Public Policy, Technological Change, and the Evolution of Educational Attainment

Rui Castro* Daniele Coen-Pirani†

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Abstract

The goal of this paper is to develop and calibrate a quantitative general equilibrium model to account for the joint evolution of educational attainment and relative wages among education groups in the U.S. over the 20th century. The key exogenous explanatory variables we consider are skill-biased technical change, U.S. education policies, and demographic changes. The main features of the model are: heterogeneity in ability among individuals; three levels of sequential educational attainment: less than high school, high school, and college; public and private colleges, frictionless asset market; endogenous schooling costs and skill prices; skill-biased technical change.

Keywords: Public and Private Education, Tuition Policy, Skill-biased Technical Change.

JEL Classification: I2, J2, O1.

*Department of Economics and CIREQ, Université de Montréal. Email: rui.castro@umontreal.ca. Web: http://www.fas.montreal.ca/sceco/castroru

†Tepper School of Business, Carnegie Mellon University. E-mail: coenp@andrew.cmu.edu. Web: http://www.andrew.cmu.edu/user/coenp/web

1 Introduction

The goal of this paper is to develop and calibrate a quantitative model to account for the joint evolution of educational attainment and relative wages among education groups in the U.S. over the 20th century. The key exogenous explanatory variables we consider are skill-biased technical change, U.S. education policies, and demographic changes. We are not only interested in matching initial and final attainment levels, but also in explaining the different medium-run variations that are observed in the data from one decade to the next.[1] In the literature so far, there has been no attempt at addressing this question in a comprehensive and systematic way.

The history of educational attainment during the 20th century can be roughly split into two parts (Goldin and Katz (2008)). The period before WWII was characterized by rapidly increasing rates of high school attendance and graduation (the high school movement). In this period, wage differentials among skill groups have tended to decline as documented by Goldin and Katz (2008) using the Iowa Census and other sources. Between 1950 and 1970, college graduation rates exhibited rapid growth accompanied by upward-trending college premia. Starting in 1970 measures of growth in educational attainment at both high school and college level show a significant slowdown. College enrollment rates fall in the 1970s, and then start rising again the 1980s but at a slower pace than in the 1950s and 1960s. High school graduation rates stagnate or even decline after 1970. The college-high school premium has notably declined during the 1970s, increased dramatically in the 1980s, and experienced a significant slowdown starting in the late-1990s (Autor, Katz, and Kearney, 2008). The post-WWII period is also characterized by faster growth in educational attainment (and labor force participation) by females than by males. While we abstract from this important development in the current version of the paper, we plan to extend the benchmark model

[1] Restuccia and Vandenbroucke (2008) construct a simple model to evaluate the specific role of educational premiums for the rise in postwar attainment. In addition to considering additional factors driving attainment, our approach differs from theirs in that we focus on the evolution of attainment over the whole 20th century (not just attainment today relative to attainment right after WWII).

to account for differences among genders in future versions. We review the evidence on educational attainment in detail in Section 3.

This evidence, if taken at face value, presents a serious challenge to the view that returns to education alone have been the driving force behind the increase in attainment over the last century. The empirical literature suggests that other factors might have played an important role. Goldin and Katz (2008) emphasize the role played by the diffusion of public secondary education during the first half of the 20th century in accounting for the rise in high school graduation rates. Similarly, the analysis in Card and Lemieux (2001); Fortin (2006); Bound and Turner (2007) points to the importance of school resources in explaining the slowdown in educational attainment after 1970. They point to the fact that large baby-boom cohorts of new students entered college beginning in the late 1960s, and extending into the early 1980s. This increase was not matched either by an increase in public spending on education, or by an increase in college tuition. College revenues have been rising only since the early 1980s.

Our starting point is a simple benchmark model, sharing some features with Heckman, Lochner, and Taber (1998) and Restuccia and Vandenbroucke (2008). The setup is an overlapping generations model with time-varying cohort size. Within each cohort, individuals differ in ability at the time when they begin attending school. This initial human capital can be either used to produce goods and earn income or to accumulate more human capital. We abstract from frictions in financial markets so that individuals make education choices to maximize the present discounted value of lifetime earnings net of direct schooling costs. At the beginning of their lives, individuals are enrolled in high school and decide whether to drop out or complete this degree, and whether to continue their education and obtain a college degree. Human capital accumulation occurs only at school, in a sequential fashion, and depends on learning ability and on teachers' human capital. We endogenize schooling costs by postulating that all teachers are college graduates hired competitively on the labor market. While high schools are assumed to be entirely publicly funded, college education can be either public or private. In the public system, expenditures per student are constrained by the amount of public funding for higher education and college tuition is subsidized out of general tax revenue. Students attending private colleges, instead, can optimally choose the quality of their education but receive no subsidy. Finally, we embed this schooling choice problem in a general equilibrium setting by using an aggregate production function to endogenize skill-specific wages.

To make this model quantitative, we plan to solve it by assuming an initial steady state in 1900 circa and then simulating a transition to a new steady state far into the future. We will then iteratively search for the set of parameters that provide a satisfactory fit to the joint evolution of educational attainment and relative wages over the course of the 20th century. The main data source for this exercise is the U.S. Census for individual data on educational attainment and earnings and publicly available data on the financing of education (expenditures and tuition) in the U.S. In the current version of the paper we illustrate its main mechanisms by focusing on a steady state, calibrating the model, and performing a series of policy experiments. We consider (i) an increase in tuition subsidies, (ii) an increase in public expenditures on education at both the high-school and the college levels, (iii) an increase in public expenditures on education at the college level only, and (iv) a technology shift favoring college-educated workers. Our current results show a negligible effect on attainment of tuition subsidies and public expenditures directed at college level. The latter arises due to fully offsetting general equilibrium effects. When public expenditures increase in all education levels, we observe an increase in high-school graduation. Finally, skill-biased technological change is able to significantly increase college graduation.

