets. Labor markets operate in stage 1. Because labor cannot move across islands, the clearing conditions for the local labor markets are as follows

$$n_i = \int_K n_{ki} dk, \forall i$$

The land market also operates in stage 1, but unlike the labor market, land claims are traded in an economy-wide market. One can think of this as follows. There is a centralized market, not located on any island, to which workers submit demand schedules for land as functions of the price, and firms submit their land endowments. A walrasian auctioneer then sets the land price such that it clears the market. The clearing condition for the land market is as follows.

$$\int_I L_i di = \int_K \int_J h_{jk} dj dk$$

Finally, the consumption good is traded in a centralized market that operates in stage 2; the clearing condition for the product market is given by

$$\int_I y_i di = \int_K C_k dk$$

Fundamentals. Each island in this economy is subject to two types of shocks: technology and land endowment shocks. I assume the island-specific productivities $A_{k,t}$ are lognormally distributed in the cross-section of islands:

$$a_i \;\equiv\; \log A_i \;=\; \bar{a} + \xi_{a,i},$$

where $\bar{a}_t$ denotes the underlying aggregate productivity shock and $\xi_{a,i}$ is a purely idiosyncratic (i.e., island-specific) productivity shock. I assume $\xi_{a,i}$ is drawn from the Gaussian distribution with mean zero and variance $\sigma^2_{A,x} \equiv 1/\kappa_{A,x}$, is orthogonal to $\bar{a}$, and is i.i.d. across islands. Similarly, the island-specific land endowments are lognormally distributed in the cross-section of islands:

$$l_i \;\equiv\; \log L_i \;=\; \bar{l} + \xi_{l,i},$$

where $\bar{l}_t$ denotes the underlying aggregate land endowment and $\xi_{l,i}$ is a purely idiosyncratic (i.e., island-specific) land shock. I assume $\xi_{l,i}$ is drawn from the Gaussian distribution with mean zero and variance $\sigma^2_{L,x} \equiv 1/\kappa_{L,x}$, is orthogonal to both $\bar{l}$, and is i.i.d. across islands.

Finally, the aggregate shock $\bar{a}$ is drawn from a Gaussian distribution with mean zero and variance $\sigma^2_{A,0} \equiv 1/\kappa_{A,0}$, while the aggregate shock $\bar{l}$ is drawn from a Gaussian distribution with mean zero and variance $\sigma^2_{L,0} \equiv 1/\kappa_{L,0}$. The two aggregate shocks, $\bar{a}$ and $\bar{l}$ are orthogonal to each other.

Information. Aggregate productivity and the aggregate land endowment are assumed to be common knowledge at stage 2, when all production materializes and consumption takes place, but not in stage 1, when the key employment, production, and asset holding choices are made. Rather, at this stage workers and firms in any given island know perfectly the productivity of their own island but not the productivities of other islands. Local productivity and local land endowments thus serve also as valuable, but noisy, private signals of the distribution of productivities, land, and information in other islands.[9] In addition to these private signals, all islands observe an exogenous public signal of aggregate productivity:

$$z = \bar{a} + \varepsilon$$

where ε is drawn from the Gaussian distribution with mean zero and variance $\sigma^2_{A,z} \equiv 1/\kappa_{A,z}$, and is orthogonal to both $\bar{a}$ and $\bar{l}$. I assume the precisions (and equivalently the variances) of all fundamentals and signals, $\kappa_{A,x}$, $\kappa_{L,x}$, $\kappa_{A,0}$, $\kappa_{L,0}$, and $\kappa_{A,z}$ are common knowledge.

Here ε is what is referred to as "noise". In terms of this model, noise is literally the error in the public signal. However, this is just a convenient modelling device for common information. More generally, one can think of noise merely as correlated errors in expectations about aggregate productivity.

Finally, firms and workers in each island observe the land asset price Q.

Remarks. The constraint imposed on firm-level expenditure in (2) is similar to those found in the larger literature on collateral constraints. In this class of models the borrowing capacity of a firm is constrained by the value of its collateral. Although this setup is described as though the firm literally takes its revenues from selling land claims and uses these revenues to pay its workers, one could allow for an intermediate step between these transactions in which a financial intermediary loans funds to the firm and secures this loan with collateral. The collateral, then, is the value of land held by the firm.

Most papers on collateral constraints provide a simple justification for this type of financial friction. Following Kiyotaki-Moore (1997), these papers interpret this type of constraint as reflecting the problem of control over assets (see Hart and Moore, 1989). The crucial feature is that the firm is needed to run or control the project but cannot precommit to repay its debt. If the firm defaults, the creditor can seize the borrower's land. Hence the firm's repayment is bounded from above by the value of its land.[10] Furthermore, the usual interpretation of the parameter χ stems from the consideration that it may be costly to liquidate seized land, hence the creditor can recoup only up to a fraction χ of the total value of the collateral assets. More generally, however, one can think of χ as parameterizing the tightness of collateral constraints in the economy. Finally, while it is true that few firms literally collateralize their loans using land, in reality there are many layers of collateralization that are ultimately grounded in a physical asset. In this sense, stylized collateral constraint environments do capture some element of reality.

The durable asset in this model plays a dual role: it serves both as an asset of value, and as

[9] The assumption that firms and workers know perfectly their own productivities and endowments is not essential; all results go through if I allow for uncertainty about local as well as aggregate productivity.

[10] This is not the same as exogenously ruling out the possibility of state-contingent contracts. As pointed out to me by Randy Wright, one could think that writing state-contingent contracts are possible in this framework so that loan markets are, in a sense, complete. However, the commitment problem of the firm makes it so that no worker nor creditor would ever want to write a state-contingent loan contract, as the firm would in every state have the incentive to run away with the money.

collateral to secure firm debt. In this way the model preserves the key feature of all collateral constraint environments. However, as already mentioned in the introduction, this feature alone does not necessarily generate amplification. One key difference between this model and Kiyotaki-Moore (1997), is that Kiyotaki-Moore allows firms to be highly levered in that they have borrowed heavily against the value of their collateral. In particular, firms enter each period already in debt, at which point these firms obtain new loans, subject to their collateral constraints, and use these loans to both repay old debt and to invest. In this environment, the response of the economy to changes in fundamentals such as productivity is amplified. In contrast, in this model I abstract from this effect and assume firms enter without any existing debt, yet must borrow in order to pay for their static operating cost of paying workers. Thus this model features a simple, static input-financing friction due to collateral constraints.[11] I show in Section 4 that in this case under complete information collateral constraints have no effect on the business cycle properties of the equilibrium. That is, the complete information benchmark is designed so that tighter constraints do not amplify the response of the economy to productivity shocks.

I assume that consumption choices in stage 2 take place in a centralized market, where information is homogeneous, and that households are "big families", with fully diversified sources of income. The big-family formulation allows for household members to be completely insured against idiosyncratic risk in consumption and hence guarantees that the economy admits a representative consumer in stage 2. This assumption is made merely to maintain high tractability in analysis despite the fact that some key economic decisions take place under heterogeneous information.

Shocks to the aggregate amount of land are introduced in this model for one reason and one reason only: so that in equilibrium the asset price is not fully-revealing of the underlying aggregate state. In other words, if I were to eliminate the shocks to the aggregate land endowment so that the only fundamental shock were that in aggregate productivity, the asset price would perfectly reveal aggregate productivity. In this case all agents would have complete information of the aggregate state of the economy. To guard against this, I introduce another disturbance in the asset price, so that as in any Noisy Rational Expectations equilibrium the mapping from the aggregate state to the asset price is not invertible. I do this by introducing shocks to the aggregate land endowment. This however could have been accomplished in a variety of ways; for example, one could incorporate an aggregate preference shock. In any case, one may think of this as a substitute for the "noise traders" that are often used in the noisy rational expectations equilibrium models in finance. However, instead of bringing in agents from outside the model, shocks to the aggregate land endowment could be considered a more micro-founded modelling device that ensures a non fully-revealing asset price.

Finally, while the benchmark of this model is a one-sector economy, the model could be further enriched to allow for a multi-sector economy in which firms from different islands specialize in the production of differentiated goods. This could be introduced here by allowing C_k to be a

[11] See Chari, Kehoe, McGrattan (2007) for an example of an input-financing friction due to differential interest rates.

CES composite of the consumption of differentiated commodities. Specifically, one could write $C_k = \left[\int_i c_{ik}^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}$ where c_{ik} denotes household k's consumption of the differentiated good produced by the firm on island i, and where $\rho > 0$ is the elasticity of substitution across these commodities. For finite values of ρ, the model nests, as a special case in which there are no collateral constraints and no trading in the land market, the environment considered in Angeletos-La'O (2008, 2009a). Angeletos-La'O (2008, 2009a) show that the specialization of goods and the strength of trade linkages across islands leads to strategic complementarity in production decisions. Here, for the bulk of this paper I omit this effect by assuming perfect substitution among goods, i.e. $\rho \to \infty$. When presenting numerical results for the full model in section 7, as an extension I return to allowing for imperfect substitution among goods and show how this additional effect interacts with the financial friction in general equilibrium.

3 Equilibrium definition and characterization

In this section I define and characterize the equilibrium of the economy.

3.1 Definition

Because of the symmetry of preferences across households and information within each island, each island admits a representative worker; that is, it is without any loss of generality to impose symmetry in the choices of workers within each island. Furthermore, because each family sends workers to every island and receives profits from every firm in the economy, each family's income is fully diversified during stage 2 and the economy admits a representative consumer in this stage.

I then use the following notation. First, let ω denote the "type" of an island in stage 1. This variable identifies the vector (a_i, l_i, z) and hence encodes all *exogenous* information available to an island about the local shocks as well as about the aggregate state. I thus let $A(\omega)$ and $L(\omega)$ denote the local productivity and land endowment in island ω, with $A(\omega) = e^{a_i}$, $L(\omega) = e^{l_i}$ for $\omega = (a_i, l_i, z)$, respectively. Next, let Ω denote the "aggregate state" of the economy. This variable denotes the realized distribution of ω in the cross-section of islands. Because Ω is a joint normal distribution with mean productivity $\bar{a}$, mean land endowment $\bar{l}$, public signal z, and a commonly known variance-covariance matrix, one may think of the vector $(\bar{a}, \bar{l}, z)$ as a sufficient statistic for the aggregate state Ω. The key informational friction in this model is that agents face uncertainty about the underlying aggregate state Ω. Finally, let $\mathcal{S}_\omega$ denote the set of possible types for each island, $\mathcal{S}_\Omega$ the set of probability distributions over $\mathcal{S}_\omega$, and $\mathcal{P}(\cdot)$ the prior probability measure for Ω over $\mathcal{S}_\Omega$.

I then formalize the information structure as follows. In stage 1, Nature draws a distribution $\Omega \in \mathcal{S}_\Omega$ using the measure $\mathcal{P}(\Omega)$. Nature then uses Ω to make independent draws of $\omega \in \mathcal{S}_\omega$, one for each island. The information that becomes available to an island in stage 1 is both ω and Q, where the asset price Q is an *endogenous* variable that in equilibrium depends on the aggregate state Ω. The island's local type ω and the asset price Q informs firms and workers on that island perfectly about their local shocks, but only imperfectly about the underlying aggregate state Ω.[12] As a result, for any given island, the labor supply and the housing demand of the local workers, the labor demand and the level of production of the local firms, and the wage that clears the local labor market, are functions of the island's ω and Q, but (conditional on Q) not on the aggregate state Ω. In stage 2, Ω becomes commonly known. Thus, the price that clears the commodity market in stage 2, and all aggregate outcomes, do depend on the current aggregate state Ω. I thus define an equilibrium as follows.

