

Bank Leverage Regulation and Macroeconomic Dynamics ^{*}

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Abstract

Regulatory constraints on bank leverage have been at the center of many policy discussions recently. One important question to emerge from these discussions has been whether these regulations should be time-dependent and how this would interact with the business cycle. We analyze this question using a dynamic stochastic general equilibrium model with banks and bank capital. In the model, bank capital emerges endogenously to solve an asymmetric information problem between banks and their creditors. The capital position of a bank thus affects its ability to attract loanable funds and, as a result, bank capital influences the business cycle through a *bank capital channel* of transmission. Government regulations on bank leverage interact with this channel. We use the model to conduct experiments on the strength of this interaction and find that regulations on bank leverage can have important effects on the shape of recessionary episodes brought about by negative shocks to technology.

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1 Introduction

Bank regulation is among the key policy issues to have emerged from the recent events in financial markets worldwide. One aspect of bank regulation that has generated intense interest is how regulatory constraints on bank leverage and balance sheets should be organized. Should they be strengthened? Should they vary with the cycle, perhaps tightening in good times and loosening when activity slows down? How should they interact with the conduct of monetary policy?

To address these important questions, we build on Holmstrom and Tirole (1997) and Meh and Moran (Forthcoming) where we construct a dynamic stochastic general equilibrium model with banks and bank capital. In the model, bank capital emerges endogenously to solve an asymmetric information problem between bankers and their creditors. As a result, the capital position of a bank affects its ability to attract loanable funds and, at a macroeconomic level, bank capital influences the transmission of shocks and business cycle through a *bank capital channel* of transmission. The model is embedded within a medium-scale version of the New Keynesian paradigm, in the spirit of Christiano et al. (2005) and Smets and Wouters (2007). Our paper thus enables this type of modeling, widely used for monetary policy analysis, to provide quantitative explorations on the likely impact of bank regulation on economic activity.

Government regulations on bank leverage and balance sheets naturally interact with this channel. For example, regulatory requirement that strengthen with economic activity would likely prevent the economy to grow as fast as it could have otherwise; loosening these requirements during recessionary episodes might in return mitigate the negative impact of such episodes on the economy. We use the model to conduct quantitative experiments on the strength of these interactions and find that regulations on bank leverage can have important effects on the shape of recessionary episodes brought about by negative shocks to technology.

2 The Model

2.1 The environment

This section describes the structure of the model and the optimization problem of the economy's agents. Time is discrete, and one model period represents a quarter. There are three classes of economic agents: households, entrepreneurs, and bankers, whose population masses are η^h , η^e and $\eta^b = 1 - \eta^h - \eta^e$, respectively. In addition there are firms producing intermediate and final goods, as well as a monetary authority.

There are three goods in the economy. First, intermediate goods are produced by monopolistically competitive firms facing nominal rigidities. Second, final goods are as-

sembled by competitive firms using the intermediate goods. Third, capital goods are produced by entrepreneurs, with a technology that uses final goods as inputs and is affected by idiosyncratic uncertainty.

Two moral hazard problems affect the production of capital goods. First, entrepreneurs can influence their technology's probability of success and may choose projects with a low probability of success, to enjoy private benefits. Bank monitoring of entrepreneurs can lessen the severity of this moral hazard problem: the more intense bank monitoring is, the less severe the moral hazard problem becomes. As an alternative to monitoring, banks can require that entrepreneurs invest their own net worth when lending to them. The choice of banks regarding the intensity with which they monitor entrepreneurs will arise as a key variable in our analysis.

A second moral hazard problem is present in the model and occurs between banks and investors, their own source of funds. Investors lack the ability to monitor entrepreneurs so they deposit funds at banks and delegate the task of monitoring entrepreneurs to their bank. However, bank monitoring is private and costly, so that banks might be tempted to monitor less than agreed, because any resulting risk in their loan portfolio would be mostly borne by investors. As a result, investors require banks to invest their own net worth (their capital) in entrepreneurs' projects. Overall, our double moral hazard framework implies that over the business cycle, the dynamics of bank capital affects how much banks can lend, the dynamics of entrepreneurial net worth affects how much entrepreneurs can borrow, and banks' monitoring intensity impacts the strength of these two net worth channels.

In addition, we consider economies where regulatory requirements limits how much banks are allowed to lend. These requirements take the form of constraints on *bank leverage*, i.e the ratio of the size of banks' balance sheets to their capital, and can be time-dependent, loosening or tightening in response to economic activity. Such regulation naturally impact the effects that bank capital has on the propagation of shocks and the business cycle and a key contribution of our analysis is to investigate quantitatively the strength of this impact.

2.2 Final good production

Competitive firms produce the final good by combining a continuum of intermediate goods indexed by $j \in (0, 1)$ using the standard Dixit-Stiglitz aggregator:

$$Y_t = \left(\int_0^1 y_{jt}^{\frac{\xi_p - 1}{\xi_p}} dj \right)^{\frac{\xi_p}{\xi_p - 1}}, \quad \xi_p > 1, \quad (1)$$

where y_{jt} denotes the time t input of the intermediate good j , and ξ_p is the constant elasticity of substitution between intermediate goods.

Profit maximization leads to the following first-order condition for the choice of y_{jt} :

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\xi_p} Y_t, \quad (2)$$

which expresses the demand for good j as a function of its relative price p_{jt}/P_t and of overall production Y_t . Imposing the zero-profit condition leads to the usual definition of the final-good price index P_t :

$$P_t = \left(\int_0^1 p_{jt}^{1-\xi_p} dj \right)^{\frac{1}{1-\xi_p}}. \quad (3)$$

2.3 Intermediate good production

Firms producing intermediate goods operate under monopolistic competition and nominal rigidities in price setting. The firm producing good j operates the technology

$$y_{jt} = \begin{cases} z_t k_{jt}^{\theta_k} h_{jt}^{\theta_h} h_{jt}^e{}^{\theta_e} h_{jt}^b{}^{\theta_b} - \Theta & \text{if } z_t k_{jt}^{\theta_k} h_{jt}^{\theta_h} h_{jt}^e{}^{\theta_e} h_{jt}^b{}^{\theta_b} \geq \Theta \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where k_{jt} and h_{jt} are the amount of capital and labor services, respectively, used by firm j at time t . In addition, h_{jt}^e and h_{jt}^b represent labor services from entrepreneurs and bankers.¹ Finally, $\Theta > 0$ represents the fixed cost of production and z_t is an aggregate technology shock that follows the autoregressive process

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_{zt}, \quad (5)$$

where $\rho_z \in (0, 1)$ and ε_{zt} is *i.i.d.* with mean 0 and standard deviation σ_z .

Minimizing production costs for a given demand solves the problem

$$\min_{\{k_{jt}, h_{jt}, h_{jt}^e, h_{jt}^b\}} r_t k_{jt} + w_t h_{jt} + w_t^e h_{jt}^e + w_t^b h_{jt}^b \quad (6)$$

with respect to the production function (4). The (real) rental rate of capital services is r_t , while w_t represents the real household wage. In addition, w_t^e and w_t^b are the compensation given entrepreneurs and banks, respectively, for their labor.

The first-order conditions of this problem with respect to k_{jt} , h_{jt} , h_{jt}^e and h_{jt}^b are respectively:

$$r_t = s_t z_t \theta_k k_{jt}^{\theta_k-1} h_{jt}^{\theta_h} h_{jt}^e{}^{\theta_e} h_{jt}^b{}^{\theta_b}; \quad (7)$$

¹Following Carlstrom and Fuerst (1997), we include labor services from entrepreneurs and bankers in the production function so that these agents always have non-zero wealth to pledge in the financial contract described below. The calibration sets the value of θ_e and θ_b so that the influence of these labor services on the model's dynamics is negligible.

$$w_t = s_t z_t \theta_h k_{jt}^{\theta_k} h_{jt}^{\theta_h - 1} h_{jt}^{e \theta_e} h_{jt}^{b \theta_b}; \quad (8)$$

$$w_t^e = s_t z_t \theta_e k_{jt}^{\theta_k} h_{jt}^{\theta_h} h_{jt}^{e \theta_e - 1} h_{jt}^{b \theta_b}; \quad (9)$$

$$w_t^b = s_t z_t \theta_b k_{jt}^{\theta_k} h_{jt}^{\theta_h} h_{jt}^{e \theta_e} h_{jt}^{b \theta_b - 1}. \quad (10)$$

In these conditions, s_t is the Lagrange multiplier on the production function (4) and represents marginal costs. Combining these conditions, one can show that total production costs, net of fixed costs, are $s_t y_{jt}$.