2 Literature Review

TBA

3 Empirical Evidence

In this section we review the main facts about the long-run evolution of educational attainment in the U.S. (see Goldin and Katz (2008) for a comprehensive review). People born in 1900 had an average of about 9 years of schooling by age 35. This number increased steadily for half a century, and people born in 1950 had a much higher average, of about 13 years of schooling. This happened during the 1970s. During this decade, we observed a dramatic and striking slowdown in educational attainment. The 1960 birth cohort had actually slightly lower attainment compared to the 1950 cohort. Educational attainment subsequently resumed its upward trend, but at a much slower rate than before. More recently we observe yet again a levelling of attainment at around 14 years of schooling.

3.1 High School

Among people born in 1900, the high school graduation rate was slightly under 30% for women and slightly above 20% for men. For both women and men born 50 years later, the graduation rate reached almost 90%. This is, once again, the beginning of the 1970s. Then, high school graduation suddenly stopped for women, and actually declined for men for over 20 years, until the 1990s. There is only a very modest recovery for the most recent cohorts. See Figure 1, based on the Census IPUMS.

3.2 College

College attainment reveals similar dynamics. For the 1900 cohort, the college graduation rate was slightly above 5% for men and slightly below 5% for women. For the 1950 cohort, 30% of men aged 30 and over graduated from college, but only about 25% of women did. During the 1970s, college graduation declined for both sexes, more so for men. In fact, the growth in graduation resumed quickly for women, while it kept stagnant for men. For the most recent cohorts, 35% of women graduate from college, while only about 27% of men do.

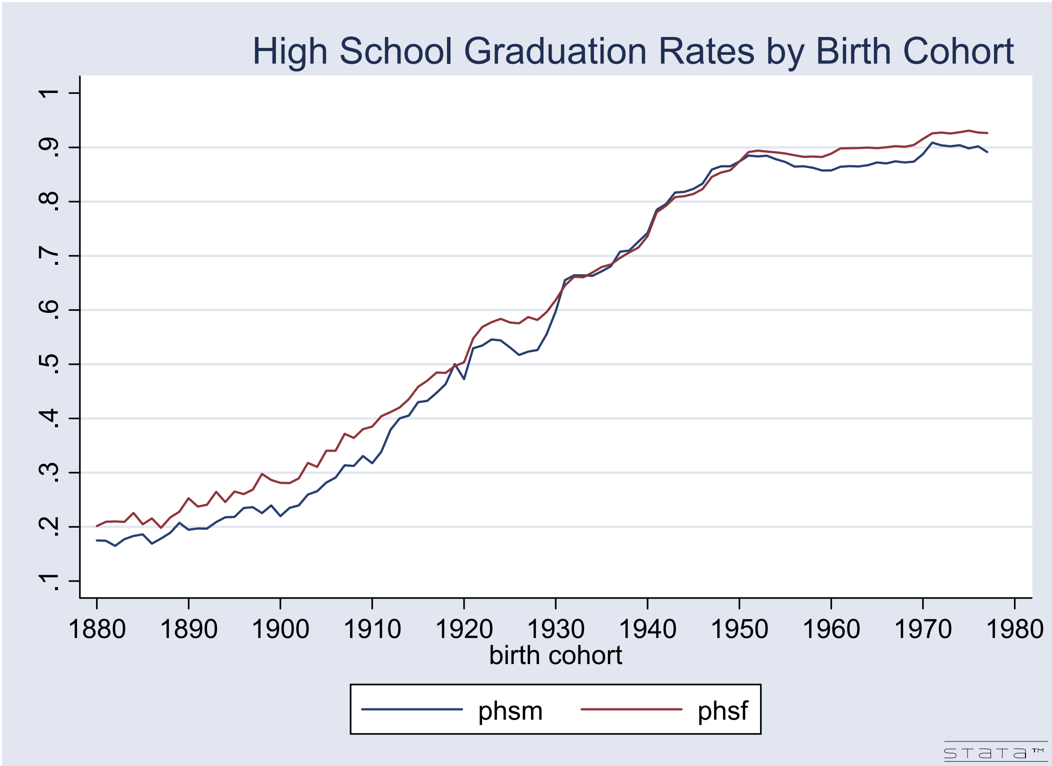

Figure 1: High-school graduation rates by birth cohort

See Figure 2, based on the Census IPUMS.[2]

Figure 3 decomposes the college graduation rate of Figure 2 according to

$$\text{Prob}(coll) = \text{Prob}(coll| \geq hs) \times \text{Prob}(\geq hs).$$

That is, the college graduation rate is the product of the college graduation rate among those with at least a high-school degree, times the high school graduation rate.[3]

This decomposition indicates whether college and high school graduation respond to the same underlying forces. If so, then most of the variation in college graduation should be accounted for by variation in high school graduation. Otherwise, increases in the college graduation rate, say, should be accounted for by more people going to and graduating from

[2]The college graduation rate is the fraction of people in a given birth cohort that has obtained a college degree by the age of 30.

