The Credit Spread Cycle with Matching Friction

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Abstract

We herein advance a contribution to the theoretical literature on financial frictions and show the significance of the matching mechanism in explaining the countercyclical behavior of interest rate spreads. We demonstrate that when matching friction is associated with a Nash bargaining solution, it provides a satisfactory explanation of the credit spread cycle in response to shocks in production technology or in the cost of lenders' resources. During periods of expansion, the credit spread experiences a tightening for two reasons. Firstly, as a result of easier access to loans, entrepreneurs have better opportunities outside a given lending relationship and can negotiate lower interest rates. Secondly, the less selective behavior of entrepreneurs and intermediaries results in the occurrence of fewer productive matches, a fall in the average productivity of matches, and a tightening of the credit spread.

Keywords: Matching Friction, Credit Market Imperfections, Credit Spread, Business Cycle.
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1 Introduction

The credit spread cycle may be thought of as the result of the effect of the business cycle on the difference between lending rates and risk-free rates, and is of fundamental importance in finance and macroeconomics. The variations in interest rate spreads that occur during the business cycle are commonly viewed as being the consequence of financial frictions[1]. Despite a large consensus regarding the empirical robustness of the countercyclical behavior of interest rate spreads[2], some disagreement still exists about the underlying theoretical mechanisms.

This article contributes to the theoretical literature on financial frictions and shows the significance of the matching mechanism in the credit spread cycle. Adopting a partial equilibrium framework, we demonstrate how matching friction, when associated with a Nash bargaining solution, explain the countercyclical behavior of the credit spread. This original approach is distinct from previous contributions on the credit spread cycle, which were mainly conducted within the agency paradigm. We herein extend the matching models of the credit market to the prediction of the credit spread cycle.

In the agency-based literature, the cyclical variation of interest rate spreads results from cyclical variations in the net worth of borrowers. The asymmetry in the flow of information between lenders and borrowers induces an inverse relationship between the borrower's net worth and the relative cost of external finance, usually measured as the spread between the rate of return on the firm's capital and a risk-free rate[3]. In our model, unlike those found in the agency-based literature, financial frictions result from the matching process of borrowers and lenders in the credit market. The borrower's net worth does not influence the financial friction mechanism. We herein focus on the cyclical variations of the credit spread[4] showing that it is linked with cyclical variations in the borrower's financing opportunities on the credit market, and the reservation productivity of the matches determined by borrowers and lenders.

Departing from the traditional agency-based literature, this paper reflects a growing interest in the ability of the matching model to explain the consequences of imperfections in the credit market.

[1] This view has been put forward most notably in the influential series of contributions of Bernanke (1983), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1996, 1999).

[2] This fact may be observed by inspecting various data on business cycles and interest rates. Gomes et al. (2003) report lead and lag correlations between two macroeconomic variables (the Total Factor Productivity and the ratio of investment to capital) and two interest rate spreads (the yield spread between Baa and Aaa rated corporate bonds and the spread between prime bank loan and a 3-month commercial paper). Guha and Hiris (2002) and Koopman and Lucas (2005) study the cyclical behavior of the spread between the yields on Baa corporate bonds and on government bonds for long periods. In their analysis, Dueker and Thornton (1997) propose an interpretation of the bank interest rate margin as being a bank markup. This contribution is particularly interesting because it investigates the countercyclical behavior of bank markup due to market imperfections.

[3] This spread is commonly called the external finance premium. There is a difference between the credit spread, which is the subject of this paper, and the external finance premium, which is usually studied in the agency-based literature. Levin et al. (2004) make the distinction between the two wedges in the well-known model of Bernanke et al. (1999) rather more explicit.

[4] The concept of the credit spread is also important since it is widely used to obtain an empirical measure of the external finance premium, which has no direct empirical counterpart, as explained by de Graeve (2009).

Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2006) constructed databases of credit flows and showed that the credit market in the United States is characterized by large and cyclical flows of credit expansions and contractions that may be explained in terms of matching friction. Den Haan et al. (2003) and Wasmer and Weil (2004) developed theoretical models to describe the powerful amplification and propagation mechanisms associated with matching friction[5]. They also provide empirical evidence in favour of the matching friction. However, these contributions are not concerned with the implications of matching friction on the cyclical behavior of credit spread. It is worth mentioning that in the model of Den Haan et al. (2003), the loan contract is still based on agency costs and not on a Nash bargaining solution. In this regard, our model is close to that of Wasmer and Weil (2004), who also consider a Nash bargaining solution. The authors emphasize the structural determinants of the credit spread in a double matching process with credit and labor markets, but do not analyse the business cycle behavior of the credit spread.

In order to address the questions outlined above, we have developed a partial equilibrium model that incorporates three separate channels, as follows. (*i*) An aggregate matching technique identifies the search-and-meet processes taking place in the credit market and then determines the flow of new matches as a function of the mass of unmatched entrepreneurs and the searching intensity of the lenders. (*ii*) The financial contract determines the credit interest rate as an outcome of a Nash bargaining solution that takes place between lenders and entrepreneurs. (*iii*) The rule of match destruction, which is the consequence of negative idiosyncratic shocks on the entrepreneur's technology production.

We show, from a qualitative perspective, that the response to technological shocks is governed by a combination of the three distinct effects described below.

1. The first effect of technological shocks on the surplus causes a procyclical credit spread. Lenders and entrepreneurs bargain to share the value of the match, which depends on the profits yielded by the production activity and the opportunity cost of the match. The positive improvement of the technological shock increases the return of the production activity and the associated profit. The lender can then negociate a highier loan rate. Since the cost of the lender's resources are independent of the technological shock, this loan's rate increase widens the credit spread. The two other effects act in the opposite sense and lead to a countercyclical credit spread.

2. The second effect is also related to the lender's appropriation of production revenues via the Nash bargaining solution. Following a positive technological shock, the average idiosyncratic productivity of matches decreases[6]. Given the higher efficiency of the aggregate technological productivity, lenders and entrepreneurs are less selective and accept matches that have a lower idiosyncratic productivity. The fall in the average idiosyncratic productivity of the matches reduces the profits from the production activity and consequently decreases the loan interest

[5] See Besci et al. (2005) and Nicolleti and Pierrard (2006) for more recent research on matching friction in the credit market.

[6] The average productivity of matches is defined as the product of the exogenous technological shock and the average idiosyncratic productivity of the matches.

rate. As a result, this downward adjustment of the productivity reservation, tightens the credit spread following a positive technological shock.

3. The third effect is a result of modifications to the external opportunities of entrepreneurs. For each agent, the ease of finding another partner determines its threat point and thus its revenues raised as a result of the bargaining process. A positive technological shock increases the average value of its matches, and stimulates the supply of loans to be matched on the credit market. This implies a shorter average delay in entrepreneurs finding loans, and reinforces their threat point in the bargaining process. Since an entrepreneur could find easily an another loan if the bargaining process were to fail, a lower interest rate on the loan may be obtained.

To assess the relative importance of these three effects, we conduct a quantitative analysis which reveals that the second and third effects dominate the first one, leading to a countercyclical credit spread. We extend the scope of the model studying an additional shock: an unanticipated exogenous movement in the short-term interest rate. Our results illustrate the countercyclical dynamic of the credit spread induced by the interest rate shock.

The remainder of this article is organized as follows. The model is described in Section 2. The equilibrium of the model is defined and studied analytically in Section 3. The results of numerical analysis of the model's predictions for the business cycle are described in Section 4. The extension to interest rate shocks is provided in Section 5. Some conclusions are given in Section 6.

2 The Model

In this section, we describe the two agents in our economy which is populated by entrepreneurs and financial intermediaries (lenders). We assume that agents interact over an infinite number of discrete periods. On the one hand, entrepreneurs are specialized in *projects management* and produce the unique final good. On the other hand, intermediaries are endowed with funds. The entrepreneurs production feasibility depends on the availability of funds owned by lenders whom supply funds because they cannot manage production projects. In the credit market, both lenders and entrepreneurs search for partners in order to form longterm relationship. Due to search and matching frictions, finding a partner on the credit market is rather time-consuming and costly.

2.1 Credit Market and Matching Frictions

Let us define E_t as the population of entrepreneurs growing at an exogenous and deterministic rate g_e. If N_t is the number of entrepreneurs that are matched with lenders, it follows that the number of unmatched entrepreneurs is given by $(E_t - N_t)$. The flow of new matches M_t is a function of the numbers of unmatched entrepreneurs, $(E_t - N_t)$, and the lenders loan supply, V_t. The search friction may be summarized by

$$M_t = m\left(V_t, E_t - N_t\right) \tag{1}$$

The function $m(\cdot,\cdot)$ increases with both arguments and is strictly concave with constant return to scale[7]. Given the assumption of constant return to scale, matching probabilities satisfy the following properties

$$q_t = q(\theta_t) = m\left(1, \frac{1}{\theta_t}\right) = \frac{m(\theta_t, 1)}{\theta_t} = \frac{p(\theta_t)}{\theta_t} = \frac{p_t}{\theta_t} \tag{2}$$

where $q_t = M_t/V_t$ is the lender's matching probability, and, $p_t = M_t/(E_t - N_t)$, the matching probability for entrepreneurs. $\theta_t = V_t/(E_t - N_t)$ represents the tightness credit market variable. Using a standard Cobb-Douglas matching function, the matching probability for entrepreneurs writes:

$$p_t = \overline{m}\theta_t^{\chi} \tag{3}$$

where $0 < \overline{m} < 1$ is the scale parameter and $0 < \chi < 1$ is the elasticity parameter of the matching function.

We define the rate of matched entrepreneurs, $n_t = N_t/E_t$. It evolves according to

$$n_{t+1}(1+g_e) = (1 - s_{t+1}) \times [n_t + m(v_t, 1-n_t)] \tag{4}$$

where s_t is the endogenous rate of separation per period. The separation rate concerns both the old matches, n_t, and the new matches, m_t.

2.2 The Financial Contract

Lenders have funds, but no projects. Entrepreneurs have projects, but no funds. Both lenders and entrepreneurs therefore search for partners. This process is time-consuming and costly. When matched, a financial contract occurs between agents. Depending on the productivity of the funded project, lenders and entrepreneurs can decide whether to maintain the match or not. If they decide to continue with the agreement, they then carry out a bargaining solution to agree the loan rate. The financial contract is then perfectly characterizing by the reservation productivity (Section 2.2.1), the Nash bargaining solution and the separation rule (Section 2.2.2).