The price-setting environment is as follows. Each period, a firm receives the signal to reoptimize its price with probability $1 - \phi_p$; with probability ϕ_p , the firm simply indexes its price to last period's aggregate inflation. After k periods with no reoptimizing, a firm's price would therefore be

$$p_{jt+k} = \prod_{s=0}^{k-1} \pi_{t+s} p_{jt}, \quad (11)$$

where $\pi_t \equiv P_t/P_{t-1}$ is the aggregate (gross) rate of price inflation.

A reoptimizing firm chooses $\tilde{p}_{jt}$ in order to maximize expected profits until the next reoptimizing signal is received. The profit maximizing problem is thus

$$\max_{\tilde{p}_{jt}} E_t \sum_{k=0}^{\infty} (\beta \phi_p)^k \lambda_{t+k} \left[\frac{p_{jt+k} y_{jt+k}}{P_{t+k}} - s_{t+k} y_{jt+k} \right], \quad (12)$$

subject to (2) and (11).²

The first-order condition for $\tilde{p}_{jt}$ leads to

$$\tilde{p}_t = \frac{\xi_p}{1 - \xi_p} \frac{E_t \sum_{k=0}^{\infty} (\beta \phi_p)^k \lambda_{t+k} s_{t+k} Y_{t+k} \pi_{t+k}^{\xi_p}}{E_t \sum_{k=0}^{\infty} (\beta \phi_p)^k \lambda_{t+k} Y_{t+k} \pi_{t+k}^{\xi_p - 1}}. \quad (13)$$

2.4 Capital good production

Each entrepreneur has access to a technology producing capital goods. The technology is subject to idiosyncratic shocks: an investment of i_t units of final goods returns $R i_t$ ($R > 1$) units of capital if the project succeeds, and zero units if it fails. The project scale i_t is variable and determined by the financial contract linking the entrepreneur and the bank (discussed below). Returns from entrepreneurial projects are publicly observable.

Different projects are available to the entrepreneurs: although they all produce the same public return R when successful, they differ in their probability of success. Without proper incentive, entrepreneurs may deliberately choose a project with low success probability, because of private benefits associated with that project.

²Time- t profits are discounted by λ_t , the marginal utility of household income.

Assume there are two classes of projects. First, the *no private benefit* project involves a high probability of success (denoted α^g) and zero private benefits. Second, there exists a *class* of projects with private benefits. These projects all have a common, lower probability of success α^b ($\alpha^b < \alpha^g$). These projects differ in the amount of private benefits they may deliver to the entrepreneurs. Denoting by i_t the size of an entrepreneur's project, the private benefits are equal to $b i_t$, where $b \in (0, +\kappa)$. If choosing a project from this class, an entrepreneur that is not monitored will thus have an incentive to select the one with the highest b possible.

Bank monitoring of entrepreneurs can reduce the private benefits associated with a bad project. By monitoring an entrepreneur with intensity μ_t , a bank reduces its potential for gaining private benefits down to the value $b(\mu_t)$. We set the following functional form for this schedule:

$$b(\mu_t) = \chi \mu_t^{-\varepsilon_b}, \quad (14)$$

with $-\varepsilon_b = b'(\mu_t)\mu_t/b(\mu_t)$ representing the elasticity of the function. One interpretation of this private benefits-monitoring schedule is that the relationship between a bank and the entrepreneur it lends to can be close and tight, resulting in low potential for entrepreneurs to shirk, or can instead be more of a “arms-lengths” relationship, with the lower intensity of monitoring leading to higher possibility of private benefits. Figure 1 below illustrates the relationship. Note that even when monitored by his bank at intensity μ_t , an entrepreneur may still choose run a project with private benefits $b(\mu_t)$. A key component of the financial contract discussed below ensures that he has the incentive to behave and choose the good, no-private benefit, project instead.

Bank monitoring is privately costly: monitoring an entrepreneur at intensity μ_t entails a monitoring cost $z\mu_t$ in final goods. The fact that bank monitoring is not publicly observable creates a second moral hazard problem, between banks and their investors. A bank that invests its own capital in entrepreneur projects, however, lessens this problem, because this bank now has a private incentive to monitor the entrepreneurs it finances as promised. This reassures bank investors and allows the bank to attract loanable funds.

The returns in the projects funded by each bank are assumed to be perfectly correlated. Correlated projects can arise because banks specialize (across sectors, regions or debt instruments) to become efficient monitors. The assumption of perfect correlation improves the model's tractability and could be relaxed at the cost of additional computational requirements.

2.5 Financial contract

An entrepreneur with net worth n_t undertaking a project of size $i_t > n_t$ needs external financing worth $l_t^d = i_t - n_t$. The bank provides this funding with a mix of deposits it

collects from the households (d_t) as well as its own net worth (capital) a_t . Considering the costs of monitoring the project ($= z\mu_t i_t$), the bank thus lends an amount $l_t^s = a_t + d_t - z\mu_t i_t$.

We concentrate on equilibria where the financial contract leads all entrepreneurs to undertake the good project; α^g thus represents the probability of success of all projects. We also assume the presence of inter-period anonymity, which restricts the analysis to one-period contracts.

The optimal financial contract is set in real terms and has the following structure. It determines an investment size (i_t), contributions to the financing from the bank (a_t) and the bank's investors (d_t), and how the project's return is shared among the entrepreneur ($R_t^e > 0$), the bank ($R_t^b > 0$) and the investors ($R_t^h > 0$). Limited liability ensures that no agent earns a negative return. The contract also specifies a monitoring intensity μ_t , to which corresponds a severity of moral hazard $b(\mu_t)$.

The financial contract maximizes the entrepreneur's expected share of the return (which is equal to $q_t \alpha^g R_t^e i_t$ if the good project is chosen) subject to a number of constraints. These constraints ensure that entrepreneurs and bankers have the incentive to behave as agreed and that the funds contributed by the banker and the household earn (market-determined) required rates of return. The contract is given by the solution to the following optimization program:

$$\max_{\{i_t, R_t^e, R_t^b, R_t^h, a_t, d_t\}} q_t \alpha^g R_t^e i_t, \quad (15)$$

subject to

$$R = R_t^e + R_t^h + R_t^b; \quad (16)$$

$$q_t \alpha^g R_t^b i_t - z\mu_t i_t \geq q_t \alpha^b R_t^b i_t; \quad (17)$$

$$q_t \alpha^g R_t^e i_t \geq q_t \alpha^b R_t^e i_t + q_t b(\mu_t) i_t; \quad (18)$$

$$q_t \alpha^g R_t^b i_t \geq (1 + r_t^a) a_t; \quad (19)$$

$$q_t \alpha^g R_t^h i_t \geq (1 + r_t^d) d_t; \quad (20)$$

$$a_t + d_t - z\mu_t i_t \geq i_t - n_t. \quad (21)$$

Equation (16) states that the shares promised to the three different agents must add up to the total return. Equation (17) is the incentive compatibility constraint for bankers, which must be satisfied in order for monitoring to occur at the specified intensity μ_t . This equation states that the expected return to the banker that monitors, net of the monitoring costs, must be at least as high as the expected return if not monitoring, a situation in which entrepreneurs would choose the project with high private benefits and the low probability of success. Equation (18) is the incentive compatibility of entrepreneurs; given that bankers monitor at intensity μ_t , entrepreneurs can at a maximum choose the project that gives them private benefits $b(\mu_t)$; the constraint details the conditions under which they will be

induced to choose the good project over that project: this is achieved by promising them an expected return that is at least as high as the one they would get, inclusive of private benefits, if they were to choose the private benefit project. Equations (19) and (20) are the participation constraints of bankers and households, respectively. They state that these agents, when engaging bank capital a_t and deposits d_t , are promised shares of the project's return that cover the (market-determined) required rates of return on bank capital and household deposits (denoted r_t^a and r_t^d , respectively). Equation (21) indicates that the loanable funds available to a banker (its own capital and the deposits it attracted), net of the monitoring costs, must be sufficient to cover the external funding requirements of the entrepreneur.

Our analysis focuses on two solutions to this contract. First we study a *market* solution, where government regulation plays no role on the financial contract. Such cases will be the solution of the maximization problem under constraints (16)-(21). In addition, we also study *regulation* solutions, where explicit government regulations about bank capitalization and leverage affect bank lending. In such equilibria, the financial contract will also have to respect an additional constraint, restricting the leverage that a bank's balance sheet can achieve over its own capital.