Definition 1. *An equilibrium consists of an employment strategy $n : \mathcal{S}_\omega \times \mathbb{R}_+ \to \mathbb{R}_+$ a production strategy $y : \mathcal{S}_\omega \times \mathbb{R}_+ \to \mathbb{R}_+$, a housing demand strategy $h : \mathcal{S}_\omega \times \mathbb{R}_+ \to \mathbb{R}_+$, a wage function $w : \mathcal{S}_\omega \times \mathbb{R}_+ \to \mathbb{R}_+$, an aggregate output function $Y : \mathcal{S}_\Omega \to \mathbb{R}_+$, an aggregate employment function $N : \mathcal{S}_\Omega \to \mathbb{R}_+$, a commodity price function $P : \mathcal{S}_\Omega \to \mathbb{R}_+$, a land price function $Q : \mathcal{S}_\Omega \to \mathbb{R}_+$, and a consumption strategy $C : \mathbb{R}_+ \to \mathbb{R}_+$, such that the following are true:*

(i) The price that clears the commodity market in stage 2 is normalized so that $P(\Omega) = 1$ for all Ω.

(ii) The quantity $C(Y)$ is the representative consumer's optimal demand for the consumption good when the aggregate output (income) is Y.

(iii) The price that clears the land market in stage 1 is $Q(\Omega)$.

(iv) When the current asset price is Q, the employment and output levels, $n(\omega, Q)$ and $y(\omega, Q)$, of the representative firm from island ω maximize the expected shareholder's value of this firm subject to its production function $y(\omega, Q) = A(\omega)n(\omega, Q)^\theta$ and collateral constraint $w(\omega, Q)n(\omega, Q) \leq \chi QL(\omega)$, taking into account that firms in other islands are behaving according to the same strategies, that the local wage is given by $w(\omega, Q)$, that the price of the commodity in stage 2 is given by $P(\Omega) = 1$, that the representative consumer will behave according to consumption strategy C, and that aggregate income will be given by $Y(\Omega)$.

(v) When the current asset price is Q, the quantity $h(\omega, Q)$ is the optimal housing demand of the typical worker in an island of type ω, and the local wage $w(\omega, Q)$ is such that the quantity $n(\omega, Q)$ is also this worker's optimal labor supply.

(vi) When the aggregate state is Ω, the aggregate output and employment indices are, respectively,

$$Y(\Omega) = \int y(\omega, Q(\Omega))d\Omega(\omega) \quad \text{and} \quad N(\Omega) = \int n(\omega, Q(\Omega))d\Omega(\omega).$$

This first condition simply states that, without any loss of generality, the numeraire for this economy is the consumption good. The rest of the conditions then represent a hybrid of the following: (i) for given production choices made in stage 1, a Walrasian equilibrium for the complete-information exchange economy that obtains in stage 2, (ii) for a given realization of the asset price,

[12] Whether they agents uncertainty about their own local shocks is immaterial for the type of effects we analyze in this paper. Merely for convenience, then, I assume that the agents of an island learn their own local shocks in stage 1.

a Bayesian-Nash equilibrium for the incomplete-information game played among different islands in stage 1, and (iii) a Noisy Rational-Expectations equilibrium in the asset market in stage 1.

Allow me to expand on this. First, once production choices are sunk in stage 1, there is simply a competive exchange economy in stage 2 so that the product market clears as in any standard Walrasian equilibrium.

Second, in stage 1 for any given realization of the asset price, there is a competive equilibrium in the labor market on each island. However, note the following. Although firms within an island, in deciding how much labor to demand, face no uncertainty in terms of the price of their good, the asset price, nor the local wage, workers on the other hand, in deciding how much labor to supply and how much land to demand, do face uncertainty about the real income of their household in stage 2. In other words, in stage 1 workers face uncertainty about the marginal value of the wealth. But this then implies that the equilibrium local wage and the equilibrium asset price, and hence the amount of goods produced on each island, depends on the anticipated realized level of real aggregate income in stage 2. Aggregate income, however, is given by the level of production in *all* islands in the economy. Therefore, through general equilibrium price effects, the equilibrium employment and production decisions in the economy can be reduced to a certain incomplete-information game played among different islands, where the incentives of firms and workers on each island depend on their expectations of the choices of firms and workers in all other islands. In Section 5 I formalize this argument, and show that the equilibrium of this economy is isomorphic to the Bayesian-Nash equilibrium for the incomplete-information game just described.

Third, consider the asset market in stage 1. A worker on an island, in deciding how much land to demand, anticipates that the asset price affects his household's budget constraint and hence, as in any standard household optimization problem, the asset price affects the worker's demand for the asset. Given these demand functions, the equilibrium asset price clears the land market. But this implies that the asset price is a function mapping the distribution of agents' private information, and thereby the aggregate state Ω, to an asset price. Thus, by observing the price, a firm or worker on any island may use the price to extract information on the aggregate state of the economy. Thus, the Noisy Rational-Expectations equilibrium in the asset market imposes a fixed-point relation between the equilibrium allocations and the equilibrium information structure, and hence captures the notion that agents learn from prices. The reason it is considered "noisy" is due to the fact that agents are making inference of two aggregate shocks, the aggregate productivity shock and the aggregate land shock, from one price. Thus the price as a function of the underlying state is not invertible and therefore cannot be fully-revealing.

3.2 Characterization

In characterizing the equilibrium, I proceed as follows. First, I solve the team-problem of the representative household and characterize the optimal consumption, labor supply, and housing demand schedules for its members. I then solve for the firm's optimal level of production. Local labor-market clearing conditions allows me to solve for the partial equilibrium within each island. Finally, using economy-wide goods and land market clearing conditions, I characterize the general equilibrium for the economy.

Household optimality. Household members make choices both in stage 1 and stage 2.

Stage 2. In stage 2 the only choice the representative household faces is its consumption of goods. As goods are perfect substitutes, there is only one good the household buys at this stage. Thus this choice is quite simple: the household merely spends all of its income (net of wages, stock dividends, and payments for land) on consumption, and the price that clears this market is given by $P(\Omega) = 1$. In equilibrium, then, $C(\Omega) = Y(\Omega)$. It follows that the equilibrium consumption strategy is given by $C(Y) = Y$. Furthermore, the marginal value of wealth of the household in this stage is given by

$$\lambda(\Omega) = (1-\gamma)\left(\frac{H(\Omega)}{C(\Omega)}\right)^{\gamma} \tag{3}$$

and is thereby decreasing in aggregate consumption.

Stage 1. I now turn to the optimal choices of the household in stage 1. Again, one can think of this as the following: the household sends off its workers to different islands with instructions on how to supply labor and buy assets as a function of the wages, asset price, and information available to each member on his respective island. The optimal labor supply of the typical worker on island ω is then given by

$$w(\omega, Q) = \frac{1}{\mathbb{E}\left[\lambda(\Omega)\middle|\, \omega, Q\right]}$$

which merely equates the wage with the marginal rate of substitution between consumption and leisure. Combining this equation with the marginal utility of wealth in (3) implies that the wage is increasing in the expected consumption of the household.

Secondly, the optimal housing demand of the typical worker on island ω is given by

$$\mathbb{E}\left[\left(\frac{\gamma}{1-\gamma}\right)\frac{C(\Omega)}{H(\Omega)}\frac{dH(\Omega)}{dh(\omega,Q)}\middle|\, \omega, Q\right] = Q(\Omega) \tag{4}$$

where

$$\frac{dH(\Omega)}{dh(\omega,Q)} = H(\Omega)^{\frac{1}{\nu}}\, h(\omega,Q)^{-\frac{1}{\nu}}$$

Condition (4) simply states that the price of land is equal to the expected marginal rate of substitution between land and consumption. Solving this for $h(\omega, Q)$ gives the following expression for the optimal demand for land of the typical worker on island ω when the asset price is Q, in terms of local expectations of aggregate output and land.

$$h(\omega,Q) = \left[\frac{1}{Q(\Omega)}\mathbb{E}\left[\left(\frac{\gamma}{1-\gamma}\right)\frac{C(\Omega)}{H(\Omega)}H(\Omega)^{\frac{1}{\nu}}\middle|\, \omega, Q\right]\right]^{\nu} \tag{5}$$

Firm Optimality. I now consider the firm's problem. In stage 1 the representative firm on island ω chooses production and employment to maximize the following objective,

$$\max \mathbb{E}\left[\lambda\left(\Omega\right)\left(y(\omega,Q)-w(\omega,Q)n(\omega,Q)\right)\middle|\,\omega,Q\right], \tag{6}$$

subject to the production function $y(\omega,Q)=A(\omega)n(\omega,Q)^{\theta}$, and the collateral constraint given by

$$w\left(\omega,Q\right)n\left(\omega,Q\right)\leq\chi QL\left(\omega\right) \tag{7}$$

If credit constraints are not binding, the production choice of the typical firm on island ω is pinned down by the standard first-order condition

$$w\left(\omega,Q\right)=\theta\frac{y\left(\omega,Q\right)}{n\left(\omega,Q\right)} \tag{8}$$

which simply states that the unconstrained firm produces where the marginal cost of labor equals its marginal product. The representative firm on island ω would always like to produce the amount dictated in (8) whenever possible, as it is the firm's profit maximizing quantity. If the collateral constraint is binding however, the firm's output is pinned down by the constraint (7), satisfied at equality.

Partial Equilibrium. In equilibrium, the local labor market within each island must clear. By combining the optimal employment choices of the firms and workers, along with market clearing, I obtain the following result.

Lemma 1. *For any asset price Q, the equilibrium production of the typical firm on island ω is given by*

$$y\left(\omega,Q\right)=\min\left\{y^{u}\left(\omega,Q\right),y^{c}\left(\omega,Q\right)\right\} \tag{9}$$

where the functions $y^{u}:\mathcal{S}_{\omega}\times\mathbb{R}_{+}\rightarrow\mathbb{R}_{+}$ and $y^{c}:\mathcal{S}_{\omega}\times\mathbb{R}_{+}\rightarrow\mathbb{R}_{+}$ are defined as follows

$$y^{u}\left(\omega,Q\right)\equiv A\left(\omega\right)^{\frac{1}{1-\theta}}\left[\frac{\theta}{w\left(\omega,Q\right)}\right]^{\frac{\theta}{1-\theta}} \tag{10}$$

$$y^{c}\left(\omega,Q\right)\equiv A\left(\omega\right)\left[\chi Q\frac{L\left(\omega\right)}{w\left(\omega,Q\right)}\right]^{\theta} \tag{11}$$

where

$$w\left(\omega,Q\right)=\mathbb{E}\left[\left(1-\gamma\right)\left(\frac{L\left(\Omega\right)}{Y\left(\Omega\right)}\right)^{\gamma}\middle|\,\omega,Q\right]^{-1} \tag{12}$$

The functions y^{u} and y^{c} defined in (10) and (11), are equivalent to conditions (8) and (7), respectively, in which I have used the production function to solve for local equilibrium output. The function $y^{c}\left(\omega,Q\right)$ denotes the amount of output produced by the firm on island ω such that its collateral constraint is binding. The representative firm on island ω thus produces unconstrained output $y^{u}\left(\omega,Q\right)$ whenever possible unless this choice violates the collateral constraint, i.e.

$y^u(\omega, Q) > y^c(\omega, Q)$, in which case the firm has no choice but to produce $y^c(\omega, Q)$. Therefore, the production of the typical firm on island ω is given by $y(\omega, Q) = \min\{y^u(\omega, Q), y^c(\omega, Q)\}$, when the asset price is Q.

Lemma 1 characterizes the equilibrium level of production on each island. One may think of this as the partial equilibrium within an island in the following sense. Given Q and local expectations of aggregate output and aggregate land endowments, the demand for labor by the firm is given by (9) while the supply of labor satisfies the wage equation given in (12). Combined, these conditions fully describe the equilibrium levels of local employment and local output on an island in terms of the realized asset price and local expectations. These last objects–the asset price and local expectations of aggregate output–are, however, determined endogenously and depend on the general equilibrium properties of the economy.

One important lesson to take from Lemma 1 is that the asset price Q directly affects the production of the constrained firm. Because the value of collateral determines how much labor a constrained firm may hire, this firm's production is increasing in Q. This is precisely the feedback from the asset price to aggregate output. While obviously not a result in and of itself, this feature is important, and is highlighted in the following remark.

Remark 1. *The production of a constrained firm is an increasing function of the asset price. That is, there is a feedback from asset prices to real output.*

In contrast, for the unconstrained firms the asset price does not affect firm output, and hence this channel is absent.

Market Clearing and General Equilibrium. Finally, market clearing conditions then pin down the general equilibrium of this economy. First aggregate output is simply given by the total sum of output produced by all firms, $Y(\Omega) = \int y(\omega, Q) d\Omega(\omega)$.