2.2.1 Reservation Productivity for Entrepreneurs and Lenders

Entrepreneurs are expected to undertake projets to produce y_t units of final good (the numeraire) using the quantity i of input according to the following constant return to scale technology

$$y_t(i;\omega) = z_t\omega i \tag{5}$$

[7] The function also satisfies $m(V_t, E_t - N_t) < \min\{V_t, E_t - N_t\}$

where z_t is the aggregate productivity level, ω the idiosyncratic productivity level. At each date, all entrepreneurs pick a new value for ω from the uniform distribution function $G(\omega)$ that satisfies

$$dG(\omega)/d\omega = 1/(\overline{\omega} - \underline{\omega}), \text{with } \overline{\omega} > \underline{\omega} \tag{6}$$

We denote $J_t(\omega)$, the entrepreneur's value function of being matched with an idiosyncratic productivity level ω. If the entrepreneur accepts the match, he gets the value function $J_t^a(\omega)$, otherwise he turns to the credit market and gets the value function V_t^e. Then, the value function $J_t(\omega)$ writes

$$J_t(\omega) = \max\{J_t^a(\omega), V_t^e\} \tag{7}$$

In the remainder, the reservation productivity level $\widetilde{\omega}_t^e$ satisfies the condition $J_t^a(\widetilde{\omega}_t^e) = V_t^e$, with

$$\max\{J_t^a(\omega), V_t^e\} = \begin{cases} J_t^a(\omega), \ \omega \geq \widetilde{\omega}_t^e \\ V_t^e, \ \omega < \widetilde{\omega}_t^e \end{cases} \tag{8}$$

For lenders the value function, $\Pi_t(\omega)$, depends also on the idiosyncratic productivity of the entrepreneur's technology, ω, which is perfectly observed by lenders unlike in agency-based models. According to the realized value of ω, a lender decides either to accept the match, and obtains the value function $\Pi_t^a(\omega)$, or to refuse it, and gets V_t^b

$$\Pi_t(\omega) = \max\left\{\Pi_t^a(\omega), V_t^b\right\} \tag{9}$$

For lenders, the reservation productivity level $\widetilde{\omega}_t^b$ satisfies the condition $\Pi_t^a\left(\widetilde{\omega}_t^b\right) = V_t^b$, with

$$\max\left\{\Pi_t^a(\omega), V_t^b\right\} = \begin{cases} \Pi_t^a(\omega_t), \ \omega \geq \widetilde{\omega}_t^b \\ V_t^b, \ \omega < \widetilde{\omega}_t^b \end{cases} \tag{10}$$

Depending on the productivity of the project, matched lenders and entrepreneurs will decide either to pursue or to sever the credit relationship. If they choose to maintain their cooperation, they will negotiate a financial contract in which a credit interest rate will be determined.

2.2.2 The Nash Bargaining Solution and The Separation Rule

The financial contract determines the credit interest rate, $R_t^\ell(\omega)$, as a function of ω, the idiosyncratic productivity of the entrepreneur's technology. The interest rate is the outcome of a Nash bargaining solution, where η is the bargaining power of the entrepreneur and $(1-\eta)$ represents the lender's bargaining power. Use of the Nash bargaining solution leads to the traditional sharing rule

$$\eta\left(\Pi_t^a(\omega) - V_t^b\right) = (1-\eta)\left(J_t^a(\omega) - V_t^e\right) \tag{11}$$

The outcome of the bargaining process ensures equality between the reservation productivity of the lender and that of the entrepreneur: $\widetilde{\omega}_t = \widetilde{\omega}_t^e = \widetilde{\omega}_t^b$. Indeed, the sharing rule (11) implies that

$$J_t^a(\omega) - V_t^e = \frac{\eta}{1-\eta}\left(\Pi_t^a(\omega) - V_t^b\right) \tag{12}$$

Equation (12) states that, for any ω, if the lender wants to pursue the relationship given by $\Pi_t^a(\omega) \geq V_t^b$, the implication is that the entrepreneur will also want to stay matched, $J_t^a(\omega) > V_t^e$. The reservation productivity threshold must satisfy

$$J_t^a(\widetilde{\omega}_t) - V_t^e = \frac{\eta}{1-\eta}\left(\Pi_t^a(\widetilde{\omega}_t) - V_t^b\right) = 0. \tag{13}$$

The endogenous separation rate, which is an increasing function of $\widetilde{\omega}_t$, is computed given the uniform distribution of ω

$$s_t = \int_{\underline{\omega}}^{\widetilde{\omega}_t} dG(\omega) = \frac{\widetilde{\omega}_t - \underline{\omega}}{\overline{\omega} - \underline{\omega}} \tag{14}$$

2.3 Value Functions

2.3.1 Entrepreneurs

We assume that entrepreneurs have no personal wealth and borrow a constant and identical amount $\ell = i$. They must borrow to produce. The value functions $J_t^a(\omega)$ and V_t^e are defined as follows

$$J_t^a(\omega) = z_t\omega\ell - \ell - R_t^\ell(\omega)\ell - x^e + \beta \mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} J_{t+1}(\omega)\, dG_{t+1}(\omega)\right\} \tag{15}$$

$$V_t^e = p_t\beta \mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} J_{t+1}(\omega)\, dG_{t+1}(\omega)\right\} + (1-p_t)\beta \mathrm{E}_t\left\{V_{t+1}^e\right\} \tag{16}$$

where β is the discount rate, $z_t\omega$ represent sales, ℓ is the cost of input, $R_t^\ell(\omega)\ell$ is the cost of the credit using $R_t^\ell(\omega)$ as the credit interest rate, and x^e represents the fixed costs of production[8]. The probability that an entrepreneur does not find a lender is given by $(1-p_t)$, in which case he must turn to the credit market in the next period. It should be noted that being matched with a lender does not necessarily ensure that the entrepreneur is awarded funds for his project. The decision to finance the project depends on the realized value of ω.

In order to solve the bargaining process, we compute the net surplus of being matched from an

[8] We introduce fixed costs of production to ensure the existence of a positive value for the productivity reservation $\widetilde{\omega}$.

entrepreneur's perspective

$$
\begin{aligned}
J_t^a(\omega) - V_t^e &= z_t\omega\ell - \left(1 + R_t^\ell(\omega)\right)\ell - x^e \\
&\quad + (1 - p_t)\,\beta \mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} \left[J_{t+1}(\omega) - V_{t+1}^e\right] dG_{t+1}(\omega)\right\}
\end{aligned} \tag{17}
$$

Equation (17) states that an entrepreneur's net surplus is the sum of the revenue per period (total sales less production and credit costs) plus the expected value of the net surplus in the next period. The entrepreneur gets the net surplus at the new period with a probability of 1 if matched and p_t if unmatched. Here, the term $(1 - p_t)$ represents the difference between the matching probabilities of the two states.

2.3.2 Financial intermediaries

The value functions $\Pi_t^a(\omega)$ and V_t^b are defined as follows

$$
\Pi_t^a(\omega) = R_t^\ell(\omega)\,\ell - R_t^h\ell - x^b + \beta \mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} \Pi_{t+1}(\omega)\, dG(\omega)\right\} \tag{18}
$$

$$
V_t^b = -d + q_t\beta \mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} \Pi_{t+1}(\omega)\, dG(\omega)\right\} + (1 - q_t)\,\beta \mathrm{E}_t\left\{V_{t+1}^b\right\} \tag{19}
$$

where $R_t^\ell(\omega)\,\ell$ denotes the revenue generated by the credit activity, $R_t^h\ell$ represents the cost of the resources, x^b is the fixed cost of managing the project, d represents the per cost of the search per period, and q_t is the matching probability for a financial intermediary. When matched to an entrepreneur, the net surplus of a financial intermediary is

$$
\begin{aligned}
\Pi_t^a(\omega) - V_t^b &= R_t^\ell(\omega)\,\ell - R_t^h\ell - x^b + d \\
&\quad + (1 - q_t)\,\beta \mathrm{E}_t\left\{\int_{\underline{\omega}}^{\overline{\omega}} \left\{\Pi_{t+1}(\omega) - V_{t+1}^b\right\} dG(\omega)\right\}
\end{aligned} \tag{20}
$$

A lender's net surplus (Equation (20)) is the sum of the per period income (the credit interests, minus the cost of ressources, the cost of the project management, plus the search cost unpaid when matched) plus the expected value of the net surplus in the next period. The lender gets the net surplus at the new period with a probability of 1 if matched and q_t if unmatched. Here, the term $(1 - q_t)$ represents the difference between the matching probabilities of the two states.

2.4 Equilibrium Decisions for the Loan Interest Rate, Separation, and Entry

We assume that the costs of searches in the credit market are completely borne by the lenders and that all unmatched entrepreneurs search for funds on the credit market. The endogenous entry of lenders determines the tightness of the credit market. The free entry condition on the credit market implies that $V_t^b = 0$. From (19), we deduce that

$$\frac{d}{q_t} = \beta \mathrm{E}_t \left\{ \int_{\underline{\omega}}^{\overline{\omega}} \Pi_{t+1}(\omega)\, dG(\omega) \right\} = \beta \mathrm{E}_t \left\{ \int_{\widetilde{\omega}_{t+1}^b}^{\overline{\omega}} \Pi_{t+1}^a(\omega)\, dG(\omega) \right\} \tag{21}$$

since $\mathrm{E}_t \left\{ V_{t+1}^b \right\} = 0$. The lenders' surplus (20) then becomes

$$\Pi_t^a(\omega) - V_t^b = \left(R_t^\ell(\omega) - R_t^h \right) \ell - x^b + \frac{d}{q_t}. \tag{22}$$

In equation (22), the new variable, d/q_t, represents the current average cost of a match. From (21), this equals the expected value of a match for the next period.

The equilibrium credit interest rate is deduced from (12), (17) and (22)

$$R_t^\ell(\omega)\,\ell = (1-\eta)\left[z_t \omega \ell - (\ell + x^e) \right] + \eta \left(x^b + R_t^h \ell - p_t \frac{d}{q_t} \right) \tag{23}$$

where $R_t^\ell(\omega)\,\ell$ represents the lender's revenues,which is equal to the product of the interest rate, $R_t^\ell(\omega)$, and the amount of the credit, ℓ. The equilibrium value of these revenues is an average of two terms weighted according to the bargaining powers represented by $(1-\eta)$ and η. The first term is the net production profit, defined as the amount of total output, $z_t\omega\ell$, minus the total cost, $(\ell + x^e)$. The second term represents the fixed cost for the lender, x^b, plus the cost of the resources, $R_t^h\ell$, minus the entrepreneur's outside opportunity, $p_t \times d/q_t$. An increase in the credit market tightness (namely $\theta_t = p_t/q_t$) improves the outside opportunity of entrepreneurs, thus diminishing the lender's payoff .

The separation rule is deduced from equations (13) and (22)

$$R_t^\ell(\widetilde{\omega}_t)\,\ell + \frac{d}{q_t} = R_t^h \ell + x^b \tag{24}$$

The term on the LHS is the value of a match for a lender, defined as the sum of the credit activity revenues, $R_t^\ell(\widetilde{\omega}_t)\,\ell$, and the expected value of a match at the next period, d/q_t. A lender would maintain a match if, and only if, its value is at least higher than the cost of the match (the term on the RHS), equal to the cost of the loan, $R_t^h\ell$, plus the cost of managing the project, x^b.