[3]The high school graduation rate is the same series as the one in Figure 1.

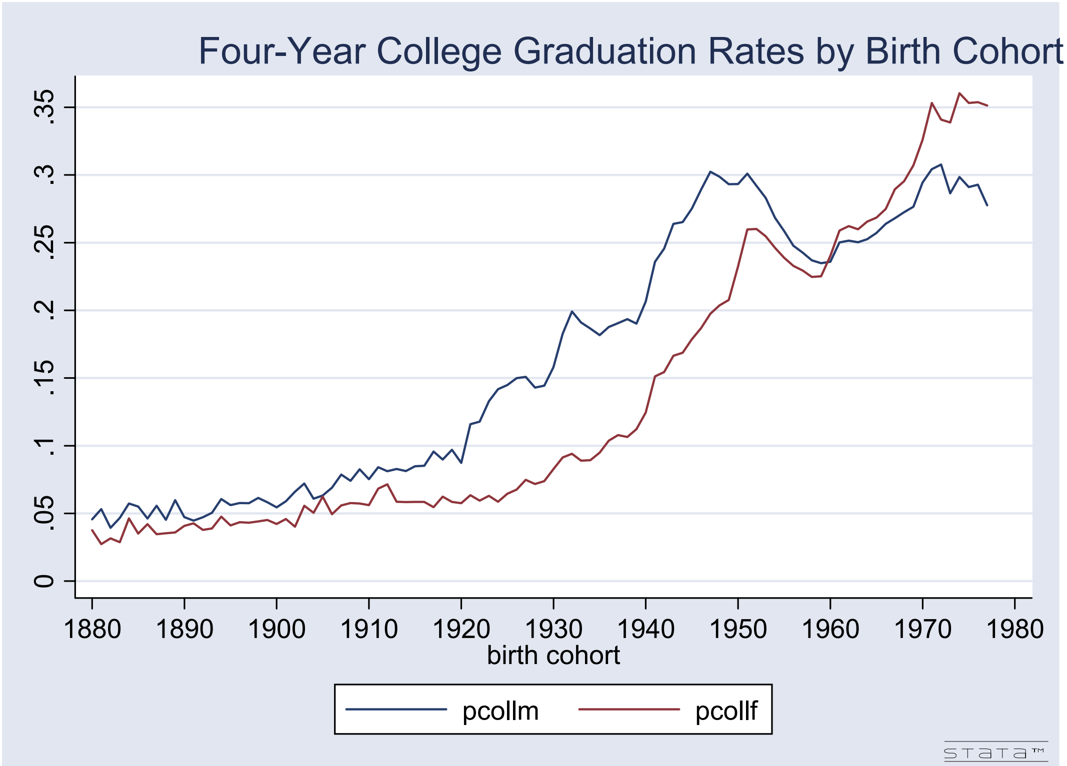

Figure 2: College graduation rates by birth cohort

college, for a given high school graduation rate. The term Prob($coll| \geq hs$) is like an intensive margin, whereas the term Prob($\geq hs$) is like an extensive margin.

In the figure, the intensive margin for males is the blue line (pcolldm) and the extensive margin the green line (phsm). The intensive margin for females is the red line (pcolldf) and the extensive margin the orange line (phsf).

Figure 3 seems to suggest three phases. First, for the cohorts born from 1930 to 1950, the increase in the college graduation rate for both men and women comes exclusively from a higher high school graduation rate. This suggests common factors driving both graduation rates for the 1930-1950 birth cohorts. Second, the decline in college graduation rate for both men and women born in 1950-1960 comes exclusively from a drop in the intensive margin. The high school graduation rate is now flat rather than increasing, but more importantly there are less men and women with a high school degree that go to and finish college. The