Both the market and the regulation solutions, however, require that incentive-compatibility, participation and budget constraints hold. In that light, imposing that the incentive-compatibility constraints (17) and (18), as well as the budget constraint (16) hold with equality, we have

$$R_t^e = \frac{b(\mu_t)}{\Delta\alpha}; \quad (22)$$

$$R_t^b = \frac{z\mu_t}{q_t\Delta\alpha}; \quad (23)$$

$$R_t^h = R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha}; \quad (24)$$

where $\Delta\alpha = \alpha^g - \alpha^b > 0$ and $R_t^j > 0$ for $j = e, b, h$.

Note from (22) and (23) that the shares allocated to the entrepreneur and the banker are affected by the severity of the two moral hazard problems. Note for example that more intense bank monitoring (a higher value of μ_t) reduces the project share that must be allocated to entrepreneurs, because the closer relationship between the bank and the entrepreneur has decreased the latter's potential for shirking ($b(\mu_t)$ decreases). However, more intense monitoring increases moral hazard between banks and their suppliers of loanable funds, as banks promise to allocate more costly resources to its monitoring, even though this activity is private. As a result, (24) shows that the per-unit share of project return that can be credibly promised to the investors supplying loanable funds to banks is limited by the extent of these two moral hazard problems. The increase in monitoring

intensity just described therefore would, on the one hand, decrease what can be paid to investors (because of the increase in μ_t) but, on the other, decrease it because of the decline in $b(\mu_t)$; the net effect depending on ε_b , the elasticity of private benefits with respect to monitoring intensity in (14).

Introducing (24) in the participation constraint of households (20) holding with equality leads to the following:

$$(1 + r_t^d)d_t = q_t\alpha^g \left(R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right) i_t, \quad (25)$$

whereas eliminating d_t from (25) by using the resource constraint (21) leads to:

$$(1 + r_t^d) [(1 + z\mu_t)i_t - a_t - n_t] = q_t\alpha^g \left(R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right) i_t. \quad (26)$$

Next, note that (19) and (23) together can be used to express a_t as

$$a_t = \frac{\alpha^g z\mu_t}{(1 + r_t^a)\Delta\alpha} i_t; \quad (27)$$

Introducing this result in (26) and solving for i_t allows us to characterize the project size as a function of entrepreneurial net worth n_t , as follows:

$$i_t = \frac{n_t}{1 + z\mu_t - \frac{\alpha^g z\mu_t}{(1+r_t^a)\Delta\alpha} - \frac{q_t\alpha^g}{1+r_t^d} \left(R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right)} = \frac{n_t}{A_1(t)}, \quad (28)$$

where $A_1(t) = 1 + z\mu_t - \frac{\alpha^g z\mu_t}{(1+r_t^a)\Delta\alpha} - \frac{q_t\alpha^g}{1+r_t^d} \left(R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right)$ is the inverse of the leverage the entrepreneur can obtain for his net worth. Equations (26)-(27)-(28) must hold whether we consider the market solution or the regulation solution.

2.6 Market solution to the financial contract

No further constraints affect the solution to the contract (15)-(21), the market solution is obtained. Recall that the entrepreneur's objective function can be rewritten as:

$$\max_{\{i_t, R_t^e, R_t^b, R_t^h, a_t, d_t\}} q_t\alpha^g R_t^e i_t = q_t\alpha^g \frac{b(\mu_t)}{\Delta\alpha} \frac{n_t}{A_1(t)}; \quad (29)$$

Since the parameters of the problems q_t , α^g , and $\Delta\alpha$ are outside of the entrepreneur's control, this objective function is thus the equivalent of choosing μ_t to maximize the ratio $\frac{b(\mu_t)}{A_1(t)}$. The first-order condition for this maximizing problem leads to the following:

$$b'(\mu_t)A_1(t) = A_1'(t)b(\mu_t); \quad (30)$$

or, recalling that $-\varepsilon_b = b'(\mu_t)\mu_t/b(\mu_t)$,

$$-\varepsilon_b A_1(t) = A_1'(t)\mu_t; \quad (31)$$

Simple algebra then yields the following solution for monitoring intensity μ_t (see Appendix for details).

$$\mu_t = \frac{1}{z} \frac{\varepsilon_b}{\varepsilon_b + 1} \frac{(q_t \alpha^g R - (1 + r_t^d))}{1 + r_t^d + \frac{\alpha^g}{\Delta \alpha} \left(\frac{r_t^a - r_t^d}{1 + r_t^a} \right)}. \quad (32)$$

Once the monitoring intensity is identified from (32) found, i_t and $A_1(t)$ are as in (28), the required participation of the bank a_t is found from (27) and the funds raised from the outside investors d_t are from (25).

Notice that the following results can be directly obtained from (32): $\frac{\partial \mu_t}{\partial q_t} > 0$ and $\frac{\partial \mu_t}{\partial r_t^a} < 0$. The first result implies that all things equal, an increase in the price of capital goods (brought in by an increase in investment demand, for example) leads to more intense bank monitoring. The second implies that in situations where bank capital is scarcer (as measured by the required return r_t^a) bank monitoring becomes scarcer. A third derivative, $\frac{\partial \mu_t}{\partial r_t^d}$ has an ambiguous sign, but is found to be negative in most of our numerical exercises: this implies that an increase in rates brought about by a monetary tightening will lead to a decrease in bank monitoring.

One important interpretation of the market solution is that banks adhere to solvency ratios determined by market discipline. This solvency ratio can be defined as a (market-determined) restriction on the leverage that a bank is allowed to achieve over its net worth a_t . Since the size of the balance sheet of a bank is $a_t + d_t$ (on its liability side) this market-determined leverage is defined as

$$\frac{a_t + d_t}{a_t} = \gamma_t^m. \quad (33)$$

The evolution of γ_t^m represents an important benchmark of comparison with the results from the regulation solution described below. We will use it in the analysis below to represent the direction in which the market would like to go, where bank regulation to be absent.

2.7 Regulation solution to the financial contract

Consider now a situation where an explicit government regulation constrains bank leverage over its net worth. This regulation is expressed as the following constraint on the leverage achieved by the financial contract:

$$\frac{a_t + d_t}{a_t} \leq \gamma_t^g. \quad (34)$$

We now show how the imposition of (34) affects the financial contract. First assume that (34) holds with equality. We can thus express the relationship between banks' own funds a_t and outside funds d_t as

$$a_t = \frac{d_t}{\gamma_t^g - 1}. \quad (35)$$

We still want to impose that the contract remain incentive-compatible. In order to ensure this, we require that expressions (25) and (27) continue to hold: the former was derived under the conditions that the two incentive-compatibility constraints as well as the participation constraint of investors held; the latter is simply the participation constraint of banks.

Combining (35) and (27), one gets

$$d_t = \frac{(\gamma_t^g - 1) \alpha^g z \mu_t i_t}{(1 + r_t^a) \Delta \alpha}, \quad (36)$$

which, once inserted into (25), yields:

$$(1 + r_t^d) \left[\frac{(\gamma_t^g - 1) \alpha^g z \mu_t i_t}{(1 + r_t^a) \Delta \alpha} \right] = q_t \alpha^g \left(R - \frac{b(\mu_t)}{\Delta \alpha} - \frac{z \mu_t}{q_t \Delta \alpha} \right) i_t, \quad (37)$$

which can be rearranged into

$$b(\mu_t) = \Delta \alpha R - \mu_t \frac{z}{q_t} \left(1 + \frac{(1 + r_t^d)}{(1 + r_t^a)} (\gamma_t^g - 1) \right). \quad (38)$$

Expression (38) synthesizes the restrictions imposed on the financial contract by the incentive-compatible constraints, the participation constraints, and the government regulation constraint. The right-hand side of (38) describes a linear function of μ_t , with an intercept of $\Delta \alpha R$ and a negative slope. The left-hand side of the equation depends on the shape of $b(\mu_t)$, which was pictured in Figure 1 above. One possible scenario is that (38) has two solutions for μ_t , a low value μ_t^l and a high value μ_t^h . This situation is depicted in Figure 2 below.