Given the equilibrium interest rate from (23), the separation rule (24) may then be written as

$$(z_t \widetilde{\omega}_t - 1)\,\ell - x^e + \left(\frac{1-\eta p_t}{1-\eta} \right) \frac{d}{q_t} = R_t^h \ell + x^b \tag{25}$$

and the free entry condition (21) is

$$\frac{d}{q_t} = \beta \mathrm{E}_t \left\{ \int_{\widetilde{\omega}_{t+1}}^{\overline{\omega}} \left[\left(R_{t+1}^\ell(\omega) - R_{t+1}^h \right) \ell - x_{t+1}^b + \frac{d}{q_{t+1}} \right] dG(\omega) \right\} \tag{26}$$

The free entry condition implies that lenders enter the credit market (bearing a cost d with a probability q_t to find a partner), and remain there until the current expected cost of matching equals the present value of the anticipated lender's surplus for the matches concerned.

2.5 Aggregate Variables

We restrict our attention to three aggregate variables, namely R_t^p, the average credit spread, Y_t, the total output, and, L_t, the total credit in the economy. This last variable is obtained by multiplying the number of financed entrepreneurs, n_t, by the value of each individual loan

$$L_t = n_t \ell \tag{27}$$

The total output is the product of the number of financed entrepreneurs, n_t, the aggregate productivity level, z_t, the value of each loan, ℓ, and the average productivity of the entrepreneurs[9]

$$Y_t = n_t \ell z_t \left(\frac{\overline{\omega} + \widetilde{\omega}_t}{2} \right). \tag{28}$$

We define the credit spread as the difference between the credit interest rate, $R_t^\ell(\omega)$, and the cost of resources for the lender, R_t^h. To account for the heterogeneity of the matches, the credit spread is defined as the average of the individual spreads[10]

$$R_t^p = (1-\eta)\left[z_t \frac{(\overline{\omega} + \widetilde{\omega}_t)}{2\ell} - \frac{x^e}{\ell} - \left(1 + R_t^h\right)\right] + \eta \left(\frac{x^b - d\theta_t}{\ell} \right) \tag{29}$$

Equation (29) is a weighted average of two terms that has coefficients representing the agents' bargaining powers $(1-\eta)$ and η. For $\eta = 1$, which corresponds to the extreme case of an absence of bargaining power for a lender, the credit spread depends on two variables, namely the lender's cost, x^b, and the credit market tightness, θ_t. In the opposite case where $\eta = 0$, entrepreneurs have no bargaining power and lenders earn all the surplus from the production process.

3 Theoretical Properties

In the following section, we reduce the model to a four-dimensional system for the variables $\{\theta_t, \widetilde{\omega}_t, R_t^p, z_t\}$. The equilibrium characterization (definition, existence ans stability) is presented in Appendix A.1. The fixed amount of the loan is normalized to unity $\ell = 1$, without loss of generality. We also

[9]The expression is derived from $Y_t = n_t z_t \ell \int_{\widetilde{\omega}_t}^{\overline{\omega}_t} \omega dH(\omega)$ where $H(\omega)$ is the distribution function of ω for the matched entrepreneurs (who have ω above $\widetilde{\omega}_t$, the productivity of reservation). This function satisfies $dH(\omega)/d(\omega) = 1/(\overline{\omega} - \widetilde{\omega}_t)$.

[10]This equation is deduced by introducing into $R_t^p = \int_{\widetilde{\omega}_t}^{\overline{\omega}} \left[R_t^\ell(\omega) - R_t^h\right] dH(\omega)$ the expression of $R_t^\ell(\omega)$ given by (23).

consider a fixed value for the interest rate $R_t^h = R^h$. Shocks to this variable are introduced in Section 5.

To discuss the theoretical properties of the credit market cycle, we log-linearized the system around its unique steady-state. The following discussion is restricted to the key endogenous variables of the model. The dynamic properties of other variables, such as the output and the total credit, are studied using model simulations in Section 4.2.1.

Definition 1 *The log-linearized version of the reduced model is*

$$(1-\chi)\widehat{\theta}_t = \mathrm{E}_t\left\{\widehat{z}_{t+1} - 2\frac{\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\widehat{\widetilde{\omega}}_{t+1}\right\} \tag{30}$$

$$z\widetilde{\omega}\widehat{\widetilde{\omega}}_t = -\left(\frac{1}{m}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\,d\theta\widehat{\theta}_t - z\widetilde{\omega}\widehat{z}_t \tag{31}$$

$$R^p\widehat{R_t^p} = (1-\eta)\left(\frac{\overline{\omega}+\widetilde{\omega}}{2}\right)z\widehat{z}_t + (1-\eta)\frac{z}{2}\widehat{\widetilde{\omega}}_t - \eta d\widehat{\theta}_t \tag{32}$$

$$\widehat{z}_t = \rho\widehat{z}_{t-1} + \varepsilon_t \tag{33}$$

where the log-deviation of the variable x is denoted $\widehat{x} = \log(x_t/x)$ for $x = \theta, \widetilde{\omega}, R^p$, and z. We denote x_z as the elasticity of the endogenous variable x_t to the shock z_t that satisfies $\widehat{x}_t = x_z \times \widehat{z}_t$ for $x = \theta, \widetilde{\omega}$, and R^p. The elasticities of $\{\theta_t, \widetilde{\omega}_t, R_t^p\}$ are

$$\theta_z = \left(\frac{\rho}{1-\chi}\right)\left(\frac{\overline{\omega}+\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\right)\left[1 - \frac{2\theta d}{z(\overline{\omega}-\widetilde{\omega})}\left(\frac{1}{m\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\rho\right]^{-1} \tag{34}$$

$$\widetilde{\omega}_z = -\left(\frac{1}{m\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\frac{(1-\chi)\,d\theta}{z\widetilde{\omega}}\theta_z - 1 \tag{35}$$

$$R^pR_z^p = (1-\eta)\left(\frac{\overline{\omega}+\widetilde{\omega}}{2}\right)z + (1-\eta)\frac{\widetilde{\omega}}{2}z\widetilde{\omega}_z - \eta d\theta\theta_z \tag{36}$$

See Appendix A.3 for the detailed calculations.

Proposition 1 *The elasticity of the credit market tightness to technological shock is positive: $\theta_z > 0$.*

We describe the full effect of technological shock on credit market tightness using three terms. Given $\widehat{\theta}_t = \theta_z\widehat{z}_t$ and $\mathrm{E}_t\{\widehat{z}_{t+1}\} = \rho\widehat{z}_t$, the three identifiable terms are[11]

$$\theta_z = \underbrace{\frac{\rho}{1-\chi}}_{\text{First term}} + \underbrace{\frac{2\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\frac{\rho}{1-\chi}}_{\text{Second term}} + \underbrace{\left(\frac{1}{m}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)\frac{2d\theta}{(\overline{\omega}-\widetilde{\omega})\,z}\rho\theta_z}_{\text{Third term}} \tag{37}$$

[11] To obtain this expression, we introduce Equation (31) into (30). The value θ_z given in Equation (34) is the solution of Equation (37).

The first term is positive and equal to $(\rho/1-\chi)$ in Equation (37). Its magnitude depends on the persistence parameter, ρ, and the elasticity parameter of the matching function, $(1-\chi)$, introduced in Equations (33) and (3), respectively. The more persistent the shock, the higher the impact of z_t on θ_t. This property results from the forward-looking characteristic of the tightness of the credit market – see Equation (30). The current entry of lenders into the credit market is driven by the expectation of future revenues that may be earned if matching occurs. In the extreme case of an absence of persistence, $\rho = 0$, since future revenues are independent of the current technology, the coefficient, θ_z, is null and the tightness of the credit market does not respond to technological shocks[12].

With regard to the second parameter $(1-\chi)$, the higher the elasticity of the matching function with respect to the mass of unmatched entrepreneurs, $(1-\chi)$, the lower the impact of the shock on the tightness of the credit market[13]. In response to the expectation of future revenues that may be associated with a positive shock, lenders accept a longer average duration of vacant positions ($1/q$ rises). In order to achieve an increase in $1/q$, θ must rise with an amplitude that depends on $(1-\chi)$. For values of χ close to unity, the average duration of vacant positions shows a slight sensitivity to θ. Consequently, large variations of θ are required in response to technological shocks (θ_z is high, indeed $\lim_{\chi\to 1}\theta_z = \infty$). At the other extreme, for χ close to zero, small variations of θ are sufficient in response to technological shocks and the tightness of the credit market is less sensitive to these shocks (θ_z is low, for $\chi = 0$).

The second and third terms are the consequence of the negative response of the productivity reservation $\widetilde{\omega}_t$ to technological shocks. A high value of $\widetilde{\omega}_t$ implies a highly selective process of matches, with a low probability of being matched on the credit market with a sufficiently productive entrepreneur[14]. Consequently, lenders are less willing to enter the credit market, since total matching costs are higher, and the tightness of the credit market θ_t falls. As explained below, productivity reservation responds negatively to technological shocks.

The third and final term identified in Equation (37) corresponds to the final term in brackets of Equation (34). In response to a positive technological shock, given a future increase in θ_{t+1} induced by a decrease in ω_{t+1}, there is a smaller current increase in θ_t. Given the stability condition (A.5), the sign of the third term in Equation (37) is strictly positive, but it can be either above or below unity. When greater than one, the third term reinforces the first and second terms (thus increasing the total effect of shocks). Under condition (38), this term is less than unity. Hence, this effect weakens the total effect of technological shocks[15].

[12] If the shocks are not persistent, any improvement of technology does not last and cannot therefore stimulate the entry of banks into the credit market.

[13] In order to gain further insight into this relationship, by inspection of Equation (A.2) it may be seen that $(1-\chi)$ may also be interpreted as the elasticity of the equilibrium duration of a vacant position for a lender with respect to the tightness of the credit market (i.e. $1/q = \theta^{1-\chi}/\overline{m}$).

[14] A high value of $\widetilde{\omega}_t$ also implies a higher average idiosyncratic productivity of matches. This productivity effect could act in the opposite sense, by stimulating the supply of loans on the credit market. However, in our model this productivity effect is strictly dominated by the effect of $\widetilde{\omega}_t$ on the probability of match acceptance described in the text.

[15] In order to understand this point, it must be emphasized that condition (38) will imply that productivity reser-

Proposition 2 *The sign of the elasticity of the productivity reservation to the technological shock is ambiguous. A sufficient (and not necessary) condition for $\widetilde{\omega}_z < 0$ is*

$$\eta \leq (1-\chi) \tag{38}$$

Equilibrium reservation productivity depends on technological shock via two mechanisms. The first follows from the perfect interchangeability of the aggregate productivity and the idiosyncratic productivity. This effect corresponds to the coefficient -1 in Equation (35). From the perspective of lenders and entrepreneurs, the aggregate productivity, z, and the idiosyncratic productivity, ω, are perfect substitutes for one another. Indeed, in the term on the LHS of Equation (25), which defines the equilibrium rule of separation, the amount of the loan is multiplied by the product of the two productivity variables ($z_t \times \widetilde{\omega}_t$). This term defines the lower production level of a match that lenders and entrepreneurs can accept[16].
The second mechanism by which technological shock affects reservation productivity arises from the response of the tightness of the credit market. This effect is represented by the coefficient of θ_z in Equation (35). The sign of this coefficient is determined by condition (38)[17]. Condition (38) implies that the reservation productivity depends inversely on the tightness of the credit market and the sign of the coefficient of θ_z, in Equation (35), is negative.