I now consider market clearing in the land market. Land market clearing imposes that the sum of the individual housing demand equations of the workers, given in (5), aggregated over all islands, must be equal to the supply of land, i.e. $L(\Omega) = \int h(\omega) d\Omega(\omega)$. This market-clearing condition determines the equilibrium asset price, $Q(\Omega)$. The equations that determine equilibrium asset price are presented in the appendix. Here, for expositional purposes, I take a log-linear approximation (which will be justified later) and present the equilibrium price in the following terms.

Lemma 2. *In equilibrium, the asset price is given by*

$$\log Q(\Omega) = \left(\overline{\mathbb{E}} \log Y(\Omega) - \overline{\mathbb{E}} \log L(\Omega)\right) - \frac{1}{\nu}\left(\log L(\Omega) - \overline{\mathbb{E}} \log L(\Omega)\right) + const \tag{13}$$

where the operator $\overline{\mathbb{E}}$ denotes the average expectation in the population.

This condition has a simple interpretation. The first term is the average expectation of the marginal rate of substitution between land and consumption. Note that under complete information the land price is given by $Q(\Omega) = \frac{\gamma}{1-\gamma}\frac{Y(\Omega)}{L(\Omega)}$, so that the asset price is exactly equal to the marginal rate of substitution between land and consumption. The second term then is merely the average forecast error of the land endowment. This is due to the fact that under dispersed information, the average expectation will not be equal to the actual amount of land.

The main property I highlight here is the fact that the asset price is increasing in the average expectation of aggregate output. In fact, the elasticity of the asset price with respect to the average expectation of aggregate output is 1. This thereby represents the feedback from expectations of aggregate output to the asset price. This point is underscored in the following remark.

Remark 2. *The asset price is increasing in the average expectation of aggregate output. That is, there is a feedback from real output to asset prices.*

Remarks 1 and 2 form the basis in this model for what one would call the two-way feedback between the financial and real sides of the economy generated by collateral constraints. One one hand the asset price is sensitive to expectations of aggregate output, while at the same time output is sensitive to the asset price when firms are constrained by collateral.

In summary, the general equilibrium of the economy, as defined in Definition 1 is fully characterized by the following lemma.

Lemma 3. *The equilibrium levels of local and aggregate output and the equilibrium asset price are given by the joint solution to the following fixed point problem*

(i) The local production strategy $y(\omega, Q)$ *satisfies all conditions in Lemma 1,* $\forall (\omega, Q)$.

(ii) Aggregate output is given by $Y(\Omega) = \int y(\omega, Q(\Omega)) d\Omega(\omega)$, $\forall \Omega$.

(ii) The asset price satisfies condition (13) in Lemma 2, $\forall \Omega$.

Condition (i) merely restates that the local production strategy must satisfy the optimality conditions of the firm and worker in the partial equilibrium within any island. Condition (ii) states that the product market must clear. Finally, condition (iii) is given by equilibrium in the asset market.

It is important to note that Q actually plays three roles in the equilibrium of this economy. First, as workers' demand functions for the asset are not completely inelastic, in equilibrium the asset price is such that it clears the land market. This is the standard function of a price in any Walrasian setting. The second role it plays is the value of collateral, and thereby directly affects the production of constrained firms. This is not a standard role of the asset price, as this effect would be absent in any frictionless business cycle setting. However, this effect would be present in any model which adds the specific friction of collateral constraints. Note that these first two roles just described form the basis for the main mechanism underlying any model with collateral constraints: it is specifically these two features of the asset price which create the two-way feedback between the financial and real sides of the economy. In the current model, too, these functions of the asset price are key to the results. Finally, the third role the asset price plays in this equilibrium is that it affects expectations by partially aggregating information in a dispersed information environment. This role would not be present in any standard complete-information environment, with or without financial frictions, but it is present in any model in which one introduces dispersed information with an observable price, i.e. a Noisy REE setting.

4 Complete Information Benchmark

In this section I solve for the general equilibrium of the economy under the assumption that information is commonly shared across islands. I therefore consider the following special case of the baseline environment

Case 1. *Firms and workers on every island have complete information about the aggregate state Ω in stage 1.*

When all information is commonly shared, aggregate output is also commonly known in equilibrium and hence all firms and workers have complete information about the aggregate shocks hitting the economy. I show that in this environment by design collateral constraints have only a level effect on the equilibrium value of aggregate output, but have no effect on the equilibrium sensitivity of output to underlying shocks.

Allow me to first state the result, and then show how it is true. The equilibrium level of aggregate output and the asset price may be characterized by the following lemma.

Lemma 4. *Suppose that information were complete as in Case 1. Then the equilibrium levels of aggregate output and the asset price are given by*

$$\begin{bmatrix} \log Y(\Omega) \\ \log Q(\Omega) \end{bmatrix} = m(\chi) + \begin{bmatrix} \phi_a \\ \phi_a \end{bmatrix} \bar{a}(\Omega) + \begin{bmatrix} \phi_{y,l} \\ \phi_{q,l} \end{bmatrix} \bar{l}(\Omega)$$

where

$$\phi_a = \frac{1}{1-\theta(1-\gamma)}, \qquad \phi_{y,l} = \frac{\theta\gamma}{1-\theta(1-\gamma)}, \qquad \phi_{q,l} = -\frac{1-\theta}{1-\theta(1-\gamma)}$$

and $m(\chi) \equiv \begin{bmatrix} m_y(\chi) & m_q(\chi) \end{bmatrix}'$ *is a* 2×1 *vector-valued function of* χ.

Consider the optimal production of each firm. Substituting the equilibrium behavior of aggregate output and the asset price into the conditions stated in Lemma 1, and using the fact that $a(\omega) = \bar{a}(\Omega) + \xi_a(\omega)$ and $l(\omega) = \bar{l}(\Omega) + \xi_l(\omega)$, I rewrite the equilibrium production on island ω as follows.

Lemma 5. *Suppose that information were complete as in Case 1. In equilibrium, the representative firm on island ω produces output*

$$\log y(\omega) = \min\{\log y^u(\omega), \log y^c(\omega)\}$$

where

$$\log y^u(\omega) = \phi_a \bar{a}(\Omega) + \phi_{y,l}\bar{l}(\Omega) + \frac{1}{1-\theta}\xi_a(\omega) - \frac{\theta}{1-\theta}\gamma m_y(\chi) + const^u \tag{14}$$

$$\log y^c(\omega) = \phi_a \bar{a}(\Omega) + \phi_{y,l}\bar{l}(\Omega) + \log\chi + \xi_a(\omega) + \theta\xi_l(\omega) + \theta(1-\gamma) m_y(\chi) + const^c \tag{15}$$

and $const^u$ and $const^c$ are constants composed entirely of the parameters θ and γ.

If there were no collateral constraints, the representative firm on island ω would always find it optimal to produce the amount dictated in (14), as it is its profit maximizing quantity. If the constraint is binding however, i.e. the amount the firm would produce if unconstrained is greater than that given by its constraint, then the firm's output is dictated by (15). The representative firm on island ω is thus constrained if and only if $\log y^u(\omega) > \log y^c(\omega)$. This implies that I may characterize the set of constrained firms in the following way.

Lemma 6. *Suppose that information were complete as in Case 1. In equilibrium, the representative firm on island ω is constrained if and only if ω is an element of the set $S_\chi \subset \mathcal{S}_\omega$, where the set S_χ is defined as*

$$S_\chi \equiv \{\omega : \xi_a(\omega) - (1-\theta)\,\xi_l(\omega) > \mu(\chi)\}$$

with

$$\mu(\chi) = \left(\frac{1-\theta}{\theta}\right)\log\chi + (1-\theta(1-\gamma))\,m(\chi) + const$$

Therefore, the constrained firms are those that have the greatest idiosyncratic shocks to their productivity and those that have the lowest idiosyncratic shocks to their land endowments. Furthermore, whether a firm is constrained or not is *independent* of the aggregate realizations of $\bar{a}(\Omega)$ and $\bar{l}(\Omega)$. This is due to the fact that in equilibrium, both constrained and unconstrained firms have the same sensitivity to aggregate shocks. Hence, an aggregate shock to productivity or land shifts all firms up or down in parallel, but does not affect the distribution of constrained and unconstrained firms relative to each other.

This property almost immediately implies that the behavior of aggregate output takes the one conjectured in Lemma 4. Let $\tilde{S}_\chi$ be the complement of S_χ, i.e. $S_\chi \cap \tilde{S}_\chi = \emptyset$ and $S_\chi \cup \tilde{S}_\chi = \mathcal{S}_\omega$; thus $\tilde{S}_\chi$ denotes the set of unconstrained firms. One can then express total output as the following

$$\log Y(\Omega) = \int_{\omega\in S_\chi} \log y^c(\omega)\,d\Omega(\omega) + \int_{\omega\in\tilde{S}_\chi} \log y^u(\omega)\,d\Omega(\omega) + disp(\chi, \sigma_{xa}, \sigma_{xl})$$

where the term *disp* captures the dispersion in production across islands, and hence is a function χ as well as the variance in idioysncratic productivity and land shocks, as well as χ. In general, the function *disp* could be either increasing or decreasing on χ.[13] From this expression it is immediate that the sensitivity of aggregate output to an innovation in aggregate productivity is ϕ_a. Thus, aggregate output indeed satisfies the equilibrium behavior posited in Lemma 4, where the functions $m_y(\chi)$ and $\mu(\chi)$ solve a fixed point problem given in the appendix.

Finally, note that the equilibrium asset price under complete information satisfies condition (13), that is,

$$\log Q(\Omega) = \log Y(\Omega) - \log L(\Omega) + const$$

thereby verifying the behavior of the asset price given in Lemma 4.

[13] I find sufficient conditions so that *disp* is a decreasing function of χ.

In Lemma 4, the coefficient ϕ_a summarizes the response of both aggregate output and the asset price to an aggregate productivity shock under complete information. Any variation in χ thereby moves the level of aggregate output and asset prices, but doesn't affect their overall sensitivity to aggregate fundamental shocks. Therefore, in this environment when information is commonly shared, tighter collateral constraints have no effect on the equilibrium behavior of the economy.

Proposition 1. *When information is complete, tighter collateral constraints have no effect on the business cycle properties of the equilibrium.*

This result provides an important benchmark for the business cycle properties of this economy under complete information. When information is commonly shared, both unconstrained and constrained firms exhibit the same sensitivity to aggregate movements in underlying fundamentals. Thus, changing the fraction of constrained firms in the economy has only a level effect on the equilibrium– constrained firms produce less than had they been unconstrained– however, it does not affect the business cycle properties of equilibrium aggregate output.

The natural question to ask in light of Proposition 1 is whether dispersed information significantly changes these results. Before moving to the general equilibrium of the full model with dispersed information, in the next section I provide intution for how and why introducing heterogeneity in information has a profound effect on the equilibrium properties of this economy.

5 Collateral Constraints and Dispersed Information

The aim of this section is to convey how the introduction of dispersed information creates a distinct role for financial frictions. In particular, I show how dispersed information interacts with the collateral constraints and delivers vastly different implications for the equilibrium of this economy than in the complete information benchmark. Towards this goal, I make a few important simplifying assumptions. These assumptions are made for purely pedagogical reasons, as the full model is solved numerically in the following sections. However, as I show here, these assumptions simplify the analysis so that the general equilibrium may be solved analytically and in closed-form. This serves to isolate the important mechanisms underlying the main results of the full model, illustrating the impact of dispersed information without any numerical complications.

Specifically, in this section I compare two economies: an economy in which all firms behave as if they were unconstrained and an economy in which all firms behave as if they were constrained. More precisely, I define the first of these economies as follows

Definition 2. *Let the "unconstrained economy" denote an economy in which the equilibrium levels of local and aggregate output and the equilibrium asset price are given by the joint solution to the following fixed point problem*

(i) The local production strategy is given by $y^u(\omega, Q)$ in condition (10), Lemma 1, $\forall(\omega, Q)$.