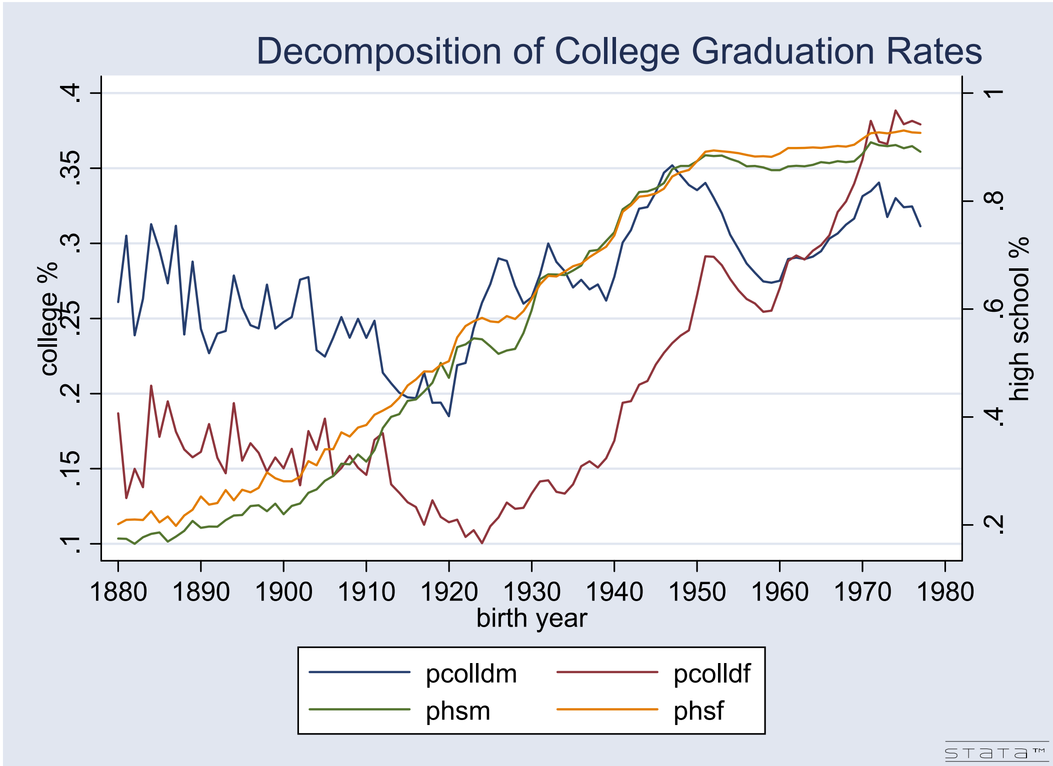

Figure 3: College graduation rate, decomposition

subsequent recovery of college graduation rates for both sexes for cohorts born after 1960 also seems to come mostly from the intensive margin. High school graduation rates have essentially plateaued, whereas it's the college graduation rate out of high school graduates that increases. We identified two turning points in the series for male college graduation (1947 and 1959 birth cohorts) and then quantified the contribution of the intensive and the extensive margins for its evolution. Table 1 displays the average growth rate of each component by birth cohort subinterval.

4 Model

We consider an overlapping generations model of schooling and education choice. Individuals differ in terms of innate ability and birth cohort. Financial markets are complete, so education choices are not constrained by individual financial resources. Education is public

		1930-47	1948-1959	1960-1977
Males	Prob($coll$)	.0382066	-.0210824	.0093043
	Prob($coll \mid \geq hs$)	.016915	-.0209112	.007148
	Prob($\geq hs$)	.0213406	-.0001712	.0021563
Females	Prob($coll$)	.0512281	.0109031	.0247218
	Prob($coll \mid \geq hs$)	.0329276	.007387	.0220007
	Prob($\geq hs$)	.0183802	.0035161	.0027212

Table 1: Decomposing the evolution of the college graduation rate

at the primary and secondary level and either private or public at the tertiary one. The model is a general equilibrium one, in which government expenditures are financed through lump-sum taxes and skill prices are determined by the usual marginal product conditions. Differently from most of the literature we assume that human capital accumulated at the tertiary level is an input in the production of new human capital. This feature of the model allows us to endogenize the cost of providing education. The inputs in the production of human capital at each schooling level are teachers' human capital, innate ability, and own human capital accumulated in the previous schooling level.

4.1 Setting

Lifetime. An individual's economic life starts at age 14 with attendance of the first grade of high school. We assume that all individuals attend s_1 years of high school. Individuals who attend s_2 years of high school are able to graduate. High school graduates can proceed with their education and attend college for another s_3 years. At the end of college individuals join the workforce. Individuals live for L periods. For convenience, we define $S_k = \sum_{j=1}^{k} s_j$ to represent the cumulative number of years of education of an agent, for $k = 1, 2, 3$, with $S_0 \equiv 0$.

Production Functions. Human capital can be of type $k = 1, 2, 3$, corresponding to less than high school, high school degree, and completed college degree. The production functions for human capital for an individual who belongs to cohort τ are:

$$\begin{aligned}
h_{0\tau} &= \theta, \\
h_{1\tau} &= g_1\left(e_{1\tau}, h_{0\tau}\right), \\
h_{2\tau} &= g_2\left(e_{2\tau}, h_{1\tau}\right), \\
h_{3\tau} &= g_3\left(e_{3\tau}, h_{2\tau}\right).
\end{aligned}$$

The inputs are: innate ability θ, a composite measure, denoted by $e_{k\tau}$, of teachers' human capital during the s_k years of school level k, and previously accumulated human capital. The output of the education process is newly produced human capital of type k, whose quantity is denoted by h_k.

Preferences and Budget Constraint. Individuals maximize the present value of their lifetime utility, discounted at the rate β:

$$\sum_{l=0}^{L-1} \beta^l u\left(c_{\tau+l}\right), \tag{1}$$

where $c_{\tau+l}$ denotes the consumption of an agent of cohort τ in year l of his life. We assume that financial markets are complete and there are no borrowing constraints, so there is one lifetime budget constraint, which takes the following form for an individual that attends school for S_k years:

$$\sum_{l=0}^{L-1} R^l c_{\tau+l} + P_k + \sum_{l=0}^{L-1} R^l T_{\tau+l} = h_k \sum_{l=S_k}^{L-1} R^l w_{k\tau+l}, \tag{2}$$

where $R \equiv (1+r)^{-1}$ and r is the exogenous interest rate, P_k denotes the present value of direct schooling costs for school level k, $T_{\tau+l}$ represents lump-sum taxes in year $\tau+l$, and $w_{k\tau+l}$ the wage per unit of human capital of type k in period l of an individual's life.

Government. The government collects lump-sum taxes and spends them on education and possibly other items. The lump-sum nature of taxes guarantees that they can always adjust to balance the government's budget without affecting schooling decisions. Thus, we can consider variation in the government's education policy without taking into account the feedback effect on taxes. Government policy is summarized in the quadruple $\left(T_t, x_t^{\text{hs}}, x_t^{\text{pb}}, \delta_t\right)$. We define x_t^{hs} to be the average units of teachers' human capital per student enrolled in high school in year t and assume that all students enrolled in high school in the same year receive the same exposure to teachers' human capital. Similarly, x_t^{pb} denotes the average teachers' human capital for students in public college in year t. Last, the government subsidizes public college expenditures at the rate δ_t, with students paying the remaining portion.