In fact, the tightness of the credit market θ_t influences the reservation productivity $\widetilde{\omega}_t$ in two different ways - see Equation (A.3). The first arises from the free entry condition. Recalling that with free entry the expected value of a match for lenders is equal to the average cost of a match[18]: $(d/\overline{m})\,\theta_t^{1-\chi}$, it follows that a high value of θ_t means that the value of a match is high. In this case, lenders and entrepreneurs are willing to accept lower idiosyncratic productivity to preserve the match ($\widetilde{\omega}_t$ decreases with θ_t).

The second way results from the bargaining process. For a fixed value of match (equal to d/q_t), the equation of the separation rule (24) implies that the equilibrium revenues for the lender, given by (23), are constant. According to Equation (23), as the tightness of the credit market increases, better external opportunities emerge for entrepreneurs, which provokes a decline in the loan interest

vation reacts negatively to a positive shock and depends inversely on the tightness of the credit market. In the intertemporal equation (30), this means that a current positive shock increases the expected value of the tightness of the credit market for the following period, which lowers the current response of the tightness of the credit market to the shock. The response is weakened, but still positive

[16] If the term on the RHS of this equation is constant (which is the case for $\theta_z = 0$), there is a one-to-one relation between z_t and $\widetilde{\omega}_t$. An increase of 1% in the aggregate productivity induces a reduction of 1% in the productivity reservation in the economy.

[17] We hereafter assume condition (38) holds. We will check the robustness of our results against this condition using numerical analysis. It is worth mentioning that this condition can be related to the famous condition of Hosios (1990) used in models with matching friction. This condition states that the trading externalities induced by matching friction are efficiently internalized by the Nash bargaining solution, provided that the bargaining powers of the agents are equal to their marginal contribution to the matching process – see also Pissarides (2000). In our setup, the Hosios (1990) condition implies $\eta = 1-\chi$.

[18] The average cost of a match is the per-period search cost, d, times the average duration of a vacant position for a bank, which is equal to the inverse of the bank matching probability $1/q_t = (d/\overline{m})\,\theta_t^{1-\chi}$.

rate. Finally, high values of θ_t make lenders more selective and the productivity reservation is also higher[19].

Condition (38) implies that the first way in which the tightness of the credit market influences the reservation productivity, strictly dominates the second. The elasticity $\widetilde{\omega}_z$ defined by (35) is then negative and less than -1 (which is the value associated with the first effect only). The positive response of the tightness of the credit market to a positive technological shock reinforces the first effect of the shock on $\widetilde{\omega}_t$. A positive technological shock increases the tightness of the credit market, hence a higher expected value of a match leads lenders and entrepreneurs to accept lower idiosyncratic productivity values.

Proposition 3 *The sign of the elasticity of the credit spread to technological shock is ambiguous. The credit spread reacts negatively to the shock ($R_z^p < 0$) if*

$$(1-\eta)\left(\frac{\overline{\omega}+\widetilde{\omega}}{2}\right)z < \eta d\theta\theta_z + (1-\eta)\frac{\widetilde{\omega}}{2}z\left(-\widetilde{\omega}_z\right) \tag{39}$$

This condition is necessary and sufficient.

We clearly identify three distinct channels by which technological shocks affects the credit spread cycle. Each channel are related to the coefficients of z, $\widetilde{\omega}_z$, and θ_z in Equation (36).

The first two depend on the entrepreneurs' profits in the production sector. Lenders earn a share $(1-\eta)$ of the profits equal to the lenders' bargaining power in the Nash bargaining solution. If they have no bargaining power ($\eta = 1$), the loan interest rate paid by the entrepreneurs only covers the costs of the lenders[20] and are independent of profits. For $\eta < 1$, the first effect of a positive technological shock increases profits for the entrepreneur: for the same costs of production, profits grow with the technological improvement (coefficient of z in Equation (36)). The lender takes advantage of this situation to negociate a highier loan rate. The size of this first effect depends on the lenders' bargaining power, and the average productivity of matches, equal to $(\overline{\omega}+\widetilde{\omega})\,z/2$. This result is quite intuitive: the greater the bargaining power of the lender, the greater the lender appropriation of the profit, and the higher the impact of the technological shock on the credit spread[21]. This first effect leads to a procyclical credit spread.

The second effect is based on the average productivity of matches, and acts in the opposite sense to the first effect, generating a countercyclical credit spread. It corresponds to the coefficient

[19] This last effect vanishes for $\eta = 0$, i.e. if entrepreneurs have no bargaining power.

[20] More precisely, the loan interests cover the fixed costs of banking activity, denoted x^b, minus the external opportunities of entrepreneurs, equal to $d\theta_t$. See Equation (29) with $\eta = 1$ and note that $d\theta_t = p_t \times (d/q_t)$, where p_t is the matching probabiliy of entrepreneurs and d/q_t is the expected value of a match on the credit market.

[21] In order to understand why the average productivity of matches forms part of the expression for R_z^p, it must be remembered that credit spread is an aggregate variable. The impact of the shock on the production of a given match depends on the idiosyncratic productivity of this match. Since the credit spread is an average of the individual credit interest rate spreads in the economy, the impact of the shock depends on the average idiosyncratic productivity of all the matches.

$(1-\eta)\, z\widetilde{\omega}$ of $\widetilde{\omega}_z$ in (36). A fall in the productivity reservation $\widetilde{\omega}_t$, in response to a positive shock, diminishes the average productivity of matches as measured by credit spread– see Equation (29) again.

The third effect is based on the threat point of entrepreneurs and, as for the second effect, acts to generate a countercyclical credit spread. This effect corresponds to the coefficient $\eta d\theta$ of θ_z in the expression of R_z^p given by (36). In order to better understand this, it should be noted that $\eta d\theta = p \times \eta \times (d/q)$. It may be observed that an entrepreneur can find another match, with probability p, and get a share η of the value of this match, which is equal to d/q, as a result of the free entry condition. Consequently, as the tightness of the credit market increases, in response to a positive shock, better external opportunities occur for entrepreneurs, which increase their threat point and provoke the credit spread to thighten up– see Equation (29).

To conclude on the credit spread, it is noted that in the extreme case of $\eta = 1$, lenders have no bargaining power. In this case, the first two effects of a shock disappear. In consequence, the credit spread is necessarily countercyclical, given the third effect. In the general case of $0 < \eta < 1$, if the second and third effects are sufficiently large to verify condition (39), the credit spread is clearly countercyclical. We now turn to numerical simulations in order to assess the plausibility of this qualitative property of our model.

4 Numerical Analysis

We use numerical analysis to clarify the ambiguous theoretical properties of the credit spread, and to describe the dynamic behavior of other aggregate variables.

4.1 Calibration

The model is calibrated by choosing the available empirical counterparts for the interest rates and the average rates of credit flows creation and destruction. Because the previous variables do not allow us to calibrate all the structural parameters, we must make additional assumptions on the values of these parameters. We restrict their range using the conditions of existence, uniqueness, and stability of the equilibrium. A unit of time corresponds to a quarter.

The calibration constraints on interest rates are as follows[22]: the quarterly interest rate on

[22] The interest rates are obtained using data generated by the Federal Reserve in their "Survey of terms of business lending". Since our model is designed for business loans, and not for loans for household or real estate, we use the series entitled "Commercial and Industrial Loan Rates Spreads over intended federal funds rate" (available on the website http://www.federalreserve.gov/releases/e2/e2chart.htm). Data are available only after 1986(3). For the period before this, we generate the spread directly, as the difference between the bank prime loan rate and the effective federal funds rate. Both these series are available from 1955 at monthly frequencies on the *FRED* website (http://research.stlouisfed.org/fred2 , Table H.15 Selected Interest Rates, series ID are MPRIME and FEDFUNDS respectively). Data are converted to quarterly frequency by taking the average for each quarter. Finally, the sample is 1955(1)-2008(4).

lender resources is $R^h = 1.020^{1/4}$ and the quarterly interest rate on loans is $R^\ell = 1.039^{1/4}$. The rate of creation and destruction of credit flows are taken from the database of commercial loan constructed by Dell'Ariccia and Garibaldi (2005) (see Table 3, p. 675). This implies a steady state for the rate of credit destruction $s = NEG = 0.0111$, which corresponds to the variable NEG of Dell'Ariccia and Garibaldi (2005). The theoretical counterpart of the variable POS of Dell'Ariccia and Garibaldi (2005) is given by $POS = [(1-s) \times m(v, 1-n) \times \ell] / L$ i.e. the flow of new credit matches divided by the total loan in the economy. We impose $POS = 0.0179$ in the calibration procedure, and assume that the matching probability of entrepreneurs is $p = 0.4$[23]. The condition of Hosios (1990) is imposed in the case of a symmetric Nash bargaining i.e. $\chi = \eta = 0.5$. The scale parameters of the production and matching technologies are set as follows: $\underline{\omega} = 0.95$, $\overline{\omega} = 1, z = 4$, and $\overline{m} = .01$. Finally, the discount rate is set to a conventional value $\beta = 0.999$.

We then deduce from steady-state restrictions the values[24] of $g, \widetilde{\omega}$, θ, q, x^e, x^b, n, Y, and L. In the following section, we provide a discussion of the sensitivity of our results to these assumptions. Before that, we describe the business cycle behavior of the model for this calibration.

4.2 The Cyclical Behavior of the Credit Spread

The previous theoretical analysis emphasizes the interactions between several mechanisms that determines the behavior of the credit spread. Some of the effects of technological shock induce a procyclical credit spread dynamic, while in contrast others lead to a countercyclical credit spread. In what follows, we perform numerical exercises to assess and to quantify the relative importance of these mechanisms according to the values of the structural parameters used.

4.2.1 IRFs

We start our numerical analysis by describing the dynamic behavior of aggregate variables. Figure 1 depicts the impulse response functions (IRFs) to a positive technological shock of the output, the credit spread, the credit market tightness, the reservation productivity, the total credit, and the average productivity of matches.

A positive technological shock leads to an expansion of the credit in the economy by two means. Firstly, the improvement of the aggregate technology leads lenders and entrepreneurs to accept a lower idiosyncratic productivity level. Since our calibration respects condition (38), the elasticity coefficient ω_z is negative and the IRF of $\widetilde{\omega}_t$ is negative. This fall in reservation productivity decreases the rate of match destruction in the economy and leads to a credit expansion. Secondly, the improvement in aggregate technology stimulates the entry of lenders into the credit market. In Proposition 1, we showed that the elasticity $\theta_z > 0$, and thus the IRF of θ_t is positive. This

[23] On average, it takes 2.5 quarters for an entrepreneur to be financed.