(ii) Aggregate output is given by $Y(\Omega) = \int y^u(\omega, Q(\Omega))d\Omega(\omega)$, $\forall\Omega$

(ii) The asset price satisfies condition (13) in Lemma 2, $\forall\Omega$.

Thus, the "unconstrained economy" equilibrium is essentially the same as the one described in Lemma 3, except for the following condition: in the "unconstrained economy" all firms follow a production strategy in which they produce their unconstrained optimal output. Conditions (ii) and (iii) are then identical to those in Lemma 3. Note that Definition 2 describes a special case of the full model in which χ approaches infinite. This definition thus describes the equilibrium of a basic one-sector static RBC economy with no collateral constraints, and in so doing, may also serve as an interesting benchmark when thinking about the role of dispersed information within the context of business cycles.

The "constrained economy" is defined in a similar manner.

Definition 3. *Let the "constrained economy" denote an economy in which the equilibrium levels of local and aggregate output and the equilibrium asset price are given by the joint solution to the following fixed point problem*

(i) The local production strategy is given by $y^c(\omega, Q)$ *in condition (11), Lemma 1,* $\forall\,(\omega, Q)$.

(ii) Aggregate output is given by $Y(\Omega) = \int y^c(\omega, Q\,(\Omega))d\Omega(\omega)$, $\forall \Omega$.

(ii) The asset price satisfies condition (13) in Lemma 2, $\forall \Omega$.

Again the equilibrium in the "constrained economy" is essentially the same as the one described in Lemma 3, except that all firms here behave as if they were constrained. That is, in the "constrained economy", all firms follow a production strategy in which they produce up to where their collateral constraint is binding.

Of course, neither equilibrium can be possible in the true general equilibrium of the full model. In the full model, for any finite χ, there will always be a non-zero measure of unconstrained firms which do not find it optimal to produce where their collateral constraint is binding, as well as a non-zero measure of constrained firms which cannot produce their unconstrained optimal amount.[14] Nevertheless, I consider these two economies here primarily for pedagogical reasons: comparing the two economies under dispersed information is instructive in understanding how dispersed information interacts with the collateral constraints. At the same time, assuming that all firms behave as if they were constrained or that all firms behave as if they were unconstrained greatly simplifies the analysis in such a way that the equilibrium fixed-point may be solved in closed form.

As for the information structure, in contrast to the complete information case presented in the last section, this section considers the impact of dispersed information. Instead of considering the full rich information structure outlined in the baseline model, however, for pedagogical reasons I again make another simplifying assumption. Recall that the asset price Q in the baseline environment serves three roles: it is at once the asset price, the value of collateral, as well as an endogenous source of information about the underlying aggregate state. The third role, in which the asset price

[14] Since i have modeled the idiosyncratic shocks to have a Gaussian structure, the support of these shocks is infinite. Thus for any χ there exists a non-zero measure of representative firms who will be unconstrained. This would not be true if the support of the idiosyncratic shocks were finite.

partially aggregates and conveys information, thereby contributes to an indirect effect on firm output. This indirect effect, however, is subsidiary to the main mechanisms underlying this model, and hence becomes an unnecessary complication in terms of understanding the interaction between dispersed information and collateral constraints.

For this reason, in this section I completely abstract from the indirect effect of the asset price to aggregate information, in order to isolate the important aspects of this model. Towards this goal I assume that all agents (in either economy) do not learn from the asset price. That is, although firms and workers on each island observe both their local information ω and the asset price Q, here I simply assume that agents only use their local information to update their priors about the aggregate state, but do *not* use the asset price. Put more simply, I assume no inference is made from observing Q. Moreover, as aggregate land endowment shocks were introduced in the baseline model solely to ensure that the asset price would not be fully-revealing, in this case they are unnecessary. Thus, I furthermore eliminate these shocks. This section therefore considers the following special information structure

Case 2. *Firms and workers on every island have incomplete and heterogenous information about the aggregate state Ω in stage 1:*

(i) The only information firms and workers use to update their priors about the aggregate state Ω in stage 1 is ω

(ii) Aggregate land is constant: $\sigma^2_{L,0} = 0$.

In later sections, I show that the exclusion of learning from the asset price in fact does not make much of a difference, i.e. the main results survive the partial aggregation of information through Q.

The Equilibrium under Dispersed Information. From Proposition 1, it is immediate that under complete information (Case 1) about the aggregate state, the equilibrium of the "unconstrained economy" and that of the "unconstrained economy" have identical business cycle properties. That is, the behavior of equilibrium aggregate output in either economy is identical up to a constant. In this sense, collateral constraints plays no role in affecting the equilibrium of the economy under complete information. I now compare this to the equilibrium in both economies under dispersed information.

Due to the Gaussian specification of shocks and information structure, one may conjecture a log-linear form for the equilibrium aggregate output and asset price. The behavior of these objects is characterized in the following lemma.

Lemma 7. *Suppose that information were dispersed as in Case 2.*

(i) In the unconstrained economy, the equilibrium aggregate output and asset price behave according to

$$\begin{bmatrix} \log Y(\Omega) \\ \log Q(\Omega) \end{bmatrix} = \phi_a \begin{bmatrix} 1 + d_{y,a} & -d_{y,\varepsilon} \\ 1 - d_{q,a} & d_{q,\varepsilon} \end{bmatrix} \begin{bmatrix} \bar{a}(\Omega) \\ \varepsilon(\Omega) \end{bmatrix} + const$$

where $d_{y,a}, d_{q,a}, d_{y,\varepsilon}, d_{q,\varepsilon}$ *are all positive.*

(ii) In the constrained economy, the equilibrium aggregate output and asset price behave according to

$$\begin{bmatrix} \log Y(\Omega) \\ \log Q(\Omega) \end{bmatrix} = \phi_a \begin{bmatrix} 1 - b_{y,a} & b_{y,\varepsilon} \\ 1 - b_{q,a} & b_{q,\varepsilon} \end{bmatrix} \begin{bmatrix} \bar{a}(\Omega) \\ \varepsilon(\Omega) \end{bmatrix} + const$$

where $b_{y,a}, b_{q,a}, b_{y,\varepsilon}, b_{q,\varepsilon}$ *are all positive and less than 1.*

This gives a closed-form solution of the equilibrium aggregate output and asset price in the two economies as a log-linear functions of the productivity shock $\bar{a}$ and the public error ε, which from now on I refer to as noise. The positive scalar b's and d's depend on the the precisions of the signals, as well as the parameters γ and θ.

First, note that aggregate output and asset prices in either economy respond disparately to an innovation in technology. Furthermore, dispersed information opens the door for possible effects of noise. While it is obvious that when one introduces incomplete information noise will have some effect, what is not obvious is how it has an effect. One stark distinction between these two economies is how equilibrium aggregate output reacts to noise: in the constrained economy output reacts positively to noise, while that in the unconstrained economy reacts negatively. I will elaborate on these distinctions shortly.

The differences in equilibrium behavior between the two economies implies that the introduction of dispersed information interacts with collateral constraints, creating a distinct role for the financial frictions. To understand how dispersed information interacts with the collateral constraints and implies results different from the complete info benchmark, it is instructive to understand the equilibrium of the economy at a game-theoretic level. In so doing, this translation explains how dispersed information is a key ingredient in this model. This translation is provided in the following subsection.

5.1 Strategic Complementarity and Positive Co-Movement

The equilibrium in either economy may be represented as the Bayesian-Nash equilibrium of a game, similar to those considered in more abstract game-theoretic settings. The literature on games with strategic complementarities (for example, Morris and Shin, 2002; Angeletos and Pavan, 2007; as well as others) considers games with a large number of players, each choosing an action in $\mathbb{R}_+$. In this game, as in the economic model, let ω identify the "types" of these players, and let Ω denote the aggregate state, i.e. the cross sectional distribution of ω. Furthermore, let the "action" of the type ω player be denoted by $\log y(\omega)$, and let $\log Y(\Omega) = \int \log y(\omega) d\Omega(\omega)$ denote the aggregate action; one can think of these respectively as the individual and aggregate levels of production. Finally, suppose the players in this game have payoffs which depend both on private fundamentals $a(\omega)$ as well as the aggregate level of production, so that players have the following "best-response functions"

$$\log y(\omega) = (1 - \alpha) \phi a(\omega) + \alpha \mathbb{E}[\log Y(\Omega) | \omega] \tag{16}$$

In this expression, the coefficient α identifies the slope of a player's best response to its expectation of the aggregate action—this is the standard definition of the degree of strategic complementarity (or strategic substitutability). If α is positive then economic decisions are said to be strategic complements; if α is negative then economic decisions are said to be strategic substitutes. Aggregating this condition over all players yields the following equilibrium condition.

$$\log Y(\Omega) = (1-\alpha)\,\phi\bar{a}(\Omega) + \alpha\overline{\mathbb{E}}\log Y(\Omega) \tag{17}$$

Thus, the Bayesian-Nash equilibrium of this game reduces to a simple fixed-point relation between the aggregate action and the average expectation of aggregate action. In the aggregate equilibrium condition (17), α identifies the sensitivity of aggregate production to the average expectation of output.

I now translate the equilibrium of the economies defined in Definitions 2 and 3 in terms of the game just described. Consider the optimal production of each firm in partial equilibrium, as characterized in Lemma 1. Aggregation over individual firm production implies that the equilibrium in the unconstrained and constrained economies can be characterized as follows.

Lemma 8. *Define the coefficients*

$$\alpha_u \equiv -\frac{\theta}{1-\theta}\gamma, \qquad \alpha_c \equiv \theta(1-\gamma)$$

so that $\alpha_u \in (-\infty, 0]$ *and* $\alpha_c \in (0,1)$

(i) In the unconstrained economy, the equilibrium level of aggregate output is given by the solution to the following fixed-point problem:

$$\log Y(\Omega) = (1-\alpha_u)\,\phi_a\bar{a}(\Omega) + \alpha_u\overline{\mathbb{E}}\log Y(\Omega) \tag{18}$$

(ii) In the constrained economy, the equilibrium level of aggregate output is given by the solution to the following fixed-point problem:

$$\log Y(\Omega) = (1-\alpha_c)\,\phi_a\bar{a}(\Omega) + \alpha_c\overline{\mathbb{E}}\log Y(\Omega) \tag{19}$$

This result establishes that the general equilibrium of either economy reduces to a simple fixed-point relation between aggregate output and the average expectation of aggregate output. It is then evident that the general equilibrium of either economy is isomorphic to the Bayesian-Nash equilibrium of the game described above. To see this, note that the conditions (18) and (19), are direct analogues of the aggregate best response in the abstract game given by (17). To further make the translation, think of an island in these economies as a "player" in the game, and the (partial) equilibrium level of output on that island as its corresponding "action". Finally, note that the "types" ω encodes the local shocks and local information sets of each island.

This game-theoretic representation for the equilibrium is useful in that facilitates a translation of some of the more abstract insights from the earlier literature. One of these abstract insights is the concept of strategic complementarity (or strategic substitutability). Note that the coefficients α_u and α_c identify the elasticity of output to variation in average expectations of aggregate output in the constrained and unconstrained economies, respectively. These coefficients are the analogues of the degree of strategic complementarity (or strategic substitutability) in the economic model. In the game described above strategic complementarity is merely an abstract idea or construct, yet here α_u and α_c are instead a result of the particular microfoundations of the economy. In what follows, I examine the micro underpinnings of these coefficients, and most importantly for the purposes of this paper, show how collateral constraints translates into a novel source of strategic complementarity.

Strategic Complementarity and Strategic Substitutability. The coefficients α_u and α_c identify the sensitivity of output to expectations of aggregate output in the unconstrained and constrained economies, respectively. Understanding the specific micro-foundations for these coefficients are crucial for understanding the economics underlying the main results of this model. Thus, before considering how the information structure affects equilibrium outcomes, I first consider where these coefficients originate. These coefficients are composed of the underlying parameters governing preferences and technology in this economy: γ and θ. Allow me to rewrite these coefficients as follows.

$$\alpha_u = \frac{-\gamma}{\frac{1}{\theta} - 1}, \qquad a_c = \frac{1-\gamma}{\frac{1}{\theta}}$$

From this, one can see that there are similarities between the two coefficients. First, note that the numerators in both expressions contain the term $-\gamma$. This term controls the elasticity of the wage to expectations of aggregate output, and is driven by the income effect on labor supply.[15] Higher aggregate income causes households to desire more consumption of both goods and leisure. Thus, if workers are optimistic about aggregate income, they will work less for any given wage. In equilibrium the inward shift in labor supply leads to a reduction in the equilibrium output produces on an island. This negative income effect on labor, which is standard in any neoclassical setting, is thereby a source of strategic substitutability in both the unconstrained and constrained economy.