Production of goods. Goods are produced using human capital of the three different types as inputs:

$$Y_t = F\left(A_{1t}H_{1t}, A_{2t}H_{2t}, A_{3t}\left(H_{3t} - H_{3t}^{\text{ed}}\right)\right),$$

where F is constant returns to scale. The inputs H_{kt} are the stocks of human capital of type k available in calendar period t and A_{kt} is a type-specific technological shift. Since human capital is also an input in the production of new human capital we subtract the human capital of type $k = 3$ employed in the education sector H_{3t}^{ed} from H_{3t}. The index t denotes time. Each aggregate human capital stock is the sum of human capital across cohorts of workers and is explicitly defined later. The stock H_{3t}^{ed} can be written as:

$$H_{3t}^{\text{ed}} = H_{3t}^{\text{pr}} + H_{3t}^{\text{pb}} + H_{3t}^{\text{hs}}, \tag{3}$$

where the terms on the right-hand side denote total teachers' human capital in private college, public college, and high school, respectively. The latter is defined as:

$$H_{3t}^{\text{hs}} = x_t^{\text{hs}}\left(\sum_{l=0}^{S_1-1} N_{t-l} + \sum_{l=S_1}^{S_2-1} N_{t-l}\left(1-\mu_{1t-l}\right)\right), \tag{4}$$

where $\left(1-\mu_{1t-l}\right)$ is the fraction of cohort born in $t-l$ that attends high school in t. Similarly, for public colleges we define:

$$H_{3t}^{\text{pb}} = x_t^{\text{pb}} \sum_{l=S_2}^{S_3-1} N_{t-l} \mu_{3t-l}^{\text{pb}}, \tag{5}$$

where μ_{3t-l}^{pb} is the fraction of the cohort born in $t-l$ that attends public college in year t. Finally, H_{3t}^{pr} is defined as:

$$H_{3t}^{\text{pr}} = \overline{x}_t^{\text{pr}} \sum_{l=S_2}^{S_3-1} N_{t-l} \mu_{3t-l}^{\text{pr}}, \tag{6}$$

where μ_{3t-l}^{pr} is the fraction of the cohort born in $t-l$ that attends private college in year t. Average units of human capital of type $k=3$ employed in private college in year t are denoted by $\overline{x}_t^{\text{pr}}$. Note that while all students attending public college are exposed to the same x_t^{pb}, students in private college can purchase human capital in the market, so they will be in general exposed to different levels of teachers' quality.

Schools. Schools operate a constant returns to scale technology that takes as input units of teachers' human capital (type $k=3$) and generates as output the composite measure $e_{k\tau}$. Specifically, $e_{k\tau}$ is a function of the sequence of teachers human capital per student per year, $x_{k\tau+l}$ in that schooling level:

$$e_{k\tau} = f_k \left(x_{k\tau+S_{k-1}}, x_{k\tau+S_{k-1}+1}, ..., x_{k\tau+S_k-1} \right) \text{ for } k = 1,2,3. \tag{7}$$

For high schools and public college, the inputs are equal to:

$$x_{k\tau+S_{k-1}+l} = x_{\tau+S_{k-1}+l}^{\text{hs}}, \text{ for } l = 0, ..., S_k - 1 - S_{k-1}, \text{ and } k = 1,2 \tag{8}$$

$$x_{k\tau+S_{k-1}+l} = x_{\tau+S_{k-1}+l}^{\text{pb}}, \text{ for } l = 0, ..., S_k - 1 - S_{k-1}, \text{ and } k = 3. \tag{9}$$

The present discounted cost of providing $e_{3\tau}^{\text{pb}}$ units of college education for public colleges is:

$$\sum_{l=S_2}^{S_3-1} R^l w_{3\tau+l} x_{\tau+l}^{\text{pb}}.$$

The present discounted value of tuition costs for public college for an individual of cohort

τ is then:

$$p_\tau^{\text{pb}} \times e_{3\tau}^{\text{pb}} = \sum_{l=S_2}^{S_3-1} R^l \left(1-\delta_{\tau+l}\right) w_{3\tau+l} x_{\tau+l}^{\text{pb}},$$

where p_τ^{pb} is the per unit tuition and $e_{3\tau}^{\text{pb}}$ denotes the number of units of college education obtained from equation (7) after replacing the inputs (9). High school expenditures are fully subsidized by the government, so $p_\tau^{\text{hs}} = 0$.

Differently from public schools, private colleges offer a menu of education units e_3^{pr} and tuition $p_\tau^{\text{pr}} \times e_3^{\text{pr}}$, where p_τ^{pr} denotes the present value of tuition per unit of college education:

$$\begin{aligned} p_\tau^{\text{pr}} &= \min_{\{x_{3\tau+l}\}} \sum_{l=S_2}^{S_3-1} R^l w_{3\tau+l} x_{3\tau+l} \\ & \quad s.t. \\ 1 &= f_3\left(x_{3\tau+S_2}, x_{3\tau+S_2+1}, ..., x_{3\tau+S_3-1}\right). \end{aligned} \tag{10}$$

Note that if an individual purchases twice as many units of private education as another individual, he pays double tuition because the production function f_3 displays constant returns to scale.