Considering these two possible solutions, which one should the financial contract focus on? To answer this question, recall that the financial contract aims to maximize the entrepreneur's expected share of the return $q_t \alpha^g R_t^e i_t$, subject to the various constraints being satisfied. Using expression (22), we know that R_t^e can be expressed as a function of monitoring intensity μ_t only. What about i_t ? Combining (21) and (34) holding at equality, one gets

$$n_t + \gamma_t^g a_t = (1 + z \mu_t) i_t, \quad (39)$$

which, with the help of the participation constraint of the bank in (27), can be used to find the following determination of i_t as a function of μ_t :

$$\left[1 + z \mu_t \left(1 - \frac{\gamma_t^g \alpha^g}{\Delta \alpha (1 + r_t^a)} \right) \right] i_t = n_t, \quad (40)$$

or

$$i_t = \frac{n_t}{1 + z\mu_t \left(1 - \frac{\gamma_t^g \alpha^g}{\Delta\alpha(1+r_t^a)}\right)}. \quad (41)$$

Combining (22) and (41) we can now show that the objective function of the entrepreneur under the regulation solution is

$$q_t \alpha^g \left[\frac{b(\mu_t)}{\Delta\alpha} \right] \left[\frac{n_t}{1 + z\mu_t \left(1 - \frac{\gamma_t^g \alpha^g}{\Delta\alpha(1+r_t^a)}\right)} \right]. \quad (42)$$

The two possible values for μ_t identified in (38) can now be compared and the one maximizing the entrepreneur's objective be singled out. In the numerical exercises conducted thus far, μ_t^l is the one that maximizes (42).

Assuming that the low value of μ_t continues to be the optimal one, Figure 2 and expression (38) allows us to expect the following about the sensitivity of μ_t to shocks: $\frac{\partial \mu_t}{\partial q_t} < 0$, $\frac{\partial \mu_t}{\partial r_t^a} < 0$, and $\frac{\partial \mu_t}{\partial \gamma_t^g} > 0$. Notice that the first result is different from what arose from the market solution, whereas the last two are similar. Our simulations below will enable us to present some evidence on the quantitative strength of these derivatives.

2.8 Regulation Policy

When the regulation solution is analyzed, regulation on bank leverage is assumed to follow the following rule:

$$\gamma_t^g = \gamma^g + \omega_y \hat{y}_t + \epsilon_t^g, \quad (43)$$

where γ^g is the steady-state leverage ratio allowed, $\hat{y}_t$ represents output deviations from steady state, and ϵ_t^g is a shock to regulation that follows the Ar(1) process

$$\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \varepsilon_{gt}, \quad (44)$$

where $\rho_g \in (0, 1)$ and ε_g is *i.i.d.* with mean 0 and standard deviation σ_g .

The form in (43) is specified at a general level to accommodate a series of scenarios about regulation. Constant regulation would set, for example, $\omega_y = \sigma_g = 0$. By contrast a regulation that loosens standards when economic activity decreases would have $\omega_y < 0$, whereas a regulation that tightens standards when economic activity decreases would have $\omega_y > 0$.

2.9 Households

There exists a continuum of households indexed by $i \in (0, \eta^h)$. Households consume, allocate their money holdings between currency and bank deposits, supply units of specialized labor, choose a capital utilization rate, and purchase capital goods.

The wage-setting environment faced by households (described below) implies that hours worked and labor earnings are different across households. We abstract from this heterogeneity by referring to the results in Erceg et al. (2000) who show, in a similar environment, that the existence of state-contingent securities makes households homogeneous with respect to consumption and saving decisions. We assume the existence of these securities and our notation below reflects their equilibrium effect, so that consumption, assets and the capital stock are not contingent on household type i .

Lifetime expected utility of household i is

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^h - \gamma c_{t-1}^h, l_{it}, M_t^c / P_t),$$

where c_t^h is consumption in period t , γ measures the importance of habit formation in consumption, l_{it} is hours worked, and M_t^c / P_t denotes the real value of currency held.

The household begins period t with money holdings M_t and receives a lump-sum money transfer X_t from the monetary authority. These monetary assets are allocated between funds invested at a bank (deposits) D_t and currency held M_t^c so that $M_t + X_t = D_t + M_t^c$. In making this decision, households weigh the tradeoff between the utility obtained from holding currency and the return from bank deposits, the risk-free rate $1 + r_t^d$.³

Households also make a capital utilization decision. Starting with beginning-of-period capital stock k_t^h , they can produce capital services $u_t k_t^h$ with u_t the utilization rate. Rental income from capital is thus $r_t u_t k_t^h$, while utilization costs are $v(u_t) k_t^h$, with $v(\cdot)$ a convex function whose calibration is discussed in Section 3 below. Finally, the household receives labor earnings $(W_{it}/P_t) l_{it}$, as well as dividends Π_t from firms producing intermediate goods.

Income from these sources is used to purchase consumption, new capital goods (priced at q_t), and money balances carried into the next period M_{t+1} , subject to the constraint

$$c_t^h + q_t i_t^h + \frac{M_{t+1}}{P_t} = (1 + r_t^d) \frac{D_t}{P_t} + r_t u_t k_t^h - v(u_t) k_t^h + \frac{W_{it}}{P_t} l_{it} + \Pi_t + \frac{M_t^c}{P_t}, \quad (45)$$

with the associated Lagrangian λ_t representing the marginal utility of income. The capital stock evolves according to the standard accumulation equation:

$$k_{t+1}^h = (1 - \delta) k_t^h + i_t^h. \quad (46)$$

The first-order conditions associated with the choice of c_t^h , M_t^c , u_t , M_{t+1} , and k_{t+1}^h are, respectively,

$$U_1(\cdot_t) - \beta \gamma E_t U_1(\cdot_{t+1}) = \lambda_t; \quad (47)$$

³To be consistent with the presence of idiosyncratic risk at the bank level, we follow Carlstrom and Fuerst (1997) and Bernanke et al. (1999) and assume that households deposit money at a large mutual fund, which in turn deposits at a cross-section of banks, diversifying away bank-level risk.

$$U_3(\cdot_t) = r_t^d \lambda_t; \quad (48)$$

$$r_t = v'(u_t); \quad (49)$$

$$\lambda_t = \beta E_t \left\{ \lambda_{t+1} (1 + r_{t+1}^d) (P_t / P_{t+1}) \right\}; \quad (50)$$

$$\lambda_t q_t = \beta E_t \left\{ \lambda_{t+1} [q_{t+1} (1 - \delta) + r_{t+1} u_{t+1} - v'(u_{t+1})] \right\}, \quad (51)$$

where $U_j(\cdot_t)$ represents the derivative of the utility function with respect to its j^{th} argument in period t .

Wage Setting

We follow Erceg et al. (2000) and Christiano et al. (2005) and assume that each household supplies a specialized labor type l_{it} , while competitive labor aggregators assemble all such types into one composite input using the technology

$$H_t \equiv \left(\int_0^{\eta^h} l_{it}^{\frac{\xi_w - 1}{\xi_w}} di \right)^{\frac{\xi_w}{\xi_w - 1}}, \quad \xi_w > 1.$$

The demand for each labor type is therefore

$$l_{it} = \left(\frac{W_{i,t}}{W_t} \right)^{-\xi_w} H_t, \quad (52)$$

where W_t is the aggregate wage (the price of one unit of composite labor input H_t). As was the case in the final-good sector, labor aggregators are competitive and make zero profits; imposing this result leads to the following determination for the economy-wide aggregate wage:

$$W_t = \left(\int_0^{\eta^h} W_{it}^{1 - \xi_w} di \right)^{\frac{1}{1 - \xi_w}}. \quad (53)$$

Households set wages according to a variant of the mechanism used in the price-setting environment above. Each period, household i receives the signal to reoptimize its nominal wage with probability $1 - \phi_w$, while with probability ϕ_w the household indexes its wage to last period's price inflation, so that $W_{i,t} = \pi_{t-1} W_{i,t-1}$. A reoptimizing worker takes into account the evolution of its wage and the demand for its labor (52) during the expected period with no reoptimization. The resulting first-order condition for wage-setting when reoptimizing ($\widetilde{W}_{it}$) yields

$$\widetilde{W}_t = P_{t-1} \frac{\xi_w}{1 - \xi_w} \frac{E_t \sum_{k=0}^{\infty} (\beta \phi_w)^k U_2(\cdot_{t+k}) H_{t+k} w_{t+k}^{\xi_w} \pi_{t+k}^{\xi_w}}{E_t \sum_{k=0}^{\infty} (\beta \phi_w)^k \lambda_{t+k} H_{t+k} w_{t+k}^{\xi_w} \pi_{t+k}^{\xi_w - 1}},$$

where $w_t \equiv W_t / P_t$ is the real aggregate wage and $U_2(\cdot_t)$ is the derivative of the utility function with respect to hours worked. Once the household's wage is set, hours worked l_{it} is determined by (52).

2.10 Entrepreneurs and Bankers

There exists a continuum of risk neutral entrepreneurs and bankers. Each period, a fraction $1 - \tau^e$ of entrepreneurs and $1 - \tau^b$ of bankers exit the economy at the end of the period's activities.⁴ Exiting agents are replaced by new ones with zero assets.