Secondly, the denominators in either expression merely reflect the fact that there are decreasing returns to scale in production: a one-percent increase in employment hired by the firm leads to only a θ-percent increase in firm output. As the unconstrained firm's production is given by a condition involving the marginal cost of production (10), whereas the firm's collateral constrains the total cost of production (11), anything that independently shifts these two conditions will have an effect on output, but subject to a scale factor.

[15] To see this clearly, note that labor is quasilinear in the consumption basket $C^{1-\gamma}L^{\gamma}$, where the share of income spent on land is γ. One can reexpress the wage as equal to the price of this consumption basket, which is given by $(1-\gamma) + \gamma Q$, where Q increases 1-for-1 with expectations of aggregate output. Thus, for a larger share of income spent on land, the price of this consumption basket increases, and in order to get workers to work, wages must increase.

Finally, the primary difference between these two coefficients is that α_c contains a term in the numerator (specifically, the 1) that is unique to the constrained economy and originates directly from the collateral constraint. This term is given by the elasticity of the asset price to expectations of aggregate output. Optimism about aggregate output, as seen in condition 13, drives up the price of land. This is precisely the feedback from aggregate output to the asset price. Moreover, recall that the collateral constraint dictates that production is constrained by the price of land. An upward shift in asset prices thereby loosens the collateral constraint, allowing constrained firms to produce more. This is precisely the feedback from the asset price to real output. In total, optimism about aggregate output increases constrained firm production. This positive collateral constraint effect on output is not standard in a neoclassical setting, and as such is absent from the unconstrained economy's sensitivity α_u. Rather, this effect is a direct manifestation of the two-way feedback between the financial market and the real economy, and is hence a distinct *source of strategic complementarity* in the constrained economy.

In summary, note that for all parameter values, α_u is always negative whereas α_c is always positive. Therefore, in total, production in the unconstrained economy are strategic substitutes, whereas production in the constrained economy are strategic complements. As I show next, these total sensitivities are important determinants of the equilibrium when information is asymmetric across islands.

Implications of Dispersed Information. I now consider how the information structure affects the aggregate properties of the equilibrium. One of the most insightful and important results from the earlier more abstract literature on these games is that strategic complementarity is an important determinant of the equilibrium if and only if information is heterogeneous among players. To illustrate this, consider the abstract game presented above with the equilibrium described by the fixed point relation in (17). When information is commonly shared, strategic complementarity (or substitutability), in this game represented by α, plays no role. In fact, one can easily check that if $\log Y(\Omega) = \overline{\mathbb{E}} \log Y(\Omega)$, then both conditions (18) and (19) reduce to

$$\log Y(\Omega) = \phi \bar{a}(\Omega)$$

and hence confirms the result from the complete information benchmark presented in Lemma 4. Specifically, the sensitivities α_u and α_c do not affect equilibrium output in either economy under complete information.

Under dispersed information however, strategic complementarity and substitutability play a critical role in determining the response of aggregate output to underlying shocks. Under the information structure spelled out above, I obtain the following characterization of aggregate output.

Lemma 9. *The equilibrium level of aggregate output in this game is given by*

$$\log Y(\Omega) = \left(1 - \frac{\alpha \kappa_{0a}}{(1-\alpha)\kappa_{xa} + \kappa_{za} + \kappa_{0a}}\right) \phi \bar{a}(\Omega) + \frac{\alpha \kappa_{za}}{(1-\alpha)\kappa_{xa} + \kappa_{za} + \kappa_{0a}} \phi \varepsilon(\Omega) + const$$

and the average expectation of aggregate output is given by

$$\overline{\mathbb{E}}\log Y(\Omega)=\left(1-\frac{\kappa_{0a}}{(1-\alpha)\,\kappa_{xa}+\kappa_{za}+\kappa_{0a}}\right)\phi\bar{a}(\Omega)+\frac{\kappa_{za}}{(1-\alpha)\,\kappa_{xa}+\kappa_{za}+\kappa_{0a}}\phi\varepsilon(\Omega)+const$$

Thus, the fixed point relation in (17) yields a closed form solution for the equilibrium level of aggregate output and the average expectation of output as log-linear functions of the fundamental and noise. This is the direct analogue of the results presented in Lemma 7, for the behavior of the aggregate output and the asset price, since $\log Q=\overline{\mathbb{E}}\log Y(\Omega)$. These expressions then confirm the results from the dispersed information presented in Lemma 7, with appropriately specified coefficients b's and d's.

From Lemma 9 we see the following: first, expectations of aggregate behavior are sensitive to the public signal. In particular, expectations of aggregate behavior in either economy respond positively to the noise in the public signal. While this would be true even if information were common but incomplete, dispersed information implies $\log Y(\Omega)\neq\overline{\mathbb{E}}\log Y(\Omega)$. Therefore, it is immediate from the fixed-point relation in (17) that noise affects equilibrium output, and, depending on the sign of α, the response of aggregate output to noise can be either positive or negative. When economic decisions are strategic substitutes ($\alpha<0$), as in the unconstrained economy, the equilibrium level of output responds negatively to expectations of aggregate output. When instead economic decisions are strategic complements ($\alpha>0$), as in the constrained economy, the equilibrium level of output responds positively to expectations of aggregate output. However, in either case, average expectations always respond positively to noise, and hence asset prices respond positively to expectations of aggregate output. Thus, a positive noise shock, or a correlated error in beliefs, can be thought of as collective optimism among agents about aggregate economic activity. Therefore, under dispersed information, the elasticity of production to expectations of aggregate economic activity then dictate how this collective optimism or pessimism manifests itself in equilibrium output.

Another observation can be made from these conditions about how much noise matters for equilibrium aggregate behavior and how this depends on the size of α. For now, I focus on the sign of the effect of noise, and how this could induce co-movement in variables, before moving to the relative contribution of noise shocks.

Positive Co-movement. In the unconstrained economy economic decisions are strategic substitutes ($\alpha_u<0$), and therefore the equilibrium level of output responds negatively to expectations of aggregate output. While the mechanics for this result has just been described, the economics behind it also is straight-forward. In a standard one-sector neoclassical setting, greater income causes households to desire both more consumption and more leisure. Anticipation of greater wealth thus makes workers less willing to work, thereby reducing their labor supply, and eventually leading to a reduction in equilibrium output. Therefore, optimism about aggregate economic activity generates a recession.

It is relatively well known (though perhaps counter-intuitive) that in a standard one-sector neoclassical model, expectations of higher income or wealth cause a recession. This fact was first pointed out by Barro and King (1984), and later emphasized by Cochrane (1994). In fact, the difficulty of the neoclassical growth model to generate a boom in response to expectations of higher future TFP has been the main obstacle for the news-driven business cycle literature. See, for example the empirical work of Beaudry and Portier (2004, 2008), the theoretical papers of Christiano et al.(2007), Jaimovich & Rebelo (2008), and Lorenzoni (2009), and finally the recent paper by Sims (2009), which offers an empirical challenge to the notion of booms being driven by news about future TFP. Although these papers focus primarily on expectations of future TFP as the driving force in dynamic representative-agent models, whereas the model considered here focuses on correlated errors in expectations among heterogeneously informed agents, the negative wealth effect on labor supply is a common denominator in any expectations-driven business cycle model.

Furthermore, it is also worth examining the cyclical behavior of aggregate consumption and employment. Aggregate consumption is given by

$$\log C\left(\Omega\right) = \log Y\left(\Omega\right) \tag{20}$$

while aggregate employment is given by

$$\log N\left(\Omega\right) = const + \frac{1}{\theta}\left(\log Y\left(\Omega\right) - \bar{a}\left(\Omega\right)\right) \tag{21}$$

It is then immediate that the response of aggregate consumption to noise is equal to that of output, while the response of employment to noise is proportional to that of aggregate output. Thus, in the unconstrained economy optimism about aggregate output not only implies that output falls, but that consumption and employment falls as well. At the same time, optimism about economic activity causes asset prices to increase, since these are directly tied to expectations of aggregate output. Therefore, not only does good news or optimism about output tend to cause a recession, but the implied negative comovement between aggregate output and asset prices seems difficult to reconcile with the procyclicality of stock prices found in the data.

On the other hand, in the constrained economy economic decisions are strategic complements ($\alpha_c > 0$), and hence the equilibrium level of output responds positively to expectations of aggregate output. Good news about aggregate output drives up the price of land, thereby increasing the value of collateral and allowing constrained firms to produce more. Therefore, in the constrained economy optimism about aggregate economic activity generates a boom in output–in this sense, expectations are partially self-fulfilling. Moreover, from conditions (20) and (21), it is apparent that in the constrained economy consumption and employment also respond positively to noise in the public signal. Good news about aggregate output thereby generates positive co-movement in output, employment and consumption and asset prices in the constrained environment. I summarize this finding in the following proposition.

Proposition 2. *Suppose that information were dispersed as in Case 2. In the constrained economy, aggregate output, employment, consumption, and asset prices exhibit positive co-movement in response to correlated errors in expectations of aggregate activity.*

Therefore, one of the main results in this paper is that collateral constraints and dispersed information imply positive co-movement over the business cycle. Moreover, this positive co-movement is caused by variation in expectations orthogonal to underlying productivity.

5.2 The Relative Contribution of Noise

The previous subsection focused on the implications of noise in terms of observable aggregate macro variables. I now show how noise may be responsible for a larger fraction of business cycle and asset price fluctuations in the constrained economy than in the unconstrained economy. Towards this goal, I consider a certain decomposition of the variance of aggregate output and the asset price into the elements driven either by noise and or by technology. Using this decomposition, I define what I mean by the relative contribution of noise.

Definition 4. *Suppose a variable x is a linear combination of I independent random variables u_i with variances σ_i^2*

$$x = \sum_{i \in I} g_i u_i$$

where g_i are the loadings on each random variable u_i. The fraction of x explained by u_j is then given by

$$R \equiv \frac{g_j^2 \sigma_j^2}{\sum_{i \in I} g_i^2 \sigma_i^2}$$

This definition is relatively simple: the fraction of x explained by u_j is merely the variance of x due to u_j, divided by the total variance of x. Applying this definition to the log of aggregate output and the log of the asset price, I find the fraction of aggregate output volatility and the fraction of asset price volatility explained by noise. This leads to the following proposition.

Proposition 3. *Suppose that information were dispersed as in Case 2. Consider the two economies in Definitions 2 and 3, both with the same parameter values.*

(i) Noise contributes to a greater fraction of asset price volatility in the constrained economy than in the unconstrained economy.

(ii) For any set of parameters $(\theta, \kappa_{0a}, \kappa_{xa}, \kappa_{za})$, there exists a $\hat{\gamma}(\theta, \kappa_{0a}, \kappa_{xa}, \kappa_{za})$ such that for all $\gamma < \hat{\gamma}$, noise contributes to a greater fraction of output volatility in the constrained economy than in the unconstrained economy.

The first part of Proposition 3 compares the contribution of noise to asset price volatility across the two economies. From Lemma 9, note the following: $d_{q,a} < b_{q,a}$ and $d_{q,\varepsilon} < b_{q,\varepsilon}$. This immediately implies that the equilibrium asset price in the constrained economy has both a lower sensitivity to the aggregate productivity shock than in the unconstrained economy, and a greater sensitivity to noise than in the unconstrained economy. First, compared to the complete information benchmark, the asset price in both economies exhibit a dampened response to the aggregate productivity shock. However, as $d_{q,a} < b_{q,a}$, the asset price in the constrained economy has an even lower sensitivity to aggregate output than the unconstrained. Second, the asset price in both economies respond positively to noise. But, as $d_{q,\varepsilon} < b_{q,\varepsilon}$, the asset price in the constrained economy is in fact more sensitive to noise than in the unconstrained economy. Together, these conditions are sufficient to prove that the fraction of asset price volatility due to noise is always larger in the constrained economy than the unconstrained economy.