[24] The values are $\widetilde{\omega} = 0.9506$, $\theta = 0.65, x^e = 1.871, x^b = 0.004, n = 0.493, Y = 1.923$, $L = 0.493$, $g = 0.006$, and $d = 0.031$.

rise in credit market tightness facilitates the financing of entrepreneurs by increasing their matching probability, and thus contributes to a credit expansion.

The IRFs of the credit market tightness and the productivity reservation return monotonically to zero as the shock disappears. However, they induce a radically different pattern for the IRFs of the total credit. In our economy, the short-run behavior of the total credit replicates the behavior of the matching rate of entrepreneurs[25], as defined by equation (4). The total credit variable adjusts rather gradually, because it takes time for lenders to find new entrepreneurs on the credit market. The IRF of the total credit depicted in Figure 1 is indeed hump-shaped with growing values during the first two years after the shock. The hump-shaped response of the total credit is very similar to the output behavior, even though the definitions of the two variables differ substantially – see Equations (27) and (28). The difference between output and total credit is the average productivity of matches, denoted ω_t^*[26]. In Figure 1, we plot the IRFs of the average productivity of matches. The response is negative and very close to zero. The weak response of this variable explains the high similarity of the IRFs of output and total loan. The sign of the response signifies that the negative values for the average idiosyncratic productivity of matches outweighs the positive values for the aggregate technological shock[27]. To conclude this Section on the IRFs, we now turn to the dynamics of the credit spread .

Figure 1 shows that the credit spread reacts negatively to positive technological shocks, implying that $R_z^p < 0$. For all horizons, the sign of the IRF of the credit spread is opposite to that of the IRF of the output. From this, we conclude that this model generates a clear countercyclical credit spread. Since the first effect of shocks induces a procyclical credit spread, it may be seen that the second and third effects described above dominate the first. To conclude, for this calibration, matching friction on the credit market supports countercyclical behavior in the credit spread.

4.2.2 Sensitive analysis

In order to assess the robustness of the previous results, we conduct a sensitivity analysis of the values of the structural parameters. For each parameter, we use the conditions of existence, uniqueness, and stability (A.4) and (A.5) to define the range of admissible values. Given the lack of empirical information for the matching rate of entrepreneurs, we also consider different steady-state values for p. For each value of p, we again apply the entire calibration procedure described in Section 4.1. We simulate the model for values within this range and compute the correlation between output and credit spread and between output and the average productivity of loans. Results are reported in Figure 2 for the two key parameters χ and η and for the variable p[28].

[25] It would be different with a variable amount of loan per match or for an endogenous arrival of entrepreneurs.

[26] More precisely, $Y_t = L_t \times \omega_t^*$ where Y_t is the output, $L_t = n_t \ell$ is the total loan variable, and $\omega_t^* = z_t \left(\overline{\omega} + \widetilde{\omega}_t\right)/2$ is the average productivity of matches.

[27] The average idiosyncratic productivity of matches is $\left(\overline{\omega} + \widetilde{\omega}_t\right)/2$, which is compared with the aggregate technological shock z_t.

[28] For parameters β, $\overline{\omega}$, $\underline{\omega}$, z, and $\overline{m}$, the coefficients of correlation do not show significative variations (the additional figures are available upon request).

The mechanism based on the market tightness is however sufficiently strong to generate a countercyclical credit spread for all the values considered. The variations of the coefficient of correlation between the credit spread and the output are very small for χ and η, but sizeable for p. However, even in the extreme case of p close to zero, the coefficient of correlation is still clearly negative (about -0.60). Finally, it is worth mentioning that even if condition (38) does not hold for all simulations, we do not observe shifts in the sign of the response $\widetilde{\omega}_t$ to technological shock. We then conclude that our main results are robust to changes in the values of parameters.

5 Extension

This final Section extends the model to the case of shocks in the short term interest rate. Up to this point, we only have considered a unique source of fluctuations, namely a technological shock. However, the literature on credit spread also focuses on interest rate shocks, specifically those involving unanticipated exogenous movement in the short term interest rate.

The following proposition characterizes the credit market cycle induced by interest rate shocks.

Proposition 4 *The elasticity of the tightness of the credit market to interest shock is negative, $\theta_{R^h} < 0$. The sign of the elasticity of the productivity reservation to interest shock is ambiguous. Condition (38) is sufficient (but not necessary) to ensure $\widetilde{\omega}_{R^h} > 0$. The credit spread reacts positively to a positive shock ($R^p_{R^h} > 0$) if*

$$(1-\eta) R^h < (1-\eta) \frac{z}{2} \widetilde{\omega}_{R^h} - \eta d \theta \theta_{R^h} \tag{40}$$

This is a necessary and sufficient condition.

Proof. *See Appendix A.4.* ■

The sign of the elasticity $R^p_{R^h}$ defined by (A.19) is ambiguous, because the effect associated with R^h acts in the opposite sense to the two effects associated with $\widetilde{\omega}_{R^h}$ and θ_{R^h}. These two last effects result from the endogenous responses of the credit market tightness and the productivity reservation. If these endogenous responses are sufficiently large (entailing high values for $\widetilde{\omega}_{R^h}$ and $-\theta_{R^h}$) to verify condition (40), the credit spread is countercyclical, since it reacts positively to a positive interest rate shock.

The first effect of a positive interest shock on the credit spread is negative, and corresponds to $(1-\eta) \times R^h$ in the elasticity $R^p_{R^h}$ defined in Equation (A.19). The credit spread thightens up, because the cost of funds for the lenders are taken into account in the bargaining process. The increase in R^h diminishes the entrepreneurs' profits and thus the loan interest paid to the lender. This effect disappears for the case of null bargaining power for the lenders $\eta = 1$.

The second and third effects of the interest rate shocks on the credit spread results from the same mechanisms as for technological shocks. Indeed the coefficients of the elasticities $\widetilde{\omega}_{R^h}$ and θ_{R^h} in the expression of $R^p_{R^h}$ given by (A.19) are identical to the coefficients of the elasticities $\widetilde{\omega}_z$ and θ_z in the expression of R^p_z given by (36). The difference originates from the signs of these coefficients and elasticities. Because Section 3 provides a detailed interpretation of the coefficients, we merely comment here on the mechanisms associated with these effects.

The second effect is related to the threat point of entrepreneurs. A positive interest rate shock lowers the tightness of the credit market ($\theta_{R^h} < 0$). As a consequence, entrepreneurs have access to poor levels of external opportunity, which weakens their threat point, and the credit spread increases since lenders can negotiate a higher interest rate on the loan. The third effect is related to the average productivity of matches. A positive interest rate shock increases the reservation productivity ($\widetilde{\omega}_{R^h} > 0$), so that the profits from final production activity are higher and the credit spread increases since lenders get a share $(1 - \eta)$ of these profits.

It should be noted that in the extreme case of the lenders having no bargaining power ($\eta = 1$), the first and third effects disappear. The credit spread is in consequence necessarily countercyclical, given the second effect via the response of the credit market tightness (the condition (40) reduces to $\theta_{R^h} < 0$). This case is naturally too restrictive to enable sensible conclusions to be drawn. To this end, we turn to numerical simulation to assess the plausibility of a countercyclical credit spread. The model simulation is carried out using the calibration described in Section 4.1 with the additional constraint $\rho_{R^h} = 0.95$ for the persistence of interest rate shocks. Figure 3 shows the IRFs produced by a negative interest rate shock for the following variables: the output, the credit spread, the credit market tightness, the reservation productivity, the total credit, and the average productivity of matches. These IRFs are very similar to the IRFs associated with technological shock, which confirms the results of the previous theoretical analysis. We perform a sensitivity analysis, for a range of parameter values identical to these of Section 4.2.2. We do not observe any cases of procyclical credit interest spread[29]. We therefore conclude that this model generates a robust countercyclical credit spread in response to both technological and interest rate shocks.

6 Conclusion

This study was motivated by evidence of the countercyclical behavior of the credit spread. We have explored the consequences of matching friction in the credit market in order to explain this evidence. To this end, we departed from the traditional view on agency costs and instead followed den Haan et al. (2003) and Wasmer and Weil (2004), who developed models of the credit market based on matching friction. We have proposed an original model with Nash bargaining on the credit interest rate, entry decisions of lenders on the credit market, and separation decisions between lenders and entrepreneurs.

[29] Corresponding figures are available upon request.

Although some of the effects of technological or interest rate shocks induce a procyclical credit spread, additional effects associated with the responses of endogenous variables (namely credit market tightness and reservation productivity) lead to countercyclical behavior. The model simulations suggest that the latter effects dominate the former, implying a robust countercyclical credit spread. Den Haan et al. (2003) and Wasmer and Weil (2004) emphasize that matching friction in the credit market induces a powerful persistence mechanism in the economy. Persistence is a long-standing puzzle in macroeconomics, which is of concern both in the literature on business cycles and on monetary policy. An interesting direction for future work is to incorporate our mechnisms in a general equilibrium model to assess its ability to generate a strong persistence in the economy in a way consistent with the business cycle dynamics of interest rates.

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A Appendix

A.1 Definition, Existence, and Stability of the equilibrium

We firstly define full equilibrium, and then its reduced form. The reduced form yields the conditions of existence, uniqueness, and stability of the equilibrium. In the following section, the fixed amount of the loan is normalized to unity $\ell = 1$, without loss of generality. We also consider a fixed value for the interest rate $R_t^h = R^h$. Shocks to this variable are introduced in Section 5.

Definition 2 *The equilibrium case is given by the set of endogenous variables* $\{Y_t, L_t, R_t^p, n_t, s_t, p_t, q_t, \theta_t, \widetilde{\omega}_t, z_t\}$ *that satisfies: (i) the definitions of aggregate variables for output* Y_t *(28), credit* L_t *(27), and the credit spread* R_t^p *(29); (ii) the law of motion of matched entrepreneurs* n_t *(4) given the rates of matching* p_t *and* q_t *given by (2) and (3), and the rate of separation* s_t *(14); (iii) the equilibrium conditions for credit market tightness* θ_t *(26) and for the productivity reservation* $\widetilde{\omega}_t$ *(25); and (iv) the following process for the exogenous and stochastic variable* z_t

$$\log(z_{t+1}) = \rho_z \log(z_t) + (1 - \rho_z) \log(z) + \varepsilon_{z,t+1} \tag{A.1}$$

where z *is the steady-state value of* z_t, ρ_z *is the persistence parameter, and* $\varepsilon_z \sim iid(0, \sigma_z^2)$ *is the innovation with variance* σ_z^2. *The set of structural parameters is* $\boldsymbol{\xi} = \{R^h, x^e, x^b, \overline{m}, \chi, \eta, \beta, d, z, \rho_z, \sigma_z\}$.

In order to establish the existence and uniqueness of the equilibrium, we reduce the equilibrium to a four-dimensional system for the variables $\{\theta_t, \widetilde{\omega}_t, R_t^p, z_t\}$. To this end, we reformulate the free entry condition and separation rule according to the following definition.