4.2 Schooling and Consumption Choices

Individuals maximize utility subject to the budget constraint. The choice problem can be split into two parts. First, the individual chooses the education level to attain in order to maximize the present discounted value of gross income net of direct schooling costs. Second, for given choice of education, the individual chooses how to allocate consumption over time.

We begin with the analysis of the optimal schooling problem. For this purpose, define the present value of earnings per unit of human capital of type k to be:

$$W_{k\tau} = \sum_{l=S_k}^{L-1} R^l w_{k\tau+l}.$$

An agent of cohort τ with ability θ compares the present values of net incomes associated with the following options:

- High school dropout:

$$V_{1\tau}(\theta) = W_{1\tau} h_{1\tau}(\theta).$$

- High school degree:

$$V_{2\tau}(\theta) = W_{2\tau} h_{2\tau}(\theta).$$

- Public college:

$$\begin{aligned} V_{3\tau}^{\mathrm{pb}}(\theta) &= W_{3\tau} h_{3\tau}^{\mathrm{pb}}(\theta) - p_{\tau}^{\mathrm{pb}} \times e_{3\tau}^{\mathrm{pb}}, \\ h_{3\tau}^{\mathrm{pb}}(\theta) &= g_3\left(e_{3\tau}^{\mathrm{pb}}, h_{2\tau}(\theta)\right). \end{aligned} \tag{11}$$

- Private college:

$$V_{3\tau}^{\mathrm{pr}}(\theta) = \max_{e_{3\tau}^{\mathrm{pr}}} \left\{ W_{3\tau} h_{3\tau}^{\mathrm{pr}}(\theta) - p_{\tau}^{\mathrm{pr}} \times e_{3\tau}^{\mathrm{pr}} \right\},$$

$$h_{3\tau}^{\mathrm{pr}}(\theta) = g_3\left(e_{3\tau}^{\mathrm{pr}}, h_{2\tau}(\theta)\right).$$

An agent (θ, τ) selects the option that gives the highest net income. For convenience we partition the set of types (θ, τ) based on their choices. Let $\Theta_{k\tau}$ for $k = 1, 2, 3$ denote the set of (θ, τ) such that option k is optimal. Note that $\Theta_{3\tau}$ includes individuals who attend either public or private college. Last denote by $h_{3\tau}(\theta)$ the human capital of a college educated individual:

$$h_{3\tau}(\theta) = \begin{cases} h_{3\tau}^{\mathrm{pb}}(\theta) \ \text{ if } V_{3\tau}^{\mathrm{pb}}(\theta) \geq V_{3\tau}^{\mathrm{pr}}(\theta) \\ h_{3\tau}^{\mathrm{pr}}(\theta) \ \text{ if } V_{3\tau}^{\mathrm{pb}}(\theta) < V_{3\tau}^{\mathrm{pr}}(\theta) \end{cases}.$$

Note that the direct schooling costs in the budget constraint (2) are $P_k = 0$ if $k = 1, 2$, $P_3 = p_{\tau}^{\mathrm{pb}} \times e_{3\tau}^{\mathrm{pb}}$ if the individual chooses a public college, and $P_3 = p_{\tau}^{\mathrm{pr}} \times e_{3}^{\mathrm{pr}}$ if he chooses a private one.

Given the education choice we can solve for the optimal consumption profile. The first order condition for consumption is:

$$u'\left(c_{\tau+l}\right) = \beta R^{-1} u'\left(c_{\tau+l+1}\right) \text{ for } l = 0, ..., L-2. \tag{12}$$

4.3 Aggregation and Equilibrium

We assume that the ability distribution is the same across all cohorts and we denote it by $G\left(\theta\right)$. We can then define the measure of agents in each cohort that chooses a given level of schooling:

$$\mu_{k\tau} = \int_{\theta \in \Theta_{k\tau}} dG\left(\theta\right) \text{ for } k = 1, 2, 3.$$

We also distinguish between college educated individuals who attend private college and college educated individuals who attend public college. The corresponding measures are $\mu_{3\tau}^{\text{pr}}$ and $\mu_{3\tau}^{\text{pb}}$.

We are now ready to define the aggregate stocks of human capital at each point in time. For each cohort τ and each education level k, we compute the quantity of human capital available (i.e. cohort is alive and not in school) for production and associated with that cohort and education level at time t:

$$\overline{H}_{k\tau t} = \begin{cases} 0 \text{ if } \left(t < \tau + S_k \text{ or } t > \tau + L - 1\right) \\ N_\tau \int_{\theta \in \Theta_{k\tau}} h_{k\tau}\left(\theta\right) G\left(\theta\right) d\theta \text{ else} \end{cases}. \tag{13}$$

Total human capital in t is then:

$$H_{kt} = \sum_{\tau=-\infty}^{t} \overline{H}_{k\tau t}. \tag{14}$$

That is we are summing over all possible cohorts from the very first one to the one just born.