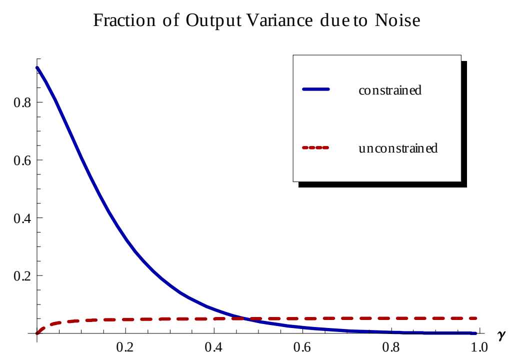
The second part of Proposition 3 compares the contribution of noise to aggregate output volatility across the two economies. While the fraction of asset price volatility due to noise is always larger in the constrained economy than the unconstrained economy, the same isn't always the case for output volatility. However, I show that for reasonable parameter values, noise contributes to a greater fraction of output variance in the constrained economy than in the unconstrained volatility.

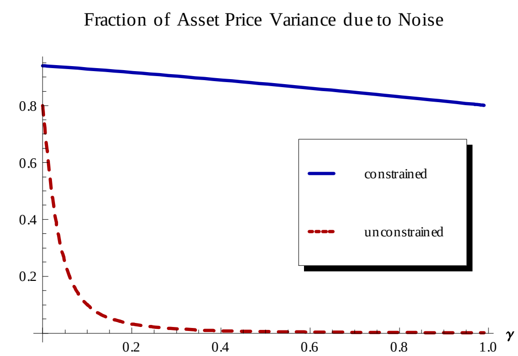
Towards this goal, I can show that the fraction of output volatility accounted for by noise in the constrained economy is decreasing in the parameter γ, whereas the fraction of output volatility accounted for by noise in the unconstrained economy is increasing γ. Thus, for γ sufficiently low the constrained economy has a greater fraction of output variance accounted for by noise than the unconstrained economy.

The mechanics for why this is true is as follows. First, from Lemma 7, note that for all parameter values, aggregate output in the constrained economy has a lower sensitivity to aggregate productivity than in the unconstrained economy. This is because stronger complementarity, i.e. $\alpha_c > \alpha_u$, raises the anchoring effect on the common prior. In other words, in the constrained economy, since economic decisions are strategic complements, production is more sensitive to public sources of information and the prior than to idiosyncratic information. On the other hand, in the unconstrained economy, since economic decisions are strategic substitutes, local output relies more on idiosyncratic information than on public sources of information and the common prior. It follows that in the economy with higher strategic complementarity, that is, the constrained economy, equilibrium output is less responsive to underlying shifts in aggregate productivity. This is one force which makes productivity contribute relatively less to aggregate output volatility in the constrained economy relative to the unconstrained economy.

What then matters is the comparison across economies of the sensitivity of aggregate output to noise. From Lemma 7 one sees that equilibrium aggregate output in the constrained economy reacts positively to noise, while equilibrium aggregate output in the unconstrained economy reacts negatively noise. Since public information helps forecast the aggregate level of output, aggregate output in the economy with strategic complementarity (the constrained economy) reacts positively to public information whereas aggregate output in the economy with strategic substitutibility (the unconstrained economy) reacts negatively. In absolute terms, either sensitivity could be higher, and the coefficients that control these sensitivities are α_c and α_u. The greater α_c and α_u are in absolute terms, the greater the sensitivity of aggregate output to noise. Recall that γ controls the income effect on labor supply, and is hence the only source of strategic substitutibility in either economy. Thus, while a lower γ implies both a greater α_c and α_u, in terms of absolute values a lower γ implies a higher $|\alpha_c|$ but a lower $|\alpha_u|$. This then explains why the fraction of output volatility accounted for by noise in the constrained economy is decreasing in the parameter γ, whereas the fraction of output volatility accounted for by noise in the unconstrained economy is increasing γ. Therefore, in order for noise to contribute to a greater fraction of output variance in the constrained economy than in the unconstrained economy, a weak income effect on labor supply will suffice.

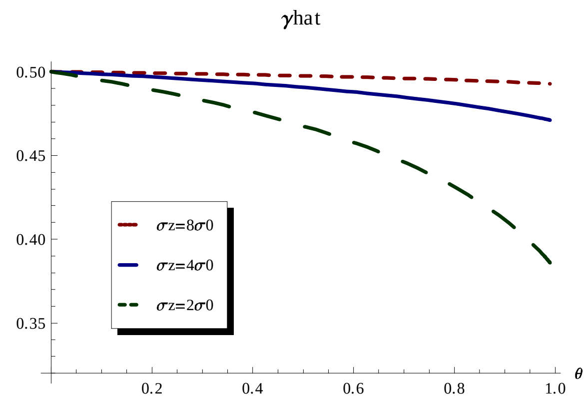
I illustrate this in the following figure. On the left-hand side I plot the fraction of output variance accounted for by noise in both the constrained and unconstrained economies. In this figure I let $\sigma_{0a} = 0.02$ for the standard deviation of the productivity innovation. I set the standard deviations of the private and public information at $\sigma_{xa} = \sigma_{za} = 4\sigma_0$. These values are arbitrary, but not implausible: when the period is interpreted as a quarter, the information about the current fundamentals and/or the current level of economic activity is likely to be very limited. Finally, I let $\theta = .99$ so that technology is near constant returns to scale– I soon show that this gives a lower bound in terms of $\hat{\gamma}$. Finally, the right hand side plots the fraction of asset volatility accounted for by noise in both the constrained and unconstrained economies for the same parameter values.





As already stated, the asset price variance due to noise is always greater in the constrained economy than in the unconstrained economy. In terms of output volatility, one sees that the fraction of output volatility accounted for by noise in the constrained economy is decreasing in the parameter γ, and that the fraction of output volatility accounted for by noise in the unconstrained economy is increasing γ. Furthermore, for $\gamma = 0$ the fraction of output volatility acounted for by noise in the unconstrained economy is zero, whereas the fraction of output volatility accounted for by noise in the constrained economy is strictly positive. Together, this suggests that there exists a cutoff $\hat{\gamma}$, here $\hat{\gamma} \approx .45$, such that for any γ lower than $\hat{\gamma}$, the relative contribution of noise to aggregate output volatility is greater in the constrained economy than in the unconstrained economy.

In the following figure, I show that for given $\sigma_{xa}, \sigma_{za}, \sigma_{0a}$, the function $\hat{\gamma}$ is decreasing in θ. Thus for any set of shocks and information structure, the case where $\theta = .99$ gives the lowest possible $\hat{\gamma}$, in other words, the lowest upper bound for γ.



Thus, if I had assumed there had been a fixed factor in production such as capital, so that θ was equal to the income share of labor, say .6, the upper bound for γ would then be higher and hence easier to satisfy. Note from this figure that for high values of the standard deviation of noise in the public signal, the function $\hat{\gamma}$ is relatively flat in terms θ.

In any case, by considering γ sufficiently low, I am effectively assuming a small income effect on labor supply. For business-cycle frequencies, it seems quite plausible that the short-run wealth effect on labor supply is weak, and many papers calibrate their models as such. For example, Jaimovich and Rebelo (2009) use preferences such that the wealth effect on labor supply can be controlled and weakened; in their preferred calibrations, γ is between 0 and .25. Woodford (2003) also uses a similar number. Thus, γ between .1 and .3 seems to be a relevant range for this parameter in terms of matching empirically plausible short-run income effects on labor supply. Moreover, in terms of this model this number implies that about 75% of household income is spent on consumption goods, while the rest is spent on housing– a decomposition of wealth that seems fairly likely. Therefore, reasonable values for γ seem to always fall within the relevant range, i.e. below the $\hat{\gamma}$ given by plausible parameterizations of θ and σ_{za}/σ_{0a} shown above.

I therefore argue that the empirically plausible case is one in which γ is sufficiently small, and hence the relative contribution of noise to both asset price and output volatility is greater in the constrained economy than in the unconstrained economy.

5.3 Movements in the Labor Wedge

Finally, I study the cyclical properties of the labor wedge. Following the literature, I define the labor wedge $1 - \tau(\Omega)$ implicitly by

$$\frac{1}{(1-\gamma)\left(\frac{L(\Omega)}{Y(\Omega)}\right)^{\gamma}} = (1-\tau(\Omega))\,\theta\frac{Y(\Omega)}{N(\Omega)} \tag{22}$$

Namely, the labor wedge is the wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor.[16] First, note that under complete information,

[16] Note that in general, the labor wedge is defined by using standard CRRA preferences for consumption. Here, however, prefences are over both consumption and land, thus if I had defined the labor wedge in the usual way, there

it is trivial to check that the labor wedge is always equal to 1 and does not vary in response to any shock. In contrast, the introduction of dispersed information changes this prediction. In this model, the equilibrium behavior of the measured labor wedge under dispersed information may be described in the following lemma.

Lemma 10. *Suppose that information were dispersed as in Case 2*

(i) In the unconstrained economy, the equilibrium labor wedge behaves according to

$$\log\left(1-\tau\left(\Omega\right)\right)=\left[\begin{array}{cc} d_{y,a} & -d_{y,\varepsilon} \end{array}\right]\left[\begin{array}{c} \bar{a}\left(\Omega\right) \\ \varepsilon\left(\Omega\right) \end{array}\right]+const$$

(ii) In the constrained economy, the equilibrium labor wedge behaves according to

$$\log\left(1-\tau\left(\Omega\right)\right)=\left[\begin{array}{cc} -b_{y,a} & b_{y,\varepsilon} \end{array}\right]\left[\begin{array}{c} \bar{a}\left(\Omega\right) \\ \varepsilon\left(\Omega\right) \end{array}\right]+const$$

The labor wedge thus exhibits very dissimilar responses to noise in the two economies. In the constrained economy, the labor wedge $1-\tau\left(\Omega\right)$ responds positively to noise, or correlated errors in expectations of aggregate output. The intution for why the labor wedge increases in response to noise is given by the following. When there is a positive noise shock, asset prices increase because agents think output will increase. As a result, firms become less constrained and increase their output, thus making the noise shock, or expectational error, somewhat self-fulfilling. In the process, though, firms hire more labor even if there has been no change in their productivity. Thus, the economy behaves as though there were a decrease in the tax on labor, distorting firms to increase employment, and thereby leading to an increase in the measured wedge between the MRS and the marginal product of labor.

In contrast, in the unconstrained economy the measured labor wedge is falls in response to noise. The intuition for this is the following. A positive noise shock causes workers to believe aggregate output will be high and thus workers would like to have more leisure and work less. This drives up wages, causing firms to reduce both employment and production. In other words, the inward shift of labor supply causes firms in equilibrium to reduce their employment even if there has been no change in their productivity. Thus, the economy behaves as though there were an increase in the labor tax, distorting firms to hire less workers, and thereby leading to a decrease in the measured labor wedge.

Proposition 4. *Suppose that information were dispersed as in Case 2. In the constrained economy, noise leads to a positive response in the measured labor wedge.*

Therefore, one of the main results of this model is that the financial friction considered here manifests itself as a time-varying labor wedge. Specifically, the interaction of collateral constraints

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would be uninteresting effects coming from the misspecification of preferences by the econometrician. Instead, I opt for specifying preferences correctly and thus isolate the results produced solely from the dispersion of information.

and dispersed information implies that the labor wedge responds positively to common errors in expectations about aggregate activity. The empirical relevance of this statement will be discussed in the following sections.

The purpose of this section was to illustrate the feedback between asset prices and output, and show how dispersed information interacts with the financial frictions. Of course I have abstracted from many important features of the full model. In general equilibrium not all firms will be constrained and furthemore agents in the economy will learn from observing the asset price. In the following sections, I solve for the general equilibrium of the full economy and demonstrate how the theoretical insights highlighted in this section are not lost in the full model.