Definition 3 *The reduced model is a set of endogenous variables* $\{\theta_t, \widetilde{\omega}_t, R_t^p, z_t\}$ *that satisfy four equations, given the set of structural parameters* $\boldsymbol{\xi}$ *defined in Definition 2. Using the interest rate definition (23), the matching probability (2), and the separation rule (25), the first equation for the free entry condition (26) becomes*

$$\frac{d}{\overline{m}} \theta_t^{1-\chi} = \frac{(1-\eta)}{2(\overline{\omega} - \underline{\omega})} \beta \mathrm{E}_t \left\{ z_{t+1} (\overline{\omega} - \widetilde{\omega}_{t+1})^2 \right\} \tag{A.2}$$

The equilibrium value of the credit market tightness θ_t *depends on the expected values for the technological shock,* $\mathrm{E}_t\{z_{t+1}\}$, *and the productivity reservation,* $\mathrm{E}_t\{\widetilde{\omega}_{t+1}\}$. *The second equation for the separation rule (25) becomes*

$$z_t \widetilde{\omega}_t \ell = x^b + x^e + (1 + R^h) - \left(\frac{1 - \eta \overline{m} \theta_t^{\chi}}{1 - \eta} \right) \frac{d}{\overline{m}} \theta_t^{1-\chi} \tag{A.3}$$

using the equations for the rates of matching given in (2) and (3). The equilibrium value for the productivity reservation $\widetilde{\omega}_t$ *depends on the current values of the technological shock,* z_t, *and the credit market tightness,* θ_t. *The third equation is (29), which gives the equilibrium value of the credit spread* *as a function of the credit market tightness, θ_t, the reservation productivity, ω_t, and the technological shock, z_t. The fourth, equation is the law of motion of the aggregate technology z_t (A.1).*

The following proposition establishes the existence and uniqueness conditions of the reduced equilibrium.

Proposition 5 *The steady-state equilibrium $\{\theta^*, \widetilde{\omega}^*, R^p, z\}$ of the reduced model defined in Definition 3 exists and is unique, if the following condition is satisfied*

$$\begin{aligned}\frac{x^b + x^e + \left(1 + R^h\right)}{z} \quad < \quad \overline{\omega} < & \frac{x^b + x^e + \left(1 + R^h\right)}{z} \\ & + \left(\frac{(1-\eta)\, z\overline{m}}{d}\right)^{\chi/(1-\chi)} \eta\overline{m} \left(\frac{\beta\Delta}{2}\right)^{1/(1-\chi)} + \left(\frac{2-\beta}{2}\right)\Delta\end{aligned} \tag{A.4}$$

in which an additional parameter has been introduced: $\Delta = \overline{\omega} - \underline{\omega}$. This condition is necessary and sufficient.

Proof. *See Appendix A.2.* ■

In the following proposition, we state the stability condition of the log-linearized equilibrium.

Proposition 6 *The log-linear equilibrium of $\left\{\widehat{\theta}_t, \widehat{\widetilde{\omega}}_t, \widehat{R^p}_t, \widehat{z}_t\right\}$ defined by equations (30)-(31)-(32)-(33) is stable if the following condition holds*

$$\left|\frac{2\theta d}{z\left(\overline{\omega} - \widetilde{\omega}\right)}\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\right| < 1 \tag{A.5}$$

This condition is sufficient.

Proof. *See Appendix A.3 .* ■

A.2 Proof of Proposition 5

In order to prove Proposition 5, we first define the function $\theta^*\left(\widetilde{\omega}; \boldsymbol{\xi}_\theta\right)$ that gives θ^* as a function of $\widetilde{\omega}$ and a set of structural parameters $\boldsymbol{\xi}_\theta = \{\beta, \eta, z, \overline{m}, \overline{\omega}, \underline{\omega}\}$

$$\theta^*\left(\widetilde{\omega}; \boldsymbol{\xi}_\theta\right) = \left[\frac{\beta\left(1-\eta\right) z\overline{m}}{2\left(\overline{\omega} - \underline{\omega}\right) d}\left(\overline{\omega} - \widetilde{\omega}\right)^2\right]^{1/(1-\chi)} \tag{A.6}$$

This function is obtained from the steady–state expression of (A.2). The limit values for θ^* are

$$\begin{aligned}\lim_{\widetilde{\omega}\to\underline{\omega}} \theta^*\left(\widetilde{\omega}; \boldsymbol{\xi}_\theta\right) &= \left.\theta^*\left(\widetilde{\omega}; \boldsymbol{\xi}_\theta\right)\right|_{\widetilde{\omega}\to\underline{\omega}} = \left[\frac{\beta\left(1-\eta\right) z\overline{m}}{2\left(\overline{\omega} - \underline{\omega}\right) d}\Delta^2\right]^{1/(1-\chi)} > 0 \\ \lim_{\widetilde{\omega}\to\overline{\omega}} \theta^*\left(\widetilde{\omega}; \boldsymbol{\xi}_\theta\right) &= 0\end{aligned}$$

If $\widetilde{\omega}^* \in]\underline{\omega}, \overline{\omega}[$ exists and is unique, the Equation (A.6) implies that θ^* exists, is unique, and satisfies $\theta^* \in \left]0, \theta^*\left(\widetilde{\omega}; \boldsymbol{\xi}_\theta\right)|_{\widetilde{\omega}\to\underline{\omega}}\right[$. In order to establish the existence and uniqueness of $\widetilde{\omega}^*$, we introduce the steady–state expression of Equation (A.3) into Equation (A.6) and deduce that $\widetilde{\omega}^* \in]\underline{\omega}, \overline{\omega}[$ is the solution of

$$T\left(\widetilde{\omega}^*; \boldsymbol{\xi}_\omega\right) = 0 \tag{A.7}$$

where $\boldsymbol{\xi}_\omega = \left(x^b, x^e, a, R^h, \ell, z, \eta, \overline{m}, d, \beta, \overline{\omega}, \underline{\omega}\right)$ is a set of structural parameters and the function $T\left(\widetilde{\omega}^*; \boldsymbol{\xi}_\omega\right)$ is

$$\begin{aligned} T\left(\widetilde{\omega}; \boldsymbol{\xi}_\omega\right) &= \frac{x^b + x^e + \left(1 + R^h\right)}{z} - \frac{\beta\left(\overline{\omega} - \widetilde{\omega}\right)^2}{2\left(\overline{\omega} - \underline{\omega}\right)} - \widetilde{\omega} \\ &\quad + \eta\overline{m}\left(\frac{\left(1-\eta\right) z\overline{m}}{d}\right)^{\chi/(1-\chi)}\left(\frac{\beta}{2\Delta}\right)^{1/(1-\chi)}\left(\overline{\omega} - \widetilde{\omega}\right)^{2/(1-\chi)} \end{aligned} \tag{A.8}$$

In order to find the solution for Equation (A.7), we first note that $T\left(\widetilde{\omega}; \boldsymbol{\xi}_\omega\right)$ is strictly decreasing with respect to $\widetilde{\omega}$. The first order derivative of $T\left(\widetilde{\omega}^*; \boldsymbol{\zeta}\right)$ with respect to $\widetilde{\omega}$ is

$$\begin{aligned} T_1\left(\widetilde{\omega}; \boldsymbol{\zeta}\right) &= -\left[1 - \beta\left(\frac{\overline{\omega} - \widetilde{\omega}}{\overline{\omega} - \underline{\omega}}\right)\right] \\ &\quad - \left(\frac{\left(1-\eta\right) z\overline{m}}{d}\right)^{\chi/(1-\chi)} \eta\overline{m}\left(\frac{\beta}{2\left(\overline{\omega} - \underline{\omega}\right)}\right)^{1/(1-\chi)} \frac{2}{1-\chi}\left(\overline{\omega} - \widetilde{\omega}\right)^{\frac{1+\chi}{1-\chi}} \end{aligned}$$

since $\beta\left(\frac{\overline{\omega}-\widetilde{\omega}}{\overline{\omega}-\underline{\omega}}\right) < 1$, the two terms of the derivative are negative and $T_1\left(\widetilde{\omega}; \boldsymbol{\zeta}\right) < 0$. Hence, the existence and uniqueness of the equilibrium value $\widetilde{\omega}^*$ requires that $\lim_{\widetilde{\omega}\to\underline{\omega}} T\left(\widetilde{\omega}; \boldsymbol{\zeta}\right) > 0$ and $\lim_{\widetilde{\omega}\to\overline{\omega}} T\left(\widetilde{\omega}; \boldsymbol{\zeta}\right) < 0$. The deduction of (A.4) is then straightforward given the two following expressions of the limits of the function $T\left(\widetilde{\omega}; \boldsymbol{\zeta}\right)$

$$\begin{aligned} \lim_{\widetilde{\omega}\to\underline{\omega}} T\left(\widetilde{\omega}; \boldsymbol{\zeta}\right) &= \frac{x^b + x^e + a + \left(1 + R^h\right)}{z} - \overline{\omega} \\ &\quad + \left(\frac{\left(1-\eta\right) z\overline{m}}{d}\right)^{\chi/(1-\chi)} \eta\overline{m}\left(\frac{\beta\left(\overline{\omega} - \underline{\omega}\right)}{2}\right)^{1/(1-\chi)} + \left(\frac{2-\beta}{2}\right)\Delta \\ \lim_{\widetilde{\omega}\to\overline{\omega}} T\left(\widetilde{\omega}; \boldsymbol{\zeta}\right) &= \frac{x^b + x^e + a + \left(1 + R^h\right)}{z} - \overline{\omega} \end{aligned}$$

A.3 Proof of the Proposition 6

In order to prove Proposition 6, we solve the recursive equilibrium of the credit market tightness. Introducing Equation (31) into Equation (30) gives

$$\left(1-\chi\right)\widehat{\theta}_t = \mathrm{E}_t\left\{\frac{\overline{\omega} + \widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\widehat{z}_{t+1} + \frac{2d/z}{\overline{\omega} - \widetilde{\omega}}\left(\frac{1-\chi}{\overline{m}\theta^\chi} - \eta\right)\theta\widehat{\theta}_{t+1}\right\}$$

The current log-deviation of the credit market tightness depends on the expected values at the next period for the technological shock and the credit market tightness. Given the autoregressive process for z_t defined in (A.1) and assuming $\mathrm{E}_t\{\varepsilon_{t+1}\} = 0$, we obtain

$$\widehat{\theta}_t = \frac{2\theta d}{z\left(\overline{\omega} - \widetilde{\omega}\right)} \left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right) \mathrm{E}_t\left\{\widehat{\theta}_{t+1}\right\} + \left(\frac{\overline{\omega} + \widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\right)\left(\frac{\rho}{1-\chi}\right)\widehat{z}_t$$

This is a standard intertemporal equation for $\widehat{\theta}_t$ that can be solved by iterating over the future period. Let us simplify the equation as follows

$$\widehat{\theta}_t = \mathbf{a}\mathrm{E}_t\left\{\widehat{\theta}_{t+1}\right\} + \mathbf{b}\widehat{z}_t$$