Entrepreneurs and bankers solve similar optimization problems: in the first part of each period, they accumulate net worth, which they invest in entrepreneurial projects later in that period. Exiting agents consume accumulated wealth while surviving agents save. These agents differ, however, with regards to their technological endowments: entrepreneurs have access to the technology producing capital goods, while bankers have the capacity to monitor entrepreneurs.

A typical entrepreneur starts period t with holdings k_t^e in capital goods, which are rented to intermediate-good producers. The corresponding rental income, combined with the value of the undepreciated capital and the small wage received from intermediate-good producers, constitute the net worth n_t available to an entrepreneur:

$$n_t = (r_t + q_t(1 - \delta)) k_t^e + w_t^e. \quad (54)$$

Similarly, a typical banker starts period t with holdings of k_t^b capital goods and rents capital services to firms producing intermediate goods. Once this bank has received all its different sources of income, it has net worth

$$a_t = (r_t + q_t(1 - \delta)) k_t^b + w_t^b. \quad (55)$$

Each entrepreneur then undertakes a capital-good producing project and invests all available net worth n_t in the project. The entrepreneur's bank also invests its own net worth a_t in the project, in addition to the funds d_t invested by households. As described above, an entrepreneur whose project is successful receives a payment of $R_t^e i_t$ in capital goods whereas the bank receives $R_t^b i_t$; unsuccessful projects have zero return. Both the banker's return R_t^b and the entrepreneur's return R_t^e depend on the choice of monitoring intensity described above.

At the end of the period, entrepreneurs and bankers associated with successful projects but having received the signal to exit the economy use their returns to buy and consume final (consumption) goods. Successful surviving agents save their entire return (retain all their earnings), which becomes their beginning-of-period real assets at the start of the subsequent period, k_{t+1}^e and k_{t+1}^b . This represents an optimal choice since these agents are risk neutral and the high return on internal funds induces them to postpone consumption. Unsuccessful agents neither consume nor save.

⁴This follows Bernanke et al. (1999). Because of financing constraints, entrepreneurs and bankers have an incentive to delay consumption and accumulate net worth until they no longer need financial markets. Assuming a constant probability of death reduces this accumulation process and ensures that a steady state with operative financing constraints exists.

2.11 Monetary policy

Monetary policy sets r_t^d , the short-term nominal interest rate, according to the following rule:

$$r_t^d = (1 - \rho_r)r^d + \rho_r r_{t-1}^d + (1 - \rho_r) [\rho_\pi(\pi_t - \bar{\pi}) + \rho_y \hat{y}_t] + \epsilon_t^{mp}, \quad (56)$$

where r^d is the steady-state rate, $\bar{\pi}$ is the monetary authority's inflation target, $\hat{y}_t$ represents output deviations from steady state, and ϵ_t^{mp} is an *i.i.d* monetary policy shock with standard deviation σ^{mp} .⁵

3 Calibration

The utility function of households is specified as

$$U(c_t^h - \gamma c_{t-1}^h, l_{i,t}, M_t^c/P_t) = \log(c_t^h - \gamma c_{t-1}^h) - \psi \frac{l_{it}^{h^{1+\eta}}}{1+\eta} + \zeta \log(M_t^c/P_t).$$

The weight on leisure ψ is set in order that steady-state work effort by households is equal to 30% of available time. One model period corresponds to a quarter, so the discount factor β is set at 0.99. Following results in Christiano et al. (2005), the parameter governing habits, γ , is fixed at 0.65 and ζ is set in order for the steady state of the model to match the average ratio of $M1$ to $M2$. The parameter η is set to 1, also following Christiano et al. (2005).

The share of capital in the production function of intermediate-good producers, θ^k , is set to the standard value of 0.36. Recall that we want to reserve a small role in production for the hours worked by entrepreneurs and bankers. To this end, we fix the share of the labor input θ^h to 0.6399 instead of $1 - 0.36 = 0.64$, and then set $\theta^e = \theta^b = 0.00005$. The parameter governing the extent of fixed costs, Θ , is chosen so that steady-state profits of the monopolists producing intermediate goods are zero. The persistence of the technology shock, ρ_z , is 0.95.

Price and wage-setting parameters are set following results in Christiano et al. (2005). Thus, the elasticity of substitution between intermediate goods (ξ_p) and the elasticity of substitution between labor types (ξ_w) are such that the steady-state markups are 20% in the goods market and 5% in the labor market. The probability of not reoptimizing for price setters (ϕ_p) is 0.60 while for wage setters (ϕ_w), it is 0.64.

To parameterize households' capital utilization decision, we first require that $u = 1$ in the steady-state, and set $v(1) = 0$. This makes steady state computations independent of $v(\cdot)$. Next, we set $\sigma_u \equiv v''(u)(u)/v'(u) = 0.01$ for $u = 1$ (Christiano et al., 2005).

⁵The targeted rate for r_t^d is achieved with appropriate injections in total money supply $X_t \equiv \bar{M}_{t+1} - \bar{M}_t$, where $\bar{M}_t$ is the total money supply at time t .

The monetary policy rule (56) is calibrated to the estimates in Clarida et al. (2000), so $\rho_r = 0.8$, $\rho_\pi = 1.5$, and $\rho_y = 0.1$. The trend rate of inflation $\bar{\pi}$ is 1.005, or 2% on a net, annualized basis.

The parameters that remain to be calibrated are α^g , $\Delta\alpha$, χ , R , ε_b , τ^e , and τ^b . These are linked to the production of capital goods. In this preliminary version of the paper, they are set to $\alpha^g = 0.99$, $\Delta\alpha = 0.2$, $\chi = 0.0075$, $\varepsilon_b = 0.5$, $z = 1.0$, $R = 1.01$, $\tau^e = 0.95$, $\tau^b = 0.95$.

Finally, regulation policy in this preliminary version is assumed to be of the form where $\omega_y = 0$. Shocks to regulation are possible, however, so that $\sigma_g > 0$ and $\rho_g = 0.9$.

4 Results

This section presents our results. We first report the characteristics of the economy's steady-state, under the market solution and under the regulation solution. Next, we describe how each of these economies respond to various shocks.

4.1 Steady State Characteristics

Table 1 below presents the steady-state characteristics of the economy. The first column relates to an economy under the market solution. Bank leverage achieved under this solution is drawn from expression (33) and is roughly 20. Recall that under this solution, bank leverage is unaffected by government regulation. Next, column 2 and column 3 present the steady-state characteristics of two economies where government regulations impose tighter leverages than the market would like to achieve: column 2 imposes leverage equal to roughly 15 times the capital base and column 3 relates to an economy where leverage cannot be more than 10 times the capital base.

Table 1: Characteristics of the Economy's Steady State

Variables	Market Solution	Regulatory Solution	Tighter Regulation
Bank Leverage (γ)	19.47	14.86	9.91
Monitoring Intensity (μ)	0.0055	0.0018	0.0016
Entrepreneurs' Moral hazard ($b(\mu)$)	0.1013	0.1770	0.1871
Capital-Asset Ratio (κ)	5.13%	6.7%	10.1%
Price of capital (q)	1.034	1.041	1.042
Entrepreneurial Leverage (I/N)	2.01	1.1429	1.08
GDP (Y)	0.8016	0.8000	0.7997
Investment-Output Ratio (I/Y)	0.2314	0.2298	0.2296

Table 1 shows that tightening bank leverage leads to higher capital-asset ratios. It also leads banks to monitor less intensively and thus moral hazard from entrepreneurs' private benefits to worsen. Tightened regulation precludes the market from efficiently creating capital goods, and thus leads to a higher relative price of capital goods q and also to a lower investment to output ratio at the level of the aggregate economy. Further, the leverage that entrepreneurs can achieve over their own net worth falls. Finally, output falls as leverage tightens.

4.2 Impulse Responses to Shocks

Technology shocks

Figure 3 presents the impulse responses to a negative technology shock in the absence of bank leverage regulation (in full lines) and in the presence of time-invariant bank leverage regulation (in dashed lines). As can be seen from the figure, the effects of a negative technology shock on output and investment occur more gradually but are more persistent when there is a bank leverage regulation. This occurs for the following reasons.

The negative shock decreases the productivity of the intermediate-good production technology, a decline that is expected to persist for several periods. This reduces the expected rental income from holding capital in future periods so that desired household investment declines, as does the price of capital q_t . In the economy with no regulation on leverage, banks react to this decrease in q_t by severely reducing the intensity with which they monitor entrepreneurs (recall that $\frac{\partial \mu_t}{\partial q_t} > 0$). This makes financing for entrepreneurs more difficult to obtain, so that bank lending and aggregate investment decreases. The decrease in aggregate investment reduces earnings for banks and entrepreneurs, leading to lower levels of net worth in the next period.