6 Equilibrium in the Full Model

In this section I describe the numerical method I use to solve for the equilibrium in the full model. Recall that in the previous section, equilibrium aggregate output could be solved for in closed-form in either economy. By imposing that all agents are either constrained or unconstrained, aggregate output in either case was log-linear in the underlying fundamentals. Combining this with the Gaussian information structure, firms and workers faced simple Bayesian inference problem, inducing log-linear production strategies and a fixed-point in equilibrium output.

In considering the full model, I now take into account that in equilibrium there are both constrained and unconstrained firms in the economy. As seen in the partial equilibrium characterization in Lemma 1, constrained and unconstrained firms react differently to their information sets. This creates a non-linearity in the aggregation of output across firms–a complication when agents must make optimal forecasts of aggregate output. Thus, it is mainly a problem of inference of aggregate behavior that prevents the equilibrium from admitting a closed-form analytical solution. In this sense, the numerical method I use is similar in spirit to that of Krusell-Smith (1998). Krusell-Smith (1998) examines an economy in which there is substantial heterogeneity in wealth, and show how the equilibrium can be approximated numerically, despite the fact that the state of the economy at any point in time is the distribution of wealth across agents–an infinite-dimensional object. Importantly, they show how a general equilibrium model with wealth heterogeneity may feature approximate aggregation, that is, aggregate endogenous variables can be described as a function of first moments, as opposed to the entire wealth distribution, without serious error.

Similarly, in this model there is heterogeneity in information, or types ω, across islands. The exogenous aggregate state variable in the economy is the realized distribution of types, Ω, in the cross-section of islands. The aggregate variables, $Y(\Omega)$ and $Q(\Omega)$, are endogenous functions of this state variable, and in theory could be highly complicated functions of the underlying distribution. I show how these aggregate endogenous variables may be approximated as simple log-linear functions of the first moments of the distribution of types. This greatly simplifies the analysis as firms and workers in stage 1 face manageable inference problems when making their employment, production, and asset demand decisions. In this sense, I follow a similar approximate aggregation technique as in Krusell-Smith (1998) in that I show how the endogenous aggregate variables can be simple functions of the distribution first moments, yet lead to small errors in the implied behavior of actual output.

I now outline my numerical strategy for computing the equilibrium. First, suppose that all agents in the economy perceive that aggregate output evolves according to a log-linear function of the first moments of Ω. Formally, I assume that agents perceive the law of motion for aggregate output Y to be given by a function F that belongs to a class $\mathcal{F}$ of log-linear functions

Definition 5. *Let $\mathcal{F}$ be the class of log-linear functions of the first moments of the aggregate state.*

$$\mathcal{F} \equiv \left\{ F : \mathcal{S}_\Omega \to \mathbb{R}_+ \left| \log F(\Omega) = \Phi_a \bar{a} + \Phi_l \bar{l} + \Phi_z z, \text{ and } \Phi_a, \Phi_l, \Phi_z \in \mathbb{R} \right. \right\}$$

This log-linear approximation of aggregate output greatly simplifies the inference problem of the firms and workers in Stage 1. I now show that combining this perceived law of motion with the Gaussian shocks and information structure generates a log-linear asset price and log-linear output strategies.

Information Structure and Asset Price First, for any F, there is a fixed-point relation between the equilibrium asset price and the equilibrium information structure. The asset price in this fixed point takes a log-linear form given in the following lemma.

Lemma 11. *For any F, the land price which clears the market is a log-linear function of the first moments of the aggregate state*

$$\log Q(\Omega; F) = \Psi_a \bar{a}(\Omega) + \Psi_l \bar{l}(\Omega) + \Psi_z z(\Omega)$$

where Ψ_a, Ψ_l, Ψ_z are functions of F.

For any asset price, there is a unique information structure generated by this price. This is because firms and workers in stage 1 observe the land price Q, and hence use the price to make inference about the underlying state. The log-linear nature of the asset price is important for tractability, as it preserves the Gaussian structure of the information. Since z is common knowledge, the asset price can be transformed into a simple Gaussian signal about the underlying aggregate productivity and land endowment:

$$\hat{q} \equiv \log Q(\Omega; F) - \Psi_{zz} z(\Omega) = \Psi_a \bar{a}(\Omega) + \Psi_l \bar{l}(\Omega)$$

I then solve the individual island's inference problem as follows. Let X_1 be the vector of aggregate fundamentals and X_2 the vector of signals observed by each island

$$X_1(\Omega) = \begin{bmatrix} \bar{a}(\Omega) \\ \bar{l}(\Omega) \end{bmatrix}, \qquad X_2(\omega) = \begin{bmatrix} a(\omega) \\ l(\omega) \\ z \\ \hat{q} \end{bmatrix}$$

and finally let $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$ be the unconditional variance-covariance matrix for these vectors. Note that while X_2 is different for each island, the matrix Σ will be the same for every island, since all signals are assumed to have the same stochastic properties. Using their signals, firms and workers update their priors and form their expectations of aggregate productivity and land holdings. Their conditional expectations are given by the following.

Lemma 12. *For any F, the expectations of firms and workers on island ω, conditional on their signals, are given by*

$$\mathbb{E}\left[X_1|\omega\right] = \Sigma_{12}\Sigma_{22}^{-1}X_2\left(\omega\right) \tag{23}$$

where Σ and $\hat{q}$ are functions of F

Thus the expectations of the aggregate state are linear functions of the signals, and are generated by the Bayesian inference made from the observed signals and the asset price. Note that this implies that the average expectation of the aggregate state is then given by

$$\mathbb{E}\left[X_1\right] = \Sigma_{12}\Sigma_{22}^{-1}\bar{X}_2$$

where $\bar{X}_2 = \begin{bmatrix} \bar{a} & \bar{l} & z & \hat{q} \end{bmatrix}'$ is simply the mean of X_2. Therefore, the average expectation is furthermore linear in the first moments of Ω.

Furthermore, for any given information structure, there exists a unique asset price. Recall that the land price which clears the market is given by condition (13). Combining this with the linear average expectations that result from the agents' inference problem, the asset price that prevails in equilibrium can in fact be expressed in the log-linear form, as stated in Lemma 11.

Thus, for any F, there is a fixed-point between the equilibrium information structure and the equilibrium asset price, as standard in rational-expectations equilibria.[17]

Individual Firm Optimality Second, for any F, I may characterize the equilibrium output produced on each island. Lemma 1 describes the partial equilibrium output produced on an island, in terms of the equilibrium asset price and expectations of the aggregate state. Moreover, Lemma 12 chactererizes the expectations of the aggregate state of firms and workers on an island. Combining these results, one may find the equilibrium output produced on any island, as a function of its information set. Thus, given the law of motion F, each island's production can be represented by a decision rule $g\left(\omega, Q; F\right)$

Lemma 13. *The optimal production of typical firm on island ω is given by*

$$\log g\left(\omega, Q; F\right) = \min\left\{\log g^u\left(\omega, Q\right),\ \ \log g^c\left(\omega, Q\right)\right\}$$

[17] Note that for any given information structure, there exists a unique asset price function, and for any asset price function there exists a unique information structure generated by this price. Nevertheless, multiplicity can originate in the fixed-point relation between the two, as often is the case with rational-expectations equilibria. While this possibility is intriguing, but I will ignore because it is orthogonal to the goals of this paper.

where the functions g^u and g^c are given by

$$\begin{aligned}\log g^u(\omega, Q) &= \phi^u_a a_i + \phi^u_l l_i + \phi^u_z z + \phi^u_q \hat{q} \\ \log g^c(\omega, Q) &= \phi^c_a a_i + \phi^c_l l_i + \phi^c_z z + \phi^c_q \hat{q}\end{aligned}$$

where the coefficients $\left\{\phi^u_a, \phi^u_l, \phi^u_q, \phi^u_z; \phi^c_a, \phi^c_l, \phi^c_q, \phi^c_z\right\}$ are functions of F.

Here, the functions g^u and g^c merely represent the production of unconstrained and constrained firms, given in (10) and (11) of Lemma 1, in which I have used the agents' perceived law of motion for aggregate output, F, and their conditional expectations of the aggregate state (given by Lemma 12). Note that conditional on being either unconstrained or constrained, output is log-linear in the islands' signals.

Aggregation Finally, given such a production strategy g for individual islands and the distribution of types ω, it is then possible to derive the implied aggregate output by aggregating the output produced in the cross-section of islands. Aggregation over island production yields and aggregate output function which I call G,

$$G(\Omega; F) \equiv \int_\omega g(\omega, Q; F)\, d\Omega(\omega)$$

Thus, given the log-linear function F agents perceive for aggregate output, the optimal decisions of firms and workers induces an aggregate output function G which is not log-linear in the underlying state. Note that the only reason that the implied aggregate output is not log-linear in $\bar{a}, \bar{l}, z$ is due to aggregation over both constrained an unconstrained firms.

I may then compare the implied aggregate output function G to the perceived aggregate output function F on which agents base their behavior, by looking at the mean-squared error between the two. The mean squared error is calculated over all realizations of Ω, weighted by its pdf. I then minimize this mean-squared error over the the class of functions $\mathcal{F}$.

$$\min_{F \in \mathcal{F}} \int_\Omega (F(\Omega) - G(\Omega; F))^2\, d\mathcal{P}(\Omega) \tag{24}$$

By this I mean that numerically I find coefficients Φ_a, Φ_l, Φ_z which minimize the mean squared error between what agents perceive and what is then induced in equilibrium. Therefore the approximate equilibrium is a function F such that, when taken as given by the agents, yields the best fit within the class $\mathcal{F}$ to the behavior of Y.

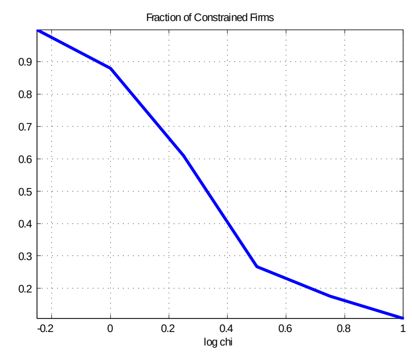
Note that had all firms been unconstrained, or all firms constrained, then the law of motion in equilibrium would in fact be log-linear. Thus, the approximation error approaches zero at either extreme: when almost all agents are constrained or when almost all agents are unconstrained. This is apparent from the previous section, in which I showed that the equilibrium could be solved analytically in either "unconstrained economy" and the "constrained economy", which implies that there is zero approximation error for the law of motion of aggregate output. I later show that it is only for the intermediate ranges where there is a non-zero, but small approximation error.

The results from this numerical approximate solution are presented in the following section.

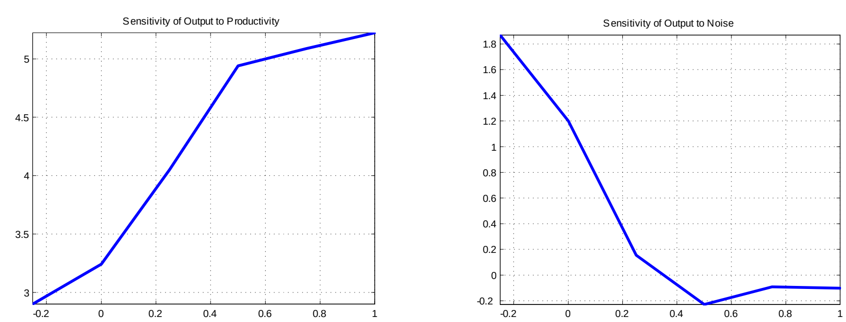
7 Numerical Results

In this section I present the numerical results to the full model. For these simulations, I use values similar to those used in Section 5. In particular, I let $\sigma_{0a} = 0.02$ for the standard deviation of the productivity innovation and set the standard deviation of land shocks to the same value $\sigma_{0l} = \sigma_{0a}$. Furthermore, $\sigma_{za} = \sigma_0$ so that the standard deviation of public information is equal to that of productivity. I set the standard deviations of the private information at $\sigma_{xa} = \sigma_{xl} = 4\sigma_0$. I let $\theta = .9$ so that technology is near constant returns to scale, and $\gamma = .1$ to ensure an empirically plausible income effect on labor supply. I set $\nu = 2$, an arbitrary number but robustness checks show that the quantitative results are not extremely sensitive to this value, as it merely controls how sensitive the asset price is to forecast errors about land shocks. Finally, I do not pick any specific value for χ, the fraction of land which is collateralizable. Rather, I focus on comparative statics with respect to χ, and study how the sensitivitiy of equilibrium output to productivity, land, and noise varies as I vary the tightness of the collateral constraints.