Profit maximization by firms producing output implies that the wage per unit of human capital is:

$$w_{kt} = A_{kt} F_k \left(A_{1t} H_{1t}, A_{2t} H_{2t}, A_{3t} \left(H_{3t} - H_{3t}^{\text{ed}} \right) \right). \tag{15}$$

We are now ready to define an equilibrium for this economy. Given the exogenous sequences of population by cohorts $\{N_t\}$, government policies $\left\{T_t, x_t^{\text{hs}}, x_t^{\text{pb}}, \delta_t\right\}$, technology $\{A_{1t}, A_{2t}, A_{3t}\}$, the interest rate r, a competitive equilibrium is represented by a sequence of human capital rental prices $\{w_{1t}, w_{2t}, w_{3t}\}$, tuition levels per unit of college education $\left\{p_t^{\text{pb}}, p_t^{\text{pr}}\right\}$, consumption choices $\{c_t(\theta)\}$, education choices, education expenditures for private college $\{e_{3t}^{\text{pr}}(\theta)\}$, aggregate stocks of human capital of each type $\{H_{1t}, H_{2t}, H_{3t}\}$, such that:

- education choices and associated expenditures are optimal given government policies, rental prices, and tuition levels;
- consumption choices satisfy the Euler equation (12) and budget constraint (2);
- private college tuition solves problem (10);
- rental prices are equal to the marginal products of human capital, as in (15);
- the aggregate human capital stocks are given by equations (13), (14), (3), (5), (6), (4);
- the government budget is balanced.

5 A Numerical Example

To investigate the qualitative properties of the model we consider a steady state with a constant population, government policies, and technology shifts.

5.1 Functional Forms

In order to calibrate the model we must specify functional forms for the production function for goods, human capital, and schools' output.

Production of goods. We start by assuming the following Cobb-Douglas production function:

$$Y = A\left(H_1 + \omega H_2\right)^{\mu}\left(H_3 - H_3^{\mathrm{ed}}\right)^{1-\mu}.$$

In this specification high school dropouts and high school graduates are perfect substitutes in production.

Schools' production functions. Schools' production functions are also isoelastic:

$$f_k\left(x_{kS_{k-1}}, x_{kS_{k-1}+1}, ..., x_{kS_k-1}\right) = \left(x_{kS_{k-1}}^{\eta_{1k}} x_{kS_{k-1}+1}^{\eta_{2k}} ... x_{kS_k-1}^{\eta_{s_k k}}\right)^{\frac{1}{\eta_{1k}+\eta_{2k}+...+\eta_{s_k k}}},$$

The school cost minimization problem (10) yields an optimal profile of teachers' human capital over the course of a student's private college education. We choose the exponents $\left\{\eta_{1k}, .., \eta_{s_k k}\right\}$ for $k = 3$ in such a way as to generate a constant input profile. For public schools the inputs are constant over time by assumption in a steady state, so the η exponents are irrelevant. However, when the economy is not in the steady state these coefficients matter. In order to calibrate their values for public schools we apply the same principle illustrated above for private ones. This procedure implies:

$$\eta_{lk} = R^{S_{k-1}+l} \text{ for } l = 0, .., s_k - 1.$$

Note that the exponents weigh later inputs less than earlier ones in the production function. This is needed to balance the effect of time discounting on the valuation of inputs' costs, as later inputs are "cheaper" from a present value perspective.

Production of human capital. The production functions for human capital are assumed to take the following isoelastic forms:

$$
\begin{aligned}
h_0 &= \theta, \\
g_1(e_1, h_0) &= e_1^{\alpha} h_0^{\gamma}, \\
g_2(e_2, h_1) &= e_2^{\alpha} h_1^{\gamma}, \\
g_3(e_3, h_2) &= e_3^{\alpha} h_2^{\gamma}.
\end{aligned}
$$

Note that the specification of the weights η's implies that $e_1 = e_2 = x^{\text{hs}}$.

Distribution of ability. We assume that $G(\theta)$ is lognormal with parameters μ_θ and σ_θ.

We focus on the schooling choices of the agents. The latter do not depend on the utility function or on the discount factor.

5.2 Parameters

The model is calibrated at a yearly frequency. We assume that the model is in a steady state in the year 1950 and use Census data and data from the Department of Education to calibrate some of the model's parameters.

The following parameters have to be calibrated:

- Population: $\overline{N}$. We normalize $\overline{N} = 1$.

- Interest rate: r. The interest rate is taken to be $r = 0.04$ or four percent per year.

- Time periods: s_1, s_2, s_3, L. We assume a duration of life $L = 52$ years after age 14, so that an individual retires/dies at age 65. The schooling periods are: $s_3 = 4$ (four year college), $s_2 = 4$ (high school degree), $s_3 = 3$ (high school dropout).

- Output production function: A, μ, ω. We normalize $A = 1$ and set $\mu = 0.893$ to match the share of total earnings of individuals without a four-year college degree. This figure comes from the 1950 Census. We set ω to match the share of high school dropouts in the working-age population. This is 58.3 percent in 1950. The resulting value of ω is 0.52.

- Ability distribution parameters: $\mu_\theta, \sigma_\theta$. See below.

- Production function for human capital: α, γ. See below.

- Government policies: $x^{\text{pb}}, x^{\text{hs}}, \delta$. We set $\delta = 0.537$. This represents an estimate of one minus the ratio of tuition to expenditures in public colleges in 1950 (Conrad and Hollis, 1955). We also set $x^{\text{hs}} = 0.489x^{\text{pb}}$ to account for the fact that expenditures per student in public high schools in 1950 are estimated to be about 49 percent of expenditures per student in public college. Finally, we set x^{pb} jointly with the other parameters not calibrated above.

We search for values of the parameters $\left(\mu_\theta, \sigma_\theta, \alpha, \gamma, x^{\text{pb}}\right)$ to make the model approximately consistent with the following features of the data:

- The share of high school graduates in 1950, 34.9 percent of the working-age population.