The assumption $|\mathbf{a}| < 1$ is sufficient to guarantee the stability of the equilibrium.We then deduce the value of the coefficient θ_z that satisfies

$$\widehat{\theta}_t = \theta_z \times \widehat{z}_t = \frac{\mathbf{b}}{1 - \mathbf{a}\rho} \times \widehat{z}_t$$

For $|\mathbf{a}\rho| < 1$, the condition $(1 - \mathbf{a}\rho) > 0$ is always satisfied and since $\mathbf{b} > 0$, we conclude that $\theta_z > 0$. The explicit expression of θ_z is

$$\theta_z = \left(\frac{\rho}{1-\chi}\right)\left(\frac{\overline{\omega} + \widetilde{\omega}}{\overline{\omega} - \widetilde{\omega}}\right) \frac{1}{1 - \frac{2\theta d}{z(\overline{\omega} - \widetilde{\omega})}\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\rho}.$$

For the reservation productivity, we assume $\widehat{\widetilde{\omega}}_t = \widetilde{\omega}_z z_t$ and deduce from (31) the following expression for $\widetilde{\omega}_z$

$$\widetilde{\omega}_z = -\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\frac{(1-\chi)\,d\theta}{z\widetilde{\omega}}\theta_z - 1$$

The case of $\widetilde{\omega}_z < 0$ requires

$$-\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\frac{(1-\chi)\,d\theta}{z\widetilde{\omega}}\theta_z < 1$$

since $\theta_z > 0$, a sufficient but not necessary condition of $\widetilde{\omega}_z < 0$ is

$$\frac{1}{\overline{m}\theta^{\chi}} > \frac{\eta}{1-\chi}.$$

A.4 Proof of the Proposition 4

This Section describes the model analysis under interest rate shocks. We first define the reduced model, and then its log-linear version.

We first define the response of the reduced model to interest rate shocks and also describe its log-linearized version.

Definition 4 *The reduced model with interest rate shocks may be described using the set of endogenous variables* $\left\{\theta_t, \widetilde{\omega}_t, R_t^p, R_t^h\right\}$ *that satisfy four equations, given the set of structural parameters* $\boldsymbol{\xi}$ *defined in Definition 2. The first equation is the free entry condition (26), which becomes*

$$\frac{d}{\overline{m}}\theta_t^{1-\chi} = \frac{(1-\eta)\, z}{2\left(\overline{\omega}-\underline{\omega}\right)}\beta \mathrm{E}_t\left\{\left(\overline{\omega}-\widetilde{\omega}_{t+1}\right)^2\right\} \tag{A.9}$$

using equations for the interest rate (23), the matching probability (2), the separation rule (25), and given the assumption $z_t = z$. *The equilibrium value of the credit market tightness* θ_t *depends on the expected values for the productivity reservation,* $\mathrm{E}_t\left\{\widetilde{\omega}_{t+1}\right\}$. *The second equation is the separation rule (25), which becomes*

$$z\widetilde{\omega}_t = x^b + x^e + \left(1+R_t^h\right) - \left(\frac{1-\eta\overline{m}\theta_t^{\chi}}{1-\eta}\right)\frac{d}{\overline{m}}\theta_t^{1-\chi} \tag{A.10}$$

using equations for the rates of matching (2) and (3) and given the assumption $z_t = z$. *The equilibrium value for the productivity reservation* $\widetilde{\omega}_t$ *depends on the current values for the interest rate,* R_t^h, *and for the credit market tightness,* θ_t. *The third equation is (29), which becomes*

$$R_t^p = (1-\eta)\left[z\frac{\left(\overline{\omega}+\widetilde{\omega}_t\right)}{2} - x^e - \left(1+R_t^h\right)\right] + \eta\left(x^b - d\theta_t\right) \tag{A.11}$$

given the assumption $z_t = z$. *The equilibrium value of the credit spread is a function of the credit market tightness,* θ_t, *the reservation productivity,* ω_t, *and the interest rate shock,* R_t^h. *The fourth, and last, equation is the law of motion of the interest rate* R_t^h

$$\log\left(R_{t+1}^h\right) = \rho_{R^h}\log\left(R_t^h\right) + \left(1-\rho_{R^h}\right)\log\left(z\right) + \varepsilon_{R^h,t+1} \tag{A.12}$$

where R^h *is the steady-state value of* R_t^h, ρ_{R^h} *the persistence parameter and* $\varepsilon_{R^h} \sim iid\left(0,\sigma_{R^h}^2\right)$ *the innovation with variance* $\sigma_{R^h}^2$.

Condition (A.4) of existence and uniqueness of the equilibrium still applies. The model is log-linearized around its unique steady-state[30].

Definition 5 *The loglinearized reduced model defined in Definition 4 may be written as*

$$(1-\chi)\,\widehat{\theta}_t = \mathrm{E}_t\left\{-2\frac{\widetilde{\omega}}{\overline{\omega}-\widetilde{\omega}}\widehat{\widetilde{\omega}}_{t+1}\right\} \tag{A.13}$$

$$z\widetilde{\omega}\widehat{\widetilde{\omega}}_t = R^h\widehat{R}_t^h - \left(\frac{1}{\overline{m}}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\, d\theta\widehat{\theta}_t \tag{A.14}$$

$$R^p\widehat{R_t^p} = -(1-\eta)\, R^h\widehat{R}_t^h + (1-\eta)\,\frac{z}{2}\widehat{\widetilde{\omega}}_t - \eta d\theta\widehat{\theta}_t \tag{A.15}$$

[30]The log-deviation of the variable x, denoted $\widehat{x}$, is $\widehat{x} = \log\left(x_t/x\right)$, for $x = \theta, \widetilde{\omega}, R^p$, and R^h.

$$\widehat{R}_t^h = \rho_{R^h}\widehat{R}_{t-1}^h + \varepsilon_{R^h,t} \tag{A.16}$$

It may be assumed that x_{R^h} is the elasticity of the endogenous variable x_t to the shock z_t that satisfies $\widehat{x}_t = x_{R^h} \times \widehat{R}_t^h$ for $x = \theta, \widetilde{\omega}$, and R^p. The elasticities of $\{\theta_t, \widetilde{\omega}_t, R_t^p\}$ are

$$\theta_{R^h} = -\left(\frac{\rho_{R^h}}{1-\chi}\right)\left(\frac{2R^h}{z\left(\overline{\omega}-\widetilde{\omega}\right)}\right)\left[1 - \frac{2d/z}{\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\right]^{-1} \tag{A.17}$$

$$z\widetilde{\omega}\widetilde{\omega}_{R^h} = R^h - \left(\frac{1}{\overline{m}}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\, d\theta\theta_{R^h} \tag{A.18}$$

$$R^p R^p_{R^h} = -\left(1-\eta\right)R^h + \left(1-\eta\right)\frac{z}{2}\widetilde{\omega}_{R^h} - \eta d\theta\theta_{R^h} \tag{A.19}$$

The condition of equilibrium stability (A.5) defined in the model with technological shock still applies with interest rate shocks.

Condition (A.5) still ensures the stability of the model. In order to obtain the values of the elasticities, we introduce Equation (A.14) into Equation (A.13)

$$\left(1-\chi\right)\widehat{\theta}_t = \mathrm{E}_t\left\{-\frac{2}{z\left(\overline{\omega}-\widetilde{\omega}\right)}R^h\widehat{R}^h_{t+1} + \frac{2d/z}{\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{1-\chi}{\overline{m}\theta^{\chi}} - \eta\right)\theta\widehat{\theta}_{t+1}\right\}$$

The current log-deviation of the credit market tightness depends on the expected values of interest rate shock and credit market tightness at the next period. Given the autoregressive process for R_t^h defined in (A.12) and assuming $\mathrm{E}_t\left\{\varepsilon_{R^h,t+1}\right\} = 0$, we obtain

$$\widehat{\theta}_t = \frac{2\theta d}{z\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\mathrm{E}_t\left\{\widehat{\theta}_{t+1}\right\} - \frac{2R^h}{z\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{\rho_{R^h}}{1-\chi}\right)\widehat{R}_t^h$$

This is a standard intertemporal equation for $\widehat{\theta}_t$ that can be solved by iterating over the next period, as for the technological shock. We simplify the equation as follows:

$$\widehat{\theta}_t = \mathbf{a}\mathrm{E}_t\left\{\widehat{\theta}_{t+1}\right\} + \mathbf{b}\widehat{z}_t$$

The assumption $|\mathbf{a}| < 1$ is sufficient to guarantee the stability of the equilibrium. The solution for θ_{R^h} such that

$$\widehat{\theta}_t = \theta_{R^h} \times \widehat{R}_t^h = \frac{\mathbf{b}}{1-\mathbf{a}\rho_{R^h}} \times \widehat{R}_t^h$$

For $|\mathbf{a}\rho| < 1$, the condition $1 - a\rho > 0$ is always satisfied and since $\mathbf{b} < 0$, $\theta_{R^h} < 0$.

$$\theta_{R^h} = -\frac{2R^h}{z\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{\rho_{R^h}}{1-\chi}\right)\left[1 - \frac{2d/z}{\left(\overline{\omega}-\widetilde{\omega}\right)}\left(\frac{1}{\overline{m}\theta^{\chi}} - \frac{\eta}{1-\chi}\right)\right]^{-1} < 0$$

For the reservation productivity, we impose $\widehat{\widetilde{\omega}}_t = \widetilde{\omega}_{R^h}\widehat{R}_t^h$ and deduce from (A.14) the following expression for $\widetilde{\omega}_{R^h}$

$$z\widetilde{\omega}\widetilde{\omega}_{R^h} = R^h - \left(\frac{1}{m}\theta^{-\chi} - \frac{\eta}{1-\chi}\right)(1-\chi)\, d\theta\theta_{R^h}$$