In the following figure I graph the fraction of constrained firms as a function of χ. This merely illustrates that for lower values of χ, i.e. tighter collateral constraints, there are more constrained firms in equilibrium.



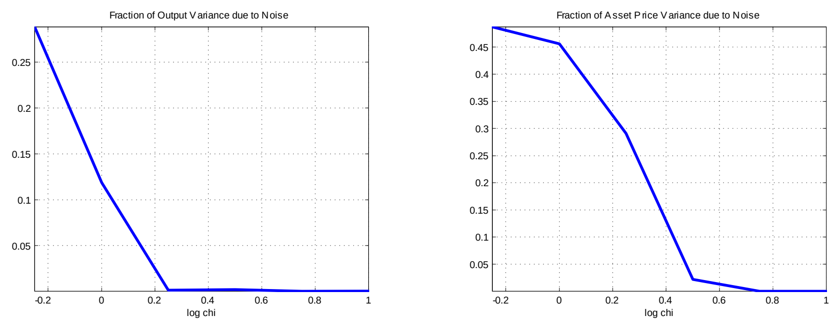
Thus, tighter collateral constraints imply that in equilibrium a greater fraction of firms are constrained. I now examine how the tightness of collateral constraints affects the response of equilibrium aggregate output and asset prices to shocks in underlying productivity and noise. The following figure graphs the sensitivity of equilibrium output to productivity and to noise, for different values of χ.



I find that tighter collateral constraints, i.e. lower values of χ, lead to a lower sensitivity of equilibrium output to productivity and a greater sensitivity to noise. Thus, when information is dispersed, tighter collateral constraints mitigate the impact of productivity yet amplify the impact of noise.

7.1 The Relative Contribution of Noise

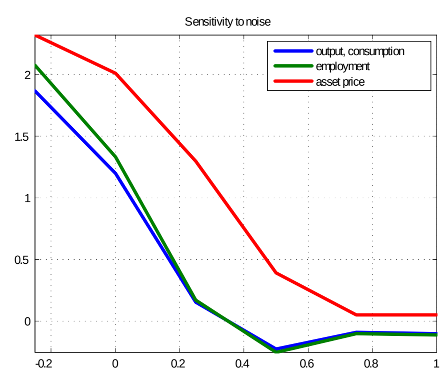
Here I show how noise may be an important source of business cycle fluctuations. Towards this goal, I define the variance decomposition of aggregate output and the asset price as in Definition 4. In the following figure I graph the fraction of output variance due to noise on the left, and the fraction of asset price variance due to noise on the right.



I find that tighter collateral constraints, i.e. lower values of χ, lead to a greater relative contribution of noise to both aggregate output and asset price fluctuations. This follows from the previous discussion on how tighter collateral constraints dampen the impact of productivity shocks, but amplify the impact of noise. Noise may therefore be an important source of asset-price and output volatility when collateral constraints are tight and information is dispersed.

7.2 Positive Co-Movement

I now examine how noise affects other observable aggregate variables; in particular I show how noise produces positive co-movement in aggregate output, employment, consumption, and asset prices. In the following figure, I graph the sensitivity of output, consumption, employment, and asset prices to noise.



Note that when collateral constraints are relatively loose, i.e. for high values of χ, positive noise or, in other words, optimism about aggregate output leads to a small increase in asset prices, since these are directly tied to expectations of aggregate output. At the same time, however, optimism causes aggregate output, consumption and employment fall. Therefore, when collateral constraints are relatively loose, good news or optimism about output tends to cause a recession.

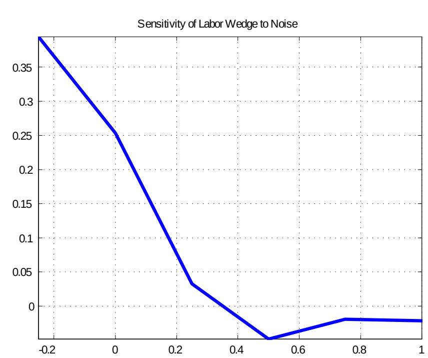
In contrast, when collateral constraints are relatively tight, i.e. for low values of χ, both the equilibrium level of output and the equilibrium asset price respond positively to common errors in expectations of aggregate output. Moreover, consumption and employment also respond positively to noise. Thus, good news about aggregate output thereby generates positive co-movement in output, consumption, employment, and asset prices as in Proposition 2.

This is important, as these co-movement properties coincide with many of the stylized facts about business cycles one observes in the data; see, for example Stock and Watson (1998). Cyclicality of the stock market and employment are standard features of business cycles. Moreover, many often think of booms as periods in which agents are optimistic about aggregate production (and likewise recessions are periods in which agents are pessimistic). In this model, when collateral constraints are tight, good news about aggregate output drives up the price of land, thereby increasing the value of collateral and allowing constrained firms to produce more. Therefore, optimism about aggregate economic activity generates a boom in output–in this sense, expectations are partially self-fulfilling.

Finally, these findings further demonstrate how the noise-driven movements found here are a possible interpretation of what many call "demand shocks". The noise-driven movements documented here resemble demand-driven fluctuations in the following sense. They have many of the same features often associated with such shocks: they contribute to positive co-movement in employment, output and consumption, yet at the same time are orthogonal to underlying movements in productivity, and are merely the product of noise in agents' information.

7.3 Movements in the Labor Wedge

I now examine the labor wedge, as defined in (22). In the following figure, I graph the sensitivity of the labor wedge to productivity and noise.



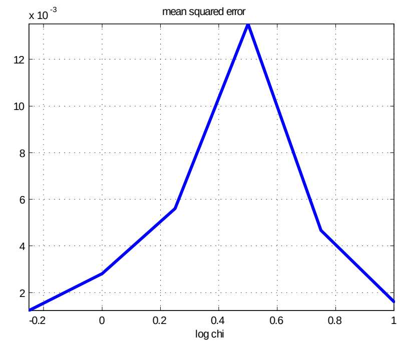
First, note that the labor wedge is sensitive to noise. Therefore, the underlying sources and financial frictions considered in this model manifest themselves as labor wedges when information is dispersed. This finding relates the noise-driven fluctuations in this model to the large body of research documenting the variation in the "labor wedge" over the business cycle (Hall, 1997; Rotemberg and Woodford, 1999; Chari, Kehoe, and McGrattan, 2007; Shimer, 2009). These researchers have documented that variation in the lavor wedge plays an important role in accounting for business-cycle fluctuations. Thus, this model may be useful for understanding certain business cycle periods, in which the labor wedge plays an important role empirically.

Second, one sees that when collateral constraints are loose, the labor wedge falls in response to noise, but when collateral constraints are tight, the labor wedge increases in response to noise. Thus, as in Proposition 4, when information is dispersed and firms are constrained, noise leads to a positive response in the measured labor wedge. Multiple authors have documented that movements in the labor wedge (as defined as the wedge, as opposed to the implicit tax) is highly cyclical and exhibits sharp decreases during recessions. For example, while productivity has stayed relatively flat during the current recession, the labor wedge has decreased significantly. The labor wedge produced by this model is thus consistent with these empirical properties: when information is dispersed and collateral constraints are tight, the observed cyclical fluctuation in the labor wedge could well be the result of small common errors in expectations.

Finally, there is evidence that a large part of business cycle fluctuations in the data can be accounted for by labor wedges as opposed to intertemporal wedges. Chari, Kehoe, and McGrattan (2007), in particular, document the empirical contribution of various types of time-varying wedges (labor, investment, efficiency) over certain business cycle episodes as well as over the entire postwar period and find that the efficiency and labor wedges together account for the bulk of fluctuations while the investment (or intertemporal) wedge plays essentially no role. Thus, they argue that models in which financial frictions show up primarily as intertemporal wedges are not promising in terms of relating to the data, while models in which financial frictions show up as efficiency or labor wedges may well be. This model is one in which the financial frictions manifest themselves as movements in the labor wedge, and hence may provide guidance in how models which feature financial frictions models could be more in line with these empirical findings.

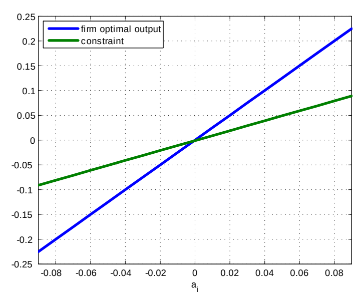
7.4 Further Results

Mean squared error. To show that the approximation method is reasonable, I present the mean squared error of the approximation method as in (24). The following figure graphs the mean-squared error for different values of χ.



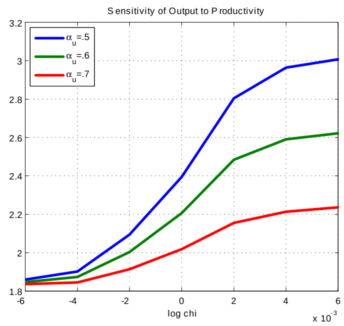
Recall that had all firms been unconstrained, or all firms constrained, then the law of motion in equilibrium would in fact be log-linear, and therefore the approximation error approaches zero at either extreme. Thus it is only for the intermediate ranges where there is a non-zero, but small approximation error.

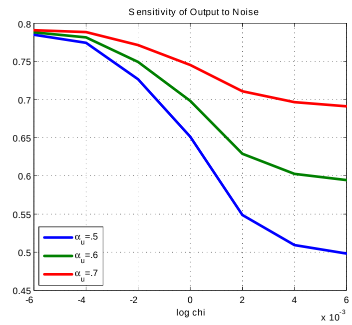
Cross sectional distribution of firms. I may also describe which types are constrained in equilibrium. I find that $\phi_a^u > \phi_a^c$, which implies that the unconstrained are more senstivite to their private idiosyncractic productivity shock than the constrained. In the following figure, I graph the optimal production of a firm who faces no constraint, and the constraint on production, both as a function of their productivity holding all other variables constant.



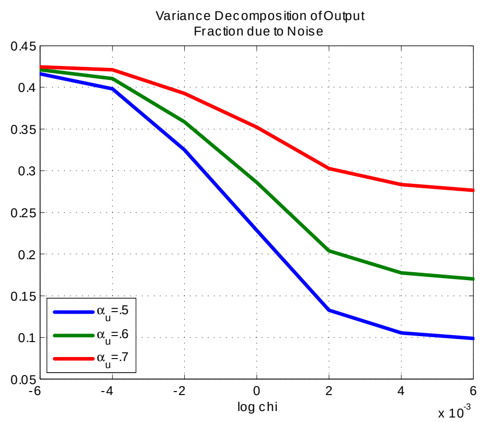
In terms of idiosyncratic productivity, since the unconstrained are more senstivite to their private idiosyncractic productivity shock than the constrained, the unconstrained cross the constrained from below. Thus, the constrained will be those with the highest productivity shocks. This occurs for two reasons. First, having higher productivity implies that a firm would like to produce more. Secondly, the constrained will be those who received the highest private signals, i.e. the ones with the highest expectations of output. This is due to the fact that the constraint depends on the asset price which is an average of all expectations of future output. Thus, firms with the greatest expectations will be the most constrained by their collateral values.

Trade Linkages and Demand Externalities. Next, I consider the effects of trade linkages across islands on the equilibrium sensitivity to the shocks.





Thus, for a given χ, lower values of ρ, the parameter that controls the substitutabiliy across goods, imply a greater sensitivity to noise and lower sensitivity to productivity.



Thus demand externalities matters only for how much unconstrained firms care about forecasting the level of economic activity relatively to forecasting the underlying economic fundamentals. This introduces an additional affect of financial frictions, as the behavior of constrained firms now affects the production choices of the unconstrained.

8 Concluding Remarks

Many economists feel that financial frictions play a role in business cycle fluctuations. This paper provides a theory of how financial frictions may imply that movements in both the business cycle and asset prices may be driven primarily by noise, as opposed to fundamentals.

9 Appendix

[to be added]

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