- The ratio of yearly earnings of college-educated workers relative to high-school educated workers. This ratio is 1.35 and corresponds to the coefficients on a college dummy in a cross-sectional regression for 1950 that controls for sex, race, and experience.

- The ratio of yearly earnings of workers with a high school degree workers relative to workers without a high-school degree. This ratio is 1.46 and corresponds to the coefficients on a high school completion dummy in a cross-sectional regression for 1950 that controls for sex, race, and experience.

- The standard deviation of log earnings in the group of high-school dropouts (after controlling for the other observables). This is 0.97.

- The standard deviation of log earnings in the group of high-school graduates (after controlling for the other observables). This is 0.90.

Our procedure yields the following parameter values:

$$\mu_\theta = 1.0,\ \sigma_\theta = 0.3,\ \alpha = 0.1,\ \gamma = 1.5,\ x^{\text{pb}} = 1.0.$$

As shown in Table 2, the model is roughly consistent with the distribution of the working-age population

5.3 Policy Experiments

The purpose of this section is to illustrate some of the properties of the model by performing a set of policy experiments. Specifically, we consider four policy experiments:

- Experiment 1. Higher tuition subsidies in public college. The parameter δ is increased from its benchmark value of 0.537 to 0.75.

- Experiment 2. Higher expenditures in high school and public college. The parameters x^{pb} and x^{hs} are both increased by 10 percent, so as to keep their ratio constant.

- Experiment 3. Higher expenditures in public college and constant expenditures in high school. The ratio $x^{\text{hs}}/x^{\text{pb}}$ is lowered by 10 percentage points from its benchmark value of 0.489 to 0.389, while keeping x^{hs} constant at its benchmark value.

- Experiment 4. A technology shift that favors skilled labor. We capture it by decreasing the production function parameter μ from its benchmark value of 0.893 to 0.5.

For each experiment we consider both a general equilibrium (GE) version in which skill prices are allowed to adjust in response to the policy change, and a partial equilibrium one (PE) in which skill prices are kept constant at their benchmark values. The following table presents the results of the benchmark model and of the counterfactual exercises of interest.

	Bench.	Exp. 1		Exp. 2		Exp. 3		Exp. 4
		GE	PE	GE	PE	GE	PE	GE
μ_1 (%)	58.3	58.3	58.3	55.8	55.8	58.3	58.3	58.3
μ_2 (%)	39.6	39.6	39.6	42.1	41.7	39.6	39.2	22.2
μ_3^{pb} (%)	2.1	2.1	2.1	2.1	2.5	2.1	2.5	19.5
μ_3^{pr} (%)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
college premium	3.951	3.949	3.949	4.050	3.952	3.950	3.851	3.042
high school premium	2.438	2.438	2.438	2.460	2.437	2.438	2.413	1.856
inequality hs dropouts (%)	28.86	28.86	28.86	28.32	28.32	28.86	28.86	28.86
inequality hs grads (%)	30.94	30.93	30.93	31.92	31.21	30.93	30.17	12.54
inequality college grads (%)	34.07	34.07	34.08	34.05	34.63	34.07	34.67	47.13
w_3/w_2 ($\times 100$)	21.625	21.622	21.625	21.154	21.625	21.139	21.625	32.122
p^{pb} ($\times 100$)	32.30	17.44	17.45	34.75	35.52	39.68	40.60	47.97
p^{pr} ($\times 100$)	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
hs cut-off (θ)	2.89	2.89	2.89	2.84	2.84	2.89	2.89	2.89
public college cut-off (θ)	4.99	4.99	4.99	5.00	4.90	4.99	4.89	3.52
private college cut-off (θ)	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Table 2: Policy Experiments

Note that the ratio w_2/w_1 is a constant equal to the parameter ω so it does not change across policy experiments. Note also, that in the steady state of the model the education choices are independent of the overall level of skill prices and they are only a function of their relative values.

We find that increasing tuition subsidies in public colleges has generally very small effects on college attendance, even in the more extreme case of zero tuition (case $\delta = 1$, not reported). Larger effects are obtained when increasing government spending on education in high school and college (experiment 2). There we observe the fraction of high school dropout decline, without any significant change in college attendance. The lack of change in college attendance is due to general equilibrium effects, as the relative skill price of college educated workers falls after the introduction of the policy. This reduces the incentives to respond to the policy shift. Experiment 3 generates similar outcomes with the feedback due to general equilibrium reducing the incentives to react to the policy shift. Finally, in response to an increase in the relative demand for college educated labor, the supply of college educated workers increases significantly at the expense of a reduction in the stock of high school graduates. Note the increase in tuition for public college that follows this technological shift. An advantage of our model is that tuition is endogenous and responds to the shocks that affect the economy.

With our current calibration none of the agents chooses to attend a private college. The calibration could be modified to achieve this result by reducing simultaneously the college subsidy δ and the quality of public colleges as indexed by x^{pb}.

6 Next Steps

Our goal is to use the model to account for the evolution of educational attainment in the U.S. in the 20th century. Our next step is to solve for the model's transition starting from an initial steady state in 1900 circa toward a final steady state in the "distant" future, say 2100. This solution would then be embedded in a loop to iteratively search the parameter space for a set of parameters yielding a satisfactory fit to the joint evolution of educational attainment and relative wages in the U.S. Once the model is calibrated it would be used to consider counterfactual experiments of the type illustrated in Table 1.

7 Conclusions

TBA

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