In addition, the reduced intensity of monitoring lessens the moral hazard associated with banks and thus the per-unit fraction of the project return that is allocated to banks (recall equation (23)). This effect, coupled with the decreases in bank lending and in aggregate investment, means that bank net worth decreases for several periods.

By contrast however, entrepreneurial net worth decreases only slightly and then starts increasing again shortly. This occurs because two forces affect this variable in opposite ways. While the decrease in aggregate investment tends to decrease entrepreneurial earnings, the decline in bank monitoring increases the per-unit fraction of the project return allocated to entrepreneurs increases. This first effect is dominant in the earlier periods of the recessionary episode, but dominates in the latter periods. This quick recovery of entrepreneurial net worth means that in latter periods, moral hazard between banks and entrepreneurs is less severe, which allows more bank lending and a speedier recovery.

Things are markedly different in the economy with time-invariant regulation on leverage. With regulation on leverage time-invariant, banks maintain a constant capital-asset ratio throughout the episode. Further, banks do not find it in their interest to reduce monitoring, because doing so would not be consistent with an unchanged leverage. Unchanged monitoring and leverage means that bank lending does not decrease as much in this economy and the initial impact of the recessionary episode are thus lessened, relative to the economy with no regulation. However, the recovery from the recession takes longer to take hold in the regulated economy. This occurs because entrepreneurial net worth, instead of being cushioned against the recession, is now falling severely. After several periods of such declines, the reduced levels of entrepreneurial net worth worsen the moral hazard between banks and entrepreneurs, and since more intense monitoring is not an option because of the regulation,

A Shock to Regulation : Allowed Bank Leverage Declines

Figure 4 presents the impulse responses following a sudden tightening in leverage regulation. This represents a negative shock to γ_t^g in a regulated economy and constrains bank to decrease their leverage, relative to their initial level.

In complying with this change of regulation, banks reduce their intensity of monitoring, which also reduces their ability to lend effectively. As a result, bank lending decreases, and so does aggregate investment.

As was the case following the negative technology shock, the decrease in aggregate investment reduces earnings for banks and entrepreneurs, leading to lower levels of net worth in the next period. This effect, coupled with the reduction in per-unit fractions of the project return allocated to banks (a result of the decrease in monitoring) means that bank net worth decreases for several periods, continuing to hinder bank lending and investment.

By contrast, entrepreneurial net worth increases throughout the episodes. Once again this occurs because of the two forces affecting this variable: while the decrease in aggregate investment tends to decrease entrepreneurial earnings, the decline in bank monitoring increases the per-unit fraction of the project return allocated to entrepreneurs increases. This second effect is dominant and as a result, entrepreneurial net worth increases, lessening moral hazard between banks and entrepreneurs and allowing for a recovery in bank lending and investment.

The effects of Counter-cyclical Bank Leverage Regulation

Figure 5 presents the effects of a negative technology shocks on two economies with regulatory requirements on bank leverage. In the first economy (full lines), allowed leverage

is constant and thus does not react to the shock. The responses of this economy are thus the same as those depicted earlier in Figure 3: although the unchanged leverage shields the economy initially from the impact of the recession, it makes the recovery more difficult to arrive, so that the negative effects of the shock are quite persistent.

In the second economy (dashed lines) regulatory requirements follow a counter-cyclical patterns, so that $\omega_y < 0$ (recall expression (43)).⁶ This means that as the recessionary episode created by the negative technology shock start affecting the economy, the regulation allows banks to increase their leverage. This policy is shown to reduce the negative effects of the shock, relative to the time-invariant regulation. This arises because the increase in leverage incites banks to up their monitoring intensity. This reduces moral hazard between entrepreneurs and their lenders and allows more lending to proceed all things equal. Aggregate investment does not decrease as much as in the time-invariant economy, therefore and the decrease in output is also muted.

5 Conclusion

To be completed.

⁶We set $\omega_y = -20$.

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Figure 1. Bank Monitoring and Private Benefits