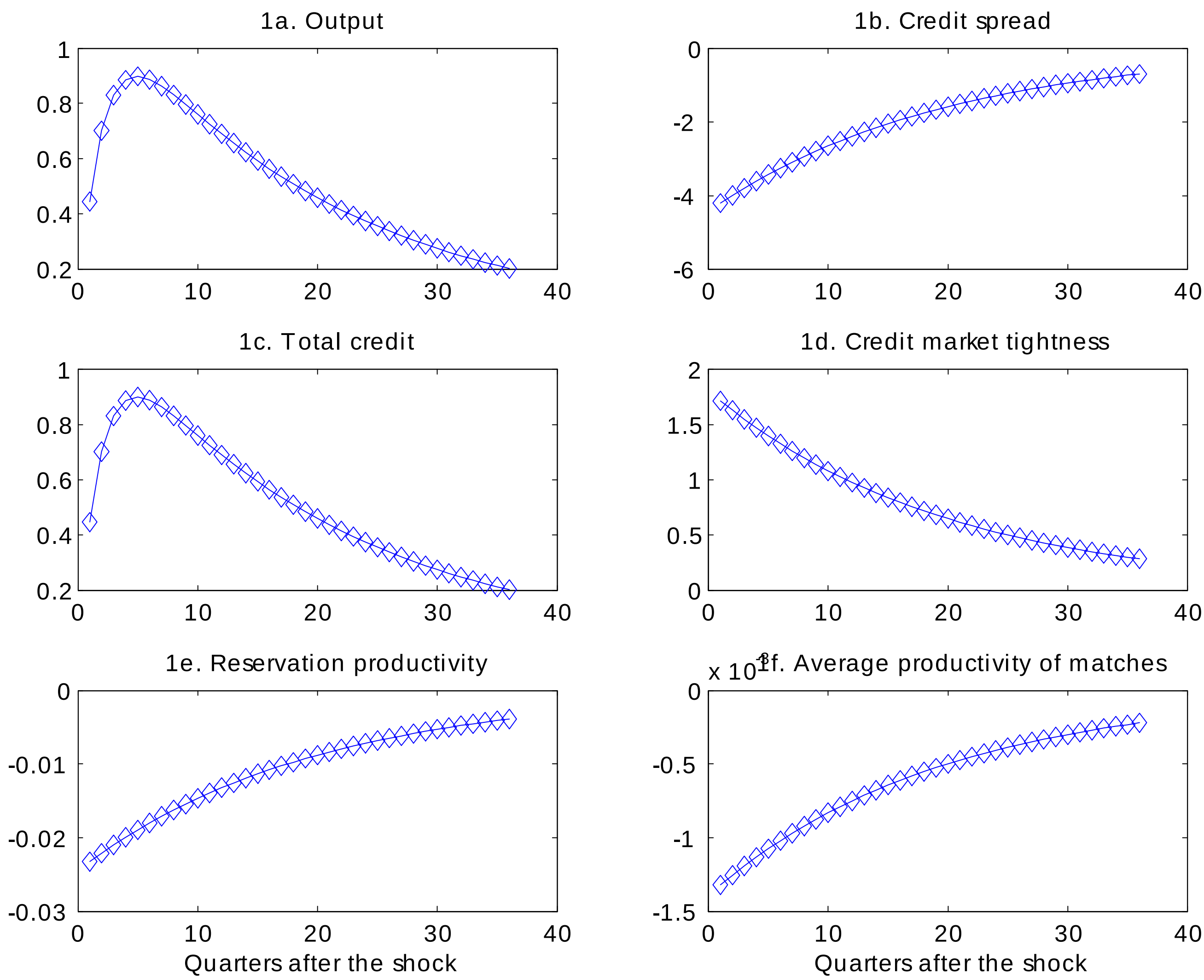
since $\theta_{R^h} < 0$, the condition (38) here implies that $\widetilde{\omega}_{R^h} > 0$.

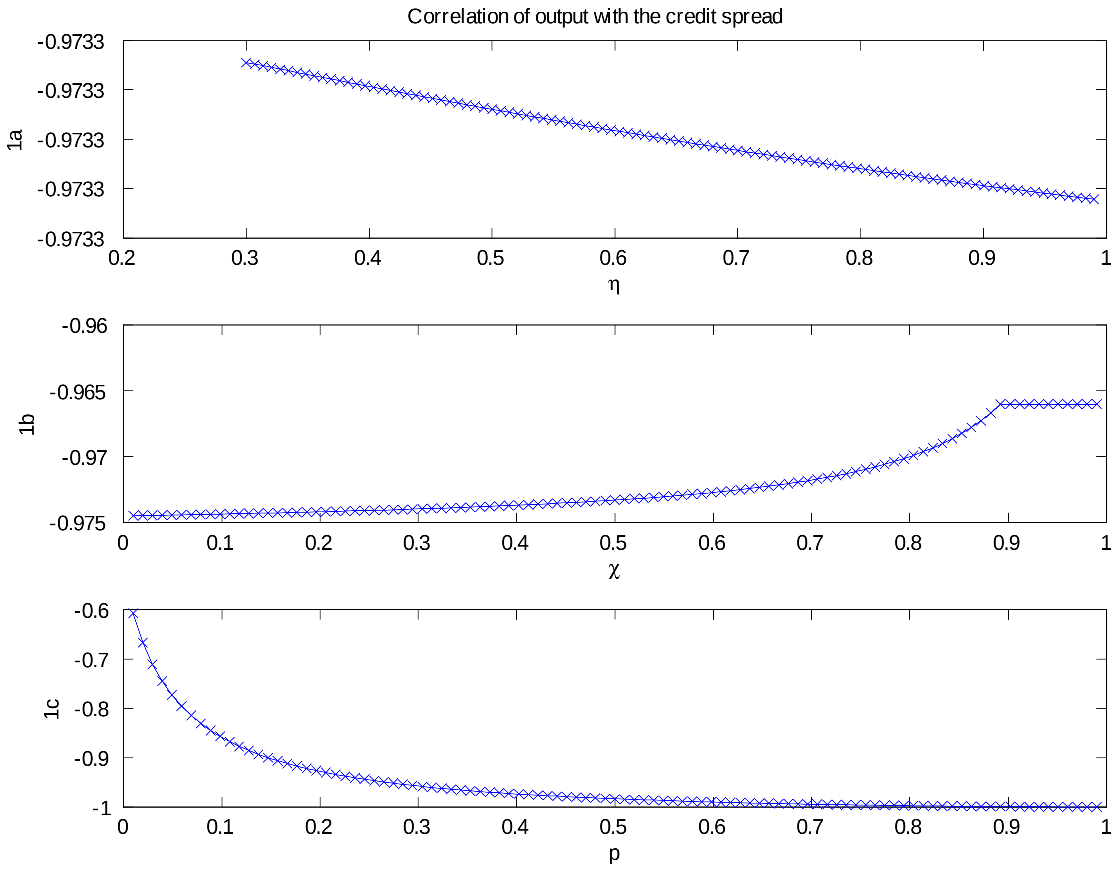
Finally, for the credit spread, we obtain

$$R^p R^p_{R^h} = -(1-\eta) R^h + (1-\eta)\frac{z}{2}\widetilde{\omega}_{R^h} - \eta d\theta\theta_{R^h}$$

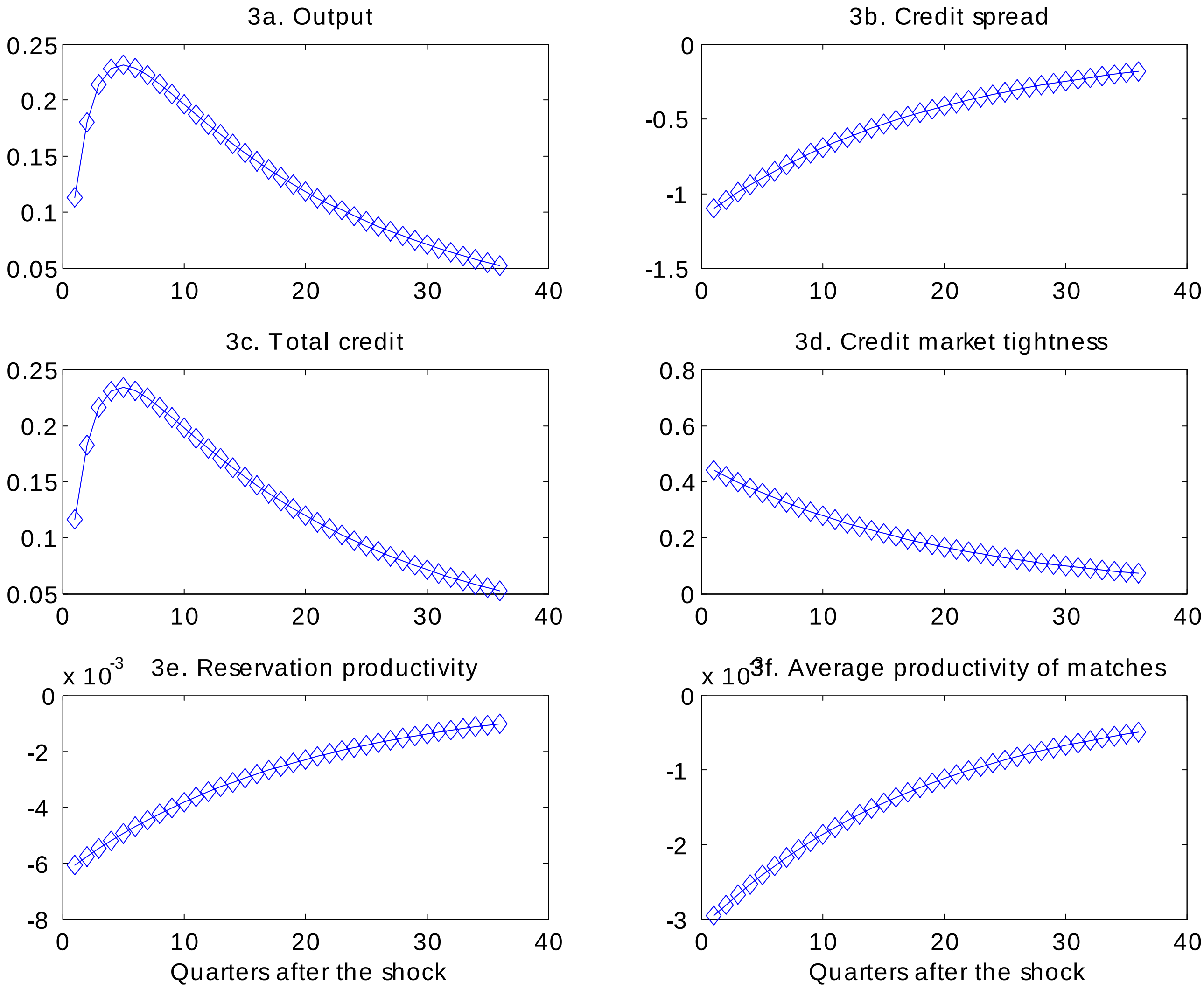
Given $\theta_{R^h} > 0$ and $R^h > 0$, condition (38) is sufficient to ensure $R^p_{R^h} > 0$, because this condition implies $\widetilde{\omega}_{R^h} < 0$.

List of Figures


1a. Output
1b. Credit spread
1c. Total credit
1d. Credit market tightness
1e. Reservation productivity
x 10^-3
1f. Average productivity of matches
Quarters after the shock
Quarters after the shock



Correlation of output with the credit spread
1a
-0.9733
-0.9733
-0.9733
-0.9733
-0.9733
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
η
1b
-0.96
-0.965
-0.97
-0.975
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
χ
1c
-0.6
-0.7
-0.8
-0.9
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
p


3a. Output
0.25
0.2
0.15
0.1
0.05
0
10
20
30
40
3b. Credit spread
0
-0.5
-1
-1.5
3c. Total credit
3d. Credit market tightness
0.8
0.6
0.4
0.2
x 10^-3
3e. Reservation productivity
-2
-4
-6
-8
Quarters after the shock
x 10^-3
3f. Average productivity of matches
-1
-2
-3
Quarters after the shock