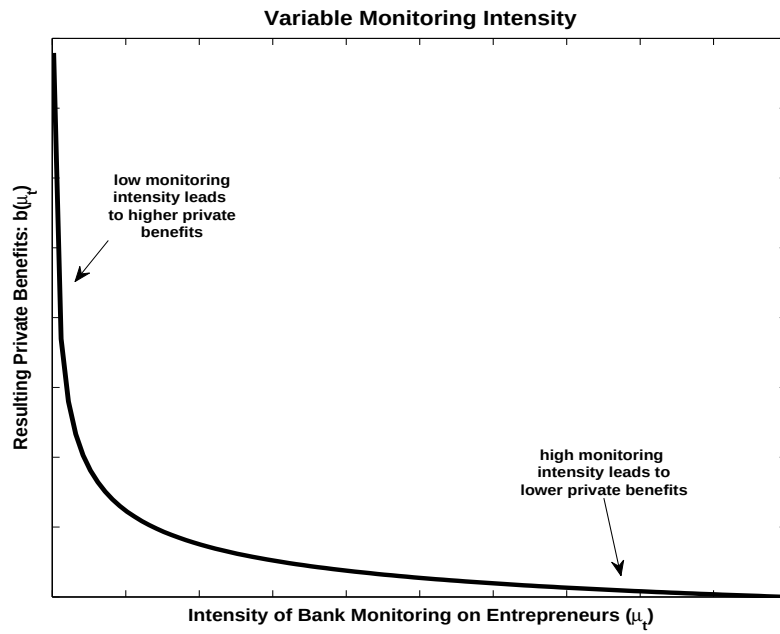


Figure 2. Expression (38): Finding μ_t under the Regulation Solution

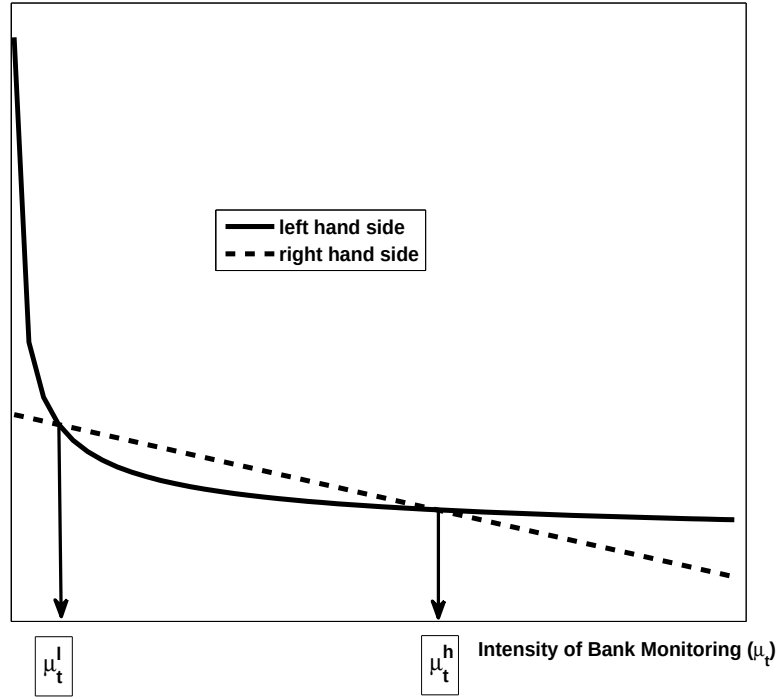


Figure 3. Responses to a Negative Technology Shock

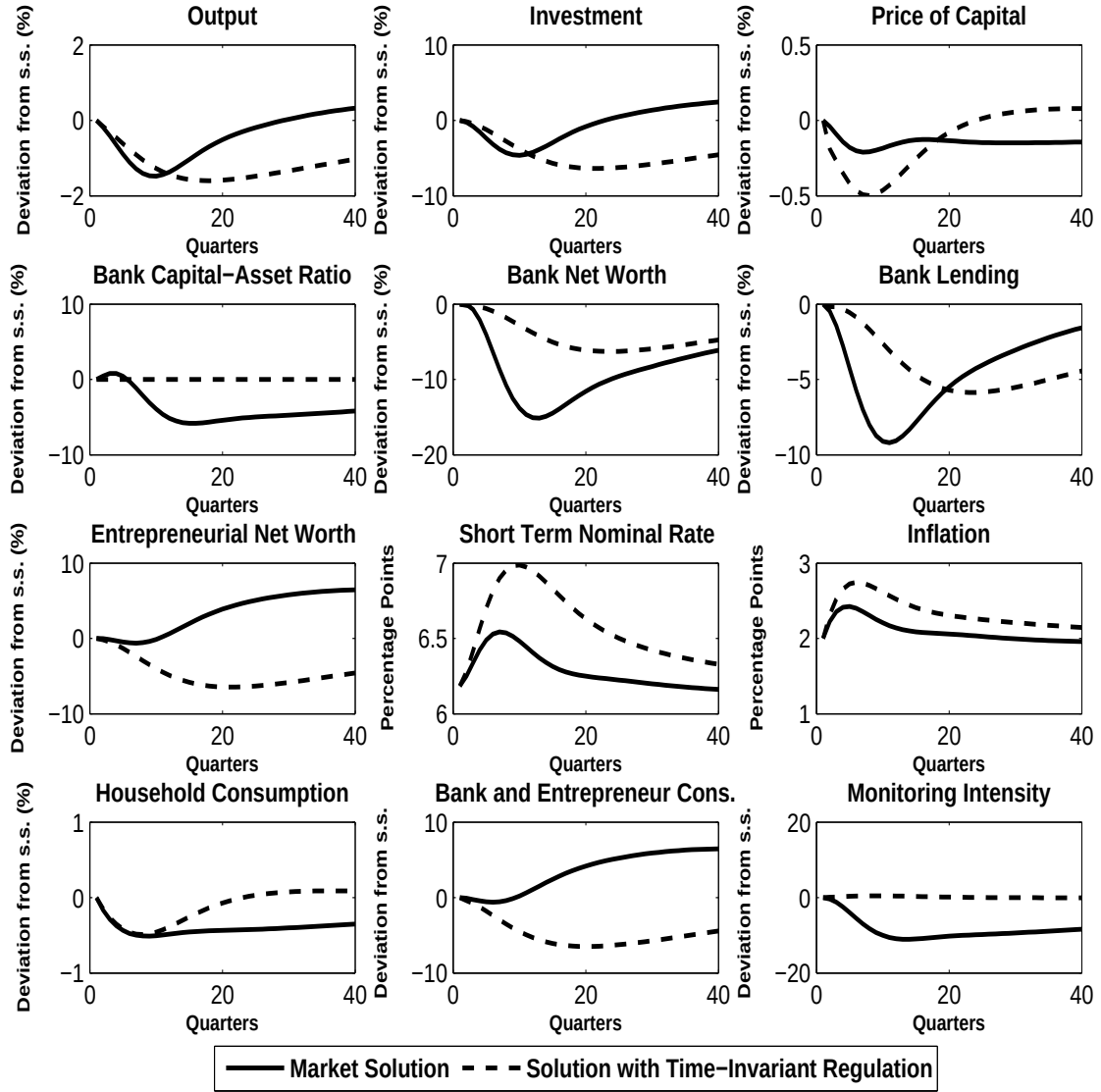


Figure 4. Regulated Economy: Responses to a Tightening of Standards
(Allowed Leverage γ^d Decreases)

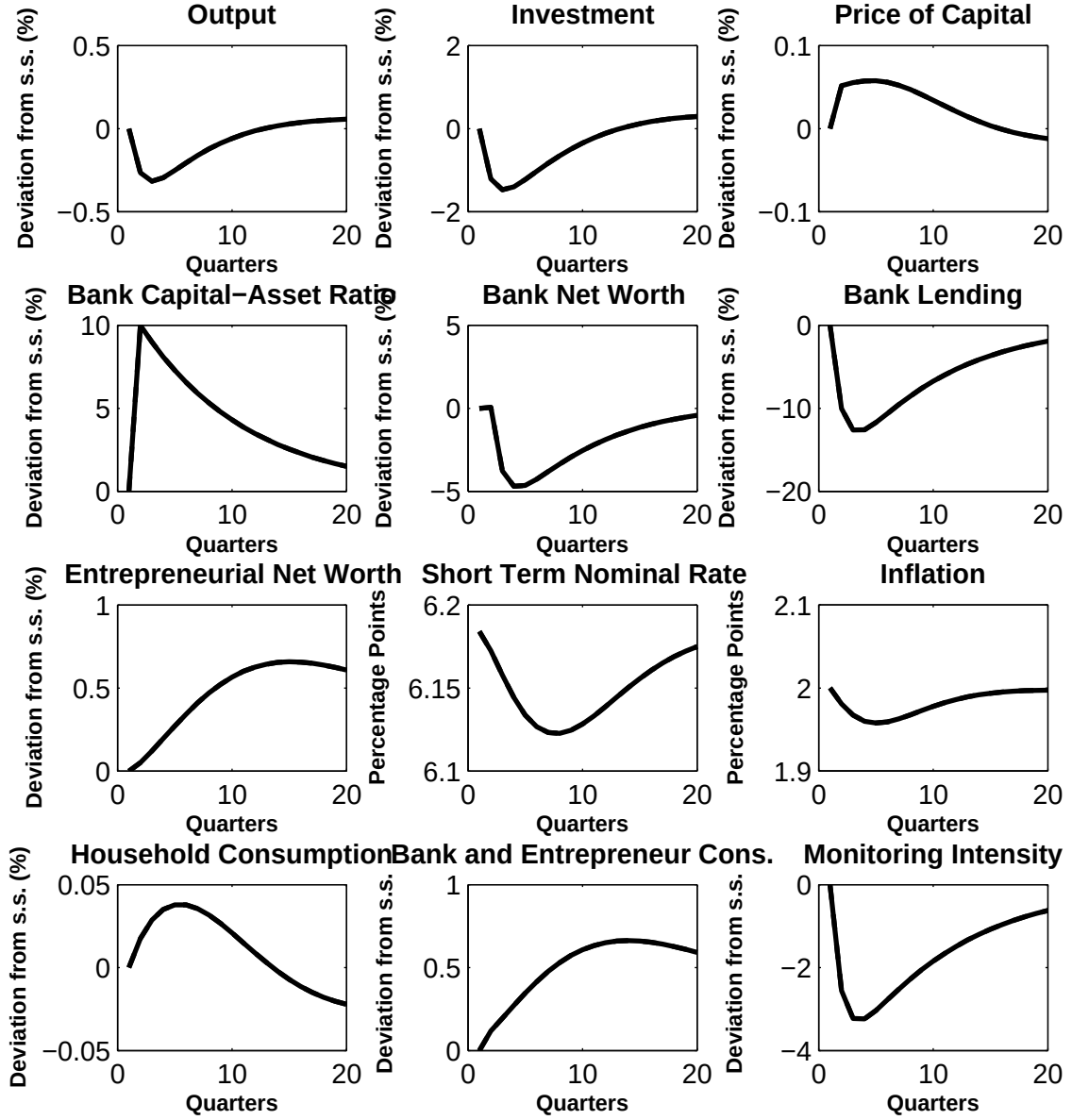


Figure 5. Negative Technology Shock: Time-Invariant versus Counter-Cyclical Regulation

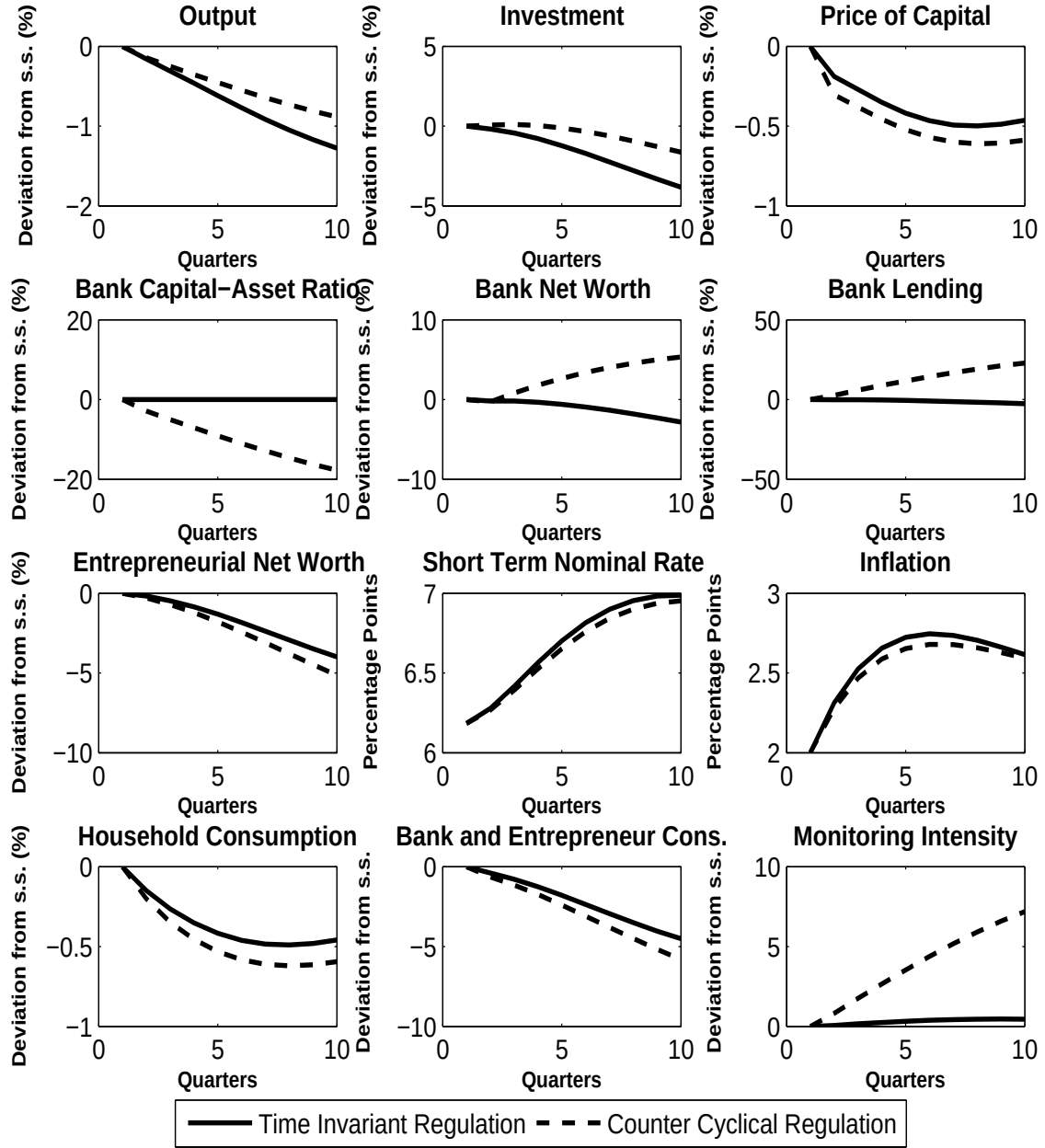
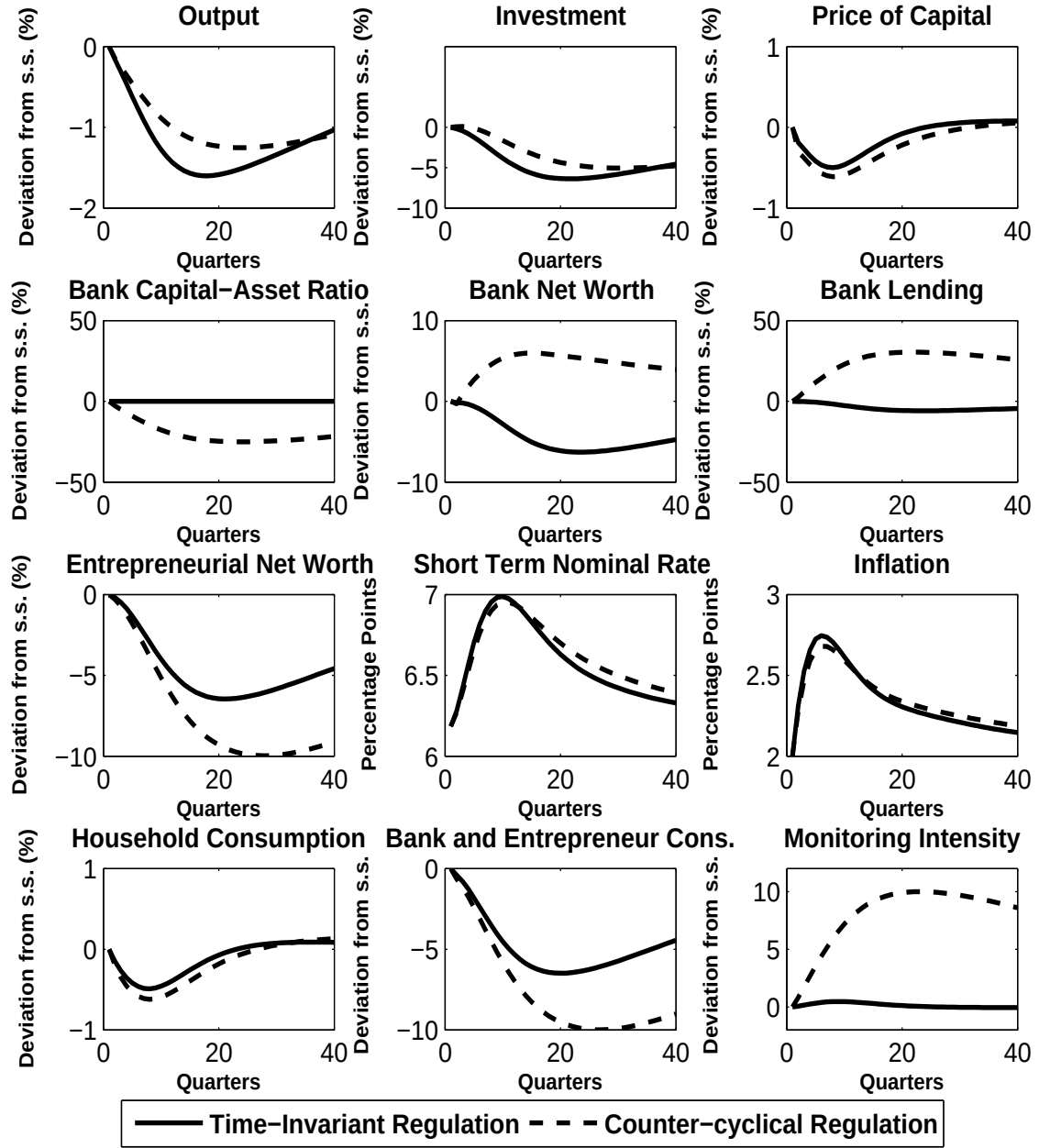


Figure 5. Negative Technology Shock: Time-Invariant versus Counter-Cyclical Regulation



A Algebra for the market-based solution for μ_t

Combining the first-order condition (31) with the definition of leverage $A_1(t)$ yields:

$$-\varepsilon_b \left[1 + z\mu_t - \frac{\alpha^g z\mu_t}{(1+r_t^a)\Delta\alpha} - \frac{q_t\alpha^g}{1+r_t^d} \left(R - \frac{b(\mu_t)}{\Delta\alpha} - \frac{z\mu_t}{q_t\Delta\alpha} \right) \right] = \mu_t \left[z - \frac{\alpha^g z}{(1+r_t^a)\Delta\alpha} + \frac{q_t\alpha^g}{1+r_t^d} \left(\frac{b'(\mu_t)}{\Delta\alpha} + \frac{z}{q_t\Delta\alpha} \right) \right], \quad (57)$$

Expanding and simplifying, this expression becomes:

$$-\varepsilon_b - \varepsilon_b z\mu_t \left[1 + \frac{\alpha^g}{\Delta\alpha} \left(\frac{1}{(1+r_t^d)} - \frac{1}{(1+r_t^a)} \right) \right] + \frac{\varepsilon_b q_t \alpha^g R}{(1+r_t^d)} - \frac{\varepsilon_b q_t \alpha^g b(\mu_t)}{\Delta\alpha(1+r_t^d)} = \mu_t z \left[1 + \frac{\alpha^g}{\Delta\alpha} \left(\frac{1}{(1+r_t^d)} - \frac{1}{(1+r_t^a)} \right) \right] - \frac{\varepsilon_b q_t \alpha^g b(\mu_t)}{\Delta\alpha(1+r_t^d)}, \quad (58)$$

Noticing that the last terms on both the left-hand side and the right-hand side cancel out, and combining terms, the expression becomes:

$$\varepsilon_b \left(\frac{q_t \alpha^g R}{(1+r_t^d)} - 1 \right) = z\mu_t(1+\varepsilon_b) \left[1 + \frac{\alpha^g}{\Delta\alpha} \left(\frac{1}{(1+r_t^d)} - \frac{1}{(1+r_t^a)} \right) \right], \quad (59)$$

or, solving for μ_t :

$$\mu_t = \frac{1}{z} \left(\frac{\varepsilon_b}{1+\varepsilon_b} \right) \frac{q_t \alpha^g R - (1+r_t^d)}{(1+r_t^d) + \frac{\alpha^g}{\Delta\alpha} \left(1 - \frac{1+r_t^d}{1+r_t^a} \right)} \quad (60)$$

as in